RISK AVERSION AND OPTIMAL TRADE DEPENDENCY*

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Abstract

This paper shows that a highly indebted country's (HIC) optimal dependency on international trade depends on the country's degree of risk aversion. Risk aversion affects optimal trade dependency directly through consumption smoothing across different states of the world (intratemporally) and indirectly through consumption smoothing across different time periods (intertemporally). When the probability of default is high (low) and the interest rate on consumption borrowing is large (small) relatively to the discount rate, optimal openness to international trade increases (decreases) with risk aversion while optimal consumption borrowing decreases (increases). Otherwise, results are uncertain.

Resumo

Neste trabalho, mostra-se que o grau ótimo de dependência externa de uma economia depende do seu grau de aversão ao risco. A aversão ao risco afeta a dependência externa ótima diretamente (intra-temporalmente) como indiretamente (inter-temporalmente). Quando a probabilidade de inadimplência é alta (baixa) e a taxa de juro em empréstimos para consumo é grande (pequena) relativamente a taxa de desconto social, a dependência externa ótima aumenta (diminui) com a aversão ao risco, enquanto que o volume ótimo de empréstimos para consumo diminui (aumenta). Nos outros casos, os resultados são incertos.

Key words: Trade dependency; openness; risk aversion.

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1. Introduction.

In international lending, unlike with private debt contracting, there is no third party which serves to enforce debt contracts and there is no formal collateral that will ensure the creditors that repayment will take place. International loan contracts, however, do take place. The reason for them must be the existence of mechanisms that rend them self-enforcing. Countries repay their debts if they have more to lose by defaulting than by repaying them.

It has been argued that countries repay because they want to be able to borrow more in the future and the only way to guarantee continued access to international credit markets is to maintain a reputation as "good debtors". However, Bulow and Rogoff (1989) show that a sovereign's desire for future access to international credit markets, retained by meeting its credit obligations, is not enough to justify repayment and argue that some sort of legal penalties are necessary. Rosenthal (1991) supports these findings with a yet stronger result: even if a defaulting country would be excluded from credit markets both as a debtor and as a creditor, which makes intertemporal consumption smoothing more difficult (and thus the cost of defaulting higher), repayment would still not be warranted.

Other authors have argued instead that a debtor's desire to maintain a "good reputation" stems from the fear of being excluded from trade markets and the consequent loss of gains from trade. Authors like Gersowitz (1983), Diwan and Donnenfeld (1986) and Bulow and Rogoff (1989), among others, have taken on the view that a debtor's creditworthiness is positively linked to the country's gains from trade which thus provide a sort of collateral on international lending. In most of this literature, however, it is assumed that the default costs (the forfeited gains from trade) are exogenous.

Yet other authors, such as Aizenman (1990, 1991) and Diwan (1990) have argued that default costs are endogenously determined by a country's trade dependency, which, in turn, depends on the investment policies. The idea behind these models is to link the debtor's investment strategy to its ability to borrow internationally. Aizenman (1990) pointed out the existence of a crucial difference between consumption borrowing and investment borrowing: while both consumption and investment borrowing raise total indebtedness, increasing the probability of default and reflecting an upward move along a given supply of credit facing the economy (the indebtedness effect), investment borrowing has the additional feature of displacing the supply curve itself. The reason for this is that production depends on the use of a necessary imported input, the channel through which creditors will impose penalties upon the debtor country in case of default. In case of default, creditors take on actions that will raise the input's price. The more the debtor country uses the input, the more it has to lose, thus making default less likely and displacing the credit supply curve downwards (the openness effect).

Aizenman (1990) uses a model with n productive activities, all similar with the exception of the intensity of use of the imported input. Changes in the intersectoral composition of investment may displace the supply of credit upwards or downwards. Aizenman shows that the socially optimal degree of openness is always larger than the competitive market yields and argues in favor of credit market policies. Aizenman (1991) reexamines the issue of strategic trade dependency in a model of partial defaults and bargaining. Diwan (1990) shows that, when credit ceilings are binding, the link between credit market conditions and trade dependency generates incentives for the debtor to either strategically increase or decrease the gains from trade by varying the allocation of investment funds.

