Essays on Finance and Macroeconomy
Axel André Simonsen

Essays on Finance and Macroeconomy

Professor Orientador: Caio Almeida
Professor Co-orientador: Marco Bonomo

Rio de Janeiro
2009
Axel André Simonsen

Essays on Finance and Macroeconomy

Tese submetida à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas como requisito de obtenção do título de Doutor em Economia

E, aprovada em 23/12/2009
Pela Banca Examinadora:

Caio Almeida (EPGE-FGV)
Marco Bonomo (EPGE-FGV)
José Valentim Vicente (Banco Central do Brasil)
Marcelo C. Medeiros (PUC)
Ricardo Brito (INSPER)

Rio de Janeiro
2009
Contents

List of Tables viii

List of Figures x

Aknowledgments xii

Introduction xiii

1 Long Term Feedback Traders 1

1.1 Introduction 2

1.2 Model Setup 5

1.2.1 Agents 5

1.2.2 Market Clearing 8

1.2.3 Equilibrium 9

1.2.4 Equilibrium Solution: A restricted VAR representation 10

1.3 Methodology 14

1.3.1 Likelihood Function 14

1.3.2 Testing the Algorithm: Simulation and Estimation with Artificial data 15

1.4 Empirical Results 18

1.4.1 Data Description 18

1.4.2 Priors and Posterior Distributions 18

1.4.3 Filtered Mispricing and Probabilities 21

1.5 Conclusion 24

2 Rebound Returns Following Extreme Downturns 26

2.1 Introduction 27
## CONTENTS

2.2 Methodology and Data ................................................. 29  
  2.2.1 Events Set Specification ........................................... 29  
  2.2.2 Data ............................................................... 31  
2.3 Empirical Findings .................................................... 33  
  2.3.1 Summary of Extreme Returns in Raw Data ...................... 33  
  2.3.2 Econometric Results .............................................. 34  
  2.3.3 Economic Significance ........................................... 37  
  2.3.4 Robustness Checks .............................................. 42  
2.4 Explanation for Reversal Effect .................................... 43  
2.5 Conclusion ............................................................. 45  

3 Effects of US Shocks on the Canadian Economy using a DSGE Model 46  
  3.1 Introduction ........................................................... 47  
  3.2 Model ................................................................. 50  
    3.2.1 Main equations .................................................. 52  
    3.2.2 Exogenous Processes ........................................... 53  
    3.2.3 Auxiliary variables ............................................. 54  
  3.3 Estimation Methodology .............................................. 55  
    3.3.1 Data description ................................................ 55  
    3.3.2 Model Solution and Likelihood Function .................... 58  
    3.3.3 Bayesian Estimation .......................................... 59  
    3.3.4 Shocks Decomposition ....................................... 60  
  3.4 Results .............................................................. 62  
    3.4.1 Models ......................................................... 62  
    3.4.2 Parameter Estimates ......................................... 62  
    3.4.3 The Role of Past Shocks on the Canadian Series ............ 65  
    3.4.4 Impulse Responses Functions ................................ 68  
    3.4.5 Counterfactual Analysis ..................................... 71  
  3.5 Conclusion ........................................................... 72  

Bibliography ............................................................. 74
CONTENTS

A Feedback traders: Derivation and estimation methodology 77
  A.1 Appendix ............................................. 77
    A.1.1 Appendix A ....................................... 77
    A.1.2 Appendix B: Steady State ....................... 78
    A.1.3 Appendix C: Solving Linear Markov Switching Rational Expectation System ............................................. 79
    A.1.4 Appendix D: State Space Markov Switching Filtering ............. 81

B Rebound after extreme events: Additional tables 84
  B.1 Additional Figures ......................................... 84

C DSGE: Derivation and estimation diagnostics 90
  C.1 Detailed Model .......................................... 90
    C.1.1 Households ......................................... 90
    C.1.2 Firms ................................................ 93
    C.1.3 Monetary Authority .................................... 95
    C.1.4 Equilibrium .......................................... 95
    C.1.5 Equilibrium with flexible prices ....................... 97
  C.2 Solving the Model ........................................ 98
    C.2.1 Log-linearization ..................................... 98
  C.3 Markov Chain Convergence Diagnostics ....................... 101
  C.4 Additional Figures ........................................ 103
# List of Tables

1.1 Coefficients estimates. There are 3 column blocks. Each block represents the parameters posterior distribution from different specifications of the baseline model. ................................................................. 20

1.2 Time series of the estimated misprice grouped by the most likely regime. . . 24

2.1 Summary of return statistics conditional to different choises of events sets. for all countries in local currency. The notation for the event sets is as follows: 
\[ [RAW] \triangleq D (r^i), [MKT] \triangleq D (r^m) \text{ and } [RAW - MKT] \triangleq D (r^i - r^m) \] .... 33

2.2 Regression of the countries estimated coefficients on risk factors. .......... 38

2.3 Summary of results of the trading strategies. ........................................ 41

2.4 Robustness checks for Strategy III. Different choices for parameter \( J \) (sample size) and \( c \) (threshold value). ................................................................. 44

3.1 Selected cross correlation of observed macroeconomic series. Quarterly data from 1986Q1 to 2008Q4 ................................................................. 49

3.2 Models specification ................................................................. 62

3.3 Parameter Estimates ................................................................. 64

3.4 Absolute Participation of Estimated Shocks ........................................ 67

3.5 Canadian Output Gap Decomposition and Forecast .............................. 72

B.1 Summary Statistics of Stock Index Returns ........................................ 84

B.2 Summary Statistics of Future Contracts ........................................... 85

B.3 Return Statistics Following the Extreme Events for the 2.5% Quantil ....... 85

B.4 Return Statistics Following the Extreme Events for the 1.0% Quantil ....... 86
B.5 Results of Regression (1) for each country. Coefficients significant at 5% are indicated by shadow cells. .......................... 86
B.6 Results of Regression (2) for each country. Coefficients significant at 5% are indicated by shadow cells. .......................... 87
B.7 Results of Regression (3) for each country. Coefficients significant at 5% are indicated by shadow cells. .......................... 87
B.8 Out-of-sample returns of strategies I and II, detailed by country ......................... 88
B.9 Out-of-sample returns of strategy III, detailed by country ............................... 88
B.10 Summary of Sub Sample Regressions ......................................................... 89
List of Figures

1.1 Impulse Response Functions. Each graph represents the effect of an unit of fundamental shock on equilibrium price over time. The top graph is the effect in the long-term only regime while the bottom graph represents the short-term only regime. ........................................... 13

1.2 Impulse Response Functions with regime switching. Each graph represents the effect of an unit of fundamental shock on equilibrium price over time. The top graph is the effect when the initial state is the non-feedback regime while the bottom graph represents the effect when the initial state is the feedback regime. ........................................... 13

1.3 Example with artificial data of price and fundamental processes under the feedback model with Markov switching. ........................................... 16

1.4 The top graph shows the price process, the estimated fundamental process and the true one. The mispricing defined as the difference between the price and the fundamental process is plotted in the bottom graph. ........... 17

1.5 Time series of the filtered fundamental value and the subsequent mispricing. 22

1.6 Estimated regimes probability time series. ........................................... 22

1.7 Time series of the estimated misprice grouped by the most likely regime. . . 23

2.1 Summary results of individual regressions (1) to (3). ........................... 36

3.1 Historical Decomposition of Canadian Output Gap ............................. 48

3.2 The dataset used in the estimation and in the empirical exercises. Quarterly data ranging from 1986:Q1 to 2008:Q4. ................................. 57
LIST OF FIGURES

3.3 Historical decomposition of Detrended output gap by model. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks. .................................................. 66

3.4 Historical relative contribution of shocks on detrended output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks. .................................................. 68

3.5 The impulse response of domestic shocks on the Canadian output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks. .................................................. 69

3.6 The impulse response of external shocks on the Canadian output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks. .................................................. 70

3.7 Effective and counterfactual output gap series in basis points. Counterfactual is defined as the series with no external neither exchange rate shocks from 2007:Q2 afterwards .................................................................................. 71

C.1 Histograms of the first third (left) and last third (right) of the Markov chain. Each row correspond to a different coefficient. .................................................. 102

C.2 The first block of 4 graphics at the top shows the Historical decompositions of Canadian Interest Rate, and the last block of 4 pictures at the bottom shows the relative contribution of shocks on Canadian Interest Rate. In each block second rows are from the models with two Phillips curve, and second columns are from the models presenting correlation with external structural shocks. 103

C.3 The first block of 4 graphics at the top shows the Historical decompositions of Canadian Inflation, and the last block of 4 pictures at the bottom shows the relative contribution of shocks on Canadian Inflation. In each block second rows are from the models with two Phillips curve, and second columns are from the models presenting correlation with external structural shocks. 104
Aknowledgments

Agradeço.
Ao meu orientador, Caio Almeida, que sempre me motivou e me deu liberdade para desenvolver novas ideias, tendo contribuído muito para meu amadurecimento acadêmico, profissional e pessoal. Também pelo enorme apoio ao longo de todo meu programa doutorado.
Ao meu co-orientador, Marco Bonomo, pelas inúmeras oportunidades que me proporcionaram, tanto acadêmicas como profissionais.
Ao professor Harrison Hong, por ter me orientado e patrocinado minha pesquisa na Princeton University.
Aos membros da banca examinadora, pela paciência na leitura dos artigos e pelos comentários feitos a este trabalho.
Ao meu pai Detoh, a minha mãe Susan e a minha irmã Chris, que sempre me apoiaram, ajudaram e encorajaram, em todos os meus projetos e desafios.
A minha namorada Joana, por seu companheirismo, carinho e amizade.
Aos meus co-autores João Marco e Bruno Lund, um agradecimento especial por nossas inúmeras e produtivas discussões.
A todos os professores da EPGE, pela crucial contribuição em minha formação.
Aos funcionários da FGV, pelo suporte indispensável.
Ao tio Neno, tia Tipi e tio Vitinho pelo amizade ao longo da minha vida.
A CAPES que patrocinou o meu programa de doutorado.
Introduction

This thesis composes of three articles with subjects on macroeconomics and finance. Each article corresponds to a chapter and is prepared in paper format. The first two chapters explore the field of behavioral finance and provide a model and evidences of market inefficiencies. The last chapter involves understanding the role of larger countries in domestic economies.

In the first chapter, I study the asset-pricing implication in an environment with a especial type of bounded rationality, the feedback trading behavior. The work assists to explain excess volatility in the market and bubbles generated by representativeness bias from some investors. The limits of arbitrage prevent the arbitrageurs from cleaning up the inefficiencies, therefore generating systematic price deviation from fundamentals.

In the second chapter, an empirical investigation is conducted to document the overreaction of stock exchange indexes in extreme market fluctuations. Using a novel characterization of extreme events, decomposing it into idiosyncratic and market events, it is possible to obtain predictability in expected returns. The predictability cannot be explained by risk differentials and or, by the size of a country. In contrast with related literature, the decomposition proposed strengthens the predictability of the excess returns and the effect is robust to bid-ask spreads.

In the third chapter, which was co-authored with Bruno Lund, we model and estimate a small open economy for the Canadian economy in a two country General Equilibrium (DSGE) framework. We show that it is important to account for the correlation between Domestic and Foreign shocks and for the Incomplete Pass-Through.
Chapter 1

Long Term Feedback Traders

Abstract I study the asset-pricing implications in an environment with feedback traders and rational arbitrageurs. Feedback traders are defined as possible naive investors who buy after a raise in prices and sell after a drop in prices. I consider two types of feedback strategies: (1) short-term (SF), motivated by institutional rules as stop-losses and margin calls and (2) long-term (LF), motivated by representativeness bias from non-sophisticated investors. Their presence in the market follows a stochastic regime swift process. Short lived assumption for the arbitrageurs prevents the correction of the misspricing generated by feedback strategies. The estimated model using US data suggests that the regime switching is able to capture the time varying autocorrelation of returns. The segregation of feedback types helps to identify the long term component that otherwise would not show up due to the large movements implied by the SF type. The paper also has normative implications for practitioners since it provides a methodology to identify mispricings driven by feedback traders.

Keywords: Feedback Traders; Underreaction; Return Anomalies; Bubble Models

JEL Classification: G10; G12; G14
CHAPTER 1. FEEDBACK TRADERS

1.1 Introduction

Positive feedback traders are defined as the investors who buy after an increase in prices or sell after a decrease in prices. This type of strategy is widely observed in most of the markets and not necessarily represents irrationality of the agents. For example, stop-loss orders and delta hedge strategies generate the feedback mechanism: A decrease in price implies sell orders that are not related to fundamental changes. The 2008 crisis is another good example of the destabilizing impact of feedback strategies, where an additional selling pressure emerged in order to meet the margin calls in times of stress (liquidity spirals).

A second type of positive feedback traders represents the less sophisticated group, who decide to step in after a trend in prices. Examples of investors with this characteristic are trend following traders, chartists, general herd behavior and decision based on psychology bias. A particular psychology bias is the representativeness, in which people tend to extrapolate the small sample information (Tversky and Kahneman (1974)), implying for example that a small sequence of good news generates optimism about future outcomes. Investors with this bias tend to follow a feedback trading rule. Moreover, bull (possible bubble) and bear (possible panic) markets can be justified by this kind of behavior.

Feedback strategies can deviate the price from fundamentals (mispricing) since part of the trading decisions is not related to the expected future cash flow of the asset. What is the reaction of arbitrageurs in an economy with these types of feedback traders? Proponents of efficient market theory based the answer on Friedman (1953) where they must stabilize the prices, i.e. bring the prices back to fundamentals. However this answer does not hold if arbitrage is risky. One concept of risky arbitrage is that if the arbitrageurs have limited capital and there exists a mispricing with stochastic time convergence, they are not going to trade aggressively towards the fundamentals (Shleifer and Vishny (1997)). As a result, a mispricing in the price dynamics can persist in equilibrium, that is, it is not going to be corrected in the short run. In the particular case of feedback traders, De Long et al. (1990) showed that arbitrageurs can also be destabilizing, in the sense that can be optimal to "jump in the bandwagon" instead of betting against it.

There is plenty empirical evidence on the feedback trading behavior on prices. Cutler et al. (2001) found the feedback behavior in a variety of asset classes using the autocorrelation implied by this story. Sentana and Wadhwani (1992) extended the logic of Cutler’s work to analyze the links between volatility and serial correlation. In particular, their model has
testable implications given by the inverse relation between autocorrelation and volatility. These findings were also confirmed by Koutmos (1997) in several foreign stock markets and are robust to alternatives theories as non-synchronous trading and bid ask spreads.

These studies interpret the effect of the high volatility on the autocorrelation as being generated by institutional reasons as stop-losses and margin calls. However, they do not take into account the time varying property of these events. For instance, aggregate selling pressures due to margin call ("fire sale") do not occur frequently. Moreover, although this event is unpredictable, once it happens, one could expect some persistent effects. The mechanism behind the persistence is the spiral generated by liquidity issues: margin calls generate selling pressure that impacts negatively the price, which generates more margin calls and so on. A similar mechanism occurs with long-term feedback strategies, where herd behavior sustain the persistence of these strategies. The stochastic arrival and the persistence effect suggest that a Markov chain process might work to characterize this type of trading reaction.

This paper develops and estimates a model for asset price dynamics in an economy where the presence of these two types of feedback traders follows a stochastic regime switch and there is a rational trader who trades against them. The rational trader lives for just one period (overlapping generation). This assumption aims to capture the limited arbitrage restriction. It is motivated by the fact that most professional investors (ex. fund managers) use other people's capital and they cannot handle a bad performance in the short run against a good performance on the long run with the risk of their capital being unwound before (see Shleifer and Vishny (1997)). As a result, without loss of generality, I specify the arbitrageurs problem as a maximization of the next period wealth, which is the same objective function as in an overlapping generation framework.

The two types of feedback traders are divided in accordance to the horizon of their demands: short-term feedback traders consider the last price variation in their demands while the long-term feedback trader consider $N > 2$ past prices. The first group is interpreted as those traders subject to institutional rules as stop-loss orders and margin requirements. The second group consists on less sophisticated investors as households with representative bias, trend followers and chartists.

The price dynamics is derived in a partial equilibrium setup resulting in a linear rational expectation system (LRES). The system solution is obtained using the Farmer et al. (2006) paper, which provide a method to solve LRES with regime switching. The solution has a
CHAPTER 1. FEEDBACK TRADERS

restricted MS-VAR\(^1\) representation with unobservable variables. I estimate the model using US stock market data. The estimation is conducted by first filtering out the latent variables and evaluating the likelihood through a regime-switching Kalman filter, and second using MCMC methods to sample the parameters from the posterior distribution.

A methodological contribution is that the number of lags \(N\) is also estimated. This parameter changes the dimension of the MS-VAR, without changing the total number of estimated parameters. For example, a reduced form first order VAR with 2 states has 7 parameters while for 10 states it has 155. In this paper, the number of parameters is invariant to the number of past prices included in the state vector. Thus, it is possible to capture long-term relations in the data without losing precision on the estimates.

In line with previous results in the literature (Sentana and Wadhwani (1992), Cutler et al. (2001), Dean and Faff (2008)), the evidence supports the short-term feedback traders. Moreover the time varying property allows a more precise estimate of the effect since they are not all the time in the market. The estimated stationary distribution points that only 16\% of the time the economy is in this regime. The feedback response is much higher than the response in absence of regime switching (which gets the average).

The main contribution, however, comes from the long-term feedback trader effect. This paper, as a best of my knowledge, is the first one to come up with this analysis. The inclusion of a second feedback type helps to disentangle the long term fluctuation. In absence of this second type, the estimated duration \((N)\) would be short since most of returns variation is associated to short-term feedbacks\(^2\). The empirical results show that the relevant coefficients are indeed significant and the estimated number of lags indicates that the average duration of this effect is 9 months. These results suggest that some herd or fad behavior is behind it. The representative psychology bias is one possible explanation.

Finally, having extracted the time series of the regimes probabilities and the estimated transition matrix, I address the question of how those regimes are connected. The main results are: (i) the regime shift structure fits better the time varying autocorrelation in returns (ii) short-term feedback (SF) traders are infrequent and more likely in distressed

\(^1\)MS-VAR is the notation for the class of Markov Switching VAR.

\(^2\)In other words, if the data generating process is mispecified assuming one feedback trader when the true one has both types, the estimated value for the lag parameter \(N\) points to the short term type (small \(N\)). The reason is that most of fluctuations are related to those types of strategies and the likelihood would drive the estimates to that.
markets, which is consistent with liquidity literature, (iii) the SF regime is more likely to appear right after the long-term feedback (LF) regimes (iv) the regime without feedback strategies ("fundamental" regime) is more likely after the SF regime, which in addition to (iii), suggests a bubble burst. The paper also has normative implications from practitioners, since the model can identify mispricing driven by feedback strategies.

The rest of the paper is organized as follows. The feedback traders model is defined in Section 2. The estimation methodology and a controlled experiment to validate the algorithm is discussed in Section 3. The empirical evidence and the analysis of the regimes relation are discussed in Section 4. Section 5 concludes.

1.2 Model Setup

The economy has one single stock. There is an infinite horizon, and at each date $t$, the stock pays a dividend of $D_t = D_{t-1} + v_t$, where $v_t \sim N(0, \sigma_v^2)$. Assuming a constant discount rate $r$, I define the stock fundamental price as the present value of the future stream of dividends.

$$ F_t = \sum_{i=0}^{\infty} \frac{E_t[D_{t+i}]}{(1+r)^i} = \frac{D_t}{1-r} = F_{t-1} + \varepsilon_t $$

(1.1)

where $\varepsilon_t = \frac{v_t}{1-r} \sim N(0, \sigma_{\varepsilon}^2)$

Thus the fundamental price $F_t$ follows a random walk process. Although this specification has the problem of allowing negatives dividends and consequently prices, one can interpret it as the log process without loss of generality$^3$.

1.2.1 Agents

I consider three groups of investors in the economy: (1) positive feedback traders (2) fundamental traders and (3) strategic arbitrageurs.

An innovation of this paper relative to related literature (Sentana and Wadhwani (1992), Cutler et al. (2001)), is that I split the feedback group in two types of traders: short-term feedback (SF) traders and long-term (LF) feedback traders. The SF represents the reaction to

$^3$Defining $D_t = D_{t-1}U_t$, $d_t = \log(D_t)$ and $\tilde{u}_t = \frac{\log(U_t)}{1-r}$, where $U_t$ is lognormal, it is possible to solve the expected value of the future cash flow resulting in $\log(F_t) = \log(F_{t-1}) + \tilde{u}_t$. \[\tilde{u}_t = \frac{v_t}{1-r} \sim N(0, \sigma_{\tilde{u}}^2)\]
prices generated by liquidity issues, like margin calls in distressed times, automatic portfolio adjustment and stop losses. The LF represents the less sophisticated investors, who decide to step in after a trend in prices. Examples are trend following traders, chartists, non sophisticated households who react after a sequence of good (or bad) news and more general herd behavior (like bubbles or panic).

Feedback traders are not necessarily irrational. SF traders can be rationalized by the liquidity literature under the main assumption of limited capital (see Brunnermeier and Pedersen (2008) for instance). For LF traders, there is an evidence that people have some psychology bias as representativeness, i.e. they tend to extrapolate the small sample information, which can justify positive feedback to past news (Tversky and Kahneman (1974)). Thus, although it is possible to build a model imposing several restrictions such as limited capital or information to rationalize such behavior, in this paper I do not address this question. I rather assume that their demands are exogenously given. This assumption simplifies the analysis without changing the main results.

Feedback traders’ demand is an exogenous function of past prices. Long-term feedback demand \( f_t^a \) is a function of \( N > 2 \) lagged prices and short-term feedback demand \( f_t^b \) is a function of the recent variation in prices. Define \( Y_{t-1}^p \triangleq \left[ P_{t-1} \cdots P_{t-N} \right]' \), then demands are given by

\[
\begin{align*}
  f_t^a &\triangleq f_t^a (Y_{t-1}^p) = \delta_a \sum_{i=1}^{N-1} (P_{t-i} - P_{t-N}) \\
  f_t^b &\triangleq f_t^b (Y_{t-1}^p) = \delta_b (P_{t-1} - P_{t-2})
\end{align*}
\]

where \( \delta_a \) and \( \delta_b \) are the feedback parameters.

Contrary to previous works, I consider a new assumption for the presence of feedback traders in the market. In my model, they do not trade every period, but at stochastic times. I postulate this structure first because of empirical evidence on the time varying role of feedback traders (Sentana and Wadhwani (1992), Cutler et al. (2001), Dean and Faff (2008)).

As an example for the short-term types, the event of a significant share of investors being triggered by stop losses or margin calls are more likely to occur at extreme price variations. The second reason is that, since I interpret the long-term types as possible non sophisticated investors (households), it is reasonable to assume that they do not trade all the times (for exogenous reasons). As an example, take the internet boom period, the share of ordinary
people on the stock market was higher than after burst period (type of herd behavior).

I model the presence of feedback traders as a stochastic regime shift given by the state variable $s_t = (s^a_t, s^b_t) \in \{(0, 0), (1, 0), (0, 1), (1, 1)\}$, where $a, b$ represents the long and short types respectively. The evolution of $s$ is governed by the transitional probability matrix of a four state Markov chain $\Gamma$, where $\Gamma_{i,j} = p(s_{t+1} = j | s_t = i)$.

Since feedback traders are not in the market every period, I have to make a distinction between the "latent" demand $(f^a_t, f^b_t)$ for the stock and the quantity "actually" held by those investors $(X^a_t, X^b_t)$ in order to ensure consistency on the market clearing. When they are present, both quantities are equal, i.e., $X^a_t = f^a_t$ if $s^a_t = 1$. In the regime of no trade ($s^a_t = 0$), the quantity held by investor at time $t$ is a fraction $\rho^a \in [0, 1]$ of the position held in the last period and the remaining fraction $(1 - \rho^a)$ is liquidated. The resulting equation for the quantity held by feedback traders at time $t$ is

$$X^a_t = s^a_t f^a_t + (1 - s^a_t) \left[ \rho^a X^a_{t-1} - (1 - \rho^a) X^a_{t-1} \right], \quad z \in \{a, b\}$$  \hspace{1cm} (1.4)

The parameter $\rho^a$ has the role of weaken the assumption that the positions taken at time $t$ have to wait till the next present regime take place to be liquidated. However, no trade in some periods is more realistic than the hypothesis that all the shares outstanding are traded at every period. For example, the average turnover\footnote{I define turnover as the ratio between the volume and the shares outstanding.} of the IBM stock (which is a very liquid stock) is around 10\% for month. This model consider an intermediary assumption. When $\rho^a = 1$, all shares are locked. In the other extreme case ($\rho^a = 0$), every position taken at time $t$ is liquidated at time $t + 1$, which nests the overlapping generations model.

