The Antitrust Mixed Logit Model

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This paper presents the Antitrust Mixed Logit Model (AMLM), a novel methodology that shows how to calibrate the parameters of a mixed-logit demand model and simulate the competitive effects of horizontal mergers. The major advantage over the simpler Logit version (the Antitrust Logit Model, ALM, developed by Werden and Froeb,1994) is flexibility, resulting in more plausible elasticities and consequently more plausible predictions of merger effects. Moreover, unlike the econometric approaches, the AMLM shares with the ALM the attributes that are particularly appealing to antitrust agencies, given time and data constraints they usually face: low data requirement and high computational speed.

Keywords- Antitrust, Mixed Logit, Merger Simulation, Competition Analysis

Este artigo apresenta o Antitrust Mixed Logit Model (AMLM), uma metodologia nova que demonstra como calibrar os parâmetros de um modelo de demanda mixed-logit e simular os efeitos competitivos decorrentes de uma fusão horizontal. A principal vantagem sobre a versão mais simples que utiliza o modelo logit (the Antitrust Logit Model, ALM, desenvolvido por Werden and Froeb,1994) é a flexibilidade, o que resulta em elasticidades mais plausíveis e, conseqüentemente, previsões mais plausíveis sobre o efeito de fusões. Ao contrário de abordagens econométricas, o AMLM compartilha com o ALM atributos interessantes para autoridades de antitruste: requer poucos dados e é rápido de processar.

Palavras-Chave- Antitruste, Mixed Logit, Simulação de Fusão, Defesa da Concorrência

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I-INTRODUCTION

Predicting the unilateral effects of horizontal mergers is one of the main objectives of competition policy. Typically, quantitative and qualitative methods are combined with other traditional methods to assess the potential loss of welfare due to the elimination of competition between two or more firms. However, in the last 10 to 15 years, due to the fast developments of new tools in the empirical industrial organization field and its increasing acceptance by antitrust agencies and courts, the application of quantitative methods has grown significantly. Among those, one of the most widely used method is the so-called merger simulation, which makes predictions of price changes due to the internalization of competition between two firms based on a structural model of demand and supply.

There many ways in which merger simulation can be framed. On the demand side the product has to be defined as a homogeneous or differentiated good. In turn, on the supply side the typical options are Bertrand (price competition) and Cournot (quantity competition). However, the simulation technique is most frequently applied to predict the competitive effects of mergers in differentiated goods industries using the Bertrand game. In this case, the common options for demand are: continuous models, such as AIDS, linear and log-linear, and discrete-choice models, such as logit, nested logit and mixed logit.

The empirical methodology to uncover the parameters of demand and supply, a necessary step to perform simulation, also present alternatives. Indeed, models can be calibrated or estimated econometrically. The econometric approach usually requires gathering a rich data set (price, quantities, shifters of demand and cost and instruments) and sometimes computationally intensive models (e.g. Nevo, 2001). The most explicit benefit is that one can assess the precision of the estimates and therefore test the parameters. The alternative is to impose more assumptions into the model and bring more external information (e.g. own and cross price elasticities) to deterministically uncover, i.e. calibrate, the parameters. In this case, in addition to the external information, only prices and shares (quantities) are needed and the computation is very fast. The drawback is that one does not have a natural way of testing the parameters and price predictions and a lot of confidence is placed on the external information. This problem is mitigated, though, using sensitivity analysis.

Within the class of the calibration approach, the Antitrust Logit Model (ALM) developed by Werden and Froeb (1994) is one of the pioneers and is certainly one of the most frequently used by antitrust agencies and representatives of merging firms to predict the competitive effects of horizontal mergers in product-differentiated industries. As a calibration method, the ALM does not place great demands on the data set and is fast to compute. These attributes make this calibration methodology particularly appealing to antitrust agencies and representatives of merging firms, given time and data constraints they usually face. However, it is well known that the Logit demand model places very restrictive limitations on own and cross price elasticities, which constitute critical economic parameters in the evaluation of merger effects[1].

