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Speculative Attacks on Debts, Dollarization and Optimum Currency Areas

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Speculative Attacks on Debts, Dollarization and Optimum Currency Areas[1]

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Abstract

The purpose of this article is to contribute to the discussion of the financial aspects of dollarization and optimum currency areas. Based on the model of self-fulfilling debt crisis developed by Cole and Kehoe [4], it is possible to evaluate the comparative welfare of economies, which either keep their local currency and an independent monetary policy, join a monetary union or adopt dollarization. In the two former monetary regimes, governments can issue debt denominated, respectively, in local and common currencies, which is completely purchased by national consumers. Given this ability, governments may decide to impose an inflation tax on these assets and use the revenues so collected to avoid an external debt crises. While the country that issues its own currency takes this decision independently, a country belonging to a monetary union depends on the joint decision of all member countries about the common monetary policy. In this way, an external debt crises may be avoided under the local and common currency regimes, if, respectively, the national and the union central banks have the ability to do monetary policy, represented by the reduction in the real return on the bonds denominated in these currencies. This resource is not available under dollarization. In a dollarized economy, the loss of control over national monetary policy does not allow adjustments for exogenous shocks that asymmetrically affect the client and the anchor countries, but credibility is strengthened. On the other hand, given the ability to inflate the local currency, the central bank may be subject to the political influence of a government not so strongly concerned with fiscal discipline, which reduces the welfare of the economy. In a similar fashion, under a common currency regime, the union central bank may also be under the influence of a group of countries to inflate the common currency, even though they do not face external restrictions. Therefore, the local and common currencies could be viewed as a way to provide welfare enhancing bankruptcy, if it is not abused. With these peculiarities of monetary regimes in mind, we simulate the levels of economic welfare for each, employing recent data for the Brazilian economy.

Keywords: dollarization, optimum currency areas, speculative attacks, debt crisis, sunspots

JEL Classification: F34, F36, F47, H63

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1 Introduction

The traditional criteria for two or more countries to establish an optimum currency area are based on the theoretical underpinnings developed in the 1960s by Mundell [18], McKinnon [15] and Kenen [14]. According to these prerequisites, the less is the need to vary the exchange rate between the local currency and that of its partner in order to adjust for imbalances between the two economies, the greater will be the advantages of a country's giving up its own currency and independent monetary policy. In general, these classic criteria indicate that the candidate countries must have certain common characteristics, such as: (i) mobile productive factors and flexibility of these markets; (ii) open trade and financial interdependence; (iii) symmetry of shocks and productive cycles; and (iv) diversified products and portfolios. In the 1990s, after the Treaty of Maastricht, new criteria for macroeconomic convergence were established for countries wishing to join the European Monetary Union. These criteria, growing out of the European experience in consolidating the EMU, include structural reforms and market liberalization along with strengthening the monetary and fiscal positions of the various national economies.

During the second half of the 1990s, most emerging economies were hit hard by financial and foreign exchange crises. The successive crises suffered by Mexico in 1994-5, Southeast Asian countries in the second half of 1997, and Russia the following year spread beyond their local origins to infect the international financial market in general. Some models, such as that of Cole and Kehoe [4] for the Mexican crisis, theorize that these crises result from a reversal of expectations by foreign investors worried about the level and maturity of government debt to international bankers. According to Radelet and Sachs [19], a central element in the Southeast Asian crisis of 1997 was a high inflow of short-term capital, making these countries vulnerable to a financial panic. Once a crisis hits a determined country, it can easily spread to others that also have weak fundamentals. These models exemplify situations in which a fixed exchange rate (much used in the 1990s) to attract foreign capital winds up increasing credit risk based on greater exposure to international liquidity caused by excessive foreign debt. This occurred not only in Mexico, Southeast Asia and Russia, but also in Brazil in 1998-9.

