Information (in) Chains

Information Transmission through Production Chains

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Abstract: We study the transmission of information in a model with a vertical input-output structure and dispersed information. Firms observe input prices with noise that endogenize the precision of information that is public within a stage but not across stages. In contrast to the case with an exogenous and overall public signal, our main result is that agents may find it optimal to rely less on public information along the chain. A direct implication is that, while information precision remains unchanged with exogenous public signals (information chains), it may decrease along the chain when semi-public signals are endogenous (information in chains).

Keywords: Complementarities, Dispersed information, Endogenous signals, Price setting, Production chains
JEL Classification: D82, D83, E31

Resumo: Neste artigo, estuda-se a transmissão de informação em um modelo com estágios de produção e informação dispersa. Firmas observam sinais sobre os preços dos insumos que endogenizam a precisão da informação que é pública dentro de um estágio, mas não entre os estágios. Em contraste com o caso com sinal público exógeno, as firmas decidem otimamente atribuir menos peso a informação pública ao longo da cadeia. Uma implicação é que a precisão da informação, que ficaria inalterada com sinais públicos exógenos, decresce ao longo da cadeia com sinais semi-públicos endógenos.

Palavras-chave: Complementaridades, Informação dispersa, Sinais endógenos, Fixação de preços, Cadeias de produção

Contents

1 Introduction

The production of a final good in modern economies usually involves multiple stages of processing. Although the notion that pricing decisions are tied to the interdependence of firms at different stages of production has been presented at least since Means (1935), how information flows along these stages are far less understood. In order to fill this gap, we study the transmission of information in a model with two features that are typical of modern economies: (a) a vertical input-output structure and (b) dispersed information. The goal of this paper is to understand how dispersed information affects not only the weight on public information but also the precision of information along the chain.

The backbone of our model is a variant of the beauty contest game of Morris and Shin (2002). We introduce an input-output structure considering a payoff function that yields the same (log-linear) optimal flexible pricing decision as the model of monopolistically competitive goods supply with production chains in Huang and Liu (2001). In this sense, we are considering *vertical* complementarities instead of *horizontal* complementarities.[1] As in Morris and Shin (2002), information is dispersed and firms observe noisy private and public signals about the underlying fundamental.

If we consider that all firms in all stages observe the same exogenous public signal, our model of vertical complementarities converges to Morris and Shin (2002) and Angeletos and Pavan (2007) models of horizontal complementarities. In this case, when actions are strategic complements, agents wish to coordinate their actions, and because public information is a relatively better predictor of others' actions, agents find it optimal to rely more on public information relative to a situation in which actions are strategically independent.

The result above is obtained with an exogenous information structure and presumes that the precision of public information remains invariant along the chain. We argue that this is unlikely to be the case when public information is endogenous through prices or other macroeconomic indicators. We extend our model to consider that firms observe input prices with noise, which endogenize the precision of information that is public within a stage but not across stages (semi-public signal).

In contrast to the case with exogenous public information, our main result is that agents may find optimal to rely less on semi-public information along the chain. The rationale of this result is that equilibrium input prices play the same role as financial prices in Grossman and Stiglitz (1976) and Angeletos and Werning (2006), aggregating disperse private information,

[1] We bring this terminology from Matsuyama (1995).

while avoiding perfect revelation due to unobservable semi-public information of the previous stage. Since the semi-public signal of a stage n includes unobservable information of all previous stages, the precision of the semi-public signal decreases along the chain. As a result, the weight on public information also decreases if the share of goods produced at stage $(n-1)$ in the production of stage n is not high enough. Finally, we show that, while information precision remains unchanged with exogenous public signals (*information chains*), it may decrease along the chain with endogenous semi–public signals (*information in chains*). This result is also a direct consequence of the semi-public signal precision's decrease.

Organization. The plan for the rest of the paper is as follows. We relate our approach with the pertinent literatures in the next section and introduce our basic model in Section 3. The core of the paper is Section 4 where, after presenting our exogenous public information model as a benchmark, we incorporate an endogenous semi-public signal and study how the information revealed by input prices affects the precision of as well as the weight on public information. Section 5 concludes. The Appendix contains proofs omitted in the main text.

2 Related Literature

To the best of our knowledge, this paper is the first to link the literatures of production chains and dispersed information, studying how pricing decisions are tied to the interdependence of firms at different stages of production when information is dispersed. We considered each literature bellow.

Production chains. The literature of production chains as a propagation mechanism has a long tradition. Means (1935), as pointed out by Basu (1995), showed that, in the Great Depression, simple goods, such as agricultural products, declined heavily in price, while their quantity was almost unchanged. Complex manufactured goods, on the other hand, showed the opposite pattern, with small price changes and consequently huge declines in the quantity of sales. Blanchard (1984) shows that a simple reduced-form model incorporating a vertical production chain with prices staggered across different stages of processing can generate patterns of price changes similar to those noted by Means (1935). Gordon (1990) considers the input–output table as an essential component in the description of price stickiness.

