Public Debt and the Limits of Fiscal Policy to Increase Economic Growth

Vladimir Kühl Teles*

Caio Cesar Mussolini†

July 15, 2011

Abstract

Research that seeks to estimate the effects of fiscal policies on economic growth has ignored the role of public debt in this relationship. This study proposes a theoretical model of endogenous growth, which demonstrates that the level of the public debt-to-gross domestic product (GDP) ratio should negatively impact the effect of fiscal policy on growth. This occurs because government indebtedness extracts part of the savings of the young to pay interest on the debts of the older generation, who are no longer saving. Therefore, the payment of debt interest assumes an allocation exchange role between generations that is similar to a pay-as-you-go pension system, which results in changes in the savings rate of the economy. The major conclusions of the theoretical model were tested using an econometric model to provide evidence for the validity of this conclusion. Our empirical analysis controls for time-invariant, country-specific heterogeneity in the growth rates. We also address endogeneity issues and allow for heterogeneity across countries in the model parameters and for cross-sectional dependence.

Key Words: Public Debt, Endogenous Growth, Fiscal Policy

JELs: O23, O41, O54.

*EESP/FGV

†EESP/FGV and Insper

1 Introduction

This article examines how the size of the public debt-to-GDP ratio limits the effects of productive government expenditures on long-term growth. We conducted this analysis by proposing a model with overlapping generations and endogenous growth, wherein the government can go into debt in order to increase its productive expenditures. Various works present models where public expenditures affect growth via a framework of endogenous growth (Barro, 1990; Chen, 2006; Devarajan et al., 1996; Glomm and Ravikumar, 1997), wherein the effect of these expenditures varies according to their nature, their composition, and the size of the tax burden. Other works demonstrate that public debt can negatively affect growth (Brauninger, 2005; Saint-Paul, 1992). In this article, we establish a link between these two classes of models and demonstrate that the effect of productive expenditures on growth is limited not only by the size of the tax burden and the rate of indebtedness, as predicted by these models, but also by the debt-to-GDP ratio.

Similar to the models of Barro (1990) and Saint-Paul (1992), the model that we propose here demonstrates that an increase in public expenditures can have three direct effects on growth. An increase in public expenditure positively affects economic productivity but at the expense of the negative effects that are associated with the increases in the tax burden and public indebtedness in order to finance the expenditures, which result in a decrease in savings; however, in our approach, we demonstrate that there is an additional indirect effect: an increase in productive expenditures results in an increase in the equilibrium interest rate due to the increase in productivity. This results in an increase in expenditures for interest rate on public debt and an additional reduction in savings. This occurs because government indebtedness extracts part of the savings of the young to pay interest on the debts of the older generation, who are no longer saving. Therefore, the payment of debt interest assumes an allocation exchange role between generations that is similar to a pay-as-you-go pension system, which results in changes in the savings rate of the economy.

In the second part of the article, we will estimate a growth equation using the specification proposed by the theoretical model for a panel of countries to provide valid evidence for the conclusions of the theoretical model. Our empirical analysis controls for time-invariant, country-specific heterogeneity in the growth rates. We also address endogeneity issues and allow for heterogeneity across countries in the model parameters and for cross-sectional dependence.

Various studies have found empirical evidence that the allocation of public funds towards education, health, and infrastructure expenses positively impact economic growth (Aschauer, 1989; Easterly and Rebelo, 1993; Gupta et al., 2005); however, there is no consensus regarding this issue, as many works find insignificant results (Agell et al., 2006; Devarajan et al., 1996). We suggest that the disagreement among these works can be reconciled by taking a closer look at the theory. We used a specified theoretical model that clarifies the proper non-linear specification of our growth regression and allows us to demonstrate that by disregarding the role of public debt in this relationship, the estimations that are used in previous studies can result in omitted variable bias. In addition, we find evidence that the magnitude of the effect of public debt in this context is considerable.

2 Theoretical Model

In this section, a simple overlapping generations model of endogenous economic growth will be developed, wherein it is established that public expenditures can affect economic productivity. The model is an extension of Barro (1990) and Glomm and Ravikumar (1997) in that the government can become indebted in order to increase its productive expenditures.

2.1 Agents

If an overlapping generations model in which each generation lives for two periods and consists of a continuum of identical individuals in the interval $(0, 1)$ is considered, the utility function of an agent who is born in the period $t = 0, 1, ...$ is defined by:

$$U = lnc_t^t + \beta lnc_{t+1}^t \tag{1}$$

where c_i^t is consumption in the period i of the individual who is born in t and $0 < \beta < 1$. The initial generation of older people is endowed with k_0 units of capital. The following generations are each endowed with one unit of labor. The youth from generation t have the following restrictions:

$$c_t^t + s_t^t \leq (1 - \tau)w_t, \tag{2}$$

$$c_{t+1}^t \leq (1 + r_{t+1})s_t^t, \tag{3}$$

$$(c_t^t, c_{t+1}^t) \geq 0, \tag{4}$$

where $s_t^t = k_{t+1} + d_{t+1}$ is savings and d_t are government bonds that are owned by private agents. Each individual takes the wage rate, w_t, the real interest rate, r_{t+1}, and the tax rate, τ, as given. Clearly, one unit of labor is inelastically supplied by each young individual and no old individual wishes to save.

2.2 Firms

There is a representative firm that attempts to maximize its profits in an environment of perfect competition, and it has the following production function:

$$y_t = Az_t^{1-\alpha}k_t^{\alpha} \tag{5}$$

where y_t is the output, z_t are the productive government expenditures, that is, the expenditures that affect the marginal product of capital, and k_t is the capital stock that is rented by the firm following the law of accumulation $k_{t+1} = (1-\delta)k_t + i_t$, where δ is the depreciation rate and i_t is the level of investment. For simplicity, let us assume $\delta = 1$. This production function is identical to Barro's (1990) if we assume that the labor supply is the same as the size of the younger generation, i.e., one. In turn, the government limits the quantity of productive expenditures as a part of the aggregate product.

$$z_t = xy_t \tag{6}$$

where x is the fraction of production that is designated for productive expenditures. Using (6) in (5), we now have

$$y_t = A^{\frac{1}{\alpha}}x^{\frac{1-\alpha}{\alpha}}k_t \tag{7}$$

Therefore, the economy's production function exhibits constant marginal returns for capital, despite the fact that the returns for firms are decreasing. This is a result of the externality of productive government expenditures. The firm does not perceive that by increasing capital stock (and consequently the product), there will be an increase in productive spending by the government, and this leads to an increase in the marginal products of labor and capital.

