The Forward-and the Equity-Premium Puzzles: Two Symptoms of the Same Illness?

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Abstract

Using information on US domestic financial data only, we build a stochastic discount factor—SDF— and check whether it accounts for foreign markets stylized facts that escape consumption based models. By interpreting our SDF as the projection of a pricing kernel from a fully specified model in the space of returns, our results indicate that a model that accounts for the behavior of domestic assets goes a long way toward accounting for the behavior of foreign assets prices. We address predictability issues associated with the forward premium puzzle by: i) using instruments that are known to forecast excess returns in the moments restrictions associated with Euler equations, and; ii) by comparing this out-of-sample results with the one obtained performing an in-sample exercise, where the return-based SDF captures sources of risk of a representative set of developed and emerging economies government bonds. Our results indicate that the relevant state variables that explain foreign-currency market asset prices are also the driving forces behind U.S. domestic assets behavior. Keywords: Equity Premium Puzzle, Forward Premium Puzzle, Return-Based Pricing Kernel. J.E.L. codes: G12; G15
1 Introduction

The Forward Premium Puzzle—henceforth, FPP—is how one calls the systematic departure from the intuitive proposition that, conditional on available information, the expected return to speculation in the forward foreign exchange market should be zero.

One of the most acknowledged puzzles in international finance, the FPP was, in its infancy, investigated by Mark (1985) within the framework of the consumption capital asset pricing model—CCAPM. Using a non-linear GMM approach, Mark estimated extremely high values for the representative agents’ risk aversion parameter, evidencing the canonical consumption model’s inability to account for its over-identifying restrictions. Similar findings in equity markets by Hansen and Singleton (1982), Hansen and Singleton (1984) and Mehra and Prescott (1985), lead to the establishment of the so called Equity Premium Puzzle—henceforth, EPP.

It may, at first, seem surprising that such similar results were never properly linked, and we may only conjecture why the literature on the FPP and the EPP drifted apart after Mark’s (1985) work.

Most probably, the existence of an early specificity for the FPP with no parallel in the case of the EPP — the predictability of returns based on interest rate differentials — may have led many early researchers to believe that even if the CCAPM was capable of accounting for the equity premium it would not solve the FPP.\footnote{We claim not that returns on equity are not predictable. In fact, it is now established that dividend-price ratios, investment-capital ratio and other variables are capable of predicting returns, e.g., Fama and French (1988, 1989), Chauvet and Potter (2000). The point is that this empirical regularity was not seen, in the early days of research with the CCAPM, as a defining feature of the EPP. Nowadays, an empirically successful model ought to take care of this (and many other) non-trivial aspects of asset behavior, as well.}

Indeed, representative of this latter point is the following quote from Engel (1996)’s very comprehensive survey: “[...] it is tempting to draw parallels between the empirical failure of models of the foreign exchange premium, and the closed-economy asset pricing models. [...] However, the forward discount puzzle is not so simple as the equity premium puzzle. International economists face not only the problem that a high degree of risk aversion is needed to account for the
estimated values of $r p_t^{re}$ [rational expectations risk premium]. There is also the question of why the forward discount is such a good predictor of $s_{t-1} - f_t$ [the difference between the (log of) next period’s spot exchange rate and the (log of) forward rate associated with the same period]. There is no evidence that the proposed solutions to the puzzles in domestic financial assets can shed light on this problem.”

In this context, the purpose of this paper is to offer evidence to the contrary; that the solution to the puzzle in domestic assets is likely to account for the analogous puzzle in foreign-asset pricing. Our take on the FPP is that it is not only an international-economics issue, but an asset-pricing problem with important repercussions to monetary economics. Due to the broad range of these two puzzles, and to the likely existence of a common explanation, we believe that the efficient approach for the profession is to join efforts in solving them. Our results invite researchers to refrain from diverting resources on the search of foreign-market specific shocks and concentrate on improving the performance of domestic asset-pricing models.

Comparing the Hansen and Jagannathan (1997) bounds for the pricing kernels, for periods of U.S. real risk-free rates lower than 2.64% per year, we need a more volatile pricing kernel to account for the EPP than the accommodate the FPP anomaly and as this real rate reduces, the foreign assets are more worthless in the extended bounds for U.S. and foreign assets.

Because a direct answer to the question of whether a consumption based model that accounted for the EPP would account for the FPP cannot be given without actually writing down such model, we devise a strategy aligned with the Chen, Roll and Ross (1986) to provide an indirect answer. While they evidenced that macroeconomic factors were able to price assets so well that one could ignore the market factors, we intend to know if a set of U.S. domestic financial market factors drive out another one, in which the factors summarize worldwide foreign government bonds information.

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2Indeed, the uncovered interest parity condition is central to many influential models in monetary economics at least since Mundell (1963). Understanding why this principle is rejected, is paramount to building better models in such an important field in economics.
What we do here is to build a pricing-kernel estimate using only U.S. domestic assets, and show that such pricing kernel generates a risk premium able to accommodate the FPP. This suggests that the relevant state variables that drive foreign-currency market puzzles are already driving the behavior of U.S. domestic assets.

The return based kernel is the unique projection of a stochastic discount factor—henceforth, SDF—on the space of returns, i.e., the SDF mimicking portfolio. One way to rationalize this SDF mimicking portfolio is to realize that it is the projection of a proper economic model (yet to be written) on the space of payoffs. Thus, the pricing properties of this projection are no worse than those of the proper model—a key insight of Hansen and Jagannathan (1991). An advantage of concentrating on the projection is that we can approximate it arbitrarily well in-sample using statistical methods and asset returns alone. Therefore, using such projection not only circumvents the non-existence of a proper consumption model but is also guaranteed not to under-perform in-sample such ideal model.

In order to estimate the SDF mimicking portfolio on this paper, we employ the unconditional linear multi-factor model, which is perhaps the dominant approach in discrete-time empirical work in finance.

Our main tests are based on Euler equations. We exploit the theoretical lack of correlation between discounted risk premia and variables in the conditioning set, or between discounted returns and their respective theoretical means, employing both discounted scaled excess-returns and discounted scaled returns in testing. We test the hypothesis that pricing deviations are statistically zero and also scaled pricing deviations using standard \( t \)-statistic, Wald, and over-identifying restriction tests in a generalized method-of-moments framework.

Our main results are clear cut: return-based pricing kernels built using U.S. assets alone, which account for domestic stylized facts, seem to account for the behavior of foreign assets as well. Hence, our SDF prices correctly the expected return to speculation in forward foreign-exchange markets for the widest group possible of developed countries with a long enough span of future exchange-rate data (Canada, Germany, Japan, Switzerland and the United Kingdom). It is important to stress the out-of-sample character of this exercise, avoiding a critique
of in-sample over-fitting often used in this literature; see, e.g., Cochrane (2001).

More specifically, we first observe that our SDF does a good job in an unconditional in-sample exercise, predicting returns on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones. In our empirical tests, we then examine the properties of our pricing kernel with regards to domestic assets related to the EPP—S&P500 and 90 day Treasury Bill. We fail to reject the null that average discounted returns are equal to unity and that excess returns are equal to zero. We, then, take this pricing kernel to try to price foreign assets. For all countries, we fail to reject the null that average discounted excess returns on currency speculation is zero and that average discounted returns are equal to one, using instruments known to having strong forecasting power for currency movements—one of the defining features of the FPP. Identical results are reported for all the over-identifying restriction tests, except for the Japanese case. We also evidence a satisfactory fitting regarding the returns on the foreign government bonds. Finally, if our statistical proxies really represents a “true pricing kernel,” it should be able to price assets in both domestic and foreign markets jointly, motivating us to include a joint test of the return-based pricing kernels, rather than estimating Euler equations separately for equity and foreign currency returns. In this strongest test, the results confirm the previous evidence, with isolated rejections only in Wald tests for Canadian and German cases.

Our empirical results are based on insignificant pricing deviations that come from reasonable values for these deviations, not higher variance, and provide a strong support for our claims. First, because the main pricing and prediction exercises are performed once more, but using a return-based SDF built in da Costa et al. (2008) that captures sources of risk of a representative set of developed and emerging economies government bonds. The results in the in-sample and out-of-sample exercises are very close. Second, the tests have no little power, as one could think! We evidence systematic rejections in all tests for all government bonds in all the same exercises, replacing our return-based for a consumption-based SDF taking into account reasonable values for risk aversion in a canonical version of the CCAPM.

To try to convey the relevance of our findings note that, as of this moment, the
question of what approach to adopt in trying to account for the FPP is still open. Recent work by Lustig and Verdelhan (2007), for example, tries to account for the forward premium behavior using a consumption based model. They build eight different foreign assets portfolios using information on differential interest rates and compute pricing errors implied by a representative agent’s Euler Equation from a linear consumption model which they calibrate by borrowing structural parameters from Yogo (2006). Burnside (2007), however, disputes their empirical procedure. He argues that one cannot reject the hypothesis that consumption risk explains none of the cross-sectional variation in the expected excess returns in their data set. The empirical evidence offered by Lustig and Verdelhan (2007) based on the low power of their statistical tests cannot support their main claim regarding the ability of consumption-based model to account for foreign asset pricing puzzles.

On the other side of the spectrum, Burnside et al. (2007) show how bid-ask spreads and price pressure may produce non-negligible effects in currency markets transactions. They argue that microstructure issues underlie the behavior of international forward premia.

Our exercise cannot tell whether a consumption model does the job, nor does it allow one to rule out the presence of significant effects of microstructure issues on foreign exchange market prices. It does, however, offer some support to the view that microstructure issues specific to foreign markets are not first order in generating the FPP, and that a single explanation (whether consumption based or not) should apply to domestic and foreign markets.\(^3\)

The remainder of the paper is organized as follows. Section 2 gives an account of the literature that tries to explain the FPP and is related to our current effort. Section 3 discusses the techniques used to estimate the SDF and the pricing tests implemented in this paper. Section 4 presents the empirical results obtained in this paper. Concluding remarks are offered in Section 5.

