FINANCIAL MODELING USING SAMPLING-IMPORTANCE RESAMPLING*

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Abstract

While Bayesian methodology has been for decades widely applied to econometric models, robust methods had just lately gained more attention of econometricians. More recently, simulation-based techniques, such as the Sampling-Importance Resampling (SIR) algorithm, have become useful and popular approaches to statistical problems. This article puts together the robust and Bayesian approaches through the SIR technique. Using financial models we show how the statistics usually obtained through the SIR technique can be enhanced by the incorporation of robust aspects. For these models we investigate the posterior inference sensitivity in relation to the change in the likelihood and prior distribution, and obtain the SIR point estimates as well as confidence intervals, which are compared to classical and robust solutions.

Resumo

Enquanto a metodologia bayesiana tem sido por décadas largamente aplicada a modelos econométricos, os métodos robustos só ultimamente ganharam a atenção dos econometristas. Mais recentemente, temos visto técnicas baseadas em simulações, tais como o algoritmo Sampling-Importance Resampling (SIR), tornarem-se abordagens úteis e populares a problemas estatísticos. Este artigo une as abordagens bayesiana e robusta através da técnica SIR. Mostramos usando modelos estatísticos em Finanças como as estimativas usualmente obtidas através da técnica SIR podem ser melhoradas através da incorporação de aspectos robustos. Para esses modelos investigamos a sensibilidade da inferência bayesiana quanto a mudanças na especificação da verossimilhança e da distribuição a priori, e obtemos intervalos de confiança e estimativas SIR, os quais são comparados a soluções clássicas e robustas.

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1. Introduction.

Simulation-based techniques have become useful and popular approaches to statistical problems (Gamerman (1997), Gilks et al. (1996)). In particular, the Sampling-Importance Resampling (SIR) algorithm (Smith and Gelfand (1992), Rubin (1988)) is a very intuitive and appealing methodology. It is basically a weighed bootstrap, where a point with higher weight in a sample from the prior distribution has higher chance to be resampled to form a sample from the posterior distribution. The advantages of having samples of the joint and marginal posterior distributions are numerous, including the computation of sample moments as well as confidence intervals, detection of influential observations, etc. In spite of being a computer-intensive statistical procedure, the SIR algorithm can be easily implemented using statistical packages such as the MINITAB (Albert (1993)) and the S-Plus (Mendes (1996)). One main point about this technique is that it permits the user to assume more elaborated specifications for the likelihood function and for the prior distributions without facing the mathematical complexity of the resulting numerical integrations.

Bayesian methodology has been widely applied to econometric models (Zellner (1971), Judge et al. (1982), Geweke (1989), among others). On the other hand, only recently robust methods have gained more attention from econometricians specially due to the globalization effect, which can create atypical economic or political extreme local events to reflect worldwide. For example, the estimation of the systematic risk of stocks under the presence of outliers was addressed by Badrinath and Chatterjee (1993). Duarte and Mendes

(1997, 1998) illustrated the use of robust estimation techniques in emerging financial markets.

This paper puts together the robust and Bayesian approaches through the SIR technique. For this purpose we study two models used in Economics and Finance and show that the statistics usually obtained can be enhanced by more “realistically” specifying the problems. This realism may be achieved, for example, by assuming probabilistic assumptions which are difficult to incorporate in classical (frequentist or Bayesian) estimation procedures. In particular, the incorporation of robust aspects (heavier tails distributions or mixture of distributions) may result in more accurate inferences. We provide examples of these situations.

In addition to this flexibility, the SIR technique is particularly interesting when finding the right solution to estimation procedures having multiple solutions such as the redescending M -estimates (Huber (1973)). The SIR technique is also useful when finding the solution of problems involving numerical iterative methods, such as the maximum likelihood estimates of data following a Cauchy distribution. A third example is finding the solution to computationally unstable problems such as the GARCH models (Bollerslev (1986)) under non usual specification. In this work, these three problems are addressed using the SIR technique.

