Central Bank Preferences and Monetary Rules under the Inflation Targeting Regime in Brazil

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Abstract
The estimated interest rate rules are reduced form equations and for that reason they do not directly reveal anything about the structural parameters of monetary policy. In this paper, we seek to further elucidate the Brazilian monetary policy under the inflation targeting regime by calibrating Central Bank preferences. More specifically, we calibrate the policymaker’s loss function by choosing the preference parameter values which minimize the deviation between the optimal and actual paths of the basic interest rate (Selic). Our results indicate that the Central Bank has adopted a flexible inflation target regime and placed some greater weight upon inflation stabilization. We also find out that the monetary authority’s concern with interest rate smoothing has been far deeper than with output stabilization.

Keywords: Central Bank Preferences, Optimal Monetary Policy, Inflation Targets.
JEL Codes: C61, E52, E58.
1. Introduction

Since the influential study by Taylor (1993), interest rate rules have been commonly used to describe the behavior of monetary policy. In Brazil, these policy rules have been estimated by several authors in order to assess the Central Bank’s response to macroeconomic variables (such as inflation, output and exchange rate), and also to indicate possible monetary policy regime shifts.\footnote{See, for instance, Silva and Portugal (2001), Minella et al. (2002, 2003), Salgado et al. (2005) and Bueno (2005).}

Theoretically, interest rate rules can be formally derived from the solution of a restricted intertemporal optimization problem in which the monetary authority tries to minimize the squared deviations of the objective variables from their respective targets.\footnote{See, for example, Rudebusch and Svensson (1999).} Following this theoretical framework, monetary policy rule coefficients are convolutions of economic model parameters that restrict the optimization problem, as well as of parameters describing the monetary authority’s preferences. As the estimated interest rate rules are just reduced form equations, the estimation of their coefficients do not directly reveal anything about the structural parameters of the monetary policy (e.g., about the policymaker’s preferences). In addition, differences in the estimated interest rate rule coefficient values should not be viewed as monetary policy regime shifts because they may result from changes in the parameters of the macroeconomic model.

Thus, a way to shed further light upon monetary policy decisions is by extracting the loss function preference parameters of the policymaker from the estimated interest rate rules. Obtaining the estimates for the monetary authority’s preferences allows:

i) knowledge of the variables that are included in the loss function; and

ii) checking whether the economic results can be reconciled with the optimal monetary policy structure.

The aim of the present paper is to estimate the preferences of the Central Bank of Brazil in the current inflation targeting regime. To achieve that, we calibrate the loss function by choosing the preference parameter values from a wide range of alternative policies, which minimize the deviation between the simulated and actual paths of the Selic rate. The advantage of the calibration exercise over maximum likelihood estimates lies in the fact that it does not depend on the assumption about the distribution of error terms found in equations that restrict the monetary authority’s optimization problem. On the other hand, the fact that it does not produce standard deviations for the policymaker’s preferences does not allow testing the statistical significance of the estimates. To circumvent this problem, we estimate both the parameters of a model that restricts the optimization
problem and the parameters of the Central Bank’s objective function that best fit
the data, using the maximum likelihood method.

Most of the economic literature that seeks to estimate the monetary authority’s
preferences and objectives has focused attention on the Federal Reserve (Fed). For
example, Salemi (1995) identifies the Fed’s loss function parameters in the post-
World War II period using a linear quadratic optimal control structure and assum-
ing that the economy can be described by a vector autoregressive (VAR) model.
Ozlale (2003) and Dennis (2006) consider that the economy can be described by
the model proposed by Rudebusch and Svensson (1999), using an infinite-horizon
quadratic loss function and estimating the Fed’s preferences by maximum likeli-
hood. Favero and Rovelli (2003) argue that the Fed is concerned only with the
effect of its decision over a quarterly time frame and uses GMM to estimate the
monetary authority’s preferences. Dennis (2004) estimates the monetary policy
objective function by using a New Keynesian sticky price model, in which families
and firms are forward-looking, in order to restrict the optimization problem. S¨oder-
lind et al. (2002) and Castelnuovo and Surico (2003) estimate the Fed’s preferences
by means of a calibration exercise. The results obtained by these studies suggest
that the Fed has placed considerable weight on interest rate smoothing and given
lesser or unsubstantial importance to the output gap during the Volcker-Greenspan
period.

Cecchetti and Ehrmann (1999), Cecchetti et al. (2001) and Collins and Sik-
los (2004) extend the analysis of monetary authority’s preferences to other coun-
tries besides the U.S. Cecchetti and Ehrmann (1999) use VAR models to capture
the economic dynamics of 23 countries (including both developed and developing
economies) and to identify the preferences of central banks by way of estimates of
inflation-output variability. Results suggest that central banks developed stronger
aversion to inflation variability in the course of the 1990s. Cecchetti et al. (2001)
estimate the preferences of central banks of countries in the European Monetary
System and demonstrate that the objective functions of these monetary authori-
ties are surprisingly alike. By utilizing a calibration strategy, Collins and Siklos
(2004) show that the central banks of Australia, Canada, U.S. and New Zealand
can be described as having an optimal inflation target, placing considerable weight
on short-term interest rate smoothing and attaching sheer weight on output vari-
ability.

The present paper contributes to the available empirical literature as it in-
trduces novel estimates for the preferences of the Central Bank of Brazil during
the inflation targeting regime. The results reveal that the Brazilian monetary
authority has adopted a flexible inflation targeting regime and attached greater
importance to inflation stabilization. We also found out that the monetary author-
ity’s concern with interest rate smoothing has been far deeper than with output
stability.
In addition to this introduction, the paper is organized into six sections. Section 2 outlines the theoretical model and the monetary authority’s intertemporal optimization problem. Section 3 describes the empirical methodology used to calibrate the policymaker’s preferences. Section 4 provides and analyzes the results for the calibration exercises. Section 5 shows the estimates obtained by maximum likelihood for the monetary authority’s weights. Section 6 concludes.

2. The Theoretical Model

2.1 Structure of the economy

In this paper, we consider a simple structural macroeconomic model for an open economy with backward-looking expectations. The model is based on Rudebusch and Svensson (1999) and Freitas and Muinhos (2001). The three equations that form the model are:\(^3\)

\[
\begin{align*}
\pi_{t+1} &= \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \alpha_5 \Delta q_t + \alpha_6 y_{t-1} + \varepsilon_{\pi,t+1} \\
y_{t+1} &= \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 r_t + \varepsilon_{y,t+1} \\
q_{t+1} &= q_t + \varepsilon_{q,t+1}
\end{align*}
\]

where \(\pi_t\) is the annualized quarterly inflation rate, \(q_t\) is the nominal exchange rate, \(y_t\) is the output gap (calculated as the gap between real and potential GDP in percentage points) and \(r_t\) is the real interest rate, defined as the difference between the nominal interest rate regarded as monetary policy instrument, \(i_t\), and the inflation rate, \(\pi_t\). The error terms \(\varepsilon_{\pi,t}, \varepsilon_{y,t}\) and \(\varepsilon_{q,t}\) – assumed to be normally distributed (N.I.I.D) with zero mean and constant variances – can be interpreted as supply shocks, demand shocks and exchange rate shocks, respectively. All variables are expressed as deviation from the mean (demeaned); therefore, no constant appears in system (1)-(3).

Phillips curve (1) shows that the current inflation rate depends on its lagged values, on the nominal exchange rate fluctuations in the previous period and on the output gap with a two-period lag.\(^4\) Equation (2) is a conventional IS curve where the output gap at time \(t\) depends on its lagged values and on the real interest rate \((r_t = i_t - \pi_t)\) in the previous period. In equation (3), we follow Freitas and Muinhos (2001) and Moreira et al. (2007) and assume that the nominal exchange rate follows a random walk. The expected signs for the responses of inflation rate

\(^3\)Following a suggestion of an anonymous referee, we estimated an alternative econometric model that tries to resemble an open-economy DSGE model. In other words, we tried to estimate both the IS and the Phillips curves including the real exchange rate and the world output as explanatory variables. The results show that these variables are not statistically significant in the model.

\(^4\)The assumption that inflation depends on the output gap at \(t - 2\) was supported by the cross-correlogram analysis.
to exchange rate fluctuations and to output gap are $\alpha_5 > 0$ and $\alpha_6 > 0$. However, the coefficient that measures the response of the output gap to the real interest rate ($\beta_3$) is expected to have a negative value.