Currently, external savings are still essential to equilibrium in the balance of payments of some countries. For an economy needing to finance public spending and whose main objective is price stability, the recommendation is to exercise fiscal discipline and a rigid monetary policy to increase credibility and therefore lower the interest rates demanded by foreign lenders. However, some economists argue that adopting the currency of an anchor country – in short, dollarization – can also provide this credibility gain (Hanke e Schuler [12]). An intermediate step between dollarization and sticking to a local currency regime is joining a monetary union in which a group of countries adopt a common currency and establish a set of rules and reciprocal benefits. This enables increased credibility without giving up all control over monetary policy, as long as the union's central bank is sufficiently independent. The elimination of the risk of devaluation and reduction of credit risk are benefits strongly expected of dollarization. Nevertheless, Chang [3], Goldfajn and Olivares [11] and Morandé and Schmidt-Hebbel [17] warn that dollarization is no guarantee of reduced credit risk.

We propose to investigate one of the aspects of the debate over dollarization, monetary union and local currency regime with independent monetary policy as it applies to countries heavily dependent on foreign capital. Our central focus is to assess the welfare of these economies under each of these arrangements, when there is a positive probability of a default on the government external debt. As the international lenders believe the government is unlikely to honor its debts, they will decide to suspend new credits which induces the borrowing country to a default. Given a benevolent government, our purpose is to compare the welfare of the residents of a country subject to an exogenous shock, that affect the mood of the international creditors about lending to the country in question. We carry out this exercise for the three monetary regimes under study, keeping in mind the characteristics of each one of them, as described below.

Under dollarization, the national government loses control over the monetary policy. Due to the high cost of later reverting to a more flexible regime, it can become powerless to react to external shocks unless there is strong symmetry between the effect of such shocks on its economy and that of the anchor country. In counterpart, dollarization provides large credibility gains because monetary policy rests with the U.S. Federal Reserve Bank, which is strongly committed to low inflation. With monetary union, each member country has some power to influence the common monetary policy through its vote in the decision-making system. It provides a lesser commitment to price stability than dollarization because it allows devaluation of the common currency to correct for external imbalances. This commitment can become weaker to the extent that the central bank may be under political pressures exerted by certain member countries. Finally, under a regime of purely local currency and independent monetary policy, the government has complete flexibility to make monetary adjustments, but this freedom can make such regime the least credible among the three types.

The welfare analysis for the three monetary regimes uses as a reference the self-fulfilling debt crisis model of Cole and Kehoe ([4], [5] and [6]). This model can be viewed as an approximation of a dollarized economy subject to a speculative attack on its public debt held by international bankers. To represent the monetary union and local currency regimes, we modify the original model by including public debt denominated in common currency and local currency respectively, which is purchased by the national consumers. The issuance of a portion of public debt in local currency or in common currency is not an option for dollarized economies. But a national government issuing local currency can decide only to pay a fraction of the real return on the bonds denominated in local currency to domestic creditors, use the revenues so raised to pay the debt to international bankers and avoid an external crisis. Also, a country belonging to a monetary union can find an alternative to an external debt crisis if it obtains extra resources through a decision of the central authority to abate the real return of the debt denominated in common currency. In a similar fashion Sims [22] stands against dollarization and favors surprise inflation as a solution to smooth situations of fiscal stress. Sargent [20], commenting on Sims′ paper, points out the lack of models that discuss the issue of dollarization. We pretend to bring some contribution to this debate.

In our work, we show that to create an inflation tax on debts denominated in local and in common currencies can be preferable in terms of the welfare of residents than to suspend payment on dollar-denominated external debt. On the one hand, a default on dollar debt can contract economic activity and exclude a country from the international credit market, in turn lowering productivity. On the other hand, a partial payment of local currency or common currency debts does not carry over to subsequent periods. It occurs one time only and thus does not reduce productivity, since domestic creditors know that this partial default is temporary and seeks only to obtain extra resources quickly to avoid the greater harm of a moratorium on the external debt. However, the alternative of imposing an inflation tax enables governments to put pressure on central banks, with the goal of resorting to a partial moratorium in order to increase public spending, even when there is no external crises to be avoided. Hence, we intend to investigate under which conditions it is most advantageous for an economy to maintain or not a flexible monetary policy.