The purpose of this paper is to analyze how differences in the debtor country's degree of risk aversion will affect its optimal open-

ness. We assume that production is affected by a random shock.¹ In order to handle the complicated analysis, the model considered has only one productive activity. In this context, more investment leads to a greater need for the imported input, and consequently results in more trade dependency, shifting down the supply of credit. It is shown that, if the probability of default is high (low) and if the consumption-borrowing interest rate is large (small) relative to the discount rate, optimal openness decreases (increases) with risk aversion, while current consumption increases (decreases). Risk aversion affects optimal trade dependency directly through consumption smoothing across different states of the world (intratemporally), and it affects it indirectly through consumption smoothing across different time periods (intertemporally).

The crucial idea that increases in debtor's aggregate investment raises creditworthiness can be found also in the literature on debt repurchases. In Froot (1991) and Cabral (1996), for example, default losses are set equal or proportional to output by assumption (and thus proportional to investment), while in this paper this proportionality is endogenously obtained as it results from creditors' actions following default.

Section 2 formulates the two-period model. Section 3 analyzes how risk aversion affects borrowing (and openness). Section 4 concludes.

2. The Adapted Aizenman's Model.

The model used is adapted from Aizenman's (1990). There are three basic differences between the two models. First, we will consider a two-period model while Aizenman (1990) considered three

¹If randomness would arise from the external interest rate instead, similar results would be obtained.

periods.² Secondly, risk aversion is introduced in the model.³ Third, and as already explained, we will work with only one productive activity.⁴ We will now recall Aizenman's (1990) model, already adapted as described.

The timing of the model is summarized as follows: The debtor country has a initial level of inherited debt B_0 . In the first period of the model there is a central planner that borrows internationally at an external interest rate r^* . He then allocates these funds between interested borrowers. Some borrowers want to borrow for consumption, others want to invest. He charges these two classes of borrowers different interest rates: r_c and r_i , respectively.⁵ Investment and first-period consumption takes place. In the beginning of the second period the "state of nature" that will determine that period's output is revealed. The central planner then decides whether to repay debt or to default. If he decides to default, he knows that creditors will retaliate by taking actions that will result in an increase in the price of the (necessary) imported input which is used in production. Second-period production and consumption takes place.

The central planner's decision on whether or not to default involves a comparison between how much there is to be lost by repayment versus by default. Repayment cost is $(1 + r^*)\bar{B}$, where $\bar{B}$ represents total indebttness in period 2, when debt repayment is due,

² Aizenman was interested not only on optimal openness but also on the issue of uncertainty over whether the central planner would remain in power for a third period. For this reason, the planner's discount factor would differ from the society's. To address optimal openness, two periods only are needed.

³ In a previous unpublished version of his 1990 paper, Aizenman (1987) used a two-period model with risk aversion and n productive activities. However, he did not analyze the question of how optimal openness is affected by the degree of debtor's risk aversion.

⁴ One may think of the case where there are several productive activities which do not differ from each other in its degree of intensity of use of the imported input.

⁵ The motivation for charging a rate different from the external interest rate will be made clear further on.

with B_c standing for consumption borrowing and I for investment-borrowing in period 1. To compute default losses one must first define second-period production in more detail. Suppose second-period output (X) is given by the following production function:

$$X = \theta K^\alpha M^\beta, \quad \alpha, \beta \geq 0, \quad \alpha + \beta < 1 \quad (1)$$

where K is the country's capital stock (given by any initial endowment of capital K_o , plus funds borrowed externally I : $K = K_o + I$), M is the amount used of the imported input, and θ is a random productivity shock. The density function for θ , $f(\theta)$, is defined on the interval $(0, 1)$.

