Selection of Minimum Variance Portfolio Using Intraday Data: An Empirical Comparison Among Different Realized Measures for BM&FBovespa Data*

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Abstract

This paper explores different covariance matrix estimators; both the conditional and the unconditional versions, obtained via intraday data and named realized measures, to the minimum variance portfolio selection problem. Intraday data are sampled in a synchronized manner as well as in an unsynchronized version. For sake of comparison, we also use daily data estimators. The major contribution of this work is of empirical nature, focused on the Brazilian scenario. We evaluate some out-of-sample performance indexes of the obtained portfolios for a set of 30 stocks traded on the São Paulo stock exchange (BM&FBovespa). The results show that the estimator of the conditional covariance matrix of returns using a scalar vt-VECH model based on higher frequency data leads to substantial earnings, reducing portfolio risk, increasing the average adjusted-by-risk return and decreasing the turnover.

Keywords: High frequency data; Realized measures; Portfolio optimization; Realized volatility; Performance assessment.

JEL Codes: C53; E43; G17.

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1. Introduction

Volatility forecasting is a relevant topic which is consolidated in the literature and in the empirical finance in areas such as portfolio selection and risk management. The search for models that better represent the asset price dynamics has brought new developments advancements in the recent literature on financial econometrics.¹ The greater availability of high frequency data has made brough about new developments in the measurement of volatility (see, for example, the surveys of Barndorff-Nielsen and Shephard (2005), Andersen et al. (2006), McAleer and Medeiros (2008), Meddahi et al. (2011), and others). The literature suggests that covariance matrix estimates based on high-frequency data produces more precise results than the estimation based on daily returns.

In recent years there have been several improvements in the estimation of the covariance matrix using intraday data. In general terms these estimators aims to estimate the quadratic variation of assets returns. Following Liu et al. (2013), we refer to the group of these quadratic variation estimators of a price process over some interval of time, as “realized measures”.

Despite the increased interested in the modeling and forecasting of multivariate volatility using intraday data, most papers focus on the development of volatility estimators and statistical measures to evaluate the performance of alternative models (Andersen et al., 2001b; Barndorff-Nielsen et al., 2011; Liu et al., 2013, and others). A smaller number of papers focus in the economic applications (Liu, 2009; Hautsch et al., 2013; Wink Junior and Pereira, 2013). As pointed out by Granger and Pesaran (2000), Pesaran and Skouras (2002) and Granger and Machina (2006), when forecasts are used in decision making, it is important to consider the decision process in the *ex post* evaluation of these forecasts, allowing the interaction between the forecasting model and the objective of the decision making.

In this context, this paper evaluates the economic benefits of high frequency data in the covariance matrix estimates for portfolio selection with 30 assets traded in the Brazilian stock market. Therefore, the major contribution of this paper is of empirical nature, focused on the Brazilian market. We consider both the conditional and unconditional approach in the estimation of the covariance matrix of returns. In turn, the realized measures are estimated using intraday data in different sampling frequencies ranging from 5 to 120 minutes, which is compared with estimators based on daily data. The portfolios constructed from different covariance matrix estimators are evaluated in terms of important indicators of the portfolio allocation, such as portfolio risk, turnover and Sharpe ratio.

The research on the volatility forecasting using intraday data was initiated by Andersen et al. (2001b) and Barndorff-Nielsen (2002). These papers designate as “realized volatility” the measure of volatility constructed by summing the squares

¹A survey of estimation and forecasting of univariate and multivariate volatility models, including considerations on empirical applications, can be found in Andersen et al. (2006).

of intraday returns. The realized variance represents a non-parametric estimator of quadratic variation (or integrated variance) of returns under the basic assumption of an arbitrage-free market (Andersen et al., 2006). Assuming that the sample path of volatility is continuous, the sampling frequency of returns approaches infinity would be able to produce arbitrarily more precise estimates of volatility. Therefore, the realized volatility converges to the quadratic variation and it is free from the measurement error (Andersen et al., 2001b; Barndorff-Nielsen, 2002; Fleming et al., 2003). With the realized volatility it is possible to define the measure of volatility without having to explicitly to define a model, making it a benchmark in terms of forecast evaluation (Andersen et al., 2006).

Following this perspective, studies such as Andersen et al. (2001a), Andersen et al. (2001b), Areal and Taylor (2002) and Koopman et al. (2005) focus on the volatility estimation using intraday data for the univariate case. However, given the importance of the volatility between several assets, especially in the empirical applications, such as portfolio selection, value at risk (VaR) and derivatives pricing, the literature extended this approach to the multivariate setting (Andersen et al., 2003; Fleming et al., 2003; Barndorff-Nielsen and Shephard, 2004; De Pooter et al., 2008; Liu, 2009 and others).

In the literature on covariance matrix estimation using high frequency data, two topics stand out: market frictions (market microstructure noise) and non-synchronous data. The debate of these questions attracts attention of researchers of both the univariate case (Barndorff-Nielsen et al., 2008; (Bandi and Russell, 2008) and multivariate case (Aït-Sahalia and Mancini, 2008; Bannouh et al., 2009; Barndorff-Nielsen et al., 2011). An inherent problem of the realized variance method stems from the frictionless market hypothesis, which, when violated produces undesirable effects on the covariance matrix estimation. Therefore, the existence of market microstructure noise can have a negative effect on the results obtained using the different realized measures.² The general objective is to select an intraday frequency that satisfies the trade-off between precision in the construction of the realized measure and the bias generated by market frictions³ (Hansen and Lunde, 2006; Bandi and Russell, 2008; Bandi et al., 2008).

One of the recent advancements in covariance matrix estimation considering the non-synchronous data is the realized kernel estimator, proposed by Barndorff-Nielsen et al. (2008) and extended to the multivariate case by Barndorff-Nielsen

²When the true price process is observable, an increase in frequency of data observation, over a fixed period of time, results in more precise estimates of the integrated variance (quadratic variation) of the prices. However, when the true price process is not observable and there is market frictions, despite the fact that the increase in return frequency increases the amount of information about the underlying integrated variance, there is an accumulation of noise that affects the bias and variance of the estimator (Bandi and Russell, 2008).