Recent studies confirm Means's observation on the patterns of price changes at different stages of production. For example, Clark (1999) studies a broad range of data sets and finds that prices at early stages of production respond more to a monetary policy shock than do prices at subsequent stages of production. Recently, Huang and Liu (2001) present a dynamic stochastic general equilibrium model embedded with a vertical input-output structure, with staggered price contracts at each stage of production. Working through the input-output relations and the timing of firms' pricing decisions, the model generates persistent fluctuations in aggregate output and the observed patterns of price dynamics following a monetary shock. None of these works, however, consider the impact of dispersed information on prices.

Dispersed information. In recent years there has been a growing interest in models that feature heterogeneous information about aggregate economic conditions and a moderate degree of complementarity in actions. Examples of this kind of models were used to capture applications such as the effects of monetary or fiscal policy, as in Woodford (2002), Lorenzoni (2009) and Angeletos and Pavan (2009), and the welfare effects of public information dissemination, as in Morris and Shin (2002), Hellwig (2005) and Angeletos and Pavan (2007). These models, however, consider a horizontal roundabout input–output structure within a single stage of production.

Endogenous public signals. The role of endogenous public information has also been considered with interest. An especially important source of endogenous public information is prices, and papers by Tarashev (2003), Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006) and Morris and Shin (2006) have pursued various methods of combining endogenous public information with coordination games. These papers, however, focus on financial prices rather than input prices. Closely related to our work is Gorodnichenko (2008), who combine menu costs with the aggregate price level in the previous period serving as an endogenous public signal to generate rigidity in price setting even when there is no real rigidity. Although the model of Gorodnichenko (2008) also features information aggregation, it focuses on how firms make state-dependent decisions on both pricing and acquisition of information across periods. We focus on how the weight on public information and information precision changes along the chain within a period.

Herding. Our model is also related to the literature on herding and information cascades that consider how firms, having observed the actions of those ahead of him, follow the behavior of the preceding firm without regard to his own information.[2] Instead, our model deals with firms deciding prices simultaneously based on an aggregate signal of its suppliers.

3 The Model

Consider a monopolistic competition model with production chains as in Huang and Liu (2001). In the model economy, the production of a final consumption good requires N stages of processing, from crude material to intermediate goods, then to more advanced goods, and so on. At each stage $n \in \{1, ..., N\}$, there is a specific continuum of monopolistically competitive firms indexed in the interval $\Omega_n = [0, 1]$ producing differentiated goods. The production at stage 1 requires only homogeneous labor services provided by a representative household, and the production at stage $n \in \{2, ..., N\}$ uses both labor and goods produced at stage $n-1$. To keep the model simple, we consider a quadratic profit function that yields the same optimal price decision as the firms of Huang and Liu's (2001) model. The resulting model is as a variant of the imperfect common knowledge models of Woodford (2002) and Morris and Shin (2002). There is a relevant fundamental, θ, representing the economic conditions of the model. Information is complete if every firm in the model observes θ. All variables are in log deviations from steady-state and all distributions are Normal. Prices under imperfect information equal the expected price under complete information.[3]

3.1 Actions and Payoffs

A firm $j \in \Omega_n = [0, 1]$ in stage $n \in \{1, ..., N\}$ sets price $p_n(j)$ to maximize the profit function

$$\Pi_n (p_n (j), p_n^*) \equiv \Pi_n^* - (p_n (j) - p_n^*)^2,$$

where $\Pi_n^* \in \mathbb{R}$ is maximum profit. The target price p_n^* is given by

$$p_n^* \equiv r P_{n-1} + (1 - r) \theta,$$

where $P_n \equiv \int_{\Omega_n} p_n (j) \, dj$ is the stage-n aggregate price, $r \in (0, 1)$ is the share of goods produced at stage $n-1$ in the production of firm j of stage n, θ is the relevant fundamental and represents exogenous nominal aggregate demand for final (N^{th} stage) goods. For

[2] The seminal papers are Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992).

[3] See Appendix for details.

simplicity, we set $P_0 = \theta$. The model differs from Woodford's (2002) as it considers the influence of stage-$(n-1)$ price level, P_{n-1}, instead of the single stage economy price level P on profits. Following the terminology in Matsuyama (1995), we are considering *vertical* complementarities instead of *horizontal* complementarities.

3.2 Timing and Information

After presenting the agents and their actions, we need to define the information structure of the model. In accordance with the pertinent literature, as in Morris and Shin (2002), we consider that, instead of observing the fundamentals, each firm $j \in \Omega_n = [0,1]$ of stage $n \in \{1,..,N\}$ receives two noisy signals about θ: (i) a private signal $x_n(j)$ and (ii) a public signal y or a semi-public signal y_n. Finally, firms simultaneously set prices based on the information they received.

We consider that each firm receives an exogenous unbiased private noisy signal $x_n(j)$ about the fundamental

$$x_n(j) = \theta + \xi_n(j),$$

where the idiosyncratic noise terms $\xi_n(j)$ are normally distributed with zero mean and variance α^{-1}, independent of θ and from one another.

