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Unemployment Insurance

An Analysis of Optimal Mechanisms Under Aggregate Shocks

Artur Bezerra de Carvalho

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Dissertação apresentada à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas, como requisito parcial para obtenção do grau de mestre em economia.

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Versão final da dissertação de mestrado, apresentada à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas, e aprovada pela banca examinadora.

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Resumo

O objetivo deste trabalho é prover uma revisão sucinta da literatura sobre o desenho ótimo de programas de seguro-desemprego, por meio da análise de alguns dos artigos mais influentes publicados nas últimas três décadas, e estender os seus principais resultados para um ambiente econômico sujeito a choques agregados. As propriedades dos contratos ótimos são discutidas à luz das hipóteses-chave usualmente adotadas em publicações teóricas nessa área. Além disso, as implicações associadas ao relaxamento dessas hipóteses também são investigadas. A análise de modelos que contemplam apenas um ciclo de desemprego começa com o trabalho de Shavell e Weiss (1979). A partir de um ambiente econômico simples e comum à maioria dos trabalhos, estudam-se as políticas de benefícios, taxas sobre os salários e o nível ótimo de esforço a ser exercido na procura por emprego. Adicionalmente, questiona-se a idéia de que as distorções no preço relativo de consumo e lazer provocadas pelo seguro-desemprego são a única explicação para alterações marginais dos incentivos à procura por emprego. Usualmente interpretada como um problema de perigo-moral causado por um efeito-substituição, a redução na oferta de trabalho causada por programas de seguro-social é discutida sob essa nova perspectiva. Apresenta-se ainda um estudo teórico sobre contratos de seguro-desemprego ótimo quando os agentes estão sujeitos a mais de um ciclo de desemprego. Finalmente, uma extensão dos modelos a um ambiente sujeito a múltiplos choques agregados é desenvolvida. O trabalho termina com um exercício numérico acerca das implicações de choques i.i.d. sobre o desenho de programas de seguro-desemprego.

Palavras-chave: Seguro-Desemprego, Esforço Individual, Probabilidade de Encontrar Emprego, Nível Ótimo de Benefícios, Choques Agregados .

Abstract

The purpose of this work is to provide a brief overview of the literature on the optimal design of unemployment insurance systems by analyzing some of the most influential articles published over the last three decades on the subject and extend the main results to a multiple aggregate shocks environment. The properties of optimal contracts are discussed in light of the key assumptions commonly made in theoretical publications on the area. Moreover, the implications of relaxing each of these hypothesis is reckoned as well. The analysis of models of only one unemployment spell starts from the seminal work of Shavell and Weiss (1979). In a simple and common setting, unemployment benefits policies, wage taxes and search effort assignments are covered. Further, the idea that the UI distortion of the relative price of leisure and consumption is the only explanation for the marginal incentives to search for a job is discussed, putting into question the reduction in labor supply caused by social insurance, usually interpreted as solely an evidence of a dynamic moral hazard caused by a substitution effect. In addition, the paper presents one characterization of optimal unemployment insurance contracts in environments in which workers experience multiple unemployment spells. Finally, an extension to multiple aggregate shocks environment is considered. The paper ends with a numerical analysis of the implications of i.i.d. shocks to the optimal unemployment insurance mechanism.

Keywords: Unemployment Insurance, Search Effort, Employment Probability, Optimal Benefits, Aggregate Shocks.

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1 Optimal Unemployment Insurance

Unemployment insurance (UI) programs are an important ingredient of social welfare policies in developed economies, accounting for a significant share of gross national product[1]. The primary purpose of UI is to insure risk-averse individuals against the loss of wage income. The necessity of UI benefits comes from the fact that agents cannot perfectly smooth consumption through time, either because credit or insurance markets are incomplete or both.

Classic empirical results in public finance however show that social insurance programs such as UI reduce labor supply. In particular, UI is commonly believed to lengthen the duration of the unemployment spell because of its effect on the effort devoted to job search. This link between unemployment benefits and durations is thus usually interpreted as a consequence of a moral hazard problem: the probability of finding a job depends on the search effort, which is costly to the agent (unemployed worker) and is not observed by the principal (e.g., government or insurance agency). Under this assumption, the problem posed to policy makers is that of providing insurance to risk-averse workers and, at the same time, providing the right incentives to search for a job.

In their seminal work on UI, Shavell and Weiss [9] focused on a setup of a single isolated spell of unemployment followed by a single spell of employment. They established that, if job search intensity affects the probability of finding a job and if this effort cannot be monitored by the principal, then benefits must decrease monotonically throughout the unemployment spell. Even though the declining sequence reduces the role of benefits as insurance, it is desirable because it provides the right incentives to search. Hopenhayn and Nicolini [3] analyze a model along the lines of Shavell and Weiss. The authors model the incentive problem created by unemployment insurance as a standard repeated moral hazard in a multiperiod setup. In a recent paper [4], from 2009, the same authors go further and analyze the properties of optimal unemployment insurance contracts in environments in which workers experience multiple unemployment spells.

The present work analyzes the above results and is divided in seven sections. Section two studies two models of UI in settings of only one spell of unemployment. The following section discusses the usual interpretation of unemployment durations as evidence of a moral hazard problem. Section four considers a model of multiple unemployment spells. Next, I present a model of unemployment insurance in the presence of aggregate shocks, followed by a numerical analysis of its main implications. I show that most of the characteristics of the previous models remain valid in this setup. However, the model also presents at least one distinctive feature in the optimal incentive scheme. Section seven concludes.

2 The one-spell of unemployment models

This section discusses the main findings on one-spell of unemployment models. It starts by addressing Shavel and Weiss [9] work and its further extension, by Hopenhayn and Nicolini [3]. Next, it points out some of the drawbacks and limitations of this kind of models by discussing Kocherlakota's [5] and Raj Chetty's [1] results.

[1] According to Hopenhayn and Nicolini [3], for the European Community countries, public expenditures on unemployment insurance averaged around 2 percent of gross national product for the second half of the 1980s.

2.1 The Shavel and Weiss's Model

Shavel and Weiss [9] model the UI problem as a principal-agent model, in which unemployed workers face exogenous credit and insurance constraints, so that an UI agency is the only source of insurance against job-loss shocks and has to determine the optimal path of benefit transfers. By assuming that the probability of employment is solely a function of agents' individual search effort, they reduce the analysis of insurance provision to a simple moral hazard setup.

For simplicity, the authors assume that the UI budget is fixed and take quit and layoff behavior as given, disregarding questions on the nature of agents' unemployment spell and the impact of UI benefits over job quits. Under this latter assumption, the UI agency objective is to maximize unemployed agents' expected discounted utility. Moreover, their model comprises identical risk-averse agents whose search behavior, as stated above, does not mutually affect their employment probability. Consequently, the planner's problem can be reduced to the tailoring of a contract between the agency and a single unemployed worker. The moral hazard feature of the model is due to the assumption of private observability of agents' search effort.

In each period, agents first collect the UI benefits, and then either find a job or remain unemployed. The probability of finding a job is a function of agents' effort and involve disutility[2] All jobs are assumed to be identical and to offer a permanent (and constant) wage over time. Further, employment is taken as an absorbing state for the worker, which means that, once the agent finds a job, the incentive problem disappears. Taken together, these two hypothesis ensure that the discounted utility of an agent who finds a job is always known, which, in its turn, simplifies enormously the calculation of reservation wages in each period. In addition, the authors avoid the intractabilities of multiple spell of unemployment setups, while still being able to make assertions on the time path of benefit transfers in a particular spell.

Shavel and Weiss assume that, once a worker has found a job, he is beyond the grasp of the insurance agency, in the sense that his future earnings cannot be taxed and the agent is free to borrow, save and consume. However, any given agent starts the unemployment spell with no wealth and is savings and borrowing constrained until he finds a job. Although this is a reasonable approximation for some of the unemployed agents, it is certainly not true for all. Nonetheless, for tractability reasons, it is assumed that unemployed agents have no other source of income but UI benefits and that the agency has direct control of their consumption stream, which precludes the occurrence of hidden savingsl[3]. UI benefit transfers are thus assumed to be the only instrument available to workers to smooth consumption over time and across states.

If search behavior were observable by the agency or if individuals did not influence their probability of receiving a job offer, the solution to the UI problem would comprise a constant sequence of benefits. A constant sequence would be desirable because it would equate marginal utilities through time, perfectly smoothing unemployed agents' consumption and therefore maximizing their expected discounted utility. In each period, the agency would simply transfer the constant unemployment benefit and prescribe a search effort level to any given unemployed worker. However, since search effort is private information by assumption, Shavell and Weiss show that a constant sequence would not be optimal, because agents would set their effort levels below the socially optimal one.

[2]One can regard this search effort as a subtraction from leisure.

[3]The possibility of hidden savings is discussed later, when we introduce Kocherlakota's [5] model.

This result is due to the fact that unemployed agents try to equate their private marginal benefit of searching to their private marginal cost, and do not take into account the social cost associated with the insurance scheme.

To induce agents to take the UI social cost into account, the authors suggest a mechanism to provide search incentives to unemployed workers. Since the only instrument available for the UI agency in this environment is the benefit transfer sequence, Shavel and Weiss show that the optimal time sequence of benefits must decline and, although always remaining positive, must tend to zero. A declining sequence is desirable because it induces individuals to get jobs sooner, at least on average, even though it reduces the role of benefits as insurance. In other words, the implicit trade-off between insurance and incentive implies that the solution to the insurance agency problem (the maximization of unemployed agent's expected discounted utility) encompass a declining replacement ratio throughout the unemployment spell.

2.2 Extension: The Hopenhayn and Nicolini's Model

Shavel and Weiss [9] showed that, to provide unemployed agents with the right incentives to search, the mechanism they considered reduced the role of insurance of the unemployed benefits. Hopenhayn and Nicolini [3] explore the welfare gains associated with a more complex mechanism, capable of providing strong incentives to search at a lower expected utility cost. They depart from Shavell and Weiss' model and broaden the set of instruments that the contract considers by introducing a novel feature on the UI contract: a wage tax. Despite not being able to monitor workers' search effort, the insurance agency can now monitor their consumption and tax their wage after they become employed.

Hopenhayn and Nicolini start by deriving some general properties of the optimal UI contract, focusing on how the duration of unemployment affects the net transfers to the worker. They show that the unemployment benefits sequence is decreasing over time while the worker remains unemployment, in accordance with Shavell and Weiss' result. Hence, the consumption of the unemployed worker must decrease over time. The authors go further and establish that the wage tax, which is the novel feature their contract considers, is not independent of the unemployment history. The analytical results suggest thus that this extra degree of freedom is not redundant, allowing the contract to provide a smoother consumption profile to the agent. Therefore, an outcome closer to that of the full-information case can be achieved. Moreover, under some conditions, it is possible to show the tax levied on the worker increases with the length of the previous unemployment spell.

The intuition behind the above results is as follows. To provide intertemporal incentives, the optimal contract punishes workers who face a continued unemployment spell by reducing their claims to future consumption. Moreover, since agents are risk-averse, their consumption should be reduced at all possible future states of nature, namely, the states in which the workers are still unemployed and those states in which they becomes employed. Therefore, not only transfers should decrease with the length of unemployment spell, but also tax on wages should increase. Of course, one must consider perverse incentive effects: the tax levied on wage could be so high that the unemployed worker would find optimal to reduce his search effort and continue to receive UI benefits. As stated in the last paragraph, there are conditions in which the *permanent income* effects dominate over this negative incentive effect.

The authors end the analysis by comparing the welfare costs of their mechanism to that of Shavel and Weiss. Some of the numerical results presented suggest that there are gains from switching to this optimal unemployment insurance scheme. According to their results, this mechanism reduces the cost of the contract (or increases the utility provided) for an agent who has limited wealth or access to alternative means to smooth consumption.

2.3 The Possibility of Hidden Savings - Kocherlakota's Model

In his paper, Kocherlakota [5] studies a variant of the Hopenhayin and Nicolini's [3] model. The author is interested in the properties of an optimal UI contract in a setting that allows the worker to *secretly* transfer consumption from one period to the next. Put differently, Kocherlakota relaxes the hypothesis that the principal can *costlessly* monitor the agent's savings and condition contractual payments on this variable.

The author argues that, in Hopenhayn and Nicolini's setting, the optimal contract has the property that the agent is *savings-constrained* when unemployed. This is due to the fact that the agent's shadow interest rate is lower than the principal's shadow interest rate, which is a common feature of contract problem in settings with repeated moral hazard, as shown by Rogerson [8]. In those environments, it is optimal to impose a sufficiently severe punishment for poor performance that the agent ends up being savings constrained, since he would like to save so as to mitigate next period's punishment.

The fact that the agent's shadow interest rate is lower than the principal's shadow interest rate stimulates savings and the optimal dynamic contracts derived in Shavel and Weiss' [9] and Hopenhayn and Nicolini's [3] models are not incentive-compatible unless the principal can costlessly monitor the unemployed worker's asset level. This assumption may be somewhat restrictive, since there a number of ways that a person can transfer resources to the future that may be hard for outsiders to observe, such as foreign bank accounts or investments in durable goods. Hence, studying the properties of optimal contracts in a setting where agents are allowed to engage in secret assets accumulation is essential.

2.3.1 The Problem

The environment is similar to the ones of the previous models. The principal has von Neumann-Morgenstern utility function

$$-\sum_{t=1}^{\infty} \beta^{t-1} c_t$$

while the agent's utility is given by

$$\sum_{t=1}^{\infty} \beta^{t-1} [u(c_t) - \upsilon(p_t)]$$

where, p_t now denotes the agent's effort in period t, and lies in the set $[0, 1]$. It is assumed that $u', -u'', \upsilon' > 0$, $\upsilon'' > 0$, and that u is bounded from above and from below.

The probability of finding a job increases with the amount of effort exerted in the following way. If an agent is unemployed at the end of period $t-1$, then the probability of becoming employed in period t is p_t, and the probability of staying unemployed is

$1 - p_t$. Unlike employment status, which is publicly observable, search effort is private information. Moreover, the agent can secretly save at rate $1/\beta - 1$. Finally, employment is an absorbing state and the paper considers, thus, only one spell of unemployment.