\(^3\)Put differently, although market microstructure issues may be important, our findings suggest that had we, as a profession been able to produce a model with the pricing kernel we use here, we would not be talking about a Forward Premium Puzzle. The order of magnitude of the failure of our current models is well above what we would see, if we had such model.
2 Critical Appraisal of Current Debate on FPP

2.1 The puzzles and the CCAPM

Fama (1984) recalls that rational expectations alone does not restrict the behavior of forward rates, since it is always possible to include a risk-premium term that reconciles the time series behavior of the associated data. The relevant question is, thus, whether a theoretically sound economic model can offer a definition of risk capable of correctly pricing the forward premium.\(^4\)

The natural candidate for a theoretically sound model for pricing risk is the CCAPM of Lucas (1978) and Breeden (1979). Assuming that the economy has an infinitely lived representative consumer, whose preferences are representable by a von Neumann-Morgenstern utility function \(u(\cdot)\), the first order conditions for his(ers) optimal portfolio choice yields

\[
1 = \beta E_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} R^i_{t+1} \right] \quad \forall i,
\]

and, consequently,

\[
0 = E_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \left( R^i_{t+1} - R^j_{t+1} \right) \right] \quad \forall i, j,
\]

where \(\beta \in (0, 1)\) is the discount factor in the representative agent’s utility function, \(R^i_{t+1}\) and \(R^j_{t+1}\) are, respectively, the real gross return on assets \(i\) and \(j\) at time \(t + 1\) and, \(C_t\) is aggregate consumption at time \(t\).

Under the CCAPM, \(\beta u'(C_{t+1}) / u'(C_t)\), is a stochastic discount factor, i.e., a random variable that discount returns in such a way that their prices are simply the assets discounted expected payoffs. We shall use \(M_{t+1}\) to denote SDF’s, be them associated with the CCAPM or not.

Let the standing representative agent be a U.S. investor who can freely trade domestic and foreign assets.\(^5\) Next, define the covered, \(R^C\), and the uncovered

\[^4\]Frankel (1979), however, argues that most exchange rate risks are diversifiable, there being no grounds for agents to be rewarded for holding foreign assets.

\[^5\]Here, we are implicitly assuming the absence of short-sale constraints or other frictions in
return, \( R^U \), on foreign government bonds trade as
\[
R^C_{t+1} = \frac{i_{F_{t+1}}(1 + i^*_t)}{S_t P_{t+1}} \quad \text{and} \quad R^U_{t+1} = \frac{S_{t+1}(1 + i^*_t)}{S_t P_{t+1}},
\]
where \( i_{F_{t+1}} \) and \( S_t \) are the forward and spot prices of foreign currency in units of domestic currency, \( P_t \) is the dollar price level and \( i^*_t \), the nominal net return on a foreign asset in terms of the foreign investor’s currency. Then, substitute \( R^C \) for \( R^i \) and \( R^U \) for \( R^j \) in (2) to get
\[
0 = \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t(1 + i^*_t)}{S_t P_{t+1}} \right].
\]
Assuming \( u(C) = C^{1-\alpha} (1 - \alpha)^{-1} \), Mark (1985) applied Hansen (1982)'s Generalized Method of Moments (GMM) to (4). He estimated a coefficient of relative risk aversion, \( \hat{\alpha} \), above 40. He then tested the over-identifying restrictions to assess the validity of the model, rejecting them when the forward premium and its lags were used as instruments.\(^6\)

It is well known that similar results obtain when one considers the excess return of equity over risk-free short term bonds. In this case, \( R^i_{t+1} = (1 + i^{SP}_{t+1})P_t/P_{t+1} \) and \( R^j_{t+1} = (1 + i^b_{t+1})P_t/P_{t+1} \), in (2), where \( i^{SP}_{t+1} \) is nominal return on S&P500 and \( i^b_{t+1} \) nominal return on the U.S. Treasury Bill. This is the EPP in a nutshell.

**Returns and Excess Returns** When we substitute \( R^C \) for \( R^i \) and \( R^U \) for \( R^j \), in (1) we get
\[
1 = \mathbb{E}_t \left[ \frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{i_{F_{t+1}}(1 + i^*_t)}{S_t P_{t+1}} \right] \quad \text{and} \quad 1 = \mathbb{E}_t \left[ \frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{S_{t+1}(1 + i^*_t)}{S_t P_{t+1}} \right].
\]
the economy. Our assumption is in contrast with that of Burnside et al. (2007) for whom bid-ask spreads’ impact on the profitability of currency speculation plays the main role in generating the FPP.

\(^6\)Similar results were reported later by Modjtabahedi (1991). Using a different, larger data set, Hodrick (1989) reported estimated values of \( \hat{\alpha} \) above 60, but did not reject the over-identifying restrictions, while Engel (1996) reported some estimated \( \hat{\alpha} \)'s in excess of 100. A more recent attempt to use Euler equations to account for the FPP is Lustig and Verdelhan (2006), where risk aversion in excess of 100 is needed to price the forward premium on portfolios of foreign currency.
In the canonical model, e.g., Hansen and Singleton (1982), Hansen and Singleton (1983) and Hansen and Singleton (1984), the parameter of risk aversion is the inverse of the inter-temporal elasticity of substitution. Even if one is willing to accept high risk aversion, one must also be prepared to accept implausibly high and volatile interest rates.

Accordingly, if one wants to identify the structural parameter $\beta$, in an econometric sense, one cannot resort to direct estimation of excess returns (e.g., 4), but rather to joint estimation of the two Euler equations for returns (e.g., 5), or to any linear rotation of them. It is, therefore, important to make a distinction between studies that test the over-identifying restrictions jointly implied by returns and those that test the ones implied by excess returns alone. For the latter no-rejection may be consistent with any value for $\beta$, including inadmissible ones.7

In our view, a successful consumption-based model must account for asset prices everywhere (domestically and abroad), as well as price returns, excess returns, and many new facts recently found in the extensive empirical research that has been in a great deal sparked by the theoretical developments of the late seventies—see, for example, Cochrane (2006).

2.2 Our strategy and main issues

Even though important progress has been made in building more successful consumption models, there is no consensus as of this moment on whether any model derived from the primitives of the economy accounts for either puzzle. The current state of the art, thus, precludes a direct answer to the question in the title of this paper. Hence, we take an indirect approach.

We extract a pricing kernel from U.S. return data alone and show that it prices both the domestic and the foreign-exchange returns and excess returns.

Following Harrison and Kreps (1979), Hansen and Richard (1987), and Hansen and Jagannathan (1991), we write the system of asset-pricing equations,

$$1 = \mathbb{E}_t \left[ M_{t+1} R^i_{t+1} \right], \quad \forall i = 1, 2, \cdots, N. \quad (6)$$

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7This is for example the case of Lustig and Verdelhan (2007), as they point out in footnote 8, p. 95.
Given free portfolio formation, the law of one price guarantees, through Riesz representation theorem, the existence of a SDF, $M_{t+1}$ satisfying (6). Since (6) applies to all assets,

$$0 = \mathbb{E}_t \left[ M_{t+1} \left( R_{t+1}^i - R_{t+1}^j \right) \right], \quad \forall i, j. \tag{7}$$

We combine statistical methods with the Asset Pricing Equation (6) to devise pricing-kernel estimates as projections of SDF’s on the space of returns, i.e., the SDF mimicking portfolio, which is unique even under incomplete markets. We denote the latter by $M_{t+1}^\ast$.

More specifically, our exercise consists in exploring a large cross-section of U.S. time-series stock returns to construct return-based pricing kernel estimates satisfying the Pricing Equation (6) for that group of assets. Then, we take these SDF estimates and use them to price assets not used in constructing them. Therefore, we perform a genuine out-of-sample forecasting exercise using SDF mimicking portfolio estimates, avoiding the in-sample over-fitting critique discussed in Cochrane (2001), for example.

We cannot overemphasize the importance of out-of-sample forecasting for our purposes. Our main point in this paper is to show that the forward- and the equity-premium puzzle are intertwined. Under the law of one price, a SDF that prices all assets exists, necessarily. Thus, an in-sample exercise would only provide evidence that the forward-premium puzzle is not simply a consequence of violations of the law of one price. We aim at showing more: a SDF can be constructed using only domestic assets, i.e., using the same source of information that guides research regarding the equity premium puzzle, and still price foreign assets. It is our view that this SDF is to capture the growth of the marginal utility of consumption in a model yet to be written.

In dispensing with consumption data, our paper parallels the works of Hansen and Hodrick (2006), Hodrick and Srivastava (1984), Cumby (1988), Huang (1989), and Lewis (1990), which implement latent variable models. They avoid the need for fully specifying a model for the pricing kernel by treating the return on a benchmark portfolio as a latent variable.\footnote{Their results met with partial success: all these papers reject the unbiasedness hypothesis but} Also, Korajczyk and Viallet (1992) apply
the arbitrage pricing theory—APT—to a large set of assets from many countries, and test whether including the factors as the prices of risk reduces the predictive power of the forward premium. They do not, however, perform out-of-sample exercises and do not try to relate the two puzzles.

Backus et al. (1995) ask whether a pricing kernel can be found that satisfies, at the same time, log-linearized versions of

$$0 = \mathbb{E}_t \left[ \frac{M^*_t (1 + i^*_t)}{S_t P_t} \right]$$

and

$$R^f_t = \frac{1}{\mathbb{E}_t (M^*_t)}$$

where $R^f_t$ is the risk-free rate of return. The nature of the question we answer is similar, albeit enlarging its scope by considering all the main domestic assets and focusing on much richer instrument sets.9

Microstructure and the FPP  Currency markets are thin and characterized by important transaction costs. Burnside et al. (2007) take this characteristic of markets seriously and show that bid-ask spreads are not negligible and that price pressures are important in understanding the persistence of apparently unexplored opportunities in those markets.

The fact that we are able to find a linear pricing functional suggests, however, that bid-ask spreads do not produce, at the frequency we examine, the magnitude of deviations from UIP that leads the empirical regularity to be considered a puzzle.

As for price pressures, the existence of a linear pricing functional does not rule out their presence. However, the fact that we construct our pricing kernel using

9Anticipating our results, we should emphasize that we do not reject (9) for any of the instruments, as well, which means that our SDF satisfies both conditions presented by Backus et al. (1995).
domestic assets poses a challenge to any explanation that relies on microstructure issues that are specific to foreign exchange markets.

