Another Look at Panel Estimates of the Elasticity of Substitution

Marcelo Mello[1]

Faculdades Ibmec/RJ

July 1st, 2011

Abstract

We estimate the elasticity of substitution between capital and labor for a large panel consisting of 100 countries over the period 1970-2007. In non-linear specifications, we find that the elasticity of substitution is unitary when the labor input is uncorrected for schooling differences across countries. However, when labor is corrected for cross-country schooling differences the elasticity of substitution is greater than one, invalidating the Cobb-Douglas assumption. In linear specifications, we find that the elasticity of substitution is lower than one, whether or not labor is corrected for cross-country differences in schooling. Furthermore, estimates from alternatives panels and additional tests, such as the estimation of a Translog production function, confirm our initial findings. These results contrast with previous panel estimates of the elasticity of substitution in the literature. Finally, we explore extensions of our dataset and methodology, such as the estimation of the elasticity of substitution between skilled and unskilled labor.

Keywords: Elasticity of Substitution, Aggregate Production Function, CES

JEL Codes: O40, O47

---

[1] Department of Economics, Ibmec/RJ, Av. Presidente Wilson 118/1101, 20030-020, Rio de Janeiro, RJ, Brazil, ph: (55-21)-4503-4161; Fax: (55-21)-4503-4168. E-mail: mmello@ibmecrj.br.
I would like to thank seminar participants at IPEA-Rio, Ibmec/RJ, and FUCAPE for helpful comments and insights. This work was initiated while I was visiting the University of Illinois at Urbana-Champaign in January of 2010 under the auspices of the Lemann Institute and the Center for Latin American Studies. The generosity of the Lemann Institute and the hospitality of the Economics Department at UIUC are gratefully acknowledged. Finally, all remaining errors are mine.

Resumo

Nós estimamos a elasticidade de substituição entre capital e trabalho num painel abrangente com 100 países no período 1970-2007. Em especificações não-lineares, nossas estimativas sugerem que a elasticidade de substituição é unitária quando o insumo trabalho não é corrigido para diferenças de escolaridade entre os países. Entretanto, quando o insumo trabalho é corrigido para diferenças de escolaridade entre os países a elasticidade de substituição é maior que a unidade, o que invalida a hipótese da Cobb-Douglas. Em especificações, lineares nós encontramos estimativas da elasticidade de substituição que são menores do que um, independente da correção por escolaridade entre os países. Ademais, estimativas de painéis alternatios e tetes adicionais, como a estimação de uma função de produção Translog, confirmam nossas estimativas iniciais. Esses resultados, contrastam com estimativas anteriores em painéis da elasticidade de substituição. Finalmente, nós exploramos extensões do nosso banco de dados e da nossa metodologia, e estimamos a elasticidade de substituição entre mão-de-obra qualificada e não-qualificada.

Palavras-Chave: Elasticidade de Substituição, Função de Produção Agregada, CES

JEL Codes: O40, O47

1. Introduction

Most applied work in economic growth assumes a Cobb-Douglas technology as representative of the aggregate technology. The Cobb-Douglas assumption is usually justified on the grounds that its property of constant capital share matches the data.

However, Bernanke and Gurkaynak (2001) have documented substantial cross-sectional variability in labor shares, and, by implication, in capital shares. Their estimates suggest that labor shares fall in the 0.6-0.8 range, with an average value of 0.65.[2] Bentolila and Saint-Paul (2003) have shown that the labor shares among 12 OECD countries varies systematically across time and space. In particular, they show that the capital-output ratio is a key determinant of the labor share.[3] Caselli and Feyer (2007) present evidence suggesting that capital shares, when appropriately measured, are negatively correlated with income per worker.

As it is well known, the choice of the Cobb-Douglas specification restricts the elasticity of substitution, henceforth denoted by σ, to be unitary. However, whether or not σ is unitary is an empirical question, and there is little consensus over econometric estimates of σ.

Berndt (1976) reports that estimates of σ generated from time series samples tend to be below unity, whereas cross-section samples generate estimates of σ around unity. He suggests that once can reconcile cross-section and time

[2] Although they tend to agree with Gollin (2002) that labor shares exhibit no systematic variation with capital per worker or GDP per worker, for the applied researcher the important question is whether capital shares varies across countries, not whether this variation is systematic or not.
[3] Note that Bentolila and Saint-Paul's results were obtained with industry-level data and therefore are not directly comparable with aggregate national data.

series estimates of σ by using higher quality data. Having done that, he finds that σ is approximately one and that the Cobb-Douglas assumption is valid.

Antràs (2004) shows that by assuming Hicks-neutral technical change, like Berndt did, estimates of σ are biased towards one. Essentially, Antràs shows that assuming Hicks neutrality when technical change is non-neutral, then one has an omitted variable problem and coefficient estimates of σ will be biased towards one. Using time series for the U.S. economy over the period 1948-1998, he shows that estimates of σ fall to approximately 0.5 when one allows for non-neutral technical change. He concludes that the Cobb-Douglas specification is not a good description of the aggregate technology for the U.S. Along the same line, with a time series sample over the period 1945-2003 for the Finish economy, Javala et alli (2006) finds that σ is approximately 0.5.

One drawback of the time series studies is that they suffer from small sample biases. This is particularly problematic given the high persistence present in growth data. For instance, Antràs's sample has only 51 observations. Similarly, the cross-section studies also suffer from small sample biases, given that the number of countries around the world with good data is small. On the other hand, the recent availability of large macroeconomic datasets made possible the construction of panel samples for the estimation of σ. Duffy and Papageorgiou (2000), to our knowledge, is the first study generating panel estimates of σ. They estimate a constant elasticity of substitution (CES) production function for a panel of 82 countries over the period 1960-1987. They estimate linear and non-linear specification of a CES technology and find evidence that the elasticity of substitution is substantially greater than one.

If the elasticity of substitution indeed differs from unity then a CES production function would better represent the aggregate technology, and the Cobb-Douglas assumption so popular in the literature should be put under scrutiny.

In this article, we estimate the elasticity of substitution between capital and labor using a broad panel of 100 countries over the period 1970-2007. Our panel is constructed with data from the Penn World Tables (PWT) version 6.3, and has a number of advantages over the previous panels used in the literature. First, our panel is broader and longer than previous panels. We have data on almost forty years for one hundred countries. Second, because our time series for individual countries is longer than in previous studies, we are able to construct higher quality estimates of the aggregate capital stock series. Third, because our panel is broad, it is representative of the world economy and it can be used to test whether the Cobb-Douglas is a good description of the aggregate technology. Fourth, our panel is constructed from what is probably the most used dataset in the growth literature, which makes any result we find potentially helpful for the literature as a whole.

We generate estimates of σ from both linear and non-linear specifications of a CES technology. Estimates generated from non-linear specifications using our benchmark panel suggest that the elasticity of substitution is approximately one if labor is uncorrected for schooling differences across countries, in contrast with previous studies in the literature. On the other hand, if labor is corrected for schooling differences across countries estimates of σ are well above unity, invalidating the Cobb-Douglas assumption.

Estimates from linear specifications of the CES technology using our benchmark panel and raw labor generate estimates of σ that are approximately one, in line with non-linear specifications. Interestingly, if labor is corrected for schooling differences across countries, estimates of σ from linear specifications are well below the unity.[4]

We use alternative panels to check for the robustness of our estimates. In particular, we use a large panel with 93 countries over the period 1970-2000 constructed by Mello (2009). In linear specifications, when labor is uncorrected for schooling differences, fixed effects estimates of σ are below unity. In addition, we use alternative country lists and/or sample period to generate estimates of σ. Our results suggest that estimates of σ are sensitive to the country list and the time period used in the estimation.

---

[4] We still have to reconcile the discrepancies between estimates from linear and non-linear specifications.

