An intra-household approach to the welfare costs of inflation

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Abstract

The literature on the welfare costs of inflation universally assumes that the many-person household can be treated as a single economic agent. This paper explores what the heterogeneity of the agents in a household might imply for such welfare analyses. First, we show that allowing for a single-unity or for a multi-unity transacting technology impacts the money demand function and, therefore, the welfare costs of inflation. Second, we derive sufficient conditions that make the welfare assessments which depart directly from the knowledge of the money demand function (as in Lucas (2000)) robust under this alternative setting. Third, we compare our general-equilibrium measure with Bailey’s (1956) partial-equilibrium one.

1 Introduction

The literature on the welfare costs of inflation almost universally assumes that the many-person household can be treated as a single economic agent.

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We shall refer to this treatment as “the unitary (U-) model”. Since households generally consist of several members, this literature can be criticized. For instance, in the cash-goods-credit-goods models (e.g., Gillman (1984), Aiyagari et al. (1998) or English (1999)) one can argue that, due to different opportunity costs of time or to different degrees of access to credit, what is a credit good to one member of the household can be a cash good to another one, and vice versa.

Alternatively, in the shopping-time approach to the welfare costs of inflation (e.g. Lucas (2000, Section 5) or Cysne (2003)), on which we shall concentrate here, it can be argued that different members of the household are likely to have different productivities either in the production of the consumption good or in transacting.

To get an intuition about this issue, let us consider the case of a shopping-time model. In such models, inflation reduces welfare because households end up holding less money and, therefore, have to allocate more of their productive time in transacting.

In the U- model, this implies a single choice of the household regarding how much to hold of real balances and how much time to allocate in transactions. The reduction in the production of the consumption good (and of welfare) keeps a one-to-one correspondence (given by the productivity of the agent) with the variation of the household’s time allocated in transacting.

However, if one considers a many-person optimizing representative household, in which the respective members share the services of money (we call this the intra-household (IH-) model), the decision procedure follows another pattern, due to the additional degrees of freedom incorporated to the problem. Besides the decision regarding real balances and transacting time, there is also the decision of how to distribute the time of each member of the household between transacting and producing the consumption good.

Taking into consideration a multi–unity household one could argue, for instance, that if one of the members of the household has a productivity in the production of the consumption good that is close to zero, but is very skilled in performing transacting services, then the welfare might not be as affected by the rise of inflation as one would conclude by using the U-model. Indeed, diverting time of this particular member from production to transacting would imply a minor impact on the output of the household, due to his high ratio of productivity in transacting, as compared to producing the consumption good. Investigating claims like these is one of our purposes here.
Our approach is theoretical. The simulations presented in the two examples in the main body of the text aim solely at providing comparisons between the welfare figures that emerge in each one of the two cases that we analyze (the U-model and the IH-model).

There is an extensive literature considering the idea of many-person households. Becker (1991) is a recent exposition of Becker’s seminal work in this area. Chiaporri (1988) and Browning and Chiaporri (1998) are other references. Our investigation here derives from such ideas. However, our approach differs from the one taken in this literature in two important aspects. First, our analysis is a general-equilibrium and dynamic one, in contrast with the usually partial-equilibrium and static modelling associated with the collective-household literature.

Second, we assume that the different members in a representative multi-person household differ with respect to their relative productivities in producing the consumption good and in transacting, but not with respect to their preferences. The members of the representative household are assumed to have the same preferences (or, equivalently, it is assumed that one of the partners can impose her (or his) preferences over the others). This hypothesis allows us to keep the traditional neoclassical approach that assumes the existence of a unique household utility function.

The work proceeds as follows. The U- and IH-models are presented in Section 2. Section 3 is dedicated to analyzing the implications, for the derivation of the money demand function and of the welfare costs of inflation, of specifying a single- or multi-unity transacting technology. The analysis is done with and without leisure as an argument of the utility function. Section 4 derives the most important conclusion of our analysis: In practical assessments, when the researcher tries to measure the welfare costs of inflation departing from the empirical knowledge of the money demand function (as in Lucas (2000)), the same welfare figures emerge, no matter which model (either the U- or the IH-framework) is being employed. Section 5 is used to show that Bailey’s (1956) partial-equilibrium measure of the welfare costs of inflation can be obtained as a first-order approximation of our general-equilibrium measure. Section 6 offers the final conclusions.