³In the literature the 5 and 20 minutes return sampling frequencies are popular suggestions to balance the benefits of intraday data and the negative effects from the market microstructure noise (Andersen et al., 2001a; Andersen et al., 2001b; Fleming et al., 2003).

et al. (2011). One of the characteristics of this estimator is its robustness to certain types of market frictions. The multivariate realized kernel, in addition to being as robust as the univariate case, is a positive semi-definite estimator and viable with non-synchronous data⁴ (Barndorff-Nielsen et al., 2011).

In empirical applications, the paper by Fleming et al. (2003) is pioneer in the analysis of the benefits of intraday data in the context of portfolio allocation. The covariance matrix estimator used is given by the rolling estimator method proposed by Foster and Nelson (1996) and Andreou and Ghysels (2002). The authors evaluate the empirical performance of conditionally mean-variance efficient portfolios that are rebalanced daily and constructed using the realized volatility. Covariance matrix estimations based on daily data are benchmarks in the study, and the results indicate that the estimations based on realized volatility present additional performance gains in the assets allocation decision in comparison to the daily estimator.

Motivated by the discussion regarding the existence of a trade-off between precision and bias in high frequency returns, De Pooter et al. (2008) presents the question of determining the optimal intraday sampling frequency in the context of minimum variance portfolio and tracking portfolio for 78 stocks of the S&P100. Using an intraday return frequency ranging from 1 to 130 minutes, the authors conclude that the conditional covariance matrix forecasts based on intraday returns improves portfolio performance when compared to daily returns. In the same way, an adequate return frequency is more important than the bias correction techniques.

In this research topic, Liu (2009) analyses the benefits of using high-frequency data from different covariance matrix estimators, carrying out an empirical analysis for minimum tracking error portfolios with 30 stocks. Similarly, with an applied approach, Hautsch et al. (2013) focus on the forecasting of the covariance matrix in high-dimensional portfolio allocation. The analysis considered 400 assets sampled in the daily and intraday frequencies. The covariance matrix forecasts based on intraday returns are obtained using the blocked realized kernel estimator developed by Hautsch et al. (2012). The authors employed benchmark approaches based on daily returns, specifically, multivariate GARCH models, sample covariance (rolling-window) and RiskMetrics, and compared those with the high-frequency forecasts. The results indicate that the covariance forecasts based on high-frequency data outperform low-frequency approaches, especially, in terms of lower portfolio volatility.

In this paper, we evaluate the gains from using intraday returns in portfolio allocation. Our analysis compares different covariance matrix estimators based on high-frequency or daily returns. In particular, we employ realized covariance, sample covariance, and the variance targeting scalar VEC model (scalar vt-VECH), and the multivariate realized kernel (MRK) estimators. Despite the widespread

⁴Barndorff-Nielsen et al. (2011) use the Refresh Time to synchronize the data.

use of the GARCH models based on daily data, the estimation using intraday data (or realized measure) is less common. In an attempt to evaluate the possible benefits of integrating the estimation of a GARCH model with high-frequency data, the scalar vt-VECH model was estimated based on a covariance estimation carried out for each sampling frequency. In turn, the MRK estimator is an important advancement in the estimation of the volatility with non-synchronous data.

The out-of-sample covariance forecasts are then used to construct global minimum variance portfolios, and those are evaluated using performance indicators such as mean return, variance, Sharpe ratio and portfolio turnover. The global minimum variance portfolio, the leftmost point in the efficiency frontier, has the unique property of the optimum weights of the assets depending solely on the assets covariance matrix, without the need to take the expected returns into consideration about the expected returns (Jagannathan and Ma, 2003, Candelon et al., 2012). Additionally, the results are compared to those obtained using the benchmark (realized covariance based on daily data). In short, different estimators are evaluated, both in a conditional and unconditional approach, in the covariance matrix estimation in the context of investment decisions on asset allocation. By considering an economic motivation given by the portfolio selection, this paper focuses on the empirical analysis. In general terms, the results indicate gains with higher frequency data when compared to the daily data used in the forecasts of the covariance matrix. Our evaluation also suggests a better performance of portfolios estimated with the conditional covariance matrix (scalar vt-VECH).

In the four year period analyzed, which covers the sample of the 30 assets with highest liquidity in the Brazilian market, the empirical evidence suggests that the covariance matrix estimators based on high frequency data outperform the estimators based on lower frequency (closing prices). The realized covariance matrix estimators following those analyzed in Liu (2009) and De Pooter et al. (2008), which include intraday data and also the overnight volatility (Liu, 2009). The intraday sampling frequency range from 5, 15, 30, 90, and 120 minutes. Additionally, a parsimonious multivariate GARCH specification was considered in order to obtain the dynamic conditional versions of the realized measures. We also implemented the multivariate realized kernel proposed in Barndorff-Nielsen et al. (2011) which allows for market microstructure noise and non-synchronous prices.

Our investigation indicates similar results to Liu (2009) and Hautsch et al. (2013), which the estimates of the global minimum variance portfolios constructed from the realized measures show substantial gains in terms of reduced portfolio risk, risk-adjusted average returns (higher Sharpe ratio) and lower turnover. Additionally, the conditional realized measures outperform their unconditional counterparts. Despite the fact that data sampled at 5 minutes are appropriate for generating portfolios with lower turnover, the 90 and 120 minutes frequency generate optimum portfolios with lower risks and higher Sharpe ratio. On the other hand, the evidences suggest that the realized measures that are robust to the mi-

crostructure effects and non-synchronous prices do not add substantial gains in terms to improve the risk-adjusted portfolio performances. Therefore, the results are similar those presented in De Pooter et al. (2008) reporting that the adequate sampling frequency is relevant for the out-of-sample portfolio performance.