2.3 Government

The government spends a fixed fraction $0 < g < 1$ of the product on its consumption and also spends a fraction x and charges a tax rate τ on the income of the agents. In addition, it borrows resources from the private sector by issuing bonds d_t that pay a remuneration of r_{t+1} in the following period, where $d_0 = 0$. Therefore, changes in government debt are expressed as the primary deficit $(g + x)y_t - \tau w_t$ added to the debt service payments $r_t d_t$.

$$d_{t+1} - d_t = r_t d_t + (g + x)y_t - \tau w_t \tag{8}$$

In order for the government to meet its budgetary restrictions, we have to assume that some component of the government budget is endogenous, serving as an adjusted variable. Thus, we will assume that the government always adjusts its indebtedness to satisfy its budgetary restriction. This is relevant because we are assuming that each time that the government decides to increase its expenditures, it also decides to increase its indebtedness. For the case of study in this manuscript, specifically, productive government expenditures, we observe that governments increase the product by raising productivity; however, their increase results in a larger deficit, such that productive expenditures have a limited effect on the economy because they are financed by increases in the public debt.

2.4 Competitive Equilibrium

Given k_0, a competitive equilibrium for this economy involves a sequence of allocations $\{k_{t+1}$, d_{t+1}, y_t, c_t^t, c_{t+1}^t, $s_t^t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t\}_{t=0}^{\infty}$, such that:

1. Given w_t and r_{t+1}, the allocation $(c_t^t, c_{t+1}^t, s_t^t)$ resolves the maximization problem for the young generation t.
2. Given w_t and r_t, the allocation (y_t, k_t) resolves the maximization problem for the representative firm,
3. $s_t^t = k_{t+1} + d_{t+1}$,
4. $y_t = c_t^t + c_t^{t-1} + k_{t+1} + gy_t + xy_t$.

The solution to the optimization problem for the younger generation in period t yields:

$$s_t^t = \beta(1 - \tau)w_t/(1 + \beta) \tag{9}$$

The profit maximization of the firms leads to:

$$\frac{\partial y_t}{\partial k_t} = \alpha A z_t^{1-\alpha} k_t^{\alpha-1} = r_t$$

Substituting (6) and (7) into this equation, we find

$$r_t = \alpha A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} \tag{10}$$

and, given that the technology exhibits constant returns to scale with the private factors, economic profit does not exist; therefore,

$$y_t - \frac{\partial y_t}{\partial k_t} k_t = (1-\alpha) A z_t^{1-\alpha} k_t^{\alpha-1} = w_t$$

and analogously

$$w_t = (1-\alpha) A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} k_t \tag{11}$$

Therefore, because productive government expenditures affect the productivity of the economy, they naturally also affect interest rates and wages. Since the present model is represented in the AK form, savings affect long-term growth. Equation (9) demonstrates that shocks to the equilibrium wage lead to greater savings, and equation (11) makes it clear that shocks to productive government expenditures(11) increase the wage level. Therefore, increases in productive expenditures lead to permanent productivity shocks and salary increases. This produces an increase in savings, investments, and subsequently, in economic growth.

However, it is important to note that in (9), the savings increase depends negatively on τ, as this decreases the net gain of the productivity increase. This result is consistent with that obtained by Barro (1990), in that increases in productive expenditures depend in a non-linear way on the size of government.

Using (7), (8), (10), and (11) in equation (9) and a bit of algebra, the capital growth rate is given by:

$$\frac{k_{t+1} - k_t}{k_t} = -1 + \frac{\beta}{1+\beta}(1-\tau)(1-\alpha) A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} - [g + x - (1-\alpha)\tau]\, A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} - (1 + \alpha A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}}) \frac{d_t}{k_t} \tag{12}$$

Initially, we will assume that there is a stable equilibrium in the economy or that $\frac{d_t}{k_t}$ is constant. In this case, equation (12) demonstrates the dynamic equilibrium of capital and, consequently, that of the product. There are three components in the right-hand side of the equation that show us the three paths by which an increase in productive government expenditures can affect growth.

The first part of the equation demonstrates that an increase in productive expenditures increases growth, which reflects their impact on the economy's productivity by increasing aggregate savings and economic growth; however, once again, the size of this effect decreases as the size of the tax burden increases.

The second component of the equation will be zero if the primary deficit in relation to the GDP $[g + x - (1-\alpha)\tau]$ is zero. This component also shows that the marginal effect of productive govern-

Figure 1:

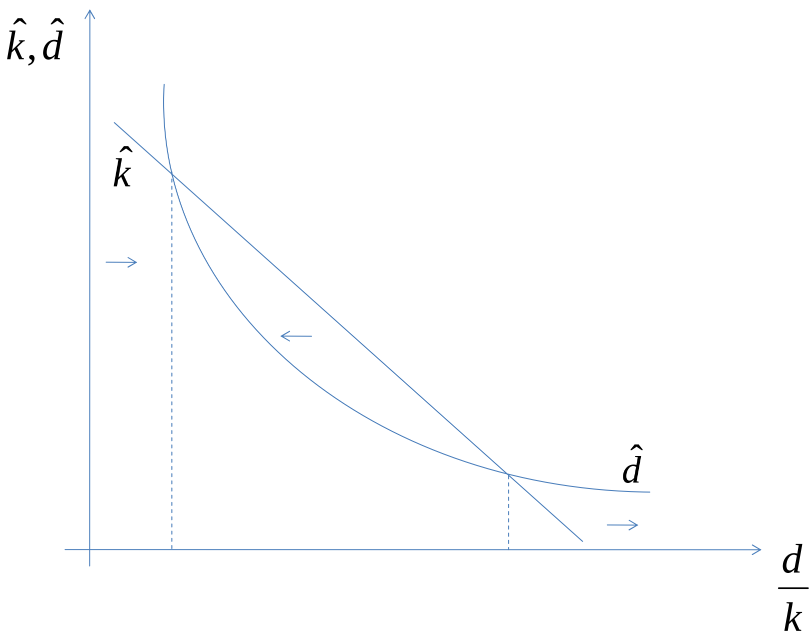


Figure 2: Increase in Government Consumption (g)