Although model (1)-(3) is parsimonious, it has two advantages:

i) it simplifies the solution to the monetary authority’s optimization problem; and

ii) it includes an important channel regarding the monetary policy transmission mechanism: the aggregate demand channel.

Specifically, the model implies that a contractionary monetary policy, implemented through a rise in $i_t$, reduces the output gap after one quarter and, consequently, reduces the inflation rate after three quarters. This assumption appears to be consistent with the macroeconomic models implemented by the Central Bank of Brazil, which predict that the aggregate demand channel of the monetary transmission mechanism takes 6 to 9 months to go into full operation (Bogdanski et al. (2000); Inflation Report issued by the Central Bank of Brazil, March 2000).

2.2 The Central Bank’s problem and the optimal monetary policy rule

Following Rudebusch and Svensson (1999), we assume that the monetary authority seeks to choose a path for the policy instrument (the nominal interest rate) so as to minimize:

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}$$

with

$$L_t = \lambda_\pi (\pi^a_t - \pi^*)^2 + \lambda_y y_t^2 + \lambda_i (i_t - i_{t-1})^2$$

where $E_t$ is the expectations operator conditional on the set of information available at $t$, $\delta$ is the discount rate ($0 < \delta < 1$), $\lambda_\pi > 0$, $\lambda_y \geq 0$ and $\lambda_i \geq 0$.

As demonstrated by Rudebusch and Svensson (1999), when $\delta \to 1$, the policymaker’s optimization problem is defined and the intertemporal loss function can be interpreted as unconditional mean of the loss function at time $t$, given by $E[L_t] = \text{var} [\pi^a_t - \pi^*] + \lambda_y \text{var} [y_t] - \lambda_i \text{var} [i_i - i_{i-1}]$, where $\text{var}$ stands for the variance.

We also considered time-varying inflation targets in the Central Bank loss function. The results concerning the optimal monetary rule and the parameters calibration are available from the authors upon request. They were presented in a previous draft of this paper, but removed from the final draft at the request of the editor to reduce the paper length.

As shown by Dennis (2006), expressing all variables as deviations from the mean (demeaned) does not change the derivation of the monetary authority’s preferences.
the output gap and the fluctuations in the interest rate are hypothetically equal to zero. The monetary authority’s preference parameters \( \lambda_\pi \), \( \lambda_y \) and \( \lambda_i \) show the importance attached to inflation stabilization, to the output gap and to interest rate smoothing. We infer that the sum of these preference parameters is equal to one, i.e., \( \lambda_\pi + \lambda_y + \lambda_i = 1 \).

The quadratic loss function (5) has been commonly used to assess optimal monetary policy for three reasons. The first one is that the quadratic loss function combined with linear restrictions implies linear decision rules. The second reason is that in addition to the stabilization of inflation and of the output gap, loss function (5) allows the monetary authority to smooth the nominal interest rate. Several reasons have been given to justify the smoothing of the nominal interest rate by central banks. Among such reasons, we highlight the following:

i) uncertainty surrounding data and coefficients in the monetary transmission mechanism;

ii) policymakers’ actions are taken only when they are sure about the results of these actions;

iii) remarkable changes in the interest rate may destabilize the financial and exchange rate markets;

iv) reversions in monetary policy actions can be noticed as errors or evidence of political inconsistency;

v) small and persistent changes in the short-term interest rate allow a large effect of monetary policy on aggregate demand without the need of excessive volatility of the policy instrument.\(^8\)

The third reason is that loss functions similar to (5) can be obtained from a second-order approximation of the intertemporal utility function of the representative agent (Woodford, 2003).

The monetary authority is supposed to minimize intertemporal loss function (4) subject to the restriction given by structural model (1)-(3). This model has an appropriate state-space representation, which can be denoted by:

\[
X_{t+1} = AX_t + B\epsilon_t + \epsilon_{t+1}
\]

where the column vector \( X_t \) of state variables, matrix \( A \), column vector \( B \) and the

\(^8\)For a theoretical and empirical analysis of interest rate smoothing, see Clarida et al. (1997), Sack (1998), Woodford (1999), Sack and Wieland (2000) and Sour (2001).
column vector of disturbances $\varepsilon_{t+1}$ are given by

$$
X_t = \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
y_t \\
y_{t-1} \\
q_t \\
q_{t-1} \\
i_t \\
i_{t-1}
\end{bmatrix}; \\
A = \begin{bmatrix}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & \alpha_6 & \alpha_5 & -\alpha_5 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_3 & 0 & 0 & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}; \\
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-\beta_3 \\
0 \\
0 \\
0 \\
1
\end{bmatrix}; \\
\varepsilon_{t+1} = \begin{bmatrix}
\varepsilon_{\pi,t+1} \\
0 \\
0 \\
0 \\
0 \\
\varepsilon_{y,t+1} \\
0 \\
0 \\
\varepsilon_{q,t+1} \\
0
\end{bmatrix}
$$

The loss function at time $t$ can be written in matrix notation. To do that, first it is necessary to express the monetary authority’s vector of objective variables as a function of the vector of state variables and of the control variable (nominal interest rate) as follows:

$$
Y_t = C_X X_t + C_i i_t
$$

where the vector of objective variables $Y_t$, matrix $C_X$ and column vector $C_i$ are given by:

$$
Y_t = \begin{bmatrix}
\pi^a \\
y_t \\
i_t - i_{t-1}
\end{bmatrix}; \\
C_X = \begin{bmatrix}
1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}; \\
C_i = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
$$

Thus, the loss function can be written as:

$$
L_t = Y_t^T K Y_t
$$

where superscript $T$ indicates the transpose of a vector (or matrix) and $K$ is a $3 \times 3$ matrix with the main diagonal equal to $(\lambda_\pi, \lambda_y, \lambda_i)$ and all remaining elements equal to zero.
The monetary authority’s optimization problem can be viewed as a stochastic linear regulator problem denoted by:

$$\min_{\{i_t\}} \mathbb{E}_0 \sum_{\tau=0}^{\infty} \delta^\tau \left[ Y_t^T K Y_t \right] = \min_{\{i_t\}} \sum_{\tau=0}^{\infty} \delta^\tau \left[ X_t^T R X_t + i_t Q i_t + 2 i_t W X_t \right]$$  \hspace{1cm} (11)

subject to

$$X_{t+1} = A X_t + B i_t + \varepsilon_{t+1}$$  \hspace{1cm} (12)

with given \(X_0, R = C_i^T K C_i\) and \(W = C_i^T K C_X\). The standard way to solve problem (12) is by dynamic programming. Let \(V(X)\) be the optimal value associated with the program starting in initial state \(X_0\). Bellman’s equation is:

$$V(X_t) = \max_{i_t} \left\{ - \left( X_t^T R X_t + i_t Q i_t + 2 i_t W X_t \right) + \delta \mathbb{E}_t V(X_{t+1}) \right\}$$  \hspace{1cm} (13)

where maximization is subject to (6). As shown in Hansen and Sargent (2004), the quadratic value function \(V(X_t)\) which satisfies Bellman’s equation is given by:

$$V(X_t) = -X_t^T P X_t - \rho$$  \hspace{1cm} (14)

where

$$\rho = \delta (1 - \delta)^{-1} tr P \sum_{\varepsilon \varepsilon}$$  \hspace{1cm} (15)

where \(tr\) is the trace of matrix \(P\) and \(\sum_{\varepsilon \varepsilon}\) is the covariance matrix of the vector of disturbances \(\varepsilon_t\). Matrix \(P\) is positive semidefinite symmetric and satisfies the algebraic matrix Riccati equation, defined as:

$$P = R + \delta A^T P A - \left( \delta A^T P B + W \right) \left( Q + \delta B^T P B \right)^{-1} \left( \delta B^T P A + W^T \right)$$  \hspace{1cm} (16)

The monetary authority’s optimal interest rate rule is denoted by:

$$i_t = f X_t$$  \hspace{1cm} (17)

with

$$f = - \left( Q + \delta B^T P B \right)^{-1} \left( \delta B^T P A + W^T \right)$$  \hspace{1cm} (18)

Equation (17) shows that the nominal interest rate at time \(t\), regarded as monetary policy instrument, is a linear function of the state variable vector, \(X_t\). The coefficients in line vector \(f = [f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8]\) represent the convolutions of the monetary authority’s preference parameters and the Phillips and IS curve parameters. Therefore, for each set of preference values, there exists a different optimal rule. Due to the assumption that the nominal exchange rate follows a random walk, we have \(f_7 = -f_8\) in the vector of parameters \(f\).

