The Macroeconomic Determinants of the Term Structure of Inflation Expectations in Brazil

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Abstract

This paper aims to analyze the dynamics of inflation expectations according to macroeconomics conditions. To this end, we extract the expected inflation curve implied by indexed bonds and then estimate a dynamic factor model. The factors correspond to the level, slope and curvature of the term structure, varying over time as a function of the exchange rate, inflation, commodities index and the CDS-implied Brazil risk. A one standard deviation shock to the exchange rate increases inflation more in the short and long ends of term structure than in the medium run. The same pattern arises after a shock to inflation. A shock to commodity prices increases inflation mostly in the short term, stabilizing at a higher level than the original curve. In contrast, a shock to the CDS shifts down the expected inflation curve in a virtually parallel manner.

Keywords: Implied Inflation, Term Structure, Macroeconomic Variables.

JEL Codes: E43, E44, G12.
1. Introduction

Inflation usually is the center of the attention and concern of central banks, regardless of whether excessively high or presenting negative rates. Perhaps, more important than actual inflation, inflation expectations are critical to the monetary authority action.

In 2004, the President of the Federal Reserve Bank, Ben Bernanke drew attention to three reasons for central banks to focus on inflation expectations. First, the pursuit of price stability is the main objective of monetary policy. In this regard, central banks employ data on asset prices and returns to back out market expectations about the private sector’s future performance.

Second, inflation expectations is a key variable for policy makers. There is a vast literature showing that successful monetary policy should anchor inflation expectations so as to avoid economic instability. Finally, Bernanke stated that the knowledge of inflation expectations enables the calculation of real interest rates. The later are important indicators not only to evaluate the current macroeconomic conditions, but also for the implementation of monetary policy.

Bernanke’s points are particularly relevant for this study because they reinforce the importance of market expectations as measured by the break-even inflation rates implied by indexed federal government bonds.

There are two approaches that stand out in the measurement of inflation expectations. The first is by conducting a survey on the expectation of inflation rates at a later date with market participants, economists, business managers, and other professionals who make decisions based on inflation expectations. In Brazil, the main survey about inflation expectations is on the weekly Focus Bulletin of the Central Bank of Brazil. The second approach to gauge market expectations about future inflation is to look at price-indexed government bonds. By deducting the rate of inflation-linked bonds (real interest rate) from the rates of fixed-income securities (nominal interest rate) with the same duration, we are able to recover the break-even inflation rate (BEIR, for short).

The literature on inflation-linked bonds in Brazil is not very extensive. With the increasing use of indexed bonds around the world and, especially, in Brazil, it is paramount to understand the relationship between different macroeconomic indicators and inflation expectations so as to enable the development of new strategies for inflation control. The goal of this study is to how changes in macroeconomic factors affect the dynamics of inflation expectations embedded in indexed government bonds. We thus estimate a model for the break-even inflation curve that controls for the macroeconomic conditions and then examine how the term structure of inflation expectations reacts to different types of macroeconomic shocks within a partial equilibrium framework.

The analysis is as follows. We first extract the term structures of nominal and real interest rates for 12 different durations. We then back out the break-even
inflation rate from the difference between the nominal and real interest rates.\footnote{We abstract from the risk and liquidity premia that may affect this difference for simplicity. See, for example, Val et al. (2010) for more details.}

Next, we estimate a factor model for the term structure of the break-even inflation rate in which the level, slope and curvature of the curve depends on the exchange rate, inflation, commodity prices (CRB) and the five-year Brazil CDS. We test the response of the break-even inflation curve to changes of one standard deviation in each one of these macroeconomic variables. The results show that, after a shock to the exchange rate, there is a upward shift in the expected inflation curve, though the increase is higher in the short term. Inflationary shocks cause a sharp increase in the short-term expected inflation, though a slight reduction in the long run. After a shock to commodity prices, the expected inflation increases rapidly in the short end, stabilizing in the long term at a higher level than the original curve. Finally, a shock to the CDS makes the break-even inflation curve move downward in a virtually parallel fashion.