We also use our methodology to estimate a simplified version of a two-level CES production function used in Mello (2008) which allow us to generate estimates of the elasticity of substitution between skilled and unskilled labor. Our estimates suggest that the elasticity of substitution between skilled and unskilled labor fall in the range 1.0-1.3, depending on how skilled labor is defined.

This article is structured as follows. In section 2, we briefly describe the theoretical properties of the CES technology and of growth models using the CES. In section 3, we review the empirical literature and the econometric issues in the estimation of the elasticity of substitution. In section 4, we discuss the construction of the dataset. In section 5, we present estimates of the elasticity of substitution from the non-linear specification. Section 6 presents estimates of σ from the linear specification. Section 7 presents estimates of a Translog production function which can be used as a specification test for the Cobb-Douglas assumption. In section 8, we generate estimates of the elasticity of substitution between skilled and unskilled labor. Section 9 concludes.

2. The CES Technology and Growth Models

We assume the following functional form for the constant elasticity of substitution (CES) technology:

$$Y_t = A_t\left[\delta K_t^{-\rho} + (1-\delta)L_t^{-\rho}\right]^{-1/\rho} \qquad (1)$$

Where Y_t is real GDP, K_t is aggregate physical capital, L_t is the number of workers, $A_t > 0$ is the technology parameter, $\delta \in (0,1)$ is the distribution parameter, and the parameter $\rho \geq -1$ determines the elasticity of substitution between capital and labor which is given by $\sigma = 1/(1+\rho)$.[5] The capital share is given by $(k) = \frac{A\delta}{\delta+(1-\delta)k_t^{\rho}}$, that is, it varies with the capital labor ratio. The above specification first appeared in Arrow et al. (1961).

The distribution parameter δ, as the name suggests, determines the distribution of income between capital and labor for a given elasticity of substitution. In particular, it can be shown that $\frac{wL}{rK} = \frac{1-\delta}{\delta}\left(\frac{K}{L}\right)^{\rho}$, that is, for a given ρ, the distribution parameter determines the ratio between labor income and capital income.

The dynamic equation for the capital per worker ratio in the Solow model is given by $\dot{k} = sf(k) - (n+\delta)k$, where k is the capital per worker ratio, s is the saving rate, $f(k)$ is the production function in per worker terms, n is population growth, and δ is the depreciation ratio. The most basic feature of the Solow model altered under a CES technology is the possibility of endogenous

[5] The restriction on ρ is needed to ensure that the elasticity of substitution is positive.

growth.[6] Note that the growth rate of capital per worker is given by $\frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n + \delta)$. It can be shown that as $k \to \infty$ the following holds:

$$lim_{k\to\infty} \frac{f(k)}{k} = lim_{k\to\infty} f^{\prime}(k) = A\delta^{-1/\rho} > \frac{(n+\delta)}{s} > 0 \qquad (2)$$

The first equality sign in the above expression says that the inverse of the capital-output ratio is equal to the marginal product of capital in the long-run. The second equality is obtained by applying L´Hopital´s rule. The first and second inequalities say that in order to obtain endogenous growth the long-run marginal product of capital has to be above a certain positive threshold.

Smetters (2003) studies the dynamic properties of the neoclassical growth model with a CES technology. He shows that if the elasticity of substitution between capital and labor is less than unity then the saving rate decreases along the transition path after the capital stock reaches a critical value, however before reaching the critical value the saving rate may increase. Alternatively, if the elasticity of substitution is less than unity the saving rate increases along the transition path after the capital stock reaches a critical value, but before reaching this value the saving rate may decrease.[7]

Azariadis (1993) shows that the Diamond (1965) model with CES technology can exhibit multiple steady-states or no equilibrium, depending on

---

[6] Endogenous growth should be understood as a situation in which the long-run equilibrium produces a positive non-zero long-run growth rate in capital per worker that depends on parameters of the model.
[7] Gómez (2008) generalizes some of the results in Smetters (2003).

whether the elasticity of substitution is greater or less than one. In particular, if the elasticity of substitution is greater than or equal to one then there always exists a unique positive steady-state. Hence, estimates of σ below unity are a necessary condition for multiple equilibria.

The value of the elasticity of substitution is also important in the development accounting literature. In particular, whether the elasticity of substitution is above or below unity matters because, as shown by Caselli (2005), if technological change is non-neutral and σ is low, i.e., below unity, inputs have a more substantial explanatory power in accounting for relative income differences across countries. On the other hand, if technological change is non-neutral and σ is high, i.e., above unity, inputs have a lower explanatory role in accounting for relative income differences across countries.

3. Empirical Estimates of the Elasticity of Substitution

Most estimates of σ are generated from cross-section or time series samples. As mentioned above, Berndt (1976) points out that estimates of σ generated from time series samples are, in general, lower than unity while estimates from cross-section samples are generally close to unity. Furthermore, Berndt (1976) suggests that one can reconcile this discrepancy by using high quality data. He concludes that when high quality data is used estimates of σ are close to unity and the Cobb-Douglas assumption is validated.[8]

Antràs (2004) shows that the assumption of Hicks neutral technical change bias estimates of σ towards unity. On the other hand, if one allows for biased technical change then one obtains estimates of σ well below unity. His bias-corrected estimates of σ are around 0.5. He concludes that the Cobb-Douglas does not describe well the U.S. economy.

Recent estimates of σ in a time series setting include Javala et al. (2006) who use cointegration techniques to estimate the elasticity of substitution between capital and labor for Finland over the period 1902-2003. Their estimates suggest that σ is approximately equal to 1 for the sample as a whole. However, for the sub-sample 1945-2003 their estimates are well below unity, in fact, close to 0.5. Their findings for the sub-sample are consistent with those in Antràs (2004) who estimates σ for the U.S. economy over the period 1948-1998.

To our knowledge, the only article which estimates the elasticity of substitution in a panel environment is Duffy and Papageorgiou (2000). They use capital stocks estimates from Nehru and Dhareshaw (1993) to construct a panel with 82 countries over the period 1960-1987. They estimate linear and non-linear specifications of the CES technology. Their estimates suggest that the elasticity of substitution is well above unity.

---

[8] Berndt's estimates are generated using U.S. manufacturing data over the period 1929-1968.

They extend their analysis by estimating the production function with physical capital and human capital. Their measure of human capital is the amount of raw labor adjusted for the mean years of schooling of individuals between the ages of 15 and 64 in the workforce. Their estimates of σ when labor is corrected for schooling differences do not change significantly.

4. Data

Our benchmark panel is constructed using PWT version 6.3. It includes 100 countries for which population is greater than or equal to 1 million in 1985. The panel covers the period 1970-2007. We call this panel PWT63_N=100.

The panel includes the following list of variables: POP, Y, K, L, yl, kl, where POP is population, Y is real GDP, K is aggregate capital, L is the number workers, yl is GDP per capita, and kl is capital per worker. Below, we explain how we construct these variables.

Following Caselli (2005), we construct the number of workers as $RGDPCH \cdot POP/RGDPWOK$, where we denote the variables by their PWT codes. Real GDP (Y) is constructed by multiplying the series RGDPWOK2 by the number of workers. The series RGDPWOK2 is given by $RGDPL2WOK = RGDPL2 * POP/Workers$, where $RGDPL2$ is an update of $RGDPL$ which is real

GDP (Laspeyre index). The reader is referred to the PWT dataset for more details on the definition of the variables.

Following Mello (2009), we use the perpetual inventory method to construct the aggregate capital stock series. The initial value of aggregate capital is set at $I_0/(g+\delta)$, where I_0 is initial investment (measured as the investment in the first year for which data is available), g is the average growth rate in investment for the first year in which data is available until 1970, and δ is the depreciation rate which is set at 6%. Once K_0 is determined, we use the aggregate capital accumulation equation, $K_t = (1-\delta)K_{t-1} + I_t$, to construct the series.