Profit maximization implies that the imported input receives a fraction β of the value of output. National income, or value added, (Y) is the remaining output – a fraction $(1 - \beta)$ of nominal output.

$$Y = (1 - \beta) pX. \quad (2)$$

Let us assume that the imported input's price is expected to be p_m if the debtor country repays debt. However, if the central planner chooses to default, creditors take retaliatory measures that result in an increase of the imported input's price to $p'_m > p_m$. In particular, we will assume that $p'_m = (1 + z)p_m$. The result is that national income will fall. This reduction in national income constitutes the "default penalty" (which we will denote by Ω henceforth) and is defined by

$$\Omega = \gamma Y, \quad \text{with } \gamma = \frac{\beta}{1 - \beta} z, \quad (3)$$

(γ represents the percentage reduction in income resulting from the terms of trade deterioration following default.)

In deciding whether or not to default, the central planner compares the default loss (Ω) with the debt repayment cost: $(1 + r^*)\bar{B}$. The central planner will be indifferent between default and repayment if the productivity shock θ assumes a value such that the repayment cost is equal to the default loss. We will denote the value of the productivity shock which renders indifference between repayment and default by θ_0 . This value is determined by the condition $(1 + r^*)\bar{B} = \Omega$.

Since the default loss (Ω) is increasing in θ , for $\theta > \theta_0$ the planner will choose to repay and for $\theta < \theta_0$ he will choose to default on debt. The probability of repayment (π) is therefore defined by the probability of θ falling above the "critical value" θ_0 :

$$\pi = \int_{\theta_0}^1 f(\theta) d\theta \quad (4)$$

Creditors can incorporate the risk of default in the interest rate charged on the loans. If creditors are risk neutral and if the random shock that the debtor country faces is idiosyncratic, the creditor charges an interest r^* on loans such that the expected return equals one plus the risk free interest rate (r_f):

$$(1 + r^*)\pi = (1 + r_f). \quad (5)$$

It should also be noted that θ_0 is increasing on the total volume of borrowing $\bar{B}$, and that therefore π will be decreasing on $\bar{B}$.

Note, from (4), that the probability of repayment (π) is decreasing with θ_0 , which in turn is increasing with total borrowing $\bar{B}$. Therefore, the external interest rate the debtor country must pay (r^*) is increasing on the total volume of borrowing. This, in other words, means that there is an upwards sloping supply of credit facing

the country,⁶ and that the debtor country has monopsony power in the international credit market.

Aizenman (1990) showed that, because the supply of credit is positively sloped, the individual cost to market borrowers differs from the social marginal cost of borrowing, since each individual does not perceive its borrowing as increasing the probability of default and the external interest rate, but instead takes the interest rate as given. Aizenman concluded that appropriate taxation and subsidization of borrowing is required to attain the social optimum.

We are now in conditions to write the default condition, which defines θ_0 :

$$(1 + r^*(\theta_0))\bar{B} = \gamma Y(\theta_0) \quad (6)$$

The competitive outcome:

There is a representative consumer with a constant risk aversion utility function. Let ρ be the Arrow-Pratt measure of relative risk aversion ($\rho = -\frac{U''}{U'} C$) and δ be the social discount rate.

$$\begin{aligned} E[U] = & \frac{1}{1-\rho} C_1^{(1-\rho)} + \frac{1}{1+\delta} \frac{1}{1-\rho} \int_{\theta_0}^1 C_{2_n}^{(1-\rho)} f(\theta) d\theta \\ & + \frac{1}{1+\delta} \frac{1}{1-\rho} \int_0^{\theta_0} C_{2_d}^{(1-\rho)} f(\theta) d\theta, \end{aligned}$$

where C_1 represents consumption in the first period of the model, and C_{2_n} and C_{2_d} are the state-contingent consumption levels in the second period (in the case of no default and of default respectively),

⁶The derivation of the supply of credit is done in Aizenman (1989) pp. 149–151.

satisfying the resource constraints given by:

$$\begin{aligned}C_1 &= E_1 + B_c, \\C_{2_n} &= Y(\theta, I) - (1 + r_c)B_c + E_2 - (1 + r_i)I + T_r, \\C_{2_d} &= (1 - \gamma)Y(\theta, I) + E_2.\end{aligned}$$

where E_1 and E_2 are first and second period's endowments, respectively, assumed to be greater than one.