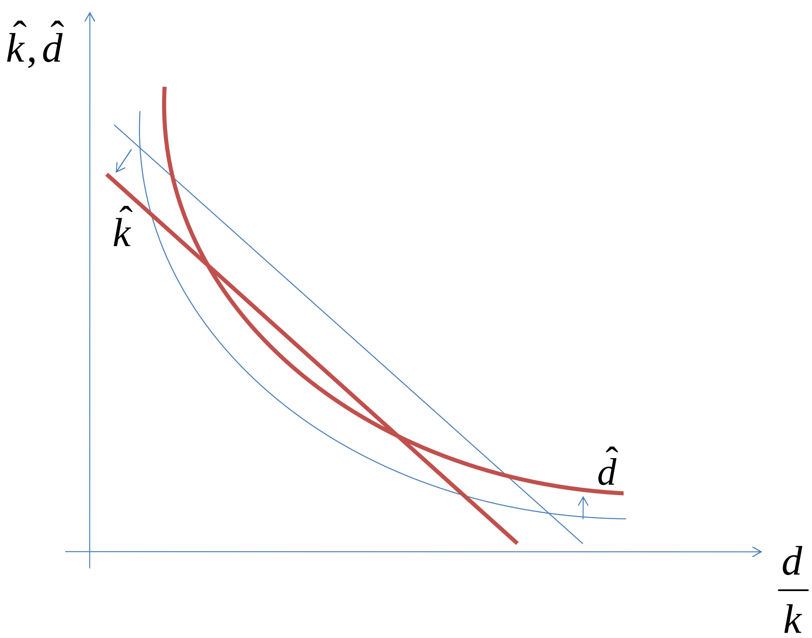
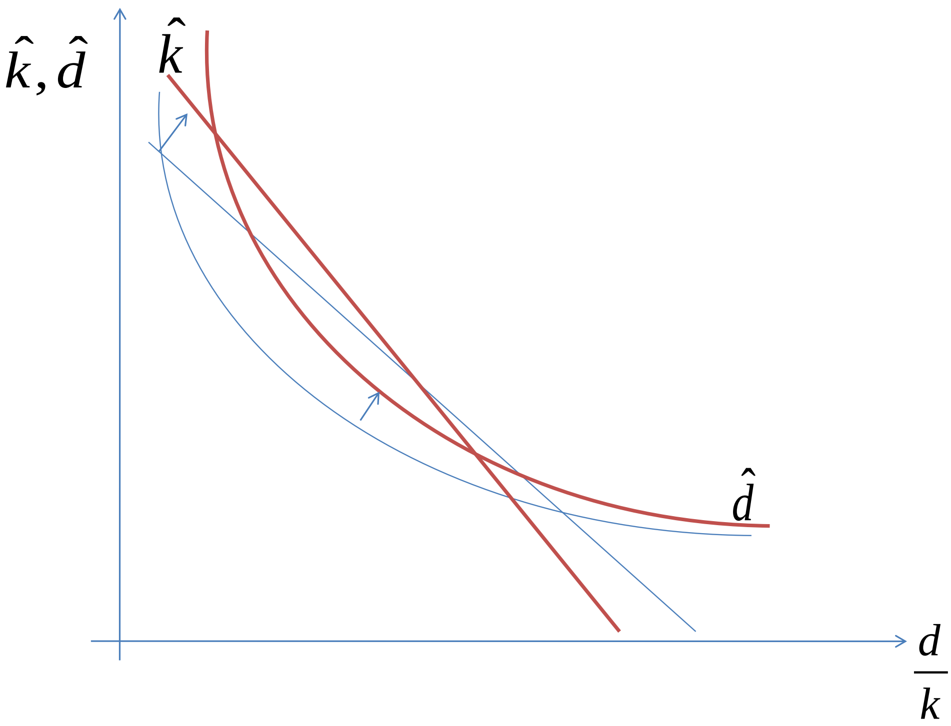


Figure 3: Increase in Productive Expenditures (x)



ment expenditures varies depending on the primary deficit. This demonstrates that when the government increases its productive expenditures, such as in education, infrastructure, or health, there is a negative effect on growth because it directly increases government indebtedness and reduces the savings that is designated for private investment.

The third component of the equation reproduces the indirect impact that productive government expenditures have on indebtedness and, therefore, on growth. Because increases in x raise the interest rate and because the government pays interest rate on public debt, an increase in productive expenditures impacts the costs of the debt, which indirectly raises the rate of government indebtedness. Thus, as the public debt increases in size, this relationship becomes more perverse such that productive expenditures will have a lower marginal effect on growth.

Models with budgetary equilibrium, as found in Barro (1990); Glomm and Ravikumar (1997), only provide results for the first part of the equation. The second component of the equation follows the same logic as the models of debt and endogenous growth that are found in Saint-Paul (1992), wherein increases in the public debt lead to greater indebtedness, which decreases the amount of savings that is allocated to capital accumulation. In our approach, we find an additional effect (the third part of the equation) where the size of the debt becomes relevant. We can understand this effect as being similar to a pay-as-you-go pension system, in that a portion of the savings of the young is reallocated to pay the older generation (through greater debt interest payments) instead of on the accumulation of capital, thereby diminishing economic growth.

Thus, the magnitude of the impact of an increase in productive expenditures on economic growth varies based on the size of the primary deficit and the public debt. In other words, it is more likely that increases in infrastructure, education, and health expenditures, or any other productive expenditures, will stimulate economic growth when the unproductive portion of the government and the debt-to-GDP ratio are lower.