The contracts in this economy specify two sequences $\{c_t^E, c_t^U\}_{t=1}^{\infty}$, where c_t^E stands for the compensation the principal pays to the agent if he is employed in period t, and c_t^U is the compensation for the unemployed worker in period t. Moreover, once an agent is employed, his compensation is constant over time: if the worker becomes employed in period t, then $c_s^E = c_t^E \; \forall \, s \geqslant t$. Since the principal and the worker have the same discount factor and there is no further incentive problem after the agent becomes employed, it is easy to see that this smooth compensation is efficient.

The principal wants to (weakly) implement a sequence of effort choices $p^* = \{p_t^*\}_{t=1}^{\infty}$ by the agent when unemployed, where $p_t^* \in (0, 1)$ for all t. The incentive-compatible contract (c^E, c^U) is such that:

$$\{S_t^*, p_t^*\}_{t=1}^{\infty} \in \underset{\{S_t, p_t\}_{t=1}^{\infty}}{\operatorname{argmax}} \sum_{t=1}^{\infty} \beta^{t-1} \prod_{s=1}^{t-1} (1 - p_s) \{p_t u(\zeta_t^E)/(1-\beta) - \upsilon(p_t) + (1 - p_t) u(\zeta_t^U)\}$$

subject to

$$\begin{aligned} \zeta_t^E &= c_t^E + S_{t-1}(1-\beta)/\beta \text{ for all } t, \\ \zeta_t^U &= c_t^U + S_{t-1}/\beta - S_t \text{ for all } t, \\ S_t,\, p_t,\, 1 - p_t,\, \zeta_t^E,\, \zeta_t^U &\geqslant 0 \text{ for all } t, \\ S_0 &= 0. \end{aligned}$$

so that it is weakly optimal for an unemployed agent to choose p_t^* in all t. Since the interest rate of the economy is $1/\beta - 1$, if an agent becomes employed in period t with savings S_{t-1}, then his optimally smoothed consumption is given by the level of consumption the contract assigns in every period (which is a constant value) plus the interest paid on the amount saved, $c_t^E + S_{t-1}(1-\beta)/\beta$, in every period thereafter[4].

Given any incentive-compatible contract, it is straightforward to show that there exists a payoff-equivalent contract $(c^{E'}, C^{U'})$ in which the agent's optimal savings sequence is zero. Restricting attention to these contracts that induce zero savings, the principal's problem is now given by the following minimization problem (UIP):

$$\min_{c^E, c^U} \sum_{t=1}^{\infty} \beta^{t-1} \prod_{s=1}^{t-1} (1 - p_s^*) \{p_t^* c_t^E/(1-\beta) + (1 - p_t^*) c_t^U\}$$

subject to

[4]Although the principal's desire of implementing an interior p^* is not formally explained, the author argues that this choice might be preferred because of search externalities. If the principal is contracting with a unit measure of agents, there may be congestion effects, in the sense that it becomes harder for a given agent to find a job when other agents are searching a lot. Thus, when designing the optimal UI, the principal internalizes this effect, and his choice of p will, for a generic class of problems, be interior.

$$
\begin{aligned}
(0, p^*) \in \underset{S \geqslant 0,\, 1 \geqslant p \geqslant 0}{\operatorname{argmax}} \sum_{t=1}^{\infty} \beta^{t-1} \prod_{s=1}^{t-1} (1-p_s) \{ & p_t u(c_t^E + S_{t-1}(1-\beta)/\beta)/(1-\beta) \\
& + (1-p_t) u(c_t^U + S_{t-1}/\beta - S_t) - v(p_t) \} \\
c_t^U + S_{t-1}\beta^{-1} - S_t \geqslant 0 & \text{ for all } t; \\
S_0 = 0; & \\
\sum_{t=1}^{\infty} \beta^{t-1} \prod_{s=1}^{t-1} (1-p_s) \{ p_t u(c_t^E)/(1-\beta) + (1-p_t) u(c_t^U) \} \geqslant u^*; & \\
c_t^E,\, c_t^U \geqslant 0 & \text{ for all } t.
\end{aligned}
$$

The objective is to find the minimal-cost incentive-compatible contracts among all that provide the agent with the ex-ante utility of at least u^*.

2.3.2 Difficulties

In this section, the difficulties underlying the solution of the UIP are discussed. Kocherlakota considers two recently developed approaches. The first consists in making the problem recursive. The models developed in the previous sections can be solved using an approach developed by Spear and Srivastava [10]. These authors showed that, without hidden savings or other hidden state variables, it is possible to model dynamic moral hazard problems recursively by introducing a one-dimensional variable: the continuation utility. The problem is solved as follows: in each period, the principal chooses current consumption and the next period's continuation utility so as to minimize his costs subject to the incentive constraints, and subject to delivering a specified amount of continuation utility to the agent.

When agents are free to bring savings into each period, their response to any given contract is different, because the presence of hidden savings essentially introduces an adverse selection problem in each date. According to Fernandes and Phelan [2], in order to deal with this kind of adverse selection problem, the principal must minimize his costs subject to delivering a given amount of continuation utility to *every* type. In the setting considered, at any given point in time, the principal would seek to minimize the expected value of his discounted costs, given that he would like to induce an agent with no assets to choose effort p^* and to choose no to save. Kocherlakota shows then that the UIP in this recursive form is computationally infeasible, beacause savings can take a continuum of values, and, therefore, the relevant state variable is a *function*, instead of a number as when only effort is hidden.

The second difficulty concerning the solution of the UIP is the fact that the first-order approach, that replaces the agent's incentive constraints with the corresponding first-order conditions, may not be valid because the agent's decision problem is intrinsically non-concave in effort and savings. With the possibility of hidden savings, the agent can experience a *second-order* gain by increasing savings above zero and lowering the search effort. The agent shirks and save, in order to compensate for the punishment in the following period. Kocherlakota proves that there is no set of known conditions in the infinite horizon problem UIP that are sufficient to guarantee that the first-order approach is valid with hidden savings. On the contrary, if the function $v(\cdot)$ has a sufficiently low curvature, it can be shown that the first-order approach is actually invalid. In other

words, the complementary nature of shirking and saving makes the agent's problem non-concave.

3 The link between benefits and the duration of unemployment spell - Is it purely due to moral hazard?

Raj Chetty [1] questions the usual interpretation of the reduction in labor supply caused by social insurance programs. Formerly, it was argued that this finding, which is one of the classic empirical results in public finance, has traditionally been interpreted as evidence of moral hazard caused by a *substitution effect*: UI distorts the relative price of leisure and consumption, reducing the marginal incentive to search for a job.

Motivated by evidence that many unemployed individuals have limited liquidity and exhibit excess sensitivity of consumption to cash-on-hand, Chetty argues that the link between UI benefits and durations is not purely due to moral hazard. Using a job search model with incomplete credit and insurance markets, he shows that, when an individual cannot smooth consumption perfectly, UI benefits affect search intensity through a "liquidity effect" in addition to the moral hazard channel emphasized in earlier works. The "liquidity effect" works as follows: with little cash on hand, households need to accept any job when UI benefits are low. When these benefits increase, they can afford to look for better matches, which lengthen the unemployment spells but leads to better outcomes.

While the substitution effect is a socially suboptimal response to the creation of a wedge between private and social marginal costs, the liquidiy effect, in contrast, is a socially beneficial response to the correction of the credit and insurance market failures. The divergent implications for the welfare consequences of UI make, therefore, the distinction between both effects crucial in designing the optimal contracts. Using data from the U.S., the author estimates that the liquidity effect accounts for 60% of the marginal effect of UI benefits on durations at current benefit rates. Further, Chetty demonstrates that this estimate implies that a benefit equalt to 50% of the pre-unemployment wage is near optimal in a UI system that pays constant benefits for six months.

4 Multiple Unemployment Spells - The Hopenhayn and Nicolini's model

Hopenhayn and Nicolini [4] are interested in examining the validity of the results from their earlier work, from 1997, in environments in which workers experience multiple unemployment spells. In most UI programs, eligibility depends on previous unemployment history and a previous period of employment is required to qualify for benefits. Additionally, coverage ratios increase with the length of previous jobs. Empirical evidence shows that higher job termination rates tend to coincide with the minimum number of periods required to qualify for unemployment benefits. The authors argue that this aspect has been neglected in theoretical work on optimal design which, as suggested in the previous sections, has focused on the simplified case of a single unemployment spell.

Unemployment insurance design is modeled in a similar fashion to Hopenhayn and Nicolini's previous work, as a repeated moral hazard problem in a setting in which search effort of unemployed workers cannot be monitored by the enforcement agency. Therefore, the insurance mechanism must trade-off incentives for job search with unemployment

duration risk. The assumptions made are fairly the same as that found in Shavel and Weiss [9] and in their own previous article. For simplicity, however, they assume that agents can choose between only two search effort levels. When a given unemployed worker exerts the lowest effort level, his probability of finding a job is zero, while, by exerting the highest level, this probability shifts to a given positive number. Of course, this number must be strictly smaller than one to characterize a moral harzard problem. Moreover, as assumed in these authors' earlier work, the insurance agency can impose a wage tax to the employed worker. To allow for a recursive formulation of the problem, it is assumed that the principal can directly control the consumption of the agent or, equivalently, monitor his wealth, as usual. The insurance contract specifies then, for each period, a net transfer to the agent and, if the agent is unemployed, a recommended action as a function of the realized history.

Hopenhayn and Nicolini begin the analysis by restricting attention to the case in which agents do not influence their employment status. When employment termination rates are exogenous, their previous results for the case of a single unemployment spell have analogues in the multiple spells case: transfers to unemployed workers decrease with the length of their unemployment spell, while re-employment taxes increase with the length of that spell. In addition, these transfer schedules decrease with previous unemployment spells. The intuition for these results is the same provided for the role of the re-employment tax in their previous work. Because risk-aversion implies that agents value consumption smoothing, to maximize unemployed agent's expected discounted utility while still providing them incentives to search, optimal mechanisms must impose permanent and not temporary reductions in consumption. Hence, the longer a worker is unemployed, the lower is his permanent consumption level.

The authors show that, under exogenous job termination, employed workers' consumption is constant along all the employment period, and so are the promised benefits in case they are fired. Because there is no information problem while the agent is working, the length of the employment spell provides no valuable information to the principal. Therefore, workers are completely insured against job loss and there is no employment dependence in the optimal unemployment insurance plan.

An important consequence of the lack of dependence of the UI contract on previous unemployment spells is the fact that the environment considered does not provide a rationale to the employment history restrictions that motivates the authors' analysis: the agency basically solves the same problem as that found in the single unemployment spell environment; given an unemployed agent previous after-tax wage, it determines the following path of insurance benefits. The previous unemployment history can thus be summarized by the current level of after-tax wage.

Another important result derived regards the changes in unemployment benefits levels when agents find a job. In this environment, if an unemployed worker becomes employed and immediately loses this job, his replacement ratio is increased; thus, finding a job is a way to upgrade unemployment benefits. If job terminations are not exogenous, this optimal contract is susceptible to opportunistic behavior, and the analysis of a virtual loophole in the contract and how insurance mechanisms can deal with it constitute the main contribution of this article to the economic theory on UI.

4.0.3 A loophole in the optimal contract - The possibility of opportunistic behavior

As mentioned above, the authors show that, if an unemployed worker finds a job and then is fired in the following period, the optimal contract will offer him a higher insurance benefit than the one he previously received at the end of his previous unemployment spell. Therefore, finding a job is a way to "upgrade" the level of unemployment benefits, while, because job termination is taken as exogenous, losing a job does not " downgrade" the coverage. As a result of the asymmetry verified in the contract's response to finding and losing a job, adverse selection problems in the decision of both creating and destroying job matches might arise.

The main concern with regards to the insurance scheme presented above is the assumption that job termination is exogenous. If the principal cannot distinguish quits from layoffs, opportunistic workers may take advantage of the UI contract. Motivated by evidence found in countries with generous unemployment insurance programs, which suggests that the UI program may induce inefficient quits, Hopenhayn and Nicolini argue that the distinction between involuntary and voluntary separations is hard to establish in practice discuss the UI design problem when quits and layoffs cannot be perfectly monitored. They identify two forms of opportunistic behavior that may arise in this setting.

The first is related to the generosity of the UI program. Usually, UI models do not consider the effort that workers must exert at work, because it does not affect qualitative results. However, this normalization is no longer valid in the multiple unemployment spells setting. In other words, disutility of working and the generosity of unemployment insurance might induce voluntary quits from socially efficient jobs. In fact, by considering the disutility of working, it is possible to show that, if the replacement ratio is high enough, workers may find optimal to quit, collect the benefits and avoid the disutility of working. To prevent this kind of behavior, one must add a "no-quit" constraint to the optimization problem, which ensures that the expected discounted utility of an employed worker is higher than his expected discounted utility in case quits. If the no-quit constraint binds at the optimum, then consumption falls when the worker loses the job and rises if the worker remains employed. Additionally, it is shown that taxes decrease with tenure and replacement ratios increase with the length of the previous employment spell. The intuition behind the result is that, to make employment an attractive state, the contract rewards tenure on a job and punishes with an incomplete replacement ratio quits and layoffs.

The second form of opportunistic behavior happens in environments where job offers are heterogeneous and the principal cannot monitor the quality of the job the worker accepts. Again, because taking a job, no matter how short lived it is, upgrades unemployment benefits, workers might *fake* employment by taking a bad job and soon quitting, just to upgrade their UI benefits. In order to study the optimal contract in this setting, the authors allow for a second type of job, the bad jobs. Bad jobs are modeled as socially inefficient because they provide a lower flow of utility: although they pay the same wage as the good jobs, they generate a higher disutility per period. Additionally, it is assumed that these jobs arrive every period with probability one, independently of the agent's effort level. Finally, model the difficulty in monitoring and distinguishing quits from layoffs, Hopenhayn and Nicolini assume that the bad jobs' termination rate is the same as that of good jobs and that the principal cannot monitor job transitions between

bad jobs.

The disutiliy of effort in bad jobs is assumed to be high enough such that, in the absence of UI, no unemployed worker would take a bad job. Moreover, while employed, the agent cannot look for another job. Bad jobs constitute thus a costly way of sending a signal of employment to the principal. Nevertheless, an UI contract with no employment dependence like the one considered in the previous section may increase the private value of these jobs, in the sense that it may be optimal to unemployed agents to accept these bad jobs in order to upgrade their continuation promises.

