INFLATION AND DEBT INDEXATION: THE EQUIVALENCE OF TWO ALTERNATIVE SCHEMES FOR THE CASE OF PERIODIC PAYMENTS

Clovis de Faro

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Abstract

The presence of inflation has induced the financial institutions to implement procedures devised to protect the real values of their loans. Two of such procedures, the floating rate scheme and the monetary correction mechanism, tend to lead to very different streams of payments. However, whenever the floating rate scheme follows the rule of strict adherence to the Fisher equation, the two procedures are financially equivalent.
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1. Introduction

The moderate to high inflation environment that has been experienced by several countries, including the United States in the late 70's and specially Brazil, even today, led their respective financial institutions to devise mechanisms aimed to protect the real values of medium to long term loans. In general, such mechanisms, broadly known as indexation procedures, are based on one of the following two alternatives. The first, which is usually adopted in the international arena and which has its theoretical foundations on the classical Fisher equation, is the so called floating rate scheme. The second, which has been officially adopted in Brazil since 1964 and has been known as the monetary correction scheme, is based on the principle of maintaining the purchasing power of the pre-specified payments.

As it could be expected, the two schemes tend to lead to very different streams of payments; both in real and in nominal terms. However, at least for the case of constant repayment of principal, the specification of the same real rate of interest and the strict adherence to the Fisher equation make the floating rate scheme financially equivalent to the monetary correction procedure.

It will also be shown that, notwithstanding the financial equivalence, the floating rate scheme implies initial payments greater than the corresponding ones under monetary correction. This peculiarity, which amounts to an amortization of the debt at a faster pace, was advanced by Dornbusch [1] and can be considered as one of the ingredients that contributed to the developing countries debt crisis of 1982. In fact, this
author pointed out that, given the real rate of interest, when higher inflation increases the nominal interest rate the effect on debtors is a shortening of the effective maturity of the debt, which may result in liquidity problems for the debtors. Our finds lend support to the hypothesis that, had the effects of inflation in the developed countries, chiefly in the U.S., be considered through the adoption of monetary correction, rather than via the floating rate scheme, the debt crisis, at least for those developing countries whose loans were taken to finance productive investments with some time to mature, could possibly have been milder.

2. The Monetary Correction Scheme

Consider the loan of $C$ units of capital, at the fixed real rate of interest $i$, per period, during $m + n$ periods; the first $m$ requiring only the payment of interest. Assuming the specification of constant repayment of principal, it follows that, as expressed in terms of prices at the date of the loan (time 0), the value of the $k$-th payment is equal to:

$$P_{k,0} = \begin{cases} 
Ci, & k = 1, \ldots, m \\
C\left\{1 + i(n + m + 1 - k)\right\}/n, & k = m + 1, \ldots, m + n 
\end{cases}$$

(1)

On the other hand, if $I_k$ denotes the value of the relevant price index at time $k$, the adoption of the monetary correction scheme implies that the actual (or current) value of the $k$-th payment is equal to

$$P_{k,k} = (I_k/I_0)P_{k,0}, \quad k = 1, \ldots, m + n$$

(2)
If $\theta_k$ denotes the inflation rate relative to the k-th period of the loan, that is

$$\theta_k = I_k / I_{k-1} - 1, \quad k = 1, ..., m + n$$ (3)

it is easily verified that

$$\sum_{k=1}^{m+n} p_{k,k} \left\{ \prod_{t=1}^{k} (1 + \theta_t) \right\}^{-1} (1 + i)^{-k} = \sum_{k=1}^{m+n} p_{k,0} (1 + i)^{-k} = C$$ (4)

In words, relation (4) reveals that the monetary correction scheme does not destroy the equivalence, at the real rate $i$, between the sequence of $m+n$ payments, when expressed at constant prices, and the lent principal $C$.