In order to verify and characterize whether there is a *steady-state* equilibrium in the economy, we have to examine the debt dynamics. Thus, by substituting (7),(10) and (11) in (8) and dividing by d_t, we have:

$$\frac{d_{t+1}-d_t}{d_t} = \alpha A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} + \left[g + x - (1-\alpha)\tau\right] A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} \frac{k_t}{d_t} \tag{13}$$

Thus, for the growth rates of k_t and d_t to be constant, or rather, so that there is an equilibrium, the ratio d_t/k_t needs to be constant, such that $k_{t+1}/k_t = d_{t+1}/d_t$. By setting equations (12) and (13) equal to one another, we obtain a quadratic equation for the equilibrium debt-capital ratio, the solution to which is defined by:

$$\frac{d}{k} = \frac{\left\{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - \alpha - [g + x - (1-\alpha)\tau]\right\} A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} - 1 \pm \sqrt{H}}{2[1 + \alpha A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}}]}$$

$$\begin{aligned} H \equiv \{[\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - \alpha - [g + x - (1-\alpha)\tau]]A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} - 1\}^2 \\ - 4(1 + \alpha A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}}))\left[g + x - (1-\alpha)\tau\right] A^{\frac{1}{\alpha}} x^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

Thus, if the solution above possesses at least one real positive root, there is a *steady-state* equilibrium. Naturally, the existence of an equilibrium depends on the values of the model parameters, and as demonstrated exhaustively by Brauninger (2005), an equilibrium is not always obtained for plausible parameter values.

In Figure 1, where $\hat{k}$ and $\hat{d}$ are the capital and debt growth rates, equations (12) and (13) are plotted in an example where we assume that the two roots are real and positive and $[g + x - (1-\alpha)\tau] > 0$.

In Figure 1, we have a result with two equilibriums: the first, which is characterized by a low debt-capital ratio, a low debt-GDP ratio, and high economic growth, is locally stable, whereas the second, which is characterized by low growth and a high debt-to-GDP ratio, is unstable.

In Figure 2, we evaluate a scenario in which an economy in equilibrium experiences an increase in unproductive government expenditures (g). In this case, the capital growth curve shifts downwards and the debt growth curve shifts upwards. If the economy is initially in a state of stable equilibrium with high growth, this leads to a decline in *steady-state* economic growth and an increase in the debt-to-GDP ratio. In an extreme situation with an excessive increase in unproductive expenditures, the two curves no longer intersect, meaning that the growth rate of debt would be larger than that of the economy and suggesting an explosive increase in debt and an absolute collapse of the economy. This would be the case if there were no *steady-state* equilibrium. This result is essentially the same as that obtained by Brauninger (2005).

However, the case that most interests us, and for which this article attempts to make a contribution, is that of increases in productive government expenditures. This case is illustrated in Figure 3. With an increase in productive expenditures (x), the slopes of the two curves change. In this example, where the economy is in a stable *steady-state*, economic growth increases, as does the debt-to-GDP ratio. This occurs because the fiscal cost of increasing this type of spending is still small in comparison to the resulting productivity gains. Consequently, the economy still has the necessary fiscal strength, and this type of policy is very successful; however, at an equilibrium with a high debt-to-GDP ratio, the effect on growth and the increase in productive expenditures will be less substantial, and the effect on the debt-capital ratio is uncertain because it will depend on the slopes of the curves, which are determined by other model parameters.

There are at least two important conclusions that we can extract from this model. The first is that contrary to the models that have been developed thus far, such as those in Saint-Paul (1992) and Brauninger (2005), increases in the debt-to-GDP ratio may be linked to increases in the growth rate under certain circumstances. The second is that the marginal effect on growth that is caused by productive expenditures, such as those on infrastructure, education, and health, depends on both the primary surplus of the government and the size of the debt. Therefore, the econometric specifications that have been used until now to examine the effects of these policies on growth, which ignore these non-linearities, may suffer from omitted variable bias.

2.5 Optimum Productive Expenditures

By differentiating equation (12) with respect to x, holding $\frac{d_t}{k_t}$ constant,[1] and setting it equal to zero, and performing some manipulations, we arrive at the optimum value x^c,

[1]This is a partial equilibrium analysis because $\frac{d_t}{k_t}$ is an endogenous variable in the model.

$$x^c = (1-\alpha)\left\{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - [g-(1-\alpha)\tau] - \alpha\frac{d_t}{k_t}\right\}. \quad (14)$$

As in Barro (1990); Glomm and Ravikumar (1997), there is an optimum size of productive government expenditures relative to GDP that maximizes the growth rate of the economy. When government expenditures are below (above) this value, an increase in these expenditures has a positive (negative) effect on growth.[2] Nevertheless, in contrast to the results that were found by the aforementioned authors, this value is not equal to the productivity parameter of public spending $(1-\alpha)$. This is a result of our assumption that only one component of government revenue is transformed into productive expenditures, and so x^c is less than $(1-\alpha)$[3]. When the economy is in a *steady-state*, this will be the same value that is needed for the economy to change and remain in equilibrium.

According to equation (14), one notes that this critical x depends on various parameters of the economy. For example, it can be observed that in countries where the agents are more patient, a larger β, and when the productivity of government expenditures $(1-\alpha)$ is higher, the critical x is higher. In contrast, for countries in which the government has more unproductive spending g or a high debt-to-capital ratio, the critical x is lower. With regard to the tax rate, the effect depends on the parameters of the economy.

3 Empirical Evidence

The theoretical model predicts that the relationship between economic growth and productive government expenditures is negatively correlated with the public debt, the tax burden, and the primary deficit. If these conclusions are correct, growth models that depend on public expenditures but do not take such interactions into account will suffer from omitted variable bias. In this section, an empirical model is suggested to provide evidence for the validity of these conclusions.

The various empirical problems that have been outlined in growth econometrics will be considered in order to provide adequate empirical evidence. In particular, it is necessary to address the likely problems of parameter heterogeneity, cross-section dependence, endogeneity, and time-invariant, country-specific heterogeneity (Bond et al., 2010; Durlauf et al., 2005).

In a certain sense, the non-linear nature of a model that includes the interactions of key variables may resolve a large part of the heterogeneity of the parameters if the theoretical model is correct; however, it is likely that other institutional aspects, such as corruption and bureaucracy, can affect the productive expenditures coefficient in the growth equation. Therefore, following the same strategy that was adopted

[2]This is valid if $\left\{\frac{\beta}{1+\beta}(1-(1-\alpha)\tau)(1-\alpha) - [g-(1-\alpha)\tau] - \alpha\frac{d_t}{k_t}\right\} \in (0,1)$.