The remainder of this study is as follows. In Section 2, we briefly review the literature. Section 3 documents the recent evolution of the Brazilian public debt market. Section 4 describes the econometric model and methodology we use. Section 5 discusses at length the empirical results we find, whereas Section 6 offers some concluding remarks.

2. Literature Review

The literature on the Brazilian inflation-linked government bonds is still very incipient as compared to the vast literature abroad. In general, this literature is mostly recent due to data limitations. The liquidity of inflation-linked fixed-income securities has increased only very recently.

The interest in inflation-linked sovereign bonds is due to the fact that it reflects the inflation rate that market participants expect to happen in the future.

Caldeira and Furlani (2011) evaluate whether the break-even inflation rates implied by indexed bonds provides relevant information to predict future inflation. They also compare whether they have greater predictive capacity than standard VAR models. They conclude that the BEIR is more informative than VAR forecasts for horizons of 3, 6 and 9 months.

Val et al. (2010) extract the break-even inflation curves taking inflation risk premium into account. They show that these implied market expectations forecast better future inflation than the market expectations published by the Focus newsletter of the Central Bank of Brazil.

Rochman and Hatisuka (2011) analyze the long-term inflation anchoring in Brazil using the expected inflation implied by indexed bonds. Using the nominal and real interest rates in the secondary market, they extract the break-even inflation rate using Svensson’s (1994) factor model. They note that the long-run...
The expected inflation rate is around 6% per annum, much higher than the inflation target of 4.5% per annum.

Chernov and Mueller (2008) examine the effectiveness of monetary policy by looking at the reaction of the term structure of inflation expectations to monetary shocks to the US. In particular, they develop a nonarbitrage model that incorporates information from market-implied and survey-implied expected inflation rates. They find evidence that the monetary policy is quite efficient, accounting for short-run bias in the expected inflation rates implied by market securities and consumer surveys.

Nelson and Siegel (1987) propose a parsimonious model to fit the forward rate curve in the US using an exponential polynomial approximation. Interestingly, they show that linearizing the polynomial by fixing the shape parameter does not affect much the goodness of fit. In particular, their model posits that the forward rate curve is such that

\[ f(\tau) = \beta_0 + \beta_1 e^{-\lambda \tau} + \beta_2 \lambda e^{-\lambda \tau} \]  

where \( \lambda \) is a shape parameter and \( \beta_0, \beta_1 \) and \( \beta_2 \) correspond to long-, short- and medium-term factors, respectively.

Diebold and Li (2006) consider the following dynamic variant of the Nelson-Siegel model:

\[ Y(t) = \beta_{1,t} + \beta_{2,t} \left( \frac{1 - e^{-\lambda t \tau}}{\lambda t \tau} \right) + \beta_{3,t} \left( \frac{1 - e^{-\lambda t \tau}}{\lambda t \tau} - e^{-\lambda t \tau} \right) \]  

where the betas are latent dynamic factors corresponding to the level, slope and curvature of the term structure. For instance, the loading on \( \beta_1 \) is constant across durations and hence it represents the level of the long-end curve. The loading on \( \beta_2 \) begins in one and then monotonically decays to zero, characterizing a slope factor given that it affects the short- and long-end in opposite fashion. The loading of \( \beta_3 \) starts at zero, increases up to a certain point and then declines to zero in the long-term. This means it mainly reflects the curvature of the term structure in view that it essentially affects only the medium term, with little impact on the short- and long-ends of the curve. Figure 6 plots the beta loadings to better illustrate the level, slope and curvature correspondence. Diebold and Li estimate daily betas using cross-sectional regressions as in (2) and then assume a VAR model to back out their evolution over time.