In addition to the benefit in the minimum variance portfolios, Hautsch et al. (2013) show that the estimators based on intraday data improves the utility function for risk-averse investors. Some authors examine the realized volatility in alternative applications, such as option pricing. For example, Christoffersen et al. (2014) developed an affine model in discrete time to model the daily returns and realized volatility jointly. The methodology is applied to S&P500 data and the results indicate that the realized volatility reduces the pricing errors in the whole moneyness spectrum, considering different maturities. Forsberg and Bollerslev (2002), Stentoft (1991) and Corsi et al. (2013) also model return and realized volatility jointly in option pricing.

For the Brazilian stock market, our study is complementary to the papers of Santos and Tessari (2012) and Rubesam and Beltrame (2013), which estimate the covariance matrix (different from the present paper) in the portfolio selection. These studies provide evidences that minimum variance portfolios constructed from more sophisticated covariance matrix estimators outperform several benchmarks widely used in the literature and in the financial market. Despite the differences in their database and the use of daily data, the results are similar to those presented in our paper and suggest performance gains with conditional covariance matrix estimates in the portfolio allocation.

The remainder of this paper is organized as follows. Section 2 describes the covariance matrix estimators based on daily and intraday returns and the portfolio optimization problem. Section 3 presents the data used and the empirical analysis. Finally, section 4 concludes.

2. Methodology: Covariance Matrix Estimators

This section describes the covariance matrix estimators based on daily and high frequency data that are used in the empirical application.

The objective of the estimators presented in this section is to extract information from the covariance between assets and use that information in the portfolio selection and to verify the existence of benefits with the high frequency data.

2.1 Covariance matrix estimators based on daily returns

2.1.1 Sample covariance

The sample covariance matrix is given by

$$H_t^1 = \frac{1}{T-1} \sum_{k=1}^T (R_{t-k} - \bar{R})' (R_{t-k} - \bar{R}), \quad (1)$$

where R_{t-k} is the return vector $1 \times i$ on day $t - k$. $\bar{R}$ is the in-sample average of returns and T is the size of the estimation window.

2.1.2 Lead and lag adjusted sample covariance matrix estimator

This specification adds the covariance matrix of the observed returns at different leads and lags to the covariance matrix of returns. The weights for the leads and lags are given by the Bartlett kernel, as suggested by Newey and West (1987), to guarantee that the covariance matrix to be positive definite. This estimator is described as

$$H_t^2 = H_t^1 + \frac{1}{T-1} \sum_{k=1}^T \sum_{m=1}^L w(m, L) \left[(R_{t-k} - \bar{R})' (R_{t-k-m} - \bar{R}) + (R_{t-k-m} - \bar{R})' (R_{t-k} - \bar{R}) \right], \quad (2)$$

where

$$w(m, L) = 1 - \frac{m}{L+1} \quad (3)$$

is the Bartlett kernel and L is the number of lags in the returns.

2.1.3 Shrinkage estimator

Proposed by Ledoit and Wolf (2003), the shrinkage estimator assumes a market model as generator of return j on day t :

$$r_{j,t} = \alpha_j + \beta_j r_{M,t} + \varepsilon_{j,t}, \quad (4)$$

where $r_{M,t}$ is the return of the market index of day t and $\varepsilon_{j,t}$ is a residual term. Considering $r_{M,t}$ and $\varepsilon_{j,t}$ uncorrelated, as well as residual returns, the covariance matrix estimator of the returns on set of assets is given by F_t :

$$F_t = s_{m,t}^2 B B' + D_t, \quad (5)$$

where B is the vector of β 's, $s_{m,t}^2$ is the sample variance of $r_{M,t}$ and D_t is the diagonal matrix of the sample variances of residuals.

The shrinkage estimator is a weighted average of two estimators, the sample covariance matrix and the market-model-based estimator, described as

$$H_t^3 = \frac{\gamma_t}{T} F_t + \left(1 - \frac{\gamma_t}{T}\right) H_t^1, \quad (6)$$

where the shrinkage estimator is given by a quadratic loss function that measures the distance between the true and the estimated covariance matrix:

$$\gamma_t = \frac{\pi_t - \rho_t}{\lambda_t}, \quad (7)$$

where γ_t is the parameter that determines the intensity of the shrinkage estimated for time t with sample size of T . Additionally, π_t denotes the sum of asymptotic variances of the entries of H_t^1 scaled by $\sqrt{T}$, ρ_t denotes the sum of the asymptotic covariances of the entries of F_t with the entries H_t^1 scaled by $\sqrt{T}$ and λ_t is a quadratic measure of the distance between the true covariance matrix and the estimated covariance matrix based on the Frobenius norm. Additionally, γ_t varies with the estimation window.

2.2 Conditional variance matrix estimator – Variance targeting Scalar VEC

The estimation using variance targeting aims to circumvent the problems found in the quasi-maximum likelihood (QML) estimation of GARCH models by using a two-step procedure (Francq et al., 2011). In the first step, the volatility equation is reparametrized so that the intercept is replaced by the unconditional variance of returns. The unconditional variance is estimated and conditionally to this measure, in the second step, the remaining parameters are estimated by QML.

The conditions for positive definiteness of conditional covariance matrix is simplified. Following Engle & Sheppard (2008b), the Variance targeting Scalar VEC (scalar-vt-VECH) specification is defined by

$$H_t^4 = C + \alpha R_t R_t' + \beta H_{t-1}^4, \quad (8)$$

where α and β are scalars. Taking C positive definite α and β not negatives, the conditional covariance will be positive definite. Assuming the stationary covariance as the objective to circumvent the curse of dimensionality and $\bar{H} = E[R_t R_t']$ the unconditional covariance of returns, we have

$$E[H_t^4] = C + \alpha \odot E[R_{t-1} R_{t-1}'] + \beta \odot E[H_{t-1}^4], \quad (9)$$

$$\bar{H} = C C + \alpha \odot \bar{H} + \beta \odot \bar{H}, \quad (10)$$

$$C = (\iota' - \alpha - \beta) \odot \bar{H}, \quad (11)$$

where ι is a vector of ones $N \times 1$. Therefore, C is not connected to and can be replaced by the estimate in 11.