For the purpose of simulating the model for the Brazilian economy in recent years and calculating the levels of welfare mentioned above, we first present the changes in the original model that allow it to incorporate local currency and partial payment of debts denominated in this currency. We specify in more detail the extension of the Cole-Kehoe model to a situation of monetary union and include the chance that the union central bank, under political pressure, will surprise the private sector with an unexpected monetary policy that particularly serves the interests of a sub-group of member countries. We further take into account the expectation of the private sector that asymmetric shocks can affect the national and central government types. If there are no antagonisms between the optimal choices of the two governments, then monetary union provides a level of welfare as good as that obtainable in case the member country had maintained an independent monetary policy.

On a more methodological ground, this paper could be viewed as a part of the literature on general equilibrium with bankruptcy, which asserts that on an incomplete markets situation the introduction of the possibility of bankruptcy can be welfare enhancing (see Dubey, Geanakoplos and Zame [9] for static economies and Araujo, Pascoa and Torres [2] in infinite horizon economies). The introduction of local currency could give rise to the possibility of a better bankruptcy technology through inflation than just the repudiation of the foreign debt which can be quite costly. However that may not be the case if the local central bank is too weak to deal with fiscal indiscipline.

2 The Cole-Kehoe Model with Local Currency

Cole and Kehoe developed a dynamic, stochastic general equilibrium model in which they consider the possibility of a self-fulfilling crisis of public debt held by international bankers occurring. Among the results, these authors describe an optimal government debt policy and show that when the debt is located in the crisis zone, a self-fulfilling crisis can occur, and that in order to leave this critical region it is optimal either to run down the debt or draw out its average maturity.

We modify the original Cole-Kehoe model in order to assess the welfare of an economy with two currencies, one local and the other the dollar. This extension of the Cole-Kehoe model brings a new perspective to the discussion of dollarization – the original model being analogous to that of a dollarized economy, and the two-currency model reproducing an economy that issues local currency and maintains an independent monetary policy. With this new version, we evaluate the expected welfare of an economy able to create an inflation tax and use the revenue so generated to avoid an external debt crisis. The local currency is added to this model with the subterfuge that the government carries public debt in local money and the inflation tax is extracted from consumers when the government decides on the maturity date to reduce the real return in local-currency debt held by the public.

Compared with the original version, we include the following variables in the model with local currency: debt in local currency, D, another sunspot variable, η, and a variable to represent the government decision whether or not to reduce the real value of local-currency debt, ϑ. In other aspects, this model closely follows that of Cole-Kehoe. There are three participants: consumers, international bankers and the government. Besides there is another public debt instrument: bonds denominated in dollars, B. We assume that dollar-based debt is only acquired by international bankers and any suspension in payment is always total, while local-currency debt is only taken up by consumers and repayment is suspended partially. Uncertainty is incorporated in the model by the two exogenous sunspot variables, ζ and η. Realization of the sunspot variable ζ indicates the willingness of international bankers to roll over their dollar credits, and realization of η reveals the type of government, whether more concerned with stability of output or prices. If the government shows itself to be more preoccupied with maintaining productivity, it prefers to extract an inflation tax rather than declare a moratorium on dollar debt. The probability international bankers will lose confidence in the government's ability to honor its obligations in dollars in the following period is denoted by π, $P[\zeta \leq \pi] = \pi$, while ξ is the probability that the government will be more concerned with avoiding recession, i.e., $P[\eta \leq \xi]$.

The model with local currency and independent monetary policy also considers the possibility that the government will declare a partial moratorium on local-currency debt even though there are no problems in financing dollar debt. International bankers roll over their loans but still there is a surprise reduction in the real value of debt in local currency for the purpose of generating extra government revenues. We suppose that the private sector attributes probability $\psi\xi$ that the central bank will act in this manner at a certain instant. When the constant ψ is equal to $1/\xi$, the private sector is certain that the central bank practices a monetary policy strongly influenced by government political interests (denoted as a weak central bank). At the other extreme, when the constant ψ is zero, the central bank is independent (denoted as strong) and resorts to a partial moratorium only to avoid an external debt crisis.