This paper is organized as follows. In Section 2 we review very briefly the SIR methodology. In Section 3 we apply the SIR technique to the Capital Asset Pricing Model (CAPM) using real data from Finance. We obtain the SIR estimates and compare the solutions to the robust Least Median of Squares estimates. Then we show that, using the SIR technique, one can easily add robustness to the analysis by detecting influential observations, by varying the specification of the error distribution, or by finding the solutions to the robust M -estimates for regression. In Section 4 we obtain SIR estimates

for the GARCH models which are compared to classical and robust solutions. For all the problems considered in this work we offer the reader some guidelines with respect to the sizes of the generated samples.

2. Sampling-Importance Resampling.

Let x be an observation from a distribution indexed by a parameter θ , $\theta \in \mathbb{R}^p$, with density (or probability mass) $l(x|\theta)$, and let $\pi(\theta)$ represent one's prior belief about the probability distribution of θ . Bayesian inference is primarily concerned with the incorporation of the information brought in by x to update the knowledge about θ , forming the posterior distribution $p(\theta|x)$. A tool to perform this task is given by the Bayes's Theorem, which states that

$$p(\theta|x) = \frac{l(x|\theta) \pi(\theta)}{\int l(x|\theta) \pi(\theta) d\theta} . \quad (1)$$

After obtaining the posterior distribution $p(\theta|x)$, one is typically interested in the marginal distributions of the elements of θ , and/or in their numerical summaries such as their moments, etc. However, when following these steps one may face several integrals which may be very difficult to calculate, even numerically (except in the simplest cases). To avoid the mathematical complexity of this problem, one is tempted to restrict his attention to more convenient forms for $l(\cdot|\theta)$ and/or $\pi(\cdot)$, which may not be the best representation of the phenomenon under study.

The SIR technique basically consists in applying the Bayes's Theorem to samples instead of densities (see Smith and Gelfand (1992) and Rubin (1988) for details). To obtain a sample from $p(\theta|x)$ using the SIR algorithm, we just need to know the functional form of $l(\cdot|\cdot)$ and to simulate a sample from $\pi(\theta)$. This idea is implemented

by sampling $(\theta_1, \theta_2, \dots, \theta_n)$ from $\pi(\theta)$ and resampling $(\theta_1^*, \theta_2^*, \dots, \theta_n^*)$ with replacement, where the probabilities to be resampled for each $\theta_i, i = 1, \dots, n$, are given by the weights

$$w_i = \frac{l(x|\theta_i)}{\sum_i l(x|\theta_i)}.$$

The SIR technique may be understood as a sample strategy which produces the empirical distribution function of the variable θ , i.e., the values θ_i and their weights w_i . The examination of the weights distribution can be very useful. Small weights spread out over several distinct values θ_i would give reliability to the resampling process. A large weight attached to a single point would reveal sampling breakdown and would alert the analyst for a better specification of the sampling distribution, in our case, taken as the prior $\pi(\theta)$.

3. The Capital Asset Pricing Model.

In this section we report the results obtained when applying the SIR method to a model which is considered a cornerstone in the modern portfolio theory: The Capital Asset Pricing Model (CAPM). The formulation of the CAPM is basically that of a simple linear regression model and it relates the expected return of a stock above the return of a risk free investment (the dependent variable) to the expected return of a market index also above the return of a risk free investment (the independent variable). Typically, the interest is in the slope of the regression line, which represents the systematic risk of the stock. In this study we apply the CAPM model to a data set containing the returns of 150 Brazilian stocks, the returns of the IBOVESPA Brazilian market index, and the returns of the risk free investment CDI-CETIP from 1994 to 1995. This data set had been previously used by Duarte and Mendes (1997).

Estimating the systematic risk of stocks is not a simple task. Robust estimation procedures had already been applied by Badrinath and Chatterjee (1993) and Duarte and Mendes (1997). Our motivation for using the SIR algorithm with the CAPM model, as in any other regression model, is mainly due to: 1) the output provided, containing the posterior samples of the regression coefficients and of the scale of the residuals which may be used for predictions, inferences, sensitivity analysis, etc; 2) the possibility of finding solutions under any specification for the prior and errors distributions without having to compute the corresponding (and probably difficult) integrals; 3) the possibility of finding solutions of problems with multiple solutions, such as the redescending M -estimates, for which there is no guarantee of convergence to the global minimum when using iterative methods on the estimating equations.

