A RATIONAL EXPECTATIONS PARADOX

MARIO HENRIQUE SIMONSEN
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1) The inflationary tax equilibrium

A well known property of the Cagan's money demand function is that it yields a Laffer curve for the inflationary tax. Money market equilibrium is described by equation:

\[ \frac{B}{P} = e^{-\alpha \pi^*} \quad (\alpha > 0) \quad (1) \]

where \( B \) stands for the monetary base, \( P \) for the general price level and \( \pi^* \) for the expected inflation rate. In steady state equilibrium the latter should coincide with the actual inflation rate:

\[ \pi = \frac{\dot{\pi}}{P} \quad (2) \]

dots indicating time derivatives. Hence, for a constant inflation rate, the inflationary tax revenue will be given by:

\[ T_{\text{inf}} = \frac{\dot{B}}{P} = \frac{d}{dt} \left( \frac{B}{P} \right) + \frac{B}{P} \pi = \frac{B}{P} \pi = \pi e^{-\alpha \pi} \quad (3) \]

increasing for \( \pi < 1/\alpha \) and decreasing thereafter, as indicated in figure 1. The inflationary tax revenue is bounded by its maximum:

\[ T_{\text{inf}} \leq \frac{1}{\alpha} e^{-1} \quad (4) \]

Now, let us assume that the real public sector deficit is kept at a constant level \( k < \frac{1}{\alpha} e^{-1} \), being fully financed by the creation of high powered money. There is no other source of expansion or contraction of the monetary base, which implies:

\[ \dot{B} = kP \quad (5) \]
The equilibrium inflation rate must equal the inflationary tax revenue to k. Now, as indicated in figure 1, because of the Laffer curve effect there are two different equilibrium inflation rates \( \pi_1 \) and \( \pi_2 \). Increasing the public sector deficit \( k \), i.e., moving up the straight line AB, increases the low equilibrium rate \( \pi_1 \) and decreases \( \pi_2 \), providing a priori support to the idea that stable equilibrium inflation rate should be \( \pi_1 \).

Hyperinflations are explained by \( k > \frac{1}{\alpha} e^{-1} \).

Dynamic hypotheses must be introduced to discuss the stability of the two possible equilibria. Assuming adaptative expectations according to Cagan's specification:

\[
\dot{\pi}^* = \beta (\pi - \pi^*) \quad (\beta > 0)
\]

one may easily prove that \( \pi_1 \) is the stable equilibrium inflation rate if and only if \( \alpha \beta < 1 \), namely, if and only if the Cagan's stability condition is met.

Now, let us turn to rational expectations. In the absence of stochastic disturbances this means perfect foresight, \( \pi^* = \pi \). In this case, taking time derivatives in both sides of equation (1)

\[
\dot{B} = e^{-\alpha \pi} (\dot{P} - \alpha \dot{P} \pi)
\]

combining with equations (2) and (5):

\[
\alpha e^{-\alpha \pi} \dot{\pi} = \pi e^{-\alpha \pi} - k
\]
This is to say that sign of $\dot{\pi}$ coincides with that of the right side of the above presented equation, that is nothing but the Laffer curve in figure 1 with the horizontal axis moved up to AB. As arrows indicate, the stable equilibrium inflation rate is now the high root $\pi_2$.

The conclusion appears to make the rational expectations hypothesis inconsistent with both empirical evidence and common sense. In fact it implies that cutting the public sector deficit (which is assumed to be fully financed by money creation) leads to an acceleration of the inflation rate, since $\pi_2$ increases when k declines.

2) Solving the paradox

What is wrong in the preceding analysis? Simply the phase diagram technique in figure 1 implicitly treats the inflation rate as a backward looking variable, as in case of adaptative expectations. The conclusion that $\pi_2$ is the stable inflation rate equilibrium assumes that the inflation rate is subject to initial condition contraints. Now, in rational expectations models both the price level and the inflation rate are forward looking variables. Since they keep no links with the past, their initial levels will jump, depending on expectations. Hence, in a rational expectations framework, the phase diagram in figure 1 can only be understood as a guide to forecasters. It simply tells that economic agents may foresee either $\pi = \pi_1$, $\pi = \pi_2$ or some variable inflation rate path either converging to $\pi_2$ or tending to minus infinity.