In the following section we make a second modification to the original Cole-Kehoe model, representing the case of a monetary union. The main characteristics of a monetary union are the adoption of a common currency, creation of a central bank for the union and a voting system for, among other purposes, adjusting the parity of the common currency in relation to the dollar. Our modifications seek to assess the welfare of an economy belonging to a monetary union, subject to a speculative attack on its dollar debt with positive probability and the possibility that this crisis can be contained by lowering the real return of the debt in the common currency. In a monetary union, a speculative attack on one member can be transmitted to the other partners through the channel of coordinated monetary policy. The monetary union has the option of avoiding a moratorium on external debt by countries in crisis and resorting to inflating the common money, depending on the relative weight each country carries in the union's voting system.

3 A Monetary Union Model

The monetary union model is still one with two currencies, the common (rather than local) one and the dollar. Each country can issue debt in these two currencies, but the recourse to partial payment of debt in common currency is under greater control than if it could issue its own money, insofar as a partial default depends on the approval of the majority of member countries.

This version incorporates I economies and a central government, equivalent to the Council of the European Union, constituted as the decision-making body for all members. Each country i, $i = 1, \ldots, I$, issues bonds denominated in the common currency, D^i, and the central government decides whether or not to reduce this debt through the variable ϑ^u, with the result subject to the votes of the national governments.

This model is similar to the extension of the Cole-Kehoe model to a country with its own local currency and independent monetary policy. We suppose that there is only one good, produced with physical capital and inelastic labor supply. The price of this reference good is normalized at one dollar, or p_t units of the common currency, in all member countries. International bankers hold dollar-denominated bonds of each country i of the union, B^i, $i = 1, \ldots, I$, and in case a determined government is unable to honor this debt, payment is always totally suspended (i.e., a country does not undertake a partial default of dollar debt). Furthermore, consumers only acquire bonds denominated in the common currency issued by the government of their own country i, D^i, and in this case a moratorium can be partial.

Additionally, for each country i, there are two sunspot variables, ζ^i and η^i. The expected value of ζ^i constitutes the expectation of international bankers regarding the decision of the government of country i to honor its dollar debt. This expectation is reflected in the bond prices foreign lenders are willing to pay. If the dollar debt level of country i is below the crisis zone, then the realization of the variable ζ^i is not relevant, because creditors are not worried about its result in order to make loans to this country. However, if the dollar debt level is in the crisis zone, then an unfavorable realization of this variable indicates a speculative attack on this debt. In this case, the external creditors do not roll over their loans because they believe that the government will fail to honor its obligations. With suspension of foreign credits, the government chooses to suspend payments and suffer the consequences, namely: falling output and lack of new dollar loans in the future. Given the positive probability of this state occurring next period, then the international bankers pay less for the government dollar debt, than they would otherwise. On the other hand, realization of the η^i variable reflects at all moments whether each national government is more worried about stability of prices or national output. This sunspot variable performs a relevant function when the national governments conjecture on inflating the common money to avoid an external crisis in their economies. Nevertheless, if no member country is under speculative attack on its dollar debt, then the realization of this variable for the member countries as a group becomes irrelevant, since there is no reason to inflate. The difference between realization of η^i for each country corresponds to the political risk the corresponding national government faces in adopting a common currency. Antagonistic types of national governments can result in different preferences regarding the conduct of a common monetary policy. This same question is analyzed by Alesina and Grilli [1], but using a different theoretical approach.

The relative influence (or weighting) of each country in the union's voting system, λ^i, and the realization of the η^i variables will characterize the type of central government, whether more concerned with stability of output or prices. The results of the weights λ^i and the realizations of η^i for all the countries is synthesized in the random variable η^u. The realization of the sunspot variables η^i and η^u indicates whether the government of country i and the central government are in harmony regarding price versus output stability. When η^i and η^u reveal that both governments are of the same type, then one can say that there is symmetry between the shocks faced by the government of country i and the union's central government. Otherwise there is asymmetry.