We also consider that each firm receives a public or a semi-public signal about θ. As our benchmark case, we first consider that each firm receives an exogenous public noisy signal y

$$y = \theta + \epsilon,$$

where the common noise term ϵ is normally distributed, independent of θ and all $\xi_n(j)' s$, with mean zero and variance β^{-1}. After, in order to investigate the role of endogenous information, we consider instead that stage-n firms observe stage-$(n-1)$ aggregate price with noise, our endogenous *semi-public signal* y_n

$$y_n = P_{n-1} + \epsilon_n,$$

where the stage noise term ϵ_n is also normally distributed, independent of θ and all $\xi_n(j)' s$, with mean zero and variance β^{-1}.

The structure of the signals, fundamental θ plus an error term, as well as the distributions of the errors are common knowledge. Because input prices depend on the underlying fundamental θ, the equilibrium stage price levels will convey information that is valuable in the coordination game.

3.3 Equilibrium

All firms j of every stage n choose prices simultaneously in order to maximize profits. We first consider equilibrium under complete information. After, we consider the dispersed information case.

3.3.1 Complete Information

The optimal pricing decision when θ is common knowledge yields the same price p_n^* for all firms of stage $n \in \{1, ..., N\}$

$$p_n^* = P_n = rP_{n-1} + (1-r)\,\theta,$$

or equivalently

$$P_n = r^n P_0 + (1 - r^n)\,\theta.$$

Because we assume $P_0 = \theta$, there is a unique optimal price for all stages that equal the fundamental

$$P^* = \theta.$$

As a result, if θ is common knowledge, the equilibrium entails $p_n(j) = \theta$ for all $j \in \Omega_n = [0, 1]$ and $n \in \{1, .., N\}$, so that information is completely transfer from one stage to another.

3.3.2 Dispersed Information

Consider now that information is dispersed. Instead of observing the fundamentals, each firm $j \in \Omega_n = [0, 1]$ of stage $n \in \{1, .., N\}$ receives two noisy signals about θ. As a result, the optimal pricing decision of firm j of stage n is

$$p_n(j) = E\left[rP_{n-1} + (1-r)\,\theta \,|\, x_n(j), y_n\right], \qquad (1)$$

where $y_n = y$ for all n if all firms in all stages observe a public instead of a semi-pubic signal.

4 Results

Now, we consider the optimal price as a function of the noisy signals received by the firms. We first consider the case of an exogenous public signal. After, we analyze the impact of endogenous semi-public information.

4.1 The Model with Exogenous Public Signal

The private posterior for a firm $j \in \Omega_n = [0,1]$ of stage $n \in \{1,..,N\}$ that receives signals $x_n(j)$ and y then becomes

$$\theta \,|\, x_n(j), y \sim N\left((1-\lambda)\, x_n(j) + \lambda y, (\alpha+\beta)^{-1}\right),$$

where the Bayesian weight on public information is

$$\lambda \equiv \frac{\beta}{\alpha+\beta} \in (0,1).$$

In this case, the optimal price is analogous to the equilibrium price of Morris and Shin (2002) or the equilibrium use of information of Angeletos and Pavan (2007)

$$p_n(j) = (1-\lambda_n)\, x_n(j) + \lambda_n y, \tag{2}$$

with the only difference that the weight on the public signal is a function of the firm's stage n

$$\lambda_n \equiv \lambda \left(\frac{1-\Gamma^n}{1-\Gamma}\right). \tag{3}$$

where

$$\Gamma \equiv r(1-\lambda) \in (0,1). \tag{4}$$

In order to clarify the characteristics of our vertical complementarity model with exogenous information, we compare it with the horizontal complementarity model with exogenous information of Angeletos and Pavan (2007). First, we show that λ_n increases monotonically with the stage n. After, we consider how the weight on public information λ_n evolves when the number of stages in the economy increases.

Result 1 (Weight on Public Information) *In our model with private and public signals, the weight on public information increases along the chain. For chains long enough, vertical* *complementarity converges to horizontal complementarity.*

$$\lambda_n > \lambda_{n-1}, \textit{ for all } n, \tag{5}$$
$$\lim_{n\to\infty} \lambda_n = \lambda + \left(\frac{\lambda r (1-\lambda)}{1 - r(1-\lambda)}\right). \tag{6}$$

As is evident from equation (3), the sensitivity of the equilibrium to private and public information depends not only on the relative precision of the two signals, captured by the Bayesian weight λ, but also on the depth of the chain until that stage, captured by $\left(\frac{1-\Gamma^n}{1-\Gamma}\right)$. When $n = 1$, the weight λ_n is simply the Bayesian weight. If $n > 1$, the public signal y contains not only information about the fundamental θ but also reveals information used for firms from previous stages. As a result, the equilibrium value of public information increases along the chain.