The diagram shows that forecasters face an indeterminacy problem as often occurs in rational expectations models. In fact, since forward looking variables are not constrained by fixed initial conditions, difference or differential equation systems usually yield infinitely many rational expectations paths. Indeterminacies are
usually removed by introducing boundness requirements on the expected paths of the variables (or their time derivatives). The problem of the inflationary tax equilibrium requires an additional assumption, as will be shown in section 3. It emerges naturally once the problem is solved according to the rational expectations methodology, namely, once the inflation rate is expressed as a function of the anticipated path of money supply.

It should be noted that a large group of rational expectations difference or differential equation systems have no stable solutions. There are privileged solutions in terms of boundness or convergence, usually the meaningful ones from the economic theory standpoint, but they are also unstable. Forward looking variables are supposed to jump whenever necessary so as to meet these solutions, disconnecting rational expectations from stability analysis.

As an example, let us take Cagan's monetary equation expressed in logs and assuming perfect foresight:

\[ b-p = -\alpha \dot{p} \] (6)

where \( b=\ln B, p=\ln P \) and, as a consequence, \( \dot{p} = \tau \). The price level, according to the rational expectations hypothesis, is determined by solving forward equation (6):

\[ p(t) = \frac{1}{\alpha} \int_{t}^{\infty} e^{\frac{t-\tau}{\alpha}} b(\tau) d\tau + ce^{\frac{t}{\alpha}} \] (7)

where \( c \) is a constant. If the monetary base expands at a constant rate \( b = \alpha \), then \( b(\tau) = b(t) + \alpha (t-t) \). Introducing this expression in the right side of (7) and calculating the integral:

\[ p(t) = b(t) + \alpha r + ce^{\frac{t}{\alpha}} \] (8)

Taking time derivatives, the inflation rate will be given by:

\[ \dot{p} = r + \frac{1}{\alpha} ce^{\frac{t}{\alpha}} \] (9)
The usual indeterminacy problem is displayed, since there are infinitely many perfect foresight paths for the inflation rate, one for each value of c. It is removed by adding a boundary condition, the expected rate of price change cannot race to infinity. This forces c=0, yielding \( \dot{p} = r \). The inflation rate equals the constant rate of expansion of the monetary base, a sensible economic result. Taking c=0 in equation (8) leads to an important conclusion: if r changes, the price level will jump.

Now, it should be made clear that \( \dot{p} = r \) is an unstable inflation rate equilibrium, as shown in figure 2. In fact, taking time derivatives in (6) and assuming \( \dot{b} = r \):

\[
\begin{align*}
r - \pi &= -\alpha \dot{\pi} \\
\end{align*}
\]

a differential equation that has no stable solution:

\[ r - \pi = -\alpha \dot{\pi} \]  

![Figure 2](image)

Another example is provided by the popular 2x2 saddle-point convergence models, where backward looking variable \( X_1 \) interacts with the forward looking \( X_2 \) according to the linear dynamic system:

\[
\begin{align*}
\dot{X}_1 &= a_{11}X_1 + a_{12}X_2 + b_1 \\
\dot{X}_2 &= a_{21}X_1 + a_{22}X_2 + b_2 \\
E_t X_2 &= a_{21}X_1 + a_{22}X_2 + b_2 \\
\end{align*}
\]

where \( \dot{X}_1 \) and \( \dot{X}_2 \) stand for the right side time derivatives of \( X_1 \) and \( X_2 \) and \( E_t \) for conditional expectation at time t. Equations (11) read as follows: i) \( X_2 \) is determined by its expected right side time derivative and by \( X_1 \) (this is possible, since \( a_{22} \neq 0 \)); ii) \( X_1 \) is a linear function of \( X_1 \) and \( X_2 \). The forward looking variable is supposed to
influence the path of the backward looking one, which requires $a_{12} \neq 0$.

Moreover, matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is assumed to have a negative determinant. Hence, its eigenvalues are $r_1 < 0 < r_2$. Since $a_{12} \neq 0$, there are two corresponding eigenvectors $y_1 = (1; k_1)$, $y_2 = (1; k_2)$, where $k_1 \neq k_2$.