To ensure the quality of capital stock estimates, we discard the initial estimates of the series. More specifically, the perpetual inventory approach has the property that even if the initial estimate is incorrect, it will converge to the true value over time. Therefore, by discarding the initial years we guard ourselves against a bad initial guess. For all countries, we initiate the series on the first year for which data is available and discard all observations until 1969. For example, the capital stock series for Argentina starts in 1950, however, the first observation we use in the series is 1970. In total, we initialize the aggregate capital stock series in 1950 for 61 countries. For the remaining 50 countries, the initial year distribution is as follows: 26 countries the initial year is 1960, and for the remaining 21 countries the initial year is in the interval 1951-1959, with the notable exception of Tunisia for which the initial year is 1961.

We also work with alternative panels as a robustness exercise. First, from our benchmark panel, we include all countries in PWT 6.3 for which data are available. This adds 11 countries to our benchmark panel, making it the broadest panel possible. We refer to this panel as PWT6.3_N=111.[9]

Our measure of human capital uses the average years of schooling for the population 25 years old or older obtained from Barro and Lee (2010) dataset. More specifically, we assume that human capital H is given by $H = e^{0.1*u}L$, where u is the average years of schooling and L is the number of workers. That is, we assume that the Mincerian coefficient of returns to education is 0.1 for all countries.

To construct measures of the amount of skilled and unskilled workers we follow Caselli and Coleman (2006). First, we obtain the Mincerian coefficients from Bils and Klenow (2004). Second, based on the Mincerian coefficients we obtain the skill-wage premium and construct the amounts of skilled and unskilled labor following Caselli and Coleman (2006).

[9] The countries added to our benchmark panel are: Barbados, Cape Verde, Comoros, Cyprus, Equatorial Guinea, Gabon, Gambia, Guinea-Bissau, Iceland, Luxembourg, and Seychelles.

5. Non-Linear Specification

We initially estimate a non-linear version of equation (1), following Duffy and Papageorgiou (2000). In particular, we estimate the following functional form:

$$Y_{it} = A_0\left[\delta K_{it}^{-\rho} + (1-\delta)L_{it}^{-\rho}\right]^{-1/\rho} e^{\lambda t+\varepsilon_{it}} \qquad (3)$$

Where we assume Hicks neutral technological progress, that is, we assume that $A_t = A_0 e^{\lambda t}$ where A_0 is the initial level of the technology and λ the growth rate of technological progress. The term ε_{it} is the random disturbance. Taking logarithms on both sides of equation (3), we obtain

$$logY_{it} = logA_0 + \lambda t - \frac{1}{\rho} log\left[\delta K_{it}^{-\rho} + (1-\delta)L_{it}^{-\rho}\right] + \varepsilon_{it} \qquad (4)$$

Initially, we estimate equation (4) by non-linear least squares, ignoring the country fixed effect. Table 1 displays estimates of the elasticity of substitution σ for three different panels, as shown below.

[Insert Table 1]

Column (1) presents estimates of σ from our benchmark panel PWT63_N=100. Non-linear estimates of σ are approximately 1.10, and the 2-standard error confidence interval includes the unity. This finding validates the Cobb-Douglas specification assumption. Column (2) presents estimates of σ from our broadest panel, the PWT63_N=111. In this case, the estimated elasticity of substitution is 1.76. The two-standard error interval (1.63, 1.88), excludes the unity, therefore, ruling out the Cobb-Douglas specification. For estimation purposes, we consider our benchmark panel, PWT63_N=100 to be more representative of the world economy than the PWT63_N=111 panel. Therefore, we prefer estimates from this panel over alternatives ones. Hence, initially our estimation exercise validates the Cobb-Douglas specification as a representation of the aggregate technology.

Column (3) displays estimates from a panel with 93 countries over the period 1970-2000 from PWT 6.1 used in Mello (2009). Like our benchmark panel, this panel only includes countries for which the population is greater than or equal to 1 million people in 1985. Estimates in column (3) suggest that σ is approximately 1.51, which is more in line with estimates in column (2). In addition, the 2-standard-error confidence interval excludes the unity.

For the sake of comparison, we present in column (4) Duffy and Papageorgiou's (2000) estimates of σ based on a panel with 82 countries over the period 1960-1987. Their estimates suggest that σ is well above the unity, around 2.3. Given that their estimate differs considerably from ours, it is instructive to further examine the data and try to pinpoint exactly why this is the case. We consider three possible sources for the differences in estimates: (i) differences in the cross-section dimension of the panel, i.e., differences in the country list; (ii) differences in the time dimension of the panel, i.e., differences in the sample time period; and (iii) differences in the quality and treatment of the data.

In this sense, we first construct a panel using their country list with data from PWT 6.3. We are able to get data on 78 countries out of Duffy and Papageorgiu's original list of 82 countries.[10] We shall refer to any panel PWT63_N=78. Column (1) in table 2 displays estimates of σ using this panel.

[Insert Table 2]

Estimates in column (1) suggests that σ is around 1.5, which is in line with estimates in table 1 using panel PWT63_N=111 but well below Duffy and Papageorgiu's estimate of 2.30. Column (2) displays estimates from PWT using the same country list and time period as Duffy and Papageorgiu (2000). In this case, estimates of σ increase to 1.69. For the sake of comparison, column (3) displays estimates of σ using the same 78 countries and time period as in column (2). Estimates in column (3) suggest that σ is close to 3, even higher than in Duffy and Papageorgiou's original panel.

Comparing our dataset with Duffy and Papageorgiou's, we observe that the series for aggregate physical capital and number of workers are highly correlated. On the other hand, we observe differences in the time series for GDP per worker. This opens a future line of investigation; in particular, it could the case that the treatment the GDP per worker series in PWT 6.3 receives to be transformed in PPP dollars might explain the differences in estimates between the two panels.

[10] The countries in Duffy and Papageorgiou's panel for which the data is not available are: West Germany, Iraq, Myanmar (Burma), and Sudan.

Columns (4) and (5) in table 2 present estimates of σ from panels in PWT 6.3 and PWT 6.1, respectively, for which all countries have time series for physical capital starting in 1950. These countries contain a better estimate of the aggregate capital stock series because for each country we discard the first 20 observations, therefore, presumably eliminating any effects of a bad guess for the initial capital stock. Estimates in columns (4) and (5) suggest that σ is 1.32 and 1.15, respectively. In both cases, the two-standard error interval excludes the unity.

[Insert Table 3]

Table 3 above displays non-linear estimates of σ with labor corrected for schooling differences across countries. Column (1) shows estimates of σ from the broadest panel for which the human capital data is available. In this case, estimates of σ are approximately 1.4, and the two-standard error interval excludes the unity. Column (2) displays estimates of σ a panel consisting of all countries in our benchmark panel for which the schooling data is available. In this case, the panel is reduced to 93 countries. Estimates of σ are around 1.5 and the two-standard error interval, again, excludes the unity. Column (3) presents estimates of σ from a panel using all countries in Duffy and Papageorgiou's country list for which data on PWT 6.3 and on schooling were available. This panel generates point estimates of σ around 1.6, and the unity is excluded from the two-standard error confidence interval. Column (4) reproduces Duffy and Papageorgiou's estimate with their measure of human capital. Their estimate of σ is lower than ours, at 1.3.[11]

Overall estimation of a non-linear specification of a CES technology suggests that estimates of σ are close to unity when labor is uncorrected for schooling differences. On the other hand, when labor is corrected for schooling differences, estimates of σ are around 1.5.