**Log-linear models** Most studies\(^{10}\) report the FPP through the finding of $\hat{\alpha}_1$ significantly smaller than zero when running the regression,

$$s_{t+1} - s_t = \alpha_0 + \alpha_1 (f_{t+1} - s_t) + u_{t+1},$$

where $s_t$ is the log of the exchange rate at time $t$, $f_{t+1}$ is the log of time $t$ forward exchange rate contract and $u_{t+1}$ is the regression error.\(^{11}\)

Notwithstanding the possible effect of Jensen inequality terms, testing the uncovered interest rate parity (UIP) is equivalent to testing the null that $\alpha_1 = 1$ and $\alpha_0 = 0$, along with the uncorrelatedness of residuals from the estimated regression. Not only is the null rejected in almost all studies, but the magnitude of the discrepancy is very large: according to Froot (1990), the average value of $\hat{\alpha}_1$ is $-0.88$ for over 75 published estimates across various exchange rates and time periods. A negative $\alpha_1$ implies an expected domestic currency appreciation when domestic nominal interest rates exceed foreign interest rates, contrary to what is needed for the UIP to hold.

Getting to (10) from first principles, however, requires stringent assumptions on the underlying asset pricing model. As we shall see, some of these additional assumptions may be behind the unexpected findings. In other words, by log-linearizing

$$1 = E_t \left[ M_{t+1} R_{t+1}^i \right] \quad \forall i = 1, 2, \cdots, N.,$$

it is possible to justify regression (10), but not without unduly strong assumptions on the behavior of discounted returns.

Although, many authors criticize the empirical use of the log-linear approximation of the Pricing Equation (11) leading to (10), as far as we know, this is the first time the criticism above is applied to the use of (10).

\(^{10}\)See the comprehensive surveys by Hodrick (1987) and Engel (1996), and the references therein.

\(^{11}\)In what follows, capital letters are used to represent variables in levels and small letters to represent the logs of these same variables.
Aligned with these theoretical considerations, we should mention recent work by da Costa et al. (2008) who propose and test a bivariate GARCH-in-mean approach, using pricing kernels built along the lines of the ones used herein. Although the omitted risk premium term is able to ‘explain’ conditional returns and excess returns, in the sense that the null is not rejected in Euler Equation tests, they find that the inclusion of (the log of) this term in the conventional regression changes neither the significance nor the magnitude of the the forward premium’s forecasting power. Their results suggest that the log-normality assumption of conditional returns may be too off the mark for one to rely on the usual regression and its extensions.\footnote{12}

**Predictability and the FPP** Another defining characteristic of the FPP is the predictability of returns on currency speculation. Because $\hat{a}_1 < 0$ and significant, given that the auto-correlation of risk premium is very persistent, interest-rate differentials predict excess returns. Although predictability in equity markets has by now been extensively documented, it was not viewed as a defining feature of the FPP, back then.

We take predictability very seriously in our tests. Because we refrain from using log-linearizations, we incorporate predictability in time series, by using forwards, price-dividend ratios, and other variables known to forecast returns as instruments.

### 3 Econometric Tests

#### 3.1 Return-Based Pricing Kernels

The basic idea behind estimating return-based pricing kernels with asymptotic techniques is that asset prices (or returns) convey information about the intertemporal marginal rate of substitution in consumption. If the Asset Pricing Equation

\footnote{12 Also, in articles similar to this one in scope, Gomes and Issler (2006) show that log linearizing the consumer Euler equation may be an explanation for the common finding of rule-of-thumb behavior in consumption decisions even when it is not present in the data.}
holds, all returns must have a common factor that can be removed by subtracting any two returns. A common factor is the SDF mimicking portfolio $M_{t+1}^*$. Because every asset return contains “a piece” of $M_{t+1}^*$, if we combine a large enough number of returns, the average idiosyncratic component of returns will vanish in limit. Then, if we choose our weights properly, we may end up with the common component of returns, i.e., the SDF mimicking portfolio.

Although the existence of a strictly positive SDF can be proved under no arbitrage, uniqueness is harder to obtain since, under incomplete markets, a continuum of SDF’s price all traded securities. Each $M_{t+1}$ can, nonetheless, be written $M_{t+1} = M_{t+1}^* + \nu_{t+1}$ for some $\nu_{t+1}$ obeying $E_t [\nu_{t+1} R_{t+1}^i] = 0$, $\forall i$ where $M_{t+1}^*$ is the unique SDF mimicking portfolio. We devise econometric techniques to estimate $M_{t+1}^*$.

We use principal-component and factor analyses, asymptotic methods that relying on weak law-of-large-numbers, if either $N \to \infty$ or $N, T \to \infty$, to provide consistent estimators of the SDF mimicking portfolio—the unique systematic portion of asset returns.\footnote{These methods can be traced back to the work of Ross (1976), developed further by Chamberlain and Rothschild (1983), and Connor and Korajczyk (1986), Connor and Korajczyk (1993). A recent additional reference is Bai (2005).} For sake of completeness, we present a more complete description of our method in Section A of the Appendix.

The assets used to estimate the multifactor version of the SDF, labelled $\widehat{M}_{t}^{\text{US}}$, is described in the subsection 4.1.

### 3.2 Prediction and pricing tests

In this subsection we explain our prediction and pricing tests.

First, were we able to approximate well enough a time series for the pricing kernel, then the estimate $\widehat{M}_{t}^{\text{US}}$ must predict, at least unconditionally, the main U.S. domestic assets used in its own extraction. Once this is assured, we perform an out-of-sample fitting exercise, taking into account the returns on foreign govern-ment bonds.
Second, we employ an Euler-equation framework, something that was missing in the forward-premium literature after Mark (1985). Since the two puzzles are present in logs and in levels, by working directly with the Pricing Equation we avoid imposing stringent auxiliary restrictions in hypothesis testing, while keeping the possibility of testing the conditional moments through the use of lagged instruments along the lines of Hansen and Singleton (1982), Hansen and Singleton (1984) and Mark (1985). We hope to have convinced our readers that the log-linearization of the Euler equation is an unnecessary and dangerous detour.\footnote{In a recent study, da Costa et al. (2008) propose and test an Asset Pricing Theory-based regression, that has the conventional one as a particular case, suggesting that the lognormality assumption of conditional returns may be to blame.}

Euler equations (6) and (7) must hold for all assets and portfolios. If we had observations on $M^*$ return data could be used to test directly whether they held. Of course, $M^*$ is a latent variable, and the best we can do is to find a consistent estimator for $M^*$. Using return data, and a large enough sample, so that $M^*$ and their estimators are “close enough,” we could still directly test the validity of these Euler equations. Note that not only are the estimators of $M^*$ functions of return data but only return data are used to investigate whether the Asset Pricing Equation holds.

Our procedure is quite similar in spirit to the one performed in da Costa et al. (2008) in the sense that, by finding a theoretical pricing kernel satisfying (6) for assets in the equity and foreign currency markets, we provide a linear functional that prices these assets. Such finding rules out microstructure explanations that generates violations of the law of one price.

Our pricing tests rely on two variants of the Pricing Equation:

$$1 = \mathbb{E}_t \left[ M_{t+1} R^i_{t+1} \right], \quad \forall i = 1, 2, \ldots, N, \text{ or,}$$

$$0 = \mathbb{E}_t \left[ M_{t+1} \left( R^i_{t+1} - R^j_{t+1} \right) \right], \quad \forall i, j. \quad (13)$$

Let $z_t$ be a vector of instrumental variables, which are all observed up to time $t$, therefore measurable with respect to $\mathbb{E}_t (\cdot)$. Employing scaled returns and scaled excess-returns—defined as $R^i_{t+1} \times z_t$ and $(R^i_{t+1} - R^j_{t+1}) \times z_t$, respectively—we are...
able to test the conditional moment restrictions associated with (12) and (13) and consequently to derive the implications from the presence of information. This is particularly important for the FPP, since, when the CCAPM was employed or when returns were not discounted, the over-identifying restriction associated with having the own current forward premium as an instrument was usually rejected: a manifestation of its predictive power. The results of the in-sample and out-of-sample exercises are discussed in the beginning of subsection 4.3.

3.2.1 In-sample exercise: The Equity Premium Puzzle

The first exercise consists in using $\hat{M}_{t}^{US}$ to price domestic asset returns: S&P500, 90-day T-bill. Its purpose is to investigate if the anomaly related to the EPP is present using our pricing-kernel estimate.

For this domestic puzzle, multiply

$$0 = \mathbb{E}_{t}\left[\hat{M}_{t}^{US}\frac{(i_{SP}^{t+1} - i_{b}^{t+1})P_{t}}{P_{t+1}}\right],$$

and

$$1 = \mathbb{E}_{t}\left[\hat{M}_{t}^{US}\frac{(1 + i_{SP}^{t+1})P_{t}}{P_{t+1}}\right],$$

by $z_{t}$ and apply the Law-of-Iterated Expectations to get

$$0 = \mathbb{E}\left\{\begin{array}{c}
\hat{M}_{t}^{US}\frac{(i_{SP}^{t+1} - i_{b}^{t+1})P_{t}}{P_{t+1}} \\
\hat{M}_{t}^{US}\frac{(1 + i_{SP}^{t+1})P_{t}}{P_{t+1}} - 1
\end{array}\right\} \otimes z_{t}. \tag{16}$$

$i_{SP}^{t+1}$ and $i_{b}^{t+1}$ are respectively the nominal returns on the S&P500 and on a U.S. government short-term bond.

