ON LONG-RUN PRICE COMOVEMENTS BETWEEN PAINTINGS AND PRINTS

Corinna Czujack, Renato Flôres e Victor Ginsburgh

Setembro de 1995
1 Introduction

The art market is the result of complex interactions, most of which can hardly be explained by economic theory. Signals such as the name of an artist, specificities of different media, reactions of art galleries, critics, museum directors and investors exert a large variety of influences on prices and values, which are difficult to model with accuracy.

It is however possible to address interesting questions, using sales prices as the basic information on values and relative scarcities. Indeed, if prices can be taken as synthesizing all the effects just mentioned, their dynamic interactions across artists and media may reveal some common patterns or, on the contrary, call attention to divergent behaviours. Both may turn out to be explainable by art history or aesthetical judgements. If so, further evidence would have been provided by a more quantitative analysis; otherwise, new directions for research could be opened.

In particular, if one accepts the idea of "a global market" for say, post-impressionist paintings, one may ask whether artists like Braque or Picasso lead or follow this global market. One may also examine whether prices of prints and paintings for a given artist follow a common pattern; if so, one may construct portfolios of prints, which are cheaper, more liquid and probably less risky than portfolios of paintings.

In this paper, we try to answer questions of this type, using the information contained in prints' and paintings' price indices for global markets and for some individual artists who have been active both as painters and as lithographers or etchers – Braque, Chagall, Ernst, Miró and Picasso.

The analysis is carried out by estimating vector autoregressive models, using the techniques developed by Johansen (1988, 1991) and Johansen and
Juselius (1990, 1992). These methods make it possible
(i) to estimate possible steady-state relations between prices and isolate
long-run from short-run behaviour, and
(ii) to test for exogenous behaviour in an error-correction model setting.

Though this methodology is known to many economists, it is much less
widespread in the field of cultural economics. Therefore, Section 2 is de­
voted to the basic framework and to the key concepts used in the paper.

Section 3 briefly discusses the construction of the price indices on which
our study is based. In Section 4, we analyze three kinds of relationships:
(i) between the market for paintings and the market for prints (Section 4.1);
(ii) between each individual artist and the global market for prints (Section
4.2);
(iii) between the print markets for the five artists (Section 4.3).

The discussion is kept informal, but the more technical details can be
found in the Appendix. Section 5 concludes the paper.

In broad terms, we find that prices of prints and paintings have a common
trend, but prints yield lower long-run returns. Relations between individual
artists and the global market for prints vary from one artist to the next,
but again, the series have common trends. Though each of the five artists
is unique by himself, a separating line can be drawn between Braque-Ernst,
and Chagall-Miró-Picasso. In particular, Picasso, as expected, plays a major
role and Chagall, though being very connected to the others, behaves quite
differently from them.

2 Cointegration and exogeneity

2.1 The error-correction model

Cointegration is loosely related to the concept of stationary linear com­
binations of nonstationary processes. Suppose that the interest lies in studying
$I$ nonstationary time series of prices $y_{i,t}$, $i = 1, 2, ..., I; t = 1, 2, ..., T$. Under
certain conditions, it is possible to estimate the coefficients of the following

--

$^{2}$Cointegration is loosely related to the concept of stationary linear com­
binations of nonstationary processes. Suppose that the interest lies in studying
$I$ nonstationary time series of prices $y_{i,t}$, $i = 1, 2, ..., I; t = 1, 2, ..., T$. Under
certain conditions, it is possible to estimate the coefficients of the following

$^{3}$See however Ginsburgh and Jeanfils (1995).

$^{4}$In this section, we present an informal discussion of the key concepts that will be
econometrically tested in this paper. The interested reader should go to the papers cited
for a rigorous view on the subject. Chapters 2 and 3 in Urbain (1993) also provide a good
insight on the issues.
system of equations:

\[
\Delta y_{i,t} = \sum_{j=1}^{J} \alpha_{ij} z_{j,t-1} + \sum_{k=1}^{K} \sum_{i'=1}^{I} \gamma_{i'k} \Delta y_{i',t-k} + \gamma_0 + \epsilon_{i,t}, \quad i = 1, 2, \ldots, I. \tag{1}
\]

One of the conditions is that the \(\Delta y_{i,t}\) are stationary. The other is that the variables:

\[
z_{j,t} = \sum_{i=1}^{I} \beta_{ij} y_{i,t}, \quad j = 1, 2, \ldots, J \leq I - 1,
\]

obtained as linear combinations of the original \(y_{i,t}\) variables, are also stationary. These last equations are called "long-run" or "steady-state" relationships.

The parameters \(\alpha_{ij}\) in (1) measure the speed at which deviations from the steady-state are corrected, so that the model is often referred to as an error correction mechanism (ECM). The vectors \((\beta_{1j}, \beta_{2j}, \ldots, \beta_{1j})\), \(j = 1, 2, \ldots, J\) are called cointegrating vectors.

The terms:

\[
\sum_{k=1}^{K} \sum_{i'=1}^{I} \gamma_{i'k} \Delta y_{i',t-k} + \gamma_0 \tag{3}
\]

describe the short-run dynamics. The number of lags \((K)\) should be chosen in such a way that the error processes \(\epsilon_{it}, i = 1, 2, \ldots, I,\) are Gaussian white noise.

Form (1) is fairly general and some special cases can readily be obtained; in particular, it may happen -(a) that no cointegrating vectors exist, (b) that some or all of the \(\alpha_{ij}\) coefficients are zero, (c) that some or all of the \(\gamma_{i'k}\) coefficients are zero. These special cases will be discussed in Section 2.2.

Note that the \(\alpha_{ij}\)'s and the \(\beta_{ij}\)'s are undetermined up to scalar multiplication; this explains why system (1) can also be written as:

\[
\Delta y_{i,t} = \sum_{i'=1}^{I} \pi_{i'i'k} y_{i',t} + \sum_{k=1}^{K} \sum_{i'=1}^{I} \gamma_{i'k} \Delta y_{i',t-k} + \gamma_0 + \epsilon_{i,t}, \quad i = 1, 2, \ldots, I, \tag{4}
\]

where the so-called impact coefficients \(\pi_{i'i'}\) are now uniquely determined.

Note also that in the case of a two-variable system, there will exist at most one cointegrating relation which, after suitable renormalization, can be written as:

\[
z_{1,t} = y_{1,t} - \beta y_{2,t}, \quad \beta > 0. \tag{5}
\]
This implies that $\alpha_{11}$ should be negative and $\alpha_{21}$ positive. The reason is that, if in (5) $z_{1,t}$ is positive, $y_{1,t}$ is "higher" than its equilibrium value, so that the error-correction mechanism acts in lowering its next value and this entails $\alpha_{11} < 0$. A similar reasoning applies to $\alpha_{21}$.

Estimating model (1) for price indices of painters or media, allows us to obtain an insight on the joint short-run dynamics of the series. As pointed out, the long-run or steady-state term, present in these equations, acts in correcting the short-run deviations back to the "ideal" long-run equilibrium. Finding cointegration, say between paintings and prints by a specific artist, is a nontrivial fact because it implies that, in the long-run, the price series do not diverge and also that their short-run variations are influenced by this long-run equilibrium. In other words, this means that the series have a common nonstationary movement, which is eliminated in the cointegrating relation.

2.2 Exogeneity and causality

The issues of exogeneity and causality often appear in econometric modelling involving several variables. Though intuitively simple, the concept of exogeneity can have different shades, as pointed out in the seminal paper by Engle et al. (1983). For our purposes, and in the context of model (1), a variable will be weakly exogenous for a set of parameters if, conditioning the model with respect to that variable, no useful sample information for inference on those parameters is lost.

In the case of the ECM-model, the concept has some fine connotations. Consider a given equation, say the first. It might happen that $\alpha_{11}$ is equal to zero; this would mean that the corresponding cointegration relationship $z_{1,t-1}$ does not play any role in explaining the behaviour of the (differenced) price index $y_{1,t}$. If all $\alpha_{1j}'s$ are zero, we say that the behaviour of $y_{1,t}$ is weakly exogenous with respect to the long-run relationships and (1) can be written as:

$$\Delta y_{1,t} = \sum_{k=1}^{K} \sum_{i'=1}^{I} \gamma_{1i',k} \Delta y_{i',t-k} + \gamma_{10} + e_{1,t}. \quad (6)$$

As a result, the system can be separated into two subsystems. The first comprises the equations corresponding to the weakly exogenous variables, while the second is formed by the remaining ones. Making the link with the

---

4This idea has received a finer treatment by Mosconi and Gianini (1993), which will not be used in this paper.
informal definition of weak exogeneity stated above, as far as the long-run relationships are concerned, we can condition the second subsystem with respect to the first one, i.e. we can study the behaviour of the indices in the second subsystem for each "fixed" set of values of the weakly exogenous variables. Note that performing the reverse would be meaningless, as the variables of the second subsystem do not convey any long-run information to the ones in the first subsystem.

Contrary to exogeneity, the concept of causality builds on ideas from common language and philosophy. In our context, causality will be defined in the lines of Granger (1969): \( x_t \) will (Granger-)cause \( w_t \) if and only if the past of \( x_t \) influences \( w_t \), but the opposite is not true. The "past" is here understood as the collection of lagged values of \( x_t \), influences being measured in terms of linear regressions.

Weak exogeneity plus Granger causality makes for what Engle et al. (1983) suggested to call "strong exogeneity." It is evident that a group of variables (here, price indices) may be weakly but not strongly exogenous. This is easily seen if one remembers that lagged differences of all the indices appear in principle in the short-run part of the equations in (1). For weakly exogenous variables to be strongly exogenous, it must be that the short-run behaviour of the other variables do not Granger-cause them, i.e. that they do not appear in their equations. For example, \( y_{1,t} \) will be strongly exogenous, if the first equation of system (1) can be written as

\[
\Delta y_{1,t} = \sum_{k=1}^{K} \gamma_{11,k} \Delta y_{1,t-k} + \gamma_{10} + e_{1,t}.
\]

No other variable than the past of \( y_{1,t} \) itself contributes in explaining \( y_{1,t} \).

**2.3 Efficiency and returns**

If, in addition to exogeneity of say, \( y_{1,t} \), we find that there is no influence of the short-run either, and that \( \gamma_{10} \) is not significantly different from zero, we are left with the following equation:

\[
\Delta y_{1,t} = e_{1,t}.
\]

and the \( \Delta y_{1,t} \) series is a random walk.

Given that \( y_{1,t} \) is a price series, (8) means that the underlying market is "weakly efficient:" in the context of the model, no variable (contemporaneous
or past) has an influence on $y_{1,t}$ and the current price is the best forecast for next period's price.

In the case of a two-equations system, there is also an interesting financial interpretation of the long-run relation. Taking the expectation of the first-difference of (5), we have:

$$E \Delta y_{1,t} = \beta E \Delta y_{2,t}. \tag{9}$$

If the price series are expressed in logarithms, the differenced series represent returns, so that (9) gives a relationship between the two expected returns. If $\beta > 1$, (long-run) returns from $y_{1,t}$ are higher than those generated by $y_{2,t}$. Since riskier assets yield higher returns, $y_{1,t}$ should probably relate to an asset that is riskier than $y_{2,t}$. Note that if $\beta = 1$, the expected returns of the two assets are equal.

Finally, consider a two-equations ECM in which the first price series is weakly efficient, while in the second equation, the long-run term is present. This means that differences in the first series are merely white noise, while in the second, some short-term forecasting is possible. It is then reasonable to expect that the first asset should provide higher returns, since it is riskier than the second one. This hypothesis can be tested on the $\beta$ coefficient, which should be larger than one.

3 The data

3.1 Principles of construction of the price indices

The price indices used in our calculations have been computed through hedonic regressions, as suggested in Chanel et al. (1992). One estimates the following equation:

$$p_{i,t} = \sum_{k=1}^{m} a_k x_{ik,t} + c(t) + \xi_{i,t}. \tag{10}$$

In this equation, $p_{i,t}$ is the (logarithm of the) price of a collectible $i$ sold at time $t$, $x_{ik,t}$ is a time-variant idiosyncratic attribute of collectible $(i, t)$; $c(t)$ is the market-wide price effect and $\xi_{i,t}$ is an error term. Model (10) can be given a convenient interpretation if one isolates in the right-hand side the time effect and the random error. The left-hand side can then be thought of as representing the price of a work $i$ sold in $t$, freed from the implicit prices of its characteristics. This "characteristic-free price" is assumed to include the
effect of time and a random error only; by averaging the characteristic-free prices of all works sold in \( t \) (or, equivalently, by regressing these prices on time dummies), one obtains the average price of a "standardized" work at time \( t \). The specification of \( c(t) \) considered in that case is:

\[
c(t) = \sum_{t=T_0}^{T_T} \phi_t \delta_t,
\]

where \([T_0, T_T]\) is the time interval over which observations are available; \( \delta_t \) is a dummy variable which takes the value one if work \( i \) is sold in period \( t \in [T_0, T_T] \), and zero otherwise; the \( \phi_t \)'s are parameters to be estimated. The sequence \([\phi_{T_0}, \phi_{T_1}, ..., \phi_{T_T}]\) is used to construct the price (or the value) index \( y_t \).

Most of the work carried out in order to compute returns on art, uses repeat sales regressions (i.e. regressions based on observations of collectibles sold at least twice) rather than hedonic regressions. The pros and cons of both methods are discussed in Chanel et al. (1992) and Ginsburgh (1994). Here, we merely note that it would be impossible to construct yearly (or half-yearly) indices using repeat sales only, since the number of resales for which prices are observed is quite limited.

### 3.2 Original data available

The data used to calculate value indices are taken from Mayer's *Annuaire des Ventes* (1963 to 1994). These yearbooks of public auctions are available since 1963 (sales of 1962); we thus cover the years 1962 to 1993. Two databases were compiled, one for paintings and one for prints. The first contains 25,073 sales for 84 well-known artists, selected in a fairly subjective way. The database for prints contains 10,028 sales for 25 artists.

In addition to global price indices based on the 84 and 25 artists respectively, we also constructed indices for five artists for whom the number of sales was large enough: Braque (359 paintings and 681 prints), Chagall (341 and 1,943), Ernst (435 and 280), Miró (288 and 1,708) and finally, Picasso (826 and 3,132).

In Mayer's compendia, each sale is described by a certain number of characteristics (see below) and by a sale number, corresponding to a specific

---

5See also Goetzmann (1990, 1993).
6This is certainly the case for paintings, but much less so for prints. See Pesando (1993).
7For more details, see de la Barre et al. (1994).
auction describing the location and the date of the sale. It is therefore possible to construct semi-annual, quarterly or even monthly price indices. The relatively small number of sales made us opt for semi-annual indices, taking the years 1962 to 1966 as base period.

The characteristics of the artworks are different for paintings and for prints. Those that describe paintings are rather limited; beside the name of the artist (and the title of the painting), the yearbooks provide only a very rough description: the size of the work (height and width), the year in which it was painted, the medium used, the place of the sale (saleroom, country) and the year of sale. Such a simple description is however hardly comprehensive enough to explain the price differences between artists and one is necessarily led to include some measure of the repute of the painter. We chose to work with dummy variables for artists.

The characteristics for prints are more numerous. Beside the name of the artist, the size of the work, the year of production, the medium used, the place of the sale (saleroom, country), and the year of the sale, Mayer provides information concerning the technique used (etching, lithography, etc.), the type of paper (Arches, Chine, etc.), the fact that the print is black and white or coloured, the signature, some specials (épreuve d’essai, épreuve d’artiste, etc.), the total number of copies and the number of the copy sold.9

Moreover, for the five individual artists considered and given that the year of production is (very often) available, it was possible to introduce dummy variables for the different “creation” periods (e.g. for Picasso, the “blue,” the “pink,” the “cubist” period, etc.), which are usually priced very differently.10

Prices are given in current French francs by Mayer, who uses the exchange rate prevailing during the day of the sale.

3.3 Indices constructed

The model we use combines (10) and (11) and is estimated on the full sample of sales and resales. Regressions were thus run for all artists together

---

8 Especially for the period 1962-1972, during which there are sometimes less than ten observation per year for a given artist.
9 See also Table 1, for more details.
10 It is obviously difficult to define these periods (painters have seldom changed their style suddenly: changes occur gradually, and painters may also simultaneously paint in different styles) so that the indications given in the various art history books do not necessarily coincide. We have used Laclotte and Cuzin (1989). The authors are respectively Chairman and Chief Curator of the paintings department at the Musée du Louvre.
to obtain the "market" index as well as individually for Braque, Chagall, Ernst, Miró, and Picasso. The indices are then deflated, using the French consumer price index. They are reproduced in Figure 1.

3.4 Overall quality of the regression results

We illustrate the results with two regressions for paintings and for prints, run on the full sample (these are the regressions used to obtain the "market indices"). As can be seen from Table 1, the fit is quite good in both cases. All the coefficients (with the exception of the "paper" medium for prints, which should probably be negative) have the expected sign.

4 Empirical results

The analysis is split into three parts. First, we look for cointegration between prices for prints and paintings for the market as a whole, as well as for each of the five artists individually. We then examine the market for prints only and test whether prices for each artist are cointegrated with the global price index. Finally, we construct a system of five equations (the price indices for the five artists') for prints and investigate their relations. Before setting up these models, we tested whether (the logarithms of) the price series used are integrated of order one, so that their first differences are stationary. The results of these tests, given in Table A1 in the Appendix, indicate that all series, with the exception of the global indexes can be considered as integrated of order one, at the 5% level. The global indexes also show evidence of a single unit root at the 10% level.

4.1 Cointegration between prints and paintings

We investigate the conjecture that the markets for paintings and prints – both globally and for each artist individually – move together in the long-run. We estimate in each case the following two-equations system:

\[ \Delta y_{i,t} = \alpha_i (\beta_1 y_{1,t-1} + \beta_2 y_{2,t-1}) + \sum_{k=1}^{K} \sum_{i'=1}^{2} \gamma_{i,i',k} \Delta y_{i',t-k} + \gamma_{i,0} + \epsilon_{i,t}, \quad i = 1, 2 \]  

where different values for K, the number of lags for short-run effects (including no lag at all), have been tried out and \( \beta_1 \) is normalised to unity.

Since only two variables are considered, there exists at most one cointegrating vector. The first and second equations concern paintings and prints,
Table 1 Some results from the hedonic regressions

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Paintings Coefficient</th>
<th>Paintings St. dev.</th>
<th>Prints Coefficient</th>
<th>Prints St. dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>.01704</td>
<td>.00042</td>
<td>Height</td>
<td>.00925</td>
</tr>
<tr>
<td>Width</td>
<td>.00482</td>
<td>.00031</td>
<td>Width</td>
<td>.00116</td>
</tr>
<tr>
<td>Surface (x 100)</td>
<td>-.00400</td>
<td>.00030</td>
<td>Surface (x 100)</td>
<td>-.00030</td>
</tr>
<tr>
<td>Canvas</td>
<td>.00000</td>
<td>-</td>
<td>Etching</td>
<td>.00000</td>
</tr>
<tr>
<td>Collage</td>
<td>-.49946</td>
<td>.04203</td>
<td>Lithography</td>
<td>-.17682</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Silkscreen</td>
<td>-.40097</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mixed techn.</td>
<td>.04434</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Arches</td>
<td>.00000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chine</td>
<td>-.18289</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Japon</td>
<td>-.19990</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ord. paper</td>
<td>.09572</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bl. and white</td>
<td>.00000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Coloured</td>
<td>.18603</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>No special</td>
<td>.00000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Epr. d'essai</td>
<td>.07445</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Epr. d'artiste</td>
<td>.21308</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bon à tirer</td>
<td>3.16567</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hors commerce</td>
<td>.23987</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total nb. of prints</td>
<td>-.00040</td>
</tr>
<tr>
<td>Artists</td>
<td>84 var.*</td>
<td></td>
<td>Artists</td>
<td>25 var.*</td>
</tr>
<tr>
<td>Salerooms</td>
<td>24 var.*</td>
<td></td>
<td>Salerooms</td>
<td>24 var.*</td>
</tr>
<tr>
<td>Semesters</td>
<td>54 var.*</td>
<td></td>
<td>Semesters</td>
<td>54 var.*</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.82</td>
<td></td>
<td>$R^2$</td>
<td>.72</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>25,073</td>
<td></td>
<td>Nb. of obs.</td>
<td>10,028</td>
</tr>
</tbody>
</table>

* For obvious reasons, we do not give the coefficients here. F-tests have been run and show that the coefficients within groups (artists, salerooms and semesters) are not equal.
respectively. The results are summarized in Table A2 in the Appendix, where a more technical discussion can also be found. The main findings follow.

The global market

For the global market, i.e. the one represented by the price indices pooling together many artists, the fitted model is:

\[
\begin{align*}
\Delta y_{1,t} &= e_{1,t} \\
\Delta y_{2,t} &= 0.37(y_{1,t-1} - 1.24y_{2,t-1}) + e_{2,t}.
\end{align*}
\]

This shows that, with respect to prints, paintings are weakly efficient: their short-run returns follow a white noise process. Therefore, as expected, their long-run returns are almost 25% higher than those of prints. Short-run returns for prints are corrected by the cointegration relationship: whenever the price for prints is higher than \(0.81\) (i.e. \(1/1.24\)) times the price for paintings, the print index falls – on average – by 37% of the difference between \(y_{1,t-1}\) and \(1.24y_{2,t-1}\).

Individual artists

For individual artists, the results follow either of two patterns. In the case of Braque and Ernst, prices of paintings are also weakly exogenous, but short-run lags appear in both equations, so that weak efficiency of paintings does not apply.

For Chagall, Miró and Picasso – though no short-run lags were found to be significant –, no index is weakly exogenous with respect to the long-run relation. For Chagall, the model is:

\[
\begin{align*}
\Delta y_{1,t} &= -0.35(y_{1,t-1} - y_{2,t-1}) + e_{1,t} \\
\Delta y_{2,t} &= 0.14(y_{1,t-1} - y_{2,t-1}) + e_{2,t}
\end{align*}
\]

showing that long-run returns for prints and paintings are identical.

For Picasso, long-run returns for prints are even higher than for paintings, as shown by his equations:

\[
\begin{align*}
\Delta y_{1,t} &= -0.39(y_{1,t-1} - 0.71y_{2,t-1}) + e_{1,t} \\
\Delta y_{2,t} &= 0.46(y_{1,t-1} - 0.71y_{2,t-1}) + e_{2,t}.
\end{align*}
\]
This result is probably a consequence of the very high prices that are already obtained for Picasso's paintings.

4.2 Prints: cointegration between individual artists and the global market

Here, we are interested in the relations between prices of prints for individual artists and the print market as a whole. We estimate a system similar to (12), where now the first equation is related to the artist and the second to the global market. Detailed results are given in Table A3 in the Appendix.

In Braque's and Ernst's cases, the global market is weakly efficient. Prints by these two artists have a lower long-run return than the one given by a notional portfolio, identical to the one used to construct the global index. In the case of Braque, for example, the model is:

\[
\Delta y_{1,t} = -0.51(y_{1,t-1} - 0.87y_{2,t-1}) + e_{1,t}, \\
\Delta y_{2,t} = e_{2,t}.
\]

For Chagall, Miró and Picasso, the cointegrating relation influences the global index, and their long-run returns are higher than those of the index. Picasso's prints are weakly efficient, and the final model fitted for him is:

\[
\Delta y_{1,t} = e_{1,t}, \\
\Delta y_{2,t} = 0.20(y_{1,t-1} - 1.39y_{2,t-1}) + e_{2,t}.
\]

Picasso prints can be thus considered as "leading" or "pushing" the market. Their long-run returns are 39% higher than those of the notional print portfolio. Moreover, if the global print index falls to levels below \(1/1.39\) times the Picasso prints index, global short-run returns will have a higher probability of being negative.

4.3 Prints: cointegration between individual artists

We now consider the system of five equations in which only prices of prints for the five artists appear. The order is Braque (subscript B), Chagall (C), Ernst (E), Miró (M) and Picasso (P). Given our previous findings, we assume that Picasso's prices are weakly exogenous and we set to zero the weakly exogenous. According to the \(\lambda_{max}\)-test, there is no cointegration at the 5% level.
\(\alpha_{P,j}\) coefficients in Picasso's equation. We then try to determine:
(i) who are the artists contributing to the long-run relationship(s) and who are the weakly exogenous ones;
(ii) are there artists who are strongly exogenous;
(iii) what is the most parsimonious model to describe the five artists' short-run behaviour.

A technical discussion is provided in the Appendix, together with Tables A4 and A5. The findings are as follows.

Chagall and Miró may be left out from the long-run cointegrating relationship. They thus follow long-run trends which are different from those of Braque, Ernst and Picasso. Therefore, it was interesting to check whether Chagall's and Miró's short-run lagged values could also be left out of the three other artists' equations. If so, this would indicate that neither the long-run nor the short-run of the former exerts any influence on the group of the latter, so that Braque, Ernst and Picasso would be strongly exogenous. However, this happens to be the case for Braque and Ernst, but not for Picasso, whose prices are partly explained by those of Miró.

The final, more parsimonious fitted model is:\(^\text{12}\)

\[
\begin{align*}
\Delta y_{B,t} &= -0.33z_{1,t} - 0.49z_{2,t} + e_{B,t} \\
\Delta y_{C,t} &= -0.17z_{1,t} + 0.06z_{2,t} - 0.58\Delta y_{C,t-1} + 0.26\Delta y_{M,t-1} + e_{C,t} \\
\Delta y_{E,t} &= 1.02z_{1,t} - 0.08z_{2,t} + e_{E,t} \\
\Delta y_{M,t} &= 0.05z_{1,t} - 0.26z_{2,t} + e_{M,t} \\
\Delta y_{P,t} &= -0.41\Delta y_{B,t-1} + 0.40\Delta y_{M,t-1} - 0.36\Delta y_{P,t} + e_{P,t}.
\end{align*}
\]

5 Conclusions

The analyses performed reveal that the prices of the five artists studied behave differently. We first have Braque and Ernst, who show a "follow the market", more classical behaviour: the global prints index is, in both cases, weakly exogenous. The same holds if the global prints index is replaced by the paintings index of the artist.

The other three artists stand out as special individualities. For all three, the paintings and prints markets are interrelated. Chagall is perhaps the most singular case. The other four are strongly exogenous w.r.t. him, in the joint, five artists' prints model. Moreover, his index for prints is the only

\(^{12}\)This parsimonious representation is based on the model of Table A5, dropping the short-run coefficient which are not significantly different from zero.
one that presents weaker evidence of being cointegrated (only at the 10% level) with the global index. Everything seems to indicate that Chagall has a very special market of his own, in which prints as well as paintings are equally rewarding in the long-run.

Picasso is, naturally, the other singularity. He alone seems to be weakly exogenous in the prints market. Indeed, his price index is weakly efficient with respect to the global index, and he is the artist whose prints generate higher long-run returns than his paintings.

Miro prints are interrelated with the market, and their formal interrelationship with those of Picasso seems to corroborate a well-known historical fact.

Finally, in financial terms, investing in prints the prices of which are weakly efficient would be a bad choice for speculators. These should go for prints whose price changes can be forecasted by exogenous variables which may signal – as fundamentals – the direction of the changes. On the other hand, the closer the indices are to efficiency, the higher is their (long-run) return (with respect to the other ones), given that they constitute riskier assets. As a consequence, buying prints by these artists is a good choice for long-run investors.

6 References


Chanel, O., L.-A. Gérard-Varet and V. Ginsburgh (1992), The relevance of hedonic price indices: the case of paintings, manuscript.


Figure 1 Deflated prices, in logarithms
(fat line: paintings; thin line: prints)
7 Appendix

7.1 Unit root testing

Table A1 presents the results for the augmented Dickey-Fuller and the Philips-Perron tests. At the 5% probability level, the first test accepts the unit root null hypothesis in all cases, except for the global prints index. The second test rejects the null hypothesis in several cases. When applied to first-order differences, the Philips-Perron test clearly rejects the second unit root in all cases; this root is however accepted for both global indices by the Dickey-Fuller test. At the 10% probability level, the unit root null hypothesis is accepted in all cases by the ADF test.

7.2 Relations between paintings and prints

Table A2 presents the results, which are now also briefly discussed.

Cointegration relationships. At the 5% probability level, cointegration is accepted by both tests in all cases. We may conclude that there exists a long-run relationship and that prices move together.

Number of short-run lags. The "optimal" number of lags is based both on the Jarque-Bera test and the existence of cointegration. As can be seen, \( K \) varies from one case to the other, but in four instances it is zero and the relationships are:

\[
\Delta y_{i,t} = \alpha_i (\beta_1 y_{1,t-1} + \beta_2 y_{2,t-1}) + \epsilon_{i,t}, \quad i = 1, 2.
\]

Weak exogeneity of paintings. To check this hypothesis, we ran a test \( H_0 : \alpha_1 = 0 \). If this assumption is accepted, the equation for paintings can be written:

\[
\Delta y_{1,t} = \sum_{k=1}^{K} \sum_{i'=1}^{2} \gamma_{i',k} \Delta y_{i',t-k} + \gamma_{10} + \epsilon_{1,t}.
\]

The hypothesis is accepted for the market as a whole, for Braque and for Ernst. It is rejected in the case of Chagall, Miró and Picasso.

Error correction mechanisms. In order for the equations to represent error correction mechanisms, we must have \( \alpha_1 \leq 0 \) and \( \alpha_2 \geq 0 \). This is verified in all cases in which \( \alpha_1 \) is significantly different from zero and means that, in
the short-run, any over- or undershooting of prices is corrected.

On the proportionality of comovements. If the hypothesis $H_0 : \beta_2 = -1$ is accepted, the ratio of prices is equal to unity and expected long-run returns are the same. This assumption is accepted only for Chagall. For the global market, as well as for Braque, Ernst and Miró, prints have smaller returns than paintings. Picasso, is an exception to this rule, and significantly so, since his $\beta_2$ is much smaller than one.\footnote{Note that if $H_0 : \alpha_1 = 0$ is accepted, we run a joint test $H_0 : \alpha_1 = 0, \beta_2 = -1$; otherwise, we only run a test $\beta_2 = -1$.}

Weak efficiency of the market for paintings. For the global market, (i) prices of paintings are weakly exogenous (i.e. $\alpha_1 = 0$) and (ii) there are no lags in the short-run part of the relation; moreover, $H_0 : \gamma_1 = 0$ is accepted, so that the global market for paintings is weakly efficient.

Returns on prints vs. returns on paintings. For the global market as well as for Braque and Ernst, the equation for prints includes the error-correction term, while for paintings it does not, so that, as discussed in Section 2.3, prints should have a smaller return than paintings and this is actually the case. Prints also have a smaller return than paintings for Chagall and Miró, while for Picasso, the opposite is true.

7.3 Prints: relations between individual artists and the global market

Table A3 gives the results for the five pairs. Cointegration is rejected at the 5% level for Chagall by the $\lambda_{\text{max}}$-test, but accepted by the Trace-test. Cointegration is accepted at the same probability level for all other artists. For the remainder, the interpretation of the results can be made along the same lines as in section 7.2.

7.4 Cointegration between the five artists

As can be checked from the results of Table A4, both tests accept cointegration.
The long-run relationship(s). The long-run cointegration relationships read:

\[ z_{j,t} = \beta_{Bj} y_{B,t} + \beta_{Cj} y_{C,t} + \beta_{Ej} y_{E,t} + \beta_{Mj} y_{M,t} + \beta_{Pj} y_{P,t}, \quad j = 1, 2, \ldots, J, \]

with \( J \leq 4 \). Both the \( \lambda_{\max} \) and the Trace-tests show that there are at most two cointegrating relations (at the 5% level), for \( 0 \leq K \leq 3 \), where \( K \) is the number of short term lags. For \( K = 4 \), there is only one such relationship.

We are interested in relationships that are parsimonious, and contain a number of individual artists that is as small as possible. We tried all combinations excluding three artists from the above long-run relationship. As is seen from Table A4, the hypothesis is strongly rejected at the 5% probability level, whatever the number of lags in the short-run part of the relations. We then turned to test the exclusion of all possible couples of artists and found that, for \( K = 1 \), the hypothesis \( \beta_{Cj} = \beta_{Mj} = 0 \) could not be rejected at the usual 5% level.\(^{14}\)

This leads to accepting the following two long-run relationships:

\[ z_{1,t} = y_{B,t} - 1.32 y_{E,t} + 0.04 y_{P,t} \]

and

\[ z_{2,t} = y_{B,t} + 0.09 y_{E,t} - 0.59 y_{P,t}, \]

in which Chagall and Miró do not contribute to the long-run.

The short-run relationship(s). In Table A5, we give the short-run behaviour for \( K = 1 \). Recall that Picasso is assumed to be weakly exogenous from the start. Given that only Braque, Ernst and Picasso enter the long-run relations, we tested for strong exogeneity of these three artists. Table A5 shows the complete results for the short-run part (with \( K = 1 \)). Though Chagall’s and Miró’s lagged price differences are not significant in the equations for Braque and Ernst – and for Miró himself – (as can be seen from the last column of Table A5, \( H_0 / \gamma_{C,1} = \gamma_{M,1} = 0 \) is accepted), Miró’s short run behaviour influences Picasso’s – which is not too surprising, given the connection between their works. This means that the four other artists are strongly exogenous for Chagall.

\(^{14}\)Note that when three artists are excluded, there can be at most one cointegration relationship, so that the \( \chi^2 \) variable used to test the null hypothesis for exclusion of three artists has 3 d.f. When two artists are excluded, there may be one or two cointegrating relations, so that the \( \chi^2 \)-test has 2 or 4 d.f.
### Table A1 Unit root tests for the (logged) price series

<table>
<thead>
<tr>
<th></th>
<th>$H_0$: I(1)</th>
<th>$H_0$: I(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A.D.F.</td>
<td>P.P.</td>
</tr>
<tr>
<td><strong>Paintings</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global</td>
<td>-2.67</td>
<td>-3.78</td>
</tr>
<tr>
<td>Braque</td>
<td>-2.55</td>
<td>-22.81</td>
</tr>
<tr>
<td>Chagall</td>
<td>-1.91</td>
<td>-24.30</td>
</tr>
<tr>
<td>Ernst</td>
<td>-1.80</td>
<td>-22.98</td>
</tr>
<tr>
<td>Miró</td>
<td>-2.77</td>
<td>-31.34</td>
</tr>
<tr>
<td>Picasso</td>
<td>-3.45</td>
<td>-22.11</td>
</tr>
<tr>
<td><strong>Prints</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global</td>
<td>-3.76</td>
<td>-22.86</td>
</tr>
<tr>
<td>Braque</td>
<td>-2.31</td>
<td>18.36</td>
</tr>
<tr>
<td>Chagall</td>
<td>-0.95</td>
<td>-14.64</td>
</tr>
<tr>
<td>Ernst</td>
<td>-2.63</td>
<td>-26.49</td>
</tr>
<tr>
<td>Miró</td>
<td>-3.39</td>
<td>-26.95</td>
</tr>
<tr>
<td>Picasso</td>
<td>-1.70</td>
<td>-16.37</td>
</tr>
<tr>
<td><strong>Crit. value</strong></td>
<td>-3.50</td>
<td>-19.80</td>
</tr>
</tbody>
</table>

A.D.F. = augmented Dickey-Fuller; P.P. = Philippe-Perron.
All tests include a constant term and a time trend.
The critical values shown refer to the 5% probability level.
<table>
<thead>
<tr>
<th></th>
<th>Global</th>
<th>Braque</th>
<th>Chagall</th>
<th>Ernst</th>
<th>Miró</th>
<th>Picasso</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nb of lags</strong></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>aₘₐₓ-test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td>15.57</td>
<td>14.65</td>
<td>15.57</td>
<td>14.06</td>
<td>28.08</td>
<td>25.28</td>
</tr>
<tr>
<td>Trace-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td>16.91</td>
<td>15.42</td>
<td>17.87</td>
<td>16.80</td>
<td>22.94</td>
<td>16.27</td>
</tr>
<tr>
<td>r ≤ 1</td>
<td>1.33</td>
<td>0.77</td>
<td>2.30</td>
<td>2.75</td>
<td>2.10</td>
<td>1.74</td>
</tr>
<tr>
<td><strong>Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β₁</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>β₂</td>
<td>-1.24</td>
<td>-1.84</td>
<td>-1.08</td>
<td>-1.73</td>
<td>-1.25</td>
<td>-0.71</td>
</tr>
<tr>
<td>α₁</td>
<td>0.07</td>
<td>0.00</td>
<td>-0.35</td>
<td>-0.03</td>
<td>-0.65</td>
<td>-0.39</td>
</tr>
<tr>
<td>α₂</td>
<td>0.37</td>
<td>0.20</td>
<td>0.14</td>
<td>0.53</td>
<td>0.16</td>
<td>0.46</td>
</tr>
<tr>
<td><strong>Hyp. tests (χ² val.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₀ : α₁ = 0</td>
<td>0.75</td>
<td>0.00</td>
<td>10.47</td>
<td>0.05</td>
<td>15.47</td>
<td>7.10</td>
</tr>
<tr>
<td>H₀ : α₂ = 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₀ : β₂ = -1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₀ : α₁ = 0; β₂ = -1</td>
<td>6.21</td>
<td>7.40</td>
<td>-</td>
<td>11.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Jarque-Bera (χ² val.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rel. 1</td>
<td>2.94</td>
<td>0.43</td>
<td>1.82</td>
<td>0.16</td>
<td>0.39</td>
<td>0.88</td>
</tr>
<tr>
<td>Rel. 2</td>
<td>1.23</td>
<td>0.79</td>
<td>0.10</td>
<td>0.97</td>
<td>0.85</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The critical values at the 5% probability level are:
14.04 for the $\lambda_{\text{max}}$-test and 15.20 and 3.96 for the Trace-test.
3.84 and 5.99 for the $\chi^2$-test with 1 and 2 d.f.
5.99 for the Jarque-Bera test ($\chi^2$ with 2 d.f.).
Table A3 Cointegration between individual artists (Rel. 1) and the global market (Rel. 2)

<table>
<thead>
<tr>
<th></th>
<th>Braque</th>
<th>Chagall</th>
<th>Ernst</th>
<th>Miró</th>
<th>Picasso</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb of lags</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{max}$-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>18.48</td>
<td>13.14</td>
<td>26.89</td>
<td>59.33</td>
<td>17.55</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>23.35</td>
<td>16.82</td>
<td>31.05</td>
<td>62.14</td>
<td>20.90</td>
</tr>
<tr>
<td>Trace-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>4.87</td>
<td>3.69</td>
<td>4.16</td>
<td>2.81</td>
<td>3.35</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.87</td>
<td>-1.22</td>
<td>-0.67</td>
<td>-1.11</td>
<td>-1.39</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.51</td>
<td>-0.11</td>
<td>-0.82</td>
<td>-0.70</td>
<td>-0.23</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.09</td>
<td>0.26</td>
<td>-0.08</td>
<td>0.46</td>
<td>0.20</td>
</tr>
<tr>
<td>Hyp. tests ($\chi^2$ val.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : \alpha_1 = 0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.90</td>
</tr>
<tr>
<td>$H_0 : \alpha_2 = 0$</td>
<td></td>
<td>0.41</td>
<td>6.47</td>
<td>0.42</td>
<td>9.77</td>
</tr>
<tr>
<td>$H_0 : \alpha_2 = 0; \beta_2 = -1$</td>
<td>3.44</td>
<td>-</td>
<td>11.50</td>
<td>-</td>
<td>12.74</td>
</tr>
<tr>
<td>$H_0 : \beta_2 = -1$</td>
<td></td>
<td>-</td>
<td>1.93</td>
<td>-</td>
<td>12.94</td>
</tr>
<tr>
<td>Jarque-Bera ($\chi^2$ val.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rel. 1</td>
<td>3.08</td>
<td>0.33</td>
<td>0.97</td>
<td>3.65</td>
<td>2.09</td>
</tr>
<tr>
<td>Rel. 2</td>
<td>1.03</td>
<td>1.32</td>
<td>1.21</td>
<td>0.71</td>
<td>0.54</td>
</tr>
</tbody>
</table>

The critical values at the 5% probability level are:
14.04 for the $\lambda_{max}$-test and 15.20 for the Trace-test.
3.84 and 5.99 for the $\chi^2$-test with 1 and 2 d.f.
5.99 for the Jarque-Bera test ($\chi^2$ with 2 d.f.).

22
Table A4 Cointegration between the five individual artists and exclusion tests for two and three artists

<table>
<thead>
<tr>
<th>Nb of lags</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_{\max})-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r = 0) vs (r = 1)</td>
<td>47.00</td>
<td>51.96</td>
<td>43.31</td>
<td>44.11</td>
<td>43.58</td>
</tr>
<tr>
<td>(r \leq 1) vs (r = 2)</td>
<td>30.15</td>
<td>30.52</td>
<td>35.70</td>
<td>33.54</td>
<td>21.17</td>
</tr>
<tr>
<td>(r \leq 2) vs (r = 3)</td>
<td>17.10</td>
<td>15.91</td>
<td>9.58</td>
<td>19.52</td>
<td>9.76</td>
</tr>
<tr>
<td>(r \leq 3) vs (r = 4)</td>
<td>7.05</td>
<td>4.89</td>
<td>5.86</td>
<td>6.18</td>
<td>1.62</td>
</tr>
<tr>
<td>Trace-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r = 0) vs (r = 1)</td>
<td>101.29</td>
<td>102.39</td>
<td>94.45</td>
<td>103.34</td>
<td>76.12</td>
</tr>
<tr>
<td>(r \leq 1) vs (r = 2)</td>
<td>54.30</td>
<td>50.43</td>
<td>51.14</td>
<td>59.24</td>
<td>32.54</td>
</tr>
<tr>
<td>(r \leq 2) vs (r = 3)</td>
<td>24.15</td>
<td>19.91</td>
<td>15.44</td>
<td>25.70</td>
<td>11.38</td>
</tr>
<tr>
<td>(r \leq 3) vs (r = 4)</td>
<td>7.05</td>
<td>4.89</td>
<td>5.86</td>
<td>6.18</td>
<td>1.62</td>
</tr>
<tr>
<td>Excl. tests ((X^2) val.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_B = \beta_C = 0)</td>
<td>17.74</td>
<td>25.31</td>
<td>24.02</td>
<td>27.60</td>
<td>21.39</td>
</tr>
<tr>
<td>(\beta_B = \beta_E = 0)</td>
<td>44.48</td>
<td>47.66</td>
<td>45.06</td>
<td>41.36</td>
<td>14.01</td>
</tr>
<tr>
<td>(\beta_B = \beta_M = 0)</td>
<td>42.25</td>
<td>42.93</td>
<td>37.61</td>
<td>37.32</td>
<td>29.44</td>
</tr>
<tr>
<td>(\beta_C = \beta_E = 0)</td>
<td>30.08</td>
<td>36.88</td>
<td>38.55</td>
<td>34.43</td>
<td>21.52</td>
</tr>
<tr>
<td>(\beta_C = \beta_M = 0)</td>
<td>15.72</td>
<td>7.75</td>
<td>20.23</td>
<td>28.71</td>
<td>26.73</td>
</tr>
<tr>
<td>(\beta_E = \beta_M = 0)</td>
<td>52.13</td>
<td>50.81</td>
<td>40.84</td>
<td>37.91</td>
<td>32.62</td>
</tr>
<tr>
<td>(\beta_B = \beta_C = \beta_E = 0)</td>
<td>23.85</td>
<td>22.55</td>
<td>18.03</td>
<td>23.97</td>
<td>21.64</td>
</tr>
<tr>
<td>(\beta_B = \beta_E = \beta_M = 0)</td>
<td>23.01</td>
<td>19.32</td>
<td>9.96</td>
<td>19.40</td>
<td>29.44</td>
</tr>
<tr>
<td>(\beta_C = \beta_E = \beta_M = 0)</td>
<td>31.07</td>
<td>26.58</td>
<td>19.35</td>
<td>21.24</td>
<td>33.82</td>
</tr>
<tr>
<td>(\beta_B = \beta_E = \beta_M = 0)</td>
<td>39.49</td>
<td>46.09</td>
<td>36.04</td>
<td>27.65</td>
<td>37.16</td>
</tr>
<tr>
<td>Jarque-Bera ((X^2) val.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rel. 1</td>
<td>0.64</td>
<td>0.35</td>
<td>0.11</td>
<td>2.87</td>
<td>11.79</td>
</tr>
<tr>
<td>Rel. 2</td>
<td>0.87</td>
<td>1.38</td>
<td>0.71</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td>Rel. 3</td>
<td>0.58</td>
<td>0.69</td>
<td>1.52</td>
<td>2.75</td>
<td>4.89</td>
</tr>
<tr>
<td>Rel. 4</td>
<td>1.87</td>
<td>5.62</td>
<td>5.48</td>
<td>23.55</td>
<td>2.54</td>
</tr>
<tr>
<td>Rel. 5</td>
<td>1.86</td>
<td>0.44</td>
<td>0.61</td>
<td>0.74</td>
<td>0.26</td>
</tr>
</tbody>
</table>

The critical values at the 5% probability level are:
27.17, 20.78, 14.04 and 3.96 for the \(\lambda_{\max}\)-test
47.18, 29.51, 15.20 and 3.96 for the Trace-test.
5.99, 7.81 and 9.49 for the \(\chi^2\)-test with 2, 3 and 4 d.f.
5.99 for the Jarque-Bera test (\(\chi^2\) with 2 d.f.).
Table A5 Cointegration between the five individual artists
Model coefficients ($K = 1$)

<table>
<thead>
<tr>
<th>Relation</th>
<th>$\alpha_{11}$</th>
<th>$\alpha_{12}$</th>
<th>$\gamma_{B,1}$</th>
<th>$\gamma_{C,1}$</th>
<th>$\gamma_{E,1}$</th>
<th>$\gamma_{M,1}$</th>
<th>$\gamma_{P,1}$</th>
<th>$H_0: \gamma_{C,1} = \gamma_{M,1} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Braque</td>
<td>-0.33</td>
<td>-0.49</td>
<td>-0.02</td>
<td>-0.19</td>
<td>-0.23</td>
<td>0.02</td>
<td>0.01</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(-0.15)</td>
<td>(-1.06)</td>
<td>(-1.86)</td>
<td>(0.17)</td>
<td>(1.0)</td>
<td>(0.17)</td>
<td>(0.1)</td>
<td></td>
</tr>
<tr>
<td>Chagall</td>
<td>-0.17</td>
<td>0.06</td>
<td>0.07</td>
<td>-0.58</td>
<td>0.05</td>
<td>0.26</td>
<td>-0.02</td>
<td>9.93</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(-4.29)</td>
<td>(0.55)</td>
<td>(2.67)</td>
<td>(-1.79)</td>
<td>(2.67)</td>
<td>(1.02)</td>
<td></td>
</tr>
<tr>
<td>Ernst</td>
<td>1.02</td>
<td>-0.08</td>
<td>-0.11</td>
<td>0.10</td>
<td>-0.12</td>
<td>0.23</td>
<td>0.02</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>(-0.52)</td>
<td>(-4.29)</td>
<td>(0.55)</td>
<td>(2.67)</td>
<td>(-1.79)</td>
<td>(2.67)</td>
<td>(1.30)</td>
<td></td>
</tr>
<tr>
<td>Miró</td>
<td>0.05</td>
<td>-0.26</td>
<td>-0.15</td>
<td>-0.32</td>
<td>0.08</td>
<td>0.16</td>
<td>0.11</td>
<td>2.49</td>
</tr>
<tr>
<td></td>
<td>(-0.85)</td>
<td>(-1.47)</td>
<td>(0.54)</td>
<td>(-1.04)</td>
<td>(0.80)</td>
<td>(1.24)</td>
<td>(0.80)</td>
<td></td>
</tr>
<tr>
<td>Picasso</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.41</td>
<td>0.11</td>
<td>0.03</td>
<td>0.40</td>
<td>-0.38</td>
<td>4.19</td>
</tr>
<tr>
<td></td>
<td>(-)</td>
<td>(-)</td>
<td>(-2.66)</td>
<td>(0.48)</td>
<td>(0.22)</td>
<td>(2.52)</td>
<td>(-2.52)</td>
<td></td>
</tr>
</tbody>
</table>

$t$-statistics are given between brackets, under the coefficients.
The statistic in the last column is an $F$-variable with 2 and 45 d.f. Its critical value at 5% is 3.21.
<table>
<thead>
<tr>
<th>Número</th>
<th>Título</th>
<th>Autor(es)</th>
<th>Data</th>
<th>Páginas</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>201.</td>
<td>HIPERINFLAÇÃO: CÂMBIO, MOEDA E ÂNCORAS NOMINAIS</td>
<td>Fernando de Holanda Barbosa</td>
<td>Novembro de 1992</td>
<td>10 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>202.</td>
<td>PREVIDÊNCIA SOCIAL: CIDADANIA E PROVISÃO</td>
<td>Clovis de Faro</td>
<td>Novembro de 1992</td>
<td>31 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>204.</td>
<td>TEORIAS ECONÔMICAS: A MEIA-VERDADE TEMPORÁRIA</td>
<td>Antonio Maria da Silveira</td>
<td>Dezembro de 1992</td>
<td>36 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>205.</td>
<td>THE RICARDIAN VICE AND THE INDETERMINATION OF SENIOR</td>
<td>Antonio Maria da Silveira</td>
<td>Dezembro de 1992</td>
<td>35 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>206.</td>
<td>HIPERINFLAÇÃO E A FORMA FUNCIONAL DA EQUAÇÃO DE DEMANDA DE MOEDA</td>
<td>Fernando de Holanda Barbosa</td>
<td>Janeiro de 1993</td>
<td>27 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>207.</td>
<td>REFORMA FINANCEIRA - ASPECTOS GERAIS E ANÁLISE DO PROJETO DA LEI</td>
<td>Rubens Penha Cysne</td>
<td>Fevereiro de 1993</td>
<td>37 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>209.</td>
<td>ELEMENTOS DE UMA ESTRATÉGIA PARA O DESENVOLVIMENTO DA</td>
<td>Antonio Salazar Pessoa Brandão e Eliseu Alves</td>
<td>Fevereiro de 1993</td>
<td>370 págs.</td>
<td></td>
</tr>
<tr>
<td>211.</td>
<td>OS SISTEMAS PREVIDENCIÁRIOS E UMA PROPOSTA PARA A REFORMULAÇÃO DO</td>
<td>Helio Portocarrero de Castro, Luiz Guilherme Schymura de Oliveira, Renato Fragelli Cardoso e Uriel de Magalhães</td>
<td>Março de 1993</td>
<td>43 págs.</td>
<td>(esgotado)</td>
</tr>
<tr>
<td>212.</td>
<td>THE INDETERMINATION OF SENIOR (OR THE INDETERMINATION OF WAGNER)</td>
<td>Antonio Maria da Silveira</td>
<td>Março de 1993</td>
<td>29 págs.</td>
<td>(esgotado)</td>
</tr>
</tbody>
</table>


221. TOPOLOGIA E CÁLCULO NO Rn - Rubens Penha Cysne e Humberto Moreira - Setembro de 1993 - 106 pág. (esgotado)


225. FINANCIAL INTEGRATION AND PUBLIC FINANCIAL INSTITUTIONS - Walter Novaes e Sérgio Ribeiro da Costa Werlang - Novembro de 1993 - 29 pág


227. A ECONOMIA BRASILEIRA NO PERÍODO MILITAR - VERSÃO REVISADA - Rubens Penha Cysne - Janeiro de 1994 - 45 pág. (esgotado)

228. THE IMPACT OF PUBLIC CAPITAL AND PUBLIC INVESTMENT ON ECONOMIC GROWTH: AN EMPIRICAL INVESTIGATION - Pedro Cavalcanti Ferreira - Fevereiro de 1994 - 37 pág. (esgotado)

229. FROM THE BRAZILIAN PAY AS YOU GO PENSION SYSTEM TO CAPITALIZATION: BAILING OUT THE GOVERNMENT - José Luiz de Carvalho e Clóvis de Faro - Fevereiro de 1994 - 24 pág.

230. ESTUDOS SOBRE A INDETERMINAÇÃO DE SENIOR - vol. II - Brena Paula Magno Fernandez, Maria Tereza Garcia Duarte, Sergio Grumbach, Antonio Maria da Silveira (Coordenador) - Fevereiro de 1994 - 51 pág.(esgotado)

231. ESTABILIZAÇÃO DE PREÇOS AGRÍCOLAS NO BRASIL: AVALIAÇÃO E PERSPECTIVAS - Clovis de Faro e José Luiz Carvalho - Março de 1994 - 33 pág. (esgotado)


234. BANDAS DE CÂMBIO: TEORIA, EVIDÊNCIA EMPÍRICA E SUA POSSÍVEL APLICAÇÃO NO BRASIL - Aloisio Pessoa de Araújo e Cypriano Lopes Feijó Filho - Abril de 1994 - 98 pág. (esgotado)


236. TESTING THE EXTERNALITIES HYPOTHESIS OF ENDOGENOUS GROWTH USING COINTEGRATION - Pedro Cavalcanti Ferreira e João Victor Issler - Abril de 1994 - 37 pág. (esgotado)


239. PUBLIC EXPENDITURES, TAXATION AND WELFARE MEASUREMENT - Pedro Cavalcanti Ferreira - Maio de 1994 - 36 pág.


241. INFLAÇÃO E O PLANO FHC - Rubens Penha Cysne - Maio de 1994 - 26 pág. (esgotado)


243. INTRODUÇÃO À INTEGRAÇÃO ESTOCÁSTICA - Paulo Klinger Monteiro - Junho de 1994 - 38 pág. (esgotado)

244. PURE ECONOMIC THEORIES: THE TEMPORARY HALF-TRUTH - Antonio M. Silveira - Junho de 1994 - 23 pág. (esgotado)

245. WELFARE COSTS OF INFLATION - THE CASE FOR INTEREST-BEARING MONEY AND EMPIRICAL ESTIMATES FOR BRAZIL - Mario Henrique Simonsen e Rubens Penha Cysne - Julho de 1994 - 25 pág. (esgotado)

246. INFRAESTRUTURA PÚBLICA, PRODUTIVIDADE E CRESCIMENTO - Pedro Cavalcanti Ferreira - Setembro de 1994 - 25 pág.


249. CUSTOS DE BEM ESTAR DA INFLAÇÃO - O CASO COM MOEDA INDEXADA E
ESTIMATIVAS EMPÍRICAS PARA O BRASIL - Mario Henrique Simonsen e Rubens
Penha Cysne - Novembro de 1994 - 28 pág. (esgotado)
250. THE ECONOMIST MACHIAVELLI - Brena P. M. Fernandez e Antonio M. Silveira -
Novembro de 1994 - 15 pág.
251. INFRAESTRUTURA NO BRASIL: ALGUNS FATOS ESTILIZADOS - Pedro Cavalcanti
Ferreira - Dezembro de 1994 - 33 pág.
252. ENTREPRENEURIAL RISK AND LABOUR'S SHARE IN OUTPUT - Renato Fragelli
Cardoso - Janeiro de 1995 - 22 pág.
253. TRADE OR INVESTMENT? LOCATION DECISIONS UNDER REGONAL
INTEGRATION - Marco Antonio F.de H. Cavalcanti e Renato G. Flôres Jr. - Janeiro de
1995 - 35 pág.
254. O SISTEMA FINANCEIRO OFICIAL E A QUEDA DAS TRANSFERÊNCIAS
255. CONVERGÊNCIA ENTRE A RENDA PER-CAPITA DOS ESTADOS BRASILEIROS -
256. A COMMENT ON "RATIONAL LEARNING LEAD TO NASH EQUILIBRIUM" BY
PROFESSORS EHUD KALAI EHUD EHUR - Alvaro Sandroni e Sergio Ribeiro da Costa
Werlang - Fevereiro de 1995 - 10 pág.
257. COMMON CYCLES IN MACROECONOMIC AGGREGATES (revised version) - João
Victor Issler e Farshid Vahid - Fevereiro de 1995 - 57 pág.
258. GROWTH, INCREASING RETURNS, AND PUBLIC INFRASTRUCTURE: TIMES
SERIES EVIDENCE (revised version) - Pedro Cavalcanti Ferreira e João Victor Issler -
Março de 1995 - 39 pág.(esgotado)
259. POLÍTICA CAMBIAL E O SALDO EM CONTA CORRENTE DO BALANÇO DE
PAGAMENTOS - Anais do Seminário realizado na Fundação Getulio Vargas no dia 08 de
dezembro de 1994 - Rubens Penha Cysne (editor) - Março de 1995 - 47 pág. (esgotado)
260. ASPECTOS MACROECONÔMICOS DA ENTRADA DE CAPITAIS - Anais do Seminário
realizado na Fundação Getulio Vargas no dia 08 de dezembro de 1994 - Rubens Penha
Cysne (editor) - Março de 1995 - 48 pág. (esgotado)
261. DIFICULDADES DO SISTEMA BANCÁRIO COM AS RESTRIÇÕES ATUAIS E
COMPULSÓRIOS ELEVADOS - Anais do Seminário realizado na Fundação Getulio
Vargas no dia 09 de dezembro de 1994 - Rubens Penha Cysne (editor) - Março de 1995 -
47 pág. (esgotado)
262. POLÍTICA MONETÁRIA: A TRANSIÇÃO DO MODELO ATUAL PARA O MODELO
CLÁSSICO - Anais do Seminário realizado na Fundação Getulio Vargas no dia 09 de
dezembro de 1994 - Rubens Penha Cysne (editor) - Março de 1995 - 54 pág. (esgotado)
263. CITY SIZES AND INDUSTRY CONCENTRATION - Afonso Arinos de Mello Franco
Neto - Maio de 1995 - 38 pág.
264. WELFARE AND FISCAL POLICY WITH PUBLIC GOODS AND INFRASTRUCTURE
(Revised Version) - Pedro Cavalcanti Ferreira - Maio de 1995 - 33 pág.
265. PROFIT SHARING WITH HETEROGENEOUS ENTREPRENEURIAL PROWESS - Renato Fragelli Cardoso - Julho de 1995 - 36 pág.

266. A DINÂMICA MONETÁRIA DA HIPERINFLAÇÃO: CAGAN REVISITADO - Fernando de Holanda Barbosa - Agosto de 1995 - 14 pág.


269. ON LONG-RUN PRICE COMOVEMENTS BETWEEN PAINTINGS AND PRINTS - Renato Flôres - Setembro de 1995 - 29 pág.