[3]Considering $g=0$ and $d=0$, it is easy to show that $x^c = (1-\alpha)$

by Bond et al. (2010); Gemmell et al. (2011); Lee et al. (1997), which involves growth regressions using a panel that is similar to ours, we will use the mean group approach of Pesaran and Smith (1995) to attempt to resolve the heterogeneity of the parameters. At the same time, as it is likely that the countries suffer from cross-section dependence in the shocks of the growth process, we will use the multifactor error structure proposed by Pesaran (2006).

These methods require the use of a series of annual country growth data. This approach is possible in our case because the growth and fiscal data vary from year to year[4]. A possible problem of utilizing annual data for growth studies is the possibility that the results reflect fluctuations in the series and not fluctuations in the long-term. Following Bond et al. (2010), we rely on dynamic econometric specifications to implicitly filter out these higher-frequency influences, while acknowledging the limitations of this approach.

Using panel data for countries, we use a specification where growth depends on its lagged values, in addition to the variables of interest, in a linear version of the parameters based on (12). Because (12) is no longer an equilibrium equation, an alternative would be to simultaneously estimate this with (13); however, given that there is no variable in (13) that is not present in (12), this approach does not help us identify the equation. Therefore, we opt to estimate (12) using a two-step GMM. Nonetheless, we recognize that this is not an equilibrium relationship but is, rather, only a verification of whether the relationship that is predicted by the model has some basis in the data. Therefore, the regression model is defined by

$$gr_{it} = \theta_0 + \sum_{s=1}^{T} \alpha_s gr_{it-s} + \gamma_0 x_{it-1} + \gamma_1 x_{it-1} * \tau_{it-1} + \gamma_2 x_{it-1} * surplus_{it-1} + \\ + \gamma_3 x_{it-1} * d_{it-1}/y_{it-1} + \gamma_4 d_{it-1}/y_{it-1} + \varepsilon_i + \epsilon_t + e_{it} \tag{15}$$

where gr_{it} is the per capita GDP growth rate, which is expressed as a percentage, of country i in year t, and T is the number of lags of the dependent variable that is included among the explanatory variables. The variables of interest are x_{it} and its interaction with the tax burden, debt, and primary surplus. The variable x_{it} is the sum of the central government expenditures for the country, such as those for education, transportation, communication, energy, and health, divided by the GDP. The variable τ is the tax burden on income as a fraction of the GDP. The variable $surplus_{it}$ is the primary surplus of the central government relative to the GDP, and the variable d_{it}/y_{it} is the debt-to-GDP ratio. All of these values are

[4]If the series that are used contain data that are not annual frequency averages (e.g., human capital), do not vary in annual frequency (e.g., political systems), do not vary over time (e.g., colonial origins), or vary little over time (e.g., income inequality), this method may not be appropriate. For further discussion, see Bond et al. (2010); Durlauf et al. (2005)

expressed as percentages. Dummy variables for the year were included to show the common effects over time among the countries, ϵ_t. Time-invariant, country-specific heterogeneity was captured by fixed-effect, cross-section dummy variables, ε_i, with the goal of controlling the effects of institutional, geographic, or other differences that did not change during the study period.

In the steady-state, with constant fiscal variables and without shocks, per capita production grows at a constant country-specific rate defined by

$$gr_i = \frac{\theta_0 + \gamma_0 x_i + \gamma_1 x_i * \tau_i + \gamma_2 x_i * surplus_i + \gamma_3 x_i * d_i/y_i + \varepsilon_i}{1 - \sum_{s=1}^{T} \alpha_s} \tag{16}$$

Therefore, the long-term effect that an increase in productive government expenditures has on the GDP will be defined by

$$\frac{\gamma_0 + \gamma_1 * \tau_i + \gamma_2 * surplus_i + \gamma_3 * d_i/y_i}{1 - \sum_{s=1}^{T} \alpha_s} \tag{17}$$

We want to show whether the interactions of x_{it} with the tax burden, debt, and primary surplus are significant in the long-term. Therefore, we should test whether the relationships $\gamma_i/(1 - \sum_{s=1}^{T} \alpha_s)$, $i = 1, 2, 3$ are significant and have the correct sign.

As expected, the dynamic structure of the model itself is such that the use of the least squares method produces biased and inconsistent estimators. Consistent estimators can be obtained if shocks e_{it} are serially uncorrelated by using the lagged values of endogenous variables that are not correlated with e_{it} and e_{it-1} as instrumental variables. In addition, the use of instrumental variables allows for the correction of the endogeneity of the current values of the fiscal variables.

3.1 Pooled Results

Equation (15) was estimated using unbalanced panel data from 74 countries during the period of 1972-2004. The data for economic growth, GDP, investment as a percentage of GDP, and trade are obtained from the Penn World Tables 6.3; inflation comes from the World Development Indicators (WDIs); productive expenditures, tax burden, and the primary surplus come from resources at the Government Financial Statistics (GFS) of the IMF; and the debt-to-GDP ratio is obtained from the Inter-American Development Bank[5].

The results of the estimations for the baseline specifications that were obtained using pooled annual data are shown in Table 1, wherein the slope parameters are imposed to be common to all of the included countries. In the GMM specifications gr_{it-1}, x_{it-1}, $x_{it-1} * \tau_{it-1}$, $x_{it-1} * surplus_{it-1}$, $x_{it-1} * d_{it-1}/y_{it-1}$

[5]Tables in the Appendix show the names of the included countries and the associated descriptive statistics