The system of orthogonality restrictions (16) can be used to assess the pricing behavior of our estimate of $M^{*}$ with respect to the components of the equity premium. Equations in this system can be tested separately or jointly. In testing, we employ a generalized method-of-moments (GMM) perspective, using (16) as a natural moment restriction to be satisfied.
The purpose of our pricing tests is to check how close the discounted payoff of the asset and its price are to each other. Under a consumption-based approach, it is usual to estimate the utility function parameters and then to test the associated system of orthogonality restrictions using GMM. But, in our two-step procedure, we have already constructed a time series for the pricing kernel, then, we do not have to estimate the utility specification parameters. The parameters to be estimated are the pricing deviations, $\mu_1$ and $\mu_2$, in the following system:

$$0 = E \left\{ \left[ \frac{M_t^{\text{US}} \left( (i_{t+1}^p - \hat{i}_{t+1})P_t \right)}{P_{t+1}} - \mu_1 \right] \otimes z_t \right\} . \quad (17)$$

In the GMM setup, each moment consists in a pricing error. In our approach, this error may be defined as the difference between the discounted payoff and its price plus the deviation multiplied by the instrument. So, the deviations $\mu_i$ are estimated minimizing any quadratic form of the sample mean of the errors for some weighting matrix. An insignificant deviation estimated may be interpreted as a first evidence that the pricing kernel used is able to approximate the discounted return on trading foreign government bond and its price are to each other. Once we have estimated the deviations that make a model fit best, testing the system of overidentifying restrictions, we can assess the overall fit of the model. We also implement a Wald test to check whether pricing deviations are jointly statistically zero.

Finally, to account for cross-section anomalies in the behavior of domestic assets, we also perform in-sample pricing tests for the six Fama and French (1993) benchmark portfolios: dynamic portfolios extracted from the Fama-French library. These results are not reported here, but they are available upon request.

Once we are confident that our pricing kernel does a good job in pricing the relevant U.S. domestic assets, we proceed to our main exercise. We test the pricing properties of the same pricing kernel for foreign assets: now, an out of sample exercise.
3.2.2 Out-of-sample exercise: The Forward Premium Puzzle

To implement an out-sample test for the foreign exchange market, we follow the standard procedure above, but considering only these conditional the moment restrictions:

\[ 0 = E_t \left[ \frac{M_t^\text{US} P_t (1 + i_{t+1}^*) [t F_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right], \quad (18) \]

and

\[ 1 = E_t \left[ \frac{M_t^\text{US} S_{t+1} (1 + i_{t+1}^*) P_t}{S_t P_{t+1}} \right], \quad (19) \]

We apply the same conditioning information procedure, multiplying (18) and (19) by \( z_t \) and applying the Law-of-Iterated Expectations, to get the following econometric test to price foreign government bonds trading:

\[
0 = E \left\{ \begin{bmatrix} \frac{M_t^\text{US} P_t (1 + i_{t+1}^*) [t F_{t+1} - S_{t+1}]}{S_t P_{t+1}} - \mu_1 \\ \frac{M_t^\text{US} S_{t+1} (1 + i_{t+1}^*) P_t}{S_t P_{t+1}} - (1 + \mu_2) \end{bmatrix} \otimes z_t \right\}. \quad (20)
\]

Again, we intend to estimate the deviations \( \mu_1 \) of the null price for the excess return and \( \mu_2 \) of the unitary price for the uncovered return, analyzing its magnitude order and significance. In order for (18) and (19) to hold, we must have \( \mu_1 = 0 \) and \( \mu_2 = 0 \), which can be jointly tested using a Wald test. Analogously to the previous procedure, we also verify whether or not the over-identifying restrictions are not rejected using the \( T \times J \) test.

Another approach for dealing with predictability in foreign government bond markets is to use dynamic portfolios built using variables with good forecasting properties regarding the returns of these bonds. We follow Lustig and Verdelhan (2007), in building eight different zero-cost foreign-currency portfolios, by selling 90 day T-bills and using the proceeds to buy government bonds of various countries. Portfolios are ranked according to interest rate differential with respect to the T-bill—that is, excess returns here in units of foreign currency over units of U.S. currency, hence measurable with respect to \( t \). These results are not reported here, but they are available upon request.
3.2.3 Main exercise: Forward and Equity Premium Puzzles jointly

If our statistical proxies really represent a “true pricing kernel,” they should be able to price assets in both domestic and foreign markets jointly. Consequently, we include a joint test of the return-based pricing kernels, rather than estimating Euler equations separately for equity and foreign currency returns, whenever feasible.

A similar procedure can be implemented for both foreign currency and domestic markets, multiplying (14), (15), (18) and (19) by \( z_t \) and applying the Law-of-Iterated Expectations. We perform a third econometric testing procedure given by

\[
0 = \mathbb{E} \left\{ \begin{align*}
\mathcal{M}^{US} P_t (1 + i_{t+1}^p) [F_{t+1} - S_{t+1}] & - \mu_1 \\
\mathcal{M}^{US} S_{t+1} (1 + i_{t+1}^p) P_t & - (1 + \mu_2) \\
\mathcal{M}^{US} (i_{t+1}^{SP} - i_{t+1}^b) P_t & - \mu_3 \\
\mathcal{M}^{US} (1 + i_{t+1}^{SP}) P_t & - (1 + \mu_4)
\end{align*} \right\} \otimes z_t
\]

(21)

to examine whether both EPP and FPP hold jointly. We test pricing deviations individually and jointly along with the restriction \( T \times J \) test.

3.3 GMM estimation setup and instruments

Since our tests are based on Euler-equation restrictions, it is natural to estimate the pricing deviations, \( \mu_i \), using GMM. As in any exercise employing GMM estimation, one must choose first how to weight the different moments used for identification of the parameters of interest. We can form second-stage estimates by choosing the inverse of the asymptotic variance-covariance matrix of first-stage sample moments, \( \hat{S}^{-1} \). These optimal weights yield the smallest sampling variation among all estimators that set linear combinations of the sample mean errors.

There is, however, a trade-off between attaining full efficiency and correctly
specifying the structure of the variance-covariance matrix. As is well known, both OLS and GLS are consistent under correct specification of the variance-covariance matrix. However, in trying to achieve full efficiency, one can render GLS estimates inconsistent if the structure chosen is incorrectly specified. OLS is a robust estimate in the sense that it does not rely on a correct choice for the weighting matrix. For that reason, most applied econometric studies use OLS in estimation and properly estimate its standard errors using the methods advanced by White (1980) and later generalized by Newey and West (1987). Similarly, using the identity matrix instead of $\hat{S}^{-1}$ may be better under the circumstances considered here.

There is an additional reason why it may be hard to achieve an optimal weighting scheme in GMM estimation, favoring the use of the identity matrix in weighting moment restrictions. Suppose that we use $N$ pricing equations with $k$ instruments to form the $N \times k$ orthogonality conditions used in GMM estimation. The variance-covariance matrix of sample moments associated with these orthogonality conditions is of order $(N \times k) \times (N \times k)$ and has $(N \times k) \times (N \times k + 1) / 2$ parameters. These must be estimated using $T$ time-series observations. If $N \times k$ is large vis-a-vis $T$, it may be infeasible to estimate the variance-covariance matrix of sample moments. Even if estimation is feasible, the estimate of this variance-covariance may be far from its asymptotic probability limit, which is a problem for asymptotic tests.

In this context, we use a different weighting matrix for each exercise.

**Pricing assets in domestic and foreign markets individually** In this paper, $T = 112$, while $k = 2, 3, \text{ or } 4$, and $N = 2 \text{ or } 4$. So, to implement the in-sample test, considering moment restrictions (17) and to account for the FPP, based on the system of orthogonality restrictions (20), we will perform an optimal GMM, since we have a reasonable number of moment conditions, 6 and 4, respectively, given our sample size. Following Hansen and Singleton (1982), our estimates are produced by an iterate procedure as discussion in Cochrane (2001).

As for the instruments, we use specific financial variables as instruments, carefully chosen according to their forecasting potential. For the EPP, we employ the dividend-price ratio and the investment-capital ratio, following Campbell and
Shiller (1988), Fama and French (1988) and Cochrane (1991), who provide evidence that these variables are good predictors of stock-market returns. For the FPP, we use the current value of the respective forward premium, \((tF_{t+1} - S_t)/S_t\), since its forecasting ability is a defining feature of this puzzle and this series is measurable with respect to the information set used by the representative consumer.

In addition to these variables, we used lagged values of returns for the assets being tested as a robustness check. Taking into account the fact that expected returns and business cycles are correlated, e.g., Fama and French (1989), we also use as instruments macroeconomic variables with documented forecasting ability regarding financial returns, such as real consumption and GDP instantaneous growth rates and the consumption-GDP ratio.

**Pricing assets in both markets jointly** In the joint tests, we have \(k = 4\) and \(N = 4\), considering the system (21) and the instruments already mentioned. This leaves us with 16 moment conditions, an extreme cases for which our 112 time-series observations cannot yield an invertible variance-covariance matrix of sample moments. Even giving up something in asymptotic efficiency, a first-stage estimate can be more robust to this statistical limitation. According to Cochrane (2001), if the number of moments is more than around 1/10 of the number of data points, the \(S\) estimates tend to become unstable and near singular. In this case, using \(\hat{S}^{-1}\) can lead us to pay more attention to probably spurious linear combinations of moments.

Since the optimal scheme is limited to a few assets and instruments, the use of asymptotic tests in is jeopardized!

We will, therefore, use the identity matrix in weighting orthogonality conditions and will correct the estimate of their asymptotic variance-covariance matrix for the presence of serial correlation and heteroskedasticity of unknown form using the techniques in Newey and West. However, if the identity matrix is used in weighting moments, it makes little sense to use Hansen’s over-identifying restrictions \(T \times J\) test to verify whether or not pricing errors are zero, since possible covariance terms between different scaled pricing errors are disregarded by using
a diagonal weighting matrix. This fact, coupled with the well known problem of the lack of power of the $T \times J$ test, suggest the use of a Wald test to check whether pricing errors are jointly statistically zero.\textsuperscript{15}

4 Empirical Results

4.1 Data and Summary Statistics

In principle, whenever econometric or statistical tests are performed, it is preferable to employ a large data set either in the time-series ($T$) or in the cross-sectional dimension ($N$). In choosing return data, we had to deal with the trade-off between $N$ and $T$. In order to get a larger $N$, one must accept a reduction in $T$: disaggregated returns are only available for smaller time spans than aggregated returns.

In terms of sample size, our main limitation for the time-series span used here is regarding FPP tests. Foreign-exchange only floated post–Bretton Woods Agreements: the Chicago Mercantile Exchange, the pioneer of the financial-futures market, only launched currency futures in 1972. In addition to that, only futures data for a few developed countries are available since then. In order to have a common sample for the largest set of countries possible, we considered here U.S. foreign-exchange data for Canada, Germany, Japan, Switzerland and the U.K., covering the period from 1977:1 to 2004:4, on a quarterly frequency, comprising 112 observations.