For further details, see Ljungqvist and Sargent (2004) and Hansen and Sargent (2004).
After obtaining optimal policy rule (17), the model dynamics is determined by:

\[ X_{t+1} = MX_t + \varepsilon_{t+1} \]  
\[ Y_t = CX_t \]

where matrices \( M \) and \( C \) are given by

\[ M = A + Bf \]
\[ C = C_X + C_if \]

3. **Empirical Calibration Strategy for Central Bank Preferences**

A broadly used strategy for identifying the monetary authority’s preferences is their consistent calibration with the data (Söderlind et al., 2002, Castelnuovo and Surico, 2003, 2004, Collins and Siklos, 2004). According to Castelnuovo and Surico (2003), the advantage of this empirical strategy over estimation methods such as maximum likelihood and GMM is that it is more robust to the misspecification of the error term. This occurs because this strategy is not reliant upon the distributions of shocks observed in the economic model that restricts the policymaker’s optimization. In addition, the calibration strategy relatively facilitates the demonstration and interpretation of the effects of changes in the calibrated parameters.

In this paper, the calibration strategy used to identify the preferences of the Central Bank of Brazil can be split into four stages. First, we estimate the parameters of each equation for structural model (1)-(3). These estimates enter system (6) and restrict the monetary authority’s intertemporal optimization problem. In the second stage, we calculate the coefficients for optimal interest rate rule (17) and solve stochastic linear regulator problem (11). Since changes in monetary authority’s preferences \( (\lambda_\pi, \lambda_y \text{ and } \lambda_i) \) yield different optimal policy rule coefficients, we solve the optimization problem for a wide array of alternative preferences. More specifically, we calculate the optimal policy rule for every possible combination of \( \lambda_\pi \) and \( \lambda_y \) on the interval [0.001 - (1 - \lambda_i - 0.001)] in steps of 0.001 for a given value of the monetary authority’s preference over the interest rate smoothing, \( \lambda_i \). We allow the preference parameter \( \lambda_i \) to vary on the interval [0-0.95] in steps of 0.05. In the third stage, we replace the values of the state variables observed in each optimal policy rule on a period-by-period basis and we calculate the nominal interest rate optimal path, given by \( i_t = f(\lambda_\pi, \lambda_y, \lambda_i)X_t \). Finally, we choose the Central Bank’s preference values \( (\lambda_\pi, \lambda_y \text{ and } \lambda_i) \) which minimize the squared deviation (SD) of the actual path from the optimal path of the nominal interest rate, i.e.,

\[ SD = \sum_{t=1}^{T} [i_t - i_t(\lambda_\pi, \lambda_y, \lambda_i)]^2 \]  

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4. Results

4.1 Macroeconomic model estimates for Brazil

As described in the previous section, the first stage necessary for the calibration of the monetary authority’s loss function parameters consists in estimating the structural macroeconomic model that restricts this policymaker’s optimization problem. As macroeconomic model (1)-(3) contains backward-looking expectations, the derivation of the optimal policy rule is based on the assumption that the parameters are invariant to policy regime shifts, thus being amenable to Lucas critique (1976). Therefore, we estimate Phillips and IS curves, shown in equations (1) and (2), by using quarterly data for the 1999:2-2007:3 period. This period seems appropriate because it includes only one exchange rate regime (floating exchange rate) and one monetary policy regime (inflation targeting regime), thus playing down the importance of Lucas critique (1976).10 We decided to use quarterly instead of monthly data for two reasons:

i) quarterly data are less susceptible to noise and measurement errors;

ii) if the effects of monetary policy actions on inflation take 2 or 3 quarters to occur, the use of monthly data implies a large number of lags in terms of explanatory variables, which partially reduces the advantage of having a sample with more observations.

The variables used, obtained from the Institute for Applied Economic Research (IPEA) and from the Central Bank of Brazil websites, are defined as follows:

i) inflation rate ($\pi_t$): annualized quarterly inflation rate measured by the Broad Consumer Price Index (IPCA), which is used as inflation target by the Central Bank;

ii) output gap ($y_t$): percentage difference between the seasonally adjusted quarterly real GDP and the potential output obtained through the Hodrick-Prescott filter;

iii) real interest rate ($r_t$): obtained by the difference between the nominal interest rate ($i_t$), defined as the quarterly mean of the Selic rate accumulated during the month and annualized, and the inflation rate ($\pi_t$);

iv) nominal exchange rate ($q_t$) and nominal exchange rate depreciation ($\Delta q_t$): variable $q_t$ is given by $100\ln(Q_t)$, where $\ln$ denotes the natural logarithm and $Q_t$ is the quarterly mean of the monthly nominal exchange rate (average selling rate). Variable $\Delta q_t$ is the percentage variation of the nominal exchange rate (or the nominal exchange rate depreciation).

10The floating exchange rate regime was adopted in Brazil in January 1999 after four years of an exchange rate crawling peg regime. The inflation targeting regime was implemented 6 months after the floating exchange rate regime was adopted.
Before estimating the structural equations, we ran ADF and Phillips-Perron tests to check for the stationarity of the variables in the model. For the ADF test, we chose the optimal number of lagged difference terms to be included in each regression, \( k \), based on the Schwarz information criterion.\(^{11}\) The maximum autoregressive order was equal to eight. For the inflation rate and the real interest rate, the tests were done using a constant, whereas for the exchange rate, a linear trend was also used. Table 1 shows the test results. Note that the ADF and Phillips-Perron tests reject, at a 5% significance level, the null hypotheses that the output gap, inflation rate and real interest rate have a unit root. For the nominal exchange rate, the tests indicate the presence of unit root for the series in the level, but not for the first-difference of the series (exchange rate depreciation).

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test</th>
<th>Phillips-Perron test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>1</td>
<td>-3.48*</td>
</tr>
<tr>
<td>( \pi_t )</td>
<td>0</td>
<td>-3.11**</td>
</tr>
<tr>
<td>( r_t )</td>
<td>1</td>
<td>-5.08*</td>
</tr>
<tr>
<td>( q_t )</td>
<td>0</td>
<td>-1.15ns</td>
</tr>
<tr>
<td>( \Delta q_t )</td>
<td>0</td>
<td>-4.60*</td>
</tr>
</tbody>
</table>

Note: *Significant at 1%.
**Significant at 5%.
nsNonsignificant.

\(^{11}\)The results did not change when we used the Akaike information criterion.

In the subsequent stage, we seek to estimate structural macroeconomic model (1)-(3). As the nominal exchange rate supposedly follows a random walk, we estimate only the Phillips and IS curves. In addition, we include the dummy variable \( D_{\pi,t}, t=1 \) for 2002:4 and 0, otherwise) in the Phillips curve in order to capture the strong increase in the inflation rate observed in the last quarter of 2002. Among the possible causes for the rise in inflation during this period are the depreciation of the exchange rate, adjustment in the monitored prices and intercrop period. To the IS curve, we added the dummy variable \( D_{y_1,t}, t=1 \) for 2001:3-4 and 0, otherwise) for the power crisis period and the dummy variable \( D_{y_2,t}, t=1 \) for 2003:1-2 and 0, otherwise) in order to capture the negative effects of adverse credit conditions and of the strong losses in workers’ real wages on economic activity in the first semester of 2003. Finally, we imposed the verticality of the Phillips curve by restricting the sum of the coefficients related to inflation in previous periods and to the fluctuations in the nominal exchange rate to 1. This implies that any exchange rate depreciation is totally transferred to prices in the long run.
Therefore, the final specifications of the Phillips and IS curves we estimated are given by:

\[
\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \alpha_5 \Delta q_{t-1} \\
+ \alpha_6 y_{t-2} + \alpha_7 D_{\pi,t} + \varepsilon_{\pi,t}
\]  \quad (24)

\[
y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 r_{t-1} + \beta_4 D_{y1,t} + \beta_5 D_{y2,t} + \varepsilon_{y,t}
\]  \quad (25)

where \(\alpha_4 = 1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_5\). The system represented by equations (24)-(25) was estimated in three different ways:

i) by ordinary least squares (OLS) for each equation in the system;

ii) by SUR (seemingly unrelated regressions); and

iii) by full information maximum likelihood (FIML).

The SUR method is more appropriate than OLS when there is contemporaneous correlation between the errors of each equation. In this case, the stronger the correlation between errors, the larger the gain in efficiency of the SUR method vis-à-vis OLS. The estimation by way of FIML is based on the assumption that contemporaneous errors have a joint normal distribution. As pointed out by Greene (2000), if the log-likelihood function is correctly specified, the FIML estimator is efficient among all other estimators that consider the joint estimation of systems of equations (such as two-stage and three-stage least squares).