Borrowing for consumption and for investment is being either taxed or subsidized (at rates τ_c and τ_i) and therefore their respective internal interest rates r_c and r_i will differ from the external interest rate r^* :

$$(1 + r_c) = (1 + r^*)(1 + \tau_c), \quad (7)$$

$$(1 + r_i) = (1 + r^*)(1 + \tau_i). \quad (8)$$

It is assumed that any government revenues arising from taxation on borrowing are redistributed to domestic residents as lump-sum transfers T_r (if there is a net subsidy, lump-sum taxes will have to be levied to balance the government's budget instead):

$$T_r = \tau_c B_c + \tau_i I.$$

The first order conditions for this problem which will yield the competitive solution outcome, B_c^* and I^* , are:

$$C_1^{-\rho} - \frac{1}{1 + \delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} (1 + r_c) f(\theta) d\theta = 0, \quad (9)$$

$$\begin{aligned}&\frac{1}{1 + \delta} \int_0^{\theta_0} C_{2_d}^{-\rho} \left(\frac{dY}{dI} (1 - \gamma) \right) f(\theta) d(\theta) \\&+ \frac{1}{1 + \delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} \left(\frac{dY}{dI} - (1 + r_i) \right) f(\theta) d(\theta) = 0.\end{aligned} \quad (10)$$

The first f.o.c. tells us that the marginal utility of a borrowed dollar in the first period must equal the expected marginal utility of the resources used to repay that borrowed dollar in the second period. The second f.o.c. tells us that the expected increase in utility due to the extra output resulting from an additional dollar invested in production must equal the expected decrease in utility as resources are used to repay that borrowed dollar, both within the second period.

The social optimum:

The central planner's problem is to maximize expected utility of the representative consumer, given the true cost of internationally borrowed funds:

$$\begin{aligned} \max E[U] = & \frac{1}{1-\rho} C_1^{(1-\rho)} + \frac{1}{1+\delta} \frac{1}{1-\rho} \int_{\theta_0}^1 C_{2_n}^{(1-\rho)} f(\theta) d\theta \\ & + \frac{1}{1+\delta} \frac{1}{1-\rho} \int_0^{\theta_0} C_{2_d}^{(1-\rho)} f(\theta) d\theta, \end{aligned}$$

second-period consumption in the repayment states is defined instead as follows:

$$C_{2_n} = Y(\theta, I) - (1 + r^*)(B_c + I) + E_2$$

First-order conditions then result to be

$$C_1^{-\rho} - \frac{1}{1+\delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} \left(1 + r^* + \frac{dr^*}{dB_c} \bar{B} \right) f(\theta) d\theta = 0, \quad (11)$$

$$\begin{aligned} & \frac{1}{1+\delta} \int_0^{\theta_0} C_{2_d}^{-\rho} \left(\frac{dY}{dI} (1 - \gamma) \right) f(\theta) d(\theta) \\ & + \frac{1}{1+\delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} \left(\frac{dY}{dI} - (1 + r^* + \frac{dr^*}{dI} \bar{B}) \right) f(\theta) d(\theta) = 0 \quad (12) \end{aligned}$$

Comparing these first-order conditions with those of the competitive market outcome, one can see that in the absence of taxation of borrowing interest rates, the two solutions would differ in that individual borrowers do not take into account the effect that their borrowing has on the external interest rate. This is captured by the $\frac{dr^*}{dB_c}$ and the $\frac{dr^*}{dI}$ terms in the social planner's optimality conditions, which are absent from the individual's. By choosing the appropriate value for the interest-rate tax variables, the central planner can guarantee that the social optimum will be attained. That involves taxing borrowing if it marginally increases the external interest rate at the optimum, and subsidizing it if the opposite is true.

3. Risk Aversion and Optimal Trade Dependency.

How will the degree of debtor risk aversion (here measured by ρ) affect optimal openness (both the competitive outcome and the social optimum)? And how is the borrowing decision affected, i.e. what are the signs of $\frac{dI_i}{d\rho}$ and $\frac{dB_c}{d\rho}$? Will a higher degree of risk aversion determine an increase or a decrease in the type of borrowing that results in higher dependency (investment borrowing) versus the type of borrowing that does not (consumption borrowing)?