Returns on covered and uncovered trading with foreign government bonds were transformed into U.S.$ real returns using the consumer price index in the U.S. The forward-rate series were extracted from the Chicago Mercantile of Exchange database, while the spot-rate series were extracted from the Bank of England and the IFS databases. To study the EPP we use the U.S.$ real returns on the S&P500 and on 90-day T-bill. The latter were extracted in nominal terms from

\textsuperscript{15}Notice that the Wald test uses a non-diagonal matrix in weighting pricing errors, even if the identity matrix is used in weighting moments.
the CRSP and the IFS database, respectively. Real returns were obtained using the U.S. CPI as deflator.

A second ingredient for testing these two puzzles is to estimate the return-based pricing kernel. $M_t^{USS}$ is a return-based pricing kernel constructed as a linear function of factors. Based on the principal-component technique, seven factors are extracted from: i) the U.S.$ real returns on all U.S. stocks traded on NYSE along the period in question, in a totality of 464 stocks, according to CRSP database and ii) the real returns on one hundred size-$BE/ME$ Fama-French portfolios. In addition to these seven factors, we also use U.S. real return on Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones as factors. All these factors are mixed in factor analyses.

$M_t^{USS}$ is completely U.S. based available at a very disaggregated level, that characterizes the stock market and other relevant benchmarks, besides capturing the size-book effects. This domestic SDF is later used to test the Pricing Equation for foreign assets—a direct response to Cochrane (2001)' critique—making our pricing tests out-of-sample in the cross-sectional dimension (assets).

All macroeconomic variables used in econometric tests were extracted from FED’s FRED database. We also employed additional forecasting financial variables that are specific to each test performed, and are listed in the appropriate tables of results.

Table 1 presents a summary statistic for real returns and excess returns on the assets to be priced over the period 1977:1 to 2004:4. The real return on covered trading of foreign bonds show means and standard deviations quite similar to that of the U.S. Treasury Bill, which is reasonable, since it reflects the covered interest rate parity in a frictionless economy. Regarding the return on covered trading, the means are ranging from 0.39% to 0.66%, while the standard deviation from 0.86% to 1.10%, except for the Swiss case with 2.06% and 7.74%, respectively.

Observing the forward rates for the Swiss franc in Chicago Mercantile Exchange database and comparing to the respective spot rates, it is possible to observe outliers in 1995. Fortunately, this forward series is not used in order to estimate the pricing kernel and for our conclusions here, this problem turned out
not to be relevant, since when we use the outlier-removed sample to check the robustness of our empirical results; they are qualitatively identical for these two different sets of data. Regarding the uncovered trading, averages range from 0.68% to 1.26%, while standard deviations range from 2.77% to 8.10%.

The median values are close to the respective means, except for covered trading with Swiss bonds and for uncovered operations with Japanese bonds. Downside risk, which takes into account only negative deviations, range from 0.58% to 1.50% and from 1.79% to 5.29%, considering covered and uncovered trading, respectively.

The real return on the S&P500 is 2.23%, while that of the 90-day T-bill is 0.44%, with a resulting quarterly excess return (equity premium) of 1.79% or 7.35% per year. As expected, real stock returns are much more volatile than the U.S. Treasury Bill return—quarterly standard deviations of 8.00% and 0.80% respectively. Regarding the other U.S. benchmark assets, Nasdaq provides higher return, 2.99%, while the mean return on Gold is 0.38%. Return on Bonds on AAA U.S. Corporations is much less volatile than the Nasdaq, 0.81% and 12.72%, respectively.

We computed the Sharpe ratio for the U.S. stock market to be 0.22, while the Sharpe ratio of the uncovered trading of foreign bonds ranges from −0.06 to 0.22, depending on the country being considered. According to Shiller (1982), Hansen and Jagannathan (1991), and Cochrane and Hansen (1992), an extremely volatile SDF is required to match the high equity Sharpe ratio of the U.S. Hence, the smoothness of aggregate consumption growth is the main reason behind the EPP. Since the higher the Sharpe ratio, the tighter the lower bound on the volatility of the pricing kernel, a natural question that arises is the following: may we regard this fact as evidence that a kernel that prices correctly the equity premium would also price correctly the forward premium? In this sense, our results can be useful to answer this question in an indirect way.

## 4.2 SDF Estimates

While some other econometric techniques require strong assumptions about distributions, under GMM we only need the stationarity of the underlying pro-
Before presenting the results for SDF estimates, we tested all returns — used either in estimating SDF’s or in the pricing test procedures — for the presence of unit roots in the auto-regressive polynomial. Because returns are prone to displaying heteroskedasticity, we used the Phillips and Perron (1988) unit-root test including an intercept in the test regression. For almost all cases we rejected the null of a unit root at the 5% level and rejected the null for all cases at the 10% level. We take this as evidence that all returns and the pricing kernels are well behaved.

The first step in constructing $M_t^{US}$ is to choose the number of factors. Following Lehmann and Modest (1988), Connor and Korajczyk (1988), Tsay (2001) and most of the empirical literature, we took a pragmatic, informal, but useful view, examining the time plot of the eigenvalues ordered from the largest to the smallest. We found that seven factors accounted for most of the variation of the returns on stocks and the Fama-French portfolios. This choice was close to the one in Connor and Korajczyk (1993), who examined returns from stocks listed on the New York Stock Exchange and the American Stock Exchange.

The estimate of $M_t^*$ is plotted in Figure 1. It ranges from −0.41 to 2.36, with mean close to 0.99 and volatility about 0.60, higher than the standard deviations observed in consumption-based pricing kernels.

In Figure 2, we plot mean-variance frontiers for the discount factors, following Hansen and Jagannathan (1997). The gray line restricts the moments of a pricing kernel needed to price unconditionally the set of U.S.$ real returns on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones. The black line restricts this set of assets and also the set comprised by returns on uncovered and covered trading with British, Canadian, German, Japanese and Swiss government bonds. Comparing the Hansen and Jagannathan (1997) bounds for the pricing kernels, for periods for which the U.S. real risk-free rates remained below 2.64% per year, a more volatile pricing kernel is needed to account for the EPP than the accommodate the FPP anomaly. As this real rate

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17Unit-root test results for all series used in the paper are available upon request.
reduces, foreign assets provide less information on valid SDF’s in the extended bounds for U.S. and foreign assets, and the difference between the gray and black line representing the bounds reduce.

4.3 Pricing-Test Results

In-sample exercise: The Equity Premium Puzzle  

The first exercise consists in verifying whether our SDF does a good job in an unconditional in-sample test, predicting returns on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones. In Figure 3, we show predicted versus realized average net real return for $M_{USS}^t$. Pricing errors are measured by the distance to the 45-degree line (dotted line). The U.S. domestic pricing kernel does a fine job in predicting these returns. The prediction errors range from 0.01% and 0.11% with 0.07% mean-square root error. We fail to reject the null that unconditional average discounted returns are equal to unity.

Table 2 presents results on the behavior of our return-based pricing kernel in pricing returns related to the EPP. These are in-sample tests. More specifically, in this table we report the results of testing the system of orthogonality restrictions (17), which accounts for the equity premium, $R_{SP}^{t+1} - R_{t+1}^b$, and for the return on S&P500, $R_{t+1}^{SP}$. We use a constant as instrument as well as $(D/P)_t$ and $(I/C)_t$.

These in-sample tests show no evidence that pricing errors are relevant: at the 5% significance level, pricing deviations are all statistically zero individually or jointly, using the Wald test. We also do not reject the null in Hansen’s overidentifying restrictions test. These results are similar considering other domestic returns mentioned here.

Beyond the high equity Sharpe ratio or the reported power of the dividend-price ratio to forecast stock-market returns, the pattern of cross-sectional returns of assets exhibit some “puzzling aspects” from the perspective of the CCAPM model. Best known among these are the “size” and the “value” effects—e.g. Fama and French (1996) and Cochrane (2006)—, i.e., the fact that small stocks and stocks with low market values—relative to book values—tend to have higher average returns than other stocks. We perform in-sample tests to check whether these
pricing anomalies occur in our data set by pricing the six Fama and French (1993) benchmark portfolios, dynamic portfolios extracted from the Fama-French library, using the Fama-French factors as instruments. Once again, the performance of $M_{t}^{USS}$ is reflected in the fact that there is no such kind of rejection.

**Out-of-sample exercise: The Forward Premium Puzzle** Table 3 presents some of our main results. In these out-of-sample pricing tests $M_{t}^{USS}$ is used to price foreign assets related to the FPP.

We report the results of tests for the system of orthogonality restrictions (20). There are two Euler equations: one with the excess return on uncovered over covered trade with foreign government bonds $(R_{t+1}^{U} - R_{t+1}^{C})$, and one with the uncovered return, $R_{t+1}^{U}$. The instrument set includes a constant and the variable that generates the FPP in regression (10), $(tF_{j,t+1} - S_{j,t})/S_{j,t}$, making the result of these tests very informative with regards to the FPP.

At the 5% significance level, all individual pricing deviations are statistically zero for all foreign currencies studied here. Wald-test results show no sign of joint significance for currency-pricing errors. Moreover, the size of the pricing deviations is relatively modest: $\hat{\mu}_2$ is comparable to returns on uncovered trading, while $\hat{\mu}_1$ is lower than the respective excess returns. Identical successful results are reported for all overidentifying restriction tests, except for Japanese case, a strong evidence about the overall fit of the model proposed.

Our conclusions come from small deviations, not higher variance.

We perform the same exercise, but now using $M_{t}^{GILS}$ — a global return-based pricing kernel constructed by da Costa et al. (2008) extracted from a data set mostly composed of aggregate US$ real returns on G7-country stock indices and short-term government bonds and returns on 22 (developed and emerging) government bonds. In addition to this data set they also use U.S.$ real returns on Lustig and Verdelhan’s (2007) eight foreign dynamic currency portfolios, which uses quarterly series of spot exchange rates and short-term foreign government bonds available in IFS database.

Since da Costa et al.’s (2008) purpose is to take into account other international
markets stylized facts that escape consumption based models, the results testing
the system of moment conditions (20), but with \( \hat{M}_t^{GLS} \) instead of \( \hat{M}_t^{USS} \), consists in
an in-sample FPP test.