Hopenhayn and Nicolini show that, to prevent workers from accepting bad jobs, the upgrade in benefits must be bounded from above by a constraint that makes agents indifferent between the upgrade in the expected discounted utility and the cost of staying at the job. As the cost of staying in the bad job is increasing with job tenure, so is the upgrade in benefits the contract can offer workers. However, because of the bound in the upgrade in benefits, as time passes, the cost of holding a bad job is high enough so that separating good from bad jobs creates no inefficiency. The idea is that no unemployed worker will find attractive to accept a bad job because it is costly and the limited increase in benefits eventually does not compensate the effort. Moreover, the optimal problem still shares most of the qualitative properties of the optimal contract in the first case of opportunistic quit behavior, with the exception of the employment dependence properties. In particular, benefits decrease and taxes increase with the length of the current and previous unemployment spells.

Despite the fact that many of the properties of the single spell of unemployment models extend to the multiple unemployment spells setting, Hopenhayn and Nicolini's work point out important limitations, as well as analytical tools to overcome them, associated with optimal UI contracts. Whether or not these tools might be regarded as feasible policies, their analysis has shed light over the intricacies of dynamic UI mechanisms by relaxing restrictive assumptions and thus allowing for a more realist environment.

5 Optimal Unemployment Insurance Under Aggregate Shocks

Since the financial crisis of 2008, unemployment rates have risen in the United States and in many other developed countries. American struggle to recover from the heavy losses in economic activity, which came in the form of financial stimulus packages approved by the Congress, has proven to be far less effective in saving and creating jobs than what the government had expected. As a consequence, American families watch with despair the increase in unemployment spells duration.

Traditionally, unemployment benefits last for just 26 weeks in the U.S. During previous economic slowdowns, the U.S. government has extended the duration of benefits to about 70 weeks. The new scenario of long-term unemployment, which may be unprecedented in the postwar United States, however has forced even more extreme temporary changes in UI policies. To prevent hundreds of thousands of unemployed workers from exhausting their benefits, Congress has approved some UI extensions that have prolonged transfers for a period of up to 99 weeks, depending on the states' unemployment rates. Still, a hard-luck group of jobless Americans has already exhausted the maximum 99 weeks of unemployment insurance. Congressmen, specially Republicans and conservative Democrats, are now resistant to the idea of providing an additional tier of unemployment

insurance, while an increasing number of families face utter economic devastation: completely out of money, due to the loss of their job income and the complete exhaustion of their savings and retirement funds.

The main arguments used to support the end of UI extensions are the following. First, since it is the government that is paying for the additional benefits, extending UI transfers would deepen the federal deficit spending. Increasing government deficit, in its turn, will possibly obstruct the approval of future tax cuts and may even imply more taxation, with possible perverse consequences to economic growth ahead. Secondly, extending UI benefits is seen as equivalent to subsidizing unemployment, encouraging people to dismiss possible interesting offers to wait for better opportunities, and therefore serving as a stimulus to longer unemployment spells.

The purpose of the present work is not to discuss the general equilibrium implications of UI programs, but rather to shed some light over the moral hazard aspects of this insurance mechanism. Therefore, I disregard questions along the lines of the first argument and focus on the design of incentive-compatible UI schemes. In times of economic recessions, how should UI benefits be adjusted? As previous works have made clear, current UI programs might not be optimal, since they establish constant benefit transfers for a limited period of time, whereas optimal mechanisms prescribe decreasing infinity sequences of replacement ratios. Although the following model does not consider heterogeneous job offers - and cannot therefore provide any insight on the impact of insurance benefits over reservation wages - it does consider different economic scenarios, to allow for the analysis of unemployed agents' responses under different employment scenarios.

The model attempts to address questions such as the recommended search effort levels under different states of natures, as well as the optimal transfer paths. If the optimal sequence of benefits is indeed decreasing over time, should the fall in benefit transfers from one period to another be the same for different states of nature? At first, one might think that continuation values should decrease more in states in which it is easier to get a job, as the fact that an unemployed worker failed to find a job would convey more information about his search efforts than an unsuccessful attempt under a harsher economic scenario. Results show however that, for a special class of probability of employment functions, continuation values do not depend on the states of nature.

5.1 The Model

The environment is fairly the same as that of the Hopenhayn and Nicoli's [3] model, except for some minor modifications to provide for the possibility of multiple states of nature.

Time is discrete. At the beginning of each period t, the economy experiences a publicly observable aggregate shock, drawn from a finite set $\mathbb{Z}$ according to an exogenous and fixed distribution. For each period t, let the aggregate shock be denoted by the random variable Z_t, and its realization, by z_t. Assume that these random variables Z_t are independent and identically distributed. Further, let z^T be the vector containing the history of shocks up to period T. Given the assumptions above, it is straightforward to show that $Prob(Z_{T+1} = z \mid z^T)$ is simply $Prob(Z_{T+1} = z)$, and that $Prob(Z_{T+1} = z) = p(z)$, for all T.

Let $\mathbb{S}$ be the finite set of states of nature and suppose that these states are given solely by the aggregate shocks, such that there exists a one-to-one function g that maps $\mathbb{Z}$ into $\mathbb{S}$. The probability of observing the shock $z \in \mathbb{Z}$ shall thus, henceforth, be referred to

simply as the probability of facing the state $s = g(z) \in \mathbb{S}$. Moreover, to each sequence z^T, there corresponds a vector s^T containing the history of states of nature up to period T, so that $s^T = (s_0, s_1, s_2, \dots, s_T)$. The set of all infinite sequences of states of nature is $\mathbb{S}^\infty$. Given the probability distribution of aggregate shocks, define $\mu_{\mathbb{S}}$ as the correspondent probability measure over the power set of $\mathbb{S}^\infty$ and assume that, at the beginning of period 0, an element $\mathfrak{s}$ of $\mathbb{S}^\infty$ is drawn according to $\mu_{\mathbb{S}}$. Finally note that $\mathfrak{s} = (s_0, s_1, s_2, \dots)$, $s_i \in \mathbb{S}$ for all i.

Agents in this economy can be either employed or unemployed, and can spend some time to find a job by exerting search effort at each period. The search effort exerted by each worker is known only to him. Preferences are defined over consumption and effort levels. Under the hypotheses of additive time separability and additive separability between consumption and leisure, agents order the stochastic processes of consumption and effort $\{c_t, a_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t] \tag{1}$$

where c_t and a_t denote consumption and effort at time t. The parameter $\beta < 1$ is the discount factor, whereas $\mathbb{E}$ is the expectation operator. Consumption takes values on $\mathbb{R}_+$, and effort can take any value on the interval $\mathcal{A}$ containing zero. The utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, strictly concave and well defined at zero.

At every period t, the probability of finding a job ϕ is a function of the search effort chosen by the agent in t and the current state of nature, such that $\phi : \mathcal{A} \times \mathbb{S} \to [0, 1)$ for every agent. Note however that this probability function is fixed through time, in the sense that, in any period in which the state of nature is s, an agent who exerts effort level a will face an employment probability that is given simply by $\phi(a, s)$. I assume further that ϕ is increasing, strictly concave and twice differentiable with respect to search effort. Furthermore, $\phi(0, s) = 0$ and $\lim_{a\to\infty} \phi(a, s) = 1$, for all $s \in \mathbb{S}$.

The only way through which aggregate shocks affect the economy is by determining the states of nature. These states, in their turn, influence agents' employment probabilities. Given the previous assumptions over the relationship between $\mathbb{Z}$ and $\mathbb{S}$, to determine the impact of aggregate shocks, it suffices to study how employment probabilities are affect by the states of nature. In order to do so, attention is restricted to shocks that affect these probabilities in an homogeneous way, by either reducing or enhancing them for every level of search effort in $\mathcal{A}$, compared to another shock. In other words, it seems appropriate to rule out cases in which a shock might enhance ϕ for every level of effort $a \in \tilde{\mathcal{A}} \subsetneq \mathcal{A}$, while reducing ϕ for $a \in \hat{\mathcal{A}} \subseteq (\mathcal{A} \setminus \tilde{\mathcal{A}})$.

Let $\succcurlyeq$ be a partial order of the elements in $\mathbb{S}$ defined by: $s \succcurlyeq s'$ if, and only if, $\phi(a, s) \geqslant \phi(a, s')$, for all $a \in \mathcal{A}$. Further, assume that every element in $\mathbb{S}$ is comparable, so that $(\mathbb{S}, \succcurlyeq)$ is a totally ordered set. Completeness ensures that, for any s and s' in $\mathbb{S}$, either $\phi(a, s) \geqslant \phi(a, s')$ or $\phi(a, s) \leqslant \phi(a, s')$ holds, for all $a \in \mathcal{A}$ ($s \succcurlyeq s'$ or $s' \succcurlyeq s$, respectively). Note that this assumption implies that, whenever $s \succcurlyeq s'$, the likelihood ratio $[\phi(a, s)/\phi(a, s')] > 1$, for all $a \in \mathcal{A}$. In this case, the state s is considered better than state s'.

At the beginning of every period t, and not before, any given worker learns the realization of the public shocks z_t and the correspondent state of nature $s = g(z_t)$. Subsequently, he chooses how much he will consume and how much effort he will exert. Thus, in t, agents first know the entire history of public shocks up to the current period

$z^t = (z_1, \ldots, z_t)$, and only then choose their consumption and effort levels in view of this history. In addition, because $\phi(0, s) = 0$ for all $s \in \mathbb{S}$, only after search effort is exerted an unemployed agent might receive a job offer.

There exists an insurance agency which offers unemployment insurance contracts. As usual, it is assumed that the principal is risk-neutral and discounts future flows at the same rate as the agents. However, since the time spent looking for a job is private information, he is not able to monitor workers' search effort. In addition, suppose that the consumption good is nonstorable and that, during agents' unemployment spells, the agency can directly control their consumption streams. This later assumption precludes the occurrence of trades such as borrowing and lending without knowledge of the principal and, as argued by Hopenhayn and Nicolini [3], is quite typical in the repeated agency literature, since it provides an upper bound on what can be achieved through an optimal contract. Therefore, let the monitoring of unemployed workers' savings be costless, so that the agency can condition contractual payments on this variable[5]. Given the assumptions above, suppose, without loss of generality, that the worker have no other source of income except the wage received when employed. As a matter of convenience, assume further that unemployed workers have zero wealth[6].

The principal has the ability to commit to a transfer policy. On the other hand, workers are free to abandon the agreement settled with the insurance agency at any moment in any given period. By doing so, however, they become ineligible for unemployment insurance for the rest of their unemployment spell. After finding a job, workers are beyond the grasp of the insurance agency, which means that the principal is unable to control their consumption or savings decision. In addition, all jobs are identical, in the sense that they require the same level of effort and offer identical wages w, assumed to be permanent and constant over time[7]. For notational convenience and because a constant term does not affect qualitative properties of the optimal contract derived below, the disutility of effort when the agent is employed is normalized to zero. Finally, assume that workers experience only one unemployment spell, or, put differently, assume that, once they find a job, they become employed for the rest of their lives.

5.1.1 The Contract

Consider the employment history of a given agent. Let $t = -1$ be the period in which this agent becomes unemployed. Still in period $t = -1$, the risk-neutral principal offers a contract to the risk-averse agent. Starting at time $t = 0$, the contract specifies unemployment insurance benefit transfers $c_t(s)$ and recommended actions $a_t(s)$ as functions of the realized aggregate shocks and the unemployment history of the agent.

Let the variable $e_t(s)$ denote the employment status of an agent at the end of period t when the realized state of nature is s, so that $e_t(s) = 1$, if the agent is employed,

[5]As discussed in section 2.3, Kocherlakota [5] has shown that allowing for the possibility of hidden savings restrains the use of recursive techniques to solve the UIP. In addition, as a second difficulty, it might invalidate the first-order approach, which is used to make incentive constraints more tractable.

[6]Under costless monitoring, this assumption imposes no restriction to the analysis, as positive wealth levels would only translate into changes in the net transfers made by the principal, in order to match agents' consumption levels to the ones they would have if they were to consume only what the agency gave them through unemployment benefits.

[7]Since the purpose of this work is to study how the provision of incentives is affected by publicly aggregate shocks that influence employment probabilities, I abstract from the cases in which workers receive different wage offers. Otherwise, if these offers could be monitored by the principal, the optimal unemployment insurance would certainly condition the payment of benefits upon this information.

and equal to zero otherwise. The employment history up to period t is denoted by e^t, a vector of $t+1$ status $\{e_\tau(s)\}_{\tau=0}^t$ containing all zeros if the agent is still unemployed at the beginning of period $t+1$ or t' zeros followed by $t-(t'-1)$ ones if the agent received a job offer at the end of period $t' < t$. Finally take $\mathbb{E}_t$ to be the set of all employment histories up to period t.

The unemployment insurance contract is a sequence of functions $\Gamma = \{\Gamma_t\}_{t=0}^\infty$ that specify benefit transfers and recommended search effort and satisfy the following:

$$\Gamma_t : \mathbb{S}^\infty \times \mathbb{E}_{t-1} \to \mathcal{A} \times \mathbb{R}_+ \tag{2}$$

$$\Gamma_t \text{ is } s^t\text{-measurable} \tag{3}$$

for every period $t = 0, 1, 2, \ldots$. The idea here is that, at each period, the principal first observes the aggregate shock and afterwards chooses benefits and effort levels according to the history of states of nature up to the current period[8]. Requiring the contract to depend on histories $\mathfrak{s} \in \mathbb{S}^\infty$ that are consistent with the so far observed history s^t of states of nature allows the principal to condition current transfers and recommended actions on the feasible future stories[9].

An expected discounted utility to the agent $V_0(\Gamma)$ and a cost, measured by the expected discounted value of net transfers to the agent, $C_0(\Gamma)$ are associated with each contract. I assume that unemployed workers act in a personally optimal way given the UI program, maximizing their expected discounted utility 1 in each period t by choosing the current search effort level $\hat{a}_t \in \mathcal{A}$. Therefore, $V_0(\Gamma)$ and $C_0(\Gamma)$ are calculated taking agents' rational responses into account. Given a level of initial promised discounted utility V for the agent, the optimal contract minimizes $C_0(\Gamma)$ subject to $V_0(\Gamma) = V$.