6. Linear Specification

From the production function in (1), $Y_t = A\left[\delta K_t^{-\rho} + (1-\delta)L_t^{-\rho}\right]^{-1/\rho}$, assuming Hicks-neutral technological progress, i.e., $A_t = A_0 e^{\lambda t}$, Kmenta (1967) showed that the Taylor expansion around $\rho = 0$ yields the following expression:

$$logY_t = a_0 + \lambda t + \delta logK_t + (1-\delta)logL_t - \frac{\rho\delta(1-\delta)}{2}[logK_t - logL_t]^2 \qquad (5)$$

Expression (5) can be written in per worker terms as follows

$$logy_t = a_0 + \lambda t + \delta logk_t - \frac{\rho\delta(1-\delta)}{2}[logk_t]^2 \qquad (6)$$

Where $a_0 = logA_0$, and $k_t = \frac{K_t}{L_t}$ is the capital per worker ratio. Equation (5) can be estimated econometrically by using the following functional form below:

$$logy_{it} = a_0 + \lambda t + \beta_1 logk_{it} + \beta_2[logk_{it}]^2 + \varepsilon_{it} \qquad (7)$$

[11] Duffy and Papageorgiou (2000) report estimates of ρ, not of σ.

Where $= \frac{-2\beta_2}{\beta_1(1-\beta_1)}$, $\delta = \beta_1$, e $A_0 = e^{a_0}$. Estimates of the standard errors of the parameters of interest can be obtained by classical methods.

Table 4 presents estimates of the Kmenta linearization, i.e., equation (7) for the panel PWT6.3_N=100. Pooled OLS estimates suggest that σ is approximately unitary. Similarly, fixed effect estimates suggest σ is about 0.93, close to the unity but not enough to validate the Cobb-Douglas. The IV-FE estimator generates an estimate of σ of 1.1, and the two-standard error interval for ρ includes the zero, that is, the Cobb-Douglas hypothesis cannot be discarded. These estimates contrast with those in Duffy and Papageorgiou (2000) who find that σ is close to 1.5 in their linear specifications, and from the non-linear specification which tends to generate larger estimates for σ.

[Insert Table 4]

Table 5 displays estimates of σ from Kmenta's linearization using panel PWT63_N=111. Columns (1) and (2) displays pooled OLS estimates. These estimates are insensitive to whether we include a linear common trend or individual specific trends, and suggest that σ is lower than one, approximately 0.75. Fixed effects estimates in columns (3) and (4), suggest that σ is around 0.9, whereas the IV fixed effects estimator in column (5) produces an estimate that is numerically close to the pooled OLS estimate. All estimates are statistically different from one. Overall, these estimates do not lend support to the Cobb-Douglas hypothesis.

[Insert Table 5]

Table 6 displays estimates of the linearized CES for the panel PWT61_N=93. We are able to generate fixed effect estimates of σ, however, estimates from other estimators fall outside the feasible parameter space and therefore are not reported. Fixed effects estimates suggest that σ is lower than unity around 0.87, and therefore invalidating the Cobb-Douglas hypothesis.

[Insert Table 6]

Table 7 displays estimates of the Kmenta linearization using data from Duffy and Papageorgiou (2000) dataset. We select 78 countries from their panel for which we can match data with our panel. Pooled OLS estimates suggest that σ is close to 1.65, and fixed effects estimates suggest that σ is within the range (1.56, 1.85). The IV-FE estimator generates an estimate that is numerically equal to the pooled OLS estimate. All point estimates are above one, and statistically different from one.

[Insert Table 7]

Table 8 displays estimates of σ from the PWT63_N=78 panel over the period 1970-2007. Estimates of σ are much lower now, within the range (0.43, 0.82), in sharp contrast with estimates obtained by Duffy and Papageorgiou (2000) shown in table 7. This finding suggests that estimates of σ are sensitive to the time period and/or the treatment of the data (e.g., correction to convert to PPP figures).

[Insert Table 8]

In table 9 we present estimates of Kmenta's linearized equation with data from PWT63_N=78 over the period 1960-1987. Estimates of σ fall within the range (0.79, 0.88) and the Cobb-Douglas assumption is rejected in all cases. In addition, these estimates are largely in line with fixed effect estimates in table 8, and in sharp contrast with the ones in table 7, generated with Duffy and Papageorgiou (2000) data. This suggests that the treatment of the data (e.g., conversion method to construct PPP figures) might have be the source of the differences in σ estimates.

[Insert Table 9]

Table 10 displays estimates of σ from two alternative panels using data from PWT 6.3 and PWT 6.1 for countries with time series available since 1950. These countries have higher quality data, and because of they have longer time series the problem of initial guess of perpetual stock method to construct the capital stock is minimized.[12]

Columns (1) and (2) in table 10 exhibit fixed effect and IV-FE estimates using data from PWT 6.3, respectively. According to fixed effect and IV-FE estimators, point estimates of σ are around 1.6. However, the two-standard error interval includes the Cobb-Douglas hypothesis in the case of fixed effects estimates. Columns (3) and (4) display fixed effects and IV-FE estimates of σ

[12] Since we discard the first 20 observations, the initial guess of the capital stock is heavily discounted, making any bad initial guess unimportant.

using data from PWT 6.1, respectively. Fixed effect estimates do not discard the Cobb-Douglas hypothesis, whereas IV-FE estimates suggest that σ is approximately 1.1, and statistically different from unity.

[Insert Table 10]

Table 11 displays estimates from Kmenta's linearization for our benchmark panel over the period 1970-2007 when labor is corrected for schooling differences across countries. In this case, point estimates of σ are quite low, falling in the range (0.45, 0.86). These estimates are lower than the ones in table 4, where there is no correction for schooling differences.

[Insert Table 11]

Overall, estimation of Kmenta's linearized equation generates point estimates of σ that are lower than one, and statistically different from unity. In particular, fixed effects estimates in tables 4-6, 8-9, and 11, suggest that σ is around 0.9. In addition, the evidence in table 10 suggests that differences in country list and the quality of the data can affect estimates of σ, as well as whether the data receives any treatment (e.g., adjustment for PPP figures).

7. The Translog production function

The Translog or transcendental logarithm production function, henceforth TL, is a flexible functional form that can be seen as a second-order approximation of an unknown production function. One possible representation of the TL production function is given by:[13]

$$logY = \alpha_0 + \alpha_L logL + \alpha_K logK + \beta_{LL}(logL)^2 - 2\beta_{Lk} logL \cdot logK + \beta_{KK}(logK)^2 \quad (8)$$

Below, we review the properties of the TL function and present panel estimates of equation (8).[14]

It can be shown that homogeneity requires $\beta_{LL} = \beta_{LK} = \beta_{KK} = \beta$, and that the degree of homogeneity is given by $\alpha_L + \alpha_K$. The Cobb-Douglas specification can be obtained by restricting the coefficients of the double and crossed derivatives to be zero, that is, imposing $\beta_{LL} = \beta_{LK} = \beta_{KK} = 0$.

The elasticity of substitution between capital and labor in the TL production function is given by

$$\sigma = \frac{1}{1 - \frac{2\beta(\alpha_L + \alpha_K)}{(\alpha_L - 2\beta logk)(\alpha_K + 2\beta logk)}} \quad (9)$$

Where k is the capital per worker ratio. As can be seen in expression (9), the elasticity of substitution in the TL function depends on k, and because of this it can be seen as a generalization of the CES technology.[15]

---

[13] In equation (8) we assume that Young's theorem holds, that is, that the crossed partial derivatives are equal.

[14] Østbye (2010) suggests the use of the Translog production function as a more flexible way to represent the aggregate technology.

[15] However, in general, the TL function does not reduce to a CES function.

We can also relate the TL function with the Kmenta approximation. In fact, it can be shown that the Kmenta approximation is a particular case of the TL function. More precisely, if we impose the restriction $\beta_{LL} = \beta_{LK} = \beta_{KK} = \beta = -\frac{\rho\delta(1-\delta)}{2}$ on equation (8), we obtain the Kmenta approximation in expression (5).