2 The Models

• The IH-Model
In this section we depart from the analysis made by Lucas (2000) by considering a two-person household, instead of the usual U-model (an extension to an \(n\)-member household can be done at no cost other than notational).

Throughout the text, we shall use subindexes 1 and 2 to denote different members of a household. Each member \(i\) of the household, \(i = 1, 2\), has a (fixed) time endowment of one unity. Time is allocated by each member of the household in shopping \((s_i, 0 \leq s_i \leq 1)\) or in the production of the consumption good. Member \(i\) is assumed to have productivity \(a_i\) (\(a_i \geq 0\), \(i = 1, 2\)) in the production of the consumption good.

\(P\) stands for the price of the consumption good. The total real product \((y = Y/P)\) is given by:

\[
y = a_1(1 - s_1) + a_2(1 - s_2)
\]  

Households gain utility from the consumption \((c = C/P)\) of a single non-storable consumption good, with preferences determined by:

\[
\int_0^\infty e^{-\gamma t} U(c)dt
\]  

where \(U(c)\) is a concave function of the consumption at instant \(t\) and \(\gamma > 0\). Households can accumulate two assets, money \((M)\) and bonds \((B)\), the latter yielding a nominal interest rate equal to \(i\). Households face the budget constraint:

\[
\dot{M} + \dot{B} = iB + P(a_1(1 - s_1) + a_2(1 - s_2) - c) + H
\]

\(H\) indicates the (exogenous) flow of money transferred by the government and the dot over the variable its time derivative. Making \(\pi = \dot{P}/P\) (inflation rate), \(m = M/P, b = B/P\), \(v = b + m\), and \(h = H/P\), the budget constraint reads:

\[
\dot{v} = a_1(1 - s_1) + a_2(1 - s_2) - c + h + (i - \pi)v - im
\]

The transacting technology is given by:

\[
c = F(m, s_1, s_2)
\]
with \( F_m > 0, F_{s_1} > 0, F_{s_2} > 0, F_{mm} < 0, F_{s_1s_1} < 0 \) and \( F_{s_2s_2} < 0 \). The transacting technology shows how each one of the inputs, shopping time of member one, shopping time of member two, and money, can be used by the household in the acquisition of the consumption good. We assume that the members of the household pool their transaction balances.

Households maximizes (2) subject to the budget constraint (3) and to the time-transacting technology (4). In the steady state, assuming interior solutions, Euler equations lead to:

\[
i = \pi + g
\]

(5)

\[
a_1 F_m = i F_{s_1}
\]

(6)

\[
a_2 F_m = i F_{s_2}
\]

(7)

The equilibrium equation:

\[
a_1(1 - s_1) + a_2(1 - s_2) = F(m, s_1, s_2)
\]

(8)

completes the model.

Given the interest rate \( i \), the variables \( m, s_1 \) and \( s_2 \) are determined by equations (6), (7) and (8). In this framework, \( a_1 s_1 + a_2 s_2 \), the time spent on transactions, weighed by the respective productivities of each member of the household, provides a measure of the welfare costs of inflation.

- **The U- Model**

The U- model versions of equations (1), (3) and (4) are given by, respectively:

\[
y = 1 - s
\]

(9)

\[
\dot{v} = 1 - s - c + h + (i - \pi) v - im
\]

(10)

and

\[
c = F^U(m, s)
\]

(11)
where the superscript $U$ in $F$ stands for the U-model.

The equations of the U-model are the same as in Lucas (2000, Section 5). The equilibrium equations describing the U-model (to be compared, respectively, with (6), (7) and (8)) are:

\begin{align}
F^U_m &= i F^U_s \\
1 - s &= F^U(m, s)
\end{align}

- **Government**

In both models, the economy is endowed with lump sum taxation, where the government can implement any given interest rate. In equilibrium, the rate of money expansion and inflation is determined so that the seigniorage matches the transfers ($h$) plus net real interest payments made by the government:

\[ h = \pi m - (i - \pi) b \]

The relation between the rate of inflation and the rate of interest is given by (5). Inflation is equal to the rate of monetary expansion, the exogenous variable of the model.