Huse (2011) proposes a similar factor model in which the betas in (2) vary over time as a function of observed state variables. The main advantage is to fully exploit the panel nature of the data by estimating the regression in (2) only once, rather than every day. The price to pay is that the betas depend exclusively on pre-specified macroeconomic state variables, instead of being latent as in the Diebold-Li model. Interestingly, Huse finds that the level, slope and curvature of the yield curve in the US respond to different types of macroeconomic shocks.
3. Government bonds in Brazil

The main source of funding for the Brazilian National Treasury to finance the budget deficit of the federal government is the issuance of government bonds in the domestic market. These are fixed-income instruments issued via public offering. The federal public debt in the domestic market (DFPD) is in Brazilian reais. It includes government bonds with either fixed or floating rates that may be indexed or not to the exchange rate or to the consumer price inflation rate. The fixed rate securities are the National Treasury Bills (LTN) and National Treasury Notes – Series F (NTN-F). The LTN is issued for short and medium maturities of up to four years and pays no coupon. The NTN-F is issued for medium and long maturities of up to ten years, paying semi-annual coupons of 10%.

The Financial Treasury Bills (LFT) is a floater, with maturity of up to five years and no coupon payment. The floating rate depends on the value of the baseline interest rate SELIC. The National Treasury Notes – A Series (NTN-A) are indexed to the exchange rate of the Brazilian real against the US dollar. The National Treasury Notes – C Series (NTN-C) and the National Treasury Notes – B Series (NTN-B) are indexed to price indexes. The NTN-C used to pay IGP-M (General Market Price Index) plus a fixed rate at the time of purchase, with seminannual coupon payments, but they no longer exists. The NTN-B is issued for maturities of up to forty years, paying semianual coupon. Their rates are indexed to the consumer price index IPCA.

The composition of the DFPD has changed significantly over the years. In January 2000, floaters and exchange-rate-indexed bonds corresponded for over 85% of the DFPD. By August 2013, fixed-rate and inflation-linked bonds represented over 75% of the DFPD (see Figure 1).

Since 2001, one of the guidelines of the National Treasury is the gradual replace-
ment of floaters with fixed-rate and inflation-linked bonds. Initially, the latter were exclusively NTN-C. After the implementation of the inflation targeting regime in 1999, the launch of NTN-B on July 4, 2001 lined the indexation to consumer price index IPCA. The first NTN-B call auctions were held in 2002, though it was only in 2005 that the participation of NTN-B as a percentage of DFPD did exceed 5%. Since then, there has been a significant increase in NTN-B issuances, so that their participation in DFPD has sharply increases (see Figure 2).

Figure 2
Absolute and relative volume of DFPD in NTN-B

Source: Brazilian National Treasury.

The increase in the DFPD duration as one of the guidelines of the National Treasury spurred even more the use of NTN-B. To buy bonds with maturities over than 10 years, the financial market participants require a real rates guarantee. As the NTN-B is indexed to inflation and has maturity of until 40 years, this was the main instrument of the National Treasury to lengthen DFPD duration.

Despite the significant increase in the volume of NTN-B emissions since 2005, the secondary market only presented a meaningful increase in liquidity after the Central Bank of Brazil unexpectedly cut off the Selic rate in August 2011 (see Figure 4).

Val et al. (2010) consider the recent increase in the traded volume of NTN-B as a fundamental factor for the development of studies about inflation expectations. This line of research has given way to real rate and implied inflation curves, thereby enabling the study of this particular market in Brazil.
Figure 3
Average Maturity of DFPD

Source: Brazilian National Treasury.

Figure 4
30-day moving average of the NTN-B trading volume in the secondary market

Source: Ativa Corretora.
4. Data Description and Methodology

4.1 Data

We back out break-even inflation curves from the difference between prefixed and real rates. The prefixed rates stem from the zero rates implied by the Brazilian government bond prices. In particular, we extract the prefixed rates from the LTN and NTN-F curves. The former refers to the short end of the curve, whereas the latter correspond to the prefixed rates for longer maturities.

Note that these federal bonds have very different coupon features: e.g., LTN does not pay coupon, NTN-F pays semiannual coupon of 10%, and NTN-B pays semiannual coupon of 6%. We thus entertain 12 different durations to build the Brazilian government curve, namely, $\tau = 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6$ and 7 (in years). We focus a bit on shorter durations because, as pointed out by Patton and Timmermann (2008a, b), expected inflation should converge to the unconditional expected inflation in the long run.