This paper presents the Antitrust Mixed Logit Model (AMLM), a new methodology whose main contribution is to show how to calibrate the parameters of a mixed-logit demand model. After this calibrating step the model follows most merger simulation models found in the industrial organization literature, i.e. it assumes Bertrand competition in order to obtain marginal costs and simulate price increases.

[1] This is a structural problem of the logit model, in the sense that, regardless of the empirical methodology employed to uncover its parameters (calibration or econometrics), the model imposes by construction an inflexible elasticity matrix (i.e. a matrix with many identical elements).

The major advantage over the logit version is the flexibility of the mixed logit demand, which generates more realistic patterns of substitution between goods and consequently more plausible predictions about merger effects. Moreover, unlike the econometric approaches to uncover the parameters of a mixed-logit demand (e.g. Berry, Levinsohn and Pakes, 1995, and Nevo, 2001), the AMLM exhibit attributes that are particularly appealing in merger investigations: low data requirement and high computational speed. Indeed, the data requirement to implement the AMLM is the same as the ALM and the computational burden is low.

This paper is organized as follows. Section II presents the simple logit demand model. The following section introduces the mixed logit. In turn, section IV exhibits the supply side, in which firms compete à la Bertrand. In section V, artificial data is used to illustrate the AMLM. Another application with real data of the ALML is presented in section VI. Finally, additional comments can be found in the last section.

II – The Calibrated Logit Demand Model

As a starting point, I present the demand side of the ALM, which proposes the calibration of the simplest discrete-choice demand model: the Logit. This model yields closed form solutions for demand and elasticities and therefore highlights the basic insights of the ALM that can be used to calibrate the more flexible model to be presented in the next section.

Consumers rank products according to their characteristics and prices. There are $N+1$ choices in the market, N inside goods and one reference good (or outside good). Consumer i chooses brand j, given price p_j, unobserved characteristics and quality (summarized by the scalar δ_j), and unobserved idiosyncratic preferences $\varepsilon_{ij,}$ according to the following utility function:

(1) $$u_{ij} = -\alpha p_j + \delta_j + \varepsilon_{ij}$$

where α is a coefficient that represents consumer *i*'s marginal utility (or disutility) of price. Moreover, δ_j can be interpreted as the mean utility (or quality) of product *j* derived from product attributes other than prices. The utility derived from the consumption of the outside good can be normalized to zero u_{i0}=0. Assuming that ε_{ij} has a Type I Extreme Value distribution, the unconditional probability of individual good *j* being chosen takes the familiar logit form

(2) $$\sigma_j(\alpha, p, \delta) = \frac{\exp(-\alpha p_j + \delta_j)}{1 + \sum_{m=1}^{N} \exp(-\alpha p_m + \delta_m)}$$

In turn, the choice probabilities conditioned on one of the inside goods being chosen (σ_{jI}), i.e. the inside good share of good j (s_{jI}) is

(3) $$s_{jI} = \sigma_{jI}(\alpha, p, \delta) = \frac{\sigma_j(\alpha, p, \delta)}{\sigma_I(\alpha, p, \delta)} = \frac{\sigma_j(\alpha, p, \delta)}{1 - \sigma_0(\alpha, p, \delta)}$$

where $\sigma_I(\alpha, p, \delta)$ is the probability of choosing one of the inside goods and $\sigma_0(\alpha, p, \delta)$ is the probability of choosing the outside good.

As in any calibration model, the ALM adds more external information to the model. This information can come from different sources, such as companies documents, other studies and opinion of industry experts. In this model, only two elasticities suffice to pin down the demand parameters. Usually, the own-price elasticity of one of the inside goods and the aggregate (industry) elasticity compose the external information set, but other combinations could be used, such as own-price elasticities for two different goods or two different cross price elasticities.

The Logit implies an analytical formula for the aggregate η_I and own-price elasticities η_{ll}. Indeed,

(4) $\eta_{ll}(\alpha, p, \delta) = -\alpha p_l [1-\sigma_l]$, and

(5) $\eta_I(\alpha, p, \delta) = -\alpha \overline{p} \sigma_0$

where, $\overline{p} = \sum_{m=1}^{N} \sigma_m p_m$ is a weighted average price. The equations above can be simplified by to the following system[2]:

(6) $$\ln[s_{jI}(1-\sigma_0)] - \ln[\sigma_0] = -\alpha p_j + \delta_j \qquad ; \quad j=1,\ldots N$$

(7) $$|\eta_l| = \frac{[\alpha \overline{p}(1-s_{lI}) + |\eta_I| s_{lI}] p_l}{\overline{p}}$$

where $\sigma_0 = \dfrac{|\eta_I|}{\alpha \overline{p}}$.