Table 1: Baseline Specifications

	(i) OLS	(ii) OLS	(iii) GMM	(iv) GMM	(v) GMM
lags included	4 lags	4 lags	4 lags	4 lags	1 lag
x_{t-1}	-4.49e-06	0.0418	-0.0002*	0.4361*	0.4522*
	(-0.35)	(0.70)	(-3.14)	(2.15)	(2.62)
d_{t-1}/y_{t-1}		-0.0081**		0.0107	0.0031
		(-1.86)		(0.26)	(0.10)
$x_{t-1} * surplus_{t-1}$		1.48e-08		1.56e-07*	1.62e-07*
		(0.69)		(2.15)	(2.62)
$x_{t-1} * \tau_{t-1}$		-1.04e-07		-1.03e-06*	-1.07e-06*
		(-0.73)		(-2.15)	(-2.61)
$x_{t-1} * (d_{t-1}/y_{t-1})$		-0.0001		-0.0011*	-0.0011*
		(-0.70)		(-2.16)	(-2.63)
Long-Run Effects					
x_{t-1}	-6.05e-06	0.0467	-0.0016	0.3745*	0.3754*
	[-0.35]	[0.70]	[-1.17]	[3.32]	[3.50]
$x_{t-1} * surplus_{t-1}$		1.65e-08		1.34e-07*	1.34e-07*
		[0.69]		[2.15]	[2.62]
$x_{t-1} * \tau_{t-1}$		-1.16e-07		-8.87e-07*	-8.89e-07*
		[-0.73]		[-2.12]	[-2.58]
$x_{t-1} * (d_{t-1}/y_{t-1})$		-0.0001		-0.0009*	-0.0009*
		[-0.71]		[-3.35]	[-3.52]
Hansen J Stat			11.198	3.322	2.217
p-value			[0.1907]	[0.5055]	[0.8184]
Test of first-order serial autocorrelation	-2.10	-1.09	-3.27	0.41	0.64
p-value	[0.03]	[0.27]	[0.001]	[0.68]	[0.52]
Test of second-order serial autocorrelation	-1.50	-0.34	1.06	1.37	1.09
p-value	[0.13]	[0.73]	[0.28]	[0.17]	[0.27]

Notes: A full set of year and time dummies were included. *t*-ratios are in parentheses. *z*-ratios of tests of the long-run effects, the *p*-values of the serial correlation tests and *p*-values of the Sargan-Hansen test of over-identifying restrictions are in square brackets.

In the GMM specifications $gr_{it-1}, x_{it-1}, x_{it-1} * \tau_{it-1}, x_{it-1} * surplus_{it-1}, x_{it-1} * d_{it-1}/y_{it-1}$ are treated as endogenous. The instrument set consists of gr_{it-5}, lags 5 and 6 of GDP per capita, lags 2-4 of investment as percentage of GDP, lags 2 and 3 of trade (sum of exports and imports) as percentage of GDP and lags 2 and 3 of inflation.

Significance levels ** 0.10, * 0.05.

are treated as endogenous. The instrument set consists of gr_{it-5}, lags 5 and 6 of the GDP per capita, lags 2-4 of investment as a percentage of the GDP, lags 2 and 3 of trade (sum of exports and imports) as a percentage of the GDP, and lags 2 and 3 of inflation. These variables are among those that have been used as instrumental variables in similar estimations in the literature, such as in Agell et al. (2006); Bond et al. (2010); Kneller et al. (1999)[6]. The validity of these instruments is not rejected by the Sargan-Hansen test for over-identifying restrictions.

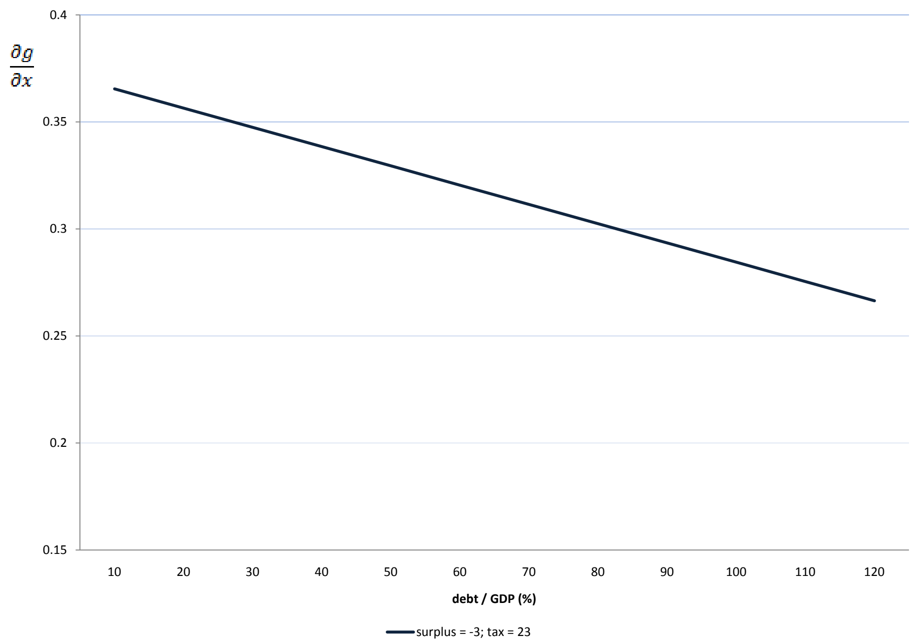
The two first columns report the results of the estimations using OLS analysis, with up to four lags and with and without the interactions of the expenditure variable with the other fiscal variables. Columns three and four depict the results for the same specifications using a two-step GMM analysis. Various specifications using different numbers of maximum lags of the dependent variable, T, were applied. We consistently found similar results, as shown in the last two columns of the Table, where it is possible to compare the results of $T = 4$ and $T = 1$.

The comparison of the results using OLS and GMM suggests that the results of the OLS considerably underestimate the degree of the growth rate persistence, and therefore, underestimate the long-term effect of the fiscal variables on growth. For this reason, we will focus on the GMM results. Additionally, the results demonstrate that the exclusion of the interactions between the productive expenditures and the tax burden, debt-GDP ratio, and primary surplus lead to omitted variable bias for the expenditure coefficients. The results without the interactions demonstrate a negative and insignificant effect on the growth of productive expenditures in both the long-term coefficient. Conversely, by including the interactions of fiscal variables, we find a positive and significant effect of public expenditures; however, this effect depends on the values of the other fiscal variables in the predicted direction of the theoretical model. Therefore, the results provide us with some evidence that the data go in the same direction as the proposed results of the theoretical model.

The size of the effect of productive expenditures on growth was calculated assuming a primary surplus of -3% and a tax burden of 23%. In this scenario, the marginal effect of an increase in productive expenditures by 1% of the GDP on long-term growth varies from 0.36% to 0.26% for values of the debt-to-GDP ratio that range between 10% and 120%, as shown in Figure 4. Changes in the values of the tax burden and the surplus shift this relationship either upwards or downwards; however, given that the coefficients of these interactions are very small, these shifts do not occur in a perceptible way. This shows that the

[6]Other estimations using the system-GMM were conducted following an alternative strategy that has been used in the literature, such as in Bond et al. (2001); Gregoriou and Ghosh (2009), and we found that the results were similar and were robust for changes in the selection of instrumental variables and in the estimation method; however, once we used the pooled annual data, the number of time series observations in the panel increases *vis-a-vis* the number of countries, making system-GMM an inadequate estimator. Therefore, we opted to pursue an analysis using the same strategy as Bond et al. (2010) and used a two-step GMM estimator.