The second group of investors, the fundamental traders, base their expected returns on prices relative to perceived fundamentals. Such behavior represents those strategies based on dividend discount models. The demand can also be derived from infinity lived rational investors who take the price as given, where the slope is a function of the risk aversion and volatility of the stock. The resulting demand $(X^a_t)$ of fundamental traders is a decreasing function of price:

$$X^a_t = \alpha (F_t - P_t), \quad \alpha > 0$$  \hspace{1cm} (1.5)

Finally, the third group, the strategic arbitrageurs, live for only one period (overlapping generations) implying that any position taken at time $t$ ($x_t$) must be liquidated at time $t + 1$.\footnote{$s^a_t = 1$ indicates that the feedback type $z \in \{a, b\}$ is in the market and if $s^a_t = 0$ he is out of the market.}
They are risk neutral and are big enough to not take the price as given. They know all the structure of the economy, including the current state \( s_t \), and they trade strategically against the feedback and fundamental investors. Let \( \Psi_{t-1} = \{Y^p_{t-1}, X^a_{t-1}, X^b_{t-1}, F_{t-1}, x_{t-1}\} \) be the state variables and \( \vartheta_t = \{s_t, \varepsilon_t\} \) the exogenous disturbances at time \( t \). The price function \( P_t(x_t, \Psi_{t-1}, \vartheta_t) \) is the result of the market clearing condition (described in next section).

At time \( t \), given \( \Psi_{t-1} \) and \( \vartheta_t \) the arbitrageurs choose the quantity \( x_t \) that maximizes their expected wealth on the next period:

\[
\max_{x_t} \left[ E_t P_{t+1} \left( X^a_{t+1}, \Psi_t, \vartheta_{t+1} \right) - P_t \left( x_t, \Psi_{t-1}, \vartheta_t \right) \right] \tag{1.6}
\]

The \( x_t \) that solves this problem is the demand of the arbitrageurs

\[
X^a_t \triangleq X^a_t \left( \Psi_{t-1}, \vartheta_t \right) \tag{1.7}
\]

### 1.2.2 Market Clearing

The mass of any type of trader is normalized to one. Total net supply is zero. The market clearing condition at time \( t \) is

\[
0 = X^L_t + X^a_t - X^a_{t-1} + X^b_t + X^b_{t-1}
\]

Note that the term \( X^a_{t-1} \) appears because strategic traders are assumed to be short living (one period overlapping generations), as a result the position held at time \( t \) must be liquidated at time \( t + 1 \). Substituting the fundamental traders demand, given by equation (1.5), into the market clearing condition and solving for the price at time \( t \) results in

\[
P_t = F_t + \frac{1}{\alpha} \left[ X^a_t + X^b_t \right] + \frac{1}{\alpha} \left( X^a_t - X^a_{t-1} \right) \tag{1.8}
\]

To give an intuition of the price function, I describe two extreme situations. First, is in absence of feedback traders the current price reduces to \( P_t = F_t + \frac{X^b_t - X^a_{t-1}}{\alpha} \). The optimal quantity chosen by strategic investors in this setup would be zero. The second is the steady state, where \( F_t = F_{t-k} = \bar{F} \), \( \forall k \). The equilibrium price is going to be \( P_t = \bar{P} = \bar{F} \) and the quantity held by strategic investor is \( x_t = \bar{x} = 0 \) (see appendix B) independent of the regime \( s \). In both situations the price would be equal to the fundamental value.
1.2.3 Equilibrium

Given the exogenous process for the fundamental value $F_t = F_{t-1} + \varepsilon_t$, the vector of past prices $Y_{t-1}^p$, the past position of feedback traders and strategic investors $(X_t^a, X_t^b, X_t^f)$ and the current regime $s_t$, the solution of the arbitrageur $X_t^s$ is given by equation (1.9), where the derivation is shown in the appendix A. Given this solution, I characterize the rational expectation equilibrium in this economy, for a given regime $s_t$, as a set of four equations and one exogenous process.

\[
\frac{X_t^s}{\lambda_{s_t}} = a\xi_{s_t} F_t + \left(q^a_s A^a + q^b_s A^b\right) Y_{t-1}^p + (\xi_{s_t} - 1) \left[X_t^a + X_t^b + X_t^s\right] + E_t X_{t+1}^s \tag{1.9}
\]

\[
P_t = F_t + \frac{1}{\alpha} \left[X_t^a + X_t^b\right] + \frac{1}{\alpha} \left(X_t^s - X_{t-1}^s\right) \tag{1.10}
\]

\[
X_t^a = s_t^a A^a Y_{t-1}^p + (1 - s_t^a) \left[p^a X_{t-1}^a - (1 - p^a) X_{t-1}^s\right] \tag{1.11}
\]

\[
X_t^b = s_t^b A^b Y_{t-1}^p + (1 - s_t^b) \left[p^b X_{t-1}^b - (1 - p^b) X_{t-1}^s\right] \tag{1.12}
\]

\[
F_t = F_{t-1} + \varepsilon_t \tag{1.13}
\]

where the restrictions over $\lambda_{s_t}$ below ensures stationarity ($\lambda_{s_t} < 1$) and second order condition ($\lambda_{s_t} > 0$)

\[
0 < \lambda_{s_t} \triangleq \frac{1}{2(2 - \xi_{s_t})} < 1
\]

\[
q^s_{s_t} \triangleq p \left(s_{t+1}^s = 1 | s_t\right)
\]

\[
\xi_{s_t} \triangleq \frac{1}{\alpha} \left[q^a_s \delta^a + q^b_s \delta^b\right]
\]

and the remaining coefficients are given by

\[
A^a_{(1 \times N)} \triangleq \delta^a \left[1 \ 1 \cdot\cdot\cdot\ 1 \ 1 - (N - 1)\right]
\]

\[
A^b_{(1 \times N)} \triangleq \delta^a \left[1 -1 0 \cdot\cdot\cdot\ 0\right]
\]

\[
\overline{A}^a_{(1 \times N)} \triangleq \delta^a \left[1 \ 1 \cdot\cdot\cdot\ 1 - (N - 1) 0\right]
\]

\[
\overline{A}^b_{(1 \times N)} \triangleq \delta^a \left[-1 0 \cdot\cdot\cdot\ 0\right]
\]

There is an issue on solving the expectation term in a closed form, i.e., characterizing the equilibrium only as a function of exogenous shocks and the state variables. I show later in
this section the solution using a numerical method. I discuss before some properties of the
equilibrium given the expectations terms.

The steady state equilibrium is characterized by $F_t = F$, $\varepsilon_t = 0$, $x_t = x$ and $P_t = P$. Simple calculation implies that $x = 0$ and $P = F$, i.e., in steady state, the arbitrageurs hold no position on the asset and the price is equal to fundamental value. Note also that the steady state holds independent of the regime in place. The reason is that if the past prices are all equal, then independent of the regime, the feedback demand is zero. In other words, the economy only departs from the steady state after fundamental shocks. To give an intuition, suppose that the economy at time $t - 1$ is in the steady state and there is an innovation on the fundamental process at time $t$ ($\varepsilon_t$). The equilibrium price at time $t$ is:

$$P_t = F_t + \frac{1}{\alpha} X_t^s = F + \left(1 + \lambda_s \left[q^a \delta^a + q^b \delta^b \right] \alpha \right) \varepsilon_t + \frac{\lambda_s}{\alpha} E_t[x_{t+1}]$$

For now, I claim (and show later) that the expectation term has a positive loading on the $\varepsilon_t$ if the number of lags is greater than the periods that the arbitrageurs lives. As pointed out by De Long et al. (1990) the presence of feedback traders implies that the arbitrageurs amplify the price fluctuation, as opposed to the standard answer (Friedman (1953)) that noise traders would be wash out by speculators. This amplification factor can be seen by the second term multiplying the $\varepsilon_t$. Thus, the arbitrageurs are willing to pay more than the fundamental value today because they expect an even bigger mispricing tomorrow, generated by the feedback traders. Since arbitrageurs have finite time, the best strategy is to "jump on the bandwagon" instead of correcting the mispricing.

1.2.4 Equilibrium Solution: A restricted VAR representation

A close form solution for the equilibrium is difficult to compute due to the regime switching and lag variables. Fortunately, the solution for linear rational expectations models is already established in the literature and the computational codes are available. The inclusion of regime switching, however, is a more recent contribution (Farmer et al. (2006)). In this subsection I discuss the main steps to obtain a solution.

First I explain the solution for a fixed regime, i.e., $s_t = s_{t+k} = s \forall k$. Let $\bar{\theta}$ be the vector collecting all the parameters of the model

$$\bar{\theta} = \left[ \alpha \quad \delta_a \quad \delta_b \quad \rho^a \quad \rho^b \quad \sigma^2 \quad N \right]^\prime$$
and define
\[ \bar{Y}_t = \begin{bmatrix} Y_t & X_t^\alpha & X_t^\beta & Y_{t-1}^\alpha & E_t [X_{t+1}] \end{bmatrix}' \]  
(1.15)

The system of equations (1.9) to (1.13), for a given regime \( s \), can be written in matrix form as
\[ g_0^s (\theta) \bar{Y}_t = g_1^s (\theta) \bar{Y}_{t-1} + \Psi^s (\theta) \varepsilon_t + \Pi^s (\theta) \eta_t \]  
(1.16)

where \( \eta_t \) is a vector of endogenous expectations errors. The solution for a fixed regime can be computed using the Sims (2002) algorithm. The algorithm returns a first order restricted VAR in the state variable:
\[ \bar{Y}_t = \Gamma^s (\theta) \bar{Y}_{t-1} + M^s (\theta) \varepsilon_t \]  
(1.17)

The solution of the rational expectations system with Markov switching is based on the method proposed by Farmer et al. (2006) and is described in detail in appendix C. The idea is to expand the fixed regime system given by equation (1.16) considering all the regimes and write it in an equivalent fixed regime representation. Roughly speaking, consists on stacking the equations (1.16) and also adding regimes shocks in the state space representation. The expanded state vector is
\[ \bar{Y}_t = \begin{bmatrix} Y_t & Y_{t-1} & \vdots & Y_{t-h} \end{bmatrix} \]
and regimes shocks are added in such a way that all the lines in the system are switched off except for those that represents the current regime. The transition matrix \( \Gamma \) is also added in the parameter vector \( \theta = [\theta' \ \text{vec}(\Gamma)'] \). Having the extended fixed regime representation, it is possible to solve using the Sims (2002) algorithm. The solution returns a first order VAR in which the coefficients are independent of the regime
\[ \tilde{Y}_t = G (\theta) \tilde{Y}_{t-1} + M (\theta) \varepsilon_t \]

Finally, it is possible to disentangle each regime and write the state equation as
\[ \bar{Y}_t = A (\theta; s_t) \bar{Y}_{t-1} + M (\theta; s_t) \varepsilon_t \]  
(1.18)

It is important to note that equation (1.17) differs from (1.18) because the former assumes that the regime \( s \) is the same forever (note that the set of parameters in this case does not
depend on the transition matrix). On the other hand, the dynamic implied by equation (1.18) consider that the rational agents take into account the regime switch possibilities (the coefficients vary with the transition probabilities).

**Impulse Response**

In this section I show the impulse response functions. In order to have a better understanding of the different model features, I discuss each one separately.

The first feature is the difference in feedback types and the consequent arbitrageurs reaction. Figure 1.1 shows the price response to a one unity of fundamental shock. The constant dashed line in both graphs represents the impulse response under no feedback regime (the innovation is reflected immediately in the price). To contrast the effect on price in an economy with rational traders (solid line), the line with a marker represents the impulse response in an economy without them. The top graph represents the regime where only long-term feedback traders are present while the bottom graph only short-term types.

There is a momentum followed by reversal effect in both pictures. The arbitrageurs do not wash out the deviation of the price from the fundamental value in the long-term case due to the short lived assumption. However, they are more effective in reducing the volatility in the short term case. For a rational short lived trader, the best strategy is to trade against the short-term trader but to ride the bubble if the current regime is the long-term traders one.

The second feature is the effect of the transition matrix on the impulse response. The simulation assumes that there is only one fundamental shock at time \( t = 1 \) and there is no regime shock (switch) afterwards. Although there is no regime switch, the impulse responses can be very different since in the model the agents are rational and take decisions based on the expected future price behavior, which is closely related to the transition matrix.

In order to simplify the analysis, consider the following example: There are only 2 regimes, one with feedback traders and the other not. Let \( \Gamma = \begin{bmatrix} a & 1-a \\ 1-a & a \end{bmatrix} \) be the transition matrix. Figure 1.2 shows the impulse responses for both regimes (initial state). The parameter "a" measures the persistence degree of each regime in a symmetric fashion.

\(^6\)The momentum effect occurs when the price variation departs from the fundamental value while the reversal occurs when the price variation goes in the fundamental value direction.
Figure 1.1: Impulse Response Functions. Each graph represents the effect of an unit of fundamental shock on equilibrium price over time. The top graph is the effect in the long-term only regime while the bottom graph represents the short-term only regime.

Figure 1.2: Impulse Response Functions with regime switching. Each graph represents the effect of an unit of fundamental shock on equilibrium price over time. The top graph is the effect when the initial state is the non-feedback regime while the bottom graph represents the effect when the initial state is the feedback regime.
If \( a = 1 \) these regimes do not change, and the impulse response is just the one that would prevail in the absence of regime switching, i.e., a flat response in the no-feedback regime and a wave response in the feedback regime (dashed blue line in both graphs). If \( a = 1/2 \), the impulse response is approximately the average between the two extreme situations \((a = 1)\) and it is less sensitive to the initial state. A transitory regime \((a < 1/2)\) generates a fluctuation around the wave generated with \( a = 1/2 \) and it is even less sensitive to the initial state. Interestingly, the fluctuation is around the wave instead of around the fundament. The intuition is that, if the regime is very transitory, arbitrageurs are willing to bet on the change in regime and thus, after a positive shock they sell if the regime is in place and buy if the regime is not.

1.3 Methodology

In this section I describe the estimation methodology and provide an example of the algorithm application using artificial data.

1.3.1 Likelihood Function

The model solution is given by the VAR representation in equation (1.18). The next step is to rewrite the VAR in a state space form for each regime by picking up the respective blocks and isolating the observable variables. The observable variables are the current \((P_t)\) and past vector of prices \((Y^p_{t-1})\). The latent variables are the remaining variables in the vector \(Y_t\) (see definition (1.15)) denoted by \(y_t\). The state space representation has the following form

\[
P_t = F(s_t) y_t + B(s_t) Y^p_{t-1} \tag{1.19}
\]

\[
y_t = A(s_t) y_{t-1} + C(s_t) Y^p_{t-1} + G(s_t) v_t \tag{1.20}
\]

Given the coefficients and the transition matrix, I use the algorithm proposed by KIM (1994) to filter out the unobservable variables and to evaluate an approximated likelihood \(^7\). The detailed filtering method is described in the appendix D.

\(^7\)The exact likelihood consists on tracking all the paths, which is computationally hard. For instance, if there are 2 regimes and 20 periods then there are around 1.000.000 paths.
1.3.2 Testing the Algorithm: Simulation and Estimation with Artificial data

In this section I generate an artificial series to illustrate the price behavior that this paper aims to investigate. Based on 200 realizations of standard normal errors, I simulate the fundamental price process using equation (1.1) and setting the initial value to zero. The fundamental process is the dashed blue line in figure 1.3. Next I simulate 200 realizations of the exogenous regime process. The regime with no feedback traders is plotted in figure 1.3 at the bottom (blue stars). The parameters chosen were $\delta_a = 0.05$, $\delta_b = 0.50$, $\rho^a = \rho^b = 0.75$ and $N = 15$. The parameters choice is motivated by illustrative reasons, they are such that the mispricing is very relevant. Having both shocks in each time and the dynamics described by the state equation given the parameters, I calculate the equilibrium price process plotted in figure 1.3 (thin green line).

Note that in the beginning of the sample, the regime with no feedback traders is in place most of the time. Thus, the observed price and the fundamental values are very similar (first 60 observations). The effect of feedback traders is clearer afterwards for two reasons. Firstly there are more realizations of feedback regime. Second the fundamental value had realizations in the same direction, which amplify the feedback effect. Feedback traders cause bubbles in the asset price dynamics (in both directions) since the realized price can deviate from fundamental value.

Now I test the estimation algorithm discussed in the last section. Suppose that the only variable the econometrist observe is the price series. Using the state space representation given by equations (1.19) and (1.20), and the Kim (1994) algorithm to extract the latent series, it is possible to evaluate the likelihood and estimate the model. The top graph in figure 1.4 shows the filtered series and the true one. The bottom graph plots the mispricing defined as the difference between the fundamental series and the observed price.

The parameter estimates were close to the true ones, and are omitted for simplicity. Interestingly the estimated fundamental process did a good job in the sense of closely matching the true one. One of the contributions of this paper is that the model can identify mispricing generated by feedback traders.
Figure 1.3: Example with artificial data of price and fundamental processes under the feedback model with Markov switching.
Figure 1.4: The top graph shows the price process, the estimated fundamental process and the true one. The mispricing defined as the difference between the price and the fundamental process is plotted in the bottom graph.
1.4 Empirical Results

In this section I estimate the model using a monthly portfolio constructed with U.S. stock market data. I use the monthly frequency for two reasons. First, this paper aims to investigate the long-term feedback behavior, thus the lower frequency is the most adequate. Second, to eliminate the alternative story that non-synchronous trading is the main explanation for the observed autocorrelation. The reason for adopting a portfolio instead of individual stocks is to avoid that idiosyncrasies affect the results.

1.4.1 Data Description

The monthly data comes from Kenneth R. French database. The data is based on portfolios of US stocks formed on size (market capitalization). The portfolios are constructed at the end of each June. The stocks included are from NYSE, AMEX, and NASDAQ available on CRSP database. The portfolios are sorted by deciles.

This paper uses the second smaller decile from Jan/1980 to Jan/2009 (349 observations). The second smaller decile is chosen because less liquid stocks are expected to be more affected by feedback trading. I do not consider the first decile because it is more likely to have outliers. To minimize the problems of non-normality and stationary, I use the de-trended log of the time series.

1.4.2 Priors and Posterior Distributions

The full vector of parameters is given by \( \theta = \left\{ N, \alpha, \delta_a, \delta_b, \rho^a, \rho^b, \sigma^2, vec(\Gamma)^T \right\} \). There are some identifications issues in estimating these parameters. Although I do not provide a closed-form solution for the model, I have performed simulation exercises that pointed out to this problem. The parameter \( \alpha \) has a direct relation to the feedbacks parameters. The intuition is that the greater is the \( \alpha \) parameter, the more aggressive are the fundamental traders to bring the price through its fundamental value (see equation (1.5)). The feedback parameters, on the other hand, are responsible for departing the price from fundamentals. As a result, the likelihood has a similar value with parameters \( (\alpha, \delta_a, \delta_b) \) and \( (k\alpha, k\delta_a, k\delta_b) \), where \( k \) is a constant. To solve this problem, I fix the value for \( \alpha = 1 \). Similar arguments hold for the persistence in old positions (\( \rho \)) and the parameters of the transition matrix.

\(^8\)http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
fix $p^a = p^b = 0.5$, which means that in the period after the feedback traders leave the market (the regime switches), there is no pressure in the price since half of them stay with the stock position and half of them unwind it (net effect is zero in each time afterwards).

Most of the priors are relatively uninformative but still imposing the relevant constraints:

$$\delta_a, \delta_b \sim U[0, 5]$$

$$\Gamma_{ij} \sim U[0, 1], \quad \sum_j \Gamma_{ij} = 1, \ i, j \in \{1, \ldots, H\}$$

$$N \sim Multinomial(30), \ p^i = \frac{1}{30}, i \in \{1, \ldots, 30\}$$

$$\sigma^2 \sim IG$$

In addition to the complete model (Regime Switching), two specifications are also estimated in order to analyze the role of the Markov switching structure and the two types of feedback traders. The Metropolis-Hasting algorithm is used to sample the parameters from the posterior distribution. I report the posterior median, and the 5 and 95 percentiles in table 1.1.

The simplest model with only one type of feedback trader (first block column) has very dispersed estimators and the number of lags suggests that the short-term feedback trader has a more important role than the long-term type. Intuitively, most of the variability in prices appear in bad times, when it is more likely that "fire sale" occurs. The model in the second column block disentangle these two types of feedbacks. By looking at the 5% and 95% quantities we note that the estimators are still very disperse but it seems that the inclusion of the second type is relevant in the sense of amplifying the response to short term fluctuations ($\delta_b = 0.29$). However, both estimators of $\sigma^2_e$ are around 5%, which is very similar to the unconditional estimated variance of the raw series (5.35%). If the time series of the misprice generated by feedback trading is relevant, one would expect that the fundamental volatility would be smaller than the volatility of the observed series (see the term multiplying $\epsilon_t$ in equation (1.14)). The first two specifications do not generate it.

The inclusion of regime switching changes drastically the estimators (third block column in table 1.1). This specification allows disentanglement of the two types of feedbacks and also their time varying effect. The parameters in the first row block have very tight posteriors, i.e., they are strongly significant. Note that the response to short term fluctuations is very
## Table 1.1: Coefficients estimates. There are 3 column blocks. Each block represents the parameters posterior distribution from different specifications of the baseline model.
high. Without regime switching, this value would violate the stationarity of the system. The variance of the fundamental values is 2.16%, which is in line with the theoretical implication of feedback trading generating high volatilities.

The model has cleaned out the effects of short-term high volatility because of the time varying structure. As a result the long term behavior has showed up. The lag parameter \( N \) is equal to 9 (months) and it is strongly significant which suggests that there is a long term feedback component on prices. The strong delay can be justified by non-sophisticated investors that take a position and are reluctant to unwind it.

Most of the transition probabilities estimates have very disperse posteriors, in particular those with lower estimates. For instance, the persistence parameter for the regime without feedback traders \( (p_{11}) \) is 0.52 (0.40-0.59) while the persistence parameter in the short-term feedback \( (p_{14}) \) is 0.03 (0.01- 0.08). The possible reason is that the sample has only 349 observations and those uncommon regimes occurs just a few times, which weakens the estimates accuracy.

### 1.4.3 Filtered Mispricing and Probabilities

The non-observable fundamental process is assumed to have a constant volatility. The time varying volatility in this model is due to feedback trading and regime switching. A by product of the model is to filter out the fundamental process. Figure 1.5 shows the filtered fundamental process and the respective mispricing.

The model has normative implications for practitioners: it is possible to identify the mispricing. For instance, in Jan/2009 the model indicates that the observable price was really below the fundamentals, a huge negative mispricing.

The probability time series is shown in figure 1.6. Price is equal to fundamentals, as defended by proponents of Efficient Market Hypothesis (EMH), most of the time (52% , green area). However, the estimation points that some deviations are possible. The brown area (regime in which both feedback are present) occurs very infrequently but when they occur, the probability is very strong (good identification). In those situations, the mispricing is very relevant.

In order to understand which regime drives the mispricing over time, I grouped the probabilities and mispricing time series in the following way. At each time, I associate the estimated mispricing to the most likely regime (has the greater probability among the
CHAPTER 1. FEEDBACK TRADERS

Figure 1.5: Time series of the filtered fundamental value and the subsequent mispricing.

Figure 1.6: Estimated regimes probability time series.
others). The results are shown in figure 1.7, where each picture represents a different regime. The number of points in each picture represents the number of times in which that regime was pointed as the most likely. Note that the regime with no feedback traders is the one with more observations and the mispricing is very close to zero. I call this regime the "fundamental" regime. The sign of the mispricing is diffuse among the regimes except for the one where both types of feedback traders are present. In this regime, most of the events are negative and with big absolute values. I call this regime the "crash" regime.

![Figure 1.7: Time series of the estimated misprice grouped by the most likely regime.](image)

How are these regimes connected? A closer look on the estimated transition matrix helps to address this question. Table 1.2 shows the median of the posterior distribution of the transition matrix. The first point is related to its stationary distribution. As pointed before, the "fundamental" regime is the most frequent among them with 52% of the time. The long-feedback type (LF) comes in second place with 35% (33% + 2%) and the short-feedback type (SF) with 16%. The "crash" regime, regime where both feedback types are present, occurs only 2% of the time. In other words, on average one out of fifty months have a "crash".

Note that the LF regime is more persistent than the SF one (37% x 20% in the diagonal of the transition matrix). This is consistent with SF being related to extreme events shocks like margin calls and stop losses, which are not expect to persist for a long time. On the other hand, the persistence of LF types weakly suggests that this group is being represented
CHAPTER 1. FEEDBACK TRADERS

Table 1.2: Time series of the estimated misprice grouped by the most likely regime.

<table>
<thead>
<tr>
<th>Stationary Distribution</th>
<th>Regime (t)</th>
<th>Regime (t+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Feedback</td>
<td>NF</td>
</tr>
<tr>
<td>51%</td>
<td>52%</td>
<td>38%</td>
</tr>
<tr>
<td>33%</td>
<td>40%</td>
<td>37%</td>
</tr>
<tr>
<td>14%</td>
<td>65%</td>
<td>13%</td>
</tr>
<tr>
<td>2%</td>
<td>49%</td>
<td>9%</td>
</tr>
</tbody>
</table>

by retail or non-sophisticated investors (they have a slow reaction when they unwind their positions).

The SF regime is more likely to come from the non-fundamental regime (third column of the matrix). A possible interpretation, consistent with stop-loss story, is that positions are more likely to be triggered when there is a mispricing (higher conditional volatility).