The calibration of demand in the ALM consists of simply solving the system of equations above for the scalar α and the N-dimensional vector $\delta = (\delta_1, \delta_2, \delta_3 \ldots \delta_N)$ given prices $p = (p_1, p_2, p_3 \ldots p_N)$, conditional shares ($s_{jI}$), and the aggregate (or industry) elasticity η_I and the elasticity of one of the inside goods η_l. This system is very simple, as we can directly solve for α from Equation (7), giving $\alpha = \dfrac{|\eta_j| \overline{p} - |\eta_I| s_{jI} p_j}{p_j \overline{p}(1-s_{jI})}$. And then, once α is determined, one can find the

[2] The system is linear in the unknowns (δ, α)

corresponding δ_j's from (6), which are given by

$$\delta_j = \ln[s_{jI}(1-\sigma_0)] - \ln[\sigma_0] + \alpha p_j \ .$$

III – Calibrating the Mixed Logit Demand Model

In this section, I shall describe the key contribution of this paper, which consists of developing a methodology to calibrate the parameters of mixed discrete-choice demand model. Consumers rank products according to their characteristics and prices. There are *N+1* choices in the market, *N* inside goods and one reference good (or outside good). Consumer *i* chooses brand *j*, given price p_j, unobserved characteristics and quality (summarized by the scalar δ_j), and unobserved idiosyncratic preferences ε_{ij}, according to the following utility function:

(8) $$u_{ij} = g(\alpha, v_i) p_j + \delta_j + \varepsilon_{ij}$$

The price coefficient $g(\alpha, v_i)$ is a random coefficient that represents consumer *i*'s marginal utility (or disutility) of price, which is a function of the parameter α and a consumer-specific term v_i. Introducing this heterogeneity in price coefficient is a natural extension of the logit model in which price changes affect all consumers in the same form.

Moreover, δ_j can be interpreted as the mean utility (or quality) of product *j* derived from product attributes other than prices. The utility derived from the consumption of the outside good can be normalized to zero u_{i0}=0. Assuming that ε_{ij} has a Type I Extreme Value distribution, the probability of individual *i* choosing good *j* (σ_{ij}) takes the familiar logit form

(9) $$\sigma_{ij}(\alpha, p, \delta, v_i) = \frac{\exp(g(\alpha, v_i) p_j + \delta_j)}{1 + \sum_{m=1}^{N} \exp(g(\alpha, v_i) p_m + \delta_m)}$$

In addition, taking the expected value of this probability with respect to the distribution of the v_i's yields the probability of good *j* being chosen (σ_j), which is given by

(10) $$\sigma_j(\alpha, p, \delta) = E_v[\sigma_{ij}(\alpha, p, \delta, v_i)]$$

In turn, the choice probabilities conditioned on one of the inside goods being chosen (σ_{jI}), i.e. the inside good share of good j (s_{jI}) is

(11) $$s_{jI} = \sigma_{jI}(\alpha, p, \delta) = \frac{\sigma_j(\alpha, p, \delta)}{\sigma_I(\alpha, p, \delta)}$$

where σ_I is the probability of one of the inside goods being chosen. Thus the model implies that the inside good share of good j depends on the parameter α, and *N*-dimensional column vectors *p* and δ, that collect all p_j*'s* and δ_j*'s* respectively.