The estimation of the systematic risk of stocks applying the SIR algorithm was carried out in two stages. In the first part we used the following specification: 1) Normal errors model; 2) Uniform or Normal prior; 3) prior and posterior samples of size 3000. The reason for using the Normal errors model was based on the fact that this assumption is related to the optimality of the Ordinary Least Squares (OLS) estimates, which are used by the vast majority of analysts in finance. The Uniform prior distribution was mainly used with less traded stocks, since there is no general agreement about the value of the systematic risk of illiquid stocks. The parameters values for the Uniform prior were set as $[-0.8, +0.8]$ for the intercept, $[-1.9, +1.9]$ for the slope, and $[0, K]$ for the scale of the residuals, where K in general was 3 times the estimate for the scale provided by the OLS procedure. The Normal prior was used with the liquid stocks since in this case most practitioners are able to guess about the magnitude of the coefficients being estimated. The mode of each posterior sample was used as a point estimate, the SIR estimate, because it is the

Generalized Maximum Likelihood estimator when a Uniform prior is used and the Bayes estimator based on a zero-one loss function. It is considered more robust because this loss function gives zero weights to values larger than some positive arbitrary constant.

Fig. 1: Comparing four estimates in the case of Pronor PNA: the SIR Normal, the SIR Cauchy, the OLS and the LMS lines. Samples of size 3000.

Figure 1 shows the scatterplot of IBOVESPA versus the stock Pronor PNA and some fits. Since Pronor PNA may be considered an illiquid stock a substantial amount of zero returns is expected. The data configuration observed for the Pronor PNA was also observed for most of the illiquid stocks in our data set. In Figure 1 the outliers and one bad leverage point tilt the OLS and the Normal errors (SIR(N)) lines, while the robust Least Median of Squares (LMS) fit

(Rousseeuw and Leroy (1987)) seems to follow the pattern defined by the majority of points. We chose the LMS estimator due to its 0.5 breakdown point, which means that this estimator can resist up to 50% of bad outliers in the sample.

Fig. 2: Posterior estimated densities of the regression slope under Normal and Cauchy errors model for Pronor PNA. Sample size 3000. Uniform prior.

In the second part, motivated by the knowledge of the lack of robustness of the estimates based on the Normal errors specification, we re-applied the algorithm specifying a Cauchy likelihood. Figure 1 also shows the Cauchy (SIR(C)) fit. Note that this line almost coincide with the LMS fit. Removing the 13 outliers detected using the standardized residuals from the SIR(C) fit and applying the SIR (SIR(N) and SIR(C)) procedures to the reduced data, we obtained

almost coincident fits with intercepts -0.2517 and -0.2509 , slopes -0.0432 and -0.0251 , and scales 0.8393 and 0.3088 , respectively. In this and in several other cases the simple change to a heavier tails likelihood was enough to provide better estimates. (For more details on the use of the SIR technique to simulated regression data to assess the sensitivity of the estimates to changes in the model specifications see Mendes (1996).) Figure 2 shows the estimated densities of the posterior samples of the slope coefficient under the two errors distributions and using the full data.

Table 1

Point estimates, 5th and 95th percentiles of the posterior samples in the cases of an illiquid and a liquid stock.