The results for the estimations of the Phillips and IS curves are shown in Table 2 (the value in brackets refers to the standard error deviation). Note that the parameter estimates obtained by OLS for the two equations are quite similar to those obtained by SUR and FIML. However, estimation by FIML yielded a larger number of statistically nonsignificant parameters comparatively to the alternative methods. This inaccuracy regarding FIML estimates may occur due to the small sample size used.
### Table 2
Parameter estimates for the macroeconomic model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>OLS</th>
<th>SUR</th>
<th>FIML</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phillips curve</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.3532*</td>
<td>0.3738*</td>
<td>0.3762n.s</td>
</tr>
<tr>
<td></td>
<td>(0.1223)</td>
<td>(0.1603)</td>
<td>(0.1603)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.0882n.s</td>
<td>-0.1090n.s</td>
<td>-0.1113n.s</td>
</tr>
<tr>
<td></td>
<td>(0.1556)</td>
<td>(0.1125)</td>
<td>(0.1810)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.3095***</td>
<td>0.3301*</td>
<td>0.3320***</td>
</tr>
<tr>
<td></td>
<td>(0.1550)</td>
<td>(0.1094)</td>
<td>(0.1722)</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.0322</td>
<td>0.0208</td>
<td>0.0196</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.3933*</td>
<td>0.3843*</td>
<td>0.3835*</td>
</tr>
<tr>
<td></td>
<td>(0.1031)</td>
<td>(0.0827)</td>
<td>(0.1136)</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>0.3244n.s</td>
<td>0.3299n.s</td>
<td>0.3294n.s</td>
</tr>
<tr>
<td></td>
<td>(0.6680)</td>
<td>(0.5885)</td>
<td>(0.8935)</td>
</tr>
<tr>
<td>$\alpha_7$</td>
<td>12.499*</td>
<td>12.723*</td>
<td>12.743n.s</td>
</tr>
<tr>
<td></td>
<td>(1.8054)</td>
<td>(2.9694)</td>
<td>(22.678)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8011</td>
<td>0.8007</td>
<td>0.8006</td>
</tr>
</tbody>
</table>

Specification tests (p values)

| LB(6)   | 0.237 | 0.222 | 0.220 |
| LB(8)   | 0.303 | 0.278 | 0.276 |
| ARCH(6) | 0.844 | 0.858 | 0.859 |
| JB      | 0.854 | 0.819 | 0.816 |

| **IS curve** |     |     |      |
| $\beta_1$ | 0.5056* | 0.5042* | 0.5038* |
|            | (0.1793) | (0.1603) | (0.1589) |
| $\beta_2$ | -0.1746n.s | -0.1885n.s | -0.1900n.s |
|            | (0.1659) | (0.1506) | (0.2114) |
| $\beta_3$ | -0.0747*** | -0.0769** | -0.0772n.s |
|            | (0.0412) | (0.0371) | (0.0593) |
| $\beta_4$ | -1.1053*** | -0.9794*** | -0.9668n.s |
|            | (0.5470) | (0.4998) | (1.7694) |
| $\beta_5$ | -2.8022* | -2.7643* | -2.7606* |
|            | (0.7965) | (0.7119) | (1.0464) |
| $R^2$ | 0.6583 | 0.6573 | 0.6571 |

Specification tests (p values)

| LB(6)   | 0.551 | 0.628 | 0.636 |
| LB(8)   | 0.380 | 0.436 | 0.442 |
| ARCH(6) | 0.323 | 0.329 | 0.330 |
| JB      | 0.659 | 0.671 | 0.671 |

Note: *Significant at 1%.
**Significant at 5%.
***Significant at 10%.
n.s.Nonsignificant.
In general, parameter estimates have the expected sign. An exception was the inflation coefficient at \( t - 2 \) in the Phillips curve, which had a negative sign but was statistically nonsignificant. The coefficient that measures the impact of the output gap on the inflation rate was not statistically significant. This result is worrying as the effect of the level of activity on inflation is an important component of the monetary policy transmission mechanism considered herein.\(^{12}\) The parameter estimate for the shift in the exchange rate regime suggests that, *ceteris paribus*, an increase by 1 percentage point in the nominal exchange rate depreciation implies an increase of around 0.38 percentage points in annualized inflation. The positive value of the coefficient related to the dummy variable \( D_{\pi,t} \) demonstrates a strong increase in the mean level of inflation, due chiefly to the exchange rate crisis in 2002. With regard to the IS curve equation, the coefficients for the output gap at \( t - 1 \), for the lagged real interest rate and for the dummy variables, estimated by OLS and SUR, were significant at 10%.

The effect of the interest rate on inflation is indirect and takes three quarters to operate fully. According to the parameter values estimated by OLS, a one-percentage point increase in the real interest rate in month \( t \) decreases the output gap by 0.0747 percentage points at \( t + 1 \). On the other hand, a one-percentage point reduction in the output gap reduces the inflation rate by 0.3244 percentage points in the subsequent period. Therefore, a rise in the real interest rate by one percentage point at \( t \) reduces the inflation rate by 0.02 percentage points at \( t + 3 \). It is imperative to show that this result should be viewed with caution due to the statistical nonsignificance of the output gap coefficient in the Phillips curve (24).

We checked for the presence of autocorrelation, autoregressive conditional heteroskedasticity (ARCH) and non-normality of errors in the Phillips and IS curves. According to the Ljung-Box (LB) test, we cannot reject the hypothesis that the residuals of both equations are serially uncorrelated. The ARCH test results do not indicate the presence of statistically significant autoregressive conditional heteroskedasticity of the residuals of the estimated equations. Finally, the Jarque-Bera test, at a 5% significance level, shows that the residuals of both equations are normally distributed. This set of results suggests that the estimated equations are well-specified.

### 4.2 Central Bank preferences in the inflation targeting regime

In this section, we seek to estimate the Central Bank’s loss function parameters by choosing the weights that cause a smaller squared deviation of the optimal path from the actual path of the Selic rate. The optimal interest rate path is obtained on a period-by-period basis by insertion of state variable actual values in the optimal monetary policy rule. Since the parameter values for the Phillips and IS curves are known, the optimal policy rule and, consequently, the optimal path for the policy

\(^{12}\)Similar results were found by Bonomo and Brito (2002) and Faria (2006).
instrument depend on the weights the monetary authority attaches to inflation and output gap stabilization and to interest rate smoothing.

To start the calibration process, we chose the parameter estimates of the macroeconomic model obtained by OLS, given that different estimation methods yielded very similar results. Following Moreira et al. (2007), we assume that the discount factor, \( \delta \), is equal to 0.98. In addition, we considered that the weight on interest rate smoothing can vary on the interval \([0-0.95]\) in steps of 0.05. For each value of \( \lambda_i \), we calculate the optimal policy rule for every possible combination of \( \lambda_\pi \) and \( \lambda_y \) on the interval \([0.001 - (1 - \lambda_i - 0.001)]\) in steps of 0.001. This calibration strategy allows us to obtain 10,480 monetary policy rules and to choose the loss function parameters that minimize the squared deviation of the optimal path from the actual path of the Selic rate.

The calibration results for the Central Bank’s loss function are presented in Table 3. For each value of \( \lambda_i \), we provide the weights \( \lambda_\pi \) and \( \lambda_y \) which produce a smaller squared deviation (SD). Initially, we noticed that when the monetary authority is supposedly not concerned with the smoothing of the monetary policy instrument, the squared deviation of the optimal interest rate from the actual interest rate is extremely large. This suggests that the Central Bank has given a positive weight to interest rate smoothing in its loss function.