3.1 Description of the market participants

(i) Consumers of Country i [1]

Each country has an infinite number of consumers who live forever and have utility function given by

$$E\sum_{t=0}^{\infty}\beta^t\left[\varrho^i c_t^i + v\left(g_t^i\right)\right]$$

with ϱ^i being the weighting of the utility of private consumption, c^i, in relation to the utility of public consumption , $v(g^i)$. The consumer budgetary constraint at time t is

$$c_t^i + k_{t+1}^i - k_t^i + q_t^i d_{t+1}^i \leq \left(1-\theta^i\right)\left[a_t^i f\left(k_t^i\right) - \delta^i k_t^i\right] + \vartheta_t^u d_t^i$$

[1] All the parameters, variables and functions that are not defined in this subsection can be found in the original articles by Cole and Kehoe ([4],[5] and [6]).

At all points in time, the consumers of country i designate a part of their savings to buy government bonds in common currency, d^i_{t+1}, at a price of q^i_t units of the good per bond and receive ϑ^u_t units of the good per bond for the total bonds acquired in the previous period, d^i_t. The common-currency debt consists of zero-coupon bonds maturing in one period, that pay one unit of the good at the price in common currency effective in the preceding period. The price of one bond at time t is $\widetilde{q}^i_t$ in units of common currency. With p_t as the price of one unit of the good in common currency at time t, each consumer pays $q^i_t = \dfrac{\widetilde{q}^i_t}{p_t}$ units of the good for each bond at this time. In the following period, if the government does not undertake a partial default, then the creditor receives a full unit of the good, the same as with a dollar-denominated bond. In this case, ϑ^u_t equals 1. However, if the government decides to abate the real value of its common-currency debt, then the consumer receives $\vartheta^u_t = \phi^u$, $0 < \phi^u < 1$ units of the good, instead of a full unit. In the model with local currency, payment for bonds in local currency is denoted by $\vartheta^i_t D^i_t$, which includes the decision variable for the national government, ϑ^i_t, rather than for the central government, ϑ^u_t.

We also assume that at the start, each consumer holds d^i_0 units of the common-currency debt of country i and k^i_0 units of the good.

(ii) International Bankers

There are an infinite number of international bankers, living forever, with utility function corresponding to

$$E \sum_{t=0}^{\infty} \beta^t x_t$$

The budgetary constraint at time t is given by

$$x_t + \sum_{i=1}^{I} q^{*i}_t b^i_{t+1} \leq \overline{x} + \sum_{i=1}^{I} z^i_t b^i_t$$

which includes purchase and redemption of dollar debt of the I countries of the monetary union. Each banker pays q^{*i}_t per dollar-denominated bond of country i at time t, b^i_{t+1}. We suppose that each bond matures in one period and pays out one unit of the good at $t+1$ if there is no suspension of debt payments. Besides this, we assume that at the initial moment, external creditors hold b_0^i bonds of the dollar debt of each country i and that the supply of credits from international bankers meets the demand for loans from these countries without competition causing a liquidity crisis.[2]

(iii) National Government

It is considered to be benevolent in that it seeks to maximize the welfare of its domestic consumers. The lack of a commitment to pay off its debt and to define a path for borrowing and spending allows multiple equilibria. At time t, the government of country i makes the following choices: a) dollar new borrowing level, B_{t+1}^i; b) common-currency new borrowing level , D_{t+1}^i; c) whether or not to partially default on its old common-currency debt, ϑ_t^u; d) whether or not to default on its old dollar debt, z_t^i; and e) current government consumption, g_t^i. The budgetary constraint at time t, expressed in real terms, equals

$$g_t^i + z_t^i B_t^i + \vartheta_t^u D_t^i \leq \theta^i \left[a_t^i f\left(K_t^i\right) - \delta^i K_t^i \right] + q_t^{*i} B_{t+1}^i + q_t^i D_{t+1}^i$$

which can be rewritten as,

$$g_t^i \leq \theta^i \left[a_t^i f\left(K_t^i\right) - \delta^i K_t^i \right] - z_t^i B_t^i + q_t^{*i} B_{t+1}^i + \left(1 - \vartheta_t^u\right) D_t^i + q_t^i D_{t+1}^i - D_t^i \quad (1)$$