We first calculate daily durations for every NTN-F and LTN. We then linearly interpolate the rates for any pre-specified durations shorter than the longest NTN-F and longer than the shortest LTN. In the absence of sufficient data to interpolating, we consider linear extrapolations. We treat the rates and durations for the NTN-B in a similar fashion, ending up with a real yield curve for the pre-specified durations.

Given the nominal and real yield curves, we compute the break-even inflation curve using the Fisher equation:

$$1 + \pi_t^{(\tau)} = \frac{1 + i_t^{(\tau)}}{1 + r_t^{(\tau)}}$$

where $\pi_t^{(\tau)}$ is the implied breakeven inflation rate (BEIR) for the duration $\tau$ at time $t$, $i_t^{(\tau)}$ is the nominal interest rate for the duration $\tau$ at time $t$ and $r_t^{(\tau)}$ is the real interest rate for the duration $\tau$ at time $t$.

Although we construct daily yield and breakeven inflation curves for the period running from January, 2 2004 to April, 30 2013, we carry out our empirical analysis using weekly curves because this is the highest frequency at which we observe enough variation in the macroeconomic data. To ensure that the macroeconomic information is predetermined, we focus on data from Thursdays given that the weekly releases of the macroeconomic information take place at most on Wednesdays.

Table 1 reports descriptive statistics of the implied inflation curve. The average curve is upward sloping for the sample period. Interestingly, the same applies to the standard deviation of the implied inflation rate. This is in stark contrast to the behavior we observe for the nominal interest rates, whose standard deviations are lower for longer durations. This happens because the longer the term the higher the
number of factors that may drive the course of inflation. Finally, we evince that the breakeven inflation rates are extremely persistent regardless of the maturity as illustrated by the high levels of autocorrelations.

Table 1
Descriptive statistics of the break-even inflation curve

<table>
<thead>
<tr>
<th>duration (in years)</th>
<th>average (% p.a.)</th>
<th>standard deviation</th>
<th>minimum</th>
<th>maximum</th>
<th>AC(1)</th>
<th>AC(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.40</td>
<td>1.44</td>
<td>2.83</td>
<td>9.70</td>
<td>0.99</td>
<td>0.75</td>
</tr>
<tr>
<td>1</td>
<td>5.44</td>
<td>1.37</td>
<td>3.13</td>
<td>9.69</td>
<td>0.99</td>
<td>0.85</td>
</tr>
<tr>
<td>1.5 anos</td>
<td>5.47</td>
<td>1.37</td>
<td>3.19</td>
<td>9.54</td>
<td>0.99</td>
<td>0.87</td>
</tr>
<tr>
<td>2 anos</td>
<td>5.51</td>
<td>1.39</td>
<td>3.32</td>
<td>11.32</td>
<td>0.99</td>
<td>0.87</td>
</tr>
<tr>
<td>2.5 anos</td>
<td>5.58</td>
<td>1.41</td>
<td>3.03</td>
<td>12.09</td>
<td>0.98</td>
<td>0.86</td>
</tr>
<tr>
<td>3 anos</td>
<td>5.67</td>
<td>1.45</td>
<td>3.11</td>
<td>12.87</td>
<td>0.98</td>
<td>0.85</td>
</tr>
<tr>
<td>3.5 anos</td>
<td>5.75</td>
<td>1.51</td>
<td>3.26</td>
<td>13.64</td>
<td>0.98</td>
<td>0.84</td>
</tr>
<tr>
<td>4 anos</td>
<td>5.82</td>
<td>1.56</td>
<td>3.30</td>
<td>14.41</td>
<td>0.98</td>
<td>0.82</td>
</tr>
<tr>
<td>4.5 anos</td>
<td>5.89</td>
<td>1.63</td>
<td>3.29</td>
<td>15.19</td>
<td>0.98</td>
<td>0.81</td>
</tr>
<tr>
<td>5 anos</td>
<td>5.97</td>
<td>1.70</td>
<td>3.29</td>
<td>16.00</td>
<td>0.98</td>
<td>0.79</td>
</tr>
<tr>
<td>6 anos</td>
<td>6.13</td>
<td>1.86</td>
<td>3.32</td>
<td>17.76</td>
<td>0.97</td>
<td>0.76</td>
</tr>
<tr>
<td>7 anos</td>
<td>6.25</td>
<td>2.04</td>
<td>3.36</td>
<td>19.53</td>
<td>0.97</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Figure 5 shows the evolution of the implied inflation rates over the sample period. It is interesting to observe that 6-month breakeven inflation rate sometimes uncouples from longer-maturity rates, possibly reflecting a faster reaction to changes in economic conditions.