A correlate question is what happens to the weigh on public information λ_n when the number of stages increases arbitrarily. The right-end side of expression (6) is exactly the weight on public information in Angeletos and Pavan (2007). As a direct consequence of this result, a relevant estimation of strategic complementary in pricing decisions may be a result of a complex-multiple stages economic structure. If vertical complementarity instead of horizontal complementarity is the major factor regarding prices, however, is out of the scope of this paper.

Our model with exogenous information is also similar to horizontal complementarity models in that dispersed information generates *price dispersion* (variation in the cross section of the population) and *non-fundamental volatility* (variation in aggregate activity around the complete-information level).

Result 2 (Dispersion and Volatility - Public Signal) *In our model with private and public signals, price dispersion, σ_n^2, decreases while non-fundamental volatility, ϱ_n^2, increases through the chain.*

$$\begin{aligned}\sigma_n^2 &\equiv Var\left[p_n(j) - P_n\right] \\ &= (1-\lambda_n)^2 \alpha^{-1} \Rightarrow \sigma_n^2 < \sigma_{n-1}^2,\end{aligned} \tag{7}$$

$$\begin{aligned}\varrho_n^2 &\equiv Var\left[\theta - P_n\right] \\ &= (\lambda_n)^2 \beta^{-1} \Rightarrow \varrho_n^2 > \varrho_{n-1}^2.\end{aligned} \tag{8}$$

Once again, our model shows that both dispersion and volatility change along the chain. On one hand, dispersion decreases along the chain because firms relies more on public information. On the other hand, the increasing relevance of public information amplifies non-fundamental volatility.

We conclude the comparative exercise of our model decomposing the information structure into its *accuracy* (the precision of the agents' forecasts about the fundamental θ) and its *commonality* (the correlation of forecast errors across agents).

Result 3 (Information Chains) *In our model with private and public signals, information precision, $V_n^{-1}(j)$, and commonality, $\chi_n(i,j)$, remain unchanged through the chain.*

$$\begin{aligned} V_n^{-1}(j) &\equiv Var\left[\upsilon_n(j)\right]^{-1} \\ &= \alpha+\beta, \end{aligned} \tag{9}$$

$$\begin{aligned} \chi_n(i,j) &\equiv Corr\left[\upsilon_n(i),\upsilon_n(j)\right] \\ &= \lambda, \end{aligned} \tag{10}$$

where $\upsilon_n(j) \equiv \theta - E\left[\theta \,|\, x_n(j), y\right]$.

Although encompasses an input-output structure, our model yields the same information precision and commonality than horizontal complementarity models when public information is exogenous. The intuition behind this result is that firms rely more on public information along the chain because a growing number of firms at previous stages are using the public sign, but not because the signs are more informative about the fundamental θ.

As the results above suggest, our model of vertical complementarities converges to Morris and Shin (2002) and Angeletos and Pavan (2007) models of horizontal complementarities with exogenous public information. The inclusion of an input-output structure *per se* is not enough to change the main conclusions that firms find it optimal to rely more on public information when pricing decisions are strategic complements and that the precision of information remains unchanged. Now, we investigate the robustness of this result when the public signal is specific of the stage (semi-public) and endogenous.

4.2 Input Prices: Endogenous Semi-public Signals

The results above presume that the precision of public information remains invariant along the chain. We argue that this is unlikely to be the case when public information is endogenous through prices or other macroeconomic indicators. To investigate the role of prices, we consider that stage-n firms observe stage-$(n-1)$ prices with noise, our endogenous semi-public signal y_n, instead of an exogenous public signal y.

Because of the linearity of the best-response condition (1) and the Gaussian specification of the information structure, the equilibrium prices are convex combinations of the signals

$$\hat{p}_n\,(j) = (1-\hat{\lambda}_n)x_n\,(j) + \hat{\lambda}_n y_n,$$

where $\hat{\lambda}_n$ is a weight to be determined.[4]

Given this function, we infer that the aggregate price level must satisfy

$$\hat{P}_n = (1-\hat{\lambda}_n)\theta + \hat{\lambda}_n y_n.$$

Firms of stage n don't observe the price level $\hat{P}_{n-1}$ nor the semi-public signal y_{n-1} of the previous stage. However, they know that $\hat{P}_{n-1}$ is a convex combination of the fundamental θ and the signal y_{n-1}. As a result, we can rewrite the semi-public signal y_n as

$$\begin{aligned} y_n &= \hat{P}_{n-1} + \epsilon_n \\ &= (1-\hat{\lambda}_{n-1})\theta + \hat{\lambda}_{n-1} y_{n-1} + \epsilon_n \\ &= \theta + \hat{\lambda}_{n-1}\,(y_{n-1} - \theta) + \epsilon_n \\ &= \theta + \hat{\epsilon}_n, \end{aligned}$$

which is an unbiased signal of θ with modified stage-n noise $\hat{\epsilon}_n$ given by

$$\hat{\epsilon}_n = \epsilon_n + \hat{\lambda}_{n-1}\hat{\epsilon}_{n-1}, \quad \hat{\epsilon}_n \sim N(0, \hat{\beta}_n^{-1}), \tag{11}$$

where $\hat{\beta}_n$ is a precision to be determined for $n \geq 2$ and $\hat{\epsilon}_1 = \epsilon_1 \sim N(0, \beta^{-1})$.