<table>
<thead>
<tr>
<th>( \lambda_i )</th>
<th>( \lambda_\pi )</th>
<th>( \lambda_y )</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.001</td>
<td>0.999</td>
<td>2496</td>
</tr>
<tr>
<td>0.05</td>
<td>0.248</td>
<td>0.702</td>
<td>81.09</td>
</tr>
<tr>
<td>0.10</td>
<td>0.445</td>
<td>0.455</td>
<td>56.40</td>
</tr>
<tr>
<td>0.15</td>
<td>0.600</td>
<td>0.250</td>
<td>51.20</td>
</tr>
<tr>
<td>0.20</td>
<td>0.727</td>
<td>0.073</td>
<td>50.13</td>
</tr>
<tr>
<td>0.25</td>
<td>0.749</td>
<td>0.001</td>
<td>50.85</td>
</tr>
<tr>
<td>0.30</td>
<td>0.699</td>
<td>0.001</td>
<td>53.54</td>
</tr>
<tr>
<td>0.35</td>
<td>0.649</td>
<td>0.001</td>
<td>56.98</td>
</tr>
<tr>
<td>0.40</td>
<td>0.599</td>
<td>0.001</td>
<td>60.57</td>
</tr>
<tr>
<td>0.45</td>
<td>0.549</td>
<td>0.001</td>
<td>64.06</td>
</tr>
<tr>
<td>0.50</td>
<td>0.499</td>
<td>0.001</td>
<td>67.39</td>
</tr>
<tr>
<td>0.55</td>
<td>0.448</td>
<td>0.002</td>
<td>70.54</td>
</tr>
<tr>
<td>0.60</td>
<td>0.399</td>
<td>0.001</td>
<td>73.45</td>
</tr>
<tr>
<td>0.65</td>
<td>0.349</td>
<td>0.001</td>
<td>76.20</td>
</tr>
<tr>
<td>0.70</td>
<td>0.299</td>
<td>0.001</td>
<td>78.80</td>
</tr>
<tr>
<td>0.75</td>
<td>0.249</td>
<td>0.001</td>
<td>81.26</td>
</tr>
<tr>
<td>0.80</td>
<td>0.198</td>
<td>0.002</td>
<td>83.64</td>
</tr>
<tr>
<td>0.85</td>
<td>0.149</td>
<td>0.001</td>
<td>85.88</td>
</tr>
<tr>
<td>0.90</td>
<td>0.098</td>
<td>0.002</td>
<td>88.12</td>
</tr>
<tr>
<td>0.95</td>
<td>0.049</td>
<td>0.001</td>
<td>90.34</td>
</tr>
</tbody>
</table>
We also realized that for $\lambda_i$ parameter values between 0 and 0.1, combinations $\{\lambda_\pi, \lambda_y\}$ which produce smaller squared deviations reveal a deeper concern of the policymaker with the output gap than with inflation. The opposite occurs for the weights placed on interest rate smoothing that are greater than 0.10. For instance, when $\lambda_i = 0.7$, the weight given to inflation is virtually equal to 0.3, whereas the weight given to the output gap is almost zero.

Finally, the results shown in Table 3 indicate that the loss function parameters that minimize the squared deviation between the optimal path and the actual path of the Selic rate are $\lambda_\pi = 0.727, \lambda_y = 0.073$ and $\lambda_i = 0.2$.\footnote{Qualitatively, the results do not change when we use the parameter estimates of the Phillips and IS curves obtained by SUR and FIML, or when we consider a model in which inflation targets vary over time. The results based on calibration, presented in a previous draft of the paper, are not shown here due to the lack of space. They are available from the authors upon request.} This demonstrates that the Central Bank of Brazil has adopted a flexible inflation targeting regime and placed a heavier weight on price stability than on output gap stability. Moreover, we perceived that the monetary authority’s concern with interest rate smoothing has been far deeper than with output stability.

**Optimal monetary policy rule**

The parameter estimates of the macroeconomic model and of the loss function imply that the optimal monetary policy, shown in equation (17), is given by:

$$i_t = 0.1388\pi_t + 0.0549\pi_{t-1} + 0.0586\pi_{t-2} + 0.0054\pi_{t-3} + 0.1153y_t + 0.0273y_{t-1} + 0.0663\Delta q_t + 0.8706i_{t-1}$$

Equation (26)

This policy rule indicates that the nominal interest rate responds contemporaneously to changes in the inflation rate, output gap, and exchange rate fluctuations. In particular, the optimal monetary rule coefficients show that: a one-percentage point increase in the inflation rate increases the Selic rate by approximately 0.14 percentage points; a one-percentage point increase in the output gap increases the Selic rate by 0.12 percentage points; and a one-percentage point increase in exchange rate depreciation increases the Selic rate by 0.07 percentage points. The monetary authority also responds to the lagged values of the inflation rate and of the output gap, although this response is weaker for inflation and for the output gap at time $t$. Another important result is concerned with the high value obtained for the autoregressive interest rate coefficient (0.87), which demonstrates the Brazilian monetary authority’s concern with smoothing the Selic rate.

Coefficients $f$’s in policy rule (26) represent the immediate effect of explanatory variables on the Selic rate. However, the state variables also have secondary effects on the interest rate due to their lagged values and to the inertial term $i_{t-1}$. We can gauge these secondary effects by expressing the optimal policy rule in the long

\footnote{The results do not show significant changes when we use different values for the discount factor, $\delta$. Again, full results can be obtained upon request.}
run, which is given by:

\[
\begin{align*}
i &= \phi_1 \pi + \phi_2 y + \phi_3 \Delta q \\
&= 1.991 \pi_t + 1.0943 y_t + 0.5122 \Delta q_t
\end{align*}
\]  

(27)

where \( \phi_1 = (f_1 + f_2 + f_3 + f_4)/(1 - f_9) \), \( \phi_2 = (f_5 + f_6)/(1 - f_9) \) and \( \phi_1 = f_7/(1 - f_9) \).

According to this policy rule, a sustained increase by one percentage point in inflation rate raises the Selic rate by 1.99 percentage points, whereas an increase by one percentage point in the output gap implies a 1.09 percentage point rise in the Selic rate. It should be highlighted that a more-than-proportional increase in the interest rate in response to inflation shows that this policy rule satisfies Taylor’s principle (1993). Thus, when inflation demonstrates a sustained rise, the Central Bank increases the nominal interest rate by such a value that allows the real interest rate to rise, the output gap to decrease and inflation to return to its target. Finally, we compare the monetary policy rule in the long run with that presented in Taylor (1993) and observe that the Central Bank responds more often to inflation (1.99 here \textit{vis-à-vis} 1.5 in Taylor) and to the output gap (1.09 here \textit{vis-à-vis} 0.5 in Taylor) than does the Federal Reserve.

4.3 Optimal path versus the actual path of the Selic rate

Figure 1 shows the optimal path of the Selic rate associated with the Central Bank preferences obtained by the calibration strategy (Selic\textsubscript{f}) and the actual path of the Selic rate (Selic).\textsuperscript{15} In general, we can observe that the optimal policy captures the major interest rate movements in the inflation targeting regime. However, some discrepancies exist. For example, we notice that a policymaker with calibrated weights would have maintained the Selic rate above the rate observed throughout the 2000:3-2001:1 period. In addition, in the third quarter of 2000, the monetary authority’s optimal decision would have been to increase the Selic rate in response to inflation pressures produced by the mark-up of the prices of agricultural products and so-called “administered items” (e.g.: telephone services, light bill, and fuel). Conversely, the Central Bank adopted an expansionary policy, making the actual Selic rate deviate further away from the optimal Selic rate.

Another major difference between the behavior of the optimal policy and that which was actually observed can be seen in the second and third quarters of 2002, in which the dispute for presidential elections was strongest. While the Central Bank exposed the Selic rate to successive decreases in this period, the policymaker, acting optimally, would have followed a contractionary monetary policy in response to exchange rate depreciation and inflation acceleration. The consequences of the expansionary policy seemingly occurred in the first quarter of 2003 when the monetary authority rose the Selic rate above that which was optimally predicted

\textsuperscript{15}As all variables are (demeaned) expressed as deviations from the mean, we added the mean of the Selic rate.
in order to reduce the increase in prices and the inflation inertia caused by the shocks in 2002.

From July to September of 2004 onwards, the Central Bank put an end to the decreasing path of the Selic rate due to new inflation pressures produced mainly by monitored prices. Nevertheless, Figure 1 shows that the increase in the Selic rate occurred at a faster pace than that which had been predicted by the optimal policy rule associated with calibrated prices. In the second quarter of 2005, the difference between the actual Selic rate and the simulated one peaked at 198 basis points and remained positive for the two subsequent quarters. This suggests a conservative behavior of the Brazilian monetary authority towards the policymaker with calibrated weights who adjusts the interest rate optimally.

Finally, we perceived that the monetary policy decisions taken ever since the second quarter of 2006 were quite close to the optimal policy, with a mean difference of -0.06% percentage points. This result seems to rule out the possibility that the Central Bank adopted a conservative behavior by reducing the interest rate during the past two years.