There is no seigniorage in the above expression. The national government raises additional revenue by lowering the real value of its common-currency debt held by consumers and not through expanding the money supply. This revenue is denoted by $(1-\vartheta_t^u)D_t^i$, where ϑ_t^u can take on two values: 1 or $\phi^u = \dfrac{1}{1+\chi^u}$, with χ^u being the rate at which the debt loses real value. Abatement of common-currency debt means fewer goods in the economy if the national government uses this recourse to settle its dollar debts and hence to avoid a default. The lesser the number of goods

[2] Fratszcher [10], Hernández and Valdés [13] and Van Rijckeghem and Weder [23] justify the presence of a common lender as one of the causes for the crises in emerging market countries during the 1990s. However, we do not include this possibility in the present model.

that can be acquired in country i implies an increase in the internal price of the goods. Therefore, the reduction in the real return of common-currency debt D_t^i is a means of taxing it, similar to an inflation tax. Based on this, we alternatively use the expressions lowering (or reducing, abating) common currency debt and inflating the common currency, although we do not explicitly describe the monetary market.

With dollarization, country i only has debts in dollars. In this case, the government of country i cannot independently choose ϑ_t^i or vote for choice ϑ_t^u. In a regime of local currency, country i issues debt in its own currency and so D_t^i is not zero as it is for dollarization.

(iv) Central Union Government

This entity, also assumed benevolent, has a different role than a national government. It has two main functions: (i) to decide on the reduction factor for debt in common currency, ϑ_t^u, weighted by the respective decisions of the member countries according to their relative influence in the voting system, λ^i, $i = 1, \ldots, I$; and (ii) to collect the revenues of all countries of the monetary union obtained by inflating the common currency and thereafter to distribute this money through transfers to the member countries. The total revenue raised is equal to $\sum_{i=1}^{I} (1 - \vartheta_t^u) D_t^i$ and its choice variable ϑ_t^u corresponds to:

$$\vartheta_t^u = \begin{cases} 1 \text{ if } \sum_j \lambda^j \geq \frac{2}{3} \sum_{i=1}^{I} \lambda^i \\ \phi^u \text{ otherwise} \end{cases}$$

with λ^j the weight attributed to a country j which is worried about price stability. If the sum of the weights of the countries that do not wish to inflate the common currency is greater than two-thirds of the total votes, then the central government chooses $\vartheta^u = 1$.

Whenever the central government decides to inflate the common currency, if no member country can disproportionately pressure the union's central bank for extra resources, then each national government receives transfers equal to the abatement of debt in common currency extracted from consumers of that country. Otherwise, the central government can give in to pressures from a certain country and earmark a relatively greater share of this inflation tax to that country, reducing the share transferred to the other members. We denote by ϖ^i the fraction of the transfers from the central to national government of country i relative to the total revenue raised in this country through lowering common-currency debt. Hence, a country with low bargaining power with the central government is defined as having $0 \leq \varpi^i < 1$. If this country has relatively greater influence over the central government, the situation is $\varpi^i > 1$, and if the central government of the union is not subject to undue pressures from any country, then $\varpi^i = 1$.[3] Thus, when $\varpi^i \neq 1$, the budgetary constraint for a national government i specified in (1) now can be rewritten as follows:

$$g_t^i \leq \theta^i \left[a_t^i f\left(K_t^i\right) - \delta^i K_t^i\right] - z_t^i B_t^i + q_t^{*i} B_{t+1}^i + \varpi^i \left(1 - \vartheta_t^u\right) D_t^i + q_t^i D_{t+1}^i - D_t^i$$