To proxy for the macroeconomic conditions, we collect the following variables at the weekly basis from Bloomberg.

- **CDS** is the 5-year credit default swap for Brazil. This is the most liquid CDS, which insures the holder against a default event from the Brazilian government. The value of the CDS essentially embeds the probability that the Brazil government will default within 5 years. It thus act as a good indicator of the “health” of the Brazilian economy.

- **IPC** is the consumer price index of the city of São Paulo as measured by FIPE. We employ this price index to compute the weekly inflation rates.

- **USDBRL** is the exchange rate of the Brazilian real (BRL) vis-à-vis the US dollar (USD). The exchange rate is a very relevant indicator in Brazil because it affects in a substantial manner the price level of tradable goods.

- **CRB** is the Thomson Reuters/CoreCommodity index of commodity prices. It conveys relevant information about the state of nature because Brazil is a heavy exporter of commodities.
**Level** is a proxy for the level factor of the nominal yield curve. In particular, as the level factor mainly relates to the long-end of the curve, we employ the 5-year nominal rate as a proxy.

**Slope** is a proxy for the steepness factor of the nominal yield curve. We use the difference between the 5-year and 6-month government rates.

To aggregate data to the weekly frequency, we consider macroeconomic releases from Thursday of week s-1 to Wednesday of week s as belonging to week s. If there is no data release until Wednesday, we keep the value of that macroeconomic variable constant (i.e., the same as in the previous week). The above procedure ensures that the macroeconomic drivers of the implied inflation curve on a given Thursday are known at least one day in advance. As before, we construct a weekly time series of each macroeconomic variable from the first week of January 2004 to the last week of April 2013. Table 2 shows their descriptive statistics.

To facilitate the interpretation of the regression coefficients, we standardize the macroeconomic variables by their average and standard deviation. Unfortunately, there is no real activity indicator at the weekly frequency and hence we could not include such a variable in the model.
Table 2
Descriptive statistics of the implied inflation curve

<table>
<thead>
<tr>
<th>Description</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>AC(1)</th>
<th>AC(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS Brazil (bps)</td>
<td>206.76</td>
<td>153.11</td>
<td>62.16</td>
<td>847.07</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td>△IPC (%)</td>
<td>0.39</td>
<td>0.29</td>
<td>-0.50</td>
<td>1.12</td>
<td>0.92</td>
<td>0.07</td>
</tr>
<tr>
<td>USDBRL</td>
<td>2.08</td>
<td>0.39</td>
<td>1.55</td>
<td>3.21</td>
<td>0.99</td>
<td>0.94</td>
</tr>
<tr>
<td>CRB</td>
<td>305.98</td>
<td>44.24</td>
<td>205.14</td>
<td>472.36</td>
<td>0.98</td>
<td>0.81</td>
</tr>
<tr>
<td>Level</td>
<td>13.60</td>
<td>2.97</td>
<td>8.81</td>
<td>26.22</td>
<td>0.99</td>
<td>0.88</td>
</tr>
<tr>
<td>Slope</td>
<td>0.91</td>
<td>1.73</td>
<td>-3.88</td>
<td>7.66</td>
<td>0.98</td>
<td>0.78</td>
</tr>
</tbody>
</table>