Qualitatively and also quantitatively speaking, the results reported in Tables 3
and 4—using \( \hat{M}_t^{USS} \) and \( \hat{M}_t^{GLS} \), respectively, in order to account for the FPP—are
very close, except for the failing in the Hansen’s test for German case in the latter table.

We also compare both pricing kernel’s performance in the unconditional pre-
diction exercise. Figure 4 displays both performances in predicting returns and
excess returns on uncovered and covered trading of British, Canadian, German,
Japanese and Swiss bonds. \( \hat{M}_t^{USS} \) and \( \hat{M}_t^{GLS} \) do both a decent job in predicting
returns and excess returns on trading with foreign government bonds. In the first
case, errors range from 0.02% and 2.05% and based on the root mean-square error
as prediction performance measure, we have 0.85%. These values are comparable
0.04%, 0.92% and 0.53%, observed using \( \hat{M}_t^{GLS} \). The main exceptions are the excess
returns on British and Swiss bonds.

**Main exercise: Forward and Equity Premium Puzzles jointly** In Table 5 we
present joint tests of the FPP and of the EPP for Canada, Germany, Japan, Switzerland
and the U.K. These pricing restrictions stack the Euler equations that char-
acterize the EPP tested in Table 2 and those tested in Table 4, i.e., the system (21).
The instrument set is the union of the sets previously considered: a constant,
\( (i_{F_{t+1}} - S_t) / S_t \), \( (D/P)_t \) and \( (1/C)_t \). In this strongest test of the pricing kernels
results confirm that return-based kernels generate no puzzles whatsoever in pricing
returns and excess returns for the U.K., Germany, and Switzerland. Moreover,
for all countries, all individual tests for pricing errors are not significant at usual
significance levels, whereas joint Wald tests reject that all pricing errors are zero
for Canada and Germany at these same levels. One way to reconcile this last set of
results is to imagine that pricing errors are indeed correlated, perhaps strongly. If
this is case, it is possible for individual tests not to reject the null while joint tests
produce the opposite result.\footnote{Hansen (1982)’s $J$-test does not reject the null because it evaluates the quadratic form at the optimal level, using the identity matrix as a weighting factor. This, of course, does not take into account the cross-correlation of the pricing errors.}

The results in Tables 5 show strong evidence that $M_{t}^{ISS}$ are able to account for most of the puzzling aspects in asset pricing, as the EPP and the FPP jointly.

Once more, intending to corroborate that the empirical results are due to lower values for the pricing deviations, we compare the results reported in Table 5 to the results in Table 6, where the da Costa et al. (2008)’s global return-based pricing kernel is used. The main difference is related to lower levels of standard errors regarding the returns.

**Some extensions**  To account for other domestic and international markets stylized facts that escape consumption based models, we also use U.S.$ real returns on the Fama and French (1993) benchmark portfolios formed on size and book-to-market, extracted from the French data library, as well as Lustig and Verdelhan (2007) eight foreign currency portfolios, constructed by da Costa et al. (2008) using quarterly series of spot exchange rates and short-term foreign government bonds available in IFS database.\footnote{The entire database of foreign government bonds is in the section B in the Appendix.} They follow Lustig and Verdelhan (2007)’s procedure, using as the foreign interest rate, the interest rate on a 3-month government security (e.g. a U.S. T-bill) or an equivalent instrument, taking into account that as data became available, new countries are added (or subtracted) to these portfolios. There is strong evidence that they price correctly all the dynamic Fama-French portfolios and all the Lustig and Verdelhan portfolios—with the exception of Portfolio1. These results are not reported here, but they are available upon request.

Our claims in this paper depend fundamentally on how strong the unconditional fitting and the out-of-sample Euler-based pricing exercises are. Besides comparing the results using $M_{t}^{ISS}$ to the results using $M_{t}^{GLS}$, we also compare to the results observed using a canonical consumption-based pricing kernel calculated with acceptable and reasonable values for the curvature parameter, accord-
ing to Mehra and Prescott (1985). We observe a generalized rejection in all tests. At the 5% significance level, we reject that all deviations of the unitary price for the returns are statistically zero individually. Most of the deviations of the null price for the excess return are also significant and the Wald test confirms these rejections. The failures in Hansen’s moments test are also systematic. The root mean-square error as prediction performance measure takes on higher values: 3.25% and 3.28% in predicting U.S. and foreign assets, respectively. It is, therefore, not true that any pricing kernel could be successful in the tests proposed here!

Finally, according to Hansen and Jagannathan (1997), "[...] pricing errors may occur either because the model is viewed formally as an approximation, [...] or because the empirical counterpart to the theoretical stochastic discount factor is error ridden". In this sense, we are able to show that \(^{\text{M}}_{\text{USS}}\), a SDF constructed to account only for domestic puzzles, not only does a good job in accounting for foreign market results, based on \(\chi^2\) statistics, but, in fact, performs better than \(^{\text{M}}_{\text{GLS}}\), according to the pricing error measure developed by these authors. The measures are 0.021 and 0.275, respectively.

**Discussion** Notwithstanding the two joint rejections on Wald tests, the favorable out-of-sample tests results obtained in Tables 3 and 5 are in sharp contrast with those formerly obtained in log-linear tests of the FPP and the results obtained using consumption-based kernels. As argued above, this may suggest the inappropriateness of log linearizing the Euler equation and/or using current consumption-based pricing kernels.

As in any empirical exercise, it is important to verify whether results are robust to changes in the environment used in testing. Here, we changed the conditioning set used through Tables 2 to 6 as well as the assets directly being tested in pricing. In both instances the basic results remain unchanged and are available upon request.

Finally, a crucial element in our analysis is showing that the returns of investments in foreign assets are accounted for by an adequate pricing kernel. This is indeed what we found when the ratio \(F_{t+1}/S_t\) was used as an instrument and the
theoretical restrictions tested were not rejected. This suggests that \( F_{t+1}/S_t \) has no predictive power for

\[
M^*_{t+1} \frac{P_t(1 + \bar{i}^*_{t+1})}{P_{t+1}} \frac{F_{t+1} - S_{t+1}}{S_t},
\]

despite having predictive power for

\[
\frac{P_t(1 + \bar{i}^*_{t+1})}{P_{t+1}} \frac{F_{t+1} - S_{t+1}}{S_t}.
\]

Hence, although the excess returns on uncovered over covered trading with foreign bonds are predictable, “risk adjusted” excess returns are not.

5 Conclusion

Previous research has cast doubt on whether a single asset pricing model was capable of correctly pricing the equity and the forward premium, which lead to the emergence of two separate literatures. We challenge this position and propose a fresh look into the relationship between the Equity and the Forward Premium Puzzles.

We do so by extracting SDF estimates using U.S. return data and showing them to be able to properly price returns and excess returns of assets that comprise the equity premium and the forward premium puzzles. We, thus, establish the common nature of the FPP and the EPP. By using consistent estimates of the SDF mimicking portfolio, we have shown that the projection of a proper CCAPM model—yet to be written—on the space of returns is able to properly price assets that comprise the EPP and the FPP. We have not found this proper model, yet. Nevertheless, we rely on the fact that its pricing properties cannot be better than those of its projection on the space of returns.

By working with a U.S. based version of SDF estimates, we were able to show that the factors contained in domestic (U.S.) market returns yield neither evidence of the EPP nor evidence of the FPP for most cases.

Our starting point is the Asset Pricing Equation, coupled with the use of consistent estimators of the SDF mimicking portfolio. We first show that, our return-based kernels constructed using domestic (U.S.) returns alone price correctly the
equity premium, the 90-day T-bill and the Fama-French portfolios. When discounted by our pricing kernels, excess returns are shown to be orthogonal to past information that is usually known to forecast undiscounted excess returns. Based on these results, we go one step further and ask whether the EPP and the FPP are but two symptoms of the same illness—the inability of standard (and augmented) consumption-based pricing kernels to price asset returns or excess-returns. In our tests, we found that the ex-ante forward premium is not a predictor of discounted excess returns, despite their being so for undiscounted excess returns.

We believe to have offered evidence that the answer to the question posed in the title of this paper is in the affirmative. A different and interesting issue is whether the forward should premium be regarded as a reward for risk taking? If we take the covariance with $M_{t+1}^*$ as the relevant measure of risk, then our answer is yes. This position is not without controversy, however. Citing (Engel (1996), p. 162): “If the [CAPM] model were found to provide a good description of excess returns in foreign exchange markets, there would be some ambiguity about whether these predicted excess returns actually represent premiums.” We do however side with the position implicit in Brandt et al. (2006) for whom the behavior of the SDF is equal to that of the intertemporal marginal rate of substitution for a model of preferences and/or market structure yet to be written. In this sense, we have provided grounds to believe that the striking similarity in the results found in trying to apply the CCAPM for the two markets is not accidental.

References


Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, “International risk sharing is better than you think, or exchange rates are too smooth,” Journal of Monetary Economics, 2006, 53, 671–698.


A Return-Based Pricing Kernels and the SDF Micking Portfolio

Factor models summarize the systematic variation of the $N$ elements of the vector $R_t = (R_{1,t}, R_{2,t}, \ldots, R_{N,t})'$ using a reduced number of $K$ factors, $K < N$. 37
Consider a $K$-factor model in $R_{i,t}$:

$$R_{i,t} = a_i + \sum_{k=1}^{K} \beta_{i,k} f_{k,t} + \eta_{i,t}, \quad (22)$$

where $f_{k,t}$ are zero-mean pervasive factors and, as is usual in factor analysis and

$$\text{plim} \frac{1}{N} \sum_{i=1}^{N} \eta_{i,t} = 0.$$  

Denote by $\Sigma_r = \mathbb{E} (R_t R_t') - \mathbb{E} (R_t) \mathbb{E} (R_t')$ the variance-covariance matrix of returns. The first principal component of the elements of $R_t$ is a linear combination $\theta' R_t$ with maximal variance subject to the normalization that $\theta$ has unit norm, i.e., $\theta' \theta = 1$. Subsequent principal components are identified if they are all orthogonal to the previous ones and are subject to the same normalization. The first $K$ principal components of $R_t$ are consistent estimates of the $f_{k,t}$'s. Factor loadings can be estimated consistently by simple OLS regressions of the form (22).