The optimal price decision for stage-n firms is then

$$\hat{p}_n\,(j) = E\left[\theta + r\hat{\lambda}_{n-1}\hat{\epsilon}_{n-1} \mid x_n\,(j)\,, y_n\right]$$

[4] This conjecture can be verified following the same argument as in Morris and Shin (2002).

and the posterior joint distribution of $(\theta, \hat{\epsilon}_{n-1})$ given the signals $(x_n(j), y_n)$ yields

$$\begin{aligned} E[\theta \,|\, x_n(j), y_n] &= (1-\delta_n)\, x_n(j) + \delta_n y_n, \\ E[\hat{\epsilon}_{n-1} \,|\, x_n(j), y_n] &= \gamma_n [y_n - x_n(j)], \end{aligned}$$

where

$$\begin{aligned} \delta_n &\equiv \frac{\beta\hat{\beta}_{n-1}}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}, \\ \gamma_n &\equiv \frac{\alpha\beta\hat{\lambda}_{n-1}}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}. \end{aligned}$$

As a result, the weight on public information has the following expression

$$\hat{\lambda}_n \equiv \delta_n + r\hat{\lambda}_{n-1}\gamma_n = \frac{\beta(\hat{\beta}_{n-1} + r\alpha\hat{\lambda}_{n-1}^2)}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}.$$

Note that $\hat{\lambda}_n \in (0,1)$ since we can rewrite this expression as

$$\hat{\lambda}_n = \frac{b_n}{a_n + b_n},$$

where

$$\begin{aligned} a_n &\equiv \alpha\left[\hat{\beta}_{n-1} + (1-r)\,\beta\hat{\lambda}_{n-1}^2\right] > 0, \\ b_n &\equiv \beta\left[\hat{\beta}_{n-1} + r\alpha\hat{\lambda}_{n-1}^2\right] > 0. \end{aligned}$$

From equation (11), we observe that precision $\hat{\beta}_n$ is recursively defined as

$$\hat{\beta}_n = \frac{\beta\hat{\beta}_{n-1}}{\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2} > 0. \tag{12}$$

We can use this last equation to obtain expressions for δ_n and $\hat{\lambda}_n$, as functions of the primitive parameters α, β, and r together with stage-n modified precision $\hat{\beta}_n$

$$\delta_n \equiv \frac{\hat{\beta}_n}{\alpha + \hat{\beta}_n}, \quad \hat{\lambda}_n = \frac{(\beta - r\alpha)\,\hat{\beta}_n + r\alpha\beta}{\beta(\alpha + \hat{\beta}_n)} \tag{13}$$

Equations (12) and (13) yield a recursive structure to obtain the *endogenous* semi-public information precision $\hat{\beta}_n$ and the weight on public information $\hat{\lambda}_n$.

To better understand how information flows through the stages, it is useful to consider first that firms observe stage-$(n-1)$ prices or, equivalently, $\beta^{-1} = 0$ for all $n \neq 1$. Abstracting for the stage-specific noise component ϵ_n, we obtain the following semi-public signals

$$y_n = \begin{cases} \theta + \epsilon_1, & n = 1, \\ \tilde{P}_{n-1} = \theta + \tilde{\epsilon}_n, & n > 1, \end{cases}$$

where

$$\tilde{\epsilon}_n \equiv \tilde{\lambda}_{n-1}\tilde{\epsilon}_{n-1} = \left(\textstyle\prod_{j=1}^{n-1} \tilde{\lambda}_j\right) \epsilon_1, \quad \tilde{\epsilon}_n \sim N(0, \tilde{\beta}_n^{-1})$$

and $\tilde{\lambda}_n \in (0,1)$ is the weight on the semi-public signal y_n. In this simpler case, the endogenous semi-public information precision $\tilde{\beta}_n$ increases along the chain

$$\tilde{\beta}_n \equiv \left(\textstyle\prod_{j=1}^{n-1} \tilde{\lambda}_j^{-2}\right) \tilde{\beta}_{n-1} > \tilde{\beta}_{n-1},$$

because the noise ϵ_1 is only partially transferred to the next stage through prices. This result is in contrast with the exogenous public information case where the precision on public information remains constant along the chain. As a result, the weight on the semi-public signal $\tilde{\lambda}_n$ also increases along the chain because the price level of a specific stage reflects more the fundamental than previous-stage price level

$$\tilde{\lambda}_n \equiv 1 - \frac{(1-r)\,\alpha}{\alpha + \tilde{\beta}_n} > \tilde{\lambda}_{n-1}.$$

We are now able to analyze how the endogenous information structure affects the results of the model when we consider the stage-specific noise components (ϵ_n). A direct consequence is that the endogenous precision $\hat{\beta}_n$ may increase or decrease along the chain.