4.2 Model
To analyze how the macroeconomic variables affect the dynamics of the breakeven inflation curve, we employ Huse’s (2011) three-factor model. The model states that

\[ y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + \beta_{3,t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right) + \text{error} \]  

(4)

in which \( \beta_{1,t}, \beta_{2,t} \) and \( \beta_{3,t} \) are affine functions of the macroeconomic variables. In particular, we assume that

\[ \beta_{1,t} = \beta_{1,0} + \beta_{1,1} USDBRL_{t-1} + \beta_{1,2} CRB_{t-1} + \beta_{1,3} CDS_{t-1} + \beta_{1,4} IPC_{t-1} \]  

(5)

\[ \beta_{2,t} = \beta_{2,0} + \beta_{2,1} USDBRL_{t-1} + \beta_{2,2} CRB_{t-1} + \beta_{2,3} CDS_{t-1} + \beta_{2,4} IPC_{t-1} \]  

(6)

\[ \beta_{3,t} = \beta_{3,0} + \beta_{3,1} USDBRL_{t-1} + \beta_{3,2} CRB_{t-1} + \beta_{3,3} CDS_{t-1} + \beta_{3,4} IPC_{t-1} \]  

(7)

To control for the distinct levels of market liquidity over the sample period, we include year dummy variables (DYear_i) among the drivers of the level factor \( \beta_{1,t} \). As market liquidity is a common factor that should affect every yield in a similar manner, it essentially affects the curve level rather than its slope and curvature. This is the reason why we do not add year dummies to \( \beta_{2,t} \) and \( \beta_{3,t} \).

The parameter \( \lambda \) establishes an exponential decay for the term structure of the breakeven inflation curve, determining at which duration the loading on the curvature factor \( \beta_{3,t} \) is highest.

We fix this parameter to the value in Diebold and Li (2006), namely, 0.0609. The reason is simple. The resulting decay entails a maximum load in the curvature factor at a 3-year maturity, which is about the average duration in the sample. Figure 6 illustrates how the factor loadings change with the duration. Apart from a constant loading on the level factor, we observe that the loading on the slope
factor decreases almost linearly with the duration, whereas the loading on the curvature factor increases as we move from the short run to the medium run and then starts slowly declining.

Figure 6
Factor loadings as a function of the duration (level in dotted line, slope in dashed line, and curvature in solid line)

Using the panel data on the breakeven inflation rates and (standardized) macroeconomic variables, we estimate Huse’s (2011) three-factor model by pooled ordinary least squares.

5. Empirical Findings

Data amount to 486 weekly observations of the breakeven inflation rate for 12 durations, adding up to 5,832 observations. Table 3 reports the coefficient estimates and their robust standard errors (and p-values). All coefficient estimates are significant at the 10% level, with the vast majority being significant even at the 1% level. The adjusted $R^2$ is 85.35%, reflecting a very good fit. Interestingly, the empirical proxies for the level and slope factors of the nominal yield curve are significant only for the level factor of the breakeven inflation curve. It has seemingly no effect on the slope and curvature factors. From the coefficient estimates, we back out the evolution of the level, slope and curvature factors of the breakeven inflation curve over time. Table 4 documents some descriptive statistics for these factors. As expected, every factor exhibits a lot of persistence, reflecting the highly persistent nature of the breakeven inflation rates.

Figures 7 to 9 compare the level, slope and curvature factors with their empirical counterparts. In particular, we proxy the curve level by the 5-year breakeven inflation rate because it relies on the most liquid long-term bonds. Figure 7 shows
that the two series are very close, displaying a correlation of 94.45%.