3. The Floating Rate Scheme

Once the real interest rate $i$ is fixed, the strict adherence to the Fisher equation implies that the current value of the nominal rate of interest that applies in the k-th period is equal to:

$$R_k = (1 + i)(1 + \theta_k) - 1, \quad k = 1, ..., m + n$$ (5)

Under this circumstance, the adoption of the floating rate scheme, for the loan considered on the previous section, implies that the current value of the k-th payment is equal to:
\[ p'_{k,k} = \begin{cases} R_k C, & k = 1, \ldots, m \\ C \left( \frac{1}{n} + R_k \left[ 1 - \frac{(k - m - 1)}{n} \right] \right), & k = m + 1, \ldots, m + n \end{cases} \tag{6} \]

As a simple numerical illustration of the differences between the two procedures, suppose a loan of 12,000 units of capital, at the real rate of interest of 10%, with \( m = 6 \) and \( n = 12 \). If the rate of inflation varies as indicated in the second column of Table 1, the corresponding values of \( p_{k,k} \) and \( p'_{k,k} \) appear in the third and second columns, respectively. Furthermore, to stress the obvious fact that they are also different, Table 1 includes, between parenthesis, the corresponding real values \( p_{k,0} \) and \( p'_{k,0} \) (deflated to time 0).

As could be expected, the figures in Table 1 provide evidence that the numerical differences between the two debt indexation schemes can be substantial.

A point that should be stressed is that, as long as we do not have deflation, we always have \( p'_{1,1} \geq p_{1,1} \) and \( p'_{s+m, s+m} \leq p_{s+m, s+m} \). This can be easily seen, since:

\[ p'_{1,1} - p_{1,1} = \begin{cases} C \theta_1, & \text{if } m \geq 1 \\ C \theta_1 (n-1)/n, & \text{if } m = 0 \end{cases} \tag{7} \]

and

\[ p'_{s+m, s+m} - p_{s+m, s+m} = \left( \frac{C}{n} \right)(1+i)(1+\theta_{s+m}) \left\{ 1 - \frac{a+m-1}{\prod_{k=1}^{\infty} (1+\theta_k)} \right\} \tag{8} \]
TABLE 1
Monetary Correction versus Floating Rate

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<tr>
<th>k</th>
<th>θk(%)</th>
<th>P_{k/0}(P_{k,0})</th>
<th>P'<em>{k/0}(P'</em>{k,0})</th>
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<td>1,320.00</td>
<td>2,520.00</td>
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<td>(1,200.00)</td>
<td>(2,290.91)</td>
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<td>1,518.00</td>
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<td></td>
<td></td>
<td>(1,200.00)</td>
<td>(2,513.83)</td>
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<tr>
<td>3</td>
<td>12</td>
<td>1,700.16</td>
<td>2,784.00</td>
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<tr>
<td></td>
<td></td>
<td>(1,200.00)</td>
<td>(1,964.99)</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>1,887.18</td>
<td>2,652.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1,200.00)</td>
<td>(1,686.33)</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>2,057.02</td>
<td>2,388.00</td>
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<td></td>
<td></td>
<td>(1,200.00)</td>
<td>(1,393.08)</td>
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<tr>
<td>6</td>
<td>8</td>
<td>2,221.59</td>
<td>2,256.00</td>
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<td>(188.94)</td>
</tr>
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<td>15,826.94</td>
<td>1,706.00</td>
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<td>(129.35)</td>
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<td></td>
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<td>(83.40)</td>
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4. **Financial Equivalence**

Despite the differences of the respective sequences of current payments, the floating rate scheme of debt indexation is financially equivalent to the monetary correction procedure. That is, as we are going to show, the present values, at the real interest rate \( i \), of each one of the two corresponding sequences of payments, when deflated to time 0, are equal.