The inputs for the calibration of the model described above are: market shares, prices, the distribution of the consumer-specific term v_i and two price elasticities (aggregate elasticity and the own-price elasticity of one the inside goods). For the demand model presented above the implied own-price elasticity for given good *l* is given by

(12) $$\eta_{ll}(\alpha, p, \delta) = \frac{p_l}{s_{lI}.\sigma_I} E_v[g(\alpha, v_i).\sigma_{il}(\alpha, p, \delta, v_i)(1 - \sigma_{il}(\alpha, p, \delta, v_i)]$$

In turn, the elasticity of the aggregate demand of all inside goods, also known as the aggregate elasticity η_I, is given by

(13) $$\eta_I(\alpha, p, \delta) = \frac{E_v[g(\alpha, v_i).P_i(\alpha, p, \delta, v_i).\sigma_{i0}(\alpha, p, \delta, v_i)]}{\sigma_I}$$

where $P_i = \sum_{m=1}^{N} (\sigma_{im}.p_m)$ and $\sigma_{i0}(\alpha, p, \delta, v_i) = \dfrac{1}{1 + \sum_{m=1}^{N} \exp(g(\alpha, v_i) p_m + \delta_m)}$ is the probability consumer *i* choosing the outside product.

Note that the system of equations formed by Equations (11), (12) e (13) can be rewritten as

$$(14) \qquad s_{jI} = \frac{|\eta_I|.\sigma_j(\alpha, p, \delta)}{E_v[|g(\alpha, v_i)|.P_i(\alpha, p, \delta, v_i).\sigma_{i0}(\alpha, p, \delta, v_i)]}; \; j=1,....N$$

$$(15) \qquad |\eta_{ll}| = \frac{|\eta_I| p_l}{s_{lI}} \frac{E_v[|g(\alpha, v_i)|.\sigma_{il}(\alpha, p, \delta, v_i)(1 - \sigma_{il}(\alpha, p, \delta, v_i))]}{E_v[|g(\alpha, v_i)|.P_i(\alpha, p, \delta, v_i).\sigma_{i0}(\alpha, p, \delta, v_i)]}$$

This system is key to the empirical strategy proposed in this paper. Indeed, notice that as assumed the researcher observes (inside) market shares s_{jI}, prices, the distribution of the consumer-specific term v_i, the aggregate elasticity η_I and the elasticity of one good η_{ll}. Therefore, since the system is formed by *N+1* equations we can uncover the *N+1* unknowns (*N*-dimensional vector δ plus the scalar α)[3].

[3] If α were a vector of dimension greater than one, and not a scalar as assumed here the system would certainly be under identified. For this reason we have to posit a mixed logit model with only one random coefficient with only one parameter. Whether this is a plausible model is largely an empirical question. Notice also that α is deterministic and therefore it does not have a standard error. The model could be easily extended to accommodate more flexible distribution by adding another elasticity and consequently another equation to the calibrating system..

IV – Supply and Merger Simulation

Uncovering demand parameters is not enough to perform merger simulation, one also has to specify how firms compete. I follow the commonly adopted assumption that firms choose prices simultaneously in a one-shot game, i.e. the market outcome is the result of a Bertrand game.

First, assume that each firm *f* produces a subset F_f of the goods sold in this market. If firms behave according to Bertrand, it can be shown that the price of product *j* produced by firm *f* at (constant) marginal cost c_j must satisfy the following equation

(16) $$\sigma_j + \sum_{r \in F_f} (p_r - c_r) \frac{\partial \sigma_r}{\partial p_j} = 0 \; ; j=1,2,\ldots,N$$

Or, equivalently,

(17) $$\sigma - (\Omega \Delta [p - c]) = 0$$

where σ, *p* and *c* are Nx1 vectors collecting σ_j's, prices e marginal costs respectively. In addition, Δ and Ω are *N*x*N* matrix whose typical element (*j,r*) is defined as follows

$$\Delta_{jr} = -\frac{\partial \sigma_r}{\partial p_j} \quad \text{and}$$

$$\Omega_{jr} \begin{cases} 1 & \text{if r e } j \text{ are produced by the same firm} \\ 0 & \text{Otherwise.} \end{cases}$$

The outside good pricing decision is assumed to be exogenous and therefore does not interact strategically with the pricing decision of the inside goods. Note that (17) is flexible enough to accommodate different market structures. The first structure is the single firm product, in which the firm can only control the price of its unique brand.