Figure 4: Marginal Effect of Productive Expenditures (x) in Economic Growth



interactions with the tax burden and with surplus, although they are significant and have the expected direction, are not large. These results suggest that the value of the size of the debt-to-GDP ratio is the most relevant variable for determining the effect of productive expenditures on growth, as it is quantitatively as well as statistically significant.

3.2 Results for Mean Group Estimates

In this section, the baseline model was re-estimated while relaxing the hypothesis that the slope parameters are common across countries. If this restriction was invalid, then the inferences that were performed in the previous sections would also be invalid. The heterogeneity of the parameters among countries is plausible because the efficiency of public expenditures can be different given alternative institutional conditions. Therefore, given the likely importance of heterogeneous coefficients, we use the Mean Group Estimator of Pesaran and Smith (1995) and follow the same strategy used by Bond et al. (2010); Gemmell et al. (2011); Lee et al. (1997) for growth regressions with a panel that is similar to ours.

In simple terms, the Mean Group Estimator individually estimates the equation (15) using a two-step GMM, using the same previously used instrument set, for each country using transformed variables by subtracting the sample mean values for the same year from the original series, to corrects the possible effects of common shocks, and then obtain the average of the estimated coefficients for the countries.

Table 2: Mean Group Estimates

Long-Run Effects	Robust Mean	Robust Mean
allowing for cross-sectional dependence	no	yes
x_{t-1}	0.0250	0.0225
	[0.01]	[0.02]
$x_{t-1} * surplus_{t-1}$	7.98e-09	9.10e-09
	[0.09]	[0.04]
$x_{t-1} * \tau_{t-1}$	-7.54e-08	-6.99e-08
	[0.02]	[0.03]
$x_{t-1} * (d_{t-1}/y_{t-1})$	-0.0001	-0.001
	[0.02]	[0.01]

Notes: Robust Means are reported. *p*-values are in square brackets.
All specifications include $gr_{it-1}, x_{it-1}, x_{it-1} * \tau_{it-1}, x_{it-1} * surplus_{it-1}, x_{it-1} * d_{it-1}/y_{it-1}$ treated as endogenous. See table 1 for the list of instruments used (here in deviations-from-means form)

In our case, we individually obtained the estimated long-term effects for the countries and estimated the robust means together with their standard errors [7].

Another possible problem for the estimations is the existence of cross-sectional dependence. This occurs if the economic growth of the country affects the growth of other countries, such that the residual e_{it} would not be independent among countries. This effect is plausible if we believe that there are spillovers that impact technology and the accumulation of physical or human capital (Conley and Ligon, 2002) or if the economic development of one country affects the terms of trade of the other countries (Acemoglu and Ventura, 2002). To correct such a problem, we follow the procedure suggested by Pesaran (2006) of including the annual average growth of the country as an explanatory variable. This method can also be seen as a way to include the more flexible tendency of the individual estimations for the countries.

Table 2 depicts the results that were generated by the Mean Group Estimator of the long-term parameters that were estimated from the variables of interest in the model. The columns depict the specifications with and without the correction for cross-sectional dependence. The results depict values that are lower in magnitude for the effect of fiscal policy on growth, showing some evidence that the pooled estimations can show bias by not considering the heterogeneity of the parameters. Even so, all of the long-term coefficients have the same signs that were previously suggested by the theoretical model and are statistically significant. Furthermore, when calculating the marginal effect of productive spending, we obtain that this effect becomes negative for a debt-GDP ratio above 22.5%.

As a result, we find that as more econometric problems are addressed, the effect of public debt on the relationship between productive government expenditures and economic growth becomes more robust, corroborating the predictions of the proposed theoretical model.

[7] This is a robust variant of the Mean Group Estimator that is suggested by Bond et al. (2010).

4 Conclusion

Research that seeks to estimate the effects of fiscal policy on economic growth has ignored the role of public debt in this relationship. This paper proposed a theoretical model of endogenous growth in which the level of the public debt-to-GDP ratio can negatively affect the effect that productive public expenditures have on growth. Therein, the main conclusions of the theoretical model were tested through an econometric model to provide evidence for the validity of this conclusion. Our empirical analysis controls for time-invariant, country-specific heterogeneity in the growth rates. We also address endogeneity issues and allow for heterogeneity across countries in model parameters and for cross-sectional dependences.

Our approach has enabled us to verify the effects that have already been predicted in the literature, such as the non-linear effects of productive expenditures on growth given the size of the tax burden, such as in Barro (1990), or given the indebtedness rate. Such effects represent negative consequences in terms of direct capital accumulation, as they lead to diminishing marginal net returns of capital or savings extracted from the economy to finance pubic expenditures.

In addition to the above effects, we were able to observe an additional effect, wherein the impact that productive expenditures have on growth depends on the size of the debt-to-GDP ratio. This occurs because an increase in the magnitude of productive expenditures leads to an increase in the productivity of the economy, and thus, to an equilibrium of interest rates, as there is no decreasing marginal return for aggregate capital in the endogenous growth models. This increase in interest rates leads to higher government spending from debt servicing, such that as the size of the debt increases, so does the impact from this increase on interest rates. This is why a higher debt-to-GDP ratio corresponds to a smaller impact of productive expenditures on economic growth.

We can also understand this effect as an income transfer between generations, specifically, from the younger generation, which has a portion of its savings invested in government securities, thus decreasing capital accumulation in order to pay the interest on the debt of the older generation, which does not save. In this sense, the observed effect is similar to that of the pay-as-you-go pension system in overlapping of generation models, where income is transferred between generations and decreases the accumulation of capital.

In addition to incorporating the effect of public debt on the relationship between productive expenditures and economic growth, the model also demonstrates that increases in the size of the debt can lead to greater economic growth, since the status quo is a healthy fiscal situation and indebtedness is associated with an increase in productive expenditures. This runs contrary to previous models, in which debt increases always lead to decreased growth. This result shows that changes in the public debt can be

Pareto optimal, leading to benefits for all generations, which is quite different from that suggested by endogenous models of debt, where expenditures are always unproductive.