5.2 The Autarky Problem

As a useful benchmark, I first describe the fate of a given unemployed worker in the absence of an insurance agency. In addition to the zero wealth assumption, suppose that unemployed agents are unable to borrow[10]. Identical job opportunities turn employment into an absorbing state: switching jobs is suboptimal because workers will receive the same wage w and exert the same level of work effort (normalized to zero). Therefore, after becoming employed, workers set search effort levels to zero and enjoy utility $u(w)$ in every subsequent period. For any employed agent, let V^e denote his expected sum of discounted utilities. It is straightforward to show that V^e is given simply by:

$$V^e = \frac{u(w)}{1-\beta} \tag{4}$$

Note that it does not depend on the states of nature, since aggregate shocks do not affect wages, neither the probability of loosing the job, which, by assumption, is always equal to zero.

[8] Remember that $s^t = (s_0, s_1, \ldots, s_t)$.

[9] This assumption allows for a more general environment, in which aggregates shocks are not independent through time. If current aggregate shocks are somehow related to previous shocks, then Γ_t might depend in potentially complicated ways on future states of nature.

[10] According to Shavel and Weiss [9], "this assumption is often realistic; for reasons of moral hazard, unemployed individuals frequently find it difficult to borrow."

Let V_s^u be the expected present value of the utility for a worker who finds himself unemployed when the state of nature is $s \in \mathbb{S}$ and chooses the current period pair of consumption and search effort (c, a) optimally. Since unemployed agents have no wealth and are borrowing-constrained, they cannot choose any other consumption level but zero, so that their problem in recursive form is given by the following Bellman equation:

$$V_s^u = \max_{\hat{a} \in \mathcal{A}} \left\{ u(0) - a(s) + \beta \left[\phi(a(s), s) V^e + (1 - \phi(a(s), s)) \sum_{s \in \mathbb{S}} p(s) V_s^u \right] \right\} \tag{5}$$

Since there is no state variable in this problem, for each state of nature, there exist a time-invariant optimal contingent search intensity $a_{at}(s)$ and an associated contingent value for being unemployed, denoted by $V_{s,at}^u$. The expected utility of an agent who becomes unemployed today and will start looking for a job tomorrow is thus $\sum_{s \in \mathbb{S}} p(s) V_{s,at}^u$.

5.3 The Full-Information Case

If the agency can costlessly monitor agents' search effort, the UIP becomes that of efficient risk sharing in which the principal bears all the risk. The optimal program gives agents a constant consumption c during the entire spell of unemployment, and prescribes contingent effort levels $a(s)$ in each period, according to the current state of nature.

Let T be the period in which a particular agent becomes employed. In every period after T, since the insurance agency cannot control his consumption anymore, the worker will consume his wage w[11]. Moreover, because every job is identical, he will exert zero effort to find another job.

As stated before, at each period t, the employment probability is solely a function of the state s and the contingent search effort $a_t(s)$. Therefore, the worker's ex-ante expected probability of remaining unemployed in period $t+1$ is given by $\sum_{s \in \mathbb{S}} p(s)(1 - \phi(a_t(s), s))$. Consequently, an agent who becomes unemployed at period 0 faces an expected probability of $\prod_{\tau=0}^{t-1} [\sum_{s \in \mathbb{S}} p(s)(1 - \phi(a_\tau(s), s))]$ of remaining unemployed until period t.

The optimal contract specifies a string of consumption and effort levels contingent on the current state of nature: $\{(c_t(s), a_t(s))_{s \in \mathbb{S}}\}_{t=0}^{\infty}$. The principal's problem is to minimize the costs associated with the unemployment insurance scheme subject to providing agents with a prespecified discounted expected utility $V \geqslant \sum_{s \in \mathbb{S}} p(s) V_{s,at}^u$.

$$\min_{\{(c_t(s), a_t(s))_{s \in \mathbb{S}}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s \in \mathbb{S}} \beta^t p(s) \left\{ c_t(s) \prod_{\tau=0}^{t-1} \left[\sum_{s \in \mathbb{S}} p(s)(1 - \phi(a_\tau(s), s)) \right] \right\} \tag{6}$$

subject to

$$\begin{aligned} \sum_{t=0}^{\infty} \sum_{s \in \mathbb{S}} \beta^t p(s) \Bigg\{ & u(w) \left[1 - \prod_{\tau=0}^{t-1} \left(\sum_{s \in \mathbb{S}} p(s)(1 - \phi(a_\tau(s), s)) \right) \right] \\ & + \Big(u(c_t(s)) - a_t(s) \Big) \left[\prod_{\tau=0}^{t-1} \left(\sum_{s \in \mathbb{S}} p(s)(1 - \phi(a_\tau(s), s)) \right) \right] \Bigg\} \geqslant V \end{aligned} \tag{7}$$

[11] Allowing for borrowing and lending by employed agents does not alter the results, as long as the interest rate in the economy is $R = (1 + r) = \beta^{-1}$, so that workers are indifferent between saving today and consuming tomorrow.

Let θ be the multiplier on the promise-keeping constraint 7. At an interior solution, the first-order condition with respect to $c_t(s)$ is given by:

$$u'(c_t(s)) = \frac{1}{\theta} \tag{8}$$

which holds for all $s \in \mathbb{S}$ and $t = 0, 1, 2, \ldots$. Since $u(\cdot)$ is strictly concave, condition 8 states that the principal offers unemployed workers a constant level of consumption c in every period of their unemployment spell, not matter the state of nature. This result is fairly intuitive and usual in contract theory: assume, by way of contradiction, that $c_t(s) < c_{t+1}(s')$ for some t and $s, s' \in \mathbb{S}$ (the case $c_t(s) > c_{t+1}(s')$ is analogous). Consider a small reduction of ε in $c_{t+1}(s')$ and a concomitant increase in $c_t(s)$ of $\{\beta[\sum_{s\in\mathbb{S}}^{\infty}(1 - \phi(a_t(s), s))]p(s')/p(s)\}\varepsilon$. This change does not alter the expected discounted cost of the insurance scheme for the principal. However, since $u'(c_t) > u'(c_{t+1})$, the unemployed worker experiences an increase in his expected discounted utility. Hence, $c_t(s) = c$ for all t and for all $s \in \mathbb{S}$. In view of this result, the UIP under full-information can be rewritten as:

$$\min_{c,\{(a_t(s))_{s\in\mathbb{S}}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t c \left\{ \prod_{\tau=0}^{t-1} \left[\sum_{s\in\mathbb{S}} p(s)(1 - \phi(a_\tau(s), s)) \right] \right\}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u(w) + \sum_{t=0}^{\infty} \beta^t \left[\Big(u(c) - u(w) \Big) - a_t(s) \right] \left\{ \prod_{\tau=0}^{t-1} \left[\sum_{s\in\mathbb{S}} p(s)(1 - \phi(a_\tau(s), s)) \right] \right\} \geqslant V$$

The prescribed levels of effort $\{(a_t(s))_{s\in\mathbb{S}}\}_{t=0}^{\infty}$ will typically depend on the states of nature and the prespecified discounted expected utility V, but not on the duration of the unemployment spell. To see this more clearly, consider the recursive approach to the optimal insurance problem below.

Let $\mathbb{V}$ be the space of promised expected discounted utilities V. Denote by $C\colon \mathbb{V} \to \mathbb{R}$ the cost function associated with the contract, so that $C(V)$ is the expected discounted cost of giving an unemployed worker expected discounted utility V. Through an argument similar to that state in section **??** and according to Ljungqvist and Sargent [6], this cost function is strictly convex, because a higher V implies a lower marginal utility for the worker[12]

Given a promise V today, the insurance agency's problem is to choose contingent first-period consumption and search effort pairs $\{c(s), a(s)\}_{s\in\mathbb{S}}$ and contingent promised continuation values $\{V_s^u\}_{s\in\mathbb{S}}$, should the worker be unsuccessful in finding a job, in order to minimize the costs associated with V. The 3-tuples $\{c(s), a(s), V_s^u\}_{s\in\mathbb{S}}$ are chosen to be functions of V satisfying the following Bellman equation:

$$C(V) = \min_{\{c(s),a(s),V_s^u\}_{s\in\mathbb{S}}} \sum_{s\in\mathbb{S}} p(s) \Big\{ c(s) + \beta[1 - \phi(a(s), s)]C(V_s^u) \Big\} \tag{9}$$

[12]Despite being able to prove that the choice set given by the promise-keeping constraint 7 is convex, I could not demonstrate that the objective function is indeed convex. I conjecture however that, for the feasible values of consumption, search effort and continuation promises, the fixed-point of the Bellman equation is a strictly convex function.

subject to the following promise-keeping constraint

$$\sum_{s\in\mathbb{S}} p(s)\Big\{u(c(s)) - a(s) + \beta\Big[\phi(a(s),s)V^e + (1-\phi(a(s),s))V_s^u\Big]\Big\} \geqslant V \tag{10}$$

where V^e is given by 4, because of the assumption that, after finding a job, workers are beyond the reach of the unemployment insurance agency. The right side of the Bellman equation is attained by policy functions that are contingent on the states of nature. For every $s \in \mathbb{S}$, the insurance agency's optimal response is given by the contingent consumption, action and continuation value policy functions:

$$\begin{aligned} c\colon \mathbb{S}\times\mathbb{V} &\to \mathbb{R}_+ & a\colon \mathbb{S}\times\mathbb{V} &\to \mathcal{A} & V^u\colon \mathbb{S}\times\mathbb{V} &\to \mathbb{V} \\ (s,V) &\to c(s,V) & (s,V) &\to a(s,V) & (s,V) &\to V_s^u(V) \end{aligned}$$

As a matter of notational convenience, I refer to $\{c(s,V), a(s,V), V_s^u(V)\}_{s\in\mathbb{S}}$ simply as $\{c(s), a(s), V_s^u\}_{s\in\mathbb{S}}$.

Let θ be the multiplier on the promise-keeping constraint 10. At an interior solution, the first-order conditions with respect to $c(s)$, $a(s)$ and V_s^u are, respectively:

$$\theta = \frac{1}{u'(c(s))} \tag{11}$$

$$C(V_s^u) = \theta\left[\frac{1}{\beta\phi_a(a(s),s)} - (V^e - V_s^u)\right] \tag{12}$$

$$C'(V_s^u) = \theta \tag{13}$$

where $\phi_a(a(s),s)$ represents the derivative of the probability function ϕ with respect to the agent's effort level evaluated at $(a(s),s) \in \mathcal{A}\times\mathbb{S}$.

Condition 11 holding for all $s \in \mathbb{S}$ implies that the consumption level does not depend on the state of nature, so that $c(s) = c$, $\forall s \in \mathbb{S}$. The envelope condition $C'(V) = \theta$ and 13, in their turn, entail that $C'(V_s^u) = C'(V)$. Strict convexity of the cost function then implies that $V_s^u = V$, for all $s \in \mathbb{S}$. Applied repeatedly over time, $V_s^u = V$ makes the continuation values remain constant during the entire spell of unemployment and across states. Equation 11 determines c, and equation 12 determines $a(s)$, both as functions of the promised expected utility V. It is straightforward to see that the first-best contract fully smooths the unemployed agent's consumption across states of nature during the entire unemployment spell. However the consumption path is not smoothed across states of employment and unemployment unless $V = V^e$. Finally, because continuation values are independent of the length of the unemployment spell, there is only one level of search effort associated with each state of nature: given V, as long as the worker has not received a job offer, he will exert the same effort $a(s)$ whenever he faces state $s \in \mathbb{S}$.

5.4 The Incentive Problem

In the preceding section, I assumed that the principal could control both consumption and search effort level of an unemployed agent. According to Hopenhayn and Nicolini [3], unemployment insurance programs have been widely criticized for their perverse effects on incentives for reemployment. Therefore, extending the analysis to settings in which the effort level is not observable by the principal becomes quite natural.

Because searching for a job involves disutility, when the individual behavior towards this action cannot be monitored, the first-best contract cannot be implemented, as agents would like to shirk by exerting an effort level that is lower than that prescribed by the insurance contract. To see this, note first that, in this setup up, the insured can freely choose the effort level, despite the agency's recommendation. Consequently, at each period, after observing the aggregate shock, he selects the level of search effort that maximizes his expected utility given his assigned contingent consumption level $c(s)$ and continuation value V_s^u as follows:

$$a^\star(s) \in \underset{\hat{a}\in\mathcal{A}}{\operatorname{argmax}}\left\{u(c(s)) - \hat{a} + \beta\left[\phi(\hat{a}, s)V^e + (1-\phi(\hat{a}, s))\, V_s^u\right]\right\} \tag{14}$$

According to 14, the level of search effort chosen by the unemployed worker depends on the sign of $V^e - V_s^u$. Moreover, $a^\star(s)$ will be strictly greater than zero if and only if $V^e > V_s^u$. The associated first-order condition is:

$$\beta\phi_a(a^\star(s), s)(V^e - V_s^u) \geqslant 1 \tag{15}$$

Since ϕ is strictly concave with respect to the agent's effort, as long as $a^\star(s)$ is an interior point of $\mathcal{A}$, a necessary and sufficient condition for it to be the optimal search effort is that the restriction above holds with equality.

It suffices to prove now that, for the first-best contingent effort levels $a(s)$ and prescribed continuation values V_s^u, the individual marginal cost associated with job searching surpasses the marginal benefits. In fact, as long as the insurance scheme is associated with costs, so that $C(V_s^u) > 0$, condition 12 implies that $\beta\phi_a(a(s), s)(V^e - V_s^u) < 1$[13], which violates 15. If the worker is free to choose his effort level, he will fulfill 15 at equality. Starting from the level of effort prescribed by the social insurance scheme, he will do so by lowering $a(s)$, increasing his job search marginal benefit, until it equals the associated marginal cost. The impossibility of implementation of first-best contracts in an environment where search effort is private information comes thus from the fact that effort involves disutility and the worker does not take into account the cost of the insurance scheme. Consequently he chooses contingent search effort levels $a^\star(s)$ below the socially optimal ones.