The second is the multi-product firm, in which the firm internalizes the price decision of different brands. A third example is a monopoly, where one firm produces all the varieties offered in the market.

One implicit assumption in merger simulation is that observed pre-merger (and also post-merger) prices are generated by the outcome of Bertrand competition between firms. Therefore, Equation (17), evaluated at pre-merger prices, is given by

(18) $$\sigma(p^{pre}) - (\Omega^{pre}\Delta(p^{pre})[p^{pre} - c]) = 0\,.$$

Hence, marginal costs can be uncovered from the following equality

(19) $$c = p^{pre} - [(\Omega^{pre}\Delta(p^{pre}))^{-1}\sigma(p^{pre})]$$

Notice that p^{pre} represents the observed pre-merger prices and that Ω^{pre} is constructed using pre-merger ownership structure. Once we have demand and supply parameters (α, δ and c) it is possible to calculate the equilibrium prices resulting from the new ownership structure arising from the merger. Indeed, the predicted post merger prices (p^{post}) is the solution of the following system of equations

(20) $$\sigma(p^{post}) - (\Omega^{post}\Delta(p^{post})[p^{post} - c]) = 0$$

where Ω^{post} is constructed using the post merger ownership structure.

V – EXAMPLE AND OTHER COMPARABLE METHODS

In this section I provide examples of the application of the AMLM and use them to compare this model to other calibration-based methodologies using both artificial and real data. The previously mentioned ALM is one of them and since it is a particular case of the discrete-choice mixed logit model (AMLM), the comparison is straightforward. Another popular model is the Proportionally Calibrated Almost Ideal Demand System (PCAIDS), developed by Epstein and Rubinfeld (2002), which belongs to another class of models that are used to uncover consumer preferences: the continuous demand models.

I begin with an artificial market with 4 brands, namely A, B, C and D, each sold by a different firm, with shares 40%, 35%, 15% and 10%, respectively. The price of good A is 9 monetary units, while B, C and D are sold for 6, 5 and 3 monetary units respectively. In addition, I follow Petrin (2002) and parameterize the consumer marginal utility for price according to the functional form given by $g(\alpha, v_i) = -\alpha v_i$, in which the term v_i follows a chi-square distribution with 3 degrees of freedom. In other to run the AMLM one still needs two elasticities: the aggregate (or industry) elasticity and the own-price elasticity of the first brand which are -1 and -2 respectively.

The first step of the AMLM is the calibration of the mixed logit demand system which consists of solving the system of five equations for the five dimensional vectors $(\alpha, \delta_1, \delta_2, \delta_3, \delta_4)$, according to system formed by (14) and (15). I find that the parameter α that composes the random price coefficient is 0.489 and the vector δ, that collects the δ_j's, is given by $(5.343, 3.921, 2.504, 0.663)$. Notice that brand A has the highest quality (5.343), D the lowest(0.663) and while B and C take intermediate values. This simple example shows that the model results are consistent with what is qualitatively suggested by the data in a differentiated good industry. Indeed, one should expect that consumers perceive brand A as superior product since, despite being the most expensive, it has the highest market share (40%). By the same reasoning one should expect brand D to be much less desired by consumers since, despite having the lowest prices, it captures the smallest fraction of the market (10%). In turn, for brand B and C which show intermediate prices and shares one should expect intermediate values for the quality index.

Another interesting outcome of the calibration of demand is the elasticity matrix. With the calibrated values for $(\alpha, \delta_1, \delta_2, \delta_3, \delta_4)$ one can use the model and data again to calculate the own price elasticity for any good j, which is given by

$$\eta_{jj}(\alpha, p, \delta) = \frac{p_j}{s_{jI}.\sigma_I} E_v[g(\alpha, v_i).\sigma_{ij}(\alpha, p, \delta, v_i)(1 - \sigma_{ij}(\alpha, p, \delta, v_i)]$$

and the cross-price elasticity of the demand of any good j with respect to the price of any other good r which is given by

$$\eta_{jr} = \frac{p_r}{s_{jI}.\sigma_I} E_v[g(\alpha, v_i).\sigma_{ir}(\alpha, p, \delta, v_i)(\sigma_{ij}(\alpha, p, \delta, v_i)]$$

Applying these formulas to this particular example I find a 4x4 matrix with the following entries

Table I

Elasticity Matrix from AMLM

	Elasticity with respect to			
Brand	P_A	P_B	P_C	P_D
A	-2.00	0.60	0.19	0.05
B	1.03	-2.39	0.28	0.10
C	0.94	0.80	-2.75	0.12
D	0.65	0.72	0.30	-2.36

Source: Author.