That is, principal components are simply linear combinations of returns. They are constructed to be orthogonal to each other, to be normalized to have a unit length and to deal with the problem of redundant returns, which is very common when a large number of assets is considered. They are ordered so that the first principal component explains the largest portion of the sample variance-covariance matrix of returns, the second one explains the next largest portion, and so on.

To understand why we need the unit-norm condition, recall that the variance of the first principal component $\theta' R_t$ is $\theta' \Sigma_r \theta$. As discussed in Dhrymes (1974), because we want to maximize this variance, the problem has no unique solution—we can make the variance as large as we want by multiplying $\theta$ by a constant $\kappa > 1$. Indeed, we are facing a scale problem, which is solved by imposing unit norm, i.e., $\theta' \theta = 1$.

When a multi-factor approach is used, the first step is to specify the number of factors by means of a statistical or theoretical method and then to consider the estimation of the model with known factors. Here, we are not particularly concerned about identifying the factors themselves. Rather, we are interested in
constructing an estimate of the unique SDF mimicking portfolio $M_t^*$. For that, we shall rely on a purely statistical model, combined with the Pricing Equation.

It is straightforward to connect principal-component and factor analyses with the Pricing Equation, delivering a consistent estimator for $M_t^*$. Given estimates of $a_i, \beta_{i,k},$ and $f_{k,t}$ in (22), one can write their respective expected-beta return expression:

$$\mathbb{E}(R_i) = \gamma + \sum_{k=1}^{K} \beta_{i,k} \lambda_k, i = 1, 2, ..., N$$

where $\lambda_k$ is interpreted as the price of the $k$-th risk factor. The fact that the zero-mean factors $f = \bar{f} - \mathbb{E}(\bar{f})$ are such that $\bar{f}$ are returns with unitary price allows us to measure the $\lambda$ coefficients directly by

$$\lambda = \mathbb{E}(\bar{f}) -$$

and consequently to estimate only $\gamma$ via a cross-sectional regression\footnote{Note that we are not assuming the existence of a risk free rate.}. Given these coefficients, one can easily get an estimate of $M_t^*$,

$$\hat{M}_t^* \equiv a + \sum_{k=1}^{K} b_{k} f_{k,t}$$

where $(a, b)$ is related to $(\lambda, \gamma)$ through

$$a \equiv \frac{1}{\gamma} \text{ and } b \equiv -\gamma \left[ \text{cov}(ff') \right]^{-1} \lambda,$$

It is easy then to see the equivalence between the beta pricing model and the linear model for the SDF. More, it is immediate that

$$\mathbb{E}(\hat{M}_t^* R_{i,t}) = 1, \ i = 1, 2, ..., N.$$ 

The number of factors used in the empirical analysis, $K$, is expected to be rather small. We followed Lehmann and Modest (1988) and Connor and Korajczyk (1988), taking the pragmatic view whereby increasing $K$ until the estimate of $M_t^*$ changed very little due to the last increment in the number of factors.

It is important to stress that we need to impose a scale whenever a factor-model is used. Here, it is implicitly imposed that $\text{VAR} (\hat{M}_t^*) = 1$; see Cochrane (2001).
Table 1
Summary statistics for the quarterly real returns and excess returns. From 1977:1 to 2004:4, 112 observations.

International data: foreign government bonds

<table>
<thead>
<tr>
<th></th>
<th>US$ real net return on the covered trading of foreign government bond</th>
<th>US$ real net return on the uncovered trading of foreign government bond</th>
<th>US$ real excess return on the uncovered over the covered trading of foreign government bond</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UK</td>
<td>Canada</td>
<td>Germany</td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.442</td>
<td>0.569</td>
<td>0.659</td>
</tr>
<tr>
<td>Median (%)</td>
<td>0.415</td>
<td>0.581</td>
<td>0.581</td>
</tr>
<tr>
<td>Stand. deviat. (%)</td>
<td>1.100</td>
<td>1.068</td>
<td>0.969</td>
</tr>
<tr>
<td>Downs. risk (%)</td>
<td>0.816</td>
<td>0.842</td>
<td>0.589</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.552</td>
<td>-2.502</td>
<td>1.078</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>2e10^-4</td>
<td>0.119</td>
<td>0.224</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>3e10^-4</td>
<td>0.151</td>
<td>0.370</td>
</tr>
</tbody>
</table>

continued on next page...
Table 1
Summary statistics for the quarterly real returns and excess returns.
From 1977:1 to 2004:4, 112 observations.

US data: stock indices and other relevant assets

<table>
<thead>
<tr>
<th></th>
<th>Stock indices</th>
<th>Other assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S&amp;P500</td>
<td>AMEX</td>
</tr>
<tr>
<td>Mean (%)</td>
<td>2.230</td>
<td>2.069</td>
</tr>
<tr>
<td>Median (%)</td>
<td>2.840</td>
<td>2.054</td>
</tr>
<tr>
<td>Downs. risk (%)</td>
<td>5.897</td>
<td>6.814</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.336</td>
<td>-0.326</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.224</td>
<td>0.175</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.303</td>
<td>0.239</td>
</tr>
</tbody>
</table>
\( \widetilde{M_t}^{\text{US$}} \) is a return-based pricing kernel constructed as a linear function of factors. Seven factors are extracted from the U.S.$ real returns on 464 stocks traded on NYSE and real returns on one hundred size-\( BE/ME \) Fama-French portfolios. In addition to these factors, we also use U.S.$ real returns on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones as factors. Data from 1977:1 to 2004:4, 112 observations.

In this figure, we plot a mean-variance frontier of all discount factors that price two sets of assets. The gray line restricts the moments of a pricing kernel in order to price unconditionally the set of U.S.$ real returns on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones. The gray line (dashed) restricts the set of pricing kernels given the returns on uncovered and covered trading with British, Canadian, German, Japanese and Swiss government bonds. The black line restricts both sets of assets. The horizontal axis contains the unconditional mean and the vertical one contains the unconditional standard deviation. Data from 1977:1 to 2004:4, 112 observations.
In this figure, we plot the realized return and its respective unconditional predictions, using $\tilde{M}^{US\$}_t$. Real return on Gold, U.S. Real Estate, Bonds on AAA U.S. Corporations, S&P500, AMEX, Nasdaq, NYSE and Dow Jones. Data from 1977:1 to 2004:4, 112 observations.

Table 2

<table>
<thead>
<tr>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = \mu_2 = 0$</td>
<td>$\tilde{\mu}_1 = 0.0052$ (0.758) [0.492]</td>
</tr>
<tr>
<td>$\chi^2$ 0.5242 [0.769]</td>
<td>$1 + \tilde{\mu}_2 = 0.9898$ (5.871) [0.862]</td>
</tr>
</tbody>
</table>

* Indicates the rejection of the null hypothesis (individually insignificant pricing deviation) at 5% level.
** Indicates the rejection of the null hypothesis (jointly insignificant pricing deviations) in Wald test at 5% level. *** Indicates the rejection of the validity of the overidentifying restrictions at 5% significance level. $^a$ Technique: Hansen’s (1982) Generalized Method of Moments is used to test Euler equations and to estimate the model parameters, over the period from 1977:1 to 2004:4, 112 observations. The GMM procedure forms a consistent estimate of the weighting matrix and then uses it to iterate the coefficient estimates until convergence. $^b$ Respective standard errors are reported in the parenthesis while p-values in the box brackets. $^c$ The standard errors are reported multiplied by 100.
Table 3
Forward-Premium Puzzle test: out-of-sample asset pricing exercise for the foreign currencies $^a, b, c, d$

System of conditional moment restrictions: $0 = \mathbb{E}\left\{ \begin{bmatrix} M_t^{US} P_t (1 + \mu_{ij}^t) [F_t^j - S_t^{ij}] - \mu_1 \\ M_t^{US} S_t^{ij} (1 + \mu_{ij}^t) P_t - (1 + \mu_2) \end{bmatrix} \otimes z_t \right\}$

$z_t = (F_{t+1}^j - S_t^j)/S_t^j$

Results for British government bonds:

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_1 = \mu_2 = 0$</td>
<td>$\hat{\mu}_1 \quad 0.0123 \quad (0.637) \quad [0.056]$</td>
<td>$J_T$ Test $0.0503$</td>
</tr>
<tr>
<td>$\chi^2$ 5.1523 [0.076]</td>
<td>$1 + \hat{\mu}_2 \quad 0.9362 \quad (7.360) \quad [0.387]$</td>
<td>P-value [0.060]</td>
</tr>
</tbody>
</table>

Results for Canadian government bonds:

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_1 = \mu_2 = 0$</td>
<td>$\hat{\mu}_1 \quad 0.0033 \quad (0.334) \quad [0.319]$</td>
<td>$J_T$ Test $0.0530$</td>
</tr>
<tr>
<td>$\chi^2$ 1.4302 [0.489]</td>
<td>$1 + \hat{\mu}_2 \quad 0.9577 \quad (7.769) \quad [0.582]$</td>
<td>P-value [0.051]</td>
</tr>
</tbody>
</table>

Results for German government bonds:

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_1 = \mu_2 = 0$</td>
<td>$\hat{\mu}_1 \quad 0.0100 \quad (0.776) \quad [0.199]$</td>
<td>$J_T$ Test $0.0473$</td>
</tr>
<tr>
<td>$\chi^2$ 2.2808 [0.320]</td>
<td>$1 + \hat{\mu}_2 \quad 0.9740 \quad (6.666) \quad [0.697]$</td>
<td>P-value [0.071]</td>
</tr>
</tbody>
</table>

Results for Japanese government bonds:

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_1 = \mu_2 = 0$</td>
<td>$\hat{\mu}_1 \quad 0.0103 \quad (0.853) \quad [0.229]$</td>
<td>$J_T$ Test $0.0747^{***}$</td>
</tr>
<tr>
<td>$\chi^2$ 2.2866 [0.319]</td>
<td>$1 + \hat{\mu}_2 \quad 0.9583 \quad (7.799) \quad [0.594]$</td>
<td>P-value [0.015]</td>
</tr>
</tbody>
</table>

Results for Swiss government bonds:

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_1 = \mu_2 = 0$</td>
<td>$\hat{\mu}_1 \quad 0.0081 \quad (0.901) \quad [0.367]$</td>
<td>$J_T$ Test $0.0247$</td>
</tr>
<tr>
<td>$\chi^2$ 0.8176 [0.665]</td>
<td>$1 + \hat{\mu}_2 \quad 1.0001 \quad (7.578) \quad [0.999]$</td>
<td>P-value [0.252]</td>
</tr>
</tbody>
</table>

* Indicates the rejection of the null hypothesis (insignificant pricing deviation) at 5% level. ** Indicates the rejection of the null hypothesis (jointly insignificant pricing deviations) in Wald test at 5% level. *** Indicates the rejection of the validity of the overidentifying restrictions at 5% significance level. $^a$ Technique: Hansen’s (1982) Generalized Method of Moments is used to test Euler equations and to estimate the model parameters, over the period from 1977:1 to 2004:4, 112 observations. The GMM procedure forms a consistent estimate of the weighting matrix and then uses it to iterate the coefficient estimates until convergence. $^b$ We use the superscript $j$ for forward, spot and interest rates in order to associate these variables to the country $j$. $^c$ Standard errors are reported in the parenthesis while p-values in the box brackets. $^d$ The standard errors are reported multiplied by 100.
Table 4
Forward-Premium Puzzle test: in-sample asset pricing exercise for the foreign currencies \(a, b, c, d\)

\[
\begin{align*}
\text{System of conditional moment restrictions: } & \quad \theta = \mathbb{E} \left\{ \left[ \frac{\tilde{M}_t^{GLS} P_t (1 + r_{t+1}^{i_j}) F_t^i - S_t^j \pi_t^{i_j}}{S_t^{i_j} P_t} - \mu_1 \right] \otimes \zeta_t \right\} \\
& \quad \zeta_t = (F_t^{i_j} - S_t^j) / S_t^j
\end{align*}
\]

**Results for British government bonds:**

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_1 & 0.0001 (0.611) \quad [0.984] \\
\chi^2 & 0.0150 & 0.993 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_2 & 1.0045 (3.771) \quad [0.905] \\
\chi^2 & 1.7436 & 0.418 \\
\end{array}
\]

**Results for German government bonds:**

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_1 & -0.0044 (0.631) \quad [0.489] \\
\chi^2 & 1.7436 & 0.418 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_2 & 0.9722 (3.627) \quad [0.444] \\
\chi^2 & 1.7436 & 0.418 \\
\end{array}
\]

**Results for Swiss government bonds:**

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_1 & -0.0106 (0.801) \quad [0.185] \\
\chi^2 & 1.7724 & 0.412 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{Wald test (deviations)} & \text{Asset pricing and deviations} & \text{Overall fit} \\
H_0: \mu_1 = \mu_2 = 0 & \hat{\mu}_2 & 1.0087 (3.779) \quad [0.819] \\
\chi^2 & 1.7724 & 0.412 \\
\end{array}
\]

**Results for Canadian government bonds:**

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6599</td>
<td>1 + \hat{\mu}_2</td>
<td>1.0187 (3.989) [0.639]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0019</td>
<td>-0.0001 (0.242) [0.440]</td>
<td>0.0441</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
\chi^2 \quad [0.719] \\
\text{P-value} \quad [0.540] \\
\end{array}
\]

**Results for Japanese government bonds:**

\[
\begin{array}{c}
\chi^2 \quad [0.711] \\
\text{P-value} \quad [0.085] \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Wald test (deviations)</th>
<th>Asset pricing and deviations</th>
<th>Overall fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0048</td>
<td>0.801 (0.664) [0.466]</td>
<td>0.1078***</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
\chi^2 \quad [0.711] \\
\text{P-value} \quad [0.002] \\
\end{array}
\]

\[
\begin{array}{c}
\chi^2 \quad [0.711] \\
\text{P-value} \quad [0.085] \\
\end{array}
\]

\[
\begin{array}{c}
\chi^2 \quad [0.711] \\
\text{P-value} \quad [0.085] \\
\end{array}
\]

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Figure 4
Multifactor pricing kernel in-sample prediction: foreign government bond markets

In this figure, we plot the realized return and its respective unconditional predictions, using $M_{t}^{US}$ and $M_{t}^{GL}$, on uncovered trading with British, Canadian, German, Japanese and Swiss government bonds (filled dots) and the excess returns (uncovered over covered) on these same bonds (empty dots). Data from 1977:1 to 2004:4, 112 observations.
Table 5

The Forward- and the Equity Premium Puzzles test: out-of-sample asset pricing exercise\textsuperscript{a, b, c, d}

\[ \begin{align*}
\mathbb{E} \left\{ \begin{array}{l}
M_t U S^S \frac{P_t (1 + r_{t+1}^S)}{S_{t+1}^t} F_{t+1}^j - S_t^j - \mu_1 \\
M_t U S^S \frac{S_{t+1}^j (1 + r_{t+1}^S)}{S_t^t} P_{t+1} - (1 + \mu_2) \\
M_t U S^S \frac{S_{t+1}^j (1 + r_{t+1}^P - b_{t+1})}{P_{t+1}^t} - \mu_3 \\
M_t U S^S \frac{(1 + r_{t+1}^S)}{P_{t+1}^t} - (1 + \mu_4)
\end{array} \right\} \otimes z_t

\end{align*} \]

System of conditional moment restrictions: \( \theta = \mathbb{E} \)

Results for British government bonds:

<table>
<thead>
<tr>
<th>Results for British government bonds:</th>
<th>Results for Canadian government bonds:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wald test (deviations)</td>
<td>Asset pricing and deviations</td>
</tr>
<tr>
<td>( H_0: \mu_i = 0, i = 1, \ldots, 4 )</td>
<td>( \hat{\mu}_1 )</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>2.6653</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_2 )</td>
<td>0.0072</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_4 )</td>
<td>1.0059</td>
</tr>
<tr>
<td>Wald test (deviations)</td>
<td>Asset pricing and deviations</td>
</tr>
<tr>
<td>( H_0: \mu_i = 0, i = 1, \ldots, 4 )</td>
<td>( \hat{\mu}_1 )</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>13.5724*</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_2 )</td>
<td>0.0072</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_4 )</td>
<td>1.0059</td>
</tr>
</tbody>
</table>

Results for German government bonds:

<table>
<thead>
<tr>
<th>Results for German government bonds:</th>
<th>Results for Japanese government bonds:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wald test (deviations)</td>
<td>Asset pricing and deviations</td>
</tr>
<tr>
<td>( H_0: \mu_i = 0, i = 1, \ldots, 4 )</td>
<td>( \hat{\mu}_1 )</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>12.1051**</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_2 )</td>
<td>0.0072</td>
</tr>
<tr>
<td>1 + ( \hat{\mu}_4 )</td>
<td>1.0059</td>
</tr>
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<tr>
<td>( \chi^2 )</td>
<td>2.0286</td>
</tr>
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<td>1 + ( \hat{\mu}_2 )</td>
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Results for Swiss government bonds:

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\* Indicates the rejection of the null hypothesis (insignificant pricing deviation) at 5% level. \** Indicates the rejection of the null hypothesis (jointly insignificant pricing deviations) in Wald test at 5% level. \*** Indicates the rejection of the validity of the overidentifying restrictions at 5% significance level. \textsuperscript{a} Technique: Hansen’s (1982) Generalized Method of Moments is used to test Euler equations and to estimate the model parameters, over the period from 1977:1 to 2004:4, 112 observations. The GMM procedure minimizes a quadratic form of the sample mean errors for the identity matrix. \textsuperscript{b} We use the superscript \( j \) for forward, spot and interest rates in order to associate these variables to the country \( j \). \textsuperscript{c} Standard errors are reported in the parenthesis while p-values in the box brackets. \textsuperscript{d} The standard errors are reported multiplied by 100.
Table 6
The Forward- and the Equity Premium Puzzles test: in-sample asset pricing exercise\textsuperscript{a, b, c, d}

System of conditional moment restrictions: $0 = \mathbb{E} \left\{ \begin{array}{l}
M_t^{GLS} \frac{p((1+i_{t+1}^j)[iF_{t+1}^j-S_{t+1}^j]}{s_{t+1}^j P_{t+1}^j} - \mu_1 \\
M_t^{GLS} \frac{S_{t+1}^j}{s_{t+1}^j P_{t+1}^j} - (1 + \mu_2) \\
M_t^{GLS} \frac{(i_{t+1}^j-P_{t+1}^j)}{s_{t+1}^j P_{t+1}^j} - \mu_3 \\
M_t^{GLS} \frac{1+i_{t+1}^j}{P_{t+1}^j} - (1 + \mu_4) 
\end{array} \right\} \otimes z_t \\
\text{with } z_t = (iF_{t+1}^j - S_{t+1}^j)/S_t^j, (D/P)_t, (I/C)_t$

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<td>$\chi^2$</td>
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</tr>
<tr>
<td>1.5111 [0.825]</td>
<td>38.7965** [0.000]</td>
<td>22.9774** [0.000]</td>
<td>1.2417 [0.871]</td>
</tr>
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<td><strong>Asset pricing and deviations</strong></td>
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</tr>
<tr>
<td>-0.0014 (0.613)</td>
<td>-0.0022 (0.247)</td>
<td>-0.0040 (0.710)</td>
<td>-0.0040 (0.710)</td>
</tr>
<tr>
<td><strong>Overall fit</strong></td>
<td><strong>Overall fit</strong></td>
<td><strong>Overall fit</strong></td>
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</tr>
<tr>
<td>$J_T$ Test 0.0024</td>
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<td>$\mu_2$</td>
<td>$\mu_2$</td>
</tr>
<tr>
<td>1.0013 (3.896)</td>
<td>1.0023 (4.084)</td>
<td>1.0080 (3.815)</td>
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</tr>
<tr>
<td>$\mu_3$</td>
<td>$\mu_3$</td>
<td>$\mu_3$</td>
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</tr>
<tr>
<td>0.0058 (0.750)</td>
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</tr>
<tr>
<td>$\mu_4$</td>
<td>$\mu_4$</td>
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<td>1.0080 (3.815)</td>
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