### 3 Single-Unity Versus Multi-Unity Transacting Technologies

In this Section we investigate the effects, on the money demand and on the welfare costs of inflation, of using a multi-unit transacting technology, vis-a-vis the usual approach of adopting a single-unit technology. This will enable us to understand, for instance, how biased can be a theoretical analysis based on a single-unity household, when the household is actually comprised of many different members. We shall see that the discrepancies among the relative productivities of the members of the household in the production of the consumption good and in transacting play an important role.

We interpret the U-model as a restricted version of the two-member-household one by making

\[ a_1 + a_2 = 1 \]
\[ F^U(m, s) = F(m, s_1, s_2) \mid s_1 = s_2 = s \] (15)

Equation (14) implies that in both models output is normalized to one when the representative household allocates the totality of its time endowments in the production of the consumption good. Equation (15) defines the transacting technology in the U- model as a restricted version of the transacting technology in the IH- approach.

To start with, note that the restriction of \( F \) to \( s_1 = s_2 \) in (15) will be binding when different members of a household have different relative productivities in transacting and in producing the consumption good. Example 1 below clarifies this point.

In Example 1 members one and two, of a given household, are treated symmetrically with respect to the transacting technology, but are allowed to have different productivities in the production of the consumption good.

**Example 1** Consider the transacting technology \( F(m, s_1, s_2) = ms_1^{0.5}s_2^{0.5} \). After using equations (6), (7) and (8), the solution for the IH- model is given by:

\[
\begin{align*}
    s_1 & = 0.5im/a_1 \\
    s_2 & = 0.5im/a_2
\end{align*}
\]

where \( m \) is determined as the positive root of

\[ g_1(m) = m^2 + \frac{1}{\alpha}m - \frac{1}{i\alpha} \]

and \( \alpha = 0.5/\sqrt{a_1a_2} \geq 1 \) (remember (14)). The total welfare cost of inflation in this case is \( a_1s_1 + a_2s_2 = im \). After using equations (12) and (13), the expression for the welfare cost of inflation in the U- model is given by \( s = im \), where \( m \) now is determined as the positive root of:

\[ g_2(m) = m^2 + m - \frac{1}{i} \]

We show in the appendix that the positive root of \( g_2(m) \) is always greater or equal than the positive root of \( g_1(m) \), and that both roots are less than \( 1/i \) (this last remark ensuring that \( a_1s_1 + a_2s_2 < 1 \) and \( s < 1 \), as required by the
The equality, between the two models, of the equilibrium demand for money and of the implied welfare costs of inflation happens iff \( a_1 = a_2 = 0.5 \) (what makes \( \alpha = 1 \)). As one would expect, this is the case in which the restriction \( s_1 = s_2 = s \) is not binding. Both members of the household are treated symmetrically with respect to the transacting technology and have the same productivity in the production of the consumption good. Table 1 and Figure 1 show how the welfare costs of inflation vary when one allows for different productivity ratios of the two members of the household.

<table>
<thead>
<tr>
<th>Yearly Interest Rate</th>
<th>Standard (U-) Model ((a_1/a_2 = 1))</th>
<th>IH- Model ( a_1/a_2 = 20 )</th>
<th>IH- Model ( a_1/a_2 = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.96</td>
<td>0.64</td>
<td>0.45</td>
</tr>
<tr>
<td>0.14</td>
<td>1.88</td>
<td>1.31</td>
<td>0.93</td>
</tr>
<tr>
<td>0.50</td>
<td>3.02</td>
<td>2.22</td>
<td>1.62</td>
</tr>
<tr>
<td>1.0</td>
<td>3.73</td>
<td>2.86</td>
<td>2.15</td>
</tr>
</tbody>
</table>

One concludes from the observation of lines one and two of the table, for instance, that a gain around 0.9% of GDP, when yearly interest rates drop from 14% to 3%, can actually be no greater than 0.48% (0.93% – 0.45%) (last column of the Table) of GDP, if one allows for different productivities among the members of a household. The bigger the difference in productivities (moving from left to right along any line), the smaller the welfare figures. The table also suggests that the higher the interest rates, the higher the discrepancies (as a percentage of GDP), between the estimates of the U-model and those of the IH-model.

### 3.1 Leisure

So far we have assumed that the members of the household did not value leisure. This subsection aims at investigating how the previous results in the first part of this section might be affected when leisure is included as one of the arguments of the utility function. Still departing from the distinct transacting technologies that characterize each model, we shall conclude, as above, that the welfare figures that emerge from the IH-model are lower than the ones associated with the U-model. The discrepancies between the two models, though, are lower when one considers that households derive utility
from leisure.