Table 12 presents panel estimates of equation (8) for our benchmark panel. The estimates are relatively stable across estimators. In addition, we test the Cobb-Douglas restriction, that is, we test $\beta_{LL} = \beta_{LK} = \beta_{KK} = 0$ and find that the Cobb-Douglas specification is overwhelmingly rejected. We also test the constant returns to scale assumption (homogeneity of degree 1). In the case of the TL production function the constant returns to scale assumption is tested by the joint hypothesis $\beta_{LL} + \beta_{LK} + 2\beta_{KK} = 0$ and $\alpha_K + \alpha_L = 1$. For the benchmark panel the evidence suggests that this restriction does not hold.

[Insert Table 12]

Table 13 presents estimates of the TL production function for the Panel_N=111, our broadest panel. Estimates displayed in table 13 are remarkably stable across estimators. Again, the Cobb-Douglas restriction is overwhelmingly rejected. The constant returns to scale restriction is not rejected at 1% significance level in effect estimates (columns 3 and 4).

[Insert Table 13]

As a robustness exercise, table 14 presents estimates of the TL production function for the panel PWT61_N=93, from PWT 6.1. The estimates are in line with the evidence presented in tables 12 and 13, with estimates stable across estimators and the Cobb-Douglas restriction is overwhelmingly rejected. Similarly, like in table 13, we find that the constant returns to scale restriction is not rejected at 1% significance level in fixed effect estimates (columns 3 and 4).

[Insert Table 14]

8. Estimates of the ES between skilled and unskilled labor

The production function used in Mello (2008), and Caselli and Coleman (2006) can also be estimated using the methodology above. More specifically, Mello (2008) presented a growth model in which the technology makes use of three inputs of production:

$$Y = AK^{\alpha}\left[\beta N^{\psi} + (1-\beta)S^{\psi}\right]^{\frac{1-\alpha}{\psi}} \qquad (7)$$

Where Y is GDP, A is the technology level, K is physical capital, α is the share of capital, and β is the distribution parameter which controls the intensity with which skilled versus unskilled labor is used in production, N is the amount of unskilled labor, and S is the amount of skilled labor. The parameter $\psi \in [-\infty, 1)$ determines the degree of substitution between skilled and unskilled labor. The elasticity of substitution between skilled and unskilled labor, denoted by θ, is defined as follows

$$\theta = \frac{\partial(S/N)}{\partial(MPL_N/MPL_S)}\frac{MPL_N/MPL_S}{S/N} = \frac{1}{1-\psi} \qquad (8)$$

The parameter of interest is the constant but less than infinity elasticity of substitution θ. From equation (7) we can take logs and Taylor-expand around ψ=0, following the Kmenta linearization and obtain the following expression:

$$\log\left(\frac{Y_{it}}{N_{it}}\right) = logA_{0i} + \lambda t + b_1 \log\left(\frac{K_{it}}{N_{it}}\right) + b_2 \log\left(\frac{S_{it}}{N_{it}}\right) + b_3\left[log\left(\frac{S_{it}}{N_{it}}\right)\right]^2 + \varepsilon_{it}$$

(9)

Where $b_1 = \alpha$, $b_2 = (1-\alpha)(1-\beta)$, $b_3 = 0.5 \cdot \psi(1-\alpha)(1-\beta)$. The standard errors can be recovered by classical methods. We estimate equation (9) using three definitions of skilled labor. First, we define skilled labor as any worker with primary education completed. Second, we consider skilled labor any worker with secondary education completed, and, third, we consider skilled labor any worker with tertiary education completed.

Table 15 displays estimates of equation (9) using primary education as the cutoff for skilled labor. Fixed effect estimates of the elasticity of substitution between skilled and unskilled labor in table 15 suggest that θ is approximately 1.3.

[Insert Table 15]

Tables 16 and 17 display estimates of equation (9) using secondary education, and tertiary education as the cutoff for skilled and unskilled labor, respectively. In both cases, estimates of θ fall within the range (1.03, 1.1). These estimates are in line with the labor economics literature.

[Insert Table 16]

[Insert Table 17]

9. Conclusion

We estimate linear and non-linear specifications of a CES production function from a broad panel with 100 countries over the period 1970-2000. Our results suggest that the estimates of the elasticity of substitution are sensitive to the specification of the estimated equation, whether it is linear or non-linear, to the country list and time period used in the estimation, and whether labor is corrected for cross-country differences in schooling.

Our estimation exercise suggests that the Cobb-Douglas assumption so popular in the growth might be justified in some cases, however, in other cases the CES technology might be a better assumption to represent the aggregate technology. Finally, further investigation in this issue is warranted.

References

Aiyar, S. and C.-J. Dalgaard, 2009, Accounting for productivity: Is it OK to assume that the world is Cobb-Douglas?, Journal of Macroeconomics 31, 290-303.

Antràs, P., 2004, Is the U.S. aggregate production function Cobb-Douglas? New estimates of the elasticity of substitution, The B.E. Journal of Macroeconomics (Contributions), vol. 4, issue 1, article 4.

Arrow, K., H. Chenery, B. Minhas, and R. Solow, 1961, Capital Labor Substitution and Economic Efficiency, Review of Economic Statistics, 43, pp. 225-250.

Barro, R. J. and Jong-Wha Lee, 2010, A new data set of educational attainment in the World, 1950-2010, NBER working paper #15902.

Bentolila, S., and G. Saint-Paul, 2003, Explaining Movements in the Labor Share, Contributions to Macroeconomics (3), 1.

Bernanke, B.S., and R.S. Gurkaynak, 2001, Is growth exogenous? Taking Mankiw, Romer, and Weil seriously. In: Bernanke, B.S., and Rogoff, K. (Eds), NBER Macroeconomics Annual, MIT press, Cambridge, Massachusetts, pp. 11-57.

Bils, M. and P. Klenow, 2000, Does schooling cause growth?, American Economic Review, vol. 90, no. 5, 1160-1183.

Berndt, E. R., 1976, Reconciling Alternative Estimates of the Elasticity of Substitution, The Review of Economics and Statistics 58(1), 59-68.

Caselli, F., 2005, Accounting for Cross-Country Income Differences. In: Aghion, P. Durlauf, S. (Eds), Handbook of Economic Growth, North-Holland, vol. 1A, 679-741.

Caselli, F., and A. Coleman, 2006, The world technology frontier, American Economic Review 96, 499-522.

Caselli, F., and J. Feyrer, 2007, The Marginal Product of Capital, Quarterly Journal of Economics 122(2), 535-568.

Duffy, J., and Papageorgiou, C.D., 2000, A cross-country empirical investigation of the aggregate production function specification, Journal of Economic Growth 5(1), 87-1220.

Duffy, J., and Papageorgiou, C.D., Perez-Sebastian, F., 2004, Capital-Skill Complementarity? Evidence from a panel of countries, Review of Economic and Statistics 86, 327-344.

Gollin, D., 2002, Getting Income Shares Right, Journal of Political Economy 110 (2), 458-474.

Heston, A., R. Summers, and B. Aten, 2004, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP).

Jalava, Jukka, Antti Ripatti, Matti Pohjola, and Jouko Vilmunen, 2006, Biased Technical change and capital-labour substitution in Finland, 1902-2003, The B. E. Journal of Macroeconomics, vol. 6, issue 1, (Topics), article 8.

Kmenta, J., 1967, On estimation of the CES production function, International Economic Review 8, 180-189.

Mello, M., 2008, Skilled Labor, Unskilled labor, and Economic Growth, Economics Letters, 100 (3), 428-431.

Mello, M., 2009, Estimates of the Marginal Product of Capital, 1970-2000, The B. E. Journal of Macroeconomics, vol. 9, issue 1 (Topics), article 16.

Nehru, V. and Dhareshwar, 1993, A new database on physical capital stock: sources, methodology and results, Revista de Análisis Econòmico 8, 37-59.

Østbye, S., 2010, The translog growth model, Journal of Macroeconomics 21, 635-640.

Solow, Robert, 1956, A contribution to the theory of economic growth, Quarterly Journal of Economics 79, 65-94.