PRONOR PNA

	Uniform Prior & Normal Model				Uniform Prior & Cauchy Model			
Variable	Median	Mode	5%	95%	Median	Mode	5%	95%
Intercept	-0.011	-0.213	-0.456	0.379	-0.396	-0.396	-0.493	-0.290
Slope	0.337	0.300	0.204	0.456	-0.021	-0.021	-0.092	0.001
Scale	4.670	4.592	4.325	4.985	0.531	0.531	0.427	0.772

TELESP PN

	Normal Prior & Normal Model				Normal Prior & Cauchy Model			
Variable	Median	Mode	5%	95%	Median	Mode	5%	95%
Intercept	-0.047	-0.153	-0.346	0.724	0.076	0.076	-0.407	0.304
Slope	1.087	1.130	0.972	1.197	1.110	1.110	0.949	1.250
Scale	3.806	3.873	3.472	4.206	1.886	1.876	1.676	2.239

The posterior samples provided by the SIR algorithm may be used for predictions about different functions of interest, to compute sample summaries, etc. Table 1 (upper part) compares for the stock Pronor PNA, the point estimates and the 5th and the 95th percentiles based on the posterior samples, for the regression coeffi-

cients and for the scale of the residuals under the Normal and the Cauchy models. We note the much more precise estimates given by the Cauchy model. Table 1 also shows (lower part) the results obtained when using a Normal(1,1) prior distribution for the slope of the regression model involving the liquid stock Telesp PN. The parameters of the Normal distribution were chosen based on information from practioners working in the Brazilian stock markets. For the intercept and scale we used the same prior previously specified for the Pronor PNA. The corresponding fits are shown in Figure 3. This is a case where a (probably) normally distributed data set contains a good leverage point. As a consequence, the three estimates — OLS, SIR(N), and SIR(C) — give roughly the same solution and there is no dramatic improvement when a more robust estimate is used.

Fig. 3: Comparing four estimates in the case of Telesp PN: The SIR Normal, the SIR Cauchy, the OLS and the LMS lines.

The sensitivity of estimates with respect to case deletion was also investigated using the SIR algorithm. We use here the same stock Pronor PNA to illustrate the procedure used to detect influential observations. Assuming a Normal likelihood, a Uniform prior distribution as specified previously, we applied the SIR method to the 176 reduced data sets obtained after the deletion of each of the 176 observations. Figure 4 shows the observations indexes plotted against the mean and the 5th and the 95th percentiles of the posterior samples of the regression slope for the stock Pronor PNA, obtained using the “leave-one-out” reduced data sets. We note that at indexes 9, 134, 135, 159, 162, 171, 173, and 175, which correspond to some of the outlying points in Figure 1, the results are quite different from the others, indicating the large influence of these data points. We would like to stress however, that just this analysis is not sufficient to detect influential observations, due to the possibility of occurrence of the masking effect (Rousseeuw and Leroy (1987)). One suggestion could be to compare these results to the results obtained when deleting pairs or triples of data points.

To complete our estimation of the risk of Brazilian stocks we now use the SIR algorithm to obtain the redescending M -estimates for regression. The M -estimates for regression (Huber (1973), Hampel et al. (1986)) may be defined as the minimizing values of a loss function $\sum \rho(r)$, where r represents a residual. They may be considered as a generalization of the maximum likelihood estimates where the (target) errors distribution is proportional to $\exp\{-\rho(r)\}$. When $\rho(\cdot)$ is a bounded nondecreasing function these estimates are called redescending. In this case they are statistically efficient and possess high breakdown point. One example of a such ρ function is the Biweighted Tukey ρ function given by

$$\rho_k(r) = \begin{cases} \frac{r^2}{2} - \frac{r^4}{2k^2} + \frac{r^6}{6k^4} & \text{se } |r| \leq k \\ \frac{k^2}{6} & \text{se } |r| > k, \end{cases} \quad (2)$$

where the tuning constant $k > 0$ is responsible for the efficiency. For example, if $k = 4.685$ this estimator attains 94.9% efficiency under normality.

Fig. 4: Analysis of sensitivity of the estimates to case deletion for the stock Pronor PNA: The 176 means and 5th and 95th percentiles of the slope posterior samples obtained for each leave-one-out data set.