Household members are assumed to maximize:

\[
\int_0^\infty e^{-gt} U(c, l) \, dt
\]

(16)

where \( l = l_1 + l_2 \) stands for leisure.

- **The IH-Model With Leisure**

Note that, similarly to the treatment given to consumption, the household does not care about the distribution of leisure between its members. Constraint (4) remains the same. However, constraint (3) is altered by the inclusion of the time dedicated to leisure:

\[
\hat{v} = a_1(1 - s_1) + a_2(1 - s_2) - c - l + h + (i - \pi) \, v - im
\]

(17)

Besides the usual first order conditions (6) and (7), one has:

\[
U_l = \frac{U_c F_{s_1}}{a_1 + F_{s_1}} = \frac{U_c F_{s_2}}{a_2 + F_{s_2}}
\]

(18)

The new equilibrium equation:

\[
1 - a_1 s_1 - a_2 s_2 - l = F(m, s_1, s_2)
\]

(19)

completes the IH-model. Equations (6), (7), (18) and (19) determine \( s_1, s_2, l \) and \( m \) as a function of the interest rate \( i \).

- **The U-Model With Leisure**

Proceeding like in Section 2, the first-order and equilibrium conditions of the usual single-member-household are now given by (12), and by:

\[
U_l = \frac{U_c F_U}{1 + F_{s_1}^U}
\]

(20)

\[
1 - s - l = F^U(m, s)
\]

(21)

Equations (12), (20) and (21) determine \( s, m \) and \( l \) as a function of the nominal interest rate in the U-model. Example 2 illustrates the quantitative aspects of this case.
Example 2 The estimates of this example are based on the same transacting technology of Example 1, and on an utility function \( u(c,l) = l^{1-\beta}c^\beta \). Using the first-order conditions and the equilibrium equation of the IH- model:

\[
\begin{align*}
    s_1 &= 0.5im/a_1 \\
    s_2 &= 0.5im/a_2 \\
    l &= \frac{im(1-\beta)}{\beta}(1+\alpha m)
\end{align*}
\]

where \( \alpha \) has the same definition as in Example 1 and now \( m \) is determined as the root of the following equation:

\[
g_{11}(m) = m^2 + \frac{m}{\alpha} - \frac{\beta}{\alpha i} = 0
\]

Alternatively, the first order and equilibrium equations (12), (20) and (21) lead to \( s = im, m \) being determined as the positive root of:

\[
g_{21}(m) = m^2 + m - \frac{\beta}{i}
\]

As in the previous example, the positive root of \( g_{21}(m) \) is greater than the positive root of \( g_{11}(m) \). Also, both roots are less than \( \frac{\beta}{i} \), ensuring \( a_1s_1 + a_2s_2 < 1 \) and \( s < 1 \) (see appendix). The equality of the welfare costs happens if \( a_1 = a_2 = 0.5 \) (what makes \( \alpha = 1 \)). Table 2 presents some welfare figures under different productivity ratios when \( \beta = 0.5 \).

<table>
<thead>
<tr>
<th>Yearly Interest Rate</th>
<th>Standard Model (( a_1/a_2 = 1 ))</th>
<th>Two-Members Model (( a_1/a_2 = 20 ))</th>
<th>Two-Members Model (( a_1/a_2 = 100 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.65</td>
<td>0.45</td>
<td>0.31</td>
</tr>
<tr>
<td>0.14</td>
<td>1.23</td>
<td>0.88</td>
<td>0.63</td>
</tr>
<tr>
<td>0.50</td>
<td>1.87</td>
<td>1.43</td>
<td>1.08</td>
</tr>
<tr>
<td>1.0</td>
<td>2.21</td>
<td>1.78</td>
<td>1.39</td>
</tr>
</tbody>
</table>

The conclusions are basically the same as those obtained from Table 1, except for the fact that the discrepancies among the welfare figures, either along each line or each column, are usually lower than before.
4 Equivalence in Practical Measurements

In empirical studies, what is usually known by the researcher is the money demand, not the transacting technology. Therefore, usual estimates of the welfare costs of inflation have to deal with the problem of recovering the welfare figures from the knowledge only of the money demand. Such a procedure is used by Lucas (2000), Simonsen and Cysne (2001), and generalized by Cysne (2003). The objective of this section is deriving sufficient conditions that make such welfare measurements (which depart from the knowledge of the money demand) robust with respect to the number of members considered in the representative household. This is an important practical issue, since this is the way how the welfare figures are generated e.g., in Lucas (2000).