To prove the above assertion, it suffices to show that:

\[
\sum_{k=1}^{m+n} p'_{k,k} \left\{ \prod_{j=1}^{k} \left( 1 + \theta_j \right) \right\}^{-1} (1 + i)^{-k} = C
\]  

(9)

That is, taking into account (6) and defining

\[
A_m = \sum_{k=1}^{m} \left\{ R_k \left( 1 + i \right)^{-k} / \prod_{j=1}^{k} (1 + \theta_j) \right\}
\]

(10)

and

\[
B_n = \left( \frac{1}{n} \right) \sum_{k=m+1}^{m+n} \left\{ \left[ 1 + R_k \left( n + m + 1 - k \right) \right] (1 + i)^{-k} / \prod_{j=1}^{k} (1 + \theta_j) \right\}
\]

(11)

we need to show that \( A_m + B_n = 1 \)

It may be easily verified that
Therefore, as proven in the Appendix, we need only to show that

\[ A_m = 1 - (1+i)^{-m} / \prod_{j=1}^{m} (1+\theta_j) \quad \text{(10')} \]

\[ B_n = (1+i)^{-m} / \prod_{j=1}^{m} (1+\theta_j) \quad \text{(11')} \]

5. Possible Breakdown

An interesting question that may be posed is the one relative to the impacts of deviations from the de Fisher equation. Deviations may happen whenever the nominal rate of interest is fixed ex-ante, instead of ex-post (after the determination of the inflation level), and there is a breakdown of rational expectations.

A general answer is not possible. In principle, one can expect that the financial equivalence between the two indexation schemes will not hold. However, focusing attention to the particular case where \( n = 2 \) and \( m = 0 \), we can show that, although for a very fortuitous set of circumstances, the equivalence may hold even when there is a breakdown of rational expectations.

Denoting by \( \theta_k^* \) the expected value of the inflation rate relative to the \( k \)-th period of the loan, it follows that, in real terms, the ex-post present value of the payments relative to the floating rate scheme can be written as

\[ V = \frac{C}{2(1+i)(1+\theta_i)} \left\{ 2(1+i)(1+\theta_i^*) - 1 + \frac{1+\theta_i^*}{1+\theta_i} \right\} \quad \text{(12)} \]
Obviously, we will have \( V = C \) if \( \theta_1^* = \theta_1 \) and \( \theta_2^* = \theta_2 \). However, we still may have \( V = C \), which implies equivalence with the monetary correction scheme, even if \( \theta_1^* \neq \theta_1 \) and \( \theta_2^* \neq \theta_2 \). This will happen whenever the following relationship holds

\[
\theta_1^* - \theta_1 = \frac{\theta_2 - \theta_2^*}{2(1+i)(1+\theta_2)}
\]  

(13)

To give a numerical example, let \( i = 8\% \) and suppose that it is estimated that the first period inflation rate is \( \theta_1^* = 11\% \), while the actual value is \( \theta_1 = 10\% \). If the second period inflation rate is estimated at the level \( \theta_2^* = 7.624\% \) and the actual value turns out to be \( \theta_2 = 10\% \), the corresponding estimation errors, which have different signs, will be compensated.

6. Conclusion

As we have indicated, the differences between the sequences of payments, in nominal terms, which are respectively associated with the floating rate scheme and that of monetary correction, can be dramatically high. Moreover, the floating rate procedure implies that the initial payments are greater than those corresponding to the monetary correction scheme, with the opposite occurring with the final ones.

Therefore, since the two procedures are financially equivalent, if the loan is being made to finance some productive investment which takes some time to mature, the borrower should always insists on having the debt indexed in accordance to the monetary correction scheme. Otherwise, as experienced with the largely indebted Third World
Countries which were led to fall in the floating rate trap, the borrower may face a initial cash flow imbalance that may result in insolvency.