The "crash" occurs in the joint regime (LF&SF). One possible explanation is that the volatility is high because there are long term investors selling (kind of panic) while at the same time short-feedback traders are triggered. Interestingly, given that this regime occurred, it is more likely that price return to fundamentals (see last row of the matrix) or at least, it is less likely that the regime with long-feedback types occurs. This finding suggests, that the "crash" is a correction of a possible bubble generated by those long-feedback types, i.e. they "get burnt".

1.5 Conclusion

This paper develops and estimates a model for asset price dynamics in an environment with two types of feedback traders and a rational investor. The presence of feedback strategies follows a stochastic regime shift.

The empirical results suggest that the regime switching is able to capture the time varying autocorrelation of returns. The inclusion of a second feedback type helps to disentangle the long term fluctuation. In absence of this second type, the estimated duration would be short since most of returns variation is associated to short-term feedbacks. The long-term type suggests that bubbles can be caused by non-sophisticated investor subject to representativeness bias and this story is partially supported by the estimated transition
The main contribution is to come up with this long-term behavior on the price dynamics and to provide an algorithm that can filter out the mispricing.
Chapter 2

Rebound Returns Following Extreme Downturns

Abstract This paper documents the overreaction of stock exchange indexes in extreme market downturns. I propose a novel characterization of the extreme events, decomposing it in idiosyncratic and market events. Using daily returns of 14 stock indexes, I found an opposite behavior of next trading day expected returns following these two types of events. On average, a positive abnormal rate of return is expected after idiosyncratic events while a negative abnormal rate of return is expected after market events. The reversal effect cannot be explained by risk differentials and neither by countries size. In contrast with related literature, the decomposition proposed in this paper strengths the reversal effect and generates profitable opportunities.

Keywords: Extreme Events; Overreaction; Price Reversals

JEL Classification: G10; G14
CHAPTER 2. REBOUND RETURNS AFTER EXTREME EVENTS

2.1 Introduction

It is common that after a large decline in stock prices we observe a reversal on the following days\(^1\). Many empirical studies have found significant reversals, especially on the subsequent day after a price decline. It has also been documented that this effect is bigger for small firms and most of its variation is explained by bid and ask bounce and other microstructure reasons (see, e.g., Bremer and Sweeney (1991), Cox and Peterson (1994)).

However, these papers focus on individual US stocks, which raises two questions not yet covered by the literature. Does the reversal effect also hold for the aggregated index and does it hold for other countries? This paper aims to fill this gap. However, at a first glance, one would expect that an analysis at stock index level is less affected by market microstructure factors and thus, it should be more difficult to find the reversal effect. Another aspect uncovered by the literature is whether the nature of price drops can be informative. For instance, take a big drop in a small country stock index. This could be due to a country specific shock or a negative shock that affected the whole market (rest of the world). If the source of price drop is informative, then the next day returns would have different distribution depending on the nature of the shock.

In this paper I investigate the one day returns following extreme events for a large set of stock indexes. Following the related literature, I define extreme events as returns realizations below (above) a fixed trigger. For example, Bremer and Sweeney (1991) and Cox and Peterson (1994) fixed a downside trigger of \(-10\%\) while Brown and Keith (1988) fixed it in \(-2.5\%). The possible caveat of this specification is that extreme events are more likely in most volatile series. If a joint test is conducted under this characterization, the results would be driven by those most volatile series. Even in individual tests, the power of each test would be different, since the number of observations depend on the volatility level. I differ from previous works in definition of trigger. The first methodological difference is that the trigger is characterized by a fixed quantil of the empirical distribution, as a result, each series under analysis have the same number of extreme events (if the sample size is the same). The second difference, most relevant, is to disentangle the nature of the trigger decomposing it in market (rest of the world) and idiosyncratic (individual country) events. The main contribution is

\(^1\) Reversal is defined as a negative autocorrelation of a process, i.e., when an innovation of a stochastic process at time \(t\) is follow, on average, by an opposite innovation at time \(t + 1\). Momentum is defined in analogous way for positive autocorrelation.
CHAPTER 2. REBOUND RETURNS AFTER EXTREME EVENTS

to show that the proposed decomposition generates reversals following idiosyncratic events while momentum following market events. In other words, given a large drop in prices, there is a positive expect return on the next day if the drop was driven by idiosyncratic shocks while there is a negative expect return if the drop was driven by market shocks. Moreover, the magnitude of reversal is higher than the reversal generated by previous works due to this trigger characterization.²

Using daily return in local currency from 14 countries, I first investigate whether there is a significant reversal effect after extreme events. The coefficients of interest tell us what is the expected percentage of rebound is on the following day. For instance, if the reversal coefficient is -0.10 and the drop on the day before was -5%, the expected positive return today is 50 basis points. Starting with idiosyncratic events, I find a significant average reversal in downturns (−0.29, t-statistic of 7.17) and in upturns (−0.14, t-statistic of 3.10). When it comes to market downturns events, the average of coefficients is positive and significant (0.14, t-statistic 2.47), indicating a momentum effect. I also perform the same exercise using abnormal returns instead of raw ones, i.e. controlling for risk factors, and the results became stronger for downturns events. The average of market coefficients increases to 0.17 (t-statistic of 3.27). For idiosyncratic events coefficients, the magnitude of downturn reversals decreases to -0.20 but they become more precise (t-statistic of 8.28). The upturn events became non-significant with an average of -0.08. The same analysis is also conducted for returns defined in US Dollar and the results are similar, except that the momentum effect increase from 0.17 to 0.26. These findings support our main hypothesis that the nature of the extreme events are informative for the next day return distribution.

Next I examine whether the magnitude of coefficients can be explained by risk differentials. Starting with a liquidity explanation, under the null of reversals driven by a liquidity premium, we would expect that smaller countries are more likely to have bigger reversals. I found that this is not the case. A second plausible justification is a simple risk premium, implying that the higher reversals must be related to higher betas. Little evidence is found in this direction. A possible caveat is the joint hypothesis problem, is not possible to be sure about the real risk model that is driving the returns, as a result the findings can be a result of misspecification in the risk model.

Having established the reversal and momentum effect, I next discuss whether these find-

²This result is intuitive since this decomposition disentangle factors with opposite effects.
³I define downturns and upturns as a negative and positive variation respectively.
 CHAPTER 2. REBOUND RETURNS AFTER EXTREME EVENTS

ings generate profitable opportunities. Using index futures contracts, a simple trading strategy that goes long following idiosyncratic events and goes short following market events is implemented. I found that accounting for bid ask spreads, the reversal strategy generates an average of 60 basis points while the momentum strategy is statistically zero. A slightly different trading strategy (and more realistic) that also consider the magnitude of the estimated coefficients raises the average profit of reversals to 120 basis points while the momentum stays non-significant.

The results of the robustness checks indicates that the reversal effect is still significant under different sub samples while the momentum effect is not. The latter effect is mainly driven by recent crisis (2007/2008). Finally, different specifications for the trading strategy are tested and the results are favorable to profitable evidence.

In sum, considering the countries under analysis, the findings point out that there is a reversal effect after drop in returns, the effect is stronger if the drop is characterized by idiosyncratic reasons, and this effect is not justified by bid and ask bounces.

The organization of the paper is as follows. Section II describes the model specification and the data. In Section III I show the empirical results, discuss the economic significance of the findings and possible theoretical explanation available in the current literature. Section IV concludes.

2.2 Methodology and Data

2.2.1 Events Set Specification

The extreme events are divided in two types: (1) market extreme events and (2) idiosyncratic extreme events. From now on, I suppress the term “extreme” to characterize the events. The events are defined as follows. Let \( r = \{ r_t \}_{t=1}^T \) be an arbitrary time series of daily return and \( F \) its empirical distribution. Each \( r_t \) is compared to a trigger. If the return is lower than this trigger, it is an extreme event. The trigger is defined as a function that maps a given quantile choice \( z \in [0,1] \), from the empirical distribution \( F \), into the respective threshold value. The set of time indices for downside events is defined as

\[
D(r) = \{ t | r_t < q(z) \}, t = 2, \ldots, T
\]

Let \( r^m \) and \( r^i \) be the daily time series returns of market and country \( i \) respectively. The
market events set is directly given by

\[ D^m \triangleq D (r^m) \]

For idiosyncratic events a careful approach must be taken. The problem in defining the extreme events using only the raw individual series, say \( D (r^i) \), is that since \( r^i \) and \( r^m \) are highly correlated, a multicollinearity problem can arise when running a regression using these sets as dummies variables. In order to disentangle the idiosyncratic events, I propose the following approach: (1) define a new series based on the difference of individual returns from market returns, and (2) exclude all remaining common events. As a result, the idiosyncratic extreme events set \( D^i \) is defined\(^4\) as

\[ D^i \triangleq \overline{D^i} / \left( D^m \cap \overline{D^i} \right) \]

where 

\[ \overline{D^i} \triangleq D (r^i - r^m) \]

I show in empirical section that, on average, the correlation between extreme events from raw and market series is in order of 50\%, which is reduced to around 20\% if the difference in returns is considered and finally, and not surprising, goes to 0\% after the exclusion of remaining common events. Note that the most important step is to take the excess return over the market, since this characterizes abnormal events. One could argue that the zero correlation can be achieve only by defining a set with the remaining events from the raw series\(^5\). The result is that, since these series are highly correlated, the remaining set would have too few observations. Another advantage in taking the excess return is that the events become sparser over the time, while the raw data events are more clustered around market events, which violates independence assumption of the realizations. Finally, it is intuitive that idiosyncratic events should be characterized by returns that are not explained by market factors, and taking only the difference avoid the problems with beta estimation, which yields a more parsimonious characterization.

The upside events are defined in a similar form

\[ U (r) = \{ t | r_t > q (1 - z) \} \]

The advantage of this specification is that both sets \( D \) and \( U \) have the same number of observations. The caveat of this specification is that once the return distribution is not

\(^4\)The notation \( A/B \) means the elements of the set \( A \) that are not in the set \( B \).

\(^5\)\( D (r^i) / (D (r^m) \cap D (r^i)) \)
symmetric, the returns associated to the time indices in set $D$ can have elements with much more absolute value than the returns associated to the time indices in $U$, as a result the comparison between the two sets is not so fair\textsuperscript{6}.

The choice of $z$ has a trade-off between how extreme are the events and the number of observation to perform the statistical test. I perform the empirical exercise with $z \in \{1\%, 2.5\\%\}$.

\subsection*{2.2.2 Data}

The daily data on stock index, futures prices and currencies come from Bloomberg database. The Morgan Stanley Capital International Word index (MSCI) is used as a proxy for the market factor, available in MSCI Barra website. The stock indexes from the following fourteen countries are included: Belgian, Brazil, Canada, Finland, France, Germany, United Kingdom, Mexico, Russia, South Africa, Spain, Sweden, Switzerland and United States. The series are based in closing value of its components. From future market, I collect the closing prices and closing bid-ask spread. As a risk-free interest rate proxy I use Federal Funds Effective Rate (FFO) available on Federal Reserve database.

The summary statistics of the index returns are presented in table B.1. Columns (2) and (3) presents the name and market capitalization of each stock index. The market capitalization is defined as the average market capitalization in USD currency for the last year of the sample\textsuperscript{7}.

Columns (4) and (5) summarizes the starting dates and the number of observations. Since the MSCI Word Index daily series is available only from 01/04/1988 (5479 observations), I choose this date as a earliest starting date for ali series. Four out of fourteen countries do not have available data from this date. The country with latest starting date is Russia, beginning in 08/01/1994 (3764 observations). The ending date is 12/23/2008 for ali series.

The remaining columns (6) through (8) report statistical properties of the series. To

\textsuperscript{6}We also test a second specification to overcome this caveat, that relies on a range criteria: $D^r = \{t | z_2 < r_{t-1} < -z_1\}$ and $U^r = \{t | z_2 > r_{t-1} > z_1\}$ where $z_2 > z_1 > 0$. Using these sets, the events are symmetrically in the sense of the range they lie on. The caveat of this specification is that the number of observations in each set can be potentially different, so the comparison test will not have the same power. The results of this alternative specification do not change our main results.

\textsuperscript{7}Note that since there are countries with more than one stock index, I choose just one index for each country, the reported market capitalization not necessarily represents the country stock market size.
minimize the problems of non-normality, I take the log of the time series. Since we focus
on returns (difference of log time series) the nonstationary problem is also minimized. All
series are in local currency in excess of risk free rate of return\(^8\). The mean of the series are
almost zero (since is a daily base) and is not reported. The average daily standard deviation
is around 145 bases points. The autocorrelation of stock index are less than 10% for all
but United Kingdom, which is 13.3%. Interestingly, the MSCI series is more autocorrelated
than any of each stock index, with 14.5%. Since one of our results indicates a positive
autocorrelation in market events, a special care must be taken with the conclusions. One
possible explanation is that the MSCI world index is composed also by Asian countries,
which have a different time zone. As a result, news in trading hours of American and
European markets are incorporated in Asian market on the next trading day. This effect
could also explain why the correlation of the countries with MSCI index are less than 80%
(last column). We provide a robustness check for this effect in last section of the paper.

The summary statistics of future contracts are presented in table B.2. The index future
contract provided by Bloomberg is a Generic Future contract for the shortest maturity
available. This Generic contract represent effective trading prices and has the advantage
to collect the time series. In additional to close price, there are also available bid and
ask closing prices, however with fewer observations. Columns (2) and (3) summarizes the
starting dates and number of observations. We chose as the earliest starting date 01/02/1995
(3624 observations) because before that all data from bid and ask were way too sparse. In
contrast to the spot prices, the futures contracts data available diverges very much among
the countries. The country with latest starting date is Mexico, beginning in 03/20/2008 (184
observations).The ending date is 12/08/2008 for all series. Columns (5) through (7) report
the averages of bid-ask spreads for all sample and for the most expensive days. Note that
the average of all samples is around 17 basis points while the 1% most expensive days are
around 121 basis points. From these numbers, it is clear that the bid-ask spread must be
considered into the analysis, specially because their magnitude can be big enough to wipe out
any profitable strategy using future contracts to explore the extreme events information.

\(^8\)Note that I use as a proxy the Fed Funds rate for all countries. This is a simplification because each
country has a different interest rate. Since this study focus on one day return, the interest rate have very
small effect in our results and could even be desconsidered in the analysis (10% a year represents 3 to 4 basis
points in one day).
2.3 Empirical Findings

2.3.1 Summary of Extreme Returns in Raw Data

In this section, the magnitude of the reversal effects in raw data are analysed for different choices of events set. As described in last section, there are four types of events set for each quantil chosen. In table B.3 and table B.4 we summarize of return statistics following these events, detailed by each country, for the quantil choices of 2.5% and 1%.

The bottom line of these tables is reported in table 2.1 for convenience. Column (3) reports the average return on the day that the return was less than the trigger (D+0) and column (4) report the average return on the next trading day (D+1). The results for the trigger 2.5% are, as expected, weaker than those for 1%. Thus, we focus our analysis henceforth only on the results related to the 1% trigger (second table).

Considering first raw events (second row), note there is an average positive expected return on D+1 of 34 basis points (column (4)) and the percentage of positive returns is 56% (column (5)). Roughly speaking, this expected return is not big enough to cover bid ask spreads. Thus, although there is a reversal, there are not profitable strategies to explore it. This is the current state of one-day-reversals literature.

<table>
<thead>
<tr>
<th>Quantil 2.5%</th>
<th>Average Number of Events</th>
<th>Average Return D+0 (bps)</th>
<th>Average Return D+1 (bps)</th>
<th>Percent</th>
<th>Cont. of market events</th>
</tr>
</thead>
<tbody>
<tr>
<td>[MKT]</td>
<td>136</td>
<td>270</td>
<td>147</td>
<td>47%</td>
<td>1.00</td>
</tr>
<tr>
<td>[RAW]</td>
<td>126</td>
<td>500</td>
<td>0.1</td>
<td>53%</td>
<td>0.51</td>
</tr>
<tr>
<td>[RAW-MKT]</td>
<td>126</td>
<td>394</td>
<td>46.5</td>
<td>58%</td>
<td>0.19</td>
</tr>
<tr>
<td>[RAW-MKT \ [MKT]</td>
<td>113</td>
<td>388</td>
<td>47.5</td>
<td>58%</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantil 1%</th>
<th>Average Number of Events</th>
<th>Average Return D+0 (bps)</th>
<th>Average Return D+1 (bps)</th>
<th>Percent</th>
<th>Cont. of market events</th>
</tr>
</thead>
<tbody>
<tr>
<td>[MKT]</td>
<td>54</td>
<td>377</td>
<td>22.1</td>
<td>43%</td>
<td>1.00</td>
</tr>
<tr>
<td>[RAW]</td>
<td>50</td>
<td>672</td>
<td>32.8</td>
<td>56%</td>
<td>0.51</td>
</tr>
<tr>
<td>[RAW-MKT]</td>
<td>50</td>
<td>520</td>
<td>101.1</td>
<td>62%</td>
<td>0.18</td>
</tr>
<tr>
<td>[RAW-MKT \ [MKT]</td>
<td>45</td>
<td>517</td>
<td>102.2</td>
<td>62%</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of return statistics conditional to different choices of events sets, for all countries in local currency. The notation for the event sets is as follows: $[RAW] \triangleq D(r^i)$, $[MKT] \triangleq D(r^m)$ and $[RAW - MKT] \triangleq D(r^i - r^m)$
CHAPTER 2. REBOUND RETURNS AFTER EXTREME EVENTS

Now consider the market events. Since the raw and market events are highly positively correlated (column (6)), it is intuitive to understand why the decomposition would strengthen the negative autocorrelation in idiosyncratic events\(^9\). The positive autocorrelation effect from market events will be reduced. The results are shown in the rows (3) and (4). Note that the average return jumps from 34 to 109 basis points on average, while the positive fraction of returns jumps to 62% from 56%. This is the main finding of this paper. Under a very simple characterization of extreme events, the idiosyncratic effect is much stronger.

2.3.2 Econometric Results

Raw Returns

In this section I implement the following regression, country by country, for the 1% quantil

\[
r_i = \alpha + \gamma_m^i \delta_1(D^{m}) \gamma^{m}_{i} + \left[ \gamma_0^i + \gamma_D^i 1(D) + \gamma_U^i 1(U) \right] r_{i-1} + u_i
\]

where \(r_i \) and \(r^{m}_{i} \) are the country and market returns, \(1(A) \) is a dummy variable for an arbitrary event set \(A\) and the innovation term \(u_i \) has a volatility process specified as a \(GARCH(1,1)\).

The coefficients of interest are \(\gamma_m^i, \gamma_D^i\) and \(\gamma_U^i\), which tell us what is the effect of an extreme event on next day return. Specifically, I test the null hypothesis of a reversal following extreme idiosyncratic events, which means a negative coefficients for \(\gamma_D\) and \(\gamma_U\). For market extreme events, the null hypothesis is that there is a momentum, which implies a positive signal for \(\gamma_m\).

The detailed results are reported in table B.5. To give an overall view of the explanation power of these regressions, Column (2) reports the \(R^2\) statistic for each country using all sample residuals while column (3) reports the \(R^2\) statistic restricted to D+1 events (the day after the extreme return). The weighted average \(R^2\) is 2.1%; such a low explanation power is expected since there are no contemporaneous risk factors as regressors. We include and discuss the risk factors on the next subsection. On the other hand, the conditional returns are very well explained by this set of regressions, accounting for an average of 47.8% of its variation (Column(3)).

\(^9\)However, this conclusion is not straightforward because the set \([RAW-MKT]\) is different from the set \([RAW]\backslash[MKT]\).
The key finding is that most of the countries in this analysis have a significant negative coefficient on the idiosyncratic downturn dummy variable, implying that there is a reversal in days after idiosyncratic events. For instance, the coefficient for the U.S. is -0.23 with a t-statistic of 2.4, implying that a 3% drop in US index on the day before, if characterized by idiosyncratic event, is expected to rebound 70 basis points on the next day. The weighted average of these coefficients among countries is -0.29. Out of 14 countries, 12 have a negative point estimate while 9 have a estimated value lower than -0.25. These findings are strongly significant and suggest that the rebound is high enough to compensate bid-ask spreads. We address this issue in the last section.

On the other hand, the upturns coefficients have very few significant elements (2 out of 14). They have also, on average, negative signs, but the average magnitude is lower than the downturn coefficients. The weighted average is -0.14. The negative sign implies that if extreme good news hit the stock market of a country, it is likely that on the next trading day the returns are going to decrease on average. Out of 14 countries, 8 have absolute value lower than 0.11. As a result, there exists a reversal pattern on idiosyncratic upturn events, but this effect is statistically weak.

When it comes to market based threshold, the results are also weak. First of all, there are less statistically significant coefficients (5 out of 14) on the market downturn dummy. In contrast to idiosyncratic dummy, most of them have positive sign and the dispersion is higher among countries. Out of 14 countries, 11 have a positive point estimate and the range of values is between -0.28 to 0.66. The high dispersion and low significance of the coefficients imply that there is no clear direction after market events.

Abnormal Returns

Having established the reversal evidence in raw return data, I next explore this effect for abnormal returns. Abnormal returns are defined as excess returns over a certain set of risk factors implied from an asset pricing model. For example, the CAPM model implies that the market return is the only risk factor. I chose the risk factor based on the findings of Harvey et al. (1994). Specifically, he measures the conditional risk for 17 countries and find that differential in risk exposures among countries explain the difference in returns\textsuperscript{10}. I use

\textsuperscript{10}Even though the countries selected in the current paper differ from his paper, there are nine out of fourteen that match. I assume that his findings also hold for the set of countries in this paper.
the return on MSCI World Index as a proxy for the risk factor denoted by $X_t$. The analysis of
abnormal return consists on running the regression (2.1) including this factor. A second
specification for returns, denoted by $\tilde{r}_t^i$, consists on converting the local currencies returns
to US Dollars. This return can be interpreted as an unhedged return, since the variation in
the currency also affects it. The following regressions are analyzed in addition to regression
(2.1):

\[
\begin{align*}
\hat{r}_t^i &= c + \beta^i X_t + \gamma^i m_1(D^m) r_{t-1}^m + \left[ \gamma_0^i + \gamma_1^i D_1(D^m) + \gamma_2^i U_1(U) \right] r_t^i + u_t^i \\
\tilde{r}_t^i &= c + \beta^i X_t + \gamma^i m_1(D^m) r_{t-1}^m + \left[ \gamma_0^i + \gamma_1^i D_1(D^m) + \gamma_2^i U_1(U) \right] \tilde{r}_t^i + u_t^i
\end{align*}
\]

The detailed results for these regressions are reported on tables B.6 and B.7 (appendix).
For a comparison among the regressions specifications, I report the key summary of results
on Figure 2.1. As expected, the inclusion of market factor has strongly increased the average
explanation power of the returns variation (from 2.1% to 35.7%). When it comes to variation
in D+1 returns, an average of 70.3% is explained by this set of regressors.

<table>
<thead>
<tr>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Figure 2.1: Summary results of individual regressions (1) to (3).

The market event dummy has, in regression (2), 9 out of 14 significant coefficients in
contrast to only 5 for regression (1). More interesting is that all 9 significant coefficients have a positive value, with an average of 0.29. This suggests an evidence of a momentum effect for market shocks. For instance, the market dummy coefficient for France is 0.23 (t-statistic 2.7) and the idiosyncratic dummy coefficient is -0.25 (t-statistic 3.4). This means that, if the France index return dropped 3% on the previous day, it is expected, on the next trading day, a positive return of 75 basis points if the drop was generated by an idiosyncratic event and a negative return of -69 basis points if the drop was generated by a market event.

The idiosyncratic downturn event dummy has a smaller average effect in regression (2) than in regression (1). The average effect decreased from -0.29 to -0.21. The number of significant coefficients dropped from 12 to 11. This is not surprising since US is the country with no more significant effect. Thus, for US, almost all variation in returns is explained by the contemporaneous MSCI (in particular, the $R^2_{D+f}$ coefficient for US is 91.3%). Most interestingly is that all idiosyncratic coefficients are negative, which strengthens the results.

In sum, the key findings discussed for raw return are also preserved controlling for risk factors, thus the reversal effect also hold for abnormal returns. Moreover, the results for abnormal returns gave us a new evidence of market events being significant and a surprising momentum effect that has not been documented yet in the literature.

### 2.3.3 Economic Significance

Having established the reversal and momentum effects, I now analyze the economic significance of these effects. I define economic significance as situations where the excess returns cannot be explained by risk differentials or if there are profitable opportunities. I start exploring the risk-based explanation for the coefficients among countries. Subsequently, I examine if a simple trading strategy is able to generate average profits controlling for bid and ask bounces.

### Risk Based Explanation

What would be possible explanations for the idiosyncratic reversals? The natural search for this answer is the liquidity premium. For instance, Cox and Peterson (1994) analyze the returns following one day large decline (however, their focus are on individual NASDAQ stocks). They argue that if temporary liquidity plays an important role in reversals, we should observe a stronger effect in less liquid markets. They find a strong relationship
for individual stocks. I use as a proxy of market liquidity the logarithm of stock index market capitalization (converted to US dollar). This measure is used as the first regressor. Secondly, I analyze whether cross section differences in risk exposures explain the magnitude of reversals. To investigate should this be the case, I include the estimated betas as a second regressor.