Proposition 1 Suppose, as in Lucas (2000), that the transacting technology to be blockwise-weakly separable with respect to money and shopping time\(^1\):

\[
F(s_1, s_2, m) = G(s_1, s_2)\beta(m)
\]

(with \(\beta'(m) > 0, G_{s_1} > 0, G_{s_2} > 0\)) and that \(G\) is first-degree homogenous. Then, welfare measurements which abstract directly from the knowledge of the money demand lead to the same results, either under the U-model or under the IH-model.

Proof. From (6), (7) and (8), the first order and equilibrium equations for the problem, in this case, are:

\[
a_1\beta'G = iG_{s_1}\beta
\]

\[
a_2\beta'G = iG_{s_2}\beta
\]

\[
1 - a_1s_1 - a_2s_2 = G\beta
\]

The homogeneity of \(G\) implies:

\[
(a_1s_1 + a_2s_2)\beta' = \beta i
\]

\(^1\)Note that this transacting technology is compatible with different elasticities of the money demand. Except for the fact that we are considering an IH-approach, with shopping time being allocated by two different economic agents, (22) can be derived from the inventory-theoretic literature (e.g. Baumol (1952), Tobin (1956), Patinkin (1965), Dvoretzky (1965) and Miller and Orr (1966)). See Lucas (1993, p.14).
Differentiating (25) with respect to $i$, and using (23) and (24):

$$-a_1 s'_1 - a_2 s'_2 = \left( \frac{a_1 s'_1 + a_2 s'_2}{i} + m' \right) G' \beta'$$

Given (25) and (26):

$$-a_1 s'_1 - a_2 s'_2 = im'(i)(1 - a_1 s_1 - a_2 s_2)$$

Denoting the welfare costs of inflation $a_1 s_1 + a_2 s_2$ by $z$,

$$z'(i) = -im'(i)(1 - z(i)), \quad z(0) = 0 \quad (27)$$

Assuming that the money demand function has been previously estimated, (27) allows for the measurement of the welfare costs of inflation $z$. The proof is complete by noticing that the counterpart of equation (22), in the case of the U-model, is given, consistently with the first degree homogeneity of $G$, by $F^U(s, m) = s \beta (m)$. Proceeding from this equation and using (12) and (13) leads to (27), with $s$ replacing $z$. ■

Despite the result established by Proposition 1, there is a subtle difference between the two analyses. In the usual U-model, the transacting technology can be recovered from equation $1 - s = s \beta (m)$, once $s$ has been calculated. However, such a procedure is not possible in the IH-model, unless additional information is available (for instance, the time allocated by each member of the household in transacting and in production).

5 A Comparison with Bailey’s Formula

Here we compare the welfare expression that emerges from our model with Bailey’s (1956) partial-equilibrium one. As it is well known, Bailey’s formula for the welfare costs of inflation ($B$) is given by:

$$B'(i) = -im'(i), \quad B(0) = 0 \quad (28)$$

**Proposition 2** Bailey’s welfare formula is an upper bound to the IH-measure of welfare costs of inflation $z(i)$ given by (27).
Proof. Suffices noticing in (27) and (28) that \(0 < 1 - z < 1^2\). ■

The comparison between \(B\) and \(z\) can be further improved by the Proposition below:

**Proposition 3** The general-equilibrium expression for the welfare costs of inflation \(z\) relates to Bailey’s formula \(B\) accordingly to:

\[
z(i) = 1 - \exp(-B)
\]

(29)

Proof. From (27):

\[
\int_0^z \frac{z'(x)}{1 - z(x)} dx = \int_0^i -im'(i) di
\]

Integrating both sides and using the initial-value conditions \(z(0) = B(0) = 0\) leads to (29). ■

**Proposition 4** Bailey’s partial-equilibrium formula is a first-degree approximation to the general-equilibrium welfare measure \(z\).