The matrix above (Table I) highlights the main advantage of using a flexible demand system, such as the mixed logit model. Indeed, this model leaves room for the data to determine the substitution pattern between brands, unlike the logit model which imposes by construction that a price increase of a good j will have the same proportional effect on the demand of any other substitute. This can also be verified empirically after we apply the calibrated logit model to the same data (Table II), which is the ALM approach to uncover demand.

Table II

Elasticity Matrix from ALM

	Elasticity with respect to			
Brand	P_A	P_B	P_C	P_D
A	-2.00	0.25	0.09	0.03
B	0.44	-1.37	0.09	0.03
C	0.44	0.25	-1.26	0.03
D	0.44	0.25	0.09	-0.77

Source: Author.

Notice that if the P_A increases 10% the demands for all other goods will respond in a uniform way, each increasing 4.44% (see column 1 of table II). The same uniform result can be found in the other columns.

The same restrictive substitution pattern is found in another well-known calibration-based methodology, the PCAIDS. This model is very attractive as it is simple and requires a very small set of information: two elasticities and sales based shares, not even prices are necessary. However, this simplicity comes at a cost. In order calibrate the demand system with such a small information set, the authors have to impose a proportionality assumption which removes the main attribute of the AIDS: the flexibility of the elasticity matrix. In fact, as in the ALM, the PCAIDS generates a rigid elasticity matrix. Another version of this model, the PCAIDS with nests (Epstein and Rubinfeld, 2004), gives better result but still shows equal cross-price elasticities within nests.

Now, I turn to the main output of merger simulation, the predictions of price increases, by proceeding in the way described in section IV and performing all possible mergers between two companies. I begin with a detailed analysis of the merger between the firms that produce goods A and B. The AMLM gives the following results:

Table III

Price Increases from the A-B Merger

Brands	Post-Merger Prices	Pre-Merger Prices	Variation(%)
A	12.36	9	37.44
B	9.31	6	55.21
C	5.44	5	8.91
D	3.25	3	8.57

Source: Author.

In average, prices after the merger are 23.16% higher. Notice also from Table III that all brands are more expensive after the merger -a typical result of merger simulation with no efficiency gains as assumed in this exercise- and that the brands that show the highest upward price variation (A with 34.4% increase and B with 55.21%) are those involved in the merger. The same pattern is found for all other possible mergers between two firms, as shown in Table IV. For instance, the merger between A and C will cause a 6.82% increase in averages prices with these brands showing the highest price increases. Notice, though, that the potential anti-competitive effects of this merger are significantly less harmful than those obtained by the A-B merger. Quantitatively similar results can be found for the B-C merger which will result in average prices 5.42% higher after the creation of the new firm. Even smaller effects are found for the other mergers: 3.16% average price increase for both the A-D and the B-D merger and 1.38% for the C-D merger.

Table IV

Simulation Results

Merging Firms	Post-Merger Average Prices	Pre-Merger Average Prices	Increase in avg. Prices (%)	Largest price variation across brands(%)	Brands with highest price increase
A/B	8.313	6.75	23.16	55.21%	*A,B*
A/C	7.211	6.75	6.82	31.191	*A,C*
A/D	6.970	6.75	3.27	20.927	*A,D*
B/C	7.116	6.75	5.42	17.092	*B,C*
B/D	6.970	6.75	3.27	17.505	*B,D*
C/D	6.844	6.75	1.39	5.744	*D,C*

Source: Author

As in other calibration-based models one way to assess the confidence in the results is to perform sensitivity analysis. On way is to run the same model, keeping all else equal, for different elasticities. Here, I take the set {-0.5,-1, -1.5} for the industry elasticity and {-2,-2.5,-3} for the own-price elasticity and pick the average price increase as the representative outcome of the merger simulation between brands A and B.