To address these questions, we use with the first-order conditions of the utility maximization problem. We start by differentiating these conditions and by writing them in matrix form:

$$\begin{pmatrix} \frac{dI}{d\rho} \\ \frac{dB_c}{d\rho} \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 U}{\partial I^2} & \frac{\partial^2 U}{\partial B_c \partial I} \\ \frac{\partial^2 U}{\partial I \partial B_c} & \frac{\partial^2 U}{\partial B_c^2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial^2 U}{\partial I \partial \rho} \\ \frac{\partial^2 U}{\partial B_c \partial \rho} \end{pmatrix}$$

We next proceed as follows: first, we sign the matrix and the vector on the RHS. This is done in subsection 3.1. In subsection 3.2 and 3.3 we proceed to the analysis of how investment and consumption borrowing, respectively, are affected by risk aversion. In subsection 3.4 the effects on total borrowing are then analyzed.

3.1 Effects of risk on indirect marginal utilities.

To sign the vector on the RHS we start by examining the effects of a change in the degree of risk aversion on the two indirect marginal utilities (with respect to investment and consumption borrowing, the two decision variables). Let $\exp E(\ln x)$ be the exponential mean of a random variable x .

Lemma 1. *If the exponential mean of consumption under repayment is larger than the exponential mean of consumption under default conditions, then $\frac{\partial^2 U}{\partial I \partial \rho} > 0$. This happens for relatively low values of the probability of default. Otherwise, $\frac{\partial^2 U}{\partial I \partial \rho} < 0$.*

The intuition behind this result is the following: if the exponential mean of consumption under repayment is larger than the exponential mean of consumption under default, if there is an increase in the degree of risk aversion, the differences between consumption levels across different states of the world within the second period will tend to be smoothed out. Thus, under a higher ρ , an increase in investment represents higher utility gains. More investment raises expected consumption in the default states while it reduces expected consumption under repayment. However, if the exponential mean of consumption under default is larger than under repayment, the opposite is true.

Proof.

$$\begin{aligned} \frac{\partial^2 U}{\partial I \partial \rho} = & \frac{-1}{1+\delta} \int_0^{\theta_0} C_{2_d}^{-\rho} \ln(C_{2_d}) \left(\frac{dY}{dI} (1-\gamma) \right) f(\theta) d\theta + \\ & + \frac{-1}{1+\delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} \ln(C_{2_n}) \left(\frac{dY}{dI} - (1+r_i) \right) f(\theta) d\theta \quad (12) \end{aligned}$$

The first-order condition implies that $\frac{dY}{dI} - (1+r_i)$ has a negative sign for some $\hat{\theta} \geq \theta_0$. Let

$$g(\theta) = C_2^{-\rho} \frac{dY}{dI} (1-\gamma) \quad \text{for} \quad \theta < \theta_0$$

and

$$g(\theta) = C_2^{-\rho} \left(\frac{dY}{dI} - (1+r_i) \right) \quad \text{for} \quad \theta > \theta_0.$$

The first order conditions tell us that $\int_0^1 g f d\theta = 0$. Now, let

$$h(\theta) = \frac{\ln C_2 f d\theta}{H},$$

where $H = \int_0^1 \ln C_2 f d\theta$. Then, h can be interpreted as a density function, provided that $C_2 \geq 1$ (as follows from $E_2 \geq 1$). Now, $\frac{\partial^2 U}{\partial I \partial \rho} > 0$ if and only if $\int_0^1 g h d\theta < 0$, or equivalently, if and only if, $\int_{\theta_0}^1 h d\theta > \pi$, that is,

$$\int_{\theta_0}^1 \ln C_{2_n} df d\theta > \pi \left(\int_0^1 \ln C_2 f d\theta \right).$$