REFERENCE

APPENDIX

To show that \( B_n = (1+i)^{-m} / \prod_{j=1}^{m}(1+\theta_j) \), let us proceed by induction on \( n \),

noticing first that \( B_n = (1+R_{m+1})(1+i)^{-m} / \prod_{j=1}^{m+1}(1+\theta_j) = (1+i)^{-m} / \prod_{j=1}^{m}(1+\theta_j) \)

Assuming now that (9) is true for \( h > 1 \), we have:

\[
B_{h+1} = \frac{1}{h+1} \sum_{k=m+1}^{m+h+1} \frac{[1+R_k(h+1+m+1-k)](1+i)^{-k}}{\prod_{j=1}^{k}(1+\theta_j)}
\]

\[
= \frac{1}{h+1} \left\{ \sum_{k=m+1}^{m+h} \frac{[1+R_k(h+1+m+1-k)](1+i)^{-k}}{\prod_{j=1}^{k}(1+\theta_j)} + \frac{(1+R_{m+h+1})(1+i)^{-m-h-1}}{\prod_{j=1}^{m+h+1}(1+\theta_j)} \right\}
\]

\[
= \frac{1}{h+1} \left\{ \sum_{k=m+1}^{m+h} \frac{[1+R_k(h+m+1-k)](1+i)^{-k}}{\prod_{j=1}^{k}(1+\theta_j)} + \frac{R_k(1+i)^{-k}}{\prod_{j=1}^{k}(1+\theta_j)} + \frac{(1+i)^{-m-h}}{\prod_{j=1}^{m+h}(1+\theta_j)} \right\}
\]

\[
= \frac{h(1+i)^{-m}}{\prod_{j=1}^{m}(1+\theta_j)} - \sum_{k=m+1}^{m+h} \frac{[(1+\theta_k)(1+i)-1](1+i)^{-k}}{\prod_{j=1}^{k}(1+\theta_j)} + \frac{(1+i)^{-m-h}}{\prod_{j=1}^{m+h}(1+\theta_j)}
\]
\[
\frac{1}{h+1} \left\{ \frac{h(1+i)^{-m}}{\pi(1+\theta_j)} + \sum_{k=m+1}^{m+h} \frac{(1+i)^{-k}}{\pi(1+\theta_j)} - \sum_{k=m+1}^{m+h} \frac{(1+i)^{-k}}{\pi(1+\theta_j)} \right\}
\]

\[
= \frac{1}{h+1} \left\{ \frac{h(1+i)^{-m}}{\pi(1+\theta_j)} + \frac{(1+i)^{-m}}{\pi(1+\theta_j)} + \frac{(1+i)^{-m-1}}{\pi(1+\theta_j)} + \frac{(1+i)^{-m-2}}{\pi(1+\theta_j)} + \ldots + \frac{(1+i)^{-m-h+1}}{\pi(1+\theta_j)} \right\}
\]

\[
- \frac{(1+i)^{-m-1}}{\pi(1+\theta_j)} - \frac{(1+i)^{-m-2}}{\pi(1+\theta_j)} - \frac{(1+i)^{-m-h}}{\pi(1+\theta_j)} - \frac{(1+i)^{-m-h+1}}{\pi(1+\theta_j)} - \frac{(1+i)^{-m-h+2}}{\pi(1+\theta_j)} - \frac{(1+i)^{-m-h+3}}{\pi(1+\theta_j)} - \ldots - \frac{(1+i)^{-m-h+1}}{\pi(1+\theta_j)}
\]

\[
= \frac{(1+i)^{-m}}{\pi(1+\theta_j)}
\]
CLOVIS DE FARO is Professor of Quantitative Methods at the Graduate School of Economics, Getulio Vargas Foundation, Rio de Janeiro. He received a B.Sc. in Civil Engineering from Universidade Federal Fluminense, and both an M.Sc. in Operations Research and a Ph.D. in Industrial Engineering from Stanford University. Besides being the Editor of Revista Brasileira de Economia, the author of 5 books and of several articles that have been published in Portuguese, his papers have also appeared in journals such as: The Engineering Economist, Journal of Financial and Quantitative Analysis, Journal of International Business Studies and OR Spektrum (in German).
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