Result 4 (Endogenous Semi-public Information Precision) *In our model with private and semi-public signals, endogenous semi-public information variance $\hat{\beta}_n$ decreases along the chain if stage-specific noise more than compensate the attenuated noises inherited from previous stages*

$$\hat{\beta}_n^{-1} = \beta^{-1} + \hat{\lambda}_{n-1}^2 \hat{\beta}_{n-1}^{-1}.$$

Result 4 highlights the importance of stage-specific noise in the information transmission process. If current stage noises are volatile enough (high β^{-1}), it may more than compensate the attenuated inherited noises from previous stages. Given α and r, there is a exogenous semi-public precision $\beta = \beta^*$ that induces a sequence of *endogenous* semi-public information precision $\hat{\beta}_n$ that decreases monotonically to a strict positive limit $\hat{\beta}^*$[5]

$$\beta^* \equiv \hat{\beta}_1 > \hat{\beta}_2 > ... > \hat{\beta}^* > 0.$$

In the results that follow, we consider this value of $\beta = \beta^*$. Next, we consider the Bayesian weight on semi-public information.

Result 5 (Bayesian Weight on Semi-public Information) *In our model with private and semi-public signals, the Bayesian weight on semi-public information, δ_n, decreases along the chain with endogenous precision $\hat{\beta}_n$*

$$\frac{\partial \delta_n}{\partial \hat{\beta}_n} = \frac{\alpha}{\left(\alpha + \hat{\beta}_n\right)^2} > 0. \tag{14}$$

This result is in contrast with the exogenous public information case, where the Bayesian weight on public information remains constant along the chain. The semi-public signal y_n reveals information about the fundamental θ that is embedded in stage-$(n-1)$ price level $\hat{P}_{n-1}$. Result 5 simply states that firms attach more weight in this piece of information as it becomes more precise (high $\hat{\beta}_n$).

Note that the same is not necessarily true for the weight $\hat{\lambda}_n$ on public information in price setting.

Result 6 (Weight on Semi-public Information) *In our model with private and semi-public signals, the weight on semi-public information in price setting, $\hat{\lambda}_n$, decreases along the chain with endogenous precision $\hat{\beta}_n$ if and only if $r < \lambda$.*

$$\frac{\partial \hat{\lambda}_n}{\partial \hat{\beta}_n} = \frac{\alpha\left[\beta - r\left(\alpha + \beta\right)\right]}{\beta(\alpha + \hat{\beta}_n)^2} > 0 \Longleftrightarrow r < \lambda. \tag{15}$$

In contrast with the exogenous public information case, the weight on semi-public information may decrease along the chain even if endogenous semi-public precision $\hat{\beta}_n$ increases. The decrease in endogenous semi-public precision along the chain translates into a decrease on the relative importance of public information if the share of goods produced at stage

[5]See Appendix for details.

$(n-1)$ in the production of stage n is lower than the original weight on public information, given by λ.

Endogenous information also affects the behavior of price dispersion and non-fundamental volatility. We just consider the case $r < \lambda$ to focus on the standard relation between the precision of a signal and the weight attached to this signal. In this situation, our next results are more direct comparable to the exogenous public signal results.

Result 7 (Dispersion and Volatility - Semi-public Signal) *In our model with private and semi-public signals and $\beta = \beta^*$, non-fundamental volatility, $\hat{\varrho}_n^2$, increases through the chain with endogenous semi-public precision*

$$\begin{aligned} \hat{\varrho}_n^2 &\equiv V\left[\theta - \hat{P}_n\right] \\ &= (\hat{\lambda}_n)^2 \hat{\beta}_n^{-1} \Rightarrow \hat{\varrho}_n^2 > \hat{\varrho}_{n-1}^2, \end{aligned} \tag{16}$$

while price dispersion, $\hat{\sigma}_n^2$, increases if and only if $r < \lambda$

$$\begin{aligned} \hat{\sigma}_n^2 &\equiv V\left[\hat{p}_n(j) - \hat{P}_n\right] \\ &= (1 - \hat{\lambda}_n)^2 \alpha^{-1} \Rightarrow \hat{\sigma}_n^2 > \hat{\sigma}_{n-1}^2 \Longleftrightarrow r < \lambda. \end{aligned} \tag{17}$$

Now, in contrast with the exogenous public information case, dispersion may increase along the chain because firms rely less on more imprecise public information when $r < \lambda$. This result is a direct consequence of the decrease of the weight on public information $\hat{\lambda}_n$ due to the decrease of endogenous public precision $\hat{\beta}_n$.

As in the exogenous public information case, however, volatility also increases. This apparent contradictory result is due to the combination of two distinct effects of endogenous information precision $\hat{\beta}_n$. First, $\hat{\beta}_n$ has a *indirect* impact through the weigh on public information $\hat{\lambda}_n$. Note that $\hat{\lambda}_n$ has a proportional impact on volatility. When $\hat{\lambda}_n$ increases (as in the exogenous public information case), volatility also increases. Alternatively, if $\hat{\lambda}_n$ decreases (as in the endogenous public information case if $r < \lambda$), volatility also decreases. Second, $\hat{\beta}_n$ has a *direct* and inversely proportional impact on volatility. In the exogenous case, this precision is invariant along the chain. In the endogenous case, however, volatility increases while public precision decreases. This second and direct effect of endogenous information precision $\hat{\beta}_n$ supplants the indirect effect on $\hat{\lambda}_n$, which establishes the result.