Table 1: Non-Linear Specification – Panel PWT 6.3, 1970-2007
Dependent variable: log(Y)

Variable	***PWT 6.3 N=100 Col. (1)***	***PWT 6.3 N=111 Col. (2)***	***PWT 6.1 N=93 Col. (3)***	***DP N=82 Col. (4)***
ρ	-0.0937	-0.4313	-0.3370	-0.5658
	(0.0818)	(0.0199)	(0.0206)	(0.0554)
δ	0.4352	0.0286	0.0860	0.05755
	(0.1942)	(0.0053)	(0.0145)	(0.0235)
time	-0.0251	-0.0014	-0.0040	-0.0116
	(0,0021)	(0.0005)	(0,0006)	(0.0009)
intercept	-4.1101	6.7131	5.8497	61.617
	(0.9290)	(0.0901)	(0.1291)	(15.787)
σ	1.1034	1.7584	1.5083	2.3076
	(0.0995)	(0.0617)	(0.0468)	(0.3088)
# Obs.	3800	4211	2873	2296

Note: Standard errors appear in parenthesis.

Table 2: Non-Linear Specification – Alternative panels
Dependent variable: log(Y)

Variable	***PWT 6.3 N=78 1970-2007 Col. (1)***	***PWT 6.3 N=78 1960-1987 Col. (2)***	***DP N=78 Col. (3)***	***PWT6.3 N=61 T=1950 Col. (4)***	***PWT 6.1 N=56 T=1950 Col. (5)***
ρ	-0.3326	-0.4076	-0.6504	-0.2469	-0.1326
	(0.0183)	(0.0236)	(0.0595)	(0.0180)	(0.0168)
δ	0.0687	0.0321	0.0308	0.1528	0.3403
	(0.0112)	(0.0068)	(0.0137)	(0.0231)	(0.0352)
time	-0.0010	-0.0031	-0.0111	0.0002	-0.0016
	(0.0005)	(0.0008)	(0.0009)	(0.0004)	(0.0006)
intercept	6.2132	6.8028	4.3222	5.4145	4.5535
	(0.1183)	(0.1017)	(0.2198)	(0.1712)	(0.1760)
σ	1.4983	1.6880	2.8604	1.3278	1.1529
	(0.0410)	(0.0672)	(0.4867)	(0.0318)	(0.0223)
# Obs	2964	2182	2184	2318	1734

Note: Standard errors appear in parenthesis.

Table 3: NLS with Schooling – Panel PWT 6.3 and DP data
Dependent variable: log(Y)

Variable	***PWT 6.3 N=99 1970-07 Col. (1)***	***PWT 6.3 N=93 POP=1985 1970-07 Col. (2)***	***PWT 6.3 N=73 1960-87 DP-data Col. (3)***	***DP N=82 1960-87 Col. (4)***
ρ	-0.2932	-0.3206	-0.3830	-0.2272
	(0.0231)	(0.0241)	(0.0303)	(0.0509)
δ	0.0925	0.0751	0.0371	0.4441
	(0.0177)	(0.0153)	(0.0097)	(0.0828)
time	-0.0044	-0.0046	-0.0042	-0.0155
	(0.0005)	(0.0004)	(0.0009)	(0.0010)
intercept	14.5765	15.0661	14.8189	6.9923
	(0.9274)	(0.9481)	(1.6928)	(2.1308)
σ	1.4148	1.4720	1.6206	1.2940
	(0.0462)	(0.0523)	(0.0796)	(--)
# Obs.	3762	3534	2044	2296

Note: Standard errors appear in parenthesis. Column (1) reports estimates from a sample with 99 countries from Panel N=111 for which data on schooling was available. Column (2) reports estimates from the sample in col. (1) excluding 6 countries for which population in 1985 was less than 1 million. Column (3) reports estimates from a sample of 73 countries listed in Duffy and Papageorgiou´s original sample using data from PWT 6.3. Finally, column (4) displays Duffy and Papageorgiu´s original estimates.

Table 4: Linear Specification – Panel PWT 63 N=100, 1970-2007
Dependent variable: log(Y/L)

Variable	*Pooled OLS Col. (1)*	*Pooled OLS Col. (2)*	*FE Col. (3)*	*FE Col. (4)*	*IVFE Col. (5)*
ln(k)	0.4707	0.4848	-1.1701	-1.0837	0.4914
	(0.1568)	(0.1571)	(0.0647)	(0.0665)	(0.1595)
$ln^2(k)$	0.0095	0.0089	0.0963	0.0931	0.0087
	(0.0082)	(0.0082)	(0.0033)	(0.0033)	(0.0083)
time	-0.0251	--	-0.0270	--	--
	(0.0021)		(0.0004)		
intercept	-4.2614	-4.3001	3.2866	2.7828	-4.3761
	(0.7331)	(0.7466)	(0.3240)	(0.3335)	(0.7585)
Estimated ρ	-0.0766	-0.0710	0.0759	0.0824	-0.0695
	(0.0686)	(0.0672)	(0.0040)	(0.0048)	(0.0675)
Implied σ	1.0830	1.0764	0.9295	0.9238	1.0747
# Obs	3800	3800	3800	3800	3700
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Note: The above panel only includes countries for which the population in 1985 was greater than or equal to 1 million people.

Table 5: Linear Specification – Panel PWT 6.3 N=111, 1970-2007
Dependent variable: log(Y/L)

Variable	***Pooled OLS*** ***Col. (1)***	***Pooled OLS*** ***Col. (2)***	***FE*** ***Col. (3)***	***FE*** ***Col. (4)***	***IVFE*** ***Col. (5)***
ln(k)	-0.2326	-0.2239	-0.5050	-0.4692	-0.2161
	(0.0415)	(0.0418)	(0.0524)	(0.0503)	(0.0427)
$\ln^2$(k)	0.0454	0.0449	0.0560	0.0548	0.0445
	(0.0021)	(0.0021)	(0.0026)	(0.0025)	(0.0021)
time	-0.0015	--	-0.0005	--	--
	(0.0005)		(0.0003)		
intercept	7.1341	7.1321	8.7476	8.5473	7.0597
	(0.2039)	(0.2066)	(0.2668)	(0.2542)	(0.2101)
Estimated ρ	0.3165	0.3281	0.1473	0.1589	0.3389
	(0.0527)	(0.0573)	(0.0138)	(0.0154)	(0.0627)
Implied σ	0.7596	0.7529	0.8716	0.8629	0.7469
# Obs	4211	4211	4211	4211	4100
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Notes: Standard errors appear in parentheses.

Table 6: Linear Specification – Panel PWT 6.1 N=93, 1970-2000
Dependent variable: log(Y/L)

Variable	*Pooled OLS*	*Pooled OLS*	*Fixed Effects*	*Fixed Effects*	*IVFE*
ln(k)	0.0489	0.0552	-0.5492	-0.5205	0.0641
	(0.0408)	(0.0412)	(0.0587)	(0.0601)	(0.0424)
$\ln^2(k)$	0.0336	0.0333	0.0609	0.0599	0.0329
	(0.0022)	(0.0022)	(0.0030)	(0.0031)	(0.0022)
time	-0.0040	--	-0.0024	--	--
	(0.0007)		(0.0004)		
intercept	5.6429	5.6356	8.7600	8.5931	5.4861
	(0.1875)	(0.1919)	(0.2851)	(0.2939)	(0.1963)
Estimated ρ	--	--	0.1432	0.1514	--
			(0.0138)	(0.0159)	
Implied σ	--	--	0.8747	0.8685	--
# Obs	2873	2873	2873	2873	2780
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Note: The above estimates were generated from a panel with 93 countries from PWT 6.1 over the period 1970-2000. As in table 4, all countries in the panel have population greater than or equal to 1 million in 1985.