The redescending M -estimating equations admit multiple solutions, obtained usually via some iterative algorithm, which in general

converges to a solution with zero breakdown point. When using the SIR algorithm we are able to obtain the global solution of the optimization problem, just by approaching this problem as a maximum likelihood estimation procedure. In other words in the SIR algorithm we take $l(x|\theta_i)$ as $\exp\{-\rho(x|\theta_i)\}$, in our notation of Section 2. We note that there is no need to compute the density normalizing constant when using SIR. To illustrate, we use the Biweighted ρ -function and consider two stocks: the previously used Telesp PN, and the illiquid stock Eberle PN. The motivation for using Eberle PN is due to the fact that this example points out the statistical and computational instability of the LMS procedure.

To apply the SIR technique we generated Uniform and Normal prior samples respectively for the illiquid and liquid stocks, as described previously. Table 2 compares the SIR estimates obtained under Normal, Cauchy and the M-Biweighted (SIR(M)) models. The table also shows the Biweighted M -estimates (M) and the LMS estimates computed using the S-Plus package. The SIR(M) method, besides providing a more complete output, seemed to have found a high breakdown global solution, at least for the data considered in this work. The algorithm used by S-Plus to compute the M -estimates needs an initial high breakdown estimate, which is in general the LMS obtained via sub-sampling. The results in the upper part of Table 2 illustrate the statistical and computational instability of the LMS procedure: a slope value of -0.66 meaning that if the market index IBOVESPA would fall, say, by 10%, we should expect the returns of the illiquid stock Eberle PN to raise by 6.6%.

On the other hand, for the TELESP PN the different methodologies provided similar results. As we have already commented, this was also true for most of the liquid stocks. We can also say that in general the SIR algorithm succeeded in finding a high breakdown solution for the redescending M -estimates, although presenting higher

variability, as exemplified by the two stocks in Table 2. All SIR estimates for any particular stock may be obtained from the author by request.

Table 2

SIR estimates and two robust estimates for the regression and scale parameters of an illiquid and a liquid stock.

	SIR(N)			SIR(C)			SIR(M)			M	LMS
	Est.	5%	95%	Est.	5%	95%	Est.	5%	95%	Est.	Est.

EBERLE PN

interc.	0.39	-0.45	0.46	-0.09	-0.44	0.27	0.12	-0.44	0.45	0.67	-3.85
slope	0.56	0.01	0.88	0.05	-0.10	0.22	0.22	-0.47	0.94	0.46	-0.66
scale	15.2	14.1	15.9	3.51	2.76	5.42	15.9	14.8	16.0		

TELESP PN

interc.	-0.15	-0.35	0.72	0.08	-0.41	0.30	0.11	-0.47	0.43	0.11	-0.02
slope	1.13	0.97	1.20	1.11	0.95	1.25	1.07	0.80	1.35	1.13	1.28
scale	3.87	3.47	4.21	1.88	1.68	2.24	3.98	3.66	3.99		

4. GARCH Models.

The Autoregressive Conditional Heteroscedastic (ARCH) model was proposed by Engle (1982) and since then have proved to be successful when modeling a variety of situations in Finance, Econometrics, etc. In its simplest form an ARCH process of order p (ARCH(p)) assumes that

$$x_i | \mathbf{x}_{i-1} \sim F(0, \sigma_i^2)$$

where $F(0, \sigma_i^2)$ represents a probability distribution with zero mean (usually the Normal distribution), $\sigma_i^2 = \alpha_0 + \alpha_1 x_{i-1}^2 + \dots + \alpha_p x_{i-p}^2$, and $\mathbf{x}_{i-1}$ represents all past information up to time i .

Maximum likelihood estimation of ARCH and GARCH (Generalized ARCH) models are already available in some statistical/econometric softwares. However, users had noticed some computational instability of the algorithms implemented by those packages, which may result, for example, in the occurrence of negative estimates. In addition, not all desirable specifications for F had been implemented in some softwares. When applying GARCH models to Brazilian financial data the choice of F is particularly important due to the high volatility of Brazilian stock markets. More important, none of the computer packages estimates GARCH models using Bayesian procedures. In such an environment where people definitely take advantage of their prior experience and recent information, the no application of the Bayesian philosophy could be considered at least naive. In the context of GARCH modeling, the ease to implement SIR technique managed to meet the need for robust specifications combined with Bayesian approach.