Table V

Sensitivity Analysis - AB merger

	Own-Price elasticity		
Ind. Elasticity	-2	-2.5	-3
-0.5	20.58	11.92	5.35
-1	23.16	15.21	8.38
-1.5	14.52	13.90	10.25

The sensitivity analysis (Table V) shows price predictions ranging from 5.36% to 23.16%, which is an apparently large interval, but the results still prove to be useful since, even in the best case scenario for the merging parties, prices exhibit a sizeable increase (5.36%), raising concerns about the merger.

VI- ANOTHER FORMULATION AND APPLICATION

The model presented in section III is general enough to accommodate other formulations. For instance, one can use information in the income distribution in order to model price coefficient or add other elasticities to the equations system in order to pin down more flexible distributions such as the normal or log-normal. Below, I apply the first approach to the ready-to-eat cereal industry in the U.S. However, it should be noticed that the objective of this application is to illustrate the methodology proposed in this paper rather than providing a detailed study of the ready-to-eat cereal industry. Nonetheless, an application of this methodology that takes into consideration all or most of the idiosyncrasies of this industry would be an interesting extension of this work.

The data set is a cross-section of fifty top selling brands in the U.S in 1992. The summary statistics are presented below[4]. The data set reports information on (inside) shares and prices. To construct the shares it is assumed that the set of inside

[4] This data set was constructed by Matt Shum, who gently allowed me to use it.

goods is composed of all the top fifty best selling brands. Thus, this implies that the outside good is representative of all other brands and other substitutes not included in the top fifty best selling list.

Table VI

Summary statistics for Ready-To-Eat Cereal Industry in the U.S – 1992

	Mean	*Std Dev*	*Variance*	*Min*	*Max*
Share	0.0152	0.0102	0.0001	0.0067	0.0567
Price ($/lb)	2.9830	0.4916	0.2416	1.7700	3.9600

Source: Descriptive statistics for variables available in the data set mentioned above.

I follow Berry, Levinsohn, and Pakes (1999) and parameterize the consumer marginal utility for price according to the functional form given by $g(\alpha, v_i) = -\frac{\alpha}{v_i}$, where the consumer-specific term v_i represents household income, whose distribution is obtained from the 1992 Current Population Survey (CPS). In order to simplify the computation of the mixed logit model, I made a few simplifications regarding this distribution. I have divided the income space into intervals of the same size (2500 USD) and computed the frequencies of each interval. Then, I discretize the distribution assuming that the average income in each interval is representative of all individuals included in this interval. In the end, we have 21 income levels and thus 21 consumer types. The discretization avoids the need for numerical integration (e.g. quadrature methods) or simulation methods (as employed by BLP) to compute the markets shares in Equation (10). This is done to reduce the computational burden. Notice that if the researcher is not willing to make these simplifications, the methodology model outlined in section III can certainly accommodate different distributional assumptions for income such that quadrature or simulation methods can be used.

In the first stage of the, I posit that $\eta_I = -0.35$ and the elasticity[5] of one inside good[6] (KG Corn Flakes) $\eta_{II} = -3$. Then we are able to uncover *N+1*-dimensional vector (δ, α). I find that α is 41979.102 , from which we can derive the distribution of the price coefficients (in absolute values) across consumers. This distribution is given by the distribution of the ratio $-\frac{\alpha}{v_i}$. We can also construct descriptive statistics for the δ_j's. These results are summarized in Table VII below.

Table VII

Summary statistics of stage 1 results

	Mean	Median	Max	Min
Price coefficient (in modulus)	2.001	0.799	16.791	0.399
Mean utilities (δ_j's)	4.157	4.168	5.614	1.758

The distribution of the price coefficient has mean 1.982 and median 0.791, implying that the distribution is not symmetric around its mean. The average (across brands) mean utility is 4.157. The distribution is approximately symmetric around the mean since the mean and the median are approximately equal.

[5] These numbers are similar to those found in Nevo (2001)
[6] These values compose external information set. I could have used other values for the aggregate and own-price elasticities to perform robustness checks. This is left for future developments of this work.