Table 7: Linear Specification – Panel N=78, DP, 1960-1987
Dependent variable: log(Y/L)

Variable	*Pooled OLS Col. (1)*	*Pooled OLS Col. (2)*	*FE Col. (3)*	*FE Col. (4)*	*IVFE Col. (5)*
ln(k)	0.2464	0.24366	0.1593	0.1251	0.2442
	(0.0598)	(0.0511)	(0.0455)	(0.0468)	(0.0519)
$ln^2(k)$	0.0363	0.0364	0.0241	0.0252	0.0363
	(0.0028)	(0.0028)	(0.0025)	(0.0026)	(0.0029)
time	-0.0115	--	-0.0010	--	--
	(0.0009)		(0.0007)		
intercept	2.9307	3.0102	4.5681	4.7396	2.9957
	(0.2223)	(0.2250)	(0.2192)	(0.2243)	(0.2311)
Estimated ρ	-0.3905	-0.3952	-0.3600	-0.4603	-0.3935
	(0.0848)	(0.0870)	(0.0143)	(0.1926)	(0.0877)
Implied σ	1.6420	1.6533	1.5626	1.8528	1.6488
R2	0.9452	0.9455	0.9421	0.9416	0.9468
# Obs	2184	2184	2184	2184	2106
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Note: The above estimates were generated using Duffy and Papageorgiou's data available at

Table 8: Linear Specification – Panel PWT 6.3 N=78, 1970-2007
Dependent variable: log(Y/L)

Variable	*Pooled OLS Col. (1)*	*Pooled OLS Col. (2)*	*FE Col. (3)*	*FE Col. (4)*	*IVFE Col. (5)*
ln(k)	-0.0659	-0.0585	-0.3260	-0.2541	-0.0519
	(0.0397)	(0.0394)	(0.0801)	(0.0791)	(0.0398)
$\ln^2(k)$	0.0363	0.0362	0.0475	0.0450	0.0359
	(0.0020)	(0.0020)	(0.0037)	(0.0037)	(0.0020)
time	-0.0010	--	-0.0006	--	--
	(0.0005)		(0.0004)		
intercept	6.3576	6.3672	7.8367	7.4166	6.3385
	(0.1960)	(0.1983)	(0.4283)	(0.4227)	(0.2005)
Estimated ρ	1.0422	1.1709	0.2198	0.2825	1.3152
	(0.6103)	(0.7689)	(0.0503)	(0.0831)	(0.9860)
Implied σ	0.4897	0.4606	0.8198	0.7797	0.4319
# Obs	2964	2964	2964	2964	2886
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Note: This table uses data from PWT 6.3.

Table 9: Linear Specification – Panel PWT 6.3 N=78, DP, 1960-1987
Dependent variable: log(Y/L)

Variable	*Pooled OLS Col. (1)*	*Pooled OLS Col. (2)*	*FE Col. (3)*	*FE Col. (4)*	*IVFE Col. (5)*
ln(k)	-0.2817	-0.2866	-0.4366	-0.4761	-0.2695
	(0.0474)	(0.0474)	(0.0422)	(0.0427)	(0.0482)
$\ln^2(k)$	0.0468	0.0470	0.0475	0.0482	0.0462
	(0.0025)	(0.0025)	(0.0022)	(0.0022)	(0.0025)
time	-0.0032	--	0.0021	--	--
	(0.0008)		(0.0007)		
intercept	7.5149	7.4958	8.8946	9.1556	7.4107
	(0.2219)	(0.2261)	(0.2141)	(0.2205)	(0.2302)
Estimated ρ	0.2593	0.2593	0.1516	0.1373	0.2700
	(0.0396)	(0.0383)	(0.0125)	(0.0105)	(0.0440)
Implied σ	0.7941	0.7969	0.8611	0.8793	0.7874
# Obs	2182	2182	2182	2182	2104
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Note: This table uses data from PWT 6.3.

Table 10: Linear Specification – PWT 6.3 N=61 1970-2007, and PWT 6.1 N=56 1970-2000
Dependent variable: log(Y/L)

Variable	***PWT 6.3 FE Col. (1)***	***PWT 6.3 IVFE Col. (2)***	***PWT 6.1 FE Col. (3)***	***PWT 6.1 IVFE Col. (4)***
ln(k)	1.0737	0.1578	0.0770	0.3712
	(0.1238)	(0.0499)	(0.0844)	(0.0545)
$ln^2(k)$	-0.0150	0.0260	0.0319	0.0144
	(0.0056)	(0.0024)	(0.0042)	(0.0028)
intercept	0.2210	5.2371	5.5956	4.8304
	(0.6787)	(0.2601)	(0.4265)	(0.2653)
Estimated ρ	-0.3793	-0.3906	-0.8983	-0.1232
	(0.5400)	(0.1363)	(1.0199)	(0.0312)
Implied σ	1.6111	1.6409	9.8311	1.1405
# Obs	2318	2257	1734	1678
FE N	Yes	Yes	Yes	Yes
FE T	Yes	Yes	Yes	Yes

Note: Estimates in columns 1 and 2 were generated from a panel with 61 countries over the period 1970-2007 using data from PWT 6.3. Estimates in columns 3 and 4 were generated from a panel with 56 countries over the period 1970-2000 using data from PWT 6.1. In both cases, only countries for which the physical capital series were initiated in 1950 were included. That is, we are discarding the first twenty observations of the physical capital series so that the data should be free of any initial guess error. Additionally, in the PWT dataset, countries with longer time series present better quality data.

Table 11: Linear Specification – Panel PWT 6.3 N=93, 1970-2007
Dependent variable: log(Y/HL)

Variable	*Pooled OLS Col. (1)*	*Pooled OLS Col. (2)*	*FE Col. (3)*	*FE Col. (4)*	*IVFE Col. (5)*
ln(k)	-0.0792	-0.0684	-0.4763	-0.4056	-0.0551
	(0.0521)	(0.0519)	(0.3869)	(0.3895)	(0.0527)
$\ln^2(k)$	0.0372	0.0336	0.0575	0.0549	0.0359
	(0.0028)	(0.0028)	(0.0199)	(0.0199)	(0.0482)
time	-0.0046	--	-0.0046	--	--
	(0.0005)		(0.0011)		
intercept	6.3754	6.3122	8.2751	7.8372	6.2479
	(0.2424)	(0.2432)	(1.8543)	(1.8751)	(0.2101)
Estimated ρ	0.8705	1.0017	0.1635	0.1926	1.2350
	(--)	(--)	(--)	(--)	(--)
Implied σ	0.5346	0.4995	0.8594	0.8385	0.4474
# Obs	3534	3534	3534	3534	3441
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Notes: Standard errors appear in parentheses.

Table 12: Translog Function – Panel N=100, T=1970-2007
Dependent variable: log(Y)

Variable	***Pooled OLS Col. (1)***	***Pooled OLS Col. (2)***	***Fixed Effects Col. (3)***	***Fixed Effects Col. (4)***	***IVFE Col. (5)***
ln(L)	0.1501	0.1444	0.1584	0.1621	0.1338
	(0.0441)	(0.0439)	(0.0956)	(0.0945)	(0.0445)
ln(K)	-0.2619	-0.2445	-0.6147	-0.5512	-0.2342
	(0.0381)	(0.0382)	(0.0595)	(0.0589)	(0.0390)
$\ln^2$(L)	0.0180	0.0175	0.0028	-0.0012	0.0174
	(0.0036)	(0.0036)	(0.0135)	(0.0132)	(0.0037)
lnK*lnL	-0.0613	-0.0605	-0.0462	-0.0425	-0.0598
	(0.0047)	(0.0047)	(0.0127)	(0.0125)	(0.0048)
$\ln^2$(K)	0.0394	0.0388	0.0425	0.0404	0.0383
	(0.0018)	(0.0018)	(0.0037)	(0.0037)	(0.0018)
time	-0.0017	--	0.0020	--	--
	(0.0005)		(0.0008)		
intercept	7.7905	7.6994	11.8374	11.1516	7.6481
	(0.3186)	(0.3204)	(0.5084)	(0.4204)	(0.3515)
Wald Test	228.98	222.77	126.19	121.72	209.86
$\beta_1=\beta_2=\beta_3=0$	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	1027.73	1012.25	16.54	14.73	965.61
CRS	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
# Obs	3800	3800	3800	3800	3700
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Table 13: Translog Function – Panel N=111, T=1970-2007
Dependent variable: log(Y)