Finally, consider the impact of endogenous semi-public information on precision and commonality.

Result 8 (Information in Chains) *In our model with private and semi-public signals and $\beta = \beta^*$, information precision, $\hat{V}_n^{-1}(j)$, and commonality, $\hat{\chi}_n(i,j)$, decreases along the chain with endogenous semi-public precision*

$$\begin{aligned} \hat{V}_n^{-1}(j) &\equiv Var\left[\upsilon_n(j)\right]^{-1} \\ &= \alpha + \hat{\beta}_n \Rightarrow \hat{V}_n^{-1}(j) < \hat{V}_{n-1}^{-1}(j), \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{\chi}_n(i,j) &\equiv Corr\left[\upsilon_n(i), \upsilon_n(j)\right] \\ &= \delta_n \Rightarrow \hat{\chi}_n(i,j) < \hat{\chi}_{n-1}(i,j), \end{aligned} \tag{19}$$

where $\upsilon_n(j) \equiv \theta - E\left[\theta \mid x_n(j), y_n\right].$

Our final result summarizes the major impacts of the introduction of an endogenous semi-public signal. Although the two measures remain unchanged with the inclusion of an input-output structure, the same is not true when we change the information structure. Precision decreases along the chain because the public signal accumulates noise or, alternatively, precision $\hat{\beta}_n$ decreases. But, when $\hat{\beta}_n$ decreases, firms rely less on public information to obtain θ, which means that δ_n decreases. As a result, commonality also decreases.

5 Discussion

We show how endogenous public information affects the weight on public information and the precision of information in a production chain. We consider that firms observe input prices with noise, which endogenize the precision of semi-public information.

Our main result is that agents may find optimal to rely less on stage-specific public information along the chain when semi-public signals are endogenous. This result is in contrast with the existing literature, which states that, when actions are strategic complements, agents wish to coordinate their actions, and because public information is a relatively better predictor of others' actions, agents find it optimal to rely more on public information relative to a situation in which actions are strategically independent.

An important implication of our main result regards information precision along the chain and highlights the importance of the information structure. We show that, while information precision remains unchanged with exogenous public signals (*information chains*), it may decrease along the chain when signals are semi-public and endogenous (*information in* *chains*). This result is also a direct consequence of the possible semi-public signal precision's decrease due to stage-specific noises.

There are various possible extensions, variations, and applications of our model. From the theoretical side, we don't consider any measure of social welfare in our model. A direct candidate is the so called "efficient use of information", proposed by Angeletos and Pavan (2007). This benchmark is the strategy that maximizes ex-ante welfare taking as given the dispersion of information in the population. It can be represented as the solution to a planner's problem, where the planner can perfectly control how an agent's action depends on his own information, but cannot transfer information from one agent to another. In contrast with standard efficiency concepts based on Mirrlees (1971) or Holmstrom and Myerson (1983) that assume costless communication and focus on incentive constraints, the efficient use of information takes decentralization of information as the main constraint, sharing with Hayek (1945) and Radner (1962) the idea that information is dispersed and can not be centralized in a planner.

The simple framework of our model can also help the understanding of diverse phenomena concerning pricing behavior. The sluggish response of consumer prices to monetary policy changes and to exchange rate devaluations are important applications.[6] How does prices are set if firms in an input-output structure observe not only our endogenous semi-public signal, the previous stage price index, but also a public signal, the nominal interest rate? And what if they observe the (public) exchange rate? Our model can be easily modified to deal with these questions as well as to its policy implications.

6 Appendix

6.1 The Model of Huang and Liu (2001)

In the model economy, the production of a final consumption good requires N stages of processing. The production of each good at stage 1 requires labor services only, with a constant-returns-to-scale technology given by $Y_1(j) = H_1(j)$, where $H_1(j)$ is the labor input and $Y_1(j)$ is the output. The production of each good at stage $n \in \{2, ..., N\}$ uses labor and

[6] For works in this line relating information and exchange rates see Bacchetta and van Wincoop (2006) and Areosa and Areosa (2011).

all goods produced at the previous stage as inputs according to the following technology

$$Y_n(j) = \left[\int_0^1 Y_{n-1}(j,z)^{\frac{1}{\mu}} dz \right]^{r\mu} H_n(j)^{1-r},$$

where $Y_n(j)$ is the output of a stage-n firm of type j, $Y_{n-1}(j,z)$ is the input supplied to j by a stage-$(n-1)$ firm of type z, $\mu > 1$ is function of the elasticity of substitution between such goods, $H_n(j)$ is the labor input used by j, and $r \in (0,1)$ is the share of composite of stage-$(n-1)$ goods in j'production. Firms behave as imperfect competitors in their output markets and are price-takers in their input markets. Given the constant-returns-to-scale technologies, the unit cost is also the marginal cost and is firm-independent. The optimal price decision under flexible prices is just a mark-up over marginal costs

$$P_n(j) = \mu \left[\bar{r} (P_{n-1})^r (W)^{1-r} \right], \tag{20}$$

where $\bar{r} \equiv r^{-r}(1-r)^{-(1-r)}$, $P_{n-1} \equiv \left[\int_0^1 P_{n-1}(z)^{\frac{1}{1-\mu}} dz \right]^{1-\mu}$ is a price index for goods produced at stage $n-1$, and W is nominal wage per hour.