Comparison with alternative weights for the loss function

It is useful to compare the monetary policy rule obtained from calibrated weights for the loss function with the rules associated with different weights. Here, we contemplate five sets of alternative weights. Table 4 summarizes the characteristics of the analyzed cases. The first case deals with the set of weights of a policymaker who adopts a strict inflation targeting regime (King, 1997). In the
second case, the weights for the loss function are those used by Rudebusch and Svensson (1999) when deriving the optimal monetary rules for the Federal Reserve and represent a flexible inflation targeting regime in which the monetary authority equally weights inflation and output stabilization and smooths the interest rate. The third case differs from the previous one because there is no concern with interest rate smoothing. This set of weights is particularly important since it was used by Almeida et al. (2003) and Moreira et al. (2007) for obtaining optimal monetary policy rules for the Brazilian economy. The fourth case is the set of weights calibrated by our calibration strategy. Cases 5 and 6 represent combinations \( \{\lambda_\pi, \lambda_y\} \) which minimize the squared deviation of the observed interest rate from the optimal interest rate for values of \( \lambda_i \) equal to 0.5 and 0.9. In the fifth case, the policymaker uses similar weights for inflation stabilization and interest rate smoothing, and a near-zero weight for the output. The sixth case deals with a monetary authority that is deeply concerned with interest rate smoothing.

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights used in the Central Bank’s loss function</td>
</tr>
<tr>
<td>Cases</td>
</tr>
<tr>
<td>1. Strict inflation targets (King, 1997)</td>
</tr>
<tr>
<td>2. Flexible inflation targets (Rudebusch and Svensson, 1999)</td>
</tr>
<tr>
<td>3. Flexible inflation targets without smoothing of it (Almeida et al., 2003, Moreira et al., 2007)</td>
</tr>
<tr>
<td>4. Calibrated weights</td>
</tr>
<tr>
<td>5. Benchmark1</td>
</tr>
<tr>
<td>6. Benchmark2</td>
</tr>
</tbody>
</table>

Table 5 shows the optimal monetary rules for the six cases described above. We can notice that values for the policy rule coefficients and for the squared deviation (SD) between the optimal and the observed interest rates are way above those found for the calibrated weights (case 4) in the set of weights in which there is no concern with interest rate smoothing.

For the set of weights considered by Rudebusch and Svensson (1999), the long-run optimal monetary policy rule implies that the interest rate is less sensitive to inflation, output gap and exchange rate depreciation than in case 4. Despite that, the squared deviation and the mean absolute error (MAE) suggest no major differences between the actual Selic rate and the optimal Selic rate in both cases.

By comparing the optimal monetary rule in case 4 with those of cases 5 and 6, we note that a greater weight on interest rate smoothing causes larger gradualism in monetary policy and, consequently, a larger squared deviation of the optimal interest rate from the interest rate which was actually observed.
Table 5
Optimal monetary rules for different weights on the Central Bank’s loss function

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_\pi$</td>
<td>1.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.727</td>
<td>0.499</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>0.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.073</td>
<td>0.001</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>0.200</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Short-run optimal monetary rule

\[ i_t = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-2} + f_4 \pi_{t-3} + f_5 y_t + f_6 y_{t-1} + f_7 \triangle q_t + f_8 i_{t-1} \]

| $f_1$    | 70.369 | 0.0977 | 2.3713 | 0.1388 | 0.0486 | 0.0067 |
| $f_2$    | 13.554 | 0.0325 | 0.5188 | 0.0549 | 0.0197 | 0.0027 |
| $f_3$    | 19.548 | 0.0386 | 0.5771 | 0.0586 | 0.0207 | 0.0029 |
| $f_4$    | 1.8467 | 0.0036 | 0.0536 | 0.0054 | 0.0019 | 0.0003 |
| $f_5$    | 24.884 | 0.1252 | 7.3949 | 0.1143 | 0.0367 | 0.0051 |
| $f_6$    | 16.267 | -0.0115 | -1.7977 | 0.0273 | 0.0118 | 0.0016 |
| $f_7$    | 22.556 | 0.0440 | 0.6543 | 0.0663 | 0.0234 | 0.0032 |
| $f_8$    | 0.0000 | 0.8405 | 0.0000 | 0.8706 | 0.9331 | 0.9793 |
| SD      | 8E+06  | 55.98  | 8512   | 50.13  | 67.39  | 88.12  |
| MAE     | 338.34 | 1.09   | 11.91  | 1.01   | 1.13   | 1.23   |

Long-run monetary rule

\[ i = \phi_1 \pi + \phi_2 y + \phi_3 \triangle q \]

| $\phi_1$ | 105.32 | 1.0809 | 3.5205 | 1.9915 | 1.3587 | 0.6087 |
| $\phi_2$ | 41.151 | 0.7129 | 5.5972 | 1.0943 | 0.7250 | 0.3237 |
| $\phi_3$ | 22.556 | 0.2759 | 0.6543 | 0.5124 | 0.3498 | 0.1546 |

The optimal path of the Selic rate in each of the cases above ($Selic_f$) is shown in Figure 2 along with the actual path of the Selic rate ($Selic$). Under the monetary policies of a policymaker who attaches exclusive importance to inflation (case 1) or with no concern with policy instrument smoothing (case 3), we found that the Selic rate would vary considerably, showing positive values with up to four digits in some periods and negative values in other periods. This strongly suggests that the Central Bank has not used any of these sets of weights for its loss function.

For those cases in which the weight on the interest rate smoothing is positive, we noted that the optimal Selic rate can adjust reasonably well to the observed Selic rate. However, simulations indicate that a policymaker whose primary goal is to smooth the interest rate would have shown larger lags in response to the rise in inflation in 2002, and also kept the interest rate persistently above that which had been observed in the past two years. On the other hand, for cases 2 and 4, we noted that the optimal paths for the interest rate were quite similar throughout the period, differing only in the last two years where calibrated weights imply an optimal Selic rate that closely resembles the observed Selic rate.

We do not impose the restriction of non-negativity on the nominal interest rate.

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Even though the visual inspection of optimal paths for the Selic rate is informative, it is not necessarily conclusive. Because of that, we used the encompassing test proposed Chong and Hendry (1986). On this test, we specify the path of the Selic rate resulting from the calibrated weights (case 4) vis-à-vis each of the five sets of weights shown in Table 4. The purpose is to decide whether the calibrated weights statistically predominate over the rival ones, i.e., whether they explain the behavior of the observed Selic rate in a more appropriate way. For that, we estimate the regression \( i_t = \phi_1 i^*_t + \phi_2 i^{**}_t + e_t \), where \( i^*_t \) is the Selic rate predicted by the calibrated weights and \( i^{**}_t \) is the Selic rate predicted by the opposing case. We discriminate between \( i^*_t \) and \( i^{**}_t \) using Wald statistics to test the null hypotheses \( H^* : \phi_1 = 1, \phi_2 = 0 \) and \( H^{**} : \phi_1 = 0, \phi_2 = 1 \). If the hypothesis
The hypothesis $H^*$: $\phi_1 = 1, \phi_2 = 0$ is not rejected and the hypothesis $H^{**}$: $\phi_1 = 0, \phi_2 = 1$ is rejected, we say that the interest rate predicted by the calibrated weights, $i^*_t$, predominates over the interest rate predicted by the opposing weights, $i^{**}_t$ (and vice versa).

The results of the encompassing tests, shown in Table 6, confirm the visual impressions obtained from Figure 2. We note that the weights of a policymaker with strict inflation targets (case 1) can be readily disregarded since they are dominated by calibrated weights. Analogously, we observe that the Central Bank does not appear to have adopted a weight equal to 0.5 for price and output gap stabilization.

<table>
<thead>
<tr>
<th></th>
<th>$H^*$</th>
<th>$H^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 4 versus Case 1</td>
<td>0.4776</td>
<td>4E+06</td>
</tr>
<tr>
<td></td>
<td>(0.7876)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 2</td>
<td>1.1167</td>
<td>4.5124</td>
</tr>
<tr>
<td></td>
<td>(0.5721)</td>
<td>(0.1047)</td>
</tr>
<tr>
<td>Case 4 versus Case 3</td>
<td>0.5953</td>
<td>4827.2</td>
</tr>
<tr>
<td></td>
<td>(0.7426)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 5</td>
<td>0.4802</td>
<td>10.281</td>
</tr>
<tr>
<td></td>
<td>(0.7846)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 6</td>
<td>0.4886</td>
<td>22.076</td>
</tr>
<tr>
<td></td>
<td>(0.7832)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Note: The $p$ values are in brackets.