In the absence of stochastic disturbances, $E_t X_2 = X_2$, i.e., rational expectations yield perfect foresight, leading to the conventional linear system:

$$\dot{X} = AX + b \quad (12)$$

where $X = (X_1, X_2)$ and $b = (b_1, b_2)$. The system is solved by:

$$X = c_1 e^{r_1 t} y_1 + c_2 e^{r_2 t} y_2 - A^{-1}b \quad (13)$$

where constants $c_1$ and $c_2$ meet the initial condition constraint:

$$X_0 = c_1 y_1 + c_2 y_2 - A^{-1}b$$

or equivalently, if $-A^{-1}b = (h_1; h_2)$:

$$X_{10} = c_1 + c_2 + h_1$$

$$X_{20} = c_1 k_1 + c_2 k_2 + h_2 \quad (14)$$

In the above equations $X_{10}$ is given by history, but $X_{20}$ can jump to any level, since $X_2$ is a forward-looking variable. To remove the indeterminacy, that allows for infinitely many perfect foresight paths, one for each $X_{20}$, the conventional extra assumption is added, that of boundness of the anticipated time derivatives: economic agents
are supposed to rule out the possibility of any component of \( E_t X = X = r_1 c_1 e^{r_1 t} y_1 + r_2 c_2 e^{r_2 t} y_2 \) racing to infinity. This requires \( c_2 = 0 \), locating the initial value of the forward looking variable at:

\[
X_{20} = k_1 (X_{10} - h_1) + h_2 \tag{15}
\]

and leading to the unique perfect foresight convergent path:

\[
X = (X_{10} - h_1) e^{r_1 t} y_1 - A^{-1} b \tag{16}
\]

where \( X \) converges to the saddle-point \(-A^{-1} b\), since \( r_1 < 0 \).

Now, once again, it should be stressed that all solutions of the differential equation system (12) are unstable, including the perfect foresight convergent path. In fact, any slight deviation between \( X_{20} \) and the expression in the right side of equation (15) will move \( X \) indefinitely away from the convergent path.

Economic literature often refers to stationary or convergent perfect foresight paths as "stable paths". The semantic confusion provides, perhaps, the heuristic explanation for the rational expectations paradox under discussion.

3) The inflationary tax equilibrium reconsidered

Let us now provide the correct rational expectations solution to the problem discussed in section 1. All economic agents share the same informations, that can be summarized by equations:

\[
b - p = -\alpha \dot{p} \tag{6}
\]

\[
b = k e^{\alpha \dot{p}} \tag{17}
\]

Equation (6) is the already presented log-linear translation of Cagan's monetary equilibrium constraint under perfect foresight.
Equation (17) was obtained dividing (5) by (1) and also assuming perfect foresight.

The price level will be a function of the projected path of the monetary base. This involves eliminating \( p \) and \( \dot{p} \) from equations (6) and (17). Simple and tedious calculations yield:

\[
\frac{\dot{b}}{b} + \dot{b} = \frac{1}{a} \ln \frac{\dot{b}}{k}
\]

or, indicating by \( \dot{r} = \dot{b} \) the rate of expansion of the monetary base:

\[
\frac{\dot{r}}{r} = \frac{1}{a} \ln \frac{\dot{r}}{k} - r \tag{18}
\]

or equivalently:

\[
\frac{\dot{r}}{r} = f(r) - \frac{1}{a} \ln k \tag{19.a}
\]

where:

\[
f(r) = \frac{1}{a} \ln r - r \tag{19.b}
\]

The stationary values of \( r \) are the solutions of the equation:

\[
f(r) = \frac{1}{a} \ln r - r = \frac{1}{a} \ln k
\]

or equivalently:

\[
re^{-ar} = k \tag{20}
\]

the roots of which, as one might expect, are the same \( \pi_1 \) and \( \pi_2 \) of section 1. The phase diagram in figure 3, a linear transformation of that in figure 1, based on equations (19.a) and (19.b) is to be understood as a simple forecaster's chart, since the projected rate of growth of the monetary base is not bound to initial condition constraints.
The usual forecasting indeterminacy problem in rational expectations models emerges again. What is new, is that it can no longer be removed by a simple boundness assumption. An additional hypothesis is needed to solve the forecaster's dilemma. The most natural one is that the rate of growth of the monetary base should eventually become positively correlated with the public sector deficit. This requires \( \frac{\dot{b}}{b} < \frac{1}{\alpha} \), namely, that \( r \) should be projected in the increasing region of the \( f(r) \) curve, which corresponds to the increasing part of the Laffer curve in figure 1. Given the boundness assumptions, forecasters have no other choice except to locate \( b = r = \pi_1 \). The inflation rate, according to equation (17) will immediately jump to this low equilibrium root.
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