To study the performance of the SIR methodology when applied to ARCH models we set up the following experiment. We generated 50 series of size 300 according to an ARCH(1) process where $\alpha_0 = 0.9$ and $\alpha_1 = 0.5$, with F being the heavy tails t-student distribution with 5 degrees of freedom (t_5). As prior distribution we used either an Uniform or a Normal distribution, but to define their parameters for each data set we had first to run a pilot experiment where the SIR algorithm was applied using diffuse priors and samples of sizes 10000. Having decided about the location of the prior distributions, at each simulation, prior samples of size 3000 were generated and held fixed while the SIR algorithm was applied based on t-student distributions with 3 ($SIR(t_3)$) and 5 ($SIR(t_5)$) degrees of freedom. This two-stage procedure was necessary to get a better picture of the posterior distribution, since if the prior sample was not sufficiently dense at the region where the posterior would be finally located, we

would probably observe a few points carrying most of the resampling probability.

For each data set, besides the results from the SIR algorithm, we also computed the t_5 -maximum likelihood estimates ($\text{MLE}(t_5)$) using the S-Plus, and the robust CM -estimates for ARCH models (Mendes (1997)). Table 3 shows the mean results from all simulations.

Table 3
Mean results from 50 simulations of an ARCH(1) process.

	SIR(t_3)				SIR(t_5)				MLE(t_5)		CM
	Mode	SD	5%	95%	Mode	SD	5%	95%	Est.	SE	Est.
α_0	0.47	0.10	0.36	0.61	0.59	0.09	0.40	0.72	0.58	0.10	0.77
α_1	0.29	0.11	0.19	0.51	0.36	0.09	0.22	0.47	0.33	0.12	0.42

The closest estimates to the true values were the robust and efficient CM -estimates. To compute these estimates we do not need to specify the underlying distribution, but their output is restricted to the point estimates. In Table 3 we observe the negative biases of the MLE estimates. We recall that the downward biases of the GARCH MLE estimates is a well known fact, and several methods for bias reduction of these estimates had been proposed in the literature (see for example, Engle et al. (1985), Linton (1997), among others). The SIR(t_3) and the SIR(t_5) estimates also showed downward biases. The estimates obtained under the t_5 specification were more accurate, showing smaller biases as well as smaller confidence intervals, although still not covering the true value. Table 3 reports the mean of the modes and the means of the 5% and the 95% percentiles of the posterior samples. We also computed the value of the Ljung-Box Q-statistic (based on sample correlations) using the squared standardized SIR residuals to verify if they were free

of volatility clusters. For most runs under the t_5 specification, we obtained a p -value of about 0.98, which strongly supported the null hypothesis that the model was correctly specified.

To investigate the influence of a more informative prior we repeated this simulation study using a Normal prior specification. In this case the posterior samples obtained provided slightly less biased estimates and shorter confidence intervals. The mean estimates for the mode, for the 5% and the 95% percentiles of α_0 and α_1 under t_5 were respectively (0.63, 0.49, 0.79) and (0.39, 0.28, 0.50). This confirms the usefulness of the SIR methodology when applied to ARCH/GARCH models whenever prior information is available.

An examination of the weights (a plot, for example) should always be done to check if the importance resampling scheme has broken down. In this study we did not find any dominant weight, which could help indicating that the resampling scheme has worked well.

Finally, we would like to comment that the selection of the best SIR estimate among the t -student distributions used was not obvious. When using the S-Plus package to estimate the number of degrees of freedom of the simulated ARCH data, we obtained surprising results including values varying from 3 to 12. This might indicate an inherent difficulty when estimating the model and/or simulating the process.

5. Conclusion.

In this paper we gave some indications on how the potential of the SIR technique may be used as a tool for performing Bayesian inferences. Sensitivity analysis of Bayesian inferences with respect to changes in the specification of the likelihood and/or prior distributions was done by including further options outside the Bayesian conjugate family of distributions. For the models considered in this

work, the SIR methodology and robustness concerns were employed as a defensive approach which produced more reliable estimates.

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