Variable	***Pooled OLS*** ***Col. (1)***	***Pooled OLS*** ***Col. (2)***	***Fixed Effects*** ***Col. (3)***	***Fixed Effects*** ***Col. (4)***	***IVFE*** ***Col. (5)***
ln(L)	1.1498	1.1424	1.2047	1.2128	1.1309
	(0.0399)	(0.0399)	(0.0908)	(0.0886)	(0.0406)
ln(K)	-0.3859	-0.3761	-0.4225	-0.3952	-0.3712
	(0.0441)	(0.0448)	(0.0621)	(0.0593)	(0.0457)
ln^2(L)	0.0274	0.0271	0.0254	0.0189	0.0267
	(0.0028)	(0.0028)	(0.0110)	(0.0106)	(0.0028)
lnK*lnL	-0.0706	-0.0699	-0.0744	-0.0692	-0.0689
	(0.0042)	(0.0042)	(0.0111)	(0.0107)	(0.0043)
ln^2(K)	0.0450	0.0446	0.0447	0.0432	0.0442
	(0.0019)	(0.0019)	(0.0036)	(0.0035)	(0.0019)
time	-0.0011	--	0.0032	--	--
	(0.0005)		(0.0008)		
intercept	8.9017	8.8830	9.8499	9.5238	8.8882
	(0.3383)	(0.3442)	(0.4447)	(0.4204)	(0.3515)
Wald Test	216.94	209.35	86.54	93.21	198.42
$\beta_1=\beta_2=\beta_3=0$	(0.00)	(0.00)	(0.00)	(0,00)	(0.00)
	271.16	261.93	4.69	4.19	248.55
CRS	(0.00)	(0.00)	(0.01)	(0.02)	(0.00)
# Obs	4211	4211	4211	4211	4100
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Table 14: Translog Function – Panel N=93, T=1970-2000
Dependent variable: log(Y)

Variable	*Pooled OLS*	*Pooled OLS*	*Fixed Effects*	*Fixed Effects*	*IVFE*
ln(L)	0.7687	0.7648	0.9848	0.9836	0.7568
	(0.0504)	(0.0509)	(0.1390)	(0.1397)	(0.0518)
ln(K)	-0.1586	-0.1504	-0.6646	-0.6409	-0.1380
	(0.0456)	(0.0461)	(0.0689)	(0.0703)	(0.0477)
ln^2(L)	0.0278	0.0276	0.0012	-0.0004	0.0279
	(0.0039)	(0.0039)	(0.0171)	(0.0171)	(0.0040)
lnK*lnL	-0.0538	-0.0535	-0.0419	-0.0406	-0.0533
	(0.0052)	(0.0052)	(0.0140)	(0.0139)	(0.0054)
ln^2(K)	0.0362	0.0359	0.0438	0.0431	0.0355
	(0.0022)	(0.0022)	(0.0038)	(0.0038)	(0.0023)
time	-0.0030	--	0.0030	--	--
	(0.0007)		(0.0008)		
intercept	8.4116	8.3752	13.2094	12.9681	8.2222
	(0.3739)	(0.3766)	(0.6473)	(0.6588)	(0.3872)
Wald Test $\beta_1=\beta_2=\beta_3=0$	113.43	109.12	116.75	109.47	99.03
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	128.32	124.46	4.09	3.89	115.28
CRS	(0.00)	(0.00)	(0.02)	(0.02)	(0.00)
# Obs	2783	2873	2873	2873	2780
FE N	No	No	Yes	Yes	Yes
FE T	No	Yes	No	Yes	Yes

Table 15: Elasticity of Substitution between Skilled and Unskilled Labor – Primary Cutoff
Dependent variable: log(Y/N)

Variable	***Pooled OLS Col. (1)***	***FE Col. (2)***	***FE Col. (3)***	***IVFE Col. (4)***
ln(K/N)	0.7166	0.6866	0.6981	0.7125
	(0.0220)	(0.0347)	(0.0364)	(0.0267)
ln(S/N)	0.1775	0.1396	0.1327	0.1844
	(0.0520)	(0.0374)	(0.0373)	(0.0666)
$\ln^2$(S/N)	0.0177	0.0440	0.0410	0.0192
	(0.0144)	(0.0084)	(0.0084)	(0.0165)
Time	-0.0139	-0.0034	--	-0.0089
	(0.0088)	(0.0061)		(0.0109)
Intercept	2.5787	2.8466	2.7480	2.5929
	(0.2525)	(0.3669)	(0.3811)	(0.3106)
Estimated ψ	0.1319	0.2309	0.2098	0.1501
	(0.0083)	(0.0070)	(0.0063)	(0.0116)
Implied θ	1.1519	1.3001	1.2655	1.1766
# Obs	319	319	319	273
FE N	No	Yes	Yes	Yes
FE T	No	No	Yes	No

Note:

Table 16: Elasticity of Substitution between Skilled and Unskilled Labor – Secondary Cutoff
Dependent variable: log(Y/N)

Variable	***Pooled OLS Col. (1)***	***FE Col. (2)***	***FE Col. (3)***	***IVFE Col. (4)***
ln(K/N)	0.7068	0.7025	0.7118	0.7032
	(0.0183)	(0.0301)	(0.0315)	(0.0222)
ln(S/N)	0.1102	0.1234	0.1123	0.1302
	(0.0404)	(0.0545)	(0.0552)	(0.0398)
$\ln^2$(S/N)	0.0055	0.0217	0.0197	0.0091
	(0.0090)	(0.0067)	(0.0068)	(0.0092)
Time	-0.0196	-0.0028	--	-0.0139
	(0.0087)	(0.0103)		(0.0103)
Intercept	2.7569	2.6607	2.5765	2.7860
	(0.2358)	(0.3519)	(0.3576)	(0.2868)
Estimated ψ	0.0251	0.1056	0.0907	0.0461
	(0.0021)	(0.0065)	(0.0052)	(0.0031)
Implied θ	1.0257	1.1181	1.0997	1.0484
# Obs	319	319	319	273
FE N	No	Yes	Yes	Yes
FE T	No	No	Yes	No

Note:

Table 17: Elasticity of Substitution between Skilled and Unskilled Labor – Tertiary Cutoff
Dependent variable: log(Y/N)

Variable	***Pooled OLS Col. (1)***	***FE Col. (2)***	***FE Col. (3)***	***IVFE Col. (4)***
ln(K/N)	0.7439	0.7038	0.7135	0.7408
	(0.0202)	(0.0301)	(0.0320)	(0.0241)
ln(S/N)	-0.0450	0.0866	0.0720	-0.0384
	(0.0708)	(0.0777)	(0.0797)	(0.0855)
$\ln^2$(S/N)	-0.0062	0.0096	0.0080	-0.0060
	(0.0085)	(0.0067)	(0.0069)	(0.0107)
Time	-0.0101	-0.0057	--	-0.0046
	(0.0086)	(0.0089)		(0.0105)
Intercept	2.0152	2.6596	2.5503	2.0383
	(0.2859)	(0.3775)	(0.3901)	(0.3357)
Estimated ψ	-0.0143	0.0385	0.0299	-0.0142
	(0.0021)	(0.0029)	(0.0023)	(0.0026)
Implied θ	0.9859	1.0401	1.0309	0.9860
# Obs	318	318	318	272
FE N	No	Yes	Yes	Yes
FE T	No	No	Yes	No

Note: