# The Role of Information in Multistage R&D Races

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**Abstract**

R&D races typically involve the development of several intermediate steps and in some industries information about the competitor's development stage is strategic. This paper defines and shows the existence of equilibrium for the incomplete information multistage R&D race model where firms need to complete a finite number of intermediate steps before the product or project is finished. In our model, there is incomplete information because firms may not know the competitor's development stage during the race.

By using numerical methods, we study the equilibrium of this race in pure Markov strategies. We show how the race duration, consumer surplus, firm value and total welfare vary with investment costs, market size and intensity of competition in the products market. We are able to assess the impact of a strong patent regime in terms of consumer surplus, firm value and total welfare.

**PRELIMINARY VERSION**

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# 1 Introduction

R&D races are inherently dynamic processes. The innovation process usually requires the development of several intermediate stages before the product is completed. In addition, in many markets, each firm may not know the other firm's development stage and this information is strategic. Secrecy surrounding the development of new products is an important characteristic of industry competition. One example is the computer industry.

We consider a two-firm multistage R&D race where a firm's success in advancing from one stage of development to the other is a stochastic process that depends on the firm's investment. As the firms invest more, it becomes more likely that they will advance to the next development stage in the next period. The cost of investing in R&D may vary across stages of development. We also allow the possibility of the firms exhibiting better abilities in R&D than their competitor's at some stages of the race.

The objective of this paper is to extend this basic multistage R&D race to study the impact of incomplete information in relation to the other firm's development stage. We suppose each firm may not be perfectly informed about the other firm's development stage. However, as the firms go along the development process, they may receive some information about the other firm's development stage and adjust their investment accordingly.

This class of multistage R&D models is general enough to study different types of information structures. Both the complete information structure, where each firm knows exactly the other firm's stage, and the private information structure, where the firm only knows its own stage can be studied using this class of models. In our model, the stage in the R&D race may be a privately informed variable that is persistent over time.

In the first part of the paper we define precisely the incomplete information R&D race and prove existence the of equilibrium in pure Markov investment strategies. We call the investment strategies Markov because they do not depend on the history of information regarding the other firm's stage, only the information at that period. The existence proof is an application of Polydoro [10]. The author proves existence of equilibrium for a general class of stochastic incomplete information games, where the law of motion in the state space is Markov.

We provide a numerical analysis of these R&D races under four different information structures. For each information structure we compare the case where there are no patents and the case where there is a strong patent regime. The information structures are linearly ordered. That is, when comparing two information structures we can identify in which structure the firms have more information.

When there are no patents at the end of the race, the first firm to complete the project gets the monopolistic profits in the product's market. Once the second firm completes the project, they share the duopolistic profits equally. In our patent regime, the first firm to complete the project gets a patent with no expiration and becomes the monopolist in the product's market forever. In both cases, intermediate developments and the whole product cannot be imitated. Firms need to develop their own product in order to sell it on the products market.

These two types of environments are good approximations for some industries. In the computing industry, when Apple launched the Ipad a patent was awarded to the product, but not to the idea. Then firms started developing their own hardware and software to make a touchscreen tablet and compete with the Ipad. In the pharmaceutical industry, once a new drug is developed a patent is awarded and the product cannot be imitated. Although a drug patent does not last forever we can think of this as an example of our patents regime.

Our comparison between information structures and patent regimes focuses on three key variables of the model. The first is the size of the product's market, the second is the cost of investing in R&D, and the third is the degree of competition in the product's market.

In our simulated model, optimal investment strategies increase as firms go along the race. When firms are closer to the end of the race it becomes more likely that they will finish the product and start receiving the monopolistic (or duopolistic) profits. Further, firms increase investment as a response to the advance of the other firm when they are leading the race and close to the end of the race. On the other hand, firms lower investment when the other firm advances if they are behind in the race.

For the class of R&D models we study in this paper, more information is good for firms and consumers. Both consumer surplus and firm value is higher when firms know more about the other firm's development stage. With more information they are able make a better assessment if a marginal investment in R&D will outweigh its cost. Furthermore, the race duration decreases when firms have more information resulting in higher consumer surplus. Firm value also increases. The difference in welfare between information structures increases when both market size and investment cost increase. The intensity of market competition, on the other hand, has the opposite effect. When firms are more able to coordinate their production in the products market duopoly, the difference in consumer surplus, firm value and total welfare is smaller across information structures.

The impact of adopting a strong patent regime where the first firm to complete the project gets an infinitely lasting patent is not the same on each information structure. Its impact depends on the combination between market size and development costs. Still, the effect of a patent regime is not necessarily good for consumer surplus. In general, when the market is small, a patent regime yields higher consumer surplus, but when the market is bigger, having no patents is better. Total welfare and firm value, on the other hand, are higher for the no patents regime.

This paper is directly related to Fershtman and Markovich [2]. The authors study the impact of different patent regimes on a complete information multistage R&D model where firm cost profiles may differ. That is, there is cost asymmetry between firms. The model presented in this paper is more general because it takes into consideration the possibility that firms may not be perfectly informed about the competitor's development stage. Still, as the objective of this paper is the to study the effect of information on the race we rule out the possibility of cost asymmetries between firms.

Multistage R&D races have a long tradition in the industrial organization literature (see for example, Fudenberg et al [3], Grossman and Shapiro [5], Harris and Vickers [6], Doraszelski [1], Judd [7] to cite a few). Unlike Fudenberg et al [3], Grossman and Shapiro [5] and Harris and Vickers [6]'s model, we don't have an information lag, we only have information constraints. Firm's may not be able to assess correctly in which part of the race the competitor is, but when they switch to the other part they receive this information. Also, both firms start the R&D race in the same stage, at the same time, and there is no leader or follower ex-ante. Firms do not accumulate experience: the probability of success of their investment in R&D is random and firms only advance one period at a time. Moreover, our model's optimal strategy does not exhibit the "leapfrogging" effect. When the firm knows that it may be behind in the race, it lowers investment.

This paper is organized as follows: in the next section we present the benchmark R&D race model and prove equilibrium existence; then, we simulate the model and analyze the impact of some key variables of the model on the race duration and welfare; and the last section concludes the paper.

## 2 Model

In this section we describe our benchmark incomplete information R&D model and prove the existence of equilibrium for this class of models.

We study a multistage R&D race model where two firms compete over the development of a new product or technology. In order to finish the product, each firm must complete $\overline{k} > 1$ intermediate steps. We denote by $\overline{k}+1$ the state in which the firm has completed the product and started selling it on the market. In this benchmark case there are no patents. That is, the other firm can keep investing in the project and once it finishes both firms share the products' market. We also consider the case where a patent with no expiration is awarded to the first firm to complete the project.

In our R&D model, each firm is perfectly informed about its development stage, but may not know the other firm's development stage. For example, suppose that in order to complete a project, firms need to complete four intermediate steps, e.g. $\overline{k} = 4$. It could be that each firm only knows whether the other is in the first half $\{1, 2\}$ or in the second half $\{3, 4\}$, but not their exact stage.

Let $K_i = \{1, \cdots, \overline{k}+1\}$ be the set of development stages of the race for firm $i = 1, 2$. We model each firm's information about the other firm's development stage as a partition of $K_{-i}$, which we denote by $P^i$. We assume that the other firm always knows whether or not the firm i has completed the project. That is, $P^i$ contains $\overline{k}+1$. Then, firm i's information set at stage $(k_i, k_{-i})$ is composed by its own stage $k_i \in K_i$ and the element $P_i(k_{-i}) \in P^i$ containing $k_{-i}$. That is, firm i's information set at $(k_i, k_{-i}) \in K_i \times K_{-i}$ is $B_i(k_i, k_{-i}) = \{k_i\} \times P_i(k_{-i})$. We denote the family of possible information sets for firm $i$ by $\mathcal{B}_i = K_i \times P^i$.

By setting $P_i$ accordingly, we can have different information structures about the stage of the R&D race. If $P_i(k_{-i}) = \{k_{-i}\}$ for each $k_{-i} \in K_{-i}$, we have a complete information setup where each firm knows the competitor's development stage. On the other hand if $P_i = \{1, \cdots, \overline{k}\}$ for each $k_{-i} \in \{1, \cdots, \overline{k}\}$ we have the private information case where each firm only knows its development stage during the race. We can also have other types of information structures. For example, if $\overline{k}$ is odd, $P_i(k_{-i}) = \{1, \frac{\overline{k}}{2}\}$ if $k_{-i} \leq \overline{k}$, $P_i(k_{-i}) = \{\frac{\overline{k}}{2}+1, \cdots, \overline{k}\}$ for $k > \frac{\overline{k}}{2}+1$ and $k \leq \overline{k}$ and $P_i(k_{-i}) = \overline{k}+1$

if $k_{-i} = \overline{k}_{-i} + 1$, firm i knows whether the other firm is in the first half, the second half of the race, or if it has already finished. Note that $P_i$ may not be the same for both firms. This framework is general enough to handle the case where one firm has more information during the race than the other.

In our setup, each firm may be uncertain about the other firm's development stage, uncertain about what the other firm thinks is their development stage, and so on. To model this interactive uncertainty we endow each firm with beliefs about the other firm's development stage for each information set in $\mathcal{B}_i$. Hence, firm i's belief[1] is a list of probability distributions $\lambda_i \in \Delta^{\mathcal{B}_i}(K_{-i})$, one for each information set in $B_i \in \mathcal{B}_i$, and it does not change over time.

Firms can only advance one stage at a time. Also, moving from development stage $k_i$ to $k_i + 1$ is a stochastic process depending on the firm's investment in R&D. Let $x_i \in [0, 1]$ denote firm i's investment. We assume that the probability of success at stage $k_i$ for firm $i$ is a linear combination between two probability distributions $\mu_{1,i}(\cdot|k_i), \mu_{2,i}(\cdot|k_i) \in \Delta(K_i)$:

$$Q_i(x_i)(k_i + 1|k_i) = x_i\mu_{1,i}(k_i + 1|k_i) + (1 - x_i)\mu_{2,i}(k_i + 1|k_i),$$

where $\mu_{1,i}(k_i + 1|k_i) = p_{1,i}$, $\mu_{1,i}(k_i|k_i) = 1 - p_{1,i}, \mu_{2,i}(k_i + 1|k_i) = p_{2,i}$ and $\mu_{2,i}(k_i + 1|k_i) = 1 - p_{2,i}$ for each $k_i < \overline{k}$. If $k_i = \overline{k} + 1$, then firm $i$ is at the end of the race and $Q_i(x_i)(\overline{k} + 1|\overline{k} + 1) = 1$ for each $x_i \in [0, 1]$.

We suppose $p_{1,i} > p_{2,i}$. That is, $\mu_{1,i}$ first order stochastically dominates $\mu_{2,i}$ at each $k_i \in K_i$. With this assumption on the R&D process, as firms invest more in R&D, it becomes more likely that they will advance to the next stage. At $\overline{k} + 1$ the product or project is already completed and the firm remains in that state forever regardless of its their investment level. If firm $i$ is unsuccessful on moving from stage $k_i$ to stage $k_i + 1$, it can try again in the next period.

The cost of investing $x_i$ to advance for the $k_i + 1 - th$ development stage is $c_{i,k}x_i^2$, where $c_{i,k}$ is a positive constant. The cost $c_{i,k}$ may vary across firms. That is, firms can have a comparative advantage over the other firm in developing the product at some stage. For example, in the pharmaceutical industry small firms have a comparative advantage in creating new drugs, but larger firms have an advantage once the drug reaches the FDA approval stage as this stage requires large amounts of investment in testing and paperwork filing. We denote firm $i$'s cost profile by $c_i = \{c_{i,1}, \cdots, c_{i,\overline{k}}\}$.

Once the first firm reaches the $\overline{k} + 1$-th stage it starts receiving the reward $\pi^M$, which is the monopolistic profit in the product's market at each period until the other firm completes the project. When both firms reach the end of the race, the market becomes a duopoly and each firm's profit is $\pi^D < \frac{\pi^M}{2}$. In the application section we consider the case where a patent that lasts forever is awarded to the first firm completing the project. Firms discount future profit at rate $\beta$.

**Definition 2.1** *A multistage incomplete information R&D race R is a tuple:* $(N = 2, \overline{k}, (P^i, \lambda_i, c_i)_{i\in N}, \pi^M, \pi^D, \beta)$.

[1]In fact, firm i's belief is a Conditional Probability System ( Myerson [8] ). That is, a list of conditional probability distributions one for each $B_i \in \mathcal{B}_i$ that is well defined even if $B_i$ has zero probability ex-ante. For details on Conditional Probabilities Systems and more specifically on the general class of games that contains the model presented in this paper see Polydoro [10].

In this paper, a firm's investment strategy depends only on its information set and is not randomized. That is, its choice of investment level does not depend on any history of observable events, e.g. its investment strategy is Markovian. A pure Markov investment strategy for firm $i$ is a function $\sigma_i : \mathcal{B}_i \to [0,1]$. The space of strategies for firm $i$ is $\Sigma_i$ and the space of strategies for both firms is $\Sigma = \times\Sigma_i$.

As the R&D race may last for many periods, it is useful to present its discounted payoff in a recursive way. First, define a value function for firm $i$ as a mapping $V_i : \mathcal{B}_i \to [0, \frac{\pi^M}{1-\beta}]$ that summarizes expected discounted payoff of future periods. Given a value function and a strategy profile we can define the expected profit by a function $h_i : \mathcal{B}_i \times \Sigma \times V_i \to [0, \frac{\pi^M}{1-\beta}]$ as follows:

$$h_i(B_i; \sigma, v_i) = \pi(B_i, \sigma_i) + \beta \sum_{k \in K} \sum_{k' \in K} Q(k'|\sigma(B_i, B_{-i}(k)), k)\lambda(k_{-i}|B_i)V_i(B_i(k')), \tag{1}$$

where $\pi(B_i, \sigma_i) = A(B_i) - c_{i,k_i}\sigma_i(B_i)^2$ and $A(B_i) = \pi^M$ if $k_i = \{\overline{k}+1\}$ and $P_i \neq \{\overline{k}+1\}$, $A(B_i) = \pi^D$ if $k_i = \{\overline{k}+1\}$ and $P_i = \{\overline{k}+1\}$ and zero otherwise. For the case where a patent is awarded to the first firm to complete the project we adjust this formulation accordingly.

An equilibrium in pure Markov investment strategies for a multistage incomplete information R&D race is a list of functions $(\sigma_i^*, V_i)_{i\in N}$ such that $\sigma_i^*(B_i) \in \arg\max_{x_i \in [0,1]} h_i(B_i; x_i, \sigma_{-i}^*, V_i)$ and $V_i(B_i) = h_i(B_i, \sigma^*, V_i)$ for each $B_i \in \mathcal{B}_i$. Equilibrium for this race is such that each firm picks an investment level that maximizes expected discounted profits given that the other firm follows their equilibrium strategy for each information set. In addition, the value function is equal to the expected profit in the equilibrium.

**Proposition 2.1** *There exists an equilibrium in pure Markov investment strategies for a dynamic R&D race with incomplete information.*

**Proof.** Fix a dynamic incomplete information R&D race $R$. To show equilibrium existence in pure Markov investment strategies for this game we apply the existence theorem in Polydoro [10]. Hence, we have to show that $R$ satisfies the theorem requirements. That is, the payoff function is twice continuously differentiable with respect to the player's actions, concave in the player's own action and has increasing differences and satisfies the strict diagonal dominance condition (Gabay and Moulin [4]). The author's existence theorem also imposes some restrictions on the belief mapping and on the stochastic process over the state space. Players' beliefs are required to have finite support[2] and the Markov law of motion is required to be a linear combination between two probability distributions over the state space for each state.

The last two sets of assumptions, on the belief mapping and on the Markov law of motion are trivially satisfied. The belief mapping has finite support because the state space is finite, and from the definition of a dynamic R&D race with incomplete information, the Markov law of motion is a linear combination between two probability distributions.

[2] The no-delusion property, which states that there is no type of other players that are considered impossible by all other players' types is trivially satisfied since there is only one belief type

It remains to show that the profit function satisfies the theorem's requirements. The profit function is clearly twice continuously differentiable in $x$ and concave in $x_i$. Next, we show that it also satisfies increasing differences $\frac{\partial^2 \pi_i}{\partial x_i \partial x_{-i}}(k, x_i) \geq 0$ and strict diagonal dominance $\left|\frac{\partial^2 \pi_i}{\partial x_i^2}(k, x_i)\right| > \left|\frac{\partial^2 \pi_i}{\partial x_i \partial x_{-i}}(k, x_i)\right|$. Taking the second order derivative of the profit function with respect to investment we get $\frac{\partial^2 \pi_i}{\partial x_i \partial x_{-i}}(k, x_i) = 0$ and $\left|\frac{\partial^2 \pi_i}{\partial x_i^2}(k, x_i)\right| = 2c_{i,k} > 0$. Therefore, both conditions are satisfied.

Now we can apply the existence theorem in Polydoro [10] to show that there exists an equilibrium in pure Markov investment strategies for the multistage incomplete information R&D race $R$. ■

# 3 Analysis of R&D races

In this section we study the impact of different information setups on a simulated multistage R&D race model. First, we present the parameters of the model. Then we present details of the numerical analysis, the interpretation of the equilibrium strategies and value function; and at the end of this section we present some comparative statistics on key parameters of the model.

## 3.1 Parameter Values

In our simulations each period corresponds to a quarter. We set the discount rate $\beta = 0.97$, which corresponds to an annual interest rate of 10%. In order to capture the effect of the dynamic of the model and allow different information structures, we set the length of the race to $\overline{k} = 6$. That is, firms need to complete 6 intermediate steps to complete the product and start selling it.

We suppose the demand function in the product's market is given by $p = \frac{20}{100} - \frac{q}{100}$ and that there are no production costs. Under this specification of the demand function, monopolistic profits are $\pi^M = 1$ and the consumer surplus at the monopolistic price is $CS^M = .5$. The duopolistic payoff is $\pi^D = \mu\pi^M$, where $0 \leq \mu < .5$ captures the intensity of duopolistic competition in the product's market. For example, if $\mu = 0$, we have a Bertrand competition. On the other hand, if $\mu = .4$, we have some type of collusive set up. The optimal quantity to be produced by both firms in the duopolistic market for a fixed $\mu$ is given by:

$$q^D(\mu) = 10 + 10\sqrt{1 - 2\mu}.$$

At the optimal duopolistic quantity $q^D(\mu)$, consumer surplus is given by $CS^D(\mu) = \frac{q^D(\mu)^2}{200}$.

We study a cost structure where firms have different abilities to invest in R&D at different stages of the race: $c_i = (\delta, \delta, \delta, 1, 1, 1)$ where $\delta \geq 1$. Under this cost structure specification firms are better at developing the product in earlier stages of the race. We suppose both firms have the same cost structure.

To capture the fact that both firms know when the race starts we add an auxiliary state that corresponds to the first development stage and denote it by 0. Then, the race state space is $K = \{0, 1, \cdots, 6, 7\}$, where the state 7 corresponds to the end of the race.

We consider four different information structures for the middle of the race. The first is a private information structure. Each firm only knows its development stage and does not have any information about the other firm's development stage: $B_i^1(k_i, k_{-i}) = \{k_i\} \times \{1, \cdots, 6\}$ for $k_{-i} \in \{1, \cdots, 6\}$. In the second information structure, each firm knows if the other firm is in the first or the second half of the race: $B_i^2(k_i, k_{-i}) = \{k_i\} \times \{1, 2, 3\}$ if $k_{-i} \in \{1, 2, 3\}$ and $B_i^2(k_i, k_{-i}) = \{k_i\} \times \{4, 5, 6\}$ if $k_{-i} \in \{4, 5, 6\}$. In the third information structure, we divide the middle of the race into three pieces: $B_i^3(k_i, k_{-i}) = \{k_i\} \times \{1, 2\}$ if $k_{-i} \in \{1, 2\}$, $B_i^3(k_i, k_{-i}) = \{k_i\} \times \{3, 4\}$ if $k_{-i} \in \{3, 4\}$ and $B_i^3(k_i, k_{-i}) = \{k_i\} \times \{5, 6\}$ if $k_{-i} \in \{5, 6\}$. In the fourth and last information structure, each firm knows exactly the other firm's development stage: $B^4(k_i, k_{-i}) = \{k_i, k_{-i}\}$. In addition, at each information structure both firms know if the other is at the beginning of the race or at the end. That is, at state $k = 0$ we have $B_i^j(k_i = 0, k_{-i} = 0) = \{0, 0\}$ and at $k_{-i} = 7$ we have $B_i^j(k_i, k_{-i} = 7) = \{k_i, 7\}$.

Each firm's belief about the other firm's development stage is the uniform distribution over the observable event $B_i$. For example, suppose $k_1 = 1$, $k_2 = 4$ and firms have the second information structure. Then, $B_1^2(1, 4) = \{1\} \times \{4, 5, 6\}$, $B_1^2(1, 4) = \{4\} \times \{1, 2, 3\}$, $\lambda_1^2(\{4, 5, 6\})(k_2) = \frac{1}{3}$ for each $k_2 \in \{4, 5, 6\}$ and $\lambda_2^2(\{1, 2, 3\})(k_2) = \frac{1}{3}$ for each $k_1 \in \{1, 2, 3\}$.

Given an investment level $x_i$, the probability that firm $i$ will advance from state $k_i$ ($k_i > 1$ and $k_i \neq 7$) to $k_i + 1$ is $Q_i(k_i + 1|x_i, k_i) = q_1 x_i + (1 - x_i) q_2$. As the state $k = 0$ only means that the race is at the beginning, the probability that firm $i$ will move to the second development stage $k = 2$ is $Q_i(2|x_i, k_i = 0) = q_1 x_i + (1 - x_i) q_2$. If the firm is at the end of the race, it remains there forever $Q_i(k_i = 7|x_i, k_i = 7) = 1$. Firms can only advance one stage at a time; therefore, the probability that the firm will remain in the same stage is $Q_i(k_i|x_i, k_i) = 1 - Q_i(k_i + 1|x_i, k_i)$ for $k_i \neq 7$ and $Q_i(1|x_i, k_i = 0) = 1 - Q_i(2|x_i, k_i = 0)$ if $k_i = 0$. We fix $q_1 = .3$ and $q_2 = \frac{q_1}{4}$, so there is a positive probability that the firm will advance to the next development stage regardless of its investment level.

Note that the Markov law of motion $Q$ and the cost structure are the same for both firms. Therefore, the only source of asymmetry between firms in our numerical exercise is their information regarding the other firm's development stage.

### 3.2 Numerical Analysis

In order to calculate equilibrium in pure Markov investment strategies for the R&D race, we adapt the Pakes and McGuire [9] iterative procedure. Before we start the calculation, we set the value function $V^0$ to the discounted monopoly profits at $k_i = 7$ and $k_{-i} \neq 7$ and the discounted duopoly profits for $k = \{7, 7\}$. Also, we set the initial investment strategy $x^0$ to zero.

In the first step we calculate the optimal strategy $x^1$ at each observable event given that the other firm follows $x^0$ and the value function is $V^0$. Then, the value function $V^1$ is equal to the expected profits when both firms employ the strategy $x^1$ and the value function is $V^0$. We repeat this procedure until $\{V^k, V^{k-1}\}$ and $\{x^{k+1}, x^k\}$ are close pointwise[3].

[3] We set the stopping criteria to $\epsilon = 10^{-6}$

### 3.3 Strategies and Value Function

We start by examining the strategic interaction between firms on each information structure for a specific set of parameters. Tables 1-4 present the equilibrium investment strategies and tables 5-8 present firms' value functions for the case where there are no patents, one table for each information structure. In this simulation, the parameters of the model are $\mu = 0.25$, $\pi^M = 1$ and $\delta = 2$. We pick this specific set of parameters to guarantee that firm strategy is not degenerate, i.e. firms make the maximum possible investment at each state, and the value of the firm changes as they go along the race. For example, if the cost of investing were too low both firms would invest the maximum possible and the value function would be roughly the same at each information structure. Therefore there would be no possibility of interpreting equilibrium strategies. In Appendix C we present the optimal investment strategies and value function for the case where a patent that lasts forever is awarded to the first firm to complete the project.

The first characteristic of the optimal strategies is that firms invest more as they become closer to the end of the race. As investing in R&D is costly, when firms become closer to the end of the race, it becomes more likely that they will complete the project and start receiving profits in the product's market. Hence, in our setup, research in the early steps receives less investment than product finishing. Note that changing the cost structure does not change this characteristic of equilibrium investment strategies because investing at later stages costs more.

In the first and second information structures, investment is decreasing in the other firm's stage. Given that the other firm is closer to the end of the race, firms lower investment, because it is less likely that they will arrive at the end first and receive the monopolistic profits. Firms take into consideration the fact that if the other firm finishes first they have to share the product's market and get the duopolistic profits.

In the third information structure, investment is not decreasing with the other firm's stage in the 6th stage. At the 6th stage the firm is already very close to the end, so if the other firm is at the beginning $k_{-i} \in \{1, 2\}$, it is better to cut investments. Still, when the other firm moves to $\{3, 4\}$, the firms increase investments. On the other hand, if the other firm is in the last event $\{5, 6\}$, it is already too close to the end. Then, firms lower investment because they know that they would probably have to share the product's market profit. In the fourth information structure, equilibrium strategy has the same characteristic, but this starts in the 4th stage.

Note that since we present the optimal strategy for the case where there are no patents and the market may end up as a duopoly, firms invest even when the other firm has already completed the project. Firms know that once they finish the project themselves, they get the duopolistic profits in the product's market. Still, investment decreases significantly.

Firm value at each observable event follows the same pattern as investment strategy. It increases when firms become closer to the end of the race, decreases significantly when the other is at the end of the race, and also decreases when the other firm is closer to the end of the race. The only exception to this pattern is in the fourth information structure when $k_i = \{6\}$. When the other firm moves from $k_{-i} = \{1\}$ to $k_{-i} = \{2\}$, firm value increases. This increase in the firm's value follows from the fact that they invest more, so it becomes more likely that they will finish the race earlier and get the monopolistic profits for a larger number of periods.

In the other case where a patent is awarded to the first firm to arrive at the end of the race, the equilibrium strategy changes. Investment is increasing in the other firm's stage when the firm is in the lead of the race. Then, it decreases when the other firm catches up and decreases further as the other firm advances toward the end of the race. This effect on strategies is driven by our "winner takes all" setup. Even if both firms complete the project, only the firm that finished first can explore the product's market. Also, firms cancel investment once they learn that the other completed the project.

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2,3,4,5,6} | {7} |
|---|---|---|---|
| 0 | 0.07 | - | - |
| 1 | - | 0.07 | 0.03 |
| 2 | - | 0.10 | 0.04 |
| 3 | - | 0.16 | 0.05 |
| 4 | - | 0.40 | 0.13 |
| 5 | - | 0.53 | 0.17 |
| 6 | - | 0.67 | 0.22 |
| 7 | - | 0.00 | 0.00 |

Table 1: Optimal Investment Strategy for Information Structure 1 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2,3} | {4,5,6} | {7} |
|---|---|---|---|---|
| 0 | 0.09 | - | - | - |
| 1 | - | 0.09 | 0.04 | 0.03 |
| 2 | - | 0.14 | 0.06 | 0.04 |
| 3 | - | 0.21 | 0.11 | 0.05 |
| 4 | - | 0.46 | 0.31 | 0.13 |
| 5 | - | 0.56 | 0.46 | 0.17 |
| 6 | - | 0.65 | 0.64 | 0.22 |
| 7 | - | 0.00 | 0.00 | 0.00 |

Table 2: Optimal Investment Strategy for Information Structure 2 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2} | {3,4} | {5,6} | {7} |
|---|---|---|---|---|---|
| 0 | 0.10 | - | - | - | - |
| 1 | - | 0.10 | 0.05 | 0.03 | 0.03 |
| 2 | - | 0.16 | 0.09 | 0.05 | 0.04 |
| 3 | - | 0.22 | 0.16 | 0.08 | 0.05 |
| 4 | - | 0.48 | 0.44 | 0.25 | 0.13 |
| 5 | - | 0.56 | 0.54 | 0.39 | 0.17 |
| 6 | - | 0.64 | 0.66 | 0.60 | 0.22 |
| 7 | - | 0.00 | 0.00 | 0.00 | 0.00 |

Table 3: Optimal Investment Strategy for Information Structure 3 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1} | {2} | {3} | {4} | {5} | {6} | {7} |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.12 | - | - | - | - | - | - | - |
| 1 | - | 0.12 | 0.09 | 0.06 | 0.04 | 0.03 | 0.03 | 0.03 |
| 2 | - | 0.18 | 0.16 | 0.11 | 0.07 | 0.05 | 0.04 | 0.04 |
| 3 | - | 0.24 | 0.23 | 0.20 | 0.13 | 0.08 | 0.06 | 0.05 |
| 4 | - | 0.48 | 0.49 | 0.48 | 0.42 | 0.28 | 0.17 | 0.13 |
| 5 | - | 0.55 | 0.56 | 0.57 | 0.56 | 0.47 | 0.28 | 0.17 |
| 6 | - | 0.64 | 0.63 | 0.64 | 0.65 | 0.63 | 0.50 | 0.22 |
| 7 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4: Optimal Investment Strategy for Information Structure 4 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2,3,4,5,6} | {7} |
|---|---|---|---|
| 0 | 2.21 | - | - |
| 1 | - | 2.17 | 1.33 |
| 2 | - | 3.40 | 1.85 |
| 3 | - | 5.34 | 2.57 |
| 4 | - | 8.32 | 3.56 |
| 5 | - | 12.06 | 4.78 |
| 6 | - | 17.01 | 6.35 |
| 7 | - | 23.34 | 8.33 |

Table 5: Firm's Value Function for Information Structure 1 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2,3} | {4,5,6} | {7} |
|---|---|---|---|---|
| 0 | 2.9 | - | - | - |
| 1 | - | 2.84 | 1.49 | 1.33 |
| 2 | - | 4.56 | 2.23 | 1.85 |
| 3 | - | 7.21 | 3.44 | 2.58 |
| 4 | - | 11.11 | 5.48 | 3.56 |
| 5 | - | 15.34 | 8.44 | 4.78 |
| 6 | - | 20.47 | 12.83 | 6.35 |
| 7 | - | 26.43 | 19.06 | 8.33 |

Table 6: Firm's Value Function for Information Structure 2 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1,2} | {3,4} | {5,6} | {7} |
|---|---|---|---|---|---|
| 0 | 2.99 | - | - | - | - |
| 1 | - | 2.92 | 1.66 | 1.37 | 1.33 |
| 2 | - | 4.83 | 2.66 | 1.96 | 1.85 |
| 3 | - | 7.79 | 4.45 | 2.89 | 2.58 |
| 4 | - | 11.87 | 7.52 | 4.44 | 3.56 |
| 5 | - | 16.32 | 11.69 | 6.86 | 4.78 |
| 6 | - | 21.44 | 16.75 | 10.72 | 6.35 |
| 7 | - | 27.30 | 22.85 | 16.68 | 8.33 |

Table 7: Firm's Value Function for Information Structure 3 - No Patents

| $(k_i, P_i(k_{-i}))$ | {0} | {1} | {2} | {3} | {4} | {5} | {6} | {7} |
|---|---|---|---|---|---|---|---|---|
| 0 | 3.30 | - | - | - | - | - | - | - |
| 1 | - | 3.30 | 2.34 | 1.69 | 1.40 | 1.34 | 1.33 | 1.33 |
| 2 | - | 5.62 | 4.14 | 2.86 | 2.10 | 1.91 | 1.86 | 1.85 |
| 3 | - | 8.96 | 7.12 | 5.07 | 3.40 | 2.79 | 2.60 | 2.58 |
| 4 | - | 13.30 | 11.40 | 8.87 | 6.01 | 4.36 | 3.69 | 3.56 |
| 5 | - | 17.70 | 15.86 | 13.35 | 10.04 | 7.12 | 5.31 | 4.78 |
| 6 | - | 22.78 | 20.97 | 18.54 | 15.28 | 11.71 | 8.19 | 6.35 |
| 7 | - | 28.60 | 26.80 | 24.38 | 21.20 | 17.62 | 13.34 | 8.33 |

Table 8: Firm's Value Function for Information Structure 4 - No Patents

### 3.4 Comparative Statics on Model Parameters

In this section we study the effect of some key parameters of the model in the summary statistics of the race. We study the effect of investment costs ($\delta$), intensity of market competition ($\mu$) and market size ($\alpha$). The market size is a constant that multiplies monopolist profit.

For the comparative statics on each parameter, we present two sets of graphs. The first is the duration of the race before the first firm finishes the race and the market becomes a monopoly and the expected time before the market becomes a duopoly. Also, we present the expected time for the market to become a monopoly for the case where there are patents.

There is also a second set of graphs with welfare statistics. We present consumer surplus, firm value at the beginning of the race, and total welfare for both the no patents and the patents case in the equilibrium calculated. In each graph we plot the summary statistics for each information structure.

Together, firm strategy and the law of motion on the stage of development, imply a stochastic process in the stage of development of both firms. Then, we can use this fact to calculate the expected time until one firm finishes the race and the expected time for both firms to finish the race (for the no patents case).

The duration of the race has a direct impact on consumer welfare. During the time where the product does not exist, consumers get zero surplus. Therefore, lowering the time to get the first invention increases consumer welfare. In addition, for the case where there are no patents, the expected time before the second firm finishes the race also affects consumer welfare. When the market becomes a duopoly, firms are not able to perfectly coordinate and share equally the monopolistic profits, hence consumer surplus increase ($\pi^D = \mu\pi^M < \frac{\pi^M}{2}$).

As an example, Table 9 presents these summary statistics for the R&D race presented in the last section.

| | No Patents | | | | Patents | | | |
|---|---|---|---|---|---|---|---|---|
| | Info 1 | Info 2 | Info 3 | Info 4 | Info 1 | Info 2 | Info 3 | Info 4 |
| Duration to Monopoly | 25.65 | 25.19 | 25.08 | 24.61 | 25.64 | 24.72 | 24.17 | 23.04 |
| Duration to Duopoly | 393.45 | 365.06 | 361.08 | 341.16 | - | - | - | - |
| Consumer Surplus | 7.63 | 7.74 | 7.76 | 7.88 | 7.63 | 7.85 | 7.98 | 8.26 |
| Firm's Value | 2.17 | 2.84 | 2.92 | 3.30 | 0.96 | 1.93 | 2.11 | 2.72 |
| Total Welfare | 9.80 | 10.58 | 10.69 | 11.18 | 8.59 | 9.78 | 10.09 | 10.98 |

Table 9: Summary Statistics for $\mu = 0.25$, $\pi^M = 1$ and $\delta = 2$.

In this specific R&D race, as firms know more about the other firm's development stage, the time before the first invention decreases. With less information about the other firm's development stage, firms invest less. Investing in R&D is costly. Hence, if it turns out that the other firm has already completed the project and there are no patents, the other firm keeps investing, but a smaller amount. Then, the time before both firms finish the product is much higher than the time it takes for the first firm to complete the product.

Lowering the time before the first firm completes the project increases consumer surplus. Therefore, as firms have more information about the other firm's stage, consumer surplus is higher. Also, firm value increases when firms have more information. More information is positive for firms because they are better able to assess the cost and benefits of investment. Hence, total welfare follows the same pattern.

Comparing consumer surplus for the no patents and the patents case, we get the standard argument in favor of patents. For this set of parameters, offering patent protection lowers the expected time to project completion. This gain in consumer welfare from lowering the expected duration of the race is higher than the benefits of having a second firm in the product's market. Still, in the patents case, firm value decreases significantly. In the strong patent regime that we consider in this paper, all investment is thrown away if the other firm finishes the project first. The overall effect on welfare of a patent regime is negative, but it decreases as firms have more information. For this example, a strong patent is worse for society, especially if firms don't have very good information about the other firm's development stage.

### 3.5 The Effect of Market Size

We now discuss the effect of market size $\alpha$ on the race's summary statistics for each information setup and patent regime. There are three sets of figures. Figure 1 presents the effect of market size on the duration of the race and Figure 2 the effect on consumer surplus, firm value and total welfare. Figure 3 presents the difference between the race statistics in the no patents and the patents case. For this calculation, we fix the other parameters of the model to $\mu = 0.25$, $\pi^M = 1$ and $\delta = 2$.

The first thing to notice is that the duration of the race is decreasing with $\alpha$ for every possible setup. When we increase the size of the market, and therefore the product's market profits, firms have a greater incentive to invest. In addition, when firms invest more in R&D ,it becomes more likely that they will finish the product earlier.

The effect of $\alpha$ on consumer surplus, firm value and total welfare is also positive. With a larger market size, firms have incentive to invest more and complete the product earlier. Then, consumers are able to buy the product earlier resulting in a greater consumer surplus. Still, in the no a patent regime, the difference between consumer surplus increases with $\alpha$.

In Figure 3 we present the difference between the race's statistics for the no patents and the patents case. For very small values of $\alpha \leq .2$ the no patents regime yields lower expected time to monopoly. When we increase $\alpha$, the patents regime yields lower race duration. The effect on consumer surplus is also not monotonic. For $\alpha < 2.1$ the patents regime yields higher consumer surplus, whereas above this value, the no patents regime is better. Also, the impact on consumer surplus of having a strong patent regime is greater the more informed firms are. Firm value increases with $\alpha$ and, in terms of total welfare, the no patents regime is better. The only exception is the complete information structure when $\alpha \in [1.4, 1.82]$.

## 3.6 The Effect of Investment Cost

We now turn to the effect of investment cost, $\delta$, on the R&D race. In Figure 4, we present the effect of $\delta$ on the duration of the race, Figure 5 presents its effect on consumer welfare, firm value and total welfare, and Figure 6 presents the difference between the summary statistics on the no patents and the patents case for each information structure. We fix the other model's parameters to $\mu = 0.25$, $\pi^M = 1$ and $\alpha = 1$.

In this race, the investment cost profile is the same for both firms and equal to $c = (\delta, \delta, \delta, 1, 1, 1)$. Hence, the parameter $\delta$ is the cost of investing in R&D in the first half of the project. A larger $\delta$ implies that both firms have higher development costs in the first half of the race, as well as a higher cost asymmetry between the first and the second half of the race.

When we increase $\delta$, we also increase the duration of the race. When we increase the cost of investing in R&D, firms lower investment, resulting in higher expected time to complete the race. Also, when firms have more information about their opponent's stage, the duration of the race is lower.

Since increasing $\delta$ results in higher expected duration of the race,it also lowers consumer surplus. Still, the impact of $\delta$ on firm value is small for all information structures. The change in total investment from a marginal increase in $\delta$ is very small when compared to the discounted monopolistic profits in the product's market. Moreover, the impact of $\delta$ on total welfare is negative.

The patents regime lowers the expected duration of the race, with the exception of the first information structure and small values of $\delta < 1.75$. Therefore, consumer surplus is also higher with a patents regime. The impact of the patent regime on the race duration and consumer surplus is greater when firms have more information and tend to decrease with $\delta$.

Firm values, on the other hand, are greater in the no patents regime and the impact of patents is greater when firms have less information. The resulting impact of the patents regime on total welfare is negative and the difference is bigger when firms are less informed.

## 3.7 The Effect of Market Competition

In the last part of this section we examine the effect of the intensity in the product's market competition $\mu$. In Figure 7 we present the effect of $\mu$ on the duration of the race and in Figure 8 the effect on consumer surplus, firm value and total welfare. We fix the other model's parameters to $\pi^M = 1$, $\alpha = 1$ and $\delta = 2$.

The parameter $\mu$ captures the intensity of competition in the product's market duopoly. A lower value of $\mu$ implies a tougher competition, where $\mu = .5$ implies a perfectly collusive setup where firms share the monopolistic profit equally. In our simple patent setup, the first firm to complete the project

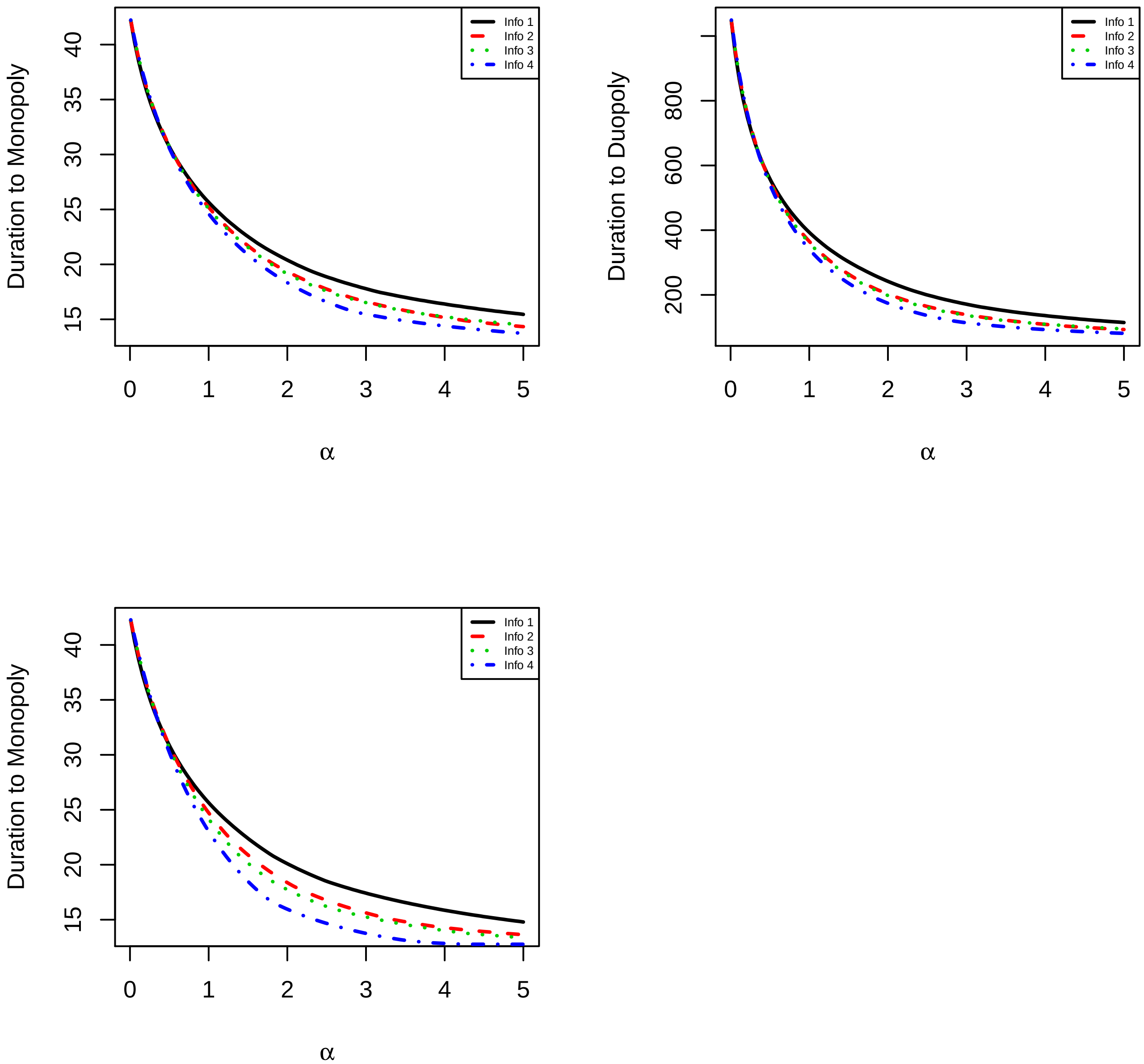


Figure 1: R&D race duration - Market Size

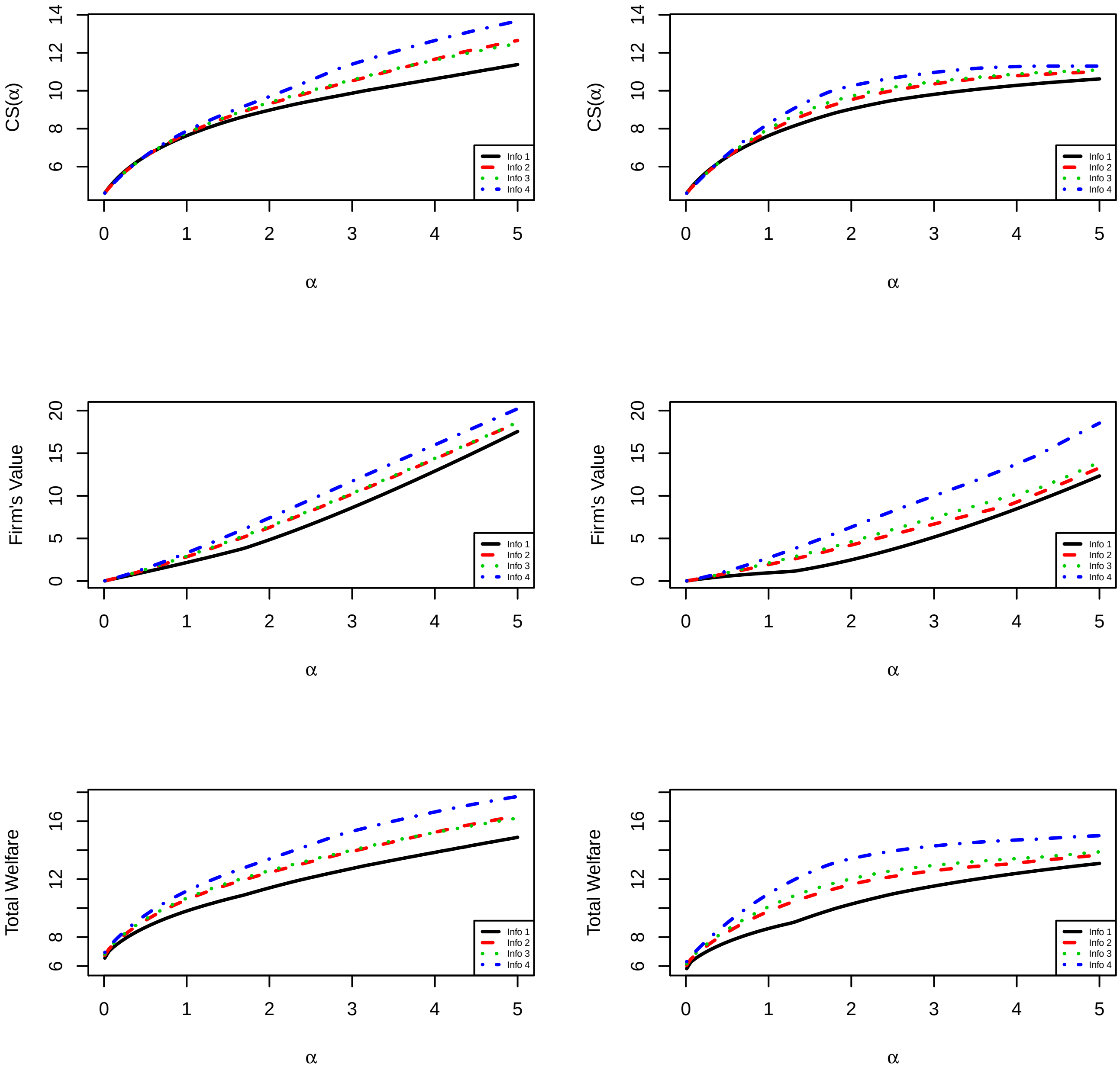


Figure 2: Consumer Surplus, Firm Value and Total Welfare - Market Size

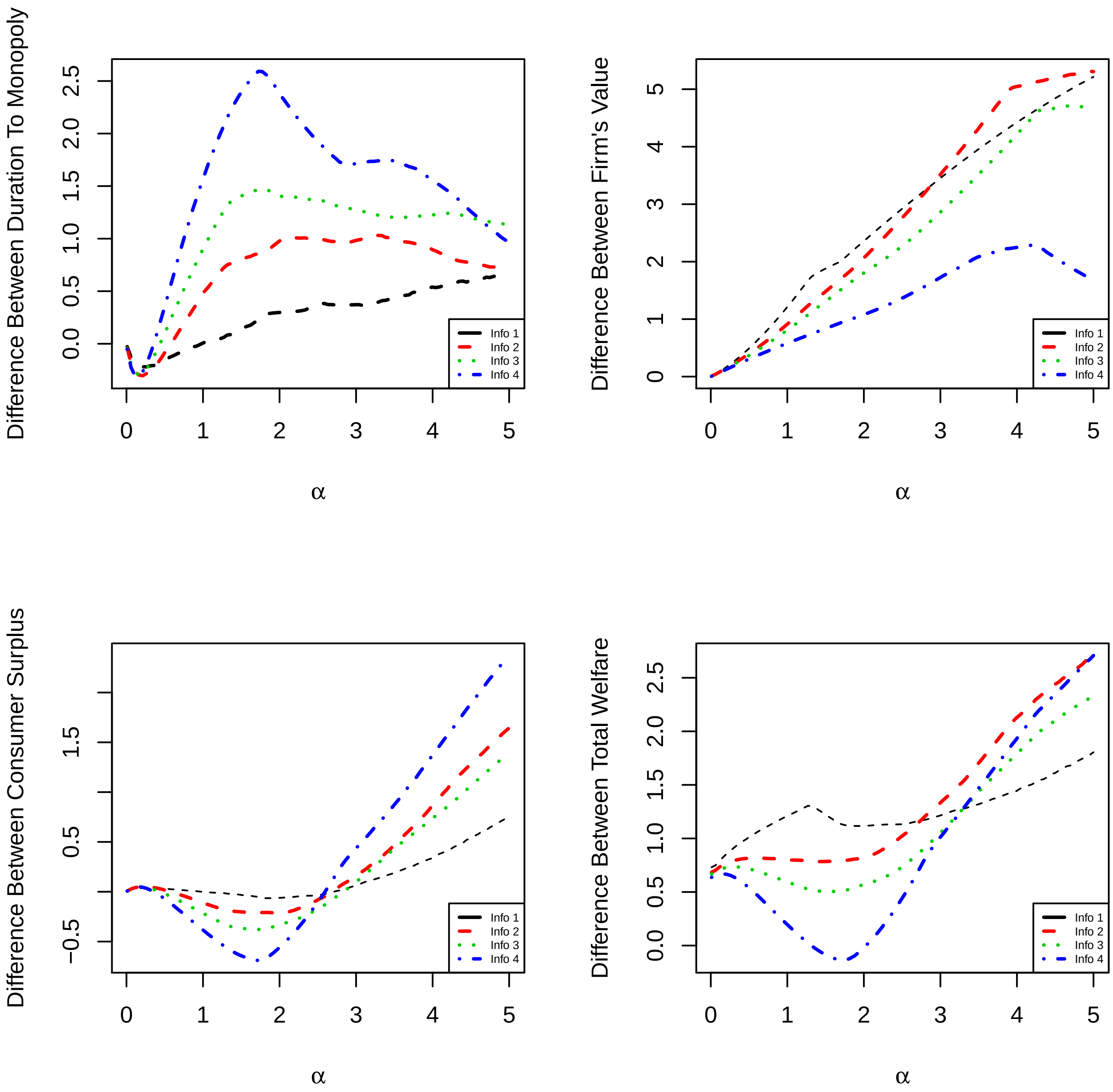


Figure 3: Difference between race's statistics by patent regime - Market Size

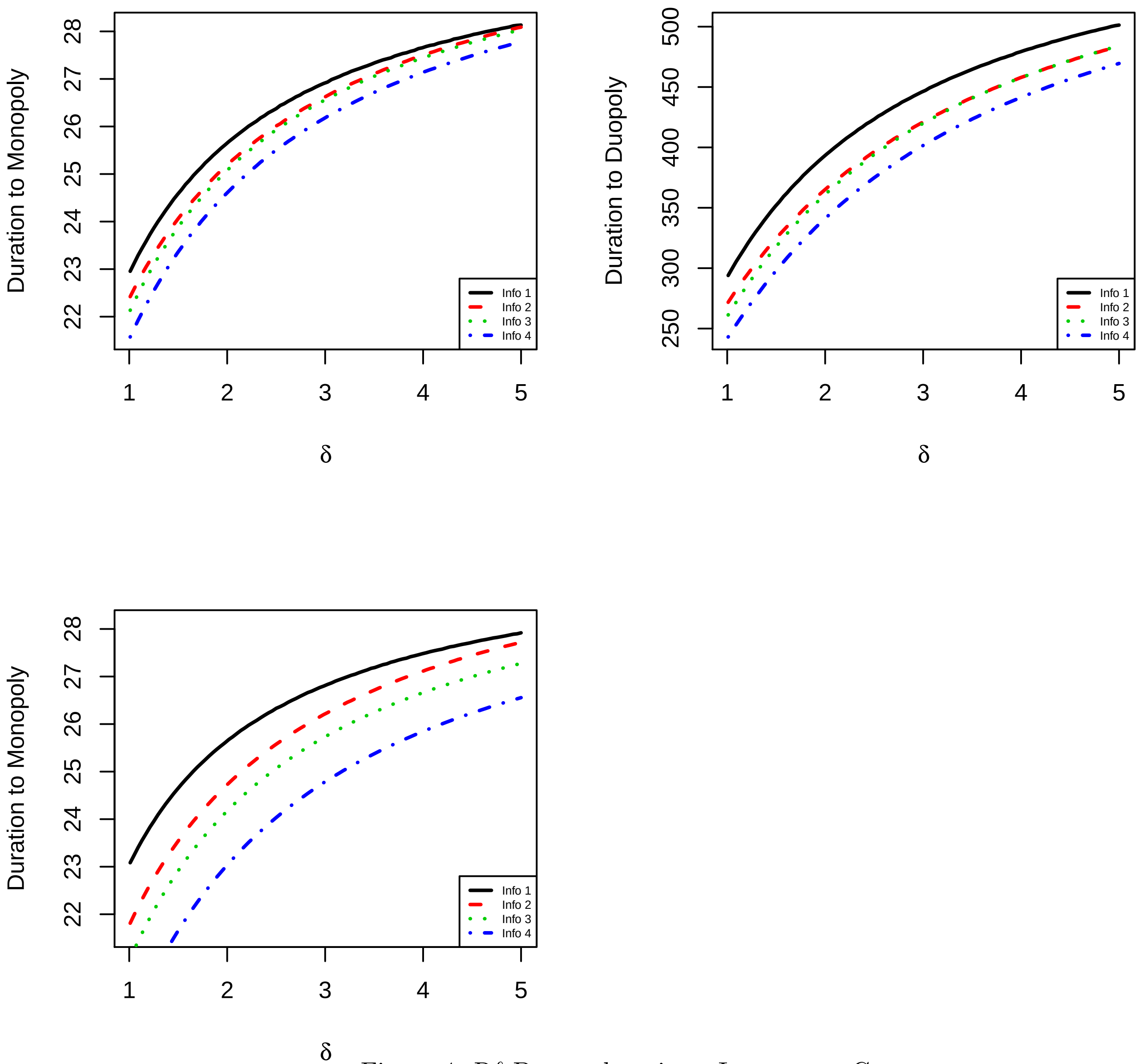


Figure 4: R&D race duration - Investment Costs

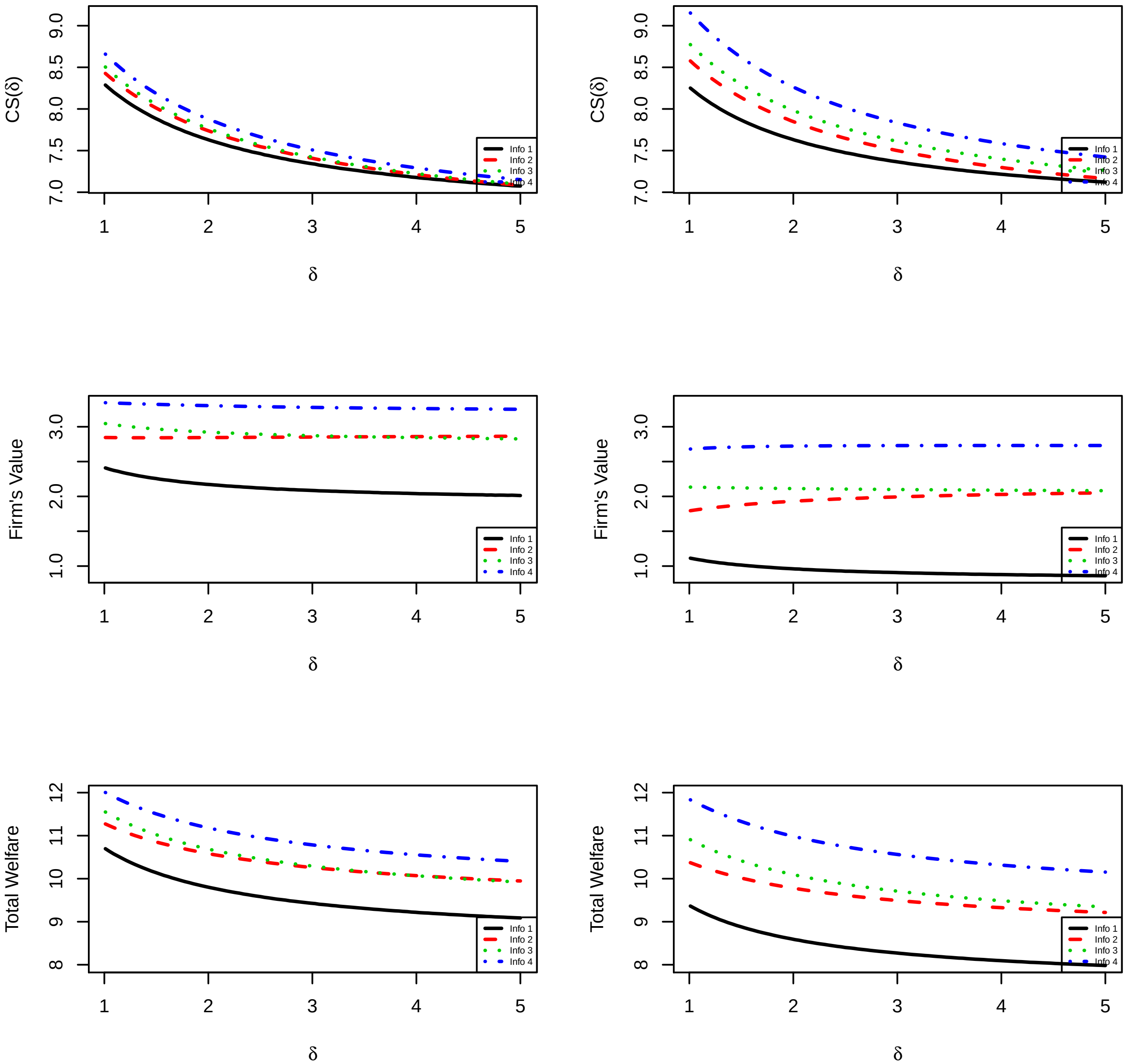


Figure 5: Consumer Surplus, Firm Value and Total Welfare - Investment Costs

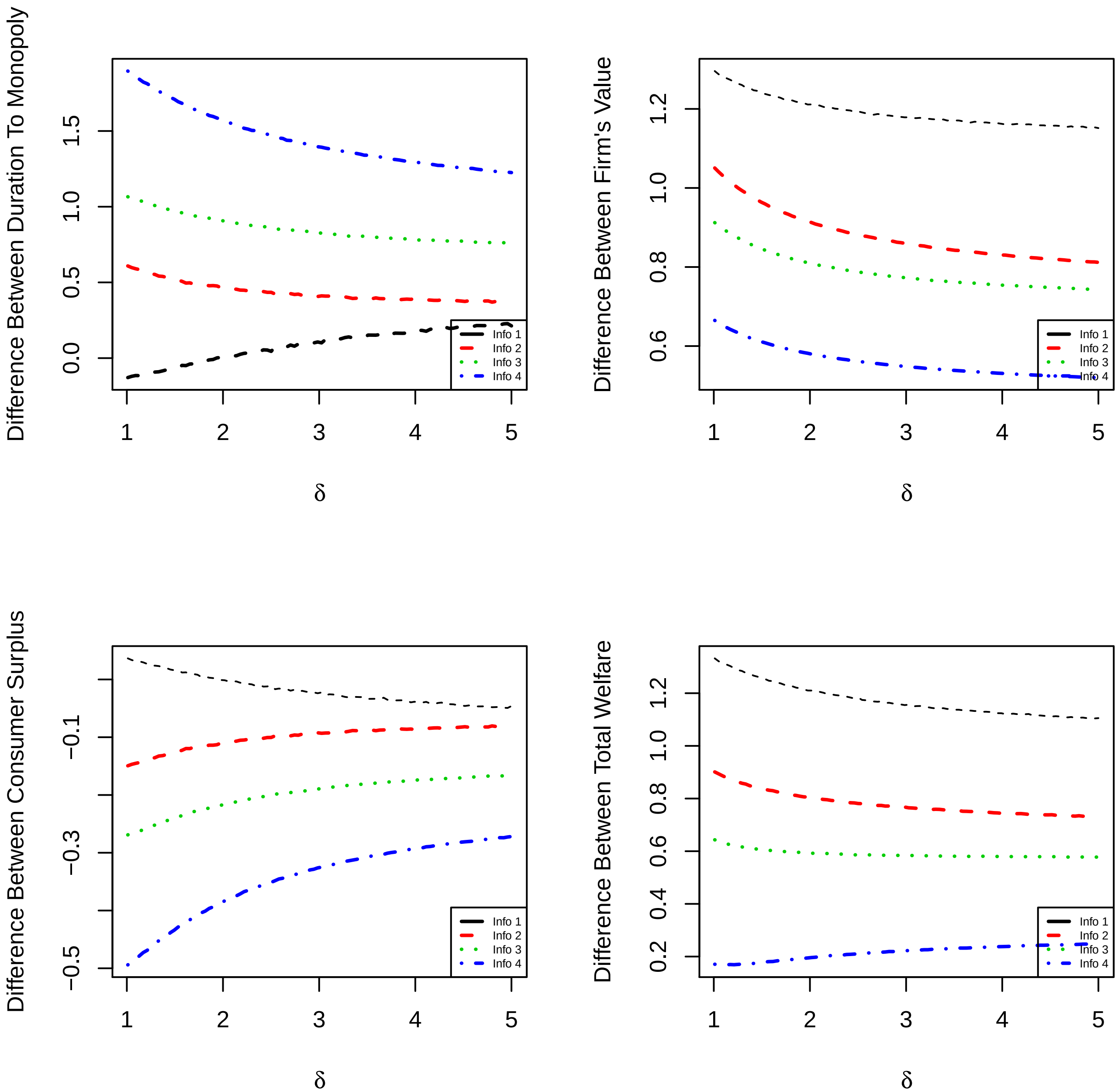


Figure 6: Difference between race's statistics by patent regime - Investment Costs

explores the market forever. Therefore it is only meaningful to study the impact of $\mu$ for the case where there are no patents.

Changing $\mu$ has two main effects on the R&D race. First it affects a firm's incentive to keep investing once the other firm has already completed the project. Hence, investment increases with $\mu$. On the other hand, when the other firm increases investment it is not necessarily optimal to increase investment too. It depends on the benefit of getting the monopolistic profits for a longer period with the higher investment cost during the race.

When we combine these two effects of $\mu$ we get a non-monotonic impact on race duration. For lower values of $\mu$ the duration of the race to reaching monopoly is increasing, but as it approaches .5 the duration of the race decreases. Also, the difference in the race's duration between information structures decreases with $\mu$.

The non-monotonic impact of $\mu$ on the race's duration translates into a non-monotonic impact on consumer surplus. Still, firm value increases with $\mu$ as firms are able to get higher profits in the duopoly in the product's market. The impact of $\mu$ on total welfare is positive. The eventual loss in consumer surplus is overcome by the increase in firm value. In addition, the difference in consumer surplus, firm value and total welfare decreases with $\mu$.

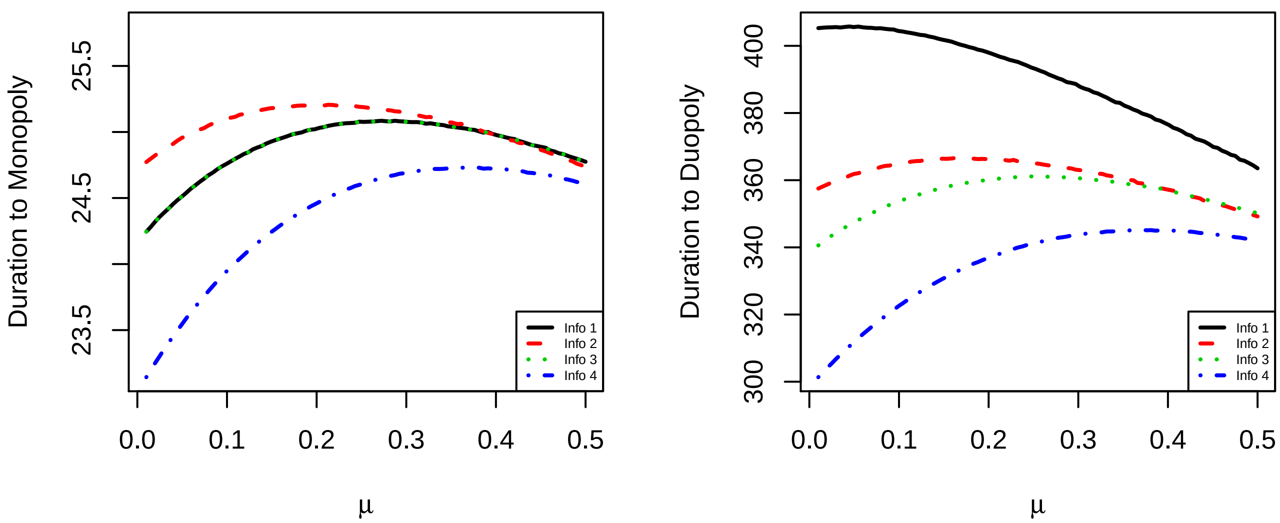


Figure 7: R&D race duration - Competition Intensity

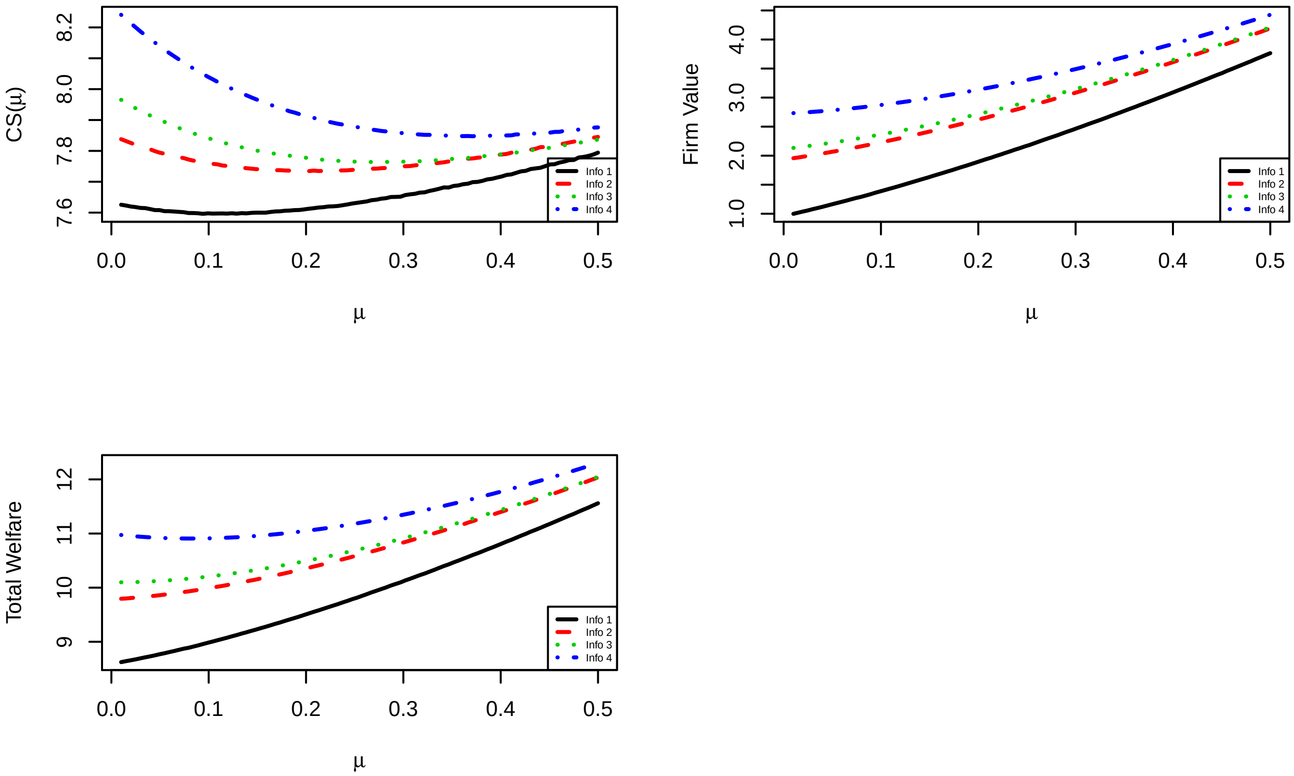


Figure 8: Consumer Surplus, Firm Value and Total Welfare - Competition Intensity

# 4 Conclusion

In this paper we present and show equilibrium existence of a multistage R&D race model where firms need to complete a finite number of intermediate steps to complete the product or project. Also, during the race, firms may not know the competitor's development stage.

We simulate a version of the model and show the impact of market size, investment costs and intensity of market competition on the race. We show that the impact of these variables is not the same for each information structure. The impact is greater when firms are more informed and the product's market is large and the impact is smaller when investment costs are high.

When we increase investment costs, race duration increases, while firm value, consumer surplus and total welfare all decrease. On the other hand, increasing the market size has the opposite effect. Lowering the intensity of competition in the product's market increases firm value and total welfare. Still, its effect on consumer surplus depends on its specific value. Having a strong patent regime is better in terms of consumer surplus only when the market is small. Firm value and total welfare decreases with patents.

In this paper we do not study the impact of cost asymmetries, information asymmetries, and other types of patent regimes (licensing etc.). These are interesting and important questions but are left to future work.

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