Rewriting this condition we get,

$$\frac{1}{\pi} \int_{\theta_0}^1 \ln C_{2_n} f d\theta > \frac{1}{(1-\pi)} \int_0^{\theta_0} \ln C_{2_d} f d\theta,$$

as claimed. $\square$

Let us now look at the other indirect marginal utility:

Lemma 2. *If the exponential mean of consumption under repayment is larger than the exponential mean of first-period consumption, then $\frac{\partial^2 U}{\partial B_c \partial \rho} < 0$. This happens when the interest rate on consumption borrowing is high enough relatively to the discount rate. Otherwise, $\frac{\partial^2 U}{\partial B_c \partial \rho} > 0$.*

The intuition behind this result is the following: increased aversion to risk will tend to smooth out the differences between consumption levels intertemporally. When the exponential mean of consumption in the repayment states of the second period is larger than first-period consumption, borrowing for consumption will represent lower utility gains. This case occurs when the expected marginal utility of consumption under repayment is low enough relatively to first-period marginal utility. By the first-order condition (10), this situation requires r_c to be high enough, relatively to δ .

Proof.

$$\frac{\partial^2 U}{\partial B_c \partial \rho} = C_1^{-\rho} \ln(C_1) + \frac{-1}{1+\delta} \int_{\theta_0}^1 C_{2_n}^{-\rho} \ln(C_{2_n}) (1+r_c) f(\theta) d\theta \quad (13)$$

Let

$$\hat{g}(\theta) = \frac{c_1^{-\rho}}{(1-\pi)} \quad \text{if} \quad \theta < \theta_0$$

and

$$\hat{g}(\theta) = -(1 + r_c) \frac{C_{2n}^{-\rho}(\theta)}{(1 + \rho)} \quad \text{if} \quad \theta > \theta_0.$$

Now $\int_0^1 f(\theta) \hat{g}(\theta) d\theta = 0$, by the first-order conditions. Let

$$\hat{h}(\theta) = \frac{(\ln C_1) f(\theta)}{\hat{H}} \quad \text{if} \quad \theta < \theta_0$$

and

$$\hat{h}(\theta) = \frac{(\ln C_{2n}(\theta)) f(\theta)}{\hat{H}} \quad \text{if} \quad \theta > \theta_0,$$

where

$$\hat{H} = (1 - \pi) \ln C_1 + \int_{\theta_0}^1 \ln C_{2n} f d\theta.$$

Now, $\hat{h}$ can be interpreted as a density function, provided that $C_1, C_{2n} \geq 1$, which is guaranteed since income in both periods exceeds one ($E_1 \geq 1$ and $E_2 \geq 1$). Then, $\frac{\partial^2 U}{\partial B_c \partial \rho} < 0$ if and only if $\int_0^1 \hat{g} \hat{h} d\theta < 0$ or, equivalently, if and only if, $\int_{\theta_0}^1 \hat{h} d\theta > \pi$. This condition can be rewritten as

$$\int_{\theta_0}^1 \ln C_{2n} f d\theta > \pi \left[(\ln C_1)(1 - \pi) + \int_{\theta_0}^1 \ln C_{2n} f d\theta \right],$$

that is,

$$\frac{1}{\pi} \int_{\theta_0}^1 \ln C_{2n} f d\theta > \ln C_1,$$

as claimed. The comment on r_c and δ follows by inspection of the first-order condition, as explained above. $\square$

Let us now look at the terms of the RHS matrix:

$$\begin{aligned}
 - \begin{pmatrix} \frac{\partial^2 U}{\partial I^2} & \frac{\partial^2 U}{\partial B_c \partial I} \\ \frac{\partial^2 U}{\partial I \partial B_c} & \frac{\partial^2 U}{\partial B_c^2} \end{pmatrix}^{-1} &= - \frac{\begin{pmatrix} \frac{\partial^2 U}{\partial B_c^2} & -\frac{\partial^2 U}{\partial B_c \partial I} \\ -\frac{\partial^2 U}{\partial I \partial B_c} & \frac{\partial^2 U}{\partial I^2} \end{pmatrix}}{\det(Hess)} \\
 &= \begin{pmatrix} A & B \\ B & C \end{pmatrix}
 \end{aligned}$$

where $Hess$ is the hessian matrix of the indirect utility function.

Lemma 3. $\frac{\partial^2 U}{\partial I^2}$, $\frac{\partial^2 U}{\partial I \partial B_c}$ and $\frac{\partial^2 U}{\partial B_c^2}$ are all negative.

Proof.

$$\frac{\partial^2 U}{\partial I \partial B_c} = \frac{\rho}{1 + \delta} \int_{\theta_0}^1 C_{2n}^{-\rho-1} \left[\frac{dY}{dI} - (1 + r_i) \right] (1 + r_c) f(\theta) d\theta.$$

From first-order condition (7), one concludes that $\frac{\partial^2 U}{\partial I \partial B_c} < 0$.

$\frac{\partial^2 U}{\partial I^2}$ and $\frac{\partial^2 U}{\partial B_c^2}$ are negative since at an interior solution the Hessian matrix of the concave indirect utility will be negative definite and therefore the principal minors must have alternating signs starting with a negative one. $\square$

Given lemma 3, and assuming existence of an interior optimum solution, the Hessian of the concave indirect utility is a negative definite matrix and the determinant ($\det(Hess)$) is positive. It can therefore be concluded that A and C are thus positive and B negative.

3.2 Effects of risk on investment and openness.

We can finally determine the total effects of risk on investment borrowing and on openness:

$$\frac{dI}{d\rho} = A \cdot \frac{\partial^2 U}{\partial I \partial \rho} + B \cdot \frac{\partial^2 U}{\partial B_c \partial \rho} \quad (14)$$

We conclude that there are direct and indirect effects determining whether investment borrowing will increase or decrease as risk aversion becomes larger. The direct effect works towards an increase (decrease) in investment as risk aversion increases, if expected consumption under no default is larger (smaller) than under default. However, there is an indirect effect through consumption borrowing given the cost of funds in equilibrium: the more borrowing there is of one type, the less is optimal to have of the other type. ($\frac{\partial^2 U}{\partial I \partial B_c} < 0$). More risk aversion will decrease (increase) consumption borrowing depending on whether r_c is low (high) enough relatively to δ . This will indirectly contribute to raise (reduce) investment borrowing. Consequently, both the direct and the indirect effects on investment borrowing can go in the same direction or in opposite directions, in which case one cannot know with certainty what happens to investment borrowing. Results are summarized in table 1:

Table 1
Investment Borrowing and Risk Aversion

	$\exp E(\ln C_{2d}) < \exp E(\ln C_{2n})$	$\exp E(\ln C_{2d}) > \exp E(\ln C_{2n})$
$\exp E(\ln C_{2n}) < \exp (\ln C_1)$	direct effect > 0 indirect effect < 0	$\frac{dI}{d\rho} < 0$
$\exp E(\ln C_{2n}) > \exp E(\ln C_1)$	$\frac{dI}{d\rho} > 0$	direct effect < 0 indirect effect > 0

3.3 Effects of risk on consumption borrowing.

The same type of analysis is conducted for consumption borrowing:

$$\frac{dB_c}{d\rho} = B \frac{\partial^2 U}{\partial I \partial \rho} + C \frac{\partial^2 U}{\partial B_c \partial \rho} \quad (15)$$

Once again, there are two separate effects: the direct effect of a higher degree of risk aversion works through intertemporal consumption smoothing: consumption borrowing will decrease (increase) if r_c is low (large) enough relatively to δ . The indirect effect works through an increase (decrease) in investment which reduces (increases) optimal consumption borrowing, given the cost of funds. This second effect depends on whether expected consumption under no default is larger (smaller) than under default. Results are summarized in table 2:

Table 2
Consumption Borrowing and Risk Aversion

	$\frac{\exp E(\ln C_{2d})}{\exp E(\ln C_{2n})} <$	$\frac{\exp E(\ln C_{2d})}{\exp E(\ln C_{2n})} >$
$\frac{\exp E(\ln C_{2n})}{\exp E(\ln C_1)} <$	direct effect > 0 indirect effect < 0	$\frac{dB_c}{d\rho} > 0$
$\frac{\exp E(\ln C_{2n})}{\exp E(\ln C_1)} >$	$\frac{dB_c}{d\rho} < 0$	direct effect < 0 indirect effect > 0

3.4 Effects of risk on total borrowing $\bar{B}$.

Finally, the effects on total borrowing arising from a change in the degree of risk aversion are:

$$\frac{d\bar{B}}{d\rho} = \frac{dI}{d\rho} + \frac{dB_c}{d\rho} \quad (16)$$

Using (14) and (15):

$$\frac{dI}{d\rho} = \frac{1}{\det(Hess)} \left\{ -\frac{\partial^2 U}{\partial I^2} \frac{\partial^2 U}{\partial I \partial \rho} + \frac{\partial^2 U}{\partial I \partial B_c} \frac{\partial^2 U}{\partial B_c \partial \rho} \right\} \quad (14')$$

$$\frac{dB_c}{d\rho} = \frac{1}{\det(Hess)} \left\{ \frac{\partial^2 U}{\partial I \partial B_c} \frac{\partial^2 U}{\partial I \partial \rho} - \frac{\partial^2 U}{\partial B_c^2} \frac{\partial^2 U}{\partial B_c \partial \rho} \right\} \quad (15')$$

$$\begin{aligned} \frac{d\bar{B}}{d\rho} = \frac{1}{\det(Hess)} & \left\{ \left\{ \frac{\partial^2 U}{\partial B_c \partial I} - \frac{\partial^2 U}{\partial B_c^2} \right\} \frac{\partial^2 U}{\partial I \partial \rho} + \right. \\ & \left. + \left\{ \frac{\partial^2 U}{\partial B_c \partial I} - \frac{\partial^2 U}{\partial I^2} \right\} \frac{\partial^2 U}{\partial B_c \partial \rho} \right\} \end{aligned} \quad (13')$$

It can be shown that $\frac{\partial^2 U}{\partial B_c \partial I} - \frac{\partial^2 U}{\partial B_c^2} > 0$ and that $\frac{\partial^2 U}{\partial B_c \partial I} - \frac{\partial^2 U}{\partial I^2} > 0$.

When both the direct effects of risk aversion on consumption and investment borrowing are of the same sign, the indirect effects cancel out, and the effect of an increase in risk aversion is known. Otherwise results are uncertain. Table 3 summarizes the results.

Table 3
Total Borrowing and Risk Aversion

	$\frac{\exp E(\ln C_{2_d})}{\exp E(\ln C_{2_n})} <$	$\frac{\exp E(\ln C_{2_d})}{\exp E(\ln C_{2_n})} >$
$\frac{\exp E(\ln C_{2_n})}{\exp E(\ln C_1)} <$	$\frac{d\bar{B}}{d\rho} > 0$	$\frac{dI}{d\rho} < 0$ $\frac{dB_c}{d\rho} > 0$
$\frac{\exp E(\ln C_{2_n})}{\exp E(\ln C_1)} >$	$\frac{dI}{d\rho} > 0$ $\frac{dB_c}{d\rho} < 0$	$\frac{d\bar{B}}{d\rho} < 0$

4. Conclusions.

The main results of the paper can be read from tables 1, 2 and 3 and can be stated as follows:

Theorem 1. *When the probability of default is high (low) and the consumption borrowing internal interest rate is small (large) enough relatively to the discount rate, optimal openness decreases (increases) with risk aversion. Otherwise, results are uncertain.*

It has been shown that optimal trade dependency is affected by the degree of risk aversion. Increased risk aversion affects optimal dependency through two channels: an intertemporal consumption smoothing effect, and an intratemporal consumption smoothing effect. The sign of these effects depends respectively on the relationship between the interest rate and the social discount rate and on the relationship between expected consumption in the no default states and expected consumption in the default states (this of course depends on the probability of default itself). These two effects can be the same sign or of opposite signs, in which case we cannot know with certainty how optimal trade openness changes.

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