2.3 Covariance matrix estimators based on intraday returns (realized measures)

The literature suggests that methods based on intraday returns increases the accuracy of the estimates and forecasts of the covariance matrix. We assume that the daily return is the sum of all the intraday returns in a day and the overnight return:

$$r_{i,t} = r_{i,t,0} + r_{i,t,1} + r_{i,t,2} + \dots + r_{i,t,N},$$

where N is the sampling frequency of the returns in the day t , $r_{i,t,n}$ is the return of the asset i in the interval $(1 \leq n \leq N)$ on day t and $r_{i,t,0}$ is the overnight return of

the asset i on the day t . We denote $R_{t,n}$ the vector of returns $1 \times i$ in the interval n . In this paper we refer to the covariance matrix estimators based on intraday returns as “realized measures”.

2.3.1 Sample covariance matrix estimator (excluding overnight returns)

Considering only the realized intraday volatility, and excluding the overnight returns, we can define an estimator based on the sum of the cross-products of the intraday returns. It is known that the covariance estimator based on daily returns is unbiased, but it is an a noisy estimator. The realized volatility, on the other hand, is asymptotically free of measurement error under the diffusion assumption.

The sample covariance matrix estimator is given by

$$H_t^5 = \frac{1}{T} \sum_{k=1}^T \sum_{n=1}^N R'_{t-k,n} R_{t-k,n}. \quad (12)$$

2.3.2 Sample covariance matrix estimator (including overnight returns)

The exclusion of overnight returns might lead to information loss and bias in the covariance matrix estimation. In the absence of overnight trading, only the closing and opening returns are observed. In this case, according to Martens (2002), the squared overnight return is an estimate with noise of the overnight volatility. Despite that, including the product of the overnight returns can increase the precision in the covariance matrix estimation.

When the product of the overnight return vector is included, from the non-trading periods in the market, as a correction term in the estimation of the realized volatility, the estimator is as follows:

$$H_t^6 = H_t^5 + \frac{1}{T} \sum_{k=1}^T (R_{t-k,0} - \bar{R}_0)' (R_{t-k,0} - \bar{R}_0), \quad (13)$$

where $\bar{R}_0$ is the average of the overnight returns in the sample.

2.3.3 Sample covariance matrix estimator (adjusted for correlations)

As the sampling frequency increases, the correlation between the returns becomes stronger, influenced by non-trading and bid-ask spreads.

One way of correcting this autocorrelation is using the sum of squared intraday returns and the product of the returns with different leads and lags in the

covariance matrix estimation. Following this procedure, the estimator is given by

$$H_t^7 = H_t^6 + \frac{1}{T} \sum_{k=1}^T \sum_{l=1}^L \sum_{n=l+1}^N (R'_{t-k,n} R_{t-k,n-l} + R'_{t-k,n-l} R_{t-k,n}), \quad (14)$$

where L is the maximum number of lead and lag covariances included.

2.3.4 Conditional covariance matrix estimator – Variance targeting Scalar VEC

The Variance targeting Scalar VEC estimator based on intraday data is described by

$$H_t^8 = C + \alpha R_t R_t' + \beta H_{t-1}^8.$$

This model follows the one presented in equations 9 through 11. However, the estimation is carried out based on a covariance estimator, in this case, the realized covariance estimation.

2.3.5 Conditional covariance matrix estimator – Multivariate realized Kernel

One of the weaknesses of the realized covariance estimators described above, based on the sum of the product of the intraday returns, is that they are not consistent in the presence of microstructure noise (see Bandi and Russell, 2005 and Bandi and Russell, 2008). Barndorff-Nielsen et al. (2011) proposes a realized multivariate kernel estimator (RMK), which is robust to certain types of market frictions and possible with non-synchronous data. Additionally, the RMK estimator is guaranteed to be positive semi-definite and consistent. The RMK estimation is a two-step procedure. Similarly to Hautsch et al (2012), the observations are synchronized using refresh time sampling (RTS).

The idea of refresh time consists in waiting until all the assets are traded at least once, in time τ_1 (for example) and, then using the last traded price of each asset (the most recent) before or at τ_1 as its price at τ_1 . The process is then restarted. We wait until all the assets are traded at least once, in time τ_2 (for example) and, again, the last (most recent) price is used for each asset as the price at τ_2 . This produces a second price vector synchronized at τ_2 . This process is repeated until all the available data on trading is synchronized. The process clearly discards a considerable part of the available trades. More formally, the first refresh time in the sample can be defined as $\tau_1 = \max(t_1^{(i)}, \dots, t_1^{(id)})$ and the subsequent refresh time as:

$$\tau_{j+1} = \max(t_{N_{\tau_j}^{(1)}+1}^{(i)}, \dots, t_{N_{\tau_j}^{(d)}+1}^{(d)})$$

Therefore, τ_1 is the time until all the assets are traded, that is, until all the prices are refreshed.

Having timed the ordinary time $\{\tau_j\}$, we can build the vectors of the returns series, on which the multivariate realized kernels are based. Be $n, m \in \mathbb{N}$, with $n - 1 + 2m = N$. Define the observation vector $X_0, X_1, \dots, X_n$ as $X_j = X(\tau_{j+m})$, $j = 1, 2, \dots, n - 1$, and

$$X_0 = \frac{1}{m} \sum_{j=1}^m X(\tau_j) \quad \text{and} \quad X_n = \frac{1}{m} \sum_{j=1}^m X(\tau_{N-m+j}),$$

where X_0 and X_n are the prices at the end points of the time period, constructed as the average of m distinct observations in the intervals $(-\delta, \delta)$ and $(n - \delta, n + \delta)$, respectively. The synchronization using refresh time allows us to define the high frequency returns vector as $x_j = X_{\tau_j} - X_{\tau_{j-1}}$, where $j = 1, 2, \dots, N$, and N is the number of observations resulting from refresh time. Finally RMK is defined as

$$RMK(X) = \sum_{h=-H}^H k\left(\frac{h}{H+1}\right) \Gamma_h, \quad (15)$$

where $k(x)$ is the ponderation function of a Parzen kernel, H is the bandwidth parameter and Γ_h is the h -th realized covariance matrix given by

$$\Gamma_h = \begin{cases} \sum_{j=|h|+1}^n x_j x'_{j-h}, & h \geq 0 \\ \sum_{j=|h|+1}^n x_{j-h} x'_j, & h < 0. \end{cases} \quad (16)$$

Barndorff-Nielsen et al. (2011) shows that the estimator is consistent, positive semi-definite and that it can deal with the market microstructure effects and data that are irregular and non-synchronous.

3. Applications in Portfolio Optimization

The economic methods of forecasting evaluation have several applications in Finance. In the Markowitz (1952) mean-variance approach the portfolio allocation depends on the mean and (co)variance of assets. The problem in obtaining a minimum variance portfolio (MVP) can be seen as a specific case of the optimum portfolio problem based on expected returns and conditional variance, the mean-variance approach Markowitz (1952). Formally, the mean-variance portfolio can be obtained by solving the the investor's utility maximization problem based on the trade-off between risk and expected return,

$$\min_w w' H_{t|t-1} w - \frac{1}{\lambda} w' \mu_{t|t-1},$$

where w is the vector of portfolio weights, $H_{t|t-1}$ is the conditional covariance matrix, $\mu_{t|t-1}$ is the vector of the conditional mean and λ is the investor's risk aversion

index. The MVP, on the other hand, corresponds to the optimal solution of the mean-variance problem for an investor with a risk aversion approaches infinity ($\lambda \rightarrow \infty$). In this case the investor is only concerned with the risk minimization of the portfolio, without taking into consideration the expected return. As previously discussed, the academic research tends to focus in minimum variance portfolios, which are subject to a smaller estimation error compared to the mean-variance portfolio (Engle and Sheppard, 2008, DeMiguel et al., 2009, Caldeira et al., 2013), because it depends only on the estimation of the covariances.

Mathematically, the problem of the optimization of the minimum variance portfolio for a group of i assets on time t is described by

$$\begin{aligned} \min_{w_t} & w_t' H_{t|t-1} w_t \\ \text{subject to} & \\ w_t' \iota &= 1 \end{aligned} \quad (17)$$

where t is vector of ones with dimension $N \times 1$. The solution for the unrestricted MVP in (17) is given by:

$$w_t = \frac{H_{t|t-1}^{-1} \iota}{\iota' H_{t|t-1}^{-1} \iota}. \quad (18)$$

3.1 Performance evaluation

The portfolio performance is evaluated in terms of returns, portfolio variance ($\hat{\sigma}^2$), Sharpe Index (SP) and turnover. These statistics are calculated as follows:

$$\begin{aligned} \hat{\sigma}^2 &= \frac{1}{\delta - 1} \sum_{t=1}^{\delta-1} \left(w_t' R_{t+1} - \hat{\mu} \right)^2, \\ IS &= \frac{\hat{\mu}}{\hat{\sigma}}, \text{ where } \hat{\mu} = \frac{1}{\delta - 1} \sum_{t=1}^{\delta-1} w_t' R_{t+1}, \\ Turnover &= \frac{1}{\delta - 1} \sum_{t=1}^{\delta-1} \sum_{j=1}^{\delta} (|w_{j,t+1} - w_{j,t}|), \end{aligned}$$

where $w_{j,t}$ is the weight of the asset j in the portfolio at time $t + 1$ before re-balancing, $w_{j,t+1}$ is the desired weight of the asset j at time $t + 1$ and δ is the out-of-sample time. The turnover can be interpreted as the mean fraction of the amount of resources allocated in the portfolio in each period.

We implement the stationary bootstrap of Politis and Romano (1994) to test the statistical significance of the difference between the variances and Sharpe ratios of the returns for two portfolios with $B = 1000$ bootstrap resamples and block size equal to $b = 5$.⁵ To compute the p -values we use the methodology proposed by Ledoit and Wolf(2008, Remark 3.2).

⁵Robustness tests were carried out to define the size of the block, using b between 5

3.2 Data description

The empirical analysis is based on the returns of 30 assets traded at BM&Bovespa between 02/01/2009 and 12/29/2011, a total of 991 daily observations. The first 250 observations are used for the initial estimation. With each trading day one more observation is added and the covariance matrix are re-estimated. The data is from the BM&FBovespa, which opens at 10 am and finishes trading at 5 pm, the trading sessions are 420 minutes long and the data is sampled in the daily and intraday frequency. The intraday data comprise the frequencies of 5, 15, 30, 60, 90 and 120 minutes. Table 1 shows the descriptive statistic of the daily returns of the 30 assets that comprise the database. We selected the most traded assets in the period from the ones in the Ibovespa index on January 2009.

Panel A of Table 2 illustrates some characteristics of the daily realized variances and covariances, obtained with the non-conditional covariance matrix estimators previously discussed. Panel B, Table 2, presents similar descriptive statistics for the daily sample variances and covariances obtained with the unconditional estimators for sampling frequency of 5 minute. Similar to De Pooter et al. (2008), some familiar patterns can be observed. For example, the average realized variance increases with the sampling frequency, except for the 5 and 15 minutes frequencies. De Pooter et al. (2008) argues that the bid-ask bounce induces negative autocorrelations in returns when prices are sampled more frequently, taking into consideration an upward bias in the realized variance. The average variance using daily returns is 6.535, which corresponds to an annualized standard deviation of 40.58%, while the average variance for 5 minutes returns is 9.739, corresponding to an annualized standard deviation of 49.54%, similar to De Pooter et al. (2008). Another result that is consistent with the findings in De Pooter et al. (2008), the average realized covariance decreases monotonically with the sampling frequency (except for the 15 minutes). The authors attribute this downward bias to non-synchronous trading.

3.3 Results

In this section we compare the out-of-sample portfolio performance constructed using different covariance matrix estimators, based on daily and high-frequency data. The portfolios are rebalanced daily, weekly and monthly, and are analyzed according to their performance in terms of average performance, standard deviation, Sharpe ratio and turnover. Tables 3 to 5 show the performance of the indicators mentioned above for the daily, weekly and monthly rebalancing, respectively.

and 250 and the results of the tests for the standard deviation and Sharpe index are similar to those reported here. The calculations were carried out using the code available at <http://www.iew.uzh.ch/chairs/wolf.html>.

Table 1
Descriptive statistics of daily returns

Note: This table presents the descriptive statistics of daily returns for the 30 assets traded at BM&FBovespa, obtained from the Economatica system. The sample covers the period from January 2nd, 2009 to December 29th, 2011, with a total of 991 daily observations. We chose the 30 assets with the greatest participation in the Ibovespa index in January, 2009.

Asset	Mean(%)	Standard deviation	Skewness	Kurtosis	1 st Autocorrelation
VALE5	0.0054	0.0255	-0.0854	3.5403	0.0447
GGBR4	-0.0610	0.0347	-2.9083	44.754	0.0847
BBDC4	-0.0200	0.0273	-1.3353	21.975	0.0207
ITSA4	0.0329	0.0269	0.6789	8.5520	0.0745
BBAS3	0.0116	0.0274	-0.0133	4.2383	0.0285
USIM5	-0.1358	0.0361	-2.6155	38.871	0.1171
PDGR3	-0.0333	0.0434	-2.9778	35.817	0.0431
PETR3	-0.0981	0.0313	-3.9446	64.196	0.0579
CYRE3	0.0304	0.0390	-0.0451	5.3453	0.1313
CSNA3	-0.1206	0.0413	-6.2920	95.865	-0.0479
GFSA3	-0.1028	0.0434	-1.6714	22.904	0.0803
MRVE3	-0.0052	0.0454	-2.8773	46.599	0.1101
RDCD3	0.0430	0.0286	0.2431	4.5618	0.0033
LAME4	0.0391	0.0304	0.3856	4.5650	0.0061
RSID3	-0.0664	0.0436	-1.6079	18.633	0.0632
CMIG4	0.0275	0.0225	-1.7542	19.595	-0.0487
MMXM3	-0.1516	0.0594	-4.4085	84.107	0.0868
JBSS3	0.0677	0.0364	0.5133	6.8655	0.0234
LREN3	0.0828	0.0322	0.4504	4.2170	-0.0419
HYPE3	-0.1112	0.0454	-11.846	24.754	0.0719
GOLL4	-0.0671	0.0341	-0.0602	3.7519	0.0914
NATU3	0.1031	0.0231	0.4302	1.9808	-0.0567
BRAP4	0.0032	0.0290	0.0774	4.0100	0.0261
ALLL11	-0.0849	0.0445	-11.308	25.886	0.1236
TNLP4	-0.0311	0.0267	0.4620	5.2406	-0.0007
BISA3	-0.0263	0.0321	0.9311	8.4590	0.0269
AMBV4	0.0339	0.0320	-15.546	38.782	-0.0083
EMBR3	-0.0236	0.0253	0.1043	3.9331	0.0849
ELET6	0.0420	0.0232	0.6590	3.8019	-0.0177
CCRO3	0.0086	0.0338	-10.982	25.025	0.0212
Median	-0.0126	0.0322	-0.7104	13.592	0.0358
Average	-0.0203	0.0341	-2.5782	22.661	0.0400
Minimum	-0.1516	0.0225	-15.546	1.9800	-0.0567
Maximum	0.1031	0.0594	0.9311	95.865	0.1313

Table 2
Mean and variance of the realized and sample (co)variances

Table 2 presents the mean and variance of the unconditional estimators of the realized and sample (co)variances at 5 minutes sampling frequency.

Frequency	Realized Variance		Realized Covariance	
	Mean	Variance	Mean	Variance
Daily	6.535	50.827	3.099	29.920
Panel A: Realized				
RMK	4.001	11.290	1.638	4.076
5 minutes	9.739	18.117	2.727	10.740
15 minutes	9.005	27.793	2.990	11.982
30 minutes	6.247	33.820	2.752	14.273
60 minutes	6.281	36.866	2.881	17.380
90 minutes	6.274	40.737	2.883	19.430
120 minutes	6.294	41.257	2.946	20.820
Panel B: Sample				
5 minutes (ex overnight)	6.688	33.181	1.896	2.036
5 minutes (in overnight)	17.738	158.249	4.964	14.432
5 minutes (adjust <i>lead</i> and <i>lag</i>)	17.141	145.445	5.133	15.936

Table 3
Minimum variance optimum portfolio performance(Daily rebalancing)

Note: The table presents the out-of-sample performance of the minimum variance portfolio using 30 assets traded at BM&FBovespa. The optimum portfolios are constructed using the unconditional and conditional covariance matrix estimators based on daily and intraday returns. The statistics of the returns, standard deviation, Sharpe ratio and turnover are based on daily returns. An asterisk (*) indicates that the difference between the standard deviation of the portfolios and the Sharpe ratio are statistically significant, at the 10% level, in relation to the estimator of the realized covariance matrix using daily data (benchmark).

	Average	Standard	Sharpe	Turnover
	return (%)	deviation	ratio	
Panel A: Unconditional covariance matrix				
Realized Covariance (daily)	-0.017	1.434	-0.012	0.852
RMK	0.026	1.517	0.017*	1.484
Realized Covariance (5-min)	0.007	1.106*	0.007	1.187
Realized Covariance (15-min)	-0.028	0.699*	-0.041	1.535
Realized Covariance (30-min)	-0.045	0.664*	-0.067	1.648
Realized Covariance (60-min)	-0.059	0.634*	-0.094	1.669
Realized Covariance (90-min)	-0.060	0.644*	-0.093	1.574
Realized Covariance (120-min)	-0.005	0.637*	-0.008*	1.459
Sample Covariance (daily)	0.006	1.129*	0.005*	0.035
Sample Covariance (Lead and Lag)	0.001	1.151*	0.008*	0,050
Optimal Shrinkage Estimator	0.005	1.127*	0.004*	0.033
Sample Covariance (5-min, ex overnight)	0.006	1.104*	0.005*	0,022
Sample Covariance (5-min, in overnight)	0.004	1.113*	0.004*	0.024
Sample Covariance (5-min, adj intraday vol)	0.003	1.106*	0.003*	0,025
Panel B: Conditional covariance matrix				
scalar vt-VECH (daily cov)	0.042	1.029*	0.040*	0.106
scalar vt-VECH (5-min)	0.007	1.078*	0.007*	0.145
scalar vt-VECH (15-min)	0.028	1.027*	0.026*	0.201
scalar vt-VECH (30-min)	0.027	1.019*	0.027*	0.185
scalar vt-VECH (60-min)	0.034	1.013*	0.033*	0.155
scalar vt-VECH (90-min)	0.030	1.011*	0.030*	0.147
scalar vt-VECH (120-min)	0.029	1.023*	0.029*	0.135
RMK (scalar vt VECH)	0.011	1.334*	0.008*	0.310

The results presents in Table 3 indicate that, when the investor rebalances the portfolio daily, the realized covariance estimated using the 60 minutes returns results in a portfolio with lower standard deviation. In terms of risk-adjusted and mean returns, the scalar vt-VECH model (daily data) presents a more satisfactory performance. It is noted that the three models with the best Sharpe ratio are those estimated using the scalar vt-VECH model, with sampling frequency of 60, 90 and 120 minutes, and the difference compared to the benchmark is statistically significant at the 10% level. The RMK, realized covariance and the sample covariance estimator perform poorly in terms of portfolio risk. According to Liu (2009), the correction for non-synchronous trading (and bid-ask spread) with lagged covariances may not be advantageous when used with daily returns of very liquid assets.

To evaluate possible performance gains of the portfolios, the assets allocation was based on different portfolio rebalancing frequencies. This approach is commonly used to evaluate the impact of transactions costs on the portfolio performance. The underlying argument is that a lower rebalancing frequency of the optimum assets weights may contribute to a more satisfactory portfolio performance, reducing the portfolio turnover. In the three rebalancing frequencies under analysis the sample covariance estimator based on 5 minutes intraday returns (excluding overnight returns) shows the best performance. The GARCH (scalar vt-VECH model) estimate using intraday data shows relevant results for portfolio performance.

Tables 4 and 5 indicate that, in of weekly and monthly rebalancing of assets weights, the portfolios built using the scalar vt-VECH model based on 120 minutes returns presents a lower risk. The best risk-adjusted return and average return are also derived from the scalar vt-VECH model, but based on daily returns. For comparison purposes, in the same out-of-sample period considered in this paper, the Ibovespa had an average return of -0.028% , standard deviation of 1.424 and risk-adjusted return of -0.020 , that is, the portfolios optimization based on the conditional estimators outperform the main index of the Brazilian stock market. The scalar vt-VECH estimator shows superiority when comparing to comparing to the realized covariance estimators and the sample covariance based on daily and intraday returns.

The economic significance of the estimation of conditional realized measures to reduce the standard deviation of minimum variation portfolio is substantial: at least 6 pp per year below the benchmark. The scalar vt-VECH model yield a systematically smaller portfolios variance than those obtained for the remaining estimators, both for the weekly and monthly rebalancing. The results also point that the use of realized measures with robustness against the effects of market microstructure and non-synchronous sample prices do not produce considerable improvements in terms of optimum portfolio performances. In Tables 2 to 4 shows that the portfolios obtained using the realized multivariate kernel (RMK), whether

conditional or unconditional, are outperformed by the other competing models in terms of risk-adjusted returns and higher portfolio turnover. De Pooter et al. (2008) argue that selecting the appropriate sampling frequency is relevant to portfolio optimization.

The realized covariance points to satisfactory results in terms of portfolio risk only in the daily rebalancing. However, the transaction costs incurred with this trading frequency may deteriorate the net portfolio performance (Caldeira et al., 2013). This effect can be reduced by rebalancing less frequently, such as on a weekly or monthly basis, which is the practice of most institutional investors. However, the lower rebalancing frequency causes a downgrading on the portfolio weights, which could cause a considerable deterioration of results. More specifically, in the weekly and monthly rebalancing, the portfolios constructed using realized covariances present a higher risk than those based on other models.

Overall, conditional covariance estimates yield better the characteristic of the data than those unconditional estimators, increasing the portfolio performance. The results in tables 2 to 4 indicates that the conditional realized measures show better performance. In fact, the portfolios implemented using these conditional realized measures presents lower risk, higher Sharpe ratio and lower turnover compared to unconditional realized measures. These evidences are favorable to conditional estimators, which suggest that the volatility timing can lead to a better risk-adjusted performance, as well as more stable portfolios.

Following the literature, the covariance matrix forecasts based on high-frequency data suggest lower portfolio volatility compared to using daily returns. This corresponds with the findings of Hautsch et al. (2013) that the high-frequency data for portfolio allocation decisions reduced portfolio volatility. In most cases, the conditional realized measures are considerably better than the unconditional estimators based on low-frequency data. The results for the different sampling frequencies are consistent with the findings of De Pooter et al. (2008) and tend to corroborate that selecting the appropriate sampling frequency is crucial to portfolio performance based on intraday data.

4. Conclusions

The covariance matrix forecast is a central aspect in the portfolio allocation. This area has seen continuous development, especially in methods based on intraday returns. The practical motivation of using intraday returns is increase the precision of covariance matrix estimates and forecasts. The performance of the covariance matrix forecast for the set of assets analyzed in this study suggests benefits in using intraday data, resulting in lower portfolio volatility. In the more plausible situation, in practical terms, in which the investors carries out a monthly or weekly adjusting, the portfolio constructed from conditional covariance matrix estimator given by the scalar vt-VECH method outperform the competing estimators and the benchmark. The return sampling frequency ranging from 15 and

Table 4
Minimum variance optimum portfolio performance (weekly rebalancing)

Note: The table presents the out-of-sample performance of the minimum variance portfolio using 30 assets traded at BM&FBovespa. The optimum portfolios are constructed using the unconditional and conditional covariance matrix estimators based on daily and intraday returns. The statistics of the returns, standard deviation, Sharpe ratio and turnover are based on daily returns. An asterisk (*) indicates that the difference between the standard deviation of the portfolios and the Sharpe ratio are statistically significant, at the 10% level, in relation to the estimator of the realized covariance matrix using daily data (benchmark).

	Average return (%)	Standard deviation	Sharpe ratio	Turnover
Panel A: Unconditional Covariance Matrix				
Realized Covariance (daily)	-0.035	1.411	-0.025	0.196
RMK	0.030	1.555*	0.019	0.306
Realized Covariance (5-min)	0.001	1.369	0.000	0.262
Realized Covariance (15-min)	-0.013	1.236*	-0.010	0.333
Realized Covariance (30-min)	0.007	1.321	0.006	0.354
Realized Covariance (60-min)	-0.004	1.420	-0.003	0.352
Realized Covariance (90-min)	-0.033	1.507	-0.022	0.347
Realized Covariance (120-min)	-0.040	1.543	-0.026	0.306
Sample Covariance (daily)	0.008	1.131*	0.007	0.027
Sample Covariance (Lead and Lag)	0.012	1.149*	0.010*	0.031
Optimal Shrinkage Estimator	0.007	1.130*	0.006*	0.026
Sample Covariance (5-min, ex overnight)	0.003	1.127*	0.003	0.020
Sample Covariance (5-min, in overnight)	0.001	1.137*	0.001	0.021
Sample Covariance (5-min, adj intraday vol)	-0.001	1.132*	-0.001	0.022
Panel B: Conditional Covariance Matrix				
scalar vt-VECH (daily cov)	0.029	1.049*	0.032*	0.062
scalar vt-VECH (5-min)	0.009	1.098*	0.009*	0.058
scalar vt-VECH (15-min)	0.024	1.048*	0.022*	0.097
scalar vt-VECH (30-min)	0.026	1.043*	0.025*	0.090
scalar vt-VECH (60-min)	0.030	1.041*	0.028*	0.080
scalar vt-VECH (90-min)	0.029	1.036*	0.028*	0.078
scalar vt-VECH (120-min)	0.027	1.035*	0.026*	0.069
RMK (scalar vt-VECH)	0.022	1.332	0.016	0.120

Table 5

Minimum variance optimum portfolio performance (monthly rebalancing)

Note: The table presents the out-of-sample performance of the minimum variance portfolio using 30 assets traded at BM&FBovespa. The optimum portfolios are constructed using the unconditional and conditional covariance matrix estimators based on daily and intraday returns. The statistics of the returns, standard deviation, Sharpe ratio and turnover are based on daily returns. An asterisk (*) indicates that the difference between the standard deviation of the portfolios and the Sharpe ratio are statistically significant, at the 10% level, in relation to the estimator of the realized covariance matrix using daily data (benchmark).

	Average	Standard	Sharpe	Turnover
	return (%)	deviation	ratio	
Panel A: Unconditional Covariance Matrix				
Realized Covariance (daily)	-0.034	1.443	-0.023	0.055
RMK	0.012	1.447	0.009*	0.092
Realized Covariance (5-min)	0.008	1.264	0.006	0.076
Realized Covariance (15-min)	0.018	1.246	0.014	0.082
Realized Covariance (30-min)	0.015	1.465	0.010	0.091
Realized Covariance (60-min)	0.010	1.428	0.007	0.089
Realized Covariance (90-min)	-0.018	1.633*	-0.011	0.096
Realized Covariance (120-min)	0.003	1.525	0.002	0.082
Sample Covariance (daily)	0.006	1.140*	0.005	0.020
Sample Covariance (Lead and Lag)	0.013	1.164*	0.011	0.021
Optimal Shrinkage Estimator	0.006	1.138*	0.005	0.019
Sample Covariance (5-min, ex overnight)	-0.001	1.135*	-0.001	0.017
Sample Covariance (5-min, in overnight)	-0.003	1.147*	-0.003	0.018
Sample Covariance (5-min, adj intraday vol)	-0.004	1.140*	-0.004	0.018
Panel B: Conditional Covariance Matrix				
scalar vt-VECH (daily cov)	0.028	1.038*	0.023	0.029
scalar vt-VECH (5-min)	0.015	1.108*	0.014	0.026
scalar vt-VECH (15-min)	0.015	1.053*	0.014	0.035
scalar vt-VECH (30-min)	0.014	1.047*	0.013	0.035
scalar vt-VECH (60-min)	0.019	1.041*	0.018	0.032
scalar vt-VECH (90-min)	0.020	1.037*	0.019	0.031
scalar vt-VECH (120-min)	0.014	1.029*	0.014	0.029
RMK (scalar vt VECH)	0.024	1.360	-0.017	0.046

120 minutes suggests better performance rather than the popular 5 minute return. This indicates portfolio performance gains from intraday data to estimate the volatility which is necessary for portfolio optimization problem.

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