Using the econometric specifications that were derived from the theoretical model, the main conclusions of the model can be established. In particular, it is clear that the omission of the interactions between productive expenditures and the tax burden, primary surplus, and public debt, can be a source of bias in the estimation of the effects of fiscal policies on growth. The results suggest that the size of the debt-to-GDP ratio is the most relevant variable in determining the effect of productive expenditures on growth, as it was found to be both quantitatively and statistically significant. Various econometric problems were addressed, wherein our findings suggest that as more econometric problems are addressed, the effect of public debt on the relationship between productive government expenditures and economic growth becomes more robust.

References

Acemoglu, D. and Ventura, J. (2002), “The world income distribution”, *The Quarterly Journal of Economics* , Vol. 117, pp. 659–694.

Agell, J., Ohlsson, H. and Thoursie, P. S. (2006), “Growth effects of government expenditure and taxation in rich countries: A comment”, *European Economic Review* , Vol. 50, pp. 211–218.

Aschauer, D. A. (1989), “Is public expenditure productive?”, *Journal of Monetary Economics* , Vol. 23, pp. 177–200.

Barro, R. J. (1990), “Government spending in a simple model of endogenous growth”, *Journal of Political Economy* , Vol. 98, pp. S103–26.

Bond, S., Leblebicioglu, A. and Schiantarelli, F. (2010), “Capital accumulation and growth: a new look at the empirical evidence”, *Journal of Applied Econometrics* , Vol. 25, pp. 1073–1099.

Bond, S. R., Hoeffler, A. and Temple, J. (2001), Gmm estimation of empirical growth models, CEPR Discussion Papers 3048, C.E.P.R. Discussion Papers.

Brauninger, M. (2005), “The budget deficit, public debt, and endogenous growth”, *Journal of Public Economic Theory* , Vol. 7, pp. 827–840.

Chen, B.-L. (2006), “Economic growth with an optimal public spending composition”, *Oxford Economic Papers* , Vol. 58, pp. 123–136.

Conley, T. G. and Ligon, E. (2002), “Economic distance and cross-country spillovers”, *Journal of Economic Growth* , Vol. 7, pp. 157–87.

Devarajan, S., Swaroop, V. and Heng-fu, Z. (1996), “The composition of public expenditure and economic growth”, *Journal of Monetary Economics* , Vol. 37, pp. 313–344.

Durlauf, S. N., Johnson, P. A. and Temple, J. R. (2005), Growth econometrics, *in* P. Aghion and S. Durlauf, eds, ‘Handbook of Economic Growth’, Vol. 1 of *Handbook of Economic Growth*, Elsevier, chapter 8, pp. 555–677.

Easterly, W. and Rebelo, S. (1993), “Fiscal policy and economic growth: An empirical investigation”, *Journal of Monetary Economics* , Vol. 32, pp. 417–458.

Gemmell, N., Kneller, R. and Sanz, I. (2011), “The timing and persistence of fiscal policy impacts on growth: Evidence from oecd countries”, *Economic Journal* , Vol. 121, pp. F33–F58.

Glomm, G. and Ravikumar, B. (1997), “Productive government expenditures and long-run growth”, *Journal of Economic Dynamics and Control* , Vol. 21, pp. 183–204.

Gregoriou, A. and Ghosh, S. (2009), “On the heterogeneous impact of public capital and current spending on growth across nations”, *Economics Letters* , Vol. 105, pp. 32–35.

Gupta, S., Clements, B., Baldacci, E. and Mulas-Granados, C. (2005), “Fiscal policy, expenditure composition, and growth in low-income countries”, *Journal of International Money and Finance* , Vol. 24, pp. 441–463.

Kneller, R., Bleaney, M. F. and Gemmell, N. (1999), “Fiscal policy and growth: evidence from oecd countries”, *Journal of Public Economics* , Vol. 74, pp. 171–190.

Lee, K., Pesaran, M. H. and Smith, R. (1997), “Growth and convergence in multi-country empirical stochastic solow model”, *Journal of Applied Econometrics* , Vol. 12, pp. 357–92.

Pesaran, M. H. (2006), “Estimation and inference in large heterogeneous panels with a multifactor error structure”, *Econometrica* , Vol. 74, pp. 967–1012.

Pesaran, M. H. and Smith, R. (1995), “Estimating long-run relationships from dynamic heterogeneous panels”, *Journal of Econometrics* , Vol. 68, pp. 79–113.

Saint-Paul, G. (1992), “Fiscal policy in an endogenous growth model”, *The Quarterly Journal of Economics* , Vol. 107, pp. 1243–59.

Table 3: Descriptive Statistics (%)

	Median	Mean	Standard Deviation	Min.	Max.
Per Capita GDP Growth	2.50	2.27	5.6	-41.11	56.40
Productive Expenditures / GDP	7.35	8.33	4.03	0.90	30.84
Income Tax / GDP	5.46	6.71	4.63	0.47	27.88
Surplus / GDP	-3.15	-3.93	6.18	-61.14	62.18
Debt / GDP	44.07	56.75	58.19	0.38	637.52

Table 4: List of Countries

South Africa	Mauricio	Slovenia
Germany	Spain	Mexico
Argentina	United States	Mongolia
Australia	Estonia	Nepal
Austria	Ethiopia	Nicaragua
Finland	Norway	Zimbabwe
Barbados	Panama	France
Belarus	Gambia	Pakistan
Belgium	Greece	Paraguay
Bolivia	Guyana	Poland
Brazil	Netherlands	UK
Bulgaria	Czech Republic	Honduras
Burundi	Rwanda	Hungary
Bhutan	Yemen	Senegal
Cameroon	Indonesia	Sri Lanka
Canada	Ireland	Sweden
Kazakhstan	Iceland	Switzerland
Chile	Israel	Suriname
Kuwait	Cyprus	Thailand
Lesotho	Tajikistan	Colombia
Costa Rica	Latvia	Tunisia
Luxembourg	Denmark	Turkey
Egypt	Maldives	Uruguay
El Salvador	Zambia	Malta
Slovakia	Morocco	