When we compare the calibrated weights with the cases in which interest rate smoothing is the Central Bank’s main goal (cases 5 and 6), the hypothesis that the calibrated weights encompass these two cases is not rejected either. Finally, the predominance of calibrated weights over the weights considered by Rudebusch and Svensson (1999) is only observed at an 11% significance level. As the optimal monetary rule for the calibrated weights produces a smaller squared deviation and mean absolute error than does case 2, it is reasonable to assume that the Central Bank has conducted the monetary policy by prioritizing inflation stabilization, but that it has brushed output stabilization and Selic rate smoothing aside.

5. Estimating Central Bank Preferences

An empirical procedure other than calibration consists in estimating the monetary authority’s preferences and the structural economic parameters by using maximum likelihood (Dennis, 2006). As pointed out in Section 2, given the values of the Phillips curve coefficients, of the IS curve coefficients and of the loss function

Table 6
Wald tests for different Central Bank’s loss function weights

<table>
<thead>
<tr>
<th></th>
<th>$H^*$</th>
<th>$H^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 4 versus Case 1</td>
<td>0.4776</td>
<td>4E+06</td>
</tr>
<tr>
<td></td>
<td>(0.7876)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 2</td>
<td>1.1167</td>
<td>4.5124</td>
</tr>
<tr>
<td></td>
<td>(0.5721)</td>
<td>(0.1047)</td>
</tr>
<tr>
<td>Case 4 versus Case 3</td>
<td>0.5953</td>
<td>4827.2</td>
</tr>
<tr>
<td></td>
<td>(0.7426)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 5</td>
<td>0.4802</td>
<td>10.281</td>
</tr>
<tr>
<td></td>
<td>(0.7846)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Case 4 versus Case 6</td>
<td>0.4886</td>
<td>22.076</td>
</tr>
<tr>
<td></td>
<td>(0.7832)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>
parameters, the dynamics of the system is determined by:

\[ X_{t+1} = AX_t + Bi_t + \varepsilon_{t+1} \quad (28) \]

\[ i_t = fX_t \quad (29) \]

After the optimal monetary policy rule has been determined, the solution to the monetary authority’s optimization problem can be expressed as follows:

\[
\begin{align*}
\pi_t & = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \alpha_5 \Delta q_{t-1} + \alpha_6 y_{t-2} + \alpha_7 D_{\pi,t} + \varepsilon_{\pi,t} \\
y_t & = \beta_1 y_{t-1} + \beta_2 y_{t-2} - \beta_3 (i_{t-1} - \pi_{t-1}) + \beta_4 D_{y1,t} + \beta_5 D_{y2,t} + \varepsilon_{y,t} \\
q_t & = q_{t-1} + \rho_1 D_{q,t} + \varepsilon_{q,t} \\
i_t & = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-3} + f_4 \pi_{t-4} + f_5 y_t + f_6 y_{t-1} + f_7 \Delta q_t + f_8 i_{t-1} + \varepsilon_{i,t}
\end{align*}
\]

where \(\alpha_4 = 1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_5\). The variable \(\varepsilon_{i,t}\) is an error term introduced in the policy rule to represent possible measurement errors. This procedure, initially suggested by Hansen and Sargent (1980), is justified by the fact that an econometrician has less available information than the monetary authority when estimating the monetary policy rule and, therefore, he could remove some variables from the model. In addition to the term \(\varepsilon_{i,t}\), we inserted dummy variables \(D_{\pi,t}, D_{y1,t}\) and \(D_{y2,t}\), described in subsection 4.1, as well as dummy variable \(D_{q,t}(=1\) for 2002:3 and 0, otherwise) in the solution to the optimization problem, for the strong exchange rate depreciation observed in the last semester of 2002.

By defining \(z_t = [\pi_{t} y_{t} i_{t}]\)'s, \(\varepsilon_t = [\varepsilon_{\pi,t} \varepsilon_{y,t} \varepsilon_{q,t} \varepsilon_{i,t}]\)' and \(D_t = [D_{y1,t} D_{q,t} D_{\pi,t} D_{y2,t}]\)', we can express the system (30)-(33) by the following structural autoregressive vector of order 4, VAR(4):

\[ A_0 z_t = A_1 z_{t-1} + A_2 z_{t-2} + A_3 z_{t-3} + A_4 z_{t-4} + A_5 D_t + \varepsilon_t \quad (34) \]

where

\[
A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -f_1 & -f_5 & -f_7 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} \alpha_1 & 0 & \alpha_5 & 0 \\ -\beta_3 & \beta_1 & 0 & \beta_5 \\ 0 & 0 & 1 & 0 \\ f_2 & f_6 & f_7 & f_8 \end{bmatrix},
\]

\[
A_2 = \begin{bmatrix} \alpha_2 & \alpha_6 & -\alpha_5 & 0 \\ 0 & \beta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ f_3 & 0 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} \alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ f_4 & 0 & 0 & 0 \end{bmatrix},
\]

\[
A_4 = \begin{bmatrix} \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad A_5 = \begin{bmatrix} 0 & 0 & \alpha_7 & 0 \\ \beta_4 & 0 & 0 & \beta_5 \\ 0 & \rho_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]
The vector of the parameters of interest is given by $\bar{\omega} = [\alpha_1, ..., \alpha_7, \beta_1, ..., \beta_5, \rho, \lambda', \lambda_y', \lambda_i']$, where $\lambda_y' = \lambda_y/\lambda_\pi$ and $\lambda_i' = \lambda_i/\lambda_\pi$ represent the relative weights attached by the monetary authority to output gap stabilization and to interest rate smoothing. We express the weights of the policymaker’s loss function relative to the weight given to inflation because this reduces the number of variables and restrictions in the parameter estimation process. We normalize $\lambda_\pi$ so that it is equal to 1. By construction, matrices $A_0, A_1, A_2, A_3, A_4$ and $A_5$ are functions of the vector of parameters $\bar{\omega}$.

By assuming that $\varepsilon_t \mid \{z_j\}_{j=1}^T \sim N(0, \sum_{\varepsilon\varepsilon}) \forall t$ and that the initial conditions are fixed, we can describe the joint probability density function (PDF) for the data as follows:

$$P(\{z_t\}_1^{T_a}; \bar{\omega}, \sum_{\varepsilon\varepsilon}) = \left[ \frac{1}{(2\pi)^{n/2}} |A_0|^{T_a/2} \right]^{\sum_{\varepsilon\varepsilon}^{-1}} \exp \left( \sum_{t=5}^{T_a} \left( -\frac{1}{2} \varepsilon_t^T \sum_{\varepsilon\varepsilon}^{-1} \varepsilon_t \right) \right)$$

$$P(\{z_t\}_1^1; \bar{\omega}, \sum_{\varepsilon\varepsilon})$$

where $T_a$ is the sample size including the initial conditions. The term $P(\{z_t\}_1^1; \bar{\omega}, \sum_{\varepsilon\varepsilon})$ is a constant because we assume that initial conditions $\{z_t\}_1^1$ are fixed. That being said, the natural logarithm of the concentrated likelihood function relative to the variance-covariance matrix of disturbances, $\sum_{\varepsilon\varepsilon}$, is given by:

$$\ln L_c(\bar{\omega}; \{z_t\}_1^{T_a}) \propto -\frac{n(T_a - 4)}{2} \ln(1 + 2\pi) + (T_a - 4) \ln |A_0| - \frac{(T_a - 4)}{2} \ln |\sum_{\varepsilon\varepsilon}|$$

where

$$\sum_{\varepsilon\varepsilon}(\bar{\omega}) = \sum_{t=5}^{T_a} \hat{\varepsilon}_t^2 / T_a - 4$$

is the maximum likelihood estimator of $\sum_{\varepsilon\varepsilon}$.

We estimate the vector of parameters by the maximization of (36) and we use (37) to find an estimate for the variance-covariance matrix $\sum_{\varepsilon\varepsilon}$. The numerical optimization was performed using the BFGS (Broyden, Fletcher, Goldfarb and Shanno) algorithm described in Gill et al. (1981).
The variance-covariance matrix of \( \widehat{\omega} \), necessary for us to infer on the estimates of structural coefficients, can be built by the inversion of Fischer information matrix:

\[
\text{var}(\widehat{\omega}) = [I(\bar{\omega})]^{-1}
\]

where

\[
I(\bar{\omega}) = -\frac{\partial^2}{\partial\omega\partial\omega'} \ln L_c(\bar{\omega}; \{z_t\}_T).
\]

### 5.1 Estimation results

The estimation results for system (34) are shown in Table 7. Except for parameter \( \alpha_2 \), the estimates had the expected signs. Once again, we observed that exchange rate depreciations remarkably affect the inflation rate, while the effect of economic activity on prices may not be considered to be different from zero. In the IS curve, we noted that the effects of changes in the real interest rate on the output gap are statistically significant. For the three estimated equations, the LB and JB tests indicate that we cannot reject the null hypotheses that the errors are serially uncorrelated and normally distributed.