An advantage of structural estimation is that, once the parameters of interest are determined, one can simulate the effect of different market environments using the structural model. The merger simulation goes as follows. Determine the demand parameters using the empirical strategy developed in this paper. Next, use the observed equilibrium prices before the merger to uncover marginal costs from Equation (19) and next find the equilibrium prices resulting from the new ownership structure using equation (20).

Tables VIII shows the results from all possible mergers between two firms. The first column indicates the firms involved in the simulated merger. The other columns present descriptive statistics of the prices changes resulting from the simulation. For instance, the model predicts that a merger between General Mills and Kelloggs would increase (share-weighted) industry average price from 2.887 to 3.315, which represents a 12.9% increase[7]. At the brand-level, the variety that presents the highest price variation (GM Triples) belongs to one of the merging firms, as expected, and exhibits a significant price increase (39.35%). The merger between *Kelloggs* and *Nabisco* would imply a moderate increase in the industry average price (1.85%). However, internalizing competition allows the new merged firm to charge a price 27% higher for the *Big Biscuit Shd* brand, which belongs originally to Nabisco. In turn, we should not expect significant anti-competitive effects from the Nabisco-Post Merger, since the industry average price increase is very small (0.53%) and no brand has its price inflated by more than 3.53%.

Table VIII

Simulation Results

Merging Firms	Post-Merger Average Prices	Pre-Merger Average Prices	Increase in avg. Prices (%)	Largest price variation across brands(%)	Brand Name
KG/GM	3.315	2.887	12.909	39.348	*GM Triples*
KG/NB	2.942	2.887	1.852	27.030	*NB Big Biscuit Shd*
KG/PT	2.997	2.887	3.653	33.305	*PT Grape Nuts*
KG/QK	2.958	2.887	2.389	36.075	*QK Popeye*
KG/RL	2.912	2.887	0.841	22.432	*RL Muesli*
GM/NB	2.938	2.887	1.725	21.926	*NB Big Biscuit Shd*
GM/PT	2.985	2.887	3.276	23.460	*PT Grape Nuts*
GM/QK	2.948	2.887	2.068	23.038	*QK 100% Natural*
GM/RL	2.911	2.887	0.826	19.453	*RL Muesli*
NB/PT	2.903	2.887	0.530	3.529	*NB Big Biscuit Shd*
NB/QK	2.897	2.887	0.345	2.163	*QK Popeye*
NB/RL	2.892	2.887	0.148	1.746	*RL Muesli*
PT/QK	2.906	2.887	0.657	4.355	*QK Popeye*
PT/RL	2.895	2.887	0.252	3.017	*RL Muesli*
RL/QK	2.892	2.887	0.157	1.606	*RL Muesli*

VII. FINAL REMARKS

The Antitrust Logit Model (ALM) developed by Werden and Froeb (1994) has been widely applied to predict the competitive effects of horizontal mergers in product-differentiated industries. The ALM does not place great demands on the data set and is fast to compute and– we only need information on prices, market shares and two exogenously given parameters (usually price elasticities). These attributes make this calibration methodology particularly appealing to antitrust agencies and merging firms, given time and data constraints they usually face. However, it is well known that the Logit demand model places very restrictive limitations on own and cross price elasticities, which constitute critical economic parameters in the evaluation of merger effects.

This paper presents the Antitrust Mixed Logit Model (AMLM), a new methodology whose main contribution is to show how to calibrate the parameter of a mixed-logit demand. After the calibration step the model follows most merger simulation models found in the industrial organization literature, i.e. it assumes Bertrand competition in order to obtain marginal costs and simulate mergers. The major advantage over the logit version is the flexibility of the mixed logit demand, which generates more realistic patterns of substitution between goods and consequently more precise predictions about merger effects. Moreover, unlike the econometric approaches to uncover the parameters of a mixed-logit demand (e.g. Berry, Levinsohn and Pakes, 1995), the AMLM shares with the ALM the attributes that are particularly appealing in merger investigations: low data requirement and high computational speed. Indeed, the data requirement to implement the AMLM is almost the same as the ALM and the computational burden is low.

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