Proof. The difference \(B - z\) is given by the series \(\sum_{k=2}^\infty B^k/k!\). Since \(0 < B < 1\), this series is convergent. ■

Note that the series \(\sum_{k=2}^\infty B^k/k!\) converges to a positive number, consistently with Proposition 2.

### 6 Conclusions

We have presented an analysis of the welfare costs of inflation that, in contrast with the usual literature, takes into consideration the fact that households are generally comprised of more than one single member.

By allowing different members in a household to have different relative productivities in the production of the consumption good and in transacting, we have seen that each model can lead to a different money demand and, therefore, to different figures concerning the welfare costs of inflation. Welfare costs are usually larger in the U-model, as compared to the IH-model, with

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\(^2\)Simonsen and Cysne (2001) present a similar conclusion with respect to another transacting technology that leads to a non-separable version of (27).
the discrepancy between the models decreasing when the utility derived from leisure is taken into consideration.

We have also concluded that under blockwise weakly separability and first degree homogeneity of the transacting technology with respect to the shopping time of each member of the household, empirical assessments of the welfare costs of inflation which depart directly from the knowledge of the money demand are lead to the same result, either considering a single unit or a multi-unit household. This point is important because it shows when welfare calculations such as those performed by Lucas (2000) are robust with respect to the modelling alternatives considered here.

Finally, we have derived a closed-form expression for the difference between Bailey’s (1956) partial-equilibrium and the general-equilibrium expression of the welfare costs of inflation (valid for any money demand function) and shown that Bailey’s measure can be interpreted as a first-order approximation of the general-equilibrium one.

Appendix

Here we show, as required in the examples 1 and 2, that the positive root of \( g_{2l}(m) \) of example 2 (or \( g_2(m) \) in example 1) is always greater or equal than the positive root of \( g_{UL}(m) \) (\( g_1(m) \) in example 1). We provide two alternative proofs:

**Proof. 1-** Starting with the second example, it suffices noticing that, if \( m_{UL} \) and \( m_{2l} \) stand, respectively, for the positive roots of \( g_{UL}(m) \) and \( g_2(m) \), then \( g_{UL}(m_{2l}) \geq 0 \). Proceeding with the calculations:

\[
 g_{UL}(m_{2l}) = A(\alpha - 1)(1 + 2\beta/i - \sqrt{1 + 4\beta/i})
\]

where \( A > 0 \). Since \( \alpha \geq 1 \) and \( 1 + 2\beta/i - \sqrt{1 + 4\beta/i} > 0 \) when \( \beta/i > 0 \), \( g_{UL}(m_{2l}) \geq 0 \). Also, observe that the equality occurs iff the productivities of both agents in the production of the consumption good is the same (\( \alpha = 1 \)). The same conclusions apply to Example 1 by making \( \beta = 1 \) above.

**Proof. 2-** Again, we start with the second example. Observe that (for \( k = 1/\alpha \)) the positive root of the family of quadratic equations \( f(x) = x^2 + kx - k\beta/i \), \( k > 0 \) is always less than \( \beta/i \). Since this root \( (x^*) \) satisfies \( x^*2 + kx^* - k\beta/i = 0 \), the application of the implicit function theorem leads to \( dx^*/dk = -(x^* - \beta/i)/\sqrt{k^2 + 4k\beta/i} > 0 \). The result follows by noticing that, since \( \alpha \geq 1 \), the value of \( k \) in \( g_{UL}(m) \), equal to \( 1/\alpha \), is lower than in
$g_2(\phi)$ (equal to one). The proof for the first example follows the same steps, by taking $\beta = 1$. ■

References


477. INADA CONDITIONS IMPLY THAT PRODUCTION FUNCTION MUST BE ASYMPOTOTICALLY COBB-DOUGLAS - Paulo Barelli; Samuel de Abreu Pessoa – Março de 2003 – 4 págs.


479. A NOTE ON COLE AND STOCKMAN - Paulo Barelli; Samuel de Abreu Pessoa – Abril de 2003 – 8 págs.


481. ON THE WELFARE COSTS OF BUSINESS CYCLES IN THE 20TH CENTURY - João Victor Issler; Afonso Arinos de Mello Franco; Osmani Teixeira de Carvalho Guillén – Maio de 2003 – 29 págs.

482. A NOTE ON COLE AND STOCKMAN - Paulo Barelli; Samuel de Abreu Pessoa – Abril de 2003 – 8 págs.


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