The values obtained for the relative weights on output stabilization and interest rate smoothing are equal to 0.0996 and 0.7036, which implies that the absolute weights are \( \lambda_\pi = 0.555, \lambda_y = 0.055 \) and \( \lambda_i = 0.39 \). However, the standard errors reveal great inaccuracy regarding the estimation of relative weights, not allowing for the rejection of the null hypotheses that these weights are equal to zero. A possible explanation for this is that the sample we used contains a small number of information so that we can extract and accurately estimate the loss function weights along with the parameters of the macroeconomic model. In addition, the statistical nonsignificance of the relative weights \( \lambda'_y \) and \( \lambda'_i \) should be viewed with extreme caution because, as we could see in Figure 1 in subsection 4.2, a policymaker with strict inflation targets would cause interest rate variability that is not compatible with the path actually observed for the Selic rate.

\[^{17}\text{To check whether the small sample size and the large number of parameters may be affecting the accuracy of estimates of relative weights, we estimate these weights by imposing a restriction that the parameters of the first three equations in the system (34) are the same as the OLS estimates shown in Table 2. The values estimated for weights \( \lambda'_y \) and \( \lambda'_i \) were 0.5804 and 0.2480, with standard errors equal to 0.6899 and 0.0640. We perceived that, although the relative weight on output is not statistically significant, the relative weight on smoothing is accurately estimated.}\]
Table 7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>S.E</th>
<th>Parameter</th>
<th>Estimate</th>
<th>S.E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phillips curve</td>
<td></td>
<td></td>
<td>IS curve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.3229</td>
<td>0.1331</td>
<td>$\beta_1$</td>
<td>0.3601</td>
<td>0.1598</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.1550</td>
<td>0.1121</td>
<td>$\beta_2$</td>
<td>0.0101</td>
<td>0.1532</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.4137</td>
<td>0.1094</td>
<td>$\beta_3$</td>
<td>-0.0946</td>
<td>0.0241</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.0106</td>
<td>-</td>
<td>$\beta_4$</td>
<td>-0.9102</td>
<td>0.5179</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.4078</td>
<td>0.0924</td>
<td>$\beta_5$</td>
<td>-2.5363</td>
<td>0.7046</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>0.7823</td>
<td>0.5262</td>
<td>$\beta_6$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_7$</td>
<td>12.340</td>
<td>3.0297</td>
<td>$\beta_7$</td>
<td>0.7890</td>
<td>-</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7890</td>
<td>-</td>
<td>$LB(6) - \text{prob}$</td>
<td>0.1780</td>
<td>-</td>
</tr>
<tr>
<td>$LB(6) - \text{prob}$</td>
<td>0.1780</td>
<td>-</td>
<td>$LB(8) - \text{prob}$</td>
<td>0.4790</td>
<td>-</td>
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<tr>
<td>$LB(8) - \text{prob}$</td>
<td>0.1870</td>
<td>-</td>
<td>JB – prob</td>
<td>0.5141</td>
<td>-</td>
</tr>
<tr>
<td>JB – prob</td>
<td>0.1870</td>
<td>-</td>
<td>Exchange rate equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda'_y$</td>
<td>0.0996</td>
<td>2.3998</td>
<td>$\lambda'_i$</td>
<td>0.0683</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda'_i$</td>
<td>0.7036</td>
<td>0.7030</td>
<td>$LB(6) - \text{prob}$</td>
<td>0.1120</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda'(y_i)$</td>
<td></td>
<td></td>
<td>$LB(8) - \text{prob}$</td>
<td>0.2280</td>
<td>-</td>
</tr>
<tr>
<td>Log-lik</td>
<td>-235.92</td>
<td>-</td>
<td>JB – prob</td>
<td>0.5455</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Standard deviations calculated from the inverse of the Hessian matrix.

The optimal monetary rule policy resulting from the estimates shown in Table 7 is given by:

\[
i_t = 0.1464\pi_t + 0.0547\pi_{t-1} + 0.0713\pi_{t-3} + 0.2534y_t + 0.1339y_{t-1} + 0.0683\Delta q_t + 0.7946\Delta i_{t-1} (39)\]

According to the estimated coefficients, the monetary authority should respond more strongly to contemporaneous changes in output gap in relation to inflation and nominal exchange rate depreciation. Again, we can see large interest rate smoothing, even though the persistence coefficient obtained herein is lower than the one obtained by the calibration strategy.

Figure 3 shows the optimal path of the Selic rate simulated by policy rule (39), $Selic_f$, the optimal path obtained from calibrated weights in subsection 4.2, $Selic_f1$, and the actual path of the Selic rate, $Selic$. Visually, the interest rates predicted by the estimated weights are relatively close to the ones predicted by the calibrated weights. The major differences lie in three periods: in the first quarter of 2001 and in the first three quarters of 2007, the policymaker with estimated weights would have adopted higher interest rates than the policymaker with calibrated weights, whereas the opposite situation would be observed in 2003. The result of these differences can be measured by the mean absolute error (MAE). For the estimated weights, the MAE between the optimal interest rate and the
observed one was 1.08 percentage points, whereas for the weights obtained via calibration, the MAE was 1.01 percentage points. This suggests that the weights obtained using our calibration strategy described the decisions made by the Central Bank more appropriately within an optimal monetary policy framework.

Finally, we test whether the Central Bank’s monetary policy decisions were taken optimally during the inflation targeting regime. To do that, we estimate the system of equations (34) leaving the monetary policy rule coefficients free. The FIML estimates for unrestricted VAR(4) indicates that the monetary policy rule is given by (standard errors in brackets):

\[
i_t = 0.0948\pi_t + 0.1544\pi_{t-1} + 0.0610\pi_{t-3} - 0.0263\pi_{t-4} - 0.0710y_t \\
\quad + 0.4576y_{t-1} + 0.0424\delta q_t + 0.6722i_{t-1} \\
\quad (0.0629) \quad (0.0558) \quad (0.0570) \quad (0.0549) \quad (0.7589) \quad (0.3310) \quad (0.0487) \quad (0.1333) \\
\]

(40)

The log-likelihood for the VAR(4) model in which the policy rule coefficients are unrestricted was equal to \(-228.21\). As the structure of equations for inflation, output gap and exchange rate remains unchanged, we can note that there are eight free parameters in the unrestricted system (the unrestricted policy rule coefficients), whereas in the restricted system there are only two (\(\lambda'_y\) and \(\lambda'_i\)). This shows that the system in which the monetary policy is adjusted optimally implies six restrictions on the unrestricted policy rule (40). The value of the likelihood ratio (LR=15.42) implies that we can reject the null hypothesis that the monetary policy was adjusted optimally to a 5% instead of to a 1% significance level.
6. Conclusions

In the past two decades, a large amount of empirical studies have assessed Central Bank actions through monetary policy estimates. However, this procedure can be problematic because the estimated reaction functions are reduced form equations whose coefficients are convolutions of the monetary authority’s preferences and of the economic behavioral patterns.

Therefore, the present paper aimed to shed further light upon the Brazilian monetary policy in the inflation targeting regime by way of calibration of Central Bank preferences. To do that, we assumed that the monetary authority solves an intertemporal optimization problem restricted to a small macroeconomic model using backward-looking expectations. Thereafter, we calibrated the policymaker’s loss function by choosing the preference parameter values that minimize the deviation of the optimal path from the observed path of the Selic rate from a wide range of alternative policies. Our results show that the Central Bank of Brazil has conducted a monetary policy that prioritizes inflation stabilization, but which has given no importance to output gap stabilization and to Selic rate smoothing. In addition, we conclude that the monetary authority’s concern with interest rate smoothing has been far deeper than with output stability.

As an alternative to the calibration strategy, we estimated the Central Bank’s objective function parameters along with the parameters of the macroeconomic model using maximum likelihood. Results indicate that the estimates for the relative weights on output gap stabilization and interest rate smoothing were qualitatively similar to those obtained by the calibration method, but statistically nonsignificant. The inaccuracy of these estimates is possibly due to the small sample size used. Moreover, the statistical nonsignificance of relative weights should not be seen as evidence in favor of a Central Bank with strict inflation targets because, under this assumption, the optimal Selic rate may vary considerably to the point that it becomes incompatible with the observed Selic rate.
References