Table 3
Regression results

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>standard error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEVEL($\beta_1(t)$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>99.921</td>
<td>0.0147</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2005</td>
<td>-0.1339</td>
<td>0.0088</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2006</td>
<td>-0.3485</td>
<td>0.0094</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2007</td>
<td>-0.2787</td>
<td>0.0130</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2008</td>
<td>-0.1522</td>
<td>0.0122</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2009</td>
<td>-0.0500</td>
<td>0.0160</td>
<td>0.0017</td>
</tr>
<tr>
<td>D2010</td>
<td>0.0604</td>
<td>0.0163</td>
<td>0.0002</td>
</tr>
<tr>
<td>D2011</td>
<td>0.1015</td>
<td>0.0152</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2012</td>
<td>0.1564</td>
<td>0.0162</td>
<td>0.0000</td>
</tr>
<tr>
<td>D2013</td>
<td>0.2042</td>
<td>0.0175</td>
<td>0.0000</td>
</tr>
<tr>
<td>USDBRL</td>
<td>0.1559</td>
<td>0.0198</td>
<td>0.0000</td>
</tr>
<tr>
<td>CRB</td>
<td>0.0274</td>
<td>0.0115</td>
<td>0.0174</td>
</tr>
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<td>IPC</td>
<td>-0.0292</td>
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</tr>
<tr>
<td>CDS</td>
<td>-0.0110</td>
<td>0.0032</td>
<td>0.0006</td>
</tr>
<tr>
<td>Level</td>
<td>0.1710</td>
<td>0.0049</td>
<td>0.0000</td>
</tr>
<tr>
<td>Slope</td>
<td>0.0124</td>
<td>0.0029</td>
<td>0.0000</td>
</tr>
<tr>
<td>SLOPE($\beta_2(t)$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.2713</td>
<td>0.0076</td>
<td>0.0000</td>
</tr>
<tr>
<td>USDBRL</td>
<td>-0.0333</td>
<td>0.0144</td>
<td>0.0212</td>
</tr>
<tr>
<td>CRB</td>
<td>0.0676</td>
<td>0.0085</td>
<td>0.0000</td>
</tr>
<tr>
<td>IPC</td>
<td>0.1299</td>
<td>0.0076</td>
<td>0.0000</td>
</tr>
<tr>
<td>CDS</td>
<td>-0.0099</td>
<td>0.0024</td>
<td>0.0000</td>
</tr>
<tr>
<td>CURVATURE($\beta_3(t)$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.3256</td>
<td>0.0292</td>
<td>0.0000</td>
</tr>
<tr>
<td>USDBRL</td>
<td>-0.2276</td>
<td>0.0555</td>
<td>0.0000</td>
</tr>
<tr>
<td>CRB</td>
<td>-0.0859</td>
<td>0.0327</td>
<td>0.0087</td>
</tr>
<tr>
<td>IPC</td>
<td>-0.0491</td>
<td>0.0294</td>
<td>0.0946</td>
</tr>
<tr>
<td>CDS</td>
<td>0.0317</td>
<td>0.0091</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

R^2 adjusted = 0.8535
Estatistic $F = 13.594,340$ P-value = 0.0000

Figure 8 compares the slope factor we estimate with the difference between the 5-year and 6-month implied inflation rates. The model seems to perform well given the moderately high correlation of 52.65% between the empirical proxy and slope factor.

Finally, in Figure 9, we compare our model-implied curvature factor with a very simple proxy based on the difference between the spread of the 5-year breakeven inflation rate over the 2.25-year rate and the spread of the latter over the 6-month rate. It is reassuring to observe that the correlation is significantly positive, even if not very high at 27.46%. However, this low correlation is not much of a concern given the sheer crudeness of the empirical proxy we employ.

We next examine the impact of each macroeconomic variable in the term structure of inflation expectations. To this end, we first construct a baseline curve for the breakeven inflation rates, imposing that all macroeconomic variables are at
Table 4
Descriptive statistics of level, slope and curvature factors

<table>
<thead>
<tr>
<th>factor</th>
<th>average</th>
<th>standard deviation</th>
<th>minimum</th>
<th>maximum</th>
<th>AC(1)</th>
<th>AC(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEVEL</td>
<td>1.930</td>
<td>0.242</td>
<td>1.533</td>
<td>3.016</td>
<td>0.986</td>
<td>0.869</td>
</tr>
<tr>
<td>SLOPE</td>
<td>-0.271</td>
<td>0.184</td>
<td>-0.679</td>
<td>0.359</td>
<td>0.953</td>
<td>0.473</td>
</tr>
<tr>
<td>CURVATURE</td>
<td>-0.326</td>
<td>0.148</td>
<td>-0.639</td>
<td>0.044</td>
<td>0.973</td>
<td>0.718</td>
</tr>
</tbody>
</table>

Figure 7
Model-implied and empirical proxy for the level factor
Figure 8
Model-implied and empirical proxy for the slope factor

Figure 9
Model-implied and empirical proxy for the curvature factor
their average values. This amounts to fixing them to zero and considering only
the estimates of \((\beta_0^0, \beta_2^0, \beta_3^0)\) in the affine specification of the level, slope and cur-
vature factors. In a second stage, we give a shock of one standard deviation in
each macroeconomic variable to verify the response of te breakeven inflation curve.
The standard deviations are 0.3931 for the exchange rate (USDBRL), 0.0419 for
the CDS, 44.1908 for the CRB index and 0.2934 for the IPC.

Figure 10 shows that a shock to the exchange rate mainly affects the market-
impied inflation in the short end of the curve. In particular, expected inflation
rises due to the higher price of tradables. This increase is smaller in the medium
term, possibly because one would expect the monetary authority to respond to the
higher inflation by upping interest rates or intervening in the FX market. However,
the curve reacts again more strongly in the long run. This could indicate either that
the current inflation targeting policy of the central bank of Brazil lacks credibility
or simply that uncertainty becomes too high.

Figure 10
Response of the breakeven inflation curve to a shock to the exchange rate

Figure 11 shows what happens after a shock to the CDS. There is a sharp drop
in the market-impied inflation rates on the short-end of the curve. The decline
in rates becomes more moderate as we move along the curve. This is not very
surprising given that an increase in the CDS signals higher nominal interest rates,
which would negatively impact the breakeven inflation curve.

Figure 12 shows a concave reaction of the expected inflation curve after a shock
to commodity prices (CRB). The increase in the expected inflation becomes less
pronounced as we move along the curve, stabilizing for longer durations. As in the
case of the CDS shock, this sharper movement in the short end of the breakeven
inflation curve could well result from overreaction.
Figure 11
Response of the breakeven inflation curve to a shock to the CDS

Figure 12
Response of the breakeven inflation curve to a shock to the CRB index
Figure 13
Response of the breakeven inflation curve to a shock to the CPI

Figure 14
Model-implied and realized breakeven inflation rates
Figure 13 focuses on the impact of a shock in the inflation rate (CPI). There is a large increase in the market-implied inflation only in the short end of the curve. The impact is indeed very small for any maturity beyond 3 years, though it is interesting to observe that it becomes slightly negative for very long maturities.

Finally, we assess the model's forecast ability. Figure 14 compares the model-implied breakeven inflation rates to their realized counterparts. The correlations between model-implied and realized rates are very high: e.g., 76.17% for the 6-month rates, 94.41% for the 2.5-year rate, 98.07% for the 5-year rates, and 94.92% for the 7-year breakeven inflation rates.

6. Conclusion

This work aims to contribute to the literature on the Brazilian fixed-income market by examining how macroeconomic conditions affect market-implied inflation expectations. To this end, we estimate Huse’s (2011) dynamic factor model for the term structure of breakeven inflation rates expectations. The model assumes that the level, slope and curvature factors are driven exclusively by observable macroeconomic variables.

As state variables, we employ the exchange rate, inflation, commodity index and the Brazilian default risk (as measured by the 5-year CDS). We show that a one-standard-deviation shock to the exchange rate increases inflation more in the short and long ends of term structure than in the medium run. The same pattern arises after a shock to inflation. A shock to commodity prices increases inflation mostly in the short term, stabilizing at a higher level than the original curve. In contrast, a shock to the CDS shifts down the expected inflation curve in a virtually parallel manner.

References