Additionally, it is possible to show that the prescribed first-best effort levels, despite not being individually optimal, are still lower than than their autarkical counterparts. From 12, $\beta\phi_a(a(s), s)(V^e - V_s^u) > 1$. By condition 15, however, $\beta\phi(a_{at}(s), s)(V^e - \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u) = 1$. It was proved in section 5.3 that $V = V_s^u$, for all $s \in \mathbb{S}$. Therefore, if $V > \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$,

$$\beta\phi_a(a(s), s)(V^e - V) > 1 = \beta\phi_a(a_{at}(s), s)\left(V^e - \sum\nolimits_{s\in\mathbb{S}} p(s)V_{s,at}^u\right)$$

Since $(V^e - V) < (V^e - \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u)$, it must be that $\phi_a(a(s), s) > \phi_a(a_{at}(s), s)$. By strict concavity of ϕ, $a(s) < a_{at}(s)$, for all $s \in \mathbb{S}$. The scheme through which $V > \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$ is delivered when effort is observable becomes quite clear now: the agency smooths the agent's consumption path by providing him with a strictly positive level c in each period, for as long as he stays unemployed, and decreases his search effort from $a_{at}(s)$ to $a(s)$.

[13]Notice that, by condition 11, the multiplier θ is equal to the inverse of the marginal utility of consumption and hence it must because strictly positive.

5.5 The Possibility of a Breach of Contract

So far, it has been assumed that unemployed agents simply accept unemployment insurance whenever it is available. It has also been shown that, when search effort is private information, they might have incentives to deviate from the first-best contract. There may be however special circumstances in which agents might opt for declining an insurance offer or not withdrawing from an already signed agreement.

Consider an agent who becomes unemployed today. Before the end of this period, the insurance agency offers him a contract. If the worker declines the offer and starts looking for a job tomorrow, his expected discounted utility in autarky at the beginning of the next period, prior to the occurrence of the aggregate shock, will be given by $\sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$, which is equal to:

$$\sum_{s\in\mathbb{S}} p(s)\left\{u(0) - a_{at}(s) + \beta\left[\phi(a_{at}(s), s)V^e + (1-\phi(a_{at}(s), s))\sum\nolimits_{s\in\mathbb{S}} p(s)V^u_{s,\,at}\right]\right\}$$

No worker will be willing to accept an insurance contract if the promised discounted utility V is less than $\sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$[14]. Therefore, $\sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$ is the minimum expected utility the insurance agency might promise and is thus the infimum of the set of promised utilities $\mathbb{V}$.

By assumption, insureds are free to abandon the contract at any moment in any period during the insurance agreement, after which they live in autarky for the rest of their unemployment spell. Whether their search effort levels are observable by the agency or not, the agency cannot intervene in this decision. Therefore, if an agent decides to walk off the contract, he will do so only after observing the aggregate shock and receiving his assigned contingent consumption for the current period. Instead of exerting the recommended search effort level $a(s)$ and being entitled to the contingent discounted expected utility V^u_s, the worker will consume $c(s)$ and select a new level of search effort to maximize his new discounted expected utility as follows:

$$a \in \operatorname*{argmax}_{\hat{a}\in\mathcal{A}} \left\{u(c(s)) - \hat{a} + \beta\left[\phi(\hat{a}, s)V^e + (1-\phi(\hat{a}, s))\sum\nolimits_{s\in\mathbb{S}} p(s)V^u_{s,\,at}\right]\right\} \tag{16}$$

By the first order condition associated with the problem above, the worker sets $a = a_{at}(s)$.

Lemma 1 below states the necessary and sufficient condition to deter the agent from abandoning the contract.

Lemma 1. *In each state $s \in \mathbb{S}$, the agency deters unemployed workers from abandoning the contract if, and only if, $V^u_s \geqslant \sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$.*

Proof. See appendix A. □

5.6 Unobservable Search Effort

Following Shavel and Weiss [9] and Hopenhayn and Nicolini [3], I now assume that the unemployment insurance agency cannot observe or enforce agents' search effort, though it can observe and control their consumption. Since the worker is free to choose a, the

[14]Notice in addition that, because in an insurance agreement an agent's assigned consumption cannot be lower than zero, $V < \sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$ implies assigned contingent efforts $a(s)$ higher than their autarkical counterparts $a_{at}(s)$.

restriction that, in every period of the unemployment spell and whatever the state of nature might be, he optimally chooses the contingent search effort $a(s)$ prescribed by the contract must nbe added to the UI problem. A modified version of condition 14 is thus needed:

$$a(s) \in \underset{\hat{a}\in\mathcal{A}}{\operatorname{argmax}}\left\{u(c(s)) - \hat{a} + \beta\left[\phi(\hat{a}, s)V^e + (1 - \phi(\hat{a}, s))\, V_s^u\right]\right\} \tag{17}$$

This is the agent's incentive-compatibility constraint, which states that the prescribed contingent search effort $a(s)$ is the optimal level of effort for the agent. As already argued, a necessary and sufficient condition for $a(s)$ to be optimal is given by the associated first-order condition of the incentive constraint:

$$\beta\phi_a(a(s), s)(V^e - V_s^u) \geqslant 1 \tag{18}$$

which must hold with equality whenever $a(s)$ belongs to the interior of $\mathcal{A}$.

Again, let $C(V)$ be the expected discounted cost for the principal associated with the optimal contract when the agent is unemployed with a continuation value V. Then $C(V)$ must satisfy the following program[15]:

$$C(V) = \min_{\{c(s),a(s),V_s^u\}_{s\in\mathbb{S}}} \sum_{s\in\mathbb{S}} p(s)\left\{c(s) + \beta[1 - \phi(a(s), s)]C(V_s^u)\right\} \tag{9}$$

subject to the promise-keeping constraint

$$\sum_{s\in\mathbb{S}} p(s)\left\{u(c(s)) - a(s) + \beta\left[\phi(a(s), s)V^e + (1 - \phi(a(s), s))V_s^u\right]\right\} \geqslant V \tag{10}$$

and the incentive-compatibility constraint's first-order condition:

$$\beta\phi_a(a(s), s)(V^e - V_s^u) \geqslant 1, \text{ for all } s \in \mathbb{S} \tag{18}$$

Similarly to the first-best problem, because the associated constraints are not linear, they generally do not define a convex set. For this reason, it becomes hard to provide conditions under which the solution to the dynamic programming problem results in a convex function. Although the introduction of lotteries can convexify the constraint set, a common finding is that optimal plans do not involve lotteries[16], because convexity of the constraint set is a sufficient but not necessary condition for convexity of the cost function. Therefore, there may be a class of problems for which lotteries are never part of the optimal contract. Following Hopenhayn and Nicolini [3], I proceed under the assumption of strict convexity of $C(V)$. In fact, in all my numerical computations in section 6.2, the function C turned out to be strictly convex, making lotteries redundant. To derive some general properties of the optimal contracts, I disregard the use of lotteries and focus on the optimal program defined above.

[15]For the sake of simplicity, I omit here the restrictions on the contingent continuation values V_s^u given by Lemma 1.

[16]Phelan and Townsend [7] have show that the use of lotteries over control variables turns the program above into a linear programming problem, which can be solved numerically. In practice, however, they have found that lotteries are often redundant, in the sense that most of the probabilities associated with the randomization of contracts are zero, and a few are one.

For each $s \in \mathbb{S}$, let η_s be the multiplier on constraint 18. Once more, let θ denote the multiplier on the promise-keeping constraint 10. At an interior solution, for each $s \in \mathbb{S}$, the first-order conditions with respect to $c(s)$, $a(s)$ and V_s^u, respectively, are:

$$\theta = \frac{1}{u'(c(s))} \tag{19}$$

$$\begin{aligned} C(V_s^u) &= \theta\left[\frac{1}{\beta\phi_a(a(s),s)} - (V^e - V_s^u)\right] - \frac{\eta_s}{p(s)}\frac{\phi_{aa}(a(s),s)}{\phi_a(a(s),s)}(V^e - V_s^u) \\ &= -\frac{\eta_s}{p(s)}\frac{\phi_{aa}(a(s),s)}{\phi_a(a(s),s)}(V^e - V_s^u) \end{aligned} \tag{20}$$

$$C'(V_s^u) = \theta - \frac{\eta_s}{p(s)}\frac{\phi_a(a(s),s)}{1-\phi(a(s),s)} \tag{21}$$

where the second equality of in equation 20 follows from the fact that the incentive-incentive constraint is binding for $a(s) > 0$. Furthermore, the envelope condition is exactly the same found in the first-best program: $C'(V) = \theta$.

Lemma 2 below shows that the incentive-compatibility constraints 18 are indeed binding and that the associated multipliers η_s are thus strictly positive.

Lemma 2. *For every $s \in \mathbb{S}$, the multiplier η_s associated with the incentive-compatibility constraint 18 is strictly positive.*

Proof. Assume, by way of contradiction, that for some $s \in \mathbb{S}$, condition 18 is not binding, so that $\eta_s = 0$. Condition 20 then implies that

$$C(V_s^u) = \theta\left[\frac{1}{\beta\phi_a(a(s),s)} - (V^e - V_s^u)\right] < 0$$

. However, as long as the insurance scheme is associated with costs, $C(V_s^u)$ cannot be negative for any $V_s^u \in \mathbb{V}$. In fact, since liquid transfers in terms of consumption good between the agency and the unemplyoed worker cannot be negative[17], the lowest cost possible for an insurance contract is zero. This level is attained when the agency promises the worker $\sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$. In this case, it can replicate autarky and set consumption transfers equal to zero for each period and each state $s \in \mathbb{S}$. Moreover, by strict convexity of the cost function, if $\eta_s = 0$, $V_s^u < \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$ and, by Lemma 1, the unemployed worker withdraws from the contract. Suppose now that $\eta_s < 0$. Once more, by strict concavity of the probability function ϕ and condition 18, $C(V_s^u) < 0$. This contradiction completes the proof. □

Equation 21 and the envelope condition allow us to conclude that $C'(V_s^u) < C'(V)$. Strict convexity of C, in its turn, implies that $V_s^u < V$, for all $s \in \mathbb{S}$. Therefore, whatever the state of nature might be tomorrow, if the agent remains unemployed, his assigned continuation value will be lower than his initial promise V. Condition 19 and strict concavity of the utility function jointly imply that the consumption of the unemployed agent must decrease over time for every state s. This result is consistent with the findings of Shavel and Weiss [9] and Hopenhayn and Nicolini [3]: to provide agents with the proper

[17]Remember that, by assumption, unemployed workers have no wealth and are insured constrained. Therefore, the expected discounted sum of all the transfers between the insurance agency and the agent cannot be negative.

incentives, contingent consumption transfers must decrease as the spell of unemployment lengthens. Another important feature of the problem is the equality of consumption across states. This finding is fairly intuitive: since the state of nature is determined exogenously, the worker is risk averse and the principal is risk-neutral, it is optimal to fully insure the agent against aggregate shocks. Therefore, by condition 19, the agency equates the contingent marginal utilities of unemployed workers.

Especially significant is the path of search effort values $a(s)$ as V_s^u falls. Note first that, if tomorrow's state of nature happens to be the same as the one verified today, then, because tomorrow's promised value is $V_s^u < V$, condition 18 holding with equality implies that next period's search effort will be higher. However, it is not possible to assert that search effort values always rise as continuation values V_s^u decrease. For instance, assume that yesterday's state was s, whereas today's state of nature is s', such that $s' \succcurlyeq s$. It is possible that the value of $a(s)$ that satisfies $\beta\phi_a(a(s), s)(V^e - V_s^u) = 1$ is higher than the value $a(s')$ that satisfies this period's incentive-compatibility constraint, $\beta\phi_a(a(s'), s')(V^e - V_{s'}^u) = 1$, as the example in section 6.2 will show. In other words, one can imagine probability functions ϕ, for which the gains in marginal probabilities from one state of nature to another more than compensates the loss in continuation values, so that the contingent search effort $a(s')$ might in fact fall from one period to the next. Therefore, search effort paths might not be monotone increasing sequences.

With regard to contingent continuation values, it is not possible to determine whether or not their differ according to the states of nature without further and perhaps fairly strong functional form assumptions. In appendix B, I show that, for a special family of functions, it is optimal to set $V_s^u = V^u$, for all $s \in \mathbb{S}$, if the associated search effort levels $a(s)$ are different from zero. In particular, the functional form used to compute the numerical results of section 6.2 satisfies the necessary conditions.

6 Quantitative Analysis

In this section, a parameterized version of the model of unemployment insurance under aggregate shocks is solved numerically following the work of Hopenhayn and Nicolini [3].

6.1 Calibration

The parameters associated with the model are those involving the utility function u, the hazard functions $\phi(\cdot, s)$, the discount factor β, the number of states of natures and the probabilities of observing each one of them, $p(s)$. Whenever possible, the functional forms adopted were those presented in Hopenhayn and Nicolini's [3] work.

The functional form chosen for the utility function was that of a constant relative risk aversion (CRRA) function: $u(c) = c^{1-\sigma}/(1-\sigma)$. The parameters β and σ were defined solely to allow for the analysis of the impact of different aggregate shocks over the insurance mechanism. Therefore, I set $\beta = 0.975$. If one considers a quarterly frequency, this value corresponds to a yearly discount factor of approximately 0.9. The elasticity of substitution σ was set to 3/4, above the value of 1/2 that Hopenhayn and Nicolini's [3] work employed to calibrate the model to weekly data. Although a higher value of σ implies a strong degree of risk aversion, augmenting the relevance of the insurance mechanism, it should be noted however that the value of the coefficient of risk aversion does not change the qualitative properties of the model. The same can be argued for the choice of the

discount factor. In fact, additional computations were made for different values of these parameters, but no qualitative change was verified[18].

To study the impact of aggregate shocks over the insurance mechanism, I set the cardinality of S equal to 3. This value introduces sufficient diversity in the model to permit thorough examinations of the trade-off between incentives and insurance. Because attention is focused on the role of contingent continuation values as instruments of provision of incentives, I set $p(s) = 1/\#\mathbb{S}$, for all $s \in \mathbb{S}$. Nevertheless, I conjecture that, because shocks are i.i.d, allowing for distinct probabilities would not change qualitative properties of the model.

The hazard rates of the model were set in an arbitrary manner: the purpose behind their choices was not to fit data to a particular frequency, but rather to generate as much contrast among the states of nature as possible. The hazard function used was that of an exponential distribution with parameter s, multiplicative of the search effort. Therefore, for each state of nature, there corresponds a scalar s that captures the impact of the aggregate shock over employment probabilities. In order to assign values to these parameters, I proceed again as in Hopenhayn and Nicolini's [3]. Since the worker has no other source of income - or borrowing and lending - except for the insurance mechanism, his consumption in autarky is zero during the unemployment spell, and equal to w, once employed. The wage w was in its turn normalized to 100. The discounted utility of a employed worker is thus $V^e = u(100)/(1-\beta)$. Letting $V^u_{s,at}$ represent the expected discounted utility of an unemployed worker facing state $s \in \mathbb{S}$ without a insurance contract, the dynamic programming system of equations for the optimal search problem is therefore:

$$\begin{cases} V^u_{s_1,at} = \max\limits_{\hat{a}\in\mathcal{A}} \left[u(0) - \hat{a} + \beta \left(\phi(\hat{a}, s_1)V^e - (1-\phi(\hat{a}, s_1)) \sum\limits_{s\in\mathbb{S}} p(s)V^u_{s,at} \right) \right] \\ V^u_{s_2,at} = \max\limits_{\hat{a}\in\mathcal{A}} \left[u(0) - \hat{a} + \beta \left(\phi(\hat{a}, s_2)V^e - (1-\phi(\hat{a}, s_2)) \sum\limits_{s\in\mathbb{S}} p(s)V^u_{s,at} \right) \right] \\ V^u_{s_3,at} = \max\limits_{\hat{a}\in\mathcal{A}} \left[u(0) - \hat{a} + \beta \left(\phi(\hat{a}, s_3)V^e - (1-\phi(\hat{a}, s_3)) \sum\limits_{s\in\mathbb{S}} p(s)V^u_{s,at} \right) \right] \end{cases}$$

The stationary contingent values $a_{at}(s)$ for the search effort levels are determined by the first order conditions:

$$\beta\phi_a(a_{at}(s), s) \left(V^e - \sum\nolimits_{s\in\mathbb{S}} p(s)V^u_{s,\,at} \right) \geqslant 1, \text{ for all s}\in \mathbb{S} \tag{15}$$

which hold with equality whenever $a_{at}(s) \in int(\mathcal{A})$. The solution to the problem above implies stationary hazard rates $\phi(a_{at}(s), s)$. Additionally, for the range of values considered, these rates are increasing functions of the parameters s. Therefore, I chose s so that the autarkical search effort values would imply the following hazard rates: 0.40, 0.25 and 0.05.

The following step consisted in writing the script for the agency problem when effort is not observable. I started by determining the range of $\mathbb{V}$. Note that, for each state

[18] I did not consider $\sigma \geqslant 1$, as this would return utility functions that are not well defined at zero, while boundedness from below constitutes a necessary condition to guarantee the existence of a solution to the autarky problem.

$s \in \mathbb{S}$, there exists a level of effort $V^u_{s,th}$ so that, for every $V \geqslant V^u_{s,th}$, the worker simply sets $a(s) = 0$. I called these levels "threshold continuation values" and they are given by:

$$V^u_{s,th} = V^e - [\beta\phi_a(0,s)]^{-1}$$

The upper bound on contingent continuation values $V^u(s)$ - and, consequently, on the set of feasible initial promises $\mathbb{V}$ - was set equal to $\max_{s\in\mathbb{S}}\{V^u_{s,th}\}$. From this level on, the worker simply choses to stay unemployed, independently of the current state of nature s, and the insurance mechanism looses its ability to provide incentives for searching.

To formulate the Bellman equation associated with the second-best UI program numerically, I used constraints 10 and 18 holding with equality to write $c(s)$ and $a(s)$ both as functions of V and V^u_s, for all $s \in \mathbb{S}$. Solving 18 for $a(s)$ and using the assumed functional form for $\phi(a,s)$ leads to:

$$a(s) = \max\left\{0, \frac{log[\beta s(V^e - V^u_s)]}{s}\right\} \tag{22}$$

Using 10 and the first-order condition of $c(s)$ 19, one can express the promise-keeping constraint as:

$$u(c) \geqslant V - \sum_{s\in\mathbb{S}} p(s)\left\{-a(s) + \beta\left[\phi(a(s),s)V^e + (1-\phi(a(s),s))V^u_s\right]\right\}$$

For the considered utility function, whenever the right side of the inequality above is negative, the promise-keeping constraint is not binding and can be satisfied with $c = 0$. Consequently, consumption was written as:

$$c = u^{-1}\left(\max\left\{0, V + \sum_{s\in\mathbb{S}} p(s)\left\{a(s) - \beta\left[\phi(a(s),s)(V^e - V^u_s) + V^u_s\right]\right\}\right\}\right) \tag{23}$$

Taken together, formulas 22 and 23 enabled the elements of the list $\{c, (a(s))_{s\in\mathbb{S}}\}$ to be expressed as functions of V and the continuation values $\{V^u_s\}_{s\in\mathbb{S}}$. Using thus these functions, the Bellman equation in $C(V)$ was written as:

$$C(V) = \min_{\{V^u_s\}_{s\in\mathbb{S}}}\left\{c + \beta\sum_{s\in\mathbb{S}} p(s)[1-\phi(a(s),s)]C(V^u_s)\right\} \tag{24}$$

where c and $a(s)$ where given by 23 and 22, respectively.

6.2 Numerical Results

Using the parameter values reported in the previous section, I computed the optimal contingent continuation values, effort levels and consumption transfers for 150 distinct initial promises. Before proceeding to the analysis of the numerical results, I first present the threshold continuation values, $V^u_{s,th}$:

Table 2 on page 40 displays the consumption and the contingent continuation values V^u_s for fifty initial promises V, as well as the differences between contingent continuation promises associated with the same initial expected discounted utility. Promises V are displayed in a ascending order. Notice first that, regardless of the current state of nature,

Table 1: Threshold Continuation Values

$V^u_{s_1,th}$	$V^u_{s_2,th}$	$V^u_{s_3,th}$
392.5698	364.2212	326.4230

contingent continuation values are smaller than initial promises, as expected. Additionally, the pair of rows $(12, 13)$ and $(33, 34)$ evidence how this particular UI mechanism responds when, for one or more state $s \in \mathbb{S}$, contingent continuation promises V^u_s exceed the corresponding threshold values.

Above line 13, continuation values are fairly the same. If the initial promise V is high enough though, it is optimal for the principal to offer a contingent continuation value higher than state s_3 threshold value $V^u_{s_3,th}$. Hence, although the worker will still exert search effort tomorrow if the state of nature is s_1 or s_2, he will not do so if s_3 is verified. As a consequence, the principal finds it optimal to differentiate contingent continuation values, offering lower continuation promises if next period's shock s belongs to $\{s_1, s_2\}$. Moreover, as long as the these contingent promises are lower than $V^u_{s_2,th}$, $V^u_{s_1} = V^u_{s_2}$, which is verified until row 33. Despite the slight difference between the threshold value $V^u_{s_2,th}$ reported on table 1 and the ones presented on row 33, previous interpretation still holds: once initial promise is high enough so that contingent continuation values exceed $V^u_{s_2,th}$, the principal differentiates $V^u_{s_1}$ from $V^u_{s_2}$. In this case, it is optimal to set $V^u_{s_2} = V^u_{s_3} > V^u_{s_1}$.

Finally, observe that contingent continuation values are all smaller than $V^u_{s_1,th}$, since this threshold is the higher initial promise considered. On the other hand, if the principal set initial promise V higher than $V^u_{s_1,th}$, the worker would simply choose not to work in the subsequent periods for as long as, given current states of nature s, his contingent continuation promise were $V^u_s \geqslant V^u_{s,th},\ s \in \mathbb{S}$.

6.3 Optimal Paths

I consider now one fifty-period random sequence of aggregate shocks. Starting from a initial promise V, I interpolate the data to find the optimal sequences of continuation values, consumption and search effort levels.

Table 3 displays the value of the initial promise V and, for each period, a list containing the following variables: current period's state of nature, consumption transfer, contingent continuation promises and recommended search effort level. The interpretation is as follows. At period $t = 0$, the principal offers V to the unemployed worker. Associated with this promise are tomorrow's benefit transfer c, contingent search effort levels $a(s)$ and contingent continuation values V^u_s. After accepting the contract, at period $t = 1$, the worker begins searching for a job and receives his first transfer c from the agency. Given the current state of nature s_2, his recommended search effort is $a(s_2) = 12.1873$, while his contingent continuation value amounts to $V^u_{s_2} = 351.1536$. Notice that this promise is lower than V. The worker arrives at period $t = 2$ with his former promise $V^u_{s_2} = 351.1536$. The economy experiences a more severe aggregate shock and faces state s_3 ($s_2 \succcurlyeq s_3$). From this point on, the distinctive features of the model added by the introduction of aggregate shocks become apparent. Given that worker's contingent continuation value $V^u_{s_3}$ is higher than the threshold value $V^u_{s_3,th}$, the principal finds it optimal to recommend zero search effort level. Moreover, precisely because $a(s_3) = 0$, it is also optimal to maintain the contingent continuation promise at the same level as that of current period's initial promise: when the principal recommends zero effort, the

incentive problem disappears, and so does the associated "punishment" in continuation values.

As long as $s = s_3$ and $V^u_{s_3} > V^u_{s_3,th}$, the principal recommends $a(s_3) = 0$ and offers contingent continuation values equal to initial promises. However, whenever s_1 or s_2 is verified, the incentive problem reappears. Since the marginal benefits of searching when facing these states are higher, the contract induces unemployed worker's search effort by reducing their discounted expected utilities in case they fail to find a job, just as in Shavel and Weiss's [9] and Hopenhayn and Nicolini's [3] models.

Figure 1 on page 42 shows the optimal path of contingent continuation values. Although it is not strictly decreasing, due to the suppression of the incentive mechanism when $s = s_3$ as discussed above, it is still a monotone decreasing sequence. Figure 2 depicts the unemployed worker's consumption path given the same particular sequence of random aggregate shocks and can be interpreted analogously. Whenever V^u_s is held constant, so is the agent's benefit transfer. Moreover, as argumented in section 5.6, c decreases whenever the worker receives a lower continuation value.

The unemployed worker's search effort path is displayed on figure 3 on page 43. To facilitate interpretation, search effort levels associated with different states of nature are depicted in different colors. Its becomes thus clear that, whithin each state $s \in \{s_1, s_2\}$, search effort levels are increasing, as a consequence of the decreasing sequence of continuation promises. However, due to the differences in the marginal benefits of searching across states, recommended effort levels are consistently higher in state s_1. Again, because $V^u_{s_3} > V^u_{s_3,th}$ during all the fifty-period spell of unemployment, $a(s) = 0$ when $s = s_3$.

7 Conclusion

This work provides an overview of some of the main articles on the optimal design of UI systems and an attempt to extend the analysis to economic settings subject to multiple aggregate shocks. By studying some of the most influential models of a single unemployment spell in which search effort is private information, the main properties of UI contracts were derived. Next, this survey approached the technical difficulties introduced by relaxing the assumption of perfect monitoring of agent's savings by the principal, notably the lack of computationally feasible procedures to attack the recursive problem and the possibility of failure of the first-order approach because of the complementary nature of shirking and saving.

The empirical evidence of reduction in labor supply caused by social insurance programs - and, in particular, by UI benefits - was briefly discussed, taking into consideration that many unemployed individuals have limited liquidity and exhibit sensitivity of consumption to cash-on-hand. It was argued that UI benefits affect search intensity not only by the traditional moral hazard channel, but also through a liquidity effect, seen as a socially beneficial response to the correction of the credit and insurance market failures. The paper also discussed the implications of multiple spells of unemployment to the characterization of the optimal contracts. Although most of the properties derived for the single unemployment spell case remain valid, additional restrictions must be made in order to avoid a loophole in the UI program.

After presenting some of main results in the literature of unemployment insurance, a model along the lines of Shavell and Weiss [9] extended the analysis to settings in

which the economy faces an aggregate shock at each period. For the i.i.d distribution of shocks considered, most of the results of previous works remain valid. Within an optimal path, search effort levels however present strong variation, due to differences in marginal benefits of searching across states of nature.

Appendices

A *Proof of Lemma 1*

I begin by abstracting the possibility of monitoring of agents' search effort and conclude by showing that, when effort is observable, the principal never promises $V_s^u < \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$.

Suppose that, after observing current period's aggregate shock, the principal assigns a given agent contingent consumption $c(s)$ and continuation value V_s^u and recommends search effort level $a(s)$. If the unemployed worker honors the contract, his expected utility will be:

$$u(c(s)) - a(s) + \beta\Big[\phi(a(s),s)V^e + (1-\phi(a(s),s))V_s^u\Big]$$

If search effort is observable, the agent can either honor the contract or withdraw from it. On the other hand, if effort is private information, he can additionally deviate from the agreement by choosing a different effort level. I show that, even in this case, if $V < \sum_{s\in\mathbb{S}} p(s)V_{s,at}^u$, it is optimal for the worker to break the unemployment insurance contract.

Assume that search effort is known only to the unemployed agent. Let $a^\star$ be the level of effort that maximizes the worker's utility in state s, so that:

$$a^\star \in \underset{\hat{a}\in\mathcal{A}}{\operatorname{argmax}}\Big\{u(c(s)) - \hat{a} + \beta\Big[\phi(\hat{a},s)V^e + (1-\phi(\hat{a},s))V_s^u\Big]\Big\}$$

If $a(s) \neq a^\star$, the worker might have incentives to deviate, since:

$$\begin{aligned} u(c(s)) - a^\star + \beta\Big[\phi(a^\star,s)V^e + (1-\phi(a^\star,s))V_s^u\Big] \geqslant \\ u(c(s)) - a(s) + \beta\Big[\phi(a(s),s)V^e + (1-\phi(a(s),s))V_s^u\Big] \end{aligned} \tag{25}$$

Alternatively, if the agent decides to withdraw from the contract, he optimally sets $a = a_{at}(s)$ and enjoys utility:

$$u(c(s)) - a_{at}(s) + \beta\Big[\phi(a_{at}(s),s)V^e + (1-\phi(a_{at}(s),s))\sum_{s\in\mathbb{S}} p(s)V_{s,at}^u\Big]$$

Of course, the unemployed worker will only take such measure if his expected utility in autarky is strictly higher than the maximum utility he can enjoy by deviating and choosing his effort level optimally under the insurance contract[19]:

$$\begin{aligned} u(c(s)) - a_{at}(s) + \beta\Big[\phi(a_{at}(s),s)V^e + (1-\phi(a_{at}(s),s))\sum_{s\in\mathbb{S}} p(s)V_{s,at}^u\Big] > \\ u(c(s)) - a^\star(s) + \beta\Big[\phi(a^\star(s),s)V^e + (1-\phi(a^\star(s),s))V_s^u\Big] \end{aligned} \tag{26}$$

[19] I assume that, whenever the worker is indifferent between breaking the contract and honoring it - even if it means to deviate and choose a different effort level - he prefers to remain insured.

Conditions 25 and 26 together imply in particular that, if the worker is better off abandoning the agreement, then:

$$u(c(s)) - a_{at}(s) + \beta\left[\phi(a_{at}(s), s)V^e + (1 - \phi(a_{at}(s), s))\sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}\right] >$$
$$u(c(s)) - a_{at}(s) + \beta\left[\phi(a_{at}(s), s)V^e + (1 - \phi(a_{at}(s), s))V^u_s\right]$$

which is equivalent to $\sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at} > V^u_s$. Consequently, the insurance agency deters unemployed workers from breaching the contract if, and only if, for all $s \in \mathbb{S}$, the assigned contingent continuation value $V^u_s \geqslant \sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$.

With regard to the setup in which the principal can observe agents' search effort, it was shown that $V^u_s = V$, for all $s \in \mathbb{S}$. The principal will only promise contingent continuation value V^u_s if he has already promised initial discounted expected utility $V = V^u_s$. Notice however that, if $V < \sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$, the worker will not accept the insurance contract. Therefore, the principal never promises $V = V^u_s < \sum_{s\in\mathbb{S}} p(s)V^u_{s,\,at}$.

B *Equal Contingent Continuation Values*

I find now a special class of probability functions ϕ for which the contingent continuation values V^u_s are equal, as long as, for each state $s \in \mathbb{S}$, the search effort values $a(s)$ associated with these discounted expected utilities are different from 0.

Consider again the UI program when search effort is unobservable:

$$C(V) = \min_{\{c(s),a(s),V^u_s\}_{s\in\mathbb{S}}} \sum_{s\in\mathbb{S}} p(s)\Big\{c(s) + \beta[1 - \phi(a(s), s)]C(V^u_s)\Big\} \tag{9}$$

subject to the promise-keeping constraint

$$\sum_{s\in\mathbb{S}} p(s)\Big\{u(c(s)) - a(s) + \beta\Big[\phi(a(s), s)V^e + (1 - \phi(a(s), s))V^u_s\Big]\Big\} \geqslant V \tag{10}$$

and the incentive-compatibility constraint's first-order condition:

$$\beta\phi_a(a(s), s)(V^e - V^u_s) \geqslant 1, \text{ for all } s \in \mathbb{S} \tag{18}$$

Assume that all continuation values V^u_s are such that their associated effort levels belong to the interior of $\mathcal{A}$[20]. Condition 18 holding with equality implies that:

$$\frac{da}{dV^u_s} = \frac{\phi_a(a(s), s)}{\phi_{aa}(a(s), s)}, \text{ for all } s \in \mathbb{S} \tag{27}$$

Consider thus a small change ε in $V^u_{s_i}$ and a concomitant increase in $V^u_{s_j}$ which preserves the equality of the promise-keeping constraint, for $s_i \neq s_j$. Differentiating 10, we get:

[20] If we consider continuation values for which the worker optimally sets $a(s) = 0$, the following calculations won't hold. The worker's expected utility in the associated states will simply be $u(c(s)) + \beta V^u_s$, whereas the cost of the principal will be given by $c(s) + \beta C(V^u_s)$.

$$p(s_i)\left\{-\frac{da}{dV^u_{s_i}}+\beta\left[\phi_a(a(s_i),s_i)\frac{da}{dV^u_{s_i}}(V^e-V^u_{s_i})+(1-\phi(a(s_i),s_i))dV^u_{s_i}\right]\right\}+$$
$$p(s_j)\left\{-\frac{da}{dV^u_{s_j}}+\beta\left[\phi_a(a(s_j),s_j)\frac{da}{dV^u_{s_j}}(V^e-V^u_{s_j})+(1-\phi(a(s_j),s_j))dV^u_{s_j}\right]\right\}=0$$

Rearranging the terms of the equation above, we arrive at:

$$p(s_i)\left\{-\left[1-\beta\phi_a(a(s_i),s_i)(V^e-V^u_{s_i})\right]\frac{da}{dV^u_{s_i}}+(1-\phi_(a(s_i),s_i))dV^u_{s_i}\right\}+$$
$$p(s_j)\left\{-\left[1-\beta\phi_a(a(s_j),s_j)(V^e-V^u_{s_j})\right]\frac{da}{dV^u_{s_j}}+(1-\phi_(a(s_j),s_j))dV^u_{s_j}\right\}=0$$

Incentive-compatibility constraints holding with equality imply that:

$$p(s_i)(1-\phi(a(s_i),s_i))dV^u_{s_i}+p(s_j)(1-\phi(a(s_j),s_j))dV^u_{s_j}=0$$

Therefore:

$$\frac{dV^u_{s_j}}{dV^u_{s_i}}=-\frac{p(s_i)(1-\phi(a(s_i),s_i))}{p(s_j)(1-\phi(a(s_j),s_j))} \tag{28}$$

If one assumes that $\{c(s),a(s),V^u_s\}_{s\in\mathbb{S}}$ is optimal, the principal must be indifferent between implementing this change in the contingent continuation values, $V^u_{s_i}$ and $V^u_{s_j}$, and carrying out the former contract. Hence, the change in the expected cost associated with this policy must be zero. Differentiating the cost function, we obtain:

$$\begin{aligned}dC(V)=&-p(s_i)\phi_a(a(s_i),s_i)C(V^u_{s_i})da_{s_i}+p(s_i)(1-\phi(a(s_i),s_i))C'(V^u_{s_i})dV^u_{s_i}\\&-p(s_j)\phi_a(a(s_j),s_j)C(V^u_{s_j})da_{s_j}+p(s_j)(1-\phi(a(s_j),s_j))C'(V^u_{s_j})dV^u_{s_j}=0\end{aligned}$$

Applying condition 27 to the equation above and rearranging its terms, we write the differential of the cost function as:

$$\begin{aligned}dC(V)=&\,p(s_i)\frac{\phi^2(a(s_i),s_i)}{\phi_{aa}(a(s_i),s_i)}\frac{C(V^u_{s_i})}{(V^e-V^u_{s_i})}dV^u_{s_i}+p(s_j)\frac{\phi^2(a(s_j),s_j)}{\phi_{aa}(a(s_j),s_j)}\frac{C(V^u_{s_j})}{(V^e-V^u_{s_j})}dV^u_{s_j}\\&+p(s_i)(1-\phi(a(s_i),s_i))C'(V^u_{s_i})dV^u_{s_i}+p(s_j)(1-\phi(a(s_j),s_j))C'(V^u_{s_j})dV^u_{s_j}=0\end{aligned}$$

By 28, we arrive at:

$$\begin{aligned}dC(V)=&-p(s_i)\left[\frac{\phi^2_a(a(s_i),s_i)}{\phi_{aa}(a(s_i),s_i)}\frac{C(V^u_{s_i})}{V^e-V^u_{s_i}}-\frac{\phi^2_a(a(s_j),s_j)}{\phi_{aa}(a(s_j),s_j)}\frac{C(V^u_{s_j})}{V^e-V^u_{s_j}}\frac{1-\phi(a_{s_i},s_i)}{1-\phi(a_{s_j},s_j)}\right]\\&+p(s_i)\left[1-\phi(a_{s_i},s_i)\right]\left[C'(V^u_{s_i})-C'(V^u_{s_j})\right]dV^u_{s_i}=0\end{aligned} \tag{29}$$

Consequently, by condition 29, for equal contingent continuation values to be optimal, it must be that the likelihood ratio of unemployment equals a function that involves the first and the second derivatives of ϕ evaluated at the optimal contingent search effort

levels $a(s_i)$ and $a(s_j)$:

$$\frac{1-\phi(a_{s_i},s_i)}{1-\phi(a_{s_j},s_j)} = \frac{\phi_a^2(a(s_i),s_i)}{\phi_{aa}(a(s_i),s_i)} \times \frac{\phi_{aa}(a(s_j),s_j)}{\phi_a^2(a(s_j),s_j)} \tag{30}$$

Notice that this result can also be achieved through the examination of the first-order conditions of the UI program. First, for $V^u_{s_i}$ to be equal to $V^u_{s_j}$, the incentive-compatibility constraints 18 require that:

$$\phi_a(a(s_i),s_i) = \phi_a(a(s_j),s_j) \tag{31}$$

holds for the optimal pair of search efforts, $\{a(s_i), a(s_j)\}$. Additionally, constraints 20 imply:

$$\frac{\eta_{s_i}}{\eta_{s_j}} = \frac{p(s_i)}{p(s_j)} \frac{\phi_a(a(s_i),s_i)}{\phi_{aa}(a(s_i),s_i)} \frac{\phi_{aa}(a(s_j),s_j)}{\phi_a(a(s_j),s_j)}$$

Using 31 in the equation above, we get:

$$\frac{\eta_{s_i}}{\eta_{s_j}} = \frac{p(s_i)}{p(s_j)} \frac{\phi_{aa}(a(s_j),s_j)}{\phi_{aa}(a(s_i),s_i)}$$

The same steps applied to 21 returns:

$$\frac{\eta_{s_i}}{\eta_{s_j}} = \frac{p(s_i)}{p(s_j)} \frac{1-\phi(a(s_i),s_i)}{1-\phi(a(s_j),s_j)}$$

Finally, the last two equations taken together provide us a second relation:

$$\frac{1-\phi(a(s_i),s_i)}{1-\phi(a(s_j),s_j)} = \frac{\phi_{aa}(a(s_j),s_j)}{\phi_{aa}(a(s_i),s_i)} \tag{32}$$

Conditions 31 and 32 then establish a system of equations that, in turn, implies 30:

$$\begin{cases} \phi_a(a(s_i),s_i) = \phi_a(a(s_j),s_j); & (i) \\ \dfrac{1-\phi(a(s_i),s_i)}{1-\phi(a(s_j),s_j)} = \dfrac{\phi_{aa}(a(s_j),s_j)}{\phi_{aa}(a(s_i),s_i)} & (ii) \end{cases}$$

If $V^u_{s_i} = V^u_{s_j}$ is optimal, then, for the associated pair of search efforts $\{a(s_i), a(s_j)\}$ given by (i), equation (ii) must hold.

8 Tables

Table 2: **Optimal Unemployment Insurance Under Aggregate Shocks**

	V	$V^u_{s_1}$	$V^u_{s_2}$	$V^u_{s_3}$	c	$V^u_{s_1}-V^u_{s_2}$	$V^u_{s_1}-V^u_{s_3}$	$V^u_{s_2}-V^u_{s_3}$
1	316.9734	317.3999	317.4014	317.4034	0.0000	-0.0015	-0.0035	-0.0020
2	317.4808	317.4908	317.4921	317.4937	0.0000	-0.0013	-0.0029	-0.0016
3	317.9881	317.9865	317.9866	317.9867	0.0000	-0.0001	-0.0002	-0.0001
4	318.4955	318.4950	318.4945	318.4948	0.0001	0.0005	0.0002	-0.0003
5	322.5544	322.5395	322.5394	322.5394	0.0133	0.0001	0.0001	0.0000
6	323.0617	323.0440	323.0438	323.0438	0.0185	0.0002	0.0002	0.0000
7	323.5691	323.5480	323.5479	323.5479	0.0252	0.0001	0.0001	0.0000
8	324.0764	324.0517	324.0517	324.0517	0.0334	0.0000	0.0000	0.0000
9	324.5838	324.5552	324.5552	324.5552	0.0434	0.0000	0.0000	0.0000
10	325.0911	325.0583	325.0583	325.0583	0.0555	0.0000	0.0000	0.0000
11	325.5985	325.5612	325.5612	325.5611	0.0697	0.0000	0.0001	0.0001
12	326.1059	326.0636	326.0636	326.0636	0.0864	0.0000	0.0000	0.0000
13	326.6132	326.5545	326.5545	326.6098	0.1041	0.0000	-0.0553	-0.0553
14	327.1206	327.0667	327.0667	327.1209	0.1245	0.0000	-0.0542	-0.0542
15	327.6279	327.5681	327.5680	327.6279	0.1492	0.0001	-0.0598	-0.0599
16	330.1647	330.0713	330.0713	330.1647	0.3278	0.0000	-0.0934	-0.0934
17	341.3266	340.9713	340.9713	341.3266	2.9820	0.0000	-0.3553	-0.3553
18	341.8340	341.4616	341.4616	341.8340	3.2051	0.0000	-0.3724	-0.3724
19	342.3413	341.9515	341.9515	342.3413	3.4392	0.0000	-0.3898	-0.3898
20	344.3708	343.9057	343.9057	344.3708	4.4916	0.0000	-0.4651	-0.4651
21	344.8781	344.3929	344.3929	344.8781	4.7848	0.0000	-0.4852	-0.4852
22	345.3855	344.8796	344.8796	345.3855	5.0905	0.0000	-0.5059	-0.5059
23	355.0253	354.0133	354.0133	355.0253	13.5537	0.0000	-1.0120	-1.0120
24	356.5474	355.4339	355.4339	356.5474	15.3965	0.0000	-1.1135	-1.1135
25	357.0547	355.9060	355.9060	357.0547	16.0438	0.0000	-1.1487	-1.1487
26	357.5621	356.3774	356.3774	357.5621	16.7078	0.0000	-1.1847	-1.1847
27	358.0694	356.8480	356.8480	358.0694	17.3887	0.0000	-1.2214	-1.2214
28	358.5768	357.3180	357.3180	358.5768	18.0865	0.0000	-1.2588	-1.2588
29	364.1577	362.4348	362.4348	364.1612	26.9091	0.0000	-1.7264	-1.7264
30	364.6651	362.9018	362.9018	364.6538	27.8311	0.0000	-1.7520	-1.7520
31	365.1725	363.3211	363.3211	365.2539	28.6755	0.0000	-1.9328	-1.9328
32	365.6798	363.2576	365.1108	365.1108	28.5464	-1.8532	-1.8532	0.0000
33	366.1872	363.3367	365.3023	366.3775	28.7072	-1.9656	-3.0408	-1.0752
34	366.6945	363.3534	366.6096	366.6096	28.7414	-3.2562	-3.2562	0.0000
35	367.2019	363.6306	367.2095	367.2095	29.3129	-3.5789	-3.5789	0.0000
36	367.7093	364.0660	367.7074	367.7074	30.2148	-3.6414	-3.6414	0.0000
37	374.3049	370.4264	374.3019	374.3019	39.0208	-3.8755	-3.8755	0.0000
38	377.3491	373.0071	377.3504	377.3504	44.8507	-4.3433	-4.3433	0.0000
39	381.4079	376.5885	381.4078	381.4078	52.4926	-4.8193	-4.8193	0.0000
40	381.9153	377.0104	381.9154	381.9154	53.6124	-4.9050	-4.9050	0.0000
41	382.4226	377.4445	382.4229	382.4229	54.6751	-4.9784	-4.9784	0.0000
42	383.4374	378.3644	383.4375	383.4375	56.5179	-5.0731	-5.0731	0.0000
43	385.4668	380.0679	385.4667	385.4667	61.0852	-5.3988	-5.3988	0.0000
44	389.5257	383.4402	389.5258	389.5258	70.7878	-6.0856	-6.0856	0.0000
45	390.0330	383.8643	390.0329	390.0329	72.0144	-6.1686	-6.1686	0.0000
46	390.5404	384.2785	390.5403	390.5403	73.3166	-6.2618	-6.2618	0.0000
47	391.0477	384.6878	391.0478	391.0478	74.6620	-6.3600	-6.3600	0.0000
48	391.5551	385.0963	391.5551	391.5551	76.0237	-6.4588	-6.4588	0.0000
49	392.0625	385.5049	392.0624	392.0624	77.3933	-6.5575	-6.5575	0.0000
50	392.5698	385.9137	392.5690	392.5690	78.7793	-6.6553	-6.6553	0.0000

Table 3: **Optimal UI given a random fifty-period sequence of aggregate shocks**

Initial Promise $V = 351.9811$				
t	State of Nature	V_s^u	c	$a(s)$
1	s_2	351.1536	10.3017	12.1873
2	s_3	351.1536	9.5145	0.0000
3	s_3	351.1536	9.5145	0.0000
4	s_2	350.3720	8.8074	12.8833
5	s_1	349.6324	8.1706	35.5017
6	s_1	348.9310	7.5950	35.9966
7	s_2	348.2649	7.0737	14.7424
8	s_1	347.6309	6.5998	36.9081
9	s_1	347.0267	6.1681	37.3292
10	s_3	347.0267	6.1681	0.0000
11	s_2	346.4501	5.7738	16.3237
12	s_3	346.4501	5.7738	0.0000
13	s_2	345.8990	5.4129	16.8003
14	s_3	345.8990	5.4129	0.0000
15	s_2	345.3716	5.0820	17.2549
16	s_3	345.3716	5.0820	0.0000
17	s_3	345.3716	5.0820	0.0000
18	s_2	344.8662	4.7778	17.6891
19	s_1	344.3815	4.4976	39.1542
20	s_3	344.3815	4.4976	0.0000
21	s_3	344.3815	4.4976	0.0000
22	s_1	343.9159	4.2392	39.4722
23	s_3	343.9159	4.2392	0.0000
24	s_2	343.4684	4.0004	18.8831
25	s_3	343.4684	4.0004	0.0000
26	s_3	343.4684	4.0004	0.0000
27	s_1	343.0377	3.7792	40.0698
28	s_3	343.0377	3.7792	0.0000
29	s_3	343.0377	3.7792	0.0000
30	s_2	342.6229	3.5741	19.6003
31	s_3	342.6229	3.5741	0.0000
32	s_2	342.2230	3.3837	19.9382
33	s_1	341.8372	3.2065	40.8814
34	s_1	341.4648	3.0417	41.1321
35	s_2	341.1048	2.8879	20.8788
36	s_3	341.1048	2.8879	0.0000
37	s_3	341.1048	2.8879	0.0000
38	s_3	341.1048	2.8879	0.0000
39	s_1	340.7567	2.7442	41.6069
40	s_2	340.4198	2.6100	21.4518
41	s_1	340.0937	2.4844	42.0498
42	s_1	339.7776	2.3666	42.2602
43	s_1	339.4712	2.2562	42.4639
44	s_3	339.4712	2.2562	0.0000
45	s_2	339.1739	2.1525	22.4880
46	s_1	338.8853	2.0550	42.8522
47	s_2	338.6051	1.9633	22.9585
48	s_1	338.3328	1.8769	43.2173
49	s_1	338.0680	1.7955	43.3917
50	s_3	338.0680	1.7955	0.0000

9 Figures

Figure 1: Promises Path

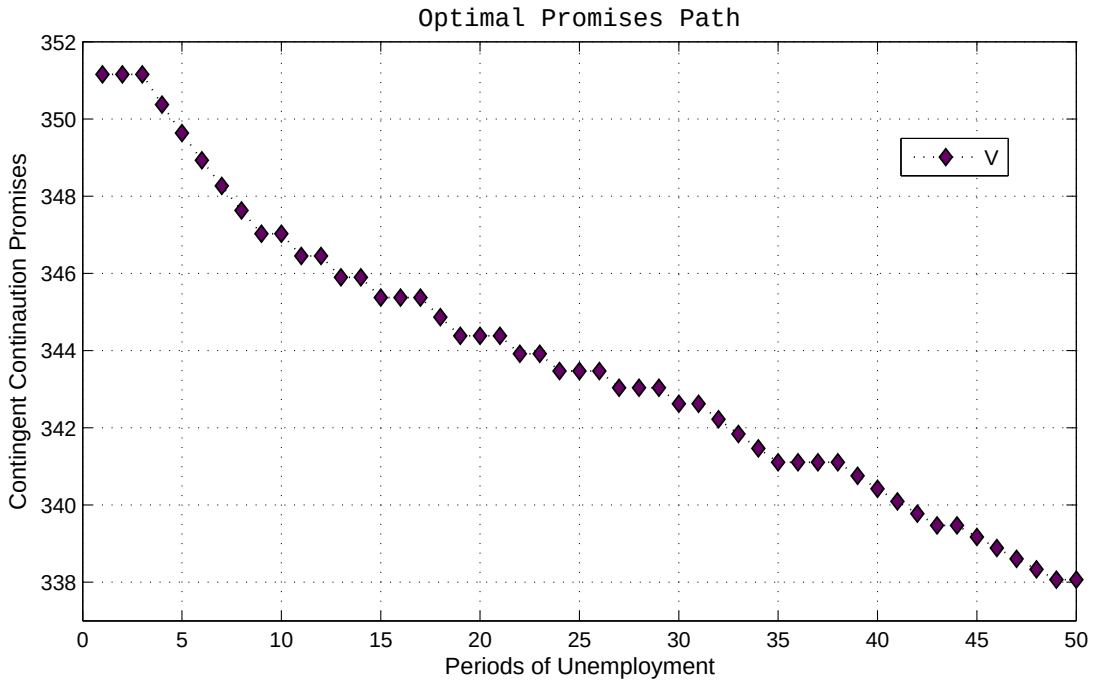


Figure 2: Consumption Path

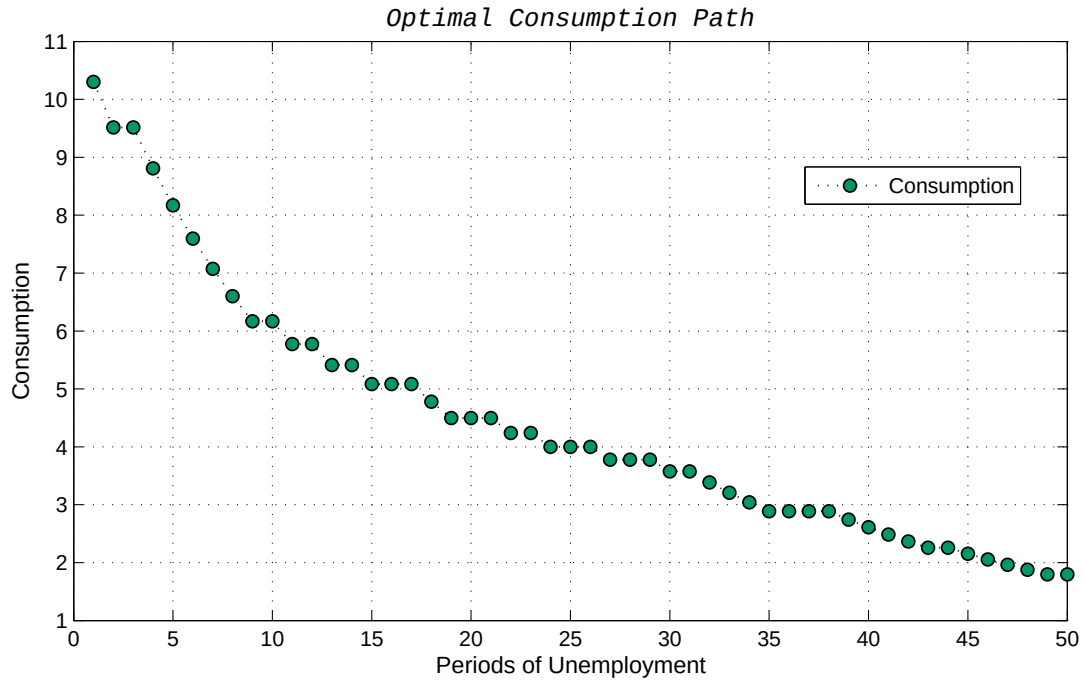
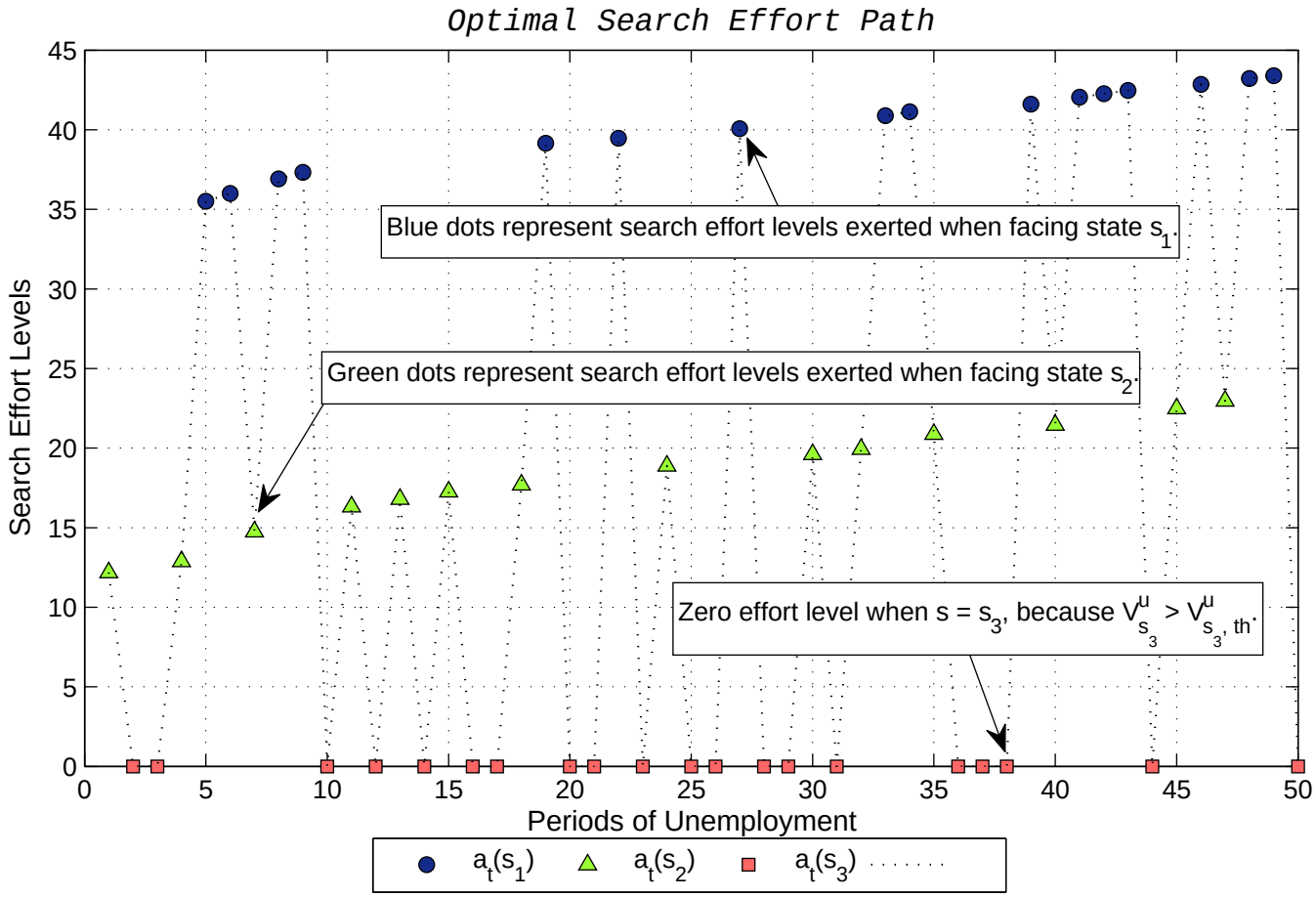


Figure 3: Search Effort Path



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