The representative household is infinitely lived and maximizes expected life utility. The following equation describes the labor supply decision of the household

$$\frac{W}{P_N} = C, \tag{21}$$

where consumption C is a Dixit and Stiglitz (1977) composite of the final-stage goods

$$C = \left[\int_0^1 Y_N(z)^{\frac{1}{\mu}} dz \right]^{\mu} \equiv Y_N, \tag{22}$$

where $Y_N(z)$ is a type z good produced at stage N. We combine equations (20), (21) and (22) to obtain

$$\frac{P_n(j)}{P_N} = \mu \left(\frac{P_{n-1}}{P_N} \right)^r (Y_N)^{1-r},$$

which log-linearized yields $p_n^* \equiv rP_{n-1} + (1-r)\theta$, where $\theta \equiv \hat{Y}_N + \hat{P}_N$ is nominal aggregate demand and a variable $\hat{Z}$ represents log-deviations of a variable Z from its steady-state value $\bar{Z}$. Pricing decisions like (1), which can be found in Woodford (2002) and Mankiw and Reis (2010), reflect the certainty-equivalence result that a price with imperfect information equals the expected price under full information.

6.2 $\hat{\beta}^*$ limit

Substitute the lag of the expression for $\hat{\lambda}_n$ at (13) in (12) to obtain a recursive expression for $\hat{\beta}_n$

$$\hat{\beta}_n = \frac{\left[\beta\left(\alpha + \hat{\beta}_{n-1}\right)\right]^2 \hat{\beta}_{n-1}}{\beta\left(\alpha + \hat{\beta}_{n-1}\right)^2 \hat{\beta}_{n-1} + \left[\left(\beta - r\alpha\right)\hat{\beta}_{n-1} + r\alpha\beta\right]^2}. \tag{23}$$

If we set $\hat{\beta}_n = \hat{\beta}_{n-1} = \hat{\beta}^*$, we obtain

$$\hat{\beta}^* = \frac{\left[\beta\left(\alpha + \hat{\beta}^*\right)\right]^2 \hat{\beta}^*}{\beta\left(\alpha + \hat{\beta}^*\right)^2 \hat{\beta}^* + \left[\left(\beta - r\alpha\right)\hat{\beta}^* + r\alpha\beta\right]^2},$$

which is a root of the third degree equation

$$f(X) \equiv (X)^3 + c_1 (X)^2 + c_2 X + c_3, \tag{24}$$

where

$$\begin{aligned} c_1 &\equiv \frac{\alpha}{\beta}\left[2\beta(1-r) + r^2\alpha\right] > 0, \\ c_2 &\equiv \alpha\left[\alpha\left(1 - 2r^2\right) - 2\beta(1-r)\right], \\ c_3 &\equiv -\alpha^2\beta\left(1 - r^2\right) < 0. \end{aligned}$$

6.3 Extreme values of r

If $r = 1$, zero is necessarily one of the roots of $f(X)$ and thus we must define $\hat{\beta}^*$ as the unique strict positive root of (24). For values of r very close to zero, we need an extra condition regarding the exogenous precisions α, β and r to guarantee monotonicity. If $r = 0$, for example, we need $\alpha \leq \frac{3}{4}\beta$. To see this, consider (24) when $r = 0$

$$f(X) \equiv (X)^3 + 2\alpha (X)^2 + \alpha(\alpha - 2\beta) X - \alpha^2\beta. \tag{25}$$

In order to $\hat{\beta}_n$ converges monotonically to $\hat{\beta}^*$, the first derivative of (23) must be non-negative on $\hat{\beta}^*$

$$\left.\frac{\partial \hat{\beta}_n}{\partial \hat{\beta}_{n-1}}\right|_{\hat{\beta}_{n-1}=\hat{\beta}^*} = \frac{\beta^2((\hat{\beta}^*)^2 - \alpha^2)}{\left[(\alpha + \hat{\beta}^*)^2 + \beta\hat{\beta}^*\right]^2} \geq 0$$

$$\Rightarrow \hat{\beta}^* \geq \alpha.$$

If we substitute $\hat{\beta}^* = \alpha$ in the third degree equation (25), we obtain the result

$$f(\alpha) = 0 \Rightarrow \alpha = \frac{3}{4}\beta.$$

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