I run a simple regression of the estimated idiosyncratic reversals coefficients ($\hat{\gamma}_D$) in a constant, a size variable and beta coefficients. The results are shown in Table 2.2. In Panel A the coefficients are estimated using local currency returns and in Panel B considering US Dollar returns.

<table>
<thead>
<tr>
<th>N. of Obs.: 14</th>
<th>Constant</th>
<th>Beta</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Local currency</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coefficient</td>
<td>(0.14)</td>
<td>(0.07)</td>
<td>0.01</td>
</tr>
<tr>
<td>t-statistic</td>
<td>0.21</td>
<td>0.39</td>
<td>0.21</td>
</tr>
</tbody>
</table>

| **Panel B: USD Currency** | | | |
| coefficient | 0.15 | (0.37) | 0.02 |
| t-statistic | 0.03 | 2.05 | 0.46 |

Table 2.2: Regression of the countries estimated coefficients on risk factors.

Before describing the results, there is a caveat in this analysis that must be noted. The conclusion is subject to the joint hypothesis problem, i.e., relies on the risk model chosen. In particular, even though it is well documented that the beta is time-varying, I use a constant beta specification because the idiosyncratic events occur in different times among countries. In order to have a measure of risk exposition comparable, I adopt this criterion. The local currency returns can be seen as hedged returns, since they are neutral from own currency variation. From Panel A, none of the coefficients are significant, implying that there is no support for a liquidity premium explanation for the reversals under hedged returns. From Panel B, the constant and size regressor are still non explicative but the difference in risk exposures, measure by the estimated beta parameter is significant at 10%. We could interpret this as a support of a world stochastic discount factor driving expected returns, what makes sense in a financially integrated market. Under this hypothesis, the expect positive reversal would be explained by the risk exposure to the world factor under unhedged returns. However, to foreshadow the robustness checks in last section, it is not possible to confirm that this coefficient is significant under different sub samples. I consider this finding weak,
therefore is not sufficient to support a risk based explanation for the reversals.

**Profitable Opportunities**

To study implementable strategies, we need to consider tradable contracts\(^\text{11}\). For this reason, I use the future contracts for this exercise. The time series are composed by futures contracts with the lowest maturity available (generally less than 1 month). These contracts are tradable and the data contains also bid and ask quotes in addition to the close price. The caveats are that the sample has fewer observations than used in the econometric section and that the prices can be slightly different from the spot market. I define reversal (momentum) trade as a long (short) position on the future contract on the actual day and a liquidation on the next day (one day holding period). Three different strategies are analyzed. Firstly, to a better understanding on the gain of events segregation, the first strategy is only a reversal trade for every extreme event. The second strategy consists on a reversal trade for idiosyncratic events and a momentum trade for market events. Finally, the third strategy is more realistic in the sense that it estimates the regressions using past data and implements the second strategy only if the reversals/momentum coefficients are relevant (signal and magnitude), otherwise it discards the trade.

Formally, let \( \tilde{r}_i = \{ r_i^{(j)} \}_{j=t-T-1}^t \) be the time series of country \( i \) raw returns for a fixed window of size \( J \). I build the sets of extreme returns dates \( (D_{i,d}, D_{i,i}) \) as defined in section (1) based on \( \tilde{r}_i \) for a chosen quantil \( z \). Let \( \tilde{\gamma}_{i,m,t} \) and \( \tilde{\gamma}_{i,D,t} \) be the estimated coefficients of the regression (2.2) using the series \( \tilde{r}_i \). The returns for each strategy are:

**Strategy (I)**: \( St_{i,\text{long}} = \{ r_{i+1}^{(t)} | t \in D_{i,d} \cup D_{i,i} \} \)

**Strategy (II)**:

\[
\begin{align*}
St_{i,\text{long}}^{\text{long}} &= \{ r_{i+1}^{(t)} | t \in D_{i,d} \} \\
St_{i,\text{short}}^{\text{short}} &= \{ -r_{i+1}^{(t)} | t \in D_{i,i} \}
\end{align*}
\]

**Strategy (III)**:

\[
\begin{align*}
St_{i,\text{long}}^{\text{long}} &= \{ r_{i+1}^{(t)} | t \in D_{i,d} \text{ and } \tilde{\gamma}_{i,D,t}^{(t)} < -c \} \\
St_{i,\text{short}}^{\text{short}} &= \{ -r_{i+1}^{(t)} | t \in D_{i,i} \text{ and } \tilde{\gamma}_{i,m,t}^{(t)} > c \}
\end{align*}
\]

\(^{11}\)There is no market for spot stock index.

\(^{12}\)The additional superscript \( t \) for the sets \( D_{i,d} \) and \( D_{i,i} \) means that these sets change over time due to the rolling window property of \( \tilde{r}_i \).
These returns can be defined as gross returns or net returns, where the net return considers the bid ask spread as follows: If the strategy in $t$ consists on a long position, buy at ask price in $t$ and sell at bid price in $t + 1$. If the strategy consists on a short position, sell at bid price in $t$ and buy at ask price in $t + 1$.

The aggregate strategy returns among countries is defined as $SZ_j = \cup_i S_i Z_i^j$, where $Z_i^j \in \{P, IP, III^f\}$ and $j \in \{long, short\}$.

Before going to the empirical results, three arbitrary parameters must be chosen. The first is the quantile $z$ for a characterization of extreme events. I chose $z=1\%$ because, as discussed before, the reversal/momentum effect are not so relevant to less extreme quantiles. For the trading strategy there are the window size $J$ and the threshold $c$. It is important to note that it is not my interest to find an optimal strategy, only to show that there exists a simple setting that generates profitable opportunities. With this in mind, I chose $J = 750$ and $c = 0.15$. The biggest series in the sample has 3624 observations, but the average is 2737. The choice of $J=750$ is justified because we need enough trading events in the sample (around 2000 trading days on average). The choice of $c = 0.15$ is that, roughly speaking, for an average downside return of 350 basis point, the reversal strategy is only implemented if the expected gross return is at least 50 basis point. Although these choices are arbitrarily, the same exercise is run for different parameters and I discuss it in the robustness section.

The results of strategies I and II are reported in table B.8. There are three main blocks, the first and second are $SP_i$ and $SIP_i$ gross returns, and the third are $SIP_i$ net returns (from bid and ask). The key point here is to understand the effect of the events decomposition in returns. For instance, take Switzerland where there are 66 events. The mean return of strategy I is almost zero. The percentage of positive returns is 52\% (34/66). Now consider the strategy II, the 66 events are split in 23 idiosyncratic and 43 market events. Using reversal trades for the former events, the mean return raises to 217 basis point with 83\% (19/23) of the returns greater than zero. A similar effect is found on market events, but using momentum trades (going short). The mean return is 116 basis point with 65\% (28/43) greater than zero. Finally, I incorporate bid and ask prices on the last block of the table. Note that since there are not available bid-ask data for every closing price, the number of events is lower. The number of remaining events for Switzerland reduces to only 18 (from 66). Considering these events, the mean return of long and short trades, with bid and ask inclusion, goes to 133 and 72 basis points. When considering all countries, the evidence goes in the same direction. Table 2.3 shows the summary of results for all countries, for each
strategy.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Number of Events</th>
<th>Average Return (bps)</th>
<th>Perc. &gt;0</th>
<th>Fraction of Countries with Ret. &gt;0</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>Long</td>
<td>855</td>
<td>9</td>
<td>51%</td>
<td>9/13</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>323</td>
<td>136</td>
<td>68%</td>
<td>10/13</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>572</td>
<td>63</td>
<td>59%</td>
<td>3/13</td>
</tr>
<tr>
<td></td>
<td>Long (bid-ask)</td>
<td>208</td>
<td>60</td>
<td>51%</td>
<td>10/13</td>
</tr>
<tr>
<td></td>
<td>Short (bid-ask)</td>
<td>376</td>
<td>(22)</td>
<td>50%</td>
<td>6/13</td>
</tr>
<tr>
<td>SII</td>
<td>Discarded</td>
<td>315</td>
<td>(19)</td>
<td>51%</td>
<td>7/13</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>192</td>
<td>180</td>
<td>71%</td>
<td>12/13</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>392</td>
<td>124</td>
<td>64%</td>
<td>0/13</td>
</tr>
<tr>
<td></td>
<td>Long (bid-ask)</td>
<td>107</td>
<td>139</td>
<td>68%</td>
<td>11/13</td>
</tr>
<tr>
<td></td>
<td>Short (bid-ask)</td>
<td>265</td>
<td>19</td>
<td>53%</td>
<td>5/13</td>
</tr>
</tbody>
</table>

Table 2.3: Summary of results of the trading strategies.

The returns from SI are very disperse among countries and on average are small. The average among countries is almost zero (9 basis points) while the percentage of returns greater than zero is 51%. For returns in excess of market factor the results are similar. I conclude that there is no profitable opportunities for strategy I. For strategy II the total of 895 events were split in 323 reversals and 572 momentum trades. This filter generates a more clear direction of the returns. The buy signal has an average of 136 basis points and 68% of gross returns are positive while the sell signal has an average of 63 basis points with 59% of gross returns positive. Considering the bid ask costs the total of 895 events reduces to 584, split in 208 and 376 between the buy and sell signals. The average returns falls to 60 and -22 basis point. In sum, three conclusion are derived from this exercise: (1) There is no reversal in raw downturn extreme events; (2) The decomposition of events in market and idiosyncratic allows positive expected gross returns, going short after a market events and going long after idiosyncratic events; (3) Only the reversal strategy (idiosyncratic events) has positive expected net returns.

Finally, although I report 60 basis point of expected net return, one could argue that this is not expressive enough. To this end, I use the strategy SIII, that is based also on estimated coefficients (using past data) before entering in a position. Although this strategy discards 392/895 signals, the reversal strategy has an expected net return of 132 basis points,
positive percentage is 62% and 12 out of 13 countries are in the positive side. In sum, this effect is very robust. The detailed values for each country is reported on Table B.9.

2.3.4 Robustness Checks

Summarizing the results so far I have found that: (1) Reversal following idiosyncratic extreme events are significant (2) Momentum following market extreme events are significant (3) Reversals defined in US Dollar are explained by risk differentials but not by size (4) Bid ask bounces eliminate profitable opportunities for momentum effect but not for reversal. In this section I conduct a number of exercises to verify the robustness of these findings.

The initial sample is split into two sub samples, namely Sub1 and Sub2. Sub1 is from 01/04/1988 to 07/03/1998 (2740 observations) and Sub2 is from 07/03/1998 to 12/23/2008 (2741 observations). I run the regression controlling for market factor and using the returns in local and US Dollar currency as in econometric section. The summary of results is reported in Table B.10. The key finding in this exercise is that the average reversal effect is robust in both sub samples but the momentum effect is not.

The difference in regressions using local currency and USD currency follows the same relative behavior as discussed for the full sample, i.e., the reversal effect is slightly smaller when the returns are unhedged. Thus, I focus the analysis on the USD returns regression for brevity. The average reversal coefficient for Sub1 is -0.17 (t-statistic of 4.04), for Sub2 is -0.16 (t-statistic 3.80) while for the full sample is -0.18 (t-statistic 5.80). I conclude that the average reversal effect is robust to sub samples. However the number of significant coefficients among countries has decreased (from 8 in full sample to 4 in Sub1 and 7 in Sub2 out of 14). Part of this decrease can be associated to the fact that the number of extreme events were reduced by half or less, decreasing the precision of the test.

When it comes to momentum effect, the results are weaker in Sub1. The average momentum coefficient for Sub1 is 0.23 (t-statistic of 1.51), for Sub2 is 0.25 (t-statistic 7.26) and for the full sample is 0.26 (t-statistic 5.65). Although the average value is very close and 11 of 14 coefficients are positive, the dispersion among countries is very high for Sub1. It seems that the full sample results are driven mostly by Sub2 events. The explanation is that the market extreme events are clustered around the end of Sub2, i.e., most of then are related to the 2007/2008 credit crisis.

I next analyze the risk based explanation for the reversal effect. There is no significance
in size and neither in betas. The value of coefficients are not even close to those reported for
the full sample, reflecting the low power of a small sample size. Using this simple analysis,
we cannot find a explanation based on risk differentials and neither in stock index size.\footnote{The detailed results are available upon request.}

Finally, I pointed out in last section that a simple reversal strategy that uses estimated
coefficients generates an average return of more than 100 basis point, considering bid and
ask spreads. In that exercise I fixed two parameter arbitrarily, the estimation sample size
($J = 750$) and the threshold value ($c = 0.15$). In order to investigate whether the results
are robust to different set of parameters, I perform the same exercise for a grid using the
combination of $J \in \{500, 750, 1000\}$ and $c \in \{0.0.1, 0.15, 0.20, 0.25\}$. I report the average
return, net of bid and ask spreads, for these strategies using each combination of parameters
values. The results are reported on table 2.4. Value in parenthesis are the number of
observations out of 899 extreme events in our sample. The shadow quantities represent
the values reported on last section. The key point is that the expected profit of 139 basis
point is not result of the arbitrarily parameters $c = 0.15$ and $J = 750$. Actually, there is an
even better result for different choices. And, at least in this sample, there is a well defined
behavior of the expected returns. As higher are the constraints (c value), higher is the
expected return. The second behavior is that higher sample size reflects in lower expected
return. This suggest that the coefficients (including betas) are time varying therefore the
estimation using a lower sample size does a better job in terms of predictability of reversals.

As stated in beginning of this section, I provide some robustness checks to verify the
four main findings of the paper. There is not enough support for the third finding,i.e., that
reversals are explained by risk differentials. The momentum effect also weakens under the
subsample analysis. The reversal effect is still robust and its implication in trading strategies
also.

### 2.4 Explanation for Reversal Effect

The literature for this sort of daily reversal is mostly empirical. Those works focus on
the puzzling evidence against the efficient market hypothesis (EMH), in which all available
information should be reflected in price, resulting in an unpredictable risk adjusted returns.
Under EMH we would not expect statistically significant reversals. The usual explanation for
CHAPTER 2. REBOUND RETURNS AFTER EXTREME EVENTS

this violation of EMH is that these reversals do not represent profitable opportunities. For instance Cox and Peterson (1994) argue that extreme decrease in price is likely to be associated with substantial selling pressure, enhancing the probability that a closing transaction is at a bid price and, in turn, leading to a reversal on next day due to bid-ask bounces. In this case, there is a significant reversal but is not possible to make profit on it. Their empirical finding point out that bid-ask bounces accounts for most of the reversal effect and the remaining residual is indicative of compensation for short term liquidity suppliers who would otherwise not trade. My findings contrast with theirs because even using a very conservative strategy that buys at ask price and sells at bid price on next day, there is still a positive expected returns under my specification of extreme events.

From theoretical works, there are a literature that is not directly for this short term reversal but helps to understand the mechanism behind it. For instance, asset pricing models with heterogeneous agents helps to explain this phenomenon as a reward required by the support buyers who provide liquidity in bad times (see, Campbell et al. (1993)). Within their model the decrease in price could be due to bad news about the fundamentals or an exogenous liquidity shock. In the later case, the prices are likely to reverse. Although they do not focus on extreme events, their framework is the most likely to explain our findings. An extreme event for a stock index is almost sure to be due to bad news. But at the same time is very likely that liquidity shocks arise together due to stop loss rules, delta hedge rebalancing

<table>
<thead>
<tr>
<th>Threshold (c)</th>
<th>Estimation Window Size (J)</th>
<th>Panel A: USD Currency</th>
<th>Panel B: Local Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500</td>
<td>750</td>
<td>1000</td>
</tr>
<tr>
<td>0.00</td>
<td>126</td>
<td>104</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>(133)</td>
<td>(133)</td>
<td>(124)</td>
</tr>
<tr>
<td>0.10</td>
<td>164</td>
<td>137</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(114)</td>
<td>(127)</td>
<td>(102)</td>
</tr>
<tr>
<td>0.15</td>
<td>196</td>
<td>139</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>(94)</td>
<td>(107)</td>
<td>(77)</td>
</tr>
<tr>
<td>0.20</td>
<td>233</td>
<td>150</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>(76)</td>
<td>(86)</td>
<td>(69)</td>
</tr>
<tr>
<td>0.25</td>
<td>231</td>
<td>195</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>(62)</td>
<td>(85)</td>
<td>(38)</td>
</tr>
</tbody>
</table>

Table 2.4: Robustness checks for Strategy III. Different choices for parameter $J$ (sample size) and $c$ (threshold value).
and panic. As a result, those investors who provide liquidity in these times require a premium for that. What is puzzling is a 100 basis point premium.

2.5 Conclusion

This paper documents the one day predictability of stock index returns followed by a large negative return on the previous trading day. Specifically, it investigates the expected return under two different specifications of negative events trigger, defined as idiosyncratic and market events. Using a set of 14 stock indexes, I found that the expected return following raw negative returns are statistically zero while the decomposition generates strong and significant reversals for idiosyncratic events and a slightly significant momentum for market events.

Possible economic explanations are explored and the result is the absence of any support of a risk differential or size effect driving the reversals. Moreover, the consensus explanation for individual stocks that the reversals do not generate bid-ask adjusted expected returns does not hold for the conditional reversals proposed in this paper.
Chapter 3

Effects of US Shocks on the Canadian Economy using a DSGE Model

Abstract We model and estimate a small open economy in a two country General Equilibrium (DSGE) framework. In our estimation, the Canada is set as the small (or domestic) economy, while the US is the big (or foreign) economy representing the rest of the world. Our main finding is that it is important to account for the correlation between Domestic and Foreign shocks and for the Incomplete Pass-Through. In a model with uncorrelated disturbances and Complete Pass-Through the US explains only 3% of the historical variability of the Canadian output. Allowing for cross-correlation and Incomplete pass-through increases this number to around 30%, which is closer to the share of US imported goods in the Canadian GDP. We also perform a counterfactual analysis using our best model to quantify the cost of the 2008 US subprime crisis. Our results suggest the US are responsible for a decrease of 1.9% in the 2008 detrended Canadian GDP.

Keywords: Bayesian Inference; DSGE Model; Open Economy

JEL Classification: E40; E47; E52
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

3.1 Introduction

The dynamic stochastic general equilibrium (DSGE) approach for modeling macro variables dynamics has engaged attention of academics and policy makers over the past 15 years. This framework, since it is derived from first principles, has attractive features relative to reduced form analysis, as it avoids the "Lucas critique". As the parameters are structural in the sense of describing preferences, production technologies, and other structural relations, they do not alter in response to changes of policy. Hence, policy makers can better understand how the dynamics of the economy is affected by particular policy decisions. Due to this appeal, lately, many central banks have adopted this approach for policy analysis.

Although the DSGE literature has greatly improved over the past few years, most of the studies that have been done in the area used a closed economy setup (see Smets and Wouters (2007), Christiano et al. (2005), Sahuc and Smets (2008), Smets and Wouters (2003), Dei Negro et al. (2007)). This is not a bad approximation should the focus is on the US or on the Euro area, but if, for instance, the interest be on emerging markets or on other small countries, the approximation no longer works well. In these economies, foreign disturbances explain an important part of the macroeconomic series variability. Because of that, recently, academics have turned their attention to the New Open Economy Macroeconomic (NOEM) models, but there is still much to be done. The aim of this paper is to contribute to this growing literature by estimating and analyzing the transmission channels of external shocks in an open economy.

We estimate an extended version of Gali and Monacelli (2005) and Lubik and Schorfheide (2005) model for the Canadian Economy. In the estimation, the US is used as a proxy for the rest of the world. The model can be considered a small scale DSGE model in the sense that there is no investment and/or capital. Our model has seven shocks: demand, supply and monetary policy in each country, and risk sharing. It also presents three frictions: firms reoptimization following a Calvo scheme, Home bias in preferences, and Incomplete Pass-Through. Contrary to the traditional NOEM approach (see Adolfson et al. (2008) and Adolfson et al. (2007)), we do not assume that the foreign economy is an exogenous process, so we are able to analyze how foreign structural disturbances affect the domestic economy.

DSGE models allow to decompose the observable macro-variables in structural shocks. We argue that this is valuable information that should be analyzed with care in order to check the model ability in describing the data, specially, for open economy estimations.
illustrate this point, we estimate a closed economy DSGE for Canada and show the historical contributions of each structural shock to the output gap in figure 3.1. Since each shock generates a different dynamics for the economy, their identification are extremely important to the Central banks. For instance, an increase in the output arisen by productivity shocks does not generate inflation pressures and thus the policy maker should be neutral. On the other hand, in the absence of shock identification, an increase in the output could be interpreted as potentially inflationary, and the policy maker could be misled to react against it.

![Canadian Output Gap](image)

Figure 3.1: Historical Decomposition of Canadian Output Gap

We now address the importance of the external shocks on the Canadian economy. By the same argument, if a negative external demand shock is misinterpreted as a domestic one, it can mislead the policy maker decision. We start by showing a simple statistics of the raw data series from Canada and US in table 3.1. Note that the cross correlation among the series are all greater than 0.6, which suggests a strong relation influence of the US dynamics on Canada.

The question is how the shock contributions in figure 3.1, would change if, instead, we have used an open-economy model. Based on the benchmark model in the NOEM literature ((Gali and Monacelli, 2005)), we estimate the Canadian dynamics using the US as a proxy for the rest of the world. It is striking that the average contribution of the US shocks on the
Canadian output gap is around 3%, which is a small number. We interpret this finding as a caveat that passed undetected by previous works.

Next we analyze two possible causes for it: (1) lack of incomplete pass-through; and (2) lack of cross-correlation. The first is motivated by a large body of literature that documents evidence of high volatility of exchange rate and low volatility of imported prices. And, the second, is motivated by Kehoe et al. (1992) that show the importance of cross-correlation for International RBC models to fit the data. The incomplete pass-through is modeled by adding a Phillips curve for the importing sector. And, the cross-correlation is just the inclusion of additional parameters.

The inclusion of a second Phillips curve raises the contribution of US shocks but the magnitude is still low (from 3% to 8% of output gap). On the other hand, the inclusion of correlated shocks raises this share to 19%, while the model with both features goes to 27%. The direction of these results also hold for the interest rate and the inflation series. We interpret that the correlation parameters might be capturing some additional effects that theoretically should spill through exchange rate. For example, there is one parameter which theoretically should be equal to the share of imported goods in the small economy. Using the benchmark model the value of this parameter is 0.01, while in our best model is 0.30, which is very close to the historical share of US imported goods over the Canadian DGP.

Impulse response (IR) analysis on the output sheds light on what are the possible causes for these results. The benchmark model has IR functions that are similar to those on closed US economy. Both IR have the domestic supply shocks with longer duration than the domestic demand ones, and the external shocks have an negligible effect. In the full model,
the IRs of US shocks have a significant impact in the Canadian output, their shapes are similar to those IRs of US shocks on the US output, and the domestic shocks of supply and demand have similar duration. We interpret this finding as a pitfall in the transmission channel of the baseline model. Possibly, the effect of external shocks are being captured by the domestic ones. When allowing for correlations and incomplete pass-through, the domestic shocks are disentangled from external influences.

Finally, based on the estimation of our full model, we quantify the impact of the recent subprime crisis into Canadian output gap and its expected duration. Counterfactual simulations suggest a decrease of 1.9% in 2008 output, and expected remaining negative effects for the years 2009 and 2010 of 0.7% and 0.15%, respectively.

The paper proceeds as follows. Section 3.2 introduces the model, based on the microfoundations of Gali and Monacelli (2005) and Monacelli (2005), allowing for habit formation, price indexation and a second Phillips curve. Section 3.3 discusses the dataset, the estimation, and the shocks decomposition methodology. Section 3.4 presents the parameter estimates, the historical contribution of structural shocks and discusses the findings through impulse response analysis. Section 3.5 concludes.

3.2 Model

We model a small open economy in a two country DSGE model framework with imperfect competition and nominal stickiness for monetary policy analysis. The first country represents the world (or Foreign) economy and is affected only by its own variables, in this sense, it can be modeled as a closed-economy. The second one is a small open (or Domestic) economy and is affected by both internal and external variables, creating in this way new sources of inflationary pressures.

In each economy there are three kinds of agents: (1) households, who seek to maximize their utility subject to a budget constraint, (2) a continuum of domestic firms and importing firms that set their goods’ prices in order to maximize its expected discounted future profits, and (3) a Monetary Authority following a Taylor type rule. The relationship between the economies is due to a risk sharing condition.

In the demand size, households seek to maximize a time-separable utility function in consumption (relative to an external habit) and labor effort over an infinite life horizon.
Each household supplies differentiated labor inputs in a perfectly elastic manner. They also earn labor income from the firms they work for, operate in an Arrow Debreu market, and receive dividends (profits) relative to the fraction of the firms they own.

In the supply side, the domestic firms demand differentiated labor to produce differentiated goods, while the importing firms demand the foreign final good to transform it in the differentiated imported good. Both type of firms set prices according to the Calvo model, ie, some firms are randomly selected to adjust prices and others not. The domestic (importing) firms that are not selected, partially readjust prices according to the domestic (imported) past inflation.

Each household likes domestic and imported goods, so the consumption bundle she cares about has both goods. The price of this bundle, the consumer price index (cpi), is a function of the domestic (or producer) price index (ppi) and the importing price index (ipi).

In the monetary side, the Central Bank sets the nominal interest rate as a function of the past interest rate; the consumer inflation rate; the growth of the consumer inflation; the level of output; and the output growth.

The model has two frictions worth highlighting: (1) home bias in the preferences, and (2) incomplete pass through. The first creates real exchange rate fluctuations, so that the purchase power parity (PPP) does not hold. The last one creates a time varying law of one price (LOP) gap in the foreign goods market.

Seven shocks drive the dynamics of the model: an aggregated domestic (foreign) technology shock, a domestic (foreign) preference shock, a domestic (foreign) monetary shock, a mark-up shock in the importing sector, and a risk sharing shock to accomodate deviations of the theoretical exchange rate from the actual data.

The derivation of the equilibrium conditions and the steady state equilibrium relationships of the model are presented in the appendix. The equilibrium of the full model is given by a system of non-linear rational expectation equations. To solve for this system we linearize the equilibrium conditions around the steady state, which is common practice in the literature. Next, we discuss the linearized version of the model.
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

3.2.1 Main equations

In what follows, $\pi_H$, $\pi_F$, $\pi$ are, respectively, the ppi, iip and cpi inflation rates, $m_{ch}$, $m_{cf}$, $m_c^*$ are, respectively, the Home producer marginal cost, the Home imported marginal cost, and the Foreign (producer) marginal cost (in particular, $m_{cf}$ is also the gap of the law of one price of the goods produced in the Foreign Economy), $c^*$ is the excess consumption, $r$ is the one period nominal interest rate, $y$ is the output, $q$ is the real exchange rate, and $e$ is the nominal exchange rate. All variables are in logs. Variables with stars are from the Foreign Economy and without stars are from the Home Economy, while variables with tilde are deviations from the steady-state (or detrended in the case of the output and the real exchange rate).

\[ \pi_{H,t} = \gamma_H \pi_{H,t-1} + \lambda_H [\tilde{mc}_{H,t}] + \beta E_t(\pi_{H,t+1} - \gamma \pi_{H,t}) \]  \hspace{1cm} (3.1)
\[ \pi_{F,t} = \gamma_F \pi_{F,t-1} + \lambda_F [\tilde{mc}_{F,t}] + \beta E_t(\pi_{F,t+1} - \gamma \pi_{F,t}) \]  \hspace{1cm} (3.2)
\[ \tilde{c}_t^* = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} [\tilde{r}_t - E_t \tilde{\pi}_{t+1} - (1 - \rho^\pi) \tilde{c}_t] \]  \hspace{1cm} (3.3)
\[ \pi_t^* = \gamma^* \pi_{t-1}^* + \lambda^* [\tilde{mc}_{t}^*] + \beta^* E_t(\pi_{t+1}^* - \gamma^* \pi_t^*) \]  \hspace{1cm} (3.4)
\[ \tilde{y}_t^* = h^* \tilde{y}_{t-1} + E_t(\tilde{y}_{t+1}^* - h^* \tilde{y}_t) - \frac{1}{\sigma^*} [\tilde{r}_t - E_t \tilde{\pi}_{t+1} - (1 - \rho^{\pi^*}) \tilde{c}_t^*] \]  \hspace{1cm} (3.5)
\[ \tilde{r}_t = \rho_r \tilde{r}_{t-1} + (1 - \rho_r) [r_{\pi} \pi_t + r_y \tilde{y}_t] + r_{dy}(\tilde{y}_t - \tilde{y}_{t-1}) \]  \hspace{1cm} + \ r_{dn}(\pi_t - \pi_{t-1}) + \tilde{c}_t^* \]  \hspace{1cm} (3.6)
\[ \tilde{r}_t^* = \rho_r^* \tilde{r}_{t-1}^* + (1 - \rho_r^*) [r_{\pi} \pi_t^* + r_y \tilde{y}_t^*] + r_{dy^*}(\tilde{y}_t^* - \tilde{y}_{t-1}^*) \]  \hspace{1cm} + \ r_{dn^*}(\pi_t^* - \pi_{t-1}^*) + \tilde{c}_t^* \]  \hspace{1cm} (3.7)

\[ \tilde{c}_t^* = \frac{1}{\sigma^*} \tilde{r}_t^* + \frac{\sigma^*}{\sigma(1 - h^*)} (\tilde{y}_t^* - h^* \tilde{y}_{t-1}^*) + \frac{1}{\sigma} (\tilde{c}_t^* - \tilde{c}_{t-1}^*) + \tilde{c}_{t-1}^* \]  \hspace{1cm} (3.8)

where the parameters $\gamma_H$, $\gamma_F$, and $\gamma^*$ denote the degree of indexation to past inflation, $\beta$ and $\beta^*$ are intertemporal discounts, $\sigma$ and $\sigma^*$ are the coefficients of relative risk aversion (or the inverse of the intertemporal elasticity of substitution), and $h$ and $h^*$ are the degree of household habits.

The Phillips Curves (PCs) 3.1, 3.2 and 3.4 say that increases in current and future expected marginal costs cause inflationary pressures, that are exacerbated by indexation to

---

$^1$ p_H, p_F, p are, respectively, the log of ppi, iip and cpi.
past inflation.

The Euler Equations (EEs) 3.3 and 3.5 say that increases in the current and future expected real interest rates increase the marginal utility of consumption, diminishing the households' demand by current consumption. A preference shock in the current period increases the marginal utility of consumption more than the expected future marginal utilities, therefore household’s will consume more. Movements in consumption are augmented due to the external habit.

The interest rates are set according to Augmented Taylor-type rules 3.6 and 3.7. The monetary authority responds to deviations of inflation, output, inflation growth and output growth from their respective steady-state levels by adjusting the short-term nominal interest rate. Actions of the Central Bank that are orthogonal to the systematic part of the rule are captured by $\epsilon'_t$.

The link between the two economies is done by the risk sharing condition 3.8, which says that differences between foreign and domestic marginal utilities of consumption are explained by adjustments in the real exchange rate or by a risk sharing shock that captures short-term deviations of the risk-sharing condition. On the other hand, shocks to the risk-sharing are absorbed by the real exchange rate.

### 3.2.2 Exogenous Processes

There are seven shocks driving the dynamics of the model divided in two types: real side and nominal side shocks. The real side shocks are autoregressive processes AR(1), while the monetary shocks are pure white noises. $a$ is a country aggregated productivity shock; $\epsilon^p$ is a preference shock that increases the consumption marginal utility; $\epsilon^m$ is a monetary policy shock that is orthogonal to the traditional policy variables; and $\epsilon^\gamma$ is a risk-sharing shock.
3.2.3 Auxiliary variables

The auxiliary variables were defined in order to simplify the main equations of the model. They are given as linear combinations of both endogenous variables and exogenous processes.

\[ \bar{\alpha}_t = \rho^* \bar{\alpha}_{t-1} + \epsilon^*_t \quad (3.9a) \]
\[ \bar{\epsilon}^*_t = \rho^* \bar{\epsilon}^*_{t-1} + \epsilon^*_{t} \quad (3.9b) \]
\[ \bar{\alpha}_t = \rho^* \bar{\alpha}_{t-1} + \epsilon^*_t \quad (3.9c) \]
\[ \bar{\epsilon}^*_{t} = \rho^* \bar{\epsilon}^*_{t-1} + \epsilon^*_{t} \quad (3.9d) \]
\[ \bar{\epsilon}^*_{t} = \rho^* \bar{\epsilon}^*_{t-1} + \epsilon^*_{t} \quad (3.9e) \]
\[ \psi_{F,t} = \rho^* \psi_{F,t-1} + \epsilon^*_{t} \quad (3.9f) \]

where \( e^k \sim N(0, (\sigma^k)^2) \) for \( k = a, pp, r, a*, pp*, r*, rsrs, F, \) with \( \text{cor}(\epsilon^v, \epsilon^{v*}) = c^v \) for \( v = a, pp, r. \)

### 3.2.3.1 Where \( \bar{\alpha}_t \), \( \bar{\epsilon}^*_t \), \( \psi_{F,t} \) are defined.

### 3.2.3.2 Equations 3.10, 3.11, and 3.14, are equilibrium results, equation 3.13 is the cpi definition, and equation 3.12 is just a rewriting of the definition.

Some other parameters used in the description of the model are functions of the structural parameters, and they are given by:
Equation 3.10 describes the dynamics of the Foreign producer marginal cost. Intuitively increases in the output that are not supported by a positive shock in productivity generate inflationary pressures.

Note that, because the Central Bank tries to stabilize output instead of the (producer) marginal cost, a trade-off arises following a technology shock. It is not possible to stabilize $\tilde{\pi}^*$, $\tilde{\gamma}^*$, $\Delta \tilde{\pi}^*$ and $\Delta \tilde{\gamma}^*$ at the same time.

The Foreign Economy marginal cost depends solely onto Foreign variables, but the Home Economy producer marginal cost depends onto both Foreign and Home variables. Hence, the trade-off between output and inflation that the Central Bank faces can be either augmented or mitigated by Foreign output, real exchange rate and/or LOP gap.

3.3 Estimation Methodology

3.3.1 Data description

We use quarterly US and Canadian data ranging from 1986:Q1 to 2008:Q4, a total of 92 observations. Seven series are used in the estimation: (1) the US CPI inflation, (2) the Canadian PPI inflation, (3) the detrended US output, (4) the detrended Canadian output, (5) the US short-term interest rate, (6) the Canadian short-term interest rate, and (7) the detrended real exchange rate.

\[
\alpha_h = \frac{1}{(1- \alpha)(1- \beta)} \tag{3.15a}
\]

\[
\omega_{\phi} = \frac{(2- \alpha)\alpha \mu}{1- \alpha} \tag{3.15b}
\]

\[
\omega_{\text{mc}} = \frac{(1- \alpha)\alpha \mu + \mu \alpha^2}{1- \alpha} \tag{3.15c}
\]

\[
\lambda_H = (1- \phi H \beta)\frac{1- \phi_H}{\phi_H} \tag{3.15d}
\]

\[
\lambda_F = (1- \phi F \beta)\frac{1- \phi_F}{\phi_F} \tag{3.15e}
\]

\[
\lambda^* = (1- \phi^* \beta^*)\frac{1- \phi^*}{\phi^*} \tag{3.15f}
\]

$\alpha$ is the degree of openness of the Home Economy, $\mu$ is the elasticity of substitution between goods produced in the Home Economy and the goods produced in the Foreign Economy. $\phi_H$, $\phi_F$ and $\phi^*$ denote the fractions of firms that do not adjust their prices in each period.
To construct the series we used the following raw data: The Canadian Non-seasonally adjusted Producer price index (PPIRNS.a_CN), the US and the Canadian Seasonally Adjusted Consumer Price Indexes (CPI_a_CN, CPL_a_US), the US and the Canadian Real Gross Domestic Product Index (GDPR.d_a_CN, GDPR.d_a_US), and the Nominal Exchange rate\(^3\) (RX_a_CN) were taken from “Global Insight International - Quarterly”, while the Canadian Overnight Money Market rate (L60B_a_C156) and the US Federal Funds rate (L60B_a_C111), from “Global Insight IMF Data - Quarterly”. The databases can be found in the Wharton Research Data Services website (http://wrds.wharton.upenn.edu/).

The construction of the dataset is done as it follows: (1) The real exchange rate \(Q = \frac{P^*}{P}\) is the nominal exchange rate adjusted by the US and the Canadian CPIs; (2) The inflation rates are the returns of the monthly means of the indexes in one quarter against the monthly means of the indexes in the quarter before; (3) The annualized short-term interest rates are quarterized; (4) Detrending of an \(x\) variable is done by regressing \(\log(x_t) = \beta_0 + \beta_1 t + \varepsilon_t\) and taking the residuals; and (5) all variables are demeaned.

The data used in our work is presented in figure 3.2. Note how the US series are correlated with the Canadian ones. According to table 3.1 the cross-correlation between US and Canadian Output, Interest Rate and Inflation are, respectively, 0.66, 0.83, and 0.84.

\(^3\)Canadian Dollar per US Dollar
Figure 3.2: The dataset used in the estimation and in the empirical exercises. Quarterly data ranging from 1986:Q1 to 2008:Q4.
3.3.2 Model Solution and Likelihood Function

In this section we describe the methodology to solve the model and the estimation technique we use in the empirical section. The first step is to represent the canonical system given by equations 3.1-3.14 in a linear rational expectation system form

\[ \Gamma_0 (\theta) x_t = \Gamma_1 (\theta) x_{t-1} + \Psi (\theta) \epsilon_t + \Pi \eta_t \]  

(3.16)

\[
\begin{align*}
    x & \triangleq \begin{bmatrix} x_{t}^{\text{endo}} & x_{t}^{\text{exot}} & x_{t}^{\text{aux}} & x_{t}^{\text{exp}} \end{bmatrix}' \\
x_{t}^{\text{endo}} & \triangleq \begin{bmatrix} \pi_{H,t} & \pi_{E,t} & \bar{y}_{t} & \pi_{t}^* & \bar{y}_{t}^* \end{bmatrix}' \\
x_{t}^{\text{exot}} & \triangleq \begin{bmatrix} x_{t}^{dlt} & \bar{r}_{t} & \bar{r}_{t}^* & q_{t} \end{bmatrix}' \\
x_{t}^{\text{exp}} & \triangleq \begin{bmatrix} E_{t} \pi_{H,t+1} & E_{t} \pi_{E,t+1} & E_{t} \bar{y}_{t+1} & E_{t} \pi_{t+1}^* & E_{t} \bar{y}_{t+1}^* \end{bmatrix}' \\
x_{t}^{\text{aux}} & \triangleq \begin{bmatrix} \bar{c}_{t} & \bar{c}_{t}^p & \bar{c}_{t}^s \pi_{t}^* & \bar{c}_{t}^{ps} & \bar{c}_{t}^{sr} \end{bmatrix}' \\
\epsilon_t & \triangleq \begin{bmatrix} \bar{c}_{t} & \bar{c}_{t}^p & \bar{c}_{t}^s \pi_{t}^* & \bar{c}_{t}^{ps} & \bar{c}_{t}^{sr} \end{bmatrix}'
\end{align*}
\]

where \( \theta \) is a vector of structural parameters and \( \eta_t \) is a vector of endogenous errors.

In order to solve the system (3.16), Sims (2002) provide a solution algorithm and sets of conditions on the matrices \( A, B, \Psi \) and \( \Pi \) under which there are solutions and verify the uniqueness thereof. When a unique solution exists it has a first order restricted VAR representation

\[ x_t = G(\theta) x_{t-1} + M(\theta) \epsilon_t \]  

(3.17)

Since there are unobservable variables, we must filter them from the data in order to evaluate the likelihood function \( L(\Upsilon_T; \theta) = \sum_{t=1}^{T} \log f(x_t | \Upsilon_{t-1}; \theta) \), where \( \Upsilon_t \) is the information set up to time \( t \). The Kalman Filter (KF) approach is suited for this task because it gives the exact likelihood function for a Gaussian ARMA process, for which our model is a special case. Given a vector of parameters \( \theta \) and the associated VAR representation (equation (3.17)), we use the KF algorithm (see Hamilton 1994 p. 372) to evaluate the \( L(\Upsilon_T; \theta) \) and it’s components. Having established the likelihood function, we turn next to the estimation procedure.
3.3.3 Bayesian Estimation

Following the growing literature in empirical DSGE models, we use the Bayesian technique to estimate the parameters of the model. This method is recommended to estimate large dimensional problems since it does not require optimization routines. Another benefit is the inclusion of previous information from the researcher (before the observation of the data) in a formal way into analysis. This characterizes the prior distribution of the parameters \( p(\theta) \). The data base \( x \) is used to update the prior distribution through the likelihood \( f(x|\theta) \), which generates the posterior distribution \( p(\theta|x) \) by the Bayes’s rule:

\[
p(\theta|x) = \frac{f(x|\theta)p(\theta)}{f(x)} \implies p(\theta|x) \propto f(x|\theta)p(\theta)
\]

In essence, this method uses numerical integration to generate the posterior distribution of the parameters through Monte Carlo simulation, i.e., it is more tractable and less sensitive to high dimensional problems.

The prior specification allows the researcher to estimate his model in a likelihood weighted parameter space. If the researcher has no prior information about the parameters, his prior will be a diffused one \( (p(\theta) = 1 \ \forall \theta \in \Theta) \). In this limit case, the posterior will be a distribution generated by the likelihood function. On the other extreme, with a degenerative prior \( (p(\theta) = 1 \text{ if } \theta = \bar{\theta}, \text{ and } p(\theta) = 0 \text{ if } \theta \neq \bar{\theta}) \) the parameters will be the same as a calibrated model. This gives flexibility to the researcher to estimate his model where the subset of parameters make sense to him. An example of this procedure is that a structural parameter representing weight must lie in the subset \([0, 1]\).

The general method is called Monte Carlo Markov Chain (MCMC). The chains encountered in MCMC have a very strong stability, namely a stationary probability distribution, which characterizes the posterior. Under the irreducibility property the initial values in which the chain starts are irrelevant, which represents an advantage related to maximum likelihood methods since their estimation procedures are sensitive to it (especially for multi-dimensional problems). The MCMC estimation procedure consists on running \( N \) steps until the convergence of the chain and burning out the beginning of the chain, say \( N/2 \). The properties that ensure the convergence and the irreducibility are shown in sources cited in notes.

We use the Metropolis-Hasting (M.H.) algorithm to estimate the posterior distribution. The M.H. algorithm is recommended when the conditional distributions are unknown or are difficult to sample from\(^5\). Let \(\pi(\theta)\) denotes the posterior distribution. To sample from \(\pi(\theta)\), the M.H. method requires a proposal distribution \(q(x|\theta)\) specification by the researcher. The candidate parameters are drawn from the proposal. Note that this density can be arbitrarily defined since the aim here is just to randomize the choice of parameters for a given parameter space. The M.H. algorithm has a rejection method, which determines whether the candidate parameter is going compose or not the empirical posterior distribution. Here we summarize the basic algorithm which involves the following steps:

Let \(Y\) be the vector of observable variables, \(p(\bullet)\) the prior and \(f(\bullet)\) the likelihood function. At the beginning of iteration \(i\) we have the vector of parameters \(\theta^{i-1}\).

1. Draw \(K^i \sim q(k|\theta^{i-1})\)
2. Set \(\theta^i = \frac{K^i}{\theta^{i-1}}\) with probability \(\rho(\theta^{i-1}, K^i)\)
   \(\theta^{i-1}\) with probability \(1 - \rho(\theta^{i-1}, K^i)\)

where

\[
\rho(\theta^i, K^i) = \min \left\{ \frac{\pi(k^i)/q(k^i|\theta^i)}{\pi(\theta^i)/q(\theta^i|k^i)} , 1 \right\}
\]

\[
\pi(\theta^i) \propto f(\theta^i|Y)p(\theta^i)
\]

3. Repeat the steps \(N\) times

The sequence \(\{\theta^i\}_{i=1}^{N+B}\) characterizes the empirical posterior distribution, where \(B\) is the burn-in threshold.

### 3.3.4 Shocks Decomposition

In this section we show the methodology for extracting the state and structural shocks. We use them to decompose the observable series in its fundamental components. The methodology is as follows. Suppose that the solution of a rational expectation model has the following VAR representation for the \(n\)-dimensional vector of states variables \(X_t\):

\[
X_t = GX_{t-1} + M\epsilon_t
\]

where \( \epsilon_t \sim N(0, S.I_k) \) represents the k-dimensional vector of structural shocks. Let \( Y_t \) be the vector observable variables. The state space representation for a normalized error \( \epsilon_t \sim N(0, I_k) \) can be rewritten by

\[
\begin{align*}
Y_t &= HX_t \\
X_t &= GX_{t-1} + Ne_t
\end{align*}
\]

where

\[
N_{n \times k} = M.S \quad , \quad \epsilon_t = S.e_t
\]

As discussed before, using KF algorithm, we can filter the series \( X_t \). The filtered state shock is \( \tilde{u}_t = \tilde{X}_t - G\tilde{X}_{t-1} \). Given a series of estimated state shocks \( \tilde{u}_t = MS\tilde{e}_t = \tilde{M}\tilde{e}_t \), we can recover the estimated structural shocks \( \tilde{c}_t \). Since the observable variables are included in the state vector, we can exactly extract the structural shocks depending on the dimensions of the observable series and structural shocks. Sorting the state vector in such a way that the observable series are in the first \( n_1 \) rows (denote by superscript "o") , the relation has the following representation

\[
\tilde{u}_t = \begin{bmatrix} \tilde{u}_t^o \\ \tilde{u}_t^{un} \end{bmatrix} = \begin{bmatrix} M^o \\ M^{un} \end{bmatrix} \tilde{c}_t
\]

Let the dimension of \( \tilde{c}_t \) be equal to \( k \). If \( n_1 = k \), the relation is direct and the solution is unique

\[
\tilde{c}_t = (M^o)^{-1} \tilde{u}_t^o
\]

If \( n_1 > k \), there are infinite solutions for the system. One alternative is to choose the projection of the state shocks on the space generate by \( M^o \)

\[
\tilde{c}_t = (M^oM^o)^{-1} M^o\tilde{u}_t^o
\]

In this paper we choose the number of structural shocks equal to the number of observable variables.i.e, we used the previous equation. Having extracted the structural shocks, the next step is to decompose the observable series as a function of those shocks. Since the model has a first order VAR representation, it can be solved backward in terms of the shocks

\[
X_t = GX_{t-1} + M\epsilon_t = \sum_{i=0}^{t-1} G^iM\epsilon_i + G^tX_0
\]

In the empirical section, we set the initial condition \( X_0 = 0 \) and adjust it on the \( \epsilon_1 \) term.
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

3.4 Results

3.4.1 Models

The question we want to address is how the structural US shocks affect the Canadian economy. To answer it we evaluate different specifications and their implied structural disturbances.

The model specifications are summarized in table 3.2. They are based on two characteristics: (1) incomplete pass-through, and (2) cross-correlation. Model I is the benchmark, and is characterized by complete pass-through and lack of cross-correlation. Whereas Model IV (the full model) has both characteristics.

<table>
<thead>
<tr>
<th>Model</th>
<th>Pass-Through</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I (bench)</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Model II</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Model III</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Model IV</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 3.2: Models specification

3.4.2 Parameter Estimates

The estimation was conducted in two steps. First we estimate the closed model for the US as a proxy for the big economy. And, second, given the estimates of US parameters, we estimate all the four open economy versions. The first reason for this procedure is that the big economy parameters cannot be affected by the small economy data (in theory). If we jointly estimate the model, the big economy parameters will generate additional degrees of freedom to fit the small economy data. The second reason is to isolate the effects of the alternative specifications (table 3.2), given the dynamics of the big economy.

We choose the same set of priors for the parameters in all models, including those of the closed-economy. We report the priors in the first column of table 3.3. Most of the priors are relatively uninformative but still imposing the relevant constraints. From a total of 22 parameters, 4 are restricted to be positive (shocks variances), and 11 have restricted
supports by definition (structural parameters that represents fraction are in the set $[0, 1]$, and correlation and autocorrelation parameters are between $[-1, 1]$). The remaining parameters have priors based on standard macroeconomic assumptions. For instance, the Taylor rule response to inflation ($r_g$) has to be greater than one in order to ensure determinacy of the system.

The Metropolis-Hasting algorithm was used to sample the parameters from the posterior distribution. Convergence diagnostics of the Markov Chain are shown in appendix C.3. We report the posterior median, and the 5 and 95 percentiles in table 3.3. The open economy models (Model I to Model IV) are in the four last blocks of the table. For comparison, we also report the estimates of the closed economy model, for US and Canada.

Model I is the benchmark for comparison. It has just one Phillips Curve in the domestic economy and the structural domestic shocks are not correlated to foreign shocks. The alternative models have additional features to improve the transmission channel of foreign shocks to the domestic economy. On average, the alternative models have more precise estimates in the sense that most of the confidence bands are smaller than the benchmark. For instance, out of 19 parameters in the benchmark model, Model II to IV have, respectively, 14, 11 and 15 parameters with tighter posterior intervals (5/95 quantiles). In order to Model comparison, we report the Bayes Factor (BF) of each model in the bottom of the table. All alternative models have a superior BF relative to the benchmark, which suggests that the features added are relevant.

The structural parameters related to the Phillips curve and the IS curve are in the first row block of the table. The parameter $\alpha$ is directly related to the share of imported goods in the theoretical model. In the data we would expect that this value should be similar to the fraction of the expenditure of imported goods in the Canadian GDP, which is around 30%. This number is in line with our full specification, whereas is only 1% in the benchmark model. The Taylor rule parameters are in the second block row of the table. The persistence parameter is the less volatile among the models, ranging from 0.93 to 0.94. The response of the Taylor rule to inflation has a wide range (1.37 to 1.96), reaching the greatest value in Model IV (which is very close to the parameter in the US estimation). In the third row block are the parameters of exogenous process. It is important to note the correlation parameters are significantly different from zero.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>US Closed</th>
<th>Canada Closed</th>
<th>Posterior</th>
<th>Posterior</th>
<th>Posterior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Median</td>
<td>Median</td>
<td>5%</td>
<td>95%</td>
<td>Median</td>
<td>5%</td>
</tr>
<tr>
<td>sig</td>
<td>GB(0.5,6)</td>
<td>3.0</td>
<td>1.0</td>
<td>1.35</td>
<td>1.09</td>
<td>0.68</td>
<td>1.92</td>
</tr>
<tr>
<td>varphi</td>
<td>GB(3,6)</td>
<td>3.0</td>
<td>1.0</td>
<td>2.13</td>
<td>2.49</td>
<td>1.26</td>
<td>4.11</td>
</tr>
<tr>
<td>gam</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.13</td>
<td>0.15</td>
<td>0.07</td>
<td>0.29</td>
</tr>
<tr>
<td>lambda</td>
<td>GB(0.2,2)</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>h</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.45</td>
<td>0.66</td>
<td>0.52</td>
<td>0.78</td>
</tr>
<tr>
<td>alpha</td>
<td>GB(0.2,2)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>mu</td>
<td>Uniform</td>
<td>1.0</td>
<td>1.0</td>
<td>1.12</td>
<td>1.52</td>
<td>0.98</td>
<td>1.69</td>
</tr>
<tr>
<td>gam_f</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>lambda_f</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>r_y</td>
<td>Beta</td>
<td>0.3</td>
<td>0.2</td>
<td>0.06</td>
<td>0.08</td>
<td>0.02</td>
<td>0.17</td>
</tr>
<tr>
<td>r_pi</td>
<td>GB(1,3)</td>
<td>1.3</td>
<td>0.3</td>
<td>1.93</td>
<td>1.64</td>
<td>1.28</td>
<td>2.38</td>
</tr>
<tr>
<td>r_dpi</td>
<td>GB(0.0,5)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.09</td>
<td>0.09</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>r_dy</td>
<td>GB(0.0,5)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.21</td>
<td>0.16</td>
<td>0.12</td>
<td>0.21</td>
</tr>
<tr>
<td>rho_r</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>0.53</td>
<td>0.53</td>
<td>0.26</td>
<td>0.91</td>
</tr>
<tr>
<td>rho_a</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>0.74</td>
<td>0.66</td>
<td>0.48</td>
<td>0.78</td>
</tr>
<tr>
<td>rho_pref</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>0.54</td>
<td>0.93</td>
<td>0.69</td>
<td>0.97</td>
</tr>
<tr>
<td>rho_rs</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>0.54</td>
<td>0.93</td>
<td>0.69</td>
<td>0.97</td>
</tr>
<tr>
<td>cor_a</td>
<td>U(-1,1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.66</td>
<td>0.66</td>
<td>0.57</td>
<td>0.74</td>
</tr>
<tr>
<td>cor_pref</td>
<td>U(-1,1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.27</td>
<td>0.27</td>
<td>0.18</td>
<td>0.40</td>
</tr>
<tr>
<td>cor_r</td>
<td>U(-1,1)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.35</td>
<td>0.35</td>
<td>0.24</td>
<td>0.48</td>
</tr>
<tr>
<td>100 x sig_a</td>
<td>son(K)</td>
<td>50</td>
<td>50</td>
<td>4.86</td>
<td>9.36</td>
<td>6.65</td>
<td>20.50</td>
</tr>
<tr>
<td>100 x sig_pref</td>
<td>son(K)</td>
<td>50</td>
<td>50</td>
<td>2.46</td>
<td>3.01</td>
<td>2.17</td>
<td>4.52</td>
</tr>
<tr>
<td>100 x sig_r</td>
<td>son(K)</td>
<td>2.0</td>
<td>2.0</td>
<td>0.13</td>
<td>0.12</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>100 x sig_rs</td>
<td>son(K)</td>
<td>5.0</td>
<td>5.0</td>
<td>4.10</td>
<td>4.10</td>
<td>4.04</td>
<td>4.21</td>
</tr>
</tbody>
</table>

Loglikelihood
3.4.3 The Role of Past Shocks on the Canadian Series

One of the empirical advantages of the DSGE framework relative to reduced-form models is that they have structural restrictions behind the estimation. Therefore, there is no need for arbitrary choices on the covariance structure, like the triangular or Cholesky decomposition, in order to recover the structural shocks. In this section we explore this advantage. As discussed in section 3.1, we estimate a closed economy model and apply the decomposition, which is reported in figure 3.1. Since the Canadian time series are strongly affected by the US economy (see table 3.1), such a decomposition is misleading. In this section we analyze how the shock decomposition in picture 3.1 would change when allowing for external shocks.

Our model is a small scale model in the sense that we have only three structural shocks for each economy and an exchange rate shock. The preference shock ($\epsilon^{P}$) is interpreted as a demand shock since it affects the Euler equation (demand side of the model). Similarly, the productivity shock ($\epsilon^{P}$) affects the Phillips curve through the marginal cost, thus is interpreted as a supply shock. The monetary policy shock ($\epsilon^{M}$) is the residual of the interest rate that is not explained by the systematic part of the Taylor rule. Finally, the exchange rate shock is a measurement error for the exchange rate (difference of the model implied exchange rate by the risk sharing condition and the observed data).

Figure 3.3 shows the decomposition of the detrended output gap. We start looking at Model I, and surprisingly the effects of Canadian shocks are almost the same as those on the closed economy estimation. In other words, the effects of external shocks (exchange and US) are irrelevant. This is in odds with the contemporaneous cross-correlation of the raw series (US and Canada) of 0.65 in the sample analyzed. This is a strong evidence that the theoretical transmission channel is not working properly. Next we evaluate possible alternatives to overcome this problem.

Model II is the same as Model I with the exception that we allow for correlation between the Canadian and US structural shocks. The average percentage explained by US raises significantly from 3% (Model I) to 19%. Although there is an improvement in explanation power of US economy, we must point out that the problem is still there. The inclusion of correlation parameters gives a degree of freedom to absorb any explanation of the effects of the US in the Canadian series. This can be caused by the “true” correlation between structural shocks added by some effect that has not been captured by the theoretical transmission channel (changes in the terms of trade).
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

There is a myriad of evidences that the transmission channel does not work very well based on the evidence of high volatility of exchange rate and low volatility on price of imported goods. To overcome this theoretical drawback, the literature has improved the models adding a new friction. The incomplete pass-through approach assume that there is an importing sector in the economy with its own nominal rigidities. This leads to a second Phillips curve in the model and allows for a more flexible channel to the transmission of external shocks. We investigate this inclusion in Models III and Model IV.

Figure 3.3: Historical decomposition of Detrended output gap by model. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks.

A direct comparison between Model I and Model III (without correlation) allows us to analyze the effect of the incomplete pass-through. Indeed, the transmission channel seems to work better, since the average absolute contribution of US shocks is around 8% (from a previous 3%). The same direction is obtained when comparing Model II and IV (with correlation) where the share goes to 27% from 19%.
Figure 3.4 shows the time series of relative contributions, instead of the average. The US contribution is the white area. Note that in model IV, the last two quarters of 2008 are mostly explained by US (over 50%). On the other hand, in the Models I to III the US explain less than 30%. Since by that time the subprime crises was taking place, Model IV seems to be working better, because is our believe that the decrease in the Canadian output was mostly driven by US spillover reasons.

Another interesting analysis is the effect of the exchange rate shocks. This disturbance is capturing short term departures from the UIP or risk-sharing condition. Despite non-structural, this shock is consistent with the evidence in Brandt et al. (2006) and Kehoe et al. (1992) that the risk-sharing condition does not hold empirically. Moreover, in our paper, it helps to understand the transmission channel. since, in the absence of cross-correlation, any US shock spills to Canada only through exchange rate. As a result, if this shock does not affect the Canadian economy, none of the US shocks will.

The exchange rate shock is the second area just above the US shock in figure 3.4. The inclusion of the second Phillips curve raises the importance of this shock (6% to 11% from Model I to Model III), so the transmission channel is working better.

The inclusion of correlation parameters reduce part of the risk-sharing error (11% to 7% from Model III to Model IV), which suggests that the model is now better specified.

We have also performed the same exercise with the Interest Rate and Inflation series. The graphics are shown in the appendix C.4. We report the summary of the results on table 3.4. The same kind of behavior discussed with the output gap is observed in the other series. We conclude that the addition of incomplete pass-through and cross-correlation in the benchmark model are extremely relevant for the estimation of an open economy model. The absence of these features can lead to omitted variable problems and, therefore biasing the estimation.

Table 3.4: Absolute Participation of Estimated Shocks.

<table>
<thead>
<tr>
<th>% Absolute Participation of Estimated Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Output Gap</td>
</tr>
<tr>
<td>Model I</td>
</tr>
<tr>
<td>Model II</td>
</tr>
<tr>
<td>Model III</td>
</tr>
<tr>
<td>Model IV</td>
</tr>
</tbody>
</table>

Output Gap, Interest Rate, Inflation.
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

Figure 3.4: Historical relative contribution of shocks on detrended output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks.

Finally, we must highlight that state of the art models (see Adolfson et al. (2008), Adolfson et al. (2007)) in the NOEM literature do not account for cross-correlation, since they take the foreign economy as an exogenous process. Our analysis indicates that their approach must be looked with skepticism, because external influences might be underestimated.

3.4.4 Impulse Responses Functions

In this section, we analyze and compare the impulse response (IR) to structural shocks among the models. The IR is computed as the effect of one standard deviation shock on the state variables dynamics. The IR analysis is interesting, because it allows a better understanding of what are the moment conditions generated by the structural model and is directly related to the identification of structural shocks. For instance, our estimated model response to a demand shock is positive in output, inflation and interest rate, with these effects steadily decreasing over time. The IRs allow us to know that these dynamics are being identified by our model and the data as a demand shock.

Figures 3.5 and 3.6 plot, respectively, the Canadian output gap IRs to domestic and
external shocks. Note that in figure 3.6 that Model I almost does not suffer influences from external shocks, just like in picture 3.4. Appendix C.4 show the results obtained to Canadian inflation and interest rate are in line with the ones obtained here. Hence, we can consider Model I as almost a closed-economy model. This conclusion will be important in the remaining of the section.

In figure 3.5, the shape of all IRs to domestic shocks are similar in all models. The IRs to demand and supply shocks are hump-shaped, while the responses to monetary policy shock have an inverted hump-shaped behavior.

In all models responses to domestic monetary shock are exactly the same. This indicates that they are not affected by the external shocks. On the other hand, the responses to both preference and productivity shocks have significant changes. Our estimation points that: (1) in models II and IV the demand shocks are more persistent than in models I and III, (2) in all models, but Model I the supply shocks have a lower impact, and (3) in models I and III there is a single-crossing between supply and demand shocks.

![Figure 3.5: The impulse response of domestic shocks on the Canadian output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks.](image)

Figure 3.6 helps to understand the change of behavior among models. Note that the addition of a second Phillips curve (Model I vs Model III) allows a small improvement in
the transmission of the US shocks. But, it is the correlation that captures most of the transmission.

Figure 3.6: The impulse response of external shocks on the Canadian output gap. Second row are the models with two Phillips curve. Second column are the models with correlation with external structural shocks.

In figure 3.5, note that in Model I there is an expressive difference in the duration of preference and technology IRs. This short-duration preference response against long-duration technology response is also verified in closed-economy estimations (see Smets and Wouters (2003)), confirming our previous belief that Model I behaves like a closed-economy. This suggests that the IR differences between Model I and IV are mostly driven by omitted information of the foreign dynamics. As the transmission mechanism is not working properly in model I, the shocks cannot reach the domestic economy. But, when the transmission mechanism is working, and external dynamics are influencing the domestic economy, internal and external shocks are now disentangled, and a different structure shows up on the Canadian IRs: domestic supply and demand shocks have the same persistence (duration) on the output gap, and there is no single-crossing.
CHAPTER 3. US SHOCKS ON THE CANADIAN ECONOMY

3.4.5 Counterfactual Analysis

One of the advantages of DSGE models is that we can undertake counterfactual experiments in order to evaluate what would have happened if a different policy had been implemented or if a different shock had hit the economy. We use this framework to investigate the effects of the 2007/2008's subprime crisis on the Canadian economy. We must highlight that contrary to US and some European countries, the Canadian banks were not that leverage on subprime mortgages and its derivatives. Thus, the crisis effects can be viewed as external effects in the Canadian series. In this sense, counterfactual analysis can work as a natural experiment of the relevance of external shocks on an open-economy.

We conduct our investigation as follows. Suppose that from the third quarter of 2007 to the fourth quarter of 2008, there were no US, neither exchange rate shocks. Now, calculate what would be dynamics of the Canadian series with the absence of these shocks. Call them counterfactual series. In figure 3.7, we plot the counterfactual as well as the effective Canadian output. Note that the decrease in the Canadian output gap in 2008 would be 0.6%. A sharp contrast with the observed fall of 2.5%.

![Figure 3.7: Effective and counterfactual output gap series in basis points. Counterfactual is defined as the series with no external neither exchange rate shocks from 2007:Q2 afterwards.](image)

According to the results shown above, our estimation suggests that the negative impact of external shocks in the Canadian output gap are in the order of 1.9% in 2008. Another
interesting exercise is to extrapolate the series and evaluate the duration of the crisis impact. We calculate the expectation of both series for the next four years. Note that the expectation of any structural shock is zero, therefore the future impact is entirely due to persistent equilibrium dynamics. The results are shown in the picture 3.7, and the end of year values are shown in table 3.5. The last column shows that the estimated negative external effect in the year 2009 and 2010 are respectively 70 and 15 basis points, vanishing completely in 2011.

Finally, although we cannot formally test these results as the “subprime crisis effect” due to simplified assumptions in the model (for instance, there is no credit market in the model), we contribute to the debate presenting this counterfactual analysis.

### 3.5 Conclusion

In this paper we show that the benchmark model fails to account for the average contribution of the US shocks on the Canadian series. One possible problem is misspecification in the exchange rate channel. We investigate this hypothesis by adding two features in the model: (1) incomplete pass-through, and (2) cross-correlation. The addition of both features improves the ability of the model to account for the external shocks spillover (see table 3.4).

Although, the correlation effect is the most important one, it only can be considered as a palliative. The reason is that the correlation parameters might be capturing some additional effects that theoretically should spill through the exchange rate. Hence, the NOEM models still face theoretical challenges in the transmission mechanisms.

In an application of the model we provide an approximated measure of the impact of the recent subprime crisis into Canadian output gap and its expected duration. The results point to a big role of the external disturbances on the last quarters of 2008, and residual

---

**Table 3.5: Canadian Output Gap Decomposition and Forecast**

<table>
<thead>
<tr>
<th>Year</th>
<th>Effective</th>
<th>Counterfactual</th>
<th>US Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>99</td>
<td>153</td>
<td>(54)</td>
</tr>
<tr>
<td>2008</td>
<td>(253)</td>
<td>(65)</td>
<td>(188)</td>
</tr>
<tr>
<td>2009</td>
<td>(102)</td>
<td>(32)</td>
<td>(70)</td>
</tr>
<tr>
<td>Forecast</td>
<td>(18)</td>
<td>(3)</td>
<td>(15)</td>
</tr>
<tr>
<td>2011</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
effects lasting for more two years (see table 3.5).

State of the art models (see Adolfson et al. (2008), Adolfson et al. (2007)) in the NOEM literature take the foreign economy as an exogenous process. In this paper we direct criticize this approach, since it is not possible to incorporate structural cross-correlations. Hence, results obtained by them must be viewed with care.
Bibliography


Appendix A

Feedback traders: Derivation and estimation methodology

A.1 Appendix

A.1.1 Appendix A

In this section I show the derivation of the strategic trader solution for problem (1.6). The price function, from the market clearing condition, is

\[ P_t \triangleq P_t(x_t, X_{t-1}, Y_{t-1}^p, s_t, \varepsilon_t) = F_t + \frac{1}{\alpha} \left[ X_t^a + X_t^b + x_t - X_{t-1}^a \right] \]

where using equations (1.2) to (1.4) the feedback quantities are

\[ X_t^a \triangleq X_t^a(Y_{t-1}^p, s_t) = \delta_a^t (P_{t-1} - P_{t-2}) + (1 - s_t) (2\rho^a - 1) X_{t-1}^a \]
\[ X_t^b \triangleq X_t^b(Y_{t-1}^p, s_t) = \delta_b^t \sum_{i=1}^{N-1} (P_{t-i} - P_{t-N}) + (1 - s_t) (2\rho^b - 1) X_{t-1}^b \]

Using the price function, we can write the expected profit per share in the period \( t + 1 \) for a given choice of \( x_t \)

\[ E_t P_{t+1} - P_t = E_t F_{t+1} + \frac{1}{\alpha} \left[ E_t X_{t+1}^a + E_t X_{t+1}^b + E_t X_{t+1}^a - x_t \right] - P_t \]

Note that the quantities \( E_t X_{t+1}^a \) and \( E_t X_{t+1}^b \) depend on the equilibrium price \( P_t \). Since the equilibrium price depends on the choice of \( x_t \), the next step is to isolate the \( P_t \) terms.
Define the quantity $X_{t+1}^z = X_{t+1} - s_t^z \delta^z P_t$, and using the fact that $E_t F_{t+1} = F_t$, we rewrite the equation

$$E_t P_{t+1} - P_t = F_t + \frac{1}{\alpha} \left[ E_t X_t^a + E_t X_t^b + E_t X_t^s - x_t \right] + \left[ \frac{1}{\alpha} \left[ q_{s}^a \delta^a + q_{s}^b \delta^b \right] - 1 \right] P_t$$

where $q_s^a \equiv p(s^a_{t+1} = 1|s_t)$.

Define $\xi_s \equiv \frac{1}{\alpha} \left[ q_{s}^a \delta^a + q_{s}^b \delta^b \right]$ and the state vector as $Y_{t-1} \equiv \left[ F_{t-1} \ X_{t-1}^s \ Y_{t-1}^p \right]$. Substituting the $P_t$ terms, the profit per share can be written as a function of state variables and the choice $x_t$ of the strategic investor.

$$V(x_t, \bullet) \equiv E_t P_{t+1} - P_t = \xi_s F_t + \frac{1}{\alpha} \left[ \left( q_{s}^a \overline{A}^a + q_{s}^b \overline{A}^b \right) Y_{t-1}^p + (\xi_s - 1) \left[ X_t^a + X_t^b - X_{t-1}^a \right] \right] + \frac{E_t X_{t+1}^s}{\alpha} + \frac{(\xi_s)}{\xi_s}$$

where

$$\overline{A}^a \equiv \delta^a \left[ \begin{array}{ccccc} 1 & \cdots & 1 & - (N-1) & 0 \\
\end{array} \right]$$

$$\overline{A}^b \equiv \delta^b \left[ \begin{array}{cccc} -1 & 0 & \cdots & 0 \\
\end{array} \right]$$

The objective function is given by $x_t V(x_t, \bullet)$. The first order condition is given by

$$\frac{x_t}{\lambda_{s_t}} = \alpha \xi_s F_t + \left( q_{s}^a \overline{A}^a + q_{s}^b \overline{A}^b \right) Y_{t-1}^p + (\xi_s - 1) \left[ X_t^a + X_t^b - X_{t-1}^a \right] + E_t X_{t+1}^s$$

where

$$\lambda_s \equiv \frac{1}{2(2 - \xi_s)}$$

### A.1.2 Appendix B: Steady State

The steady state equilibrium is characterized by $F_t = \overline{F}$, $\varepsilon_t = 0$, $x_t = x_t$ and $\overline{P} = P_t$. Substituting these conditions into the equilibrium system given by equations (1.9) to (??), it is straightforward to get

$$\frac{\overline{x}}{\lambda_{s_t}} = \alpha \xi_s \overline{F} + \left( q_{s}^a \overline{A}^a + q_{s}^b \overline{A}^b \right) \overline{Y}^p + (2 - \xi_s) \overline{x} \quad \text{(A.1)}$$

$$F_t = \overline{F}; P_t = \overline{F}; X_t^a = 0; X_t^b = 0$$

Since in the steady state $\overline{Y}^p = \epsilon_N x \overline{F}$, then $\alpha \xi_s \overline{F} + \left( q_{s}^a \overline{A}^a + q_{s}^b \overline{A}^b \right) \overline{Y}^p = 0$. Hence, the only value for $x_t$ to solve equation (A.1) is $x_t = 0$. 
A.1.3 Appendix C: Solving Linear Markov Switching Rational Expectation System

This section describes briefly the method to solve Linear Rational Expectations Models proposed by Farmer et al. (2006)(FWZ). Consider the following generalization of the constant parameter model given in equation (A.1):

\[
\begin{bmatrix}
A(s_t) \\
\alpha_1(s_t) \\
\alpha_2
\end{bmatrix}_{n \times l} x_t = \begin{bmatrix}
\beta(s_t) \\
\beta_1(s_t) \\
\beta_2
\end{bmatrix}_{n \times l} x_{t-1} + \begin{bmatrix}
\Psi(s_t) \\
\psi(s_t)
\end{bmatrix}_{k \times l} \varepsilon_t + \begin{bmatrix}
\Pi \\
0 \\
\pi
\end{bmatrix}_{l \times l} \eta_t, \quad x_0 = \bar{x}_0 \quad (A.2)
\]

where \( s_t \) follows an \( h \)-state Markov chain, \( h \in H \triangleq \{1,...,h\} \), with a stationary transition matrix \( \Gamma \). If a solution for (A.2) exists, it will have the form:

\[ x_t = g_1(s_t) x_{t-1} + g_2(s_t) \varepsilon_t \]

In other words, the aim of the algorithm is to solve for the matrices \( g_1(s_t) \) and \( g_2(s_t) \), which consists on a restricted VAR representation, allowing the estimation of the model by standard techniques.

**Expanding the System**

The first step consists of expanding the original system in a constant parameter representation (stacking all the regimes together in one system) of the form

\[ AX_t = BX_{t-1} + \Psi u_t + \Pi \eta_t \quad (A.3) \]

where

\[
X_t = \begin{bmatrix}
t_{(\xi_t=1)} x_t \\
\vdots \\
t_{(\xi_t=h)} x_t
\end{bmatrix}_{n \times 1}
\]

We are interested in the solution to (A.2) and for this purpose equation (A.3) is a useful way of representing the model because it has constant parameter matrices, \( A, B, \Psi \) and \( \Pi \) and because the shocks \( u_t \) and \( \eta_t \) have zero means. These properties allow us to use known techniques to compute a solution (see Sims 2002).
The construction of these constant matrices is not trivial and it is one of the contributions of FWZ’s paper. They provide necessary and sufficient conditions for the proposed class of minimal state variable solutions (MSV) to be unique. They proposed an algorithm to build these matrices and ensure that these conditions hold. The central role of the algorithm is to find a matrix $\Phi$ that ensures the boundedness of the stochastic process that solves the model.

They begin assuming the existence of a family of matrices $\{\phi_i\}_{i=2}^h$ where each $\phi_i$ has dimension $l \times n$ and has a full rank. We described later how to express $\{\phi_i\}_{i=2}^h$ as the fixed point of a system of nonlinear equations. Define the matrix $\Phi$ as follows,

$$\Phi = \begin{bmatrix} \mathbf{e}_2'^{\otimes} \otimes \phi_2 \\ \vdots \\ \mathbf{e}_h'^{\otimes} \otimes \phi_h \end{bmatrix}$$ (A.4)

and let the matrices $A$, $B$ and $\Pi$ be given by

$$A = \begin{bmatrix} \text{diag}(a_1(1), \ldots, a_1(h)) \\ a_2 & \cdots & a_2 \end{bmatrix}$$ (A.5)

$$B = \begin{bmatrix} \text{diag}(b_1(1), \ldots, b_1(h)) (\Gamma \otimes \mathbf{I}_n) \\ b_2 & \cdots & b_2 \\ 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix}$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix and $\mathbf{e}_i$ is the $i^{th}$ column of $\mathbf{I}_n$.

**Definition of the Shocks**

Let $\mathbf{1}_h$ be the $h$-dimensional column vector of ones and $X_t \in \mathbb{R}^{nh}$ be an arbitrary vector. Next define $h$ matrices $S_{\xi_t}$ for $\xi_t \in \{1, \ldots, h\}$, the vector of error terms $u_t$ and the matrix $\Psi$ as follows

$$S_{\xi_t} = (\text{diag} \{b_1(1), \ldots, b_1(h)\}) \left[(\mathbf{e}_1'^{\otimes} \otimes \Gamma - \Gamma) \otimes \mathbf{I}_n\right]$$
The error term \( u_t \) contains two kinds of shock. The first block is the "switching" shocks and the second block the "fundamental" shocks. This construction ensures that \( E_{t-1}[u_t] = 0 \).

The Algorithm to Compute the Solution

We describe the iterative procedure to find the fixed point for the matrix \( \Phi \). Let superscript \( j \) denotes the \( j^{th} \) step of an iterative procedure.

Beginning with a set of full rank matrices \( \{\phi_i^0\}_{i=2}^{h} \), using (A.4) and (A.5) build the matrix \( A^0 \). Next compute the QZ-decomposition of \( \{A^0, B\} \) where \( Q^0 S^0 Z^0 = A^0 \) and \( Q^0 T^0 Z^0 = B \) and the upper triangular matrices \( S^0 = (s_{i,j}) \) and \( T^0 = (t_{i,j}) \) have been arranged in such a way that \( t_{i,j}/s_{i,j} \) are in increasing order. Let \( q \in \{1, 2, \ldots, h\} \) be the integer such that \( t_{i,j}/s_{i,j} < 1 \) if \( i \leq q \) and \( t_{i,j}/s_{i,j} > 1 \) if \( i > q \). Let \( Z_u \), partitioned as \( Z_u = \begin{bmatrix} Z_1 \cdots Z_h \end{bmatrix} \), be the last \( nk - q \) rows of \( Z^0 \) and set \( \phi_z^j = z_i \). Repeat the procedure until convergence.

A.1.4 Appendix D : State Space Markov Switching Filtering

In this section I describe the method of filtering the state variables from a state space representation with Markov switching. The algorithm is the method proposed by Kim (1994) and I briefly describe the main steps.

Consider the following state-space representation of a dynamic linear model

\[
\begin{align*}
y_t &= F(s_t)x_t + B(s_t)z_t + c_t \\
x_t &= A(s_t)x_{t-1} + C(s_t)z_t + G(s_t)v_t
\end{align*}
\]

where

\[
\begin{bmatrix} c_t \\ v_t \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix} \end{bmatrix} \right)
\]

The stochastic process for the unobservable process \( s_t \) follows a discrete-state Markov chain with transition probability matrix \( \Gamma \). Let the \( i \) and \( j \) denote the indexes of past and
current state, i.e, \( s_{t-1} = i \) and \( s_t = j \). For a given state \( x_{t-1} \), and all the coefficients in equations (A.6) and (A.7), the algorithm is basically a Kalman filter for each pair \( (i,j) \), as follows:

\[
\begin{align*}
\hat{x}_{t|i}^{(i,j)} &= A_j \hat{x}_{t-1|i-1} + C_j z_t \\
\hat{P}_{t|i}^{(i,j)} &= A_j \hat{P}_{t-1|i-1} A_j' + G_j Q G_j' \\
\hat{y}_{t|i}^{(i,j)} &= y_t - F_j \hat{x}_{t-1|i-1} - B_j z_t \\
H_{t|i}^{(i,j)} &= F_j \hat{P}_{t-1|i-1} F_j' + R \\
K_{t|i}^{(i,j)} &= \hat{P}_{t|i}^{(i,j)} F_j' \left[ H_{t|i}^{(i,j)} \right]^{-1} \\
\hat{x}_{t|i}^{(i,j)} &= \hat{x}_{t|i-1}^{(i,j)} + K_{t|i}^{(i,j)} \hat{y}_{t|i}^{(i,j)} \\
\hat{P}_{t|i}^{(i,j)} &= \left( I - K_{t|i}^{(i,j)} F_j \right) \hat{P}_{t|i-1}^{(i,j)}
\end{align*}
\]

where \( \hat{x}_{t-1|i-1} \) is an inference about \( x_{t-1} \) based on information up to time \( t - 1 \); \( \hat{x}_{t|i}^{(i,j)} \) is an inference about \( x_t \), based on information up to time \( t - 1 \); \( \hat{y}_{t|i}^{(i,j)} \) is the conditional forecast error of \( y_t \) based on information up to time \( t - 1 \); \( \hat{H}_{t|i}^{(i,j)} \) is the conditional variance of forecast error \( \hat{y}_{t|i}^{(i,j)} \); and \( K_{t|i}^{(i,j)} \) is the Kalman gain. The indexes \( i \) and \( j \) represents information given \( s_{t-1} = i \) and \( s_t = j \) respectively.

The exact likelihood function requires tracking all the possible regimes. This calculation is computational intractable. For instance, a two regime Markov chain have around 1.000.000 paths after 20 steps. The approach proposed by KIM (1994) is to approximate the likelihood by reducing the \( H \times H \) posteriors in each step \( \left( x_{t|i}^{(i,j)}, P_{t|i}^{(i,j)} \right) \) into \( H \) to complete the Kalman filter. The approximate posteriors are given by

\[
\begin{align*}
\hat{x}_{t|i}^j &= \frac{\sum_{i=1}^{H} \hat{p}(s_t = j, s_{t-1} = i|\Psi_t) \hat{x}_{t|i}^{(i,j)}}{\hat{p}(s_t = j|\Psi_t)} \\
\hat{P}_{t|i}^j &= \frac{\sum_{i=1}^{H} \hat{p}(s_t = j, s_{t-1} = i|\Psi_t) \left\{ \left( F_{t|i}^{(i,j)} \right) + \left( \hat{x}_{t|i}^j - \hat{x}_{t|i}^{(i,j)} \right) \left( \hat{x}_{t|i}^j - \hat{x}_{t|i}^{(i,j)} \right)' \right\}}{\hat{p}(s_t = j|\Psi_t)}
\end{align*}
\]

where \( \Psi_t \) is the information set up to time \( t \) and the probabilities calculations are described below.

\[
\hat{p}(s_t = j, s_{t-1} = i|\Psi_t) = \frac{f(y_t|s_t = j, s_{t-1} = i, \Psi_{t-1}) \hat{p}(s_t = j, s_{t-1} = i|\Psi_{t-1})}{\int f(y_t|\Psi_{t-1})}
\]
where

\[
f (y_t | s_t = j, s_{t-1} = i, \Psi_{t-1}) = (2\pi)^{-N/2} \left| H_{t}^{(i,j)} \right|^{-\frac{1}{2}} \exp \left( \eta_{t|t-1}^{(i,j)} \left( H_{t}^{(i,j)} \right)^{-1} \eta_{t|t-1}^{(i,j)} \right)
\]

\[
f (y_t | \Psi_{t-1}) = \sum_{i=1}^{H} \sum_{j=1}^{H} f (y_t | s_t = j, s_{t-1} = i, \Psi_{t-1})
\]

\[
p (s_t = j, s_{t-1} = i | \Psi_{t-1}) = \Gamma_{i,j} \sum_{k=1}^{H} p (s_{t-1} = i, s_{t-2} = k | \Psi_{t-1})
\]

As a by product of running the above Kalman filter, the conditional log-likelihood function can be obtained by

\[
LL = \log (f (y_T, y_{T-1}, \cdots | \Psi_0)) = \sum_{t=1}^{T} \log (f (y_t | \Psi_{t-1}))
\]
Appendix B

Rebound after extreme events: Additional tables

B.1 Additional Figures

Table B.1: Summary Statistics of Stock Index Returns
### Table B.2: Summary Statistics of Future Contracts

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of Events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Number of Events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>117</td>
<td>(35)</td>
<td>(40)</td>
<td>49%</td>
<td>(80)</td>
<td>45</td>
<td>65%</td>
<td>0.44</td>
<td>113</td>
<td>(38)</td>
<td>53</td>
<td>65%</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>57</td>
<td>(39)</td>
<td>55%</td>
<td>52%</td>
<td>(80)</td>
<td>48</td>
<td>60%</td>
<td>0.44</td>
<td>127</td>
<td>(20)</td>
<td>59</td>
<td>68%</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>136</td>
<td>(131)</td>
<td>57%</td>
<td>65%</td>
<td>(99)</td>
<td>59</td>
<td>67%</td>
<td>0.54</td>
<td>117</td>
<td>(47)</td>
<td>105</td>
<td>83%</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>195</td>
<td>(185)</td>
<td>55%</td>
<td>54%</td>
<td>(99)</td>
<td>65</td>
<td>67%</td>
<td>0.60</td>
<td>127</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>136</td>
<td>(131)</td>
<td>57%</td>
<td>65%</td>
<td>(99)</td>
<td>59</td>
<td>67%</td>
<td>0.54</td>
<td>117</td>
<td>(47)</td>
<td>105</td>
<td>83%</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>195</td>
<td>(185)</td>
<td>55%</td>
<td>54%</td>
<td>(99)</td>
<td>65</td>
<td>67%</td>
<td>0.60</td>
<td>127</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>101</td>
<td>(101)</td>
<td>57%</td>
<td>65%</td>
<td>(99)</td>
<td>59</td>
<td>67%</td>
<td>0.54</td>
<td>117</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>195</td>
<td>(185)</td>
<td>55%</td>
<td>54%</td>
<td>(99)</td>
<td>65</td>
<td>67%</td>
<td>0.60</td>
<td>127</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td>395</td>
<td>(395)</td>
<td>55%</td>
<td>54%</td>
<td>(99)</td>
<td>65</td>
<td>67%</td>
<td>0.60</td>
<td>127</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>101</td>
<td>(101)</td>
<td>57%</td>
<td>65%</td>
<td>(99)</td>
<td>59</td>
<td>67%</td>
<td>0.54</td>
<td>117</td>
<td>(21)</td>
<td>113</td>
<td>83%</td>
<td>0.62</td>
<td></td>
</tr>
</tbody>
</table>

### Table B.3: Return Statistics Following the Extreme Events for the 2.5% Quantil

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Events</td>
<td>Mean Return</td>
</tr>
<tr>
<td>Belgium</td>
<td>117</td>
<td>(35)</td>
</tr>
<tr>
<td>Brazil</td>
<td>57</td>
<td>(39)</td>
</tr>
<tr>
<td>Canada</td>
<td>136</td>
<td>(131)</td>
</tr>
<tr>
<td>Finland</td>
<td>195</td>
<td>(185)</td>
</tr>
<tr>
<td>France</td>
<td>136</td>
<td>(131)</td>
</tr>
<tr>
<td>Germany</td>
<td>195</td>
<td>(185)</td>
</tr>
<tr>
<td>Mexico</td>
<td>101</td>
<td>(101)</td>
</tr>
<tr>
<td>Russia</td>
<td>195</td>
<td>(185)</td>
</tr>
<tr>
<td>South Africa</td>
<td>395</td>
<td>(395)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>101</td>
<td>(101)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Number of Events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Number of Events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
<th>Mean Return</th>
<th>Mean Return</th>
<th>Corr. w/ market events</th>
</tr>
</thead>
</table>
Table B.4: Return Statistics Following the Extreme Events for the 1.0% Quantile

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Return (%)</td>
<td>Mean Return</td>
<td>Corr. w/ market</td>
<td>Mean Return (%)</td>
</tr>
<tr>
<td></td>
<td>D-1 (2%)</td>
<td>D-1 (2%)</td>
<td>events</td>
<td>D-1 (2%)</td>
</tr>
<tr>
<td>Belgium</td>
<td>48</td>
<td>(490)</td>
<td>(90)</td>
<td>52%</td>
</tr>
<tr>
<td>Brazil</td>
<td>39</td>
<td>(381)</td>
<td>230</td>
<td>62%</td>
</tr>
<tr>
<td>Canada</td>
<td>54</td>
<td>(450)</td>
<td>25</td>
<td>67%</td>
</tr>
<tr>
<td>Finland</td>
<td>54</td>
<td>(483)</td>
<td>18</td>
<td>75%</td>
</tr>
<tr>
<td>France</td>
<td>54</td>
<td>(325)</td>
<td>71</td>
<td>85%</td>
</tr>
<tr>
<td>Germany</td>
<td>54</td>
<td>(370)</td>
<td>70</td>
<td>88%</td>
</tr>
<tr>
<td>United kingdom</td>
<td>54</td>
<td>(441)</td>
<td>18</td>
<td>67%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>54</td>
<td>(582)</td>
<td>55</td>
<td>76%</td>
</tr>
<tr>
<td>United States</td>
<td>54</td>
<td>(457)</td>
<td>72</td>
<td>47%</td>
</tr>
</tbody>
</table>

Table B.5: Results of Regression (1) for each country. Coefficients significant at 5% are indicated by shadow cells.
Table B.6: Results of Regression (2) for each country. Coefficients significant at 5% are indicated by shadow cells.

<table>
<thead>
<tr>
<th>Country</th>
<th>Average Number of Sign.*</th>
<th>Coeff. at 5%</th>
<th>AR(1) Coeff. t-stat</th>
<th>MSCI Factor Coeff. t-stat</th>
<th>Imkt Coeff. t-stat</th>
<th>Id Coeff. t-stat</th>
<th>It Coeff. t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>39%</td>
<td>67.3%</td>
<td>0.05</td>
<td>0.82</td>
<td>0.33</td>
<td>0.29</td>
<td>0.14</td>
</tr>
<tr>
<td>Brazil</td>
<td>25%</td>
<td>43.7%</td>
<td>0.11</td>
<td>1.33</td>
<td>0.19</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>Canada</td>
<td>44%</td>
<td>78.9%</td>
<td>0.01</td>
<td>0.80</td>
<td>0.24</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Finland</td>
<td>25%</td>
<td>74.1%</td>
<td>0.05</td>
<td>0.95</td>
<td>0.24</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>Germany</td>
<td>43%</td>
<td>74.3%</td>
<td>0.28</td>
<td>1.07</td>
<td>0.23</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>46%</td>
<td>72.7%</td>
<td>0.05</td>
<td>0.91</td>
<td>0.35</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>Mexico</td>
<td>26%</td>
<td>84.7%</td>
<td>0.08</td>
<td>1.03</td>
<td>0.72</td>
<td>0.52</td>
<td>0.16</td>
</tr>
<tr>
<td>Russia</td>
<td>15%</td>
<td>67.5%</td>
<td>0.14</td>
<td>1.09</td>
<td>0.72</td>
<td>0.54</td>
<td>0.22</td>
</tr>
<tr>
<td>South Africa</td>
<td>19%</td>
<td>41.7%</td>
<td>0.08</td>
<td>0.71</td>
<td>0.45</td>
<td>0.17</td>
<td>0.49</td>
</tr>
<tr>
<td>Spain</td>
<td>37%</td>
<td>69.4%</td>
<td>0.01</td>
<td>0.92</td>
<td>0.23</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>Sweden</td>
<td>37%</td>
<td>60.3%</td>
<td>0.01</td>
<td>1.08</td>
<td>0.23</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>Switzerland</td>
<td>35%</td>
<td>66.0%</td>
<td>0.07</td>
<td>0.83</td>
<td>0.23</td>
<td>0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>United States</td>
<td>61%</td>
<td>91.6%</td>
<td>0.19</td>
<td>0.98</td>
<td>0.26</td>
<td>0.14</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Average 5.5% 69.9% 0.02 0.97 0.26 (0.13) (0.08)

Table B.7: Results of Regression (3) for each country. Coefficients significant at 5% are indicated by shadow cells.
### Table B.8: Out-of-sample returns of strategies I and II, detailed by country.

<table>
<thead>
<tr>
<th>Country</th>
<th>N. Events Total</th>
<th>Average (Apr.)</th>
<th>Alpha (%)</th>
<th>Average (Sep.)</th>
<th>Alpha (%)</th>
<th>Average (Dec.)</th>
<th>Alpha (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>52</td>
<td>152.40%</td>
<td>142.40%</td>
<td>138.54%</td>
<td>122.85%</td>
<td>113.67%</td>
<td>105.35%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>74</td>
<td>67.40%</td>
<td>70.90%</td>
<td>108.90%</td>
<td>104.54%</td>
<td>113.18%</td>
<td>108.75%</td>
</tr>
<tr>
<td>Finland</td>
<td>56</td>
<td>197.50%</td>
<td>211.50%</td>
<td>132.35%</td>
<td>141.57%</td>
<td>126.48%</td>
<td>118.57%</td>
</tr>
<tr>
<td>France</td>
<td>85</td>
<td>26.40%</td>
<td>34.50%</td>
<td>122.43%</td>
<td>229.82%</td>
<td>78.78%</td>
<td>121.22%</td>
</tr>
<tr>
<td>Germany</td>
<td>55</td>
<td>29.50%</td>
<td>23.50%</td>
<td>156.54%</td>
<td>206.83%</td>
<td>186.83%</td>
<td>170.80%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>105</td>
<td>27.50%</td>
<td>22.50%</td>
<td>259.58%</td>
<td>177.78%</td>
<td>59.82%</td>
<td>137.42%</td>
</tr>
<tr>
<td>Russia</td>
<td>19</td>
<td>36.50%</td>
<td>18.50%</td>
<td>179.45%</td>
<td>177.82%</td>
<td>135.42%</td>
<td>197.93%</td>
</tr>
<tr>
<td>South Africa</td>
<td>83</td>
<td>120.40%</td>
<td>122.40%</td>
<td>130.55%</td>
<td>115.79%</td>
<td>131.33%</td>
<td>124.38%</td>
</tr>
<tr>
<td>Spain</td>
<td>63</td>
<td>7.50%</td>
<td>24.50%</td>
<td>129.54%</td>
<td>152.72%</td>
<td>90.53%</td>
<td>143.53%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>27</td>
<td>61.40%</td>
<td>13.50%</td>
<td>3.22%</td>
<td>297.12%</td>
<td>16.64%</td>
<td>3.21%</td>
</tr>
<tr>
<td>United States</td>
<td>66</td>
<td>117.50%</td>
<td>5.50%</td>
<td>23.40%</td>
<td>217.82%</td>
<td>15.8%</td>
<td>121.92%</td>
</tr>
<tr>
<td>Sweden</td>
<td>65</td>
<td>29.50%</td>
<td>56.50%</td>
<td>128.88%</td>
<td>143.88%</td>
<td>109.44%</td>
<td>120.33%</td>
</tr>
</tbody>
</table>

**Weights-Weighted Average**

| N. Events Weighed Average | 89.5 | 15.76% | 14.56% | 132.35% | 110.68% | 83.53% | 28.33% |

### Table B.9: Out-of-sample returns of strategy III, detailed by country.

<table>
<thead>
<tr>
<th>Country</th>
<th>N. Events Total</th>
<th>Dec. Gross Returns</th>
<th>Gross Returns</th>
<th>Net Returns (Bid asks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>91</td>
<td>142.78%</td>
<td>127.92%</td>
<td>127.92%</td>
</tr>
<tr>
<td>Brazil</td>
<td>77</td>
<td>36.41%</td>
<td>27.35%</td>
<td>21.84%</td>
</tr>
<tr>
<td>Canada</td>
<td>92</td>
<td>123.30%</td>
<td>94.80%</td>
<td>94.80%</td>
</tr>
<tr>
<td>France</td>
<td>51</td>
<td>30.40%</td>
<td>20.83%</td>
<td>18.50%</td>
</tr>
<tr>
<td>Germany</td>
<td>85</td>
<td>15.50%</td>
<td>12.85%</td>
<td>11.23%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>122</td>
<td>22.50%</td>
<td>18.50%</td>
<td>16.35%</td>
</tr>
<tr>
<td>Russia</td>
<td>19</td>
<td>40.80%</td>
<td>22.76%</td>
<td>19.50%</td>
</tr>
<tr>
<td>South Africa</td>
<td>83</td>
<td>136.50%</td>
<td>130.55%</td>
<td>126.55%</td>
</tr>
<tr>
<td>Spain</td>
<td>60</td>
<td>22.50%</td>
<td>19.79%</td>
<td>17.80%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>27</td>
<td>152.50%</td>
<td>138.55%</td>
<td>128.55%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>66</td>
<td>117.50%</td>
<td>90.53%</td>
<td>70.53%</td>
</tr>
<tr>
<td>Sweden</td>
<td>65</td>
<td>29.50%</td>
<td>20.50%</td>
<td>9.50%</td>
</tr>
<tr>
<td>United States</td>
<td>66</td>
<td>117.50%</td>
<td>50.50%</td>
<td>30.50%</td>
</tr>
</tbody>
</table>

**Weights-Weighted Average**

| N. Events Weighed Average | 89.5 | 15.76% | 14.56% | 132.35% | 110.68% | 83.53% | 28.33% |


### Table B.10: Summary of Sub Sample Regressions.

<table>
<thead>
<tr>
<th>Sub1</th>
<th>Coefficients</th>
<th>B2</th>
<th>AR(1)</th>
<th>Market Factor</th>
<th>Im</th>
<th>Id</th>
<th>Iu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg. (2)</td>
<td>Weighted Averages</td>
<td>20.1%</td>
<td>73.9%</td>
<td>0.08</td>
<td>0.71</td>
<td>0.22</td>
<td>(0.21)</td>
</tr>
<tr>
<td></td>
<td>t-statistic</td>
<td>2.57</td>
<td>11.62</td>
<td>3.51</td>
<td>4.80</td>
<td>2.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff at 5%</td>
<td>15/24</td>
<td>14/14</td>
<td>2/14</td>
<td>5/14</td>
<td>1/14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff&lt;0</td>
<td>2/2</td>
<td>5/5</td>
<td>0/0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reg. (3)</td>
<td>Weighted Averages</td>
<td>20.1%</td>
<td>75.8%</td>
<td>0.06</td>
<td>0.76</td>
<td>0.21</td>
<td>(0.17)</td>
</tr>
<tr>
<td>USD</td>
<td>t-statistic</td>
<td>2.57</td>
<td>11.98</td>
<td>3.51</td>
<td>4.81</td>
<td>2.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff at 5%</td>
<td>13/24</td>
<td>14/14</td>
<td>2/14</td>
<td>4/14</td>
<td>1/14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff&lt;0</td>
<td>2/2</td>
<td>5/5</td>
<td>0/0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub2</th>
<th>Coefficients</th>
<th>B2</th>
<th>AR(1)</th>
<th>Market Factor</th>
<th>Im</th>
<th>Id</th>
<th>Iu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg. (2)</td>
<td>Weighted Averages</td>
<td>45.0%</td>
<td>71.2%</td>
<td>(0.03)</td>
<td>0.98</td>
<td>0.14</td>
<td>(0.17)</td>
</tr>
<tr>
<td></td>
<td>t-statistic</td>
<td>1.31</td>
<td>22.25</td>
<td>3.89</td>
<td>4.88</td>
<td>2.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff at 5%</td>
<td>8/14</td>
<td>24/14</td>
<td>5/14</td>
<td>7/14</td>
<td>1/14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff&lt;0</td>
<td>1/6</td>
<td>7/7</td>
<td>1/1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reg. (3)</td>
<td>Weighted Averages</td>
<td>44.4%</td>
<td>68.2%</td>
<td>(0.03)</td>
<td>1.05</td>
<td>0.25</td>
<td>(0.16)</td>
</tr>
<tr>
<td>USD</td>
<td>t-statistic</td>
<td>1.31</td>
<td>24/14</td>
<td>7.26</td>
<td>8.82</td>
<td>2.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff at 5%</td>
<td>8/14</td>
<td>24/14</td>
<td>6/14</td>
<td>7/14</td>
<td>2/14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Sign. Coeff&lt;0</td>
<td>2/6</td>
<td>7/7</td>
<td>2/2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix C

DSGE: Derivation and estimation diagnostics

C.1 Detailed Model

Now we describe the full model in detail, as well as the steady-state and the loglinearized equilibrium.

C.1.1 Households

Preferences

Each household living in the domestic country seeks to maximize the lifetime utility function:

\[ U = \sum_{t=0}^{\infty} \beta^t E_0 \varepsilon_t^P \left( \frac{1}{1-\sigma} (C_t - H_t)^{1-\sigma} - \frac{1}{1+\varphi} (L_t^\delta)^{1+\varphi} \right) \]  

(C.1)

Utility depends positively on the consumption, \( C_t \), relative to an external habit, \( H_t \), and negatively on labour supply \( L_t^\delta \). \( \sigma \) is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution; \( \varphi \) represents the inverse of the elasticity of work effort with respect to the real wage; and \( \beta \), the intertemporal discount. Furthermore, there is a general preference shock, \( \varepsilon_t^P \), that affects the households' intertemporal substitution.
We assume that $0 < \beta < 1$, $\varphi > 1$, $\sigma > 0$ and the log of the preference shock, $\log (\varepsilon_t^{\mu}) = \varepsilon_t^p$, follows the process:

$$c_t^p = (1 - \rho^p)\mu^p + \rho^p c_{t-1}^p + \varepsilon_t^p$$ \hspace{1cm} (C.2)

where $c_t^p \sim N(0,(\sigma^p)^2)$. The external habit stock is assumed to be proportional to aggregate past consumption:

$$H_t^e = hC_{t-1}$$ \hspace{1cm} (C.3)

The variable $C_t$ is defined as a CES consumption index:

$$C_t \triangleq \left[(1 - \alpha)^{1/\mu} (C_{H,t})^{\frac{\mu-1}{\alpha}} + \alpha^{1/\mu} (C_{F,t})^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$$ \hspace{1cm} (C.4)

where $\mu$ is the elasticity of intratemporal substitution between a bundle of Home goods $C_{H,t}$ and a bundle of Foreign goods $C_{F,t}$, while $\alpha$ measures degree of openness, and it is defined as the share of the imported goods on the Home Household consumption expenditure. We assume that $0 < \alpha < 1$ and $\mu > 0$.

The variables $C_{H,t}$ and $C_{F,t}$ are defined respectively by the CES indexes:

$$C_{k,t} \triangleq \left[\int_0^1 C_{k,t}(i) \left(\frac{\varepsilon_{k,t}}{\varepsilon_{k,t} - 1}\right)^{\frac{\varepsilon_{k,t} - 1}{\varepsilon_{k,t} - 1}} di\right]^{\frac{\varepsilon_{k,t} - 1}{\varepsilon_{k,t} - 1}}$$ \hspace{1cm} (C.5)

where $k = H, F$, and $C_{H,t}(i)$ and $C_{F,t}(i)$ are respectively the levels of consumption of the domestic and imported goods $i \in [0, 1]$, and $\varepsilon_{k,t}$ is the elasticity of intratemporal substitution between the Home goods and the imported goods. We assume that $\varepsilon_{k,t} > 0$.

We suppose that $\varepsilon_{k,t} = \varepsilon_k$ (or, the mark-up $\Psi_{k,t} = \Psi_k$) is constant for $k = H, F$.

The preference of each household of the foreign economy is analogous, except by the consumption index:

$$C_t^* \triangleq \left[(\alpha^*)^{1/\mu^*} (C_{H,t}^*)^{\frac{\mu^*-1}{\alpha^*}} + (1 - \alpha^*)^{1/\mu^*} (C_{F,t}^*)^{\frac{\mu^*-1}{\mu^*}}\right]^{\frac{\mu^*}{\mu^*-1}}$$ \hspace{1cm} (C.7)

\footnote{\(\varepsilon_{k,t}\) relates itself with the mark-up, $\Psi_{k,t}$, according to:}

$$\Psi_{k,t} = \left(\frac{\varepsilon_{k,t}}{\varepsilon_{k,t} - 1}\right)$$ \hspace{1cm} (C.6)
where $\alpha^*$ is the share of imported goods on the Foreign Household consumption expenditure, with $\alpha^* \neq \alpha$ generally.

**The intratemporal consumption choice**

The intratemporal consumption choice is done in a two-step fashion. First, each household take as given the price of the Home goods and the imported goods, denoted by $P_{H,i,t}(i)$ and $P_{F,i}(i)$, $i \in [0,1]$, and the levels of $C_{H,i}$ and $C_{F,i}$ to choose $C_{H,i}(i)$ and $C_{F,i}(i)$. In the second step, given $P_{H,t}$, $P_{F,t}$ and the Home household choice of $C_t$, they choose their allocation between Home and Foreign goods.

$$C_{k,t}(i) = \left( \frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\xi_k,t} C_{k,t}$$

where $k = H, F$.

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\mu} C_t$$

$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\mu} C_t$$

In our model the LOP is not satisfied, because of incomplete pass-through, and the PPP does not hold due to the home bias in the preferences:

$$MC_{F,t} \Delta \frac{c_{F,t}}{P_{F,t}} \neq 1$$

$$MC_{H,t}^* \Delta \frac{P_{H,t}}{c_{H,t}} \neq 1$$

$$Q_t \Delta \frac{c_t}{P_t} \neq 1$$

where $MC_{F,t}$ and $MC_{H,t}^*$ are LOP gaps (or "real" marginal costs of the importing sector), $Q_t$ is the real exchange rate, and $c_t$ is the nominal exchange rate.

---

2 the definitions of ppi, ipi, and cpi are given by

$$P_{k,t} \Delta \int_0^1 P_{k,i}(i)^{1-\xi_k,t} di \; ; \; k = H, F$$

$$P_t \Delta \left( (1 - \alpha) P_{H,t}^{1-\mu} + \alpha P_{F,t}^{1-\mu} \right)^{1/\mu}$$

and, for the Foreign Economy we have analogous results.
The budget constraint

Given the CPI index $P_t$, the period $t$ budget constraint of each agent is written as:

\[
C_t + \sum_{s=1}^{\infty} \sum_{h_{t+s} \in H_{t+s}} \frac{Z_t(h_{t+s})}{P_t} (B_t(h_{t+s}) - B_{t-1}(h_{t+s})) = \frac{W_t}{P_t} L_t^i + \sum_{k=H,F} x_{k,t-1} \cdot \frac{D V_{k,t}}{P_t} + \sum_{k=H,F} (x_{k,t-1} - x_{k,t}) \cdot \frac{V_{k,t}}{P_t}
\]

where $W_t$ is the domestic nominal wage, $V_t(i)$ is the nominal value of the $i$-th domestic firm, $D V_t(i)$ is the nominal dividends paid out by the $i$-th domestic firm. $x_t(i)$ is the share on the $i$-th domestic firms, $B_t(h_{t+s})$ is the household holdings of the history $h_{t+s}$-contingent claim Arrow-Debreu security, $Z_t(h_{t+s})$ is the nominal price of this asset, and the $\cdot$ denotes inner product $(a \cdot b = \int_0^1 a(i)b(i)di)$. All the nominal values are measured in Home currency. The period $t$ budget constraint of the Foreign representative household is analogous.

C.1.2 Firms

Each monopolistic-competitive firm sets the price, when allowed - as it will be discussed - of its differentiated good facing an isoelastic and downward-sloping labor demand curve, and subject to a technological constraint. The labor market is competitive.

Domestic Firms’ Technology

All Home firms operate a constant return of scale technology, use only labor as input for production and there is no investment.

\[
Y_t(i) = A_t L_t(i)
\]

where $Y_t(i)$ is the Home firm $i$’s output, $L_t(i)$ is the Home firm $i$’s labor demand and $A_t$ is and aggregate technological shock. The logarithm of this technological shock, $\alpha_t = \log (A_t)$, follows an AR(1) process:

\[
a_t = (1 - \rho^a)\alpha + \rho^a \alpha_{t-1} + \epsilon_t^a
\]
where \( 0 < \rho \theta < 1 \) and \( \epsilon_t^a \sim N(0,(\sigma^a)^2) \). All foreign firms operate a similar technology, except for the fact that \( \rho^a \) is possibly different from \( \rho^\theta \).

The technologies constraints imply that the Home and Foreign firm's labor demand are given respectively by \( L_t(i) = \frac{Y_t(i)}{A^i} \) and \( L_t^*(i) = \frac{Y_t^*(i)}{A^i} \) for each \( i \in [0,1] \), the real marginal costs are defined respectively by,

\[
MC_{H,t} \triangleq \frac{W_t}{A_t P_{H,t}}; \quad MC_{F,t}^* \triangleq \frac{W_t^*}{A_t P_{F,t}^*}
\]

and the aggregate labor demands are given by

\[
L^d_t = \frac{Y_t}{A} U_t; \quad L^d_t^* = \frac{Y_t^*}{A^* U_t^*}
\]

where \( Y_t \equiv \int_0^1 Y_t(i) \frac{\varepsilon_{H,t}}{\varepsilon_{H,t} + \varepsilon_{F,t}} di \), \( L_t = \int_0^1 L_t(i) di = \frac{Y_t}{A} U_t \) and \( U_t \equiv \int_0^1 \frac{Y_t(i)}{Y_t} di \) is a dispersion measure. Note that when prices are flexibles all firms located in the same country have the same output, implying that \( U_t = 1 \). For the Foreign Economy we have analogous definitions.

**Importing Firms’ Technology**

Each importing firm \( i \) has the technology of transforming the final good produced in the Foreign country \( Y_{F,t}^* \) in the differentiated imported good \( i \) for the domestic consumers \( Y_{F,t}(i) \) for \( i \in (0,1) \).

**Price setting**

Each firm sets price in a staggering way, as in Calvo (1983). In each period \( t \), it is allowed to a proportion \( (1 - \phi_k) \) of firms randomly selected to set a new price \( P_{k,t} \). The remaining \( \phi_k \) firms partially adjust prices according to the last inflation \( \frac{P_{k,t-1}}{P_{k,t-2}} \). The individual firm’s probability of readjusting each period \( t \) is independent of the time elapsed since its last adjustment.

A firm \( i \) selected to adjust price in period \( t \) sets a new price in order to maximize the present value of its stream of expected future profits. All the profits are distributed to the stockholders in the form of dividends according to their share of stocks. This is equivalent
of having the households as managers of the firm $i$, and choosing $\bar{p}_{k,t}$, in order to maximize his utility.

$$V_k,t(i) = \sum_{s=0}^{\infty} \phi_k^s E_t [D_{t,t+s} DV_{k,t+s}(i)]$$  \hspace{1cm} (C.19)

where $D_{t,t+s} = \beta^s \left( \frac{\Lambda_{t+s}}{\Lambda_t} \right) \frac{F_t}{P_{t+s}}$ is the $s$-period Home SDF at period $t$, $\Lambda_{t+s} = (C_{t+s} - hC_{t+s-1})^{-\gamma} \varepsilon_{t+s}^P$ is the $(t+s)$-period marginal utility of consumption.

$$DV_{k,t+s}(i) = [Y_{k,t+s}(i) - MC_{k,t+s}] P_{k,t+s}(i)$$  \hspace{1cm} (C.20)

is the profit at period $t+s$, $Y_{k,t+s}(i) = \left( \frac{P_{k,t+s}(i)}{\bar{p}_{k,t+s}(i)} \right)^{\gamma_k} Y_{k,t+s}$ is the demand of the good $i$ at period $t+s$, and $P_{k,t+s}(i) = P_{k,t}(i) \left( \frac{P_{k,t+s-1}}{P_{k,t-1}} \right)^{\gamma_k}$ is the price of the good $i$ at period $t+s$.

Given the firms' behavior above the dynamics of producer price index is written (remember equation (C.8)):

$$P_{k,t} = \left[ (1 - \phi_k) \bar{p}_{k,t}^{1-\epsilon_k} + \phi_k \left\{ P_{k,t-1} \left( \frac{P_{k,t-1}}{P_{k,t-2}} \right)^{\gamma_k} \right\}^{1-\epsilon_k} \right]^{1/\epsilon_k}$$  \hspace{1cm} (C.21)

for $k = H, F$.

### C.1.3 Monetary Authority

The Monetary Authority is an agent that sets the interest rate following an exogenous Taylor type rule.

$$R_t = R \left( \frac{P_t}{P_{t-1}}, Y_t, R_{t-1}, \zeta_t \right)$$  \hspace{1cm} (C.22)

where $R_t$ is the one period nominal interest rate of a one-period maturity Home bond and $\zeta_t$ is the vector of exogenous disturbances. The rule the Foreign Economy is analogous.

### C.1.4 Equilibrium

In equilibrium all agents solve their problems subject to their constraints and all markets clear.
From the Home household problem we have the following first order conditions:

\[
\frac{1}{R_t} = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right] \quad (C.23)
\]

\[
W_t = (\Lambda_t)^{-1} (L_t^s)^\varphi \quad (C.24)
\]

Each firm \(i \in [0, 1]\) that is randomly selected has the following F.O.C.:

\[
0 = \sum_{s=0}^{\infty} \phi^s_t E_t D_{t,t+s} \left[ (Y_{k,t+s}(i) - MC_{k,t+s}) \frac{\partial P_{k,t+s}(i)}{\partial P_{k,t}(i)} + \frac{\partial Y_{k,t+s}(i)}{\partial P_{k,t}(i)} P_{k,t+s}(i) \right] \quad (C.25)
\]

Remember that the price that solves the above F.O.C is \(p_{k,i} = P_{k,i}(j)\). So that the dynamics of the ppi and ipi are given by equation (C.21).

And, the Home Monetary Authority sets the interest rate according to (C.22):

All of the Home decisions rules hold in a similar form for the Foreign Economy.

The relationship between the Home and Foreign economies is given by the risk-sharing equation that is derived by matching the Euler equations for both economies with the additional assumption that markets are complete:

\[
(\Lambda_t)^{-1/\sigma} = \vartheta Q_0^{1/\sigma} (\Lambda^*_t)^{-1/\sigma} \varepsilon_{t}^{RS} \quad (C.26)
\]

where \(\varepsilon_t\) is the nominal exchange rate (the value of one unit of Foreign currency in terms of Home currency), \(\vartheta = Q_0 \lambda_{0}^{\lambda_{0}}\) is a constant that depends on the initial conditions from both economies, and \(\varepsilon_{t}^{RS}\) is a risk-sharing shock captures short-term deviations of the risk-sharing condition. \(\varepsilon_{t}^{RS} = \log(\varepsilon_{t}^{RS})\) is supposed to have the following process:

\[
\varepsilon_{t}^{RS} = (1 - \rho_{rs}^{\varepsilon}) \mu_{\varepsilon}^{rs} + \rho_{rs}^{\varepsilon} \varepsilon_{t-1}^{RS} + \sigma_{t}^{RS} \quad (C.27)
\]

where \(\sigma_{t}^{RS} \sim N(0, (\sigma_{rs})^2)\).

In equilibrium all markets must clear:
\begin{align*}
Y_t(i) &= C_{H,t}(i) + C^*_{H,t}(i) : Y_t^*(i) = C_{F,t}(i) + C^*_{F,t}(i) \\
J_t^*(i) &= L^d_t(i) : J_t^*(i) = L^d_t(i) \\
\int_0^1 x(t)^d(i) \, dj &= 1 : \int_0^1 x(t)^d(i) \, dj = 1 \\
\int_0^1 B(h_{t+s})^d(i) \, dj &= 0 : \int_0^1 B(h_{t+s})^d(i) \, dj = 0
\end{align*}

for \( i \in (0,1) \).

**Definition 1** (Rational Expectations Equilibrium). Fix a probability space \((\Omega, \mathcal{F}, \mathbb{P})\) and a filtration \((\mathcal{F}_t)\), satisfying the usual conditions, and suppose that the vector of exogenous disturbances \( \zeta_t \) is a Markov process in some state space \( D \subset \mathbb{R}^n \), which evolves according to the following VAR(1):

\begin{equation}
\zeta_t = \mu + \Phi \zeta_{t-1} + \sigma u_t
\end{equation}

where \( u_t \) is an \((\mathcal{F}_t)\)-measurable function in \( \mathbb{R}^n \).

A rational expectation equilibrium of the model is a vector of endogenous processes

\[
\{ P_{H,t}, P_{F,t}, Y_t, R_t, P^*_{H,t}, P^*_{F,t}, Y^*_t, R^*_t, Q_t \}
\]

that satisfies equations (C.23) to (C.31), (C.9), (C.13), (C.17), (C.21), (and the analogous equations for the Foreign Economy) in every period \( t \geq 0 \), given the vector of exogenous disturbances \( \zeta_t \) and the initial condition for the Economy, and all the markets clear.

**C.1.5 Equilibrium with flexible prices**

The rational expectation equilibrium with flexible prices defines the natural variables. This equilibrium is exactly the same as above, except for the decision rule of the firm that reoptimize every period according to the equation:

\begin{equation}
MC_{k,t} = \frac{1}{\Psi_{k,t}}
\end{equation}

All the firms set prices in order to equalize the real marginal cost to the real marginal benefit.
C.2 Solving the Model

C.2.1 Log-linearization

To solve the model, we log-linearize all the equilibrium equations around the steady state (SS) of the economy, with the hypothesis:

- \( \alpha^* = 0 \);
- \( \mu = \mu^* \);
- The dispersions \( U_t = U \) and \( U_t^* = U^* \) are constants;
- The SS Net Foreign Exports (NX) are equal to zero;
- The SS real exchange rate and LOP gaps are \( Q = MC_H^* = MC_F = 1 \);
- The SS Prices are equal: \( P = P_H = P_F = P^* = P_H^* = P_F^* = 1 \); and
- The SS of the exogenous processes is defined as the unconditional expectation.

Household FOCs

\[
\begin{align*}
\tilde{c}_t^e &= \tilde{E}_t \tilde{c}_{t+1}^e - \frac{1}{\sigma} \left[ \tilde{r}_t - E_t \tilde{\pi}_{t+1} - (1 - \rho^p) \tilde{c}_t^p \right] \quad (C.34a) \\
\tilde{w}_t &= \tilde{p}_t + \tilde{\phi}_t^e + \sigma \tilde{c}_t^e \\[10pt]
\text{where,} \\
\tilde{c}_t^e &= \frac{1}{1 - \rho} (\tilde{c}_t - \tilde{h} \tilde{c}_{t-1}) \quad (C.35)
\end{align*}
\]

Firms

\[
\tilde{p}_{k,t}(j) = (1 - \phi_k \beta) \sum_{s=0}^{\infty} (\phi_k \beta)^s \left\{ \log \left( \frac{P_{k,t+s}}{P_{k,t}} \right) - \gamma \log \left( \frac{P_{k,t-1+s}}{P_{k,t-1}} \right) + \tilde{\psi}_{k,t+s} + \tilde{m}_{k,t+s}(j) \right\} \quad (C.36)
\]

where \( k = H, F \) and \( \tilde{p}_{k,t}(j) = \log \left( \frac{P_{k,t}(j)}{P_{k,t}} \right) \).

The equation above holds for all firms \( j \in (0, 1) \) adjusting prices in period \( t \), so we have that \( \tilde{p}_{k,t}(j) = \tilde{p}_{k,t} \).

\[
p_{k,t} = \phi_k (p_{k,t-1} + \gamma \pi_{k,t-1}) + (1 - \phi_k) \tilde{p}_{k,t} \quad (C.37)
\]
\( \pi_{k,t} = \frac{\gamma_k}{1 + \beta_k} \pi_{k,t-1} + \frac{\lambda_k}{1 + \beta_k} (\hat{m}c_{k,t} + \hat{\psi}_{k,t}) + \frac{\beta}{1 + \beta_k} F_k \sigma_{k,t+1} \)  

where \( \lambda_k = \left(1 - \phi_k/\beta\right) \frac{1 - \phi_k}{\phi_k} \).

Log-linearizing the definition of the real marginal costs and the firm’s demand of labor we obtain:

\[ \begin{align*}
\hat{m}c_{H,t} &= \hat{w}_t - \hat{p}_{H,t} - \hat{a}_t \quad (C.39a) \ \\
\hat{m}c_{F,t} &= \hat{c}_t + \hat{p}_{F,t} - \hat{p}_{F,t} \quad (C.39b) \\
\hat{m}c_{F,t}^* &= \hat{w}_t^* - \hat{p}_{F,t}^* - \hat{a}_t^* \quad (C.39c) \\
\hat{m}c_{H,t}^* &= \hat{p}_{H,t} - \hat{c}_t - \hat{p}_{H,t}^* \quad (C.39d) \\
\hat{t}_t &= \hat{y}_t - \hat{a}_t \quad (C.39e) \\
\hat{t}_t^* &= \hat{y}_t^* - \hat{a}_t^* \quad (C.39f)
\end{align*} \]

**Price index, exchange rate and terms of trade**

The Home country term of trade is defined as the ratio of imported goods price to domestic goods price, ie, \( S_t = \frac{P_{H,t}}{P_{H,t}} \).

Log-linearizing the domestic and foreign CPI around the steady-state we have:

\[ \begin{align*}
p_t &= (1 - \alpha)p_{H,t} + \alpha p_{F,t} \quad (C.40a) \\
p_t^* &= \alpha^* p_{H,t}^* + (1 - \alpha^*) p_{F,t}^* \quad (C.40b)
\end{align*} \]

Substituting the log-linearized terms of trade into the equations above and noticing that \( s_t = -s_t^* - mc_{H,t} - mc_{F,t} \) we get:

\[ \begin{align*}
p_t &= p_{H,t} + \alpha s_t \quad (C.41) \\
p_t &= p_{F,t} - (1 - \alpha) s_t \quad (C.42) \\
p_t^* &= p_{H,t}^* + (1 - \alpha^*)(s_t + mc_{F,t} + mc_{H,t}) \quad (C.43) \\
p_t^* &= p_{F,t}^* - \alpha^*(s_t + mc_{F,t} + mc_{H,t}) \quad (C.44)
\end{align*} \]

Taking the first difference of equation \((C.41), (C.44)\) we get the cpi inflation as a function of ppi inflation, terms of trade, and LOP gaps:
\[
\begin{align*}
\pi_t &= \pi_{H,t} + \alpha \Delta s_t \\
\pi_t^* &= \pi_{H,t}^* - \alpha^* \Delta(s_t + \bar{m}c_{F,t} + \bar{m}c_{H,t}^*)
\end{align*}
\] (C.45, C.46)

We can use the real exchange rate definition in order to obtain a log-linearized relationship between the terms of trade and the exchange rate.

\[
Q_t = \frac{c_t P_t^*}{P_t}
\] (C.47)

\[
q_t = \bar{m}c_{F,t} + (1 - \alpha - \alpha^*)s_t - \alpha^*(\bar{m}c_{F,t} + \bar{m}c_{H,t}^*)
\] (C.48)

Note that if the PPP were satisfied, then we would have \(Q_t = 1\).

**Taylor type rule**

\[
\begin{align*}
\tilde{r}_t &= \rho_r \tilde{r}_{t-1} + (1 - \rho_r) [r_x \pi_t + r_y \tilde{y}_t] + r_{ds}(\tilde{y}_t - \tilde{y}_{t-1}) \\
&+ r_{ds}(\pi_t - \pi_{t-1}) + \tilde{c}_t
\end{align*}
\] (C.49a)

\[
\tilde{r}_t^* = \rho_r \tilde{r}_t^* + (1 - \rho_r) [r_x \pi_t^* + r_y \tilde{y}_t^*] + r_{ds}(\tilde{y}_t^* - \tilde{y}_{t-1}^*) \\
&+ r_{ds}(\pi_t^* - \pi_{t-1}^*) + \tilde{c}_t^*
\] (C.49b)

**Risk Sharing**

\[
\tilde{c}_t = \frac{1}{\sigma} q_t + \frac{\sigma^*}{\sigma} \tilde{c}_t^* + \frac{1}{\sigma}(\tilde{c}_t^p - \tilde{c}_t^p + \tilde{c}_t^{aq})
\] (C.50)

**Market clearing**

Now we are going to use the conditions \(\alpha = 0\), \(\mu = \mu^*\) and \(NX = 0\). From the goods market clearing we have:

\[
\begin{align*}
\tilde{y}_t &= (1 - \alpha)\tilde{H}_{H,t} + \alpha \tilde{c}_{H,t} \\
\tilde{y}_t^* &= \tilde{c}_t^*
\end{align*}
\] (C.52a, C.52b)

---

\(^3\) The nominal GDP can be written as:

\[
P_H Y + (P_F - P_F^*)C_F = P_H C_H + P_F C_F + NX
\] (C.51)

If \(NX = 0\), then we have that \(P_F^* C_F = P_H C_H^*\). Hence, in the SS, \(\frac{C_H}{Y} = \alpha\) and \(\frac{C_F}{Y} = 1 - \alpha\).
APPENDIX C. DSGE: DERIVATION AND ESTIMATION DIAGNOSTICS

Log-linearizing the intra-temporal household FOCs:

\[
\begin{align*}
\tilde{c}_{h,t} &= -\mu(p_{H,t} - p_t) + \tilde{c}_t \\
\tilde{c}^\ast_{h,t} &= -\mu^*(p^\ast_{H,t} - p^\ast_t) + \tilde{c}^\ast_t
\end{align*}
\] (C.53a, C.53b)

Applying the equations above to the good market clearing equations:

\[
\tilde{y}_t = \omega^g_q q_t - \omega^g_{mc} \bar{m}_{cF,t} + (1 - \alpha) \tilde{c}_t + \alpha \tilde{y}_t^\ast
\] (C.54)

where, \( \omega^g_q = \frac{(2-\alpha)\mu}{1-\alpha} \) and \( \omega^g_{mc} = \frac{(1-\alpha)\mu + \rho \alpha^2}{1-\alpha} \). We can write (C.54) in terms of \( \tilde{c}_t^\ast \):

\[
\tilde{c}_t^\ast = \alpha_b[(\tilde{y}_t - h\tilde{y}_{t-1}) - \alpha(\tilde{y}_t^\ast - h\tilde{y}_{t-1}^\ast)] - \omega^g_q(q_t - hq_{t-1}) + \omega^g_{mc}(\bar{m}_{cF,t} - h\bar{m}_{cF,t-1})
\] (C.55)

where \( \alpha_b = \frac{1}{(1-\alpha)(1-h)} \)

From the labor market clearing we have that \( \tilde{\ell}_t^l = \tilde{\ell}_t^\ast \), and \( \tilde{\ell}_t^{l\ast} = \tilde{\ell}_t^{l\ast} \).

Now is possible to write the model in the canonical form.

C.3 Markov Chain Convergence Diagnostics

Figure C.1 report histograms, means and standard deviations for the first and last thirds of the posterior draws for the model IV. Having the simulated chain converged to its stationary distribution, we should expect to obtain the same marginal distribution for different subsample from the posterior draws. In this sense, the marginal distributions in the graphs do not present evidence of failure of convergence. Similar results are obtained for the remaining models, and hence they are omitted. They are available upon request.
Figure C.1: Histograms of the first third (left) and last third (right) of the Markov chain. Each row corresponds to a different coefficient.
C.4 Additional Figures

Figure C.2: The first block of 4 graphics at the top shows the Historical decompositions of Canadian Interest Rate, and the last block of 4 pictures at the bottom shows the relative contribution of shocks on Canadian Interest Rate. In each block second rows are from the models with two Phillips curve, and second columns are from the models presenting correlation with external structural shocks.
ERROR: IOError
OFFENDING COMMAND: image
STACK:
- dictionary-
- mark-
- savelevel-