We also assume that at the initial period for each country i the supply of dollar debt B_0^i is equal to its demand for this debt, b_0^i; the supply of common-currency debt D_0^i is equal to the demand for this type of debt, d_0^i; and the aggregate capital stock per worker, K_0^i, is equal to the individual capital stock, k_0^i. We also maintain the hypothesis of Cole-Kehoe that if the government of country i fails to pay its dollar debt in the current period, then output will fall to α^i, with $0 < \alpha^i < 1$, and remain at this level thereafter. Similarly, we suppose that if the central union government decides to pay only $0 < \phi^u < 1$ units of the good per bond instead of a full unit, then output does not fall, consumers receive ϕ^u units of the good per common-currency bond, and believe that the government will only henceforth start paying this quantity of goods per bond.[4]

[3]Cooper and Kempf [8] study two regional power configurations. In the first, two groups are able to pressure the central monetary authority to inflate the common currency, and in the second, only one of the regions benefits from seigniorage. Their work inspired the inclusion of countries with different bargaining power levels over the central bank in this extension of the Cole-Kehoe model.

[4]This latter hypothesis means that at the time of a partial default, the return on a common-currency bond is less than in prior periods. As will be seen below, the return is ϕ^u/β and not $1/\beta$ or $(1 - \xi^u + \phi^u \xi^u)/\beta$ as previously. At later times, the price which consumers pay for this bond is $\phi^u \beta$ and it pays back ϕ^u units of the good.

3.2 An equilibrium

Following the Cole-Kehoe model, we define a recursive equilibrium for country i belonging to a monetary union, with a strong central bank (independent of political pressures). Suppose the weighting λ^i of each nation in the voting system of the union is known to all and invariable.

3.2.1 Timing

- The sunspot variables ζ_t^i, η_t^i and η_t^u are realized and the aggregate state of economy i is $s_t^i = (K_t^i,\, B_t^i,\, D_t^i,\, a_{t-1}^i,\, \vartheta_{t-1}^u,\, \zeta_t^i,\, \eta_t^i,\, \eta_t^u)$;

- The government of each country i, taking the dollar bond price schedule $q_t^{*i} = q^{*i}(s_t^i,\, B_{t+1}^i)$ as given, chooses the level of new dollar debt to offer, B_{t+1}^i;

- The government of each country i, taking the common-currency bond price schedule, $q_t^i = q^i(s_t^i,\, B_{t+1}^1,\, \ldots,\, B_{t+1}^I)$ as given, chooses the level of new common-currency debt to offer, D_{t+1}^i, and each country knows the supply and demand of common-currency bonds for other member countries;

- International bankers, taking as given q_t^{*i} for each country i and ϑ_t^u, choose the level of dollar bonds to acquire, b_{t+1}^i, $i = 1,\, \ldots,\, I$;

- Consumers, considering q_t^{*i}, q_t^i and ϑ_t^u as given, choose the level of common-currency bonds issued by their country that they want to buy, d_{t+1}^i;

- The central government decides whether or not to inflate the common currency, ϑ_t^u;

- The national governments decide whether or not to default on their dollar debts, z_t^i, and choose their current consumption, g_t^i ;

- Consumers, taking $a_t^i = a^i(a_{t-1}^i,\, z_t^i)$ as given, choose c_t^i and k_{t+1}^i.

3.2.2 States and value functions of agents from country i

Consumers make decisions at two points in time: first, they choose the level of common-currency bonds, d'^i, and then they decide on c^i and k'^i. We assume that in choosing d'^i, consumers know their state variables k^i, d^i, s^i, B'^i and D'^i. Also, they know the aggregate state, s^i, B'^i and λ^i for all countries. Upon deciding d'^i, the consumers' state is (k^i, d^i, s^i, B'^1, ..., B'^I, D'^i). Given s^i and λ^i, for $i = 1$, ..., I, the private sector knows the type of central government and thus whether or not inflation is a recourse that will be used to avoid an external debt crisis for the union in the current period. If more than two-thirds of the countries have dollar debts in the crisis zone, then depending on the realization of η^u, the central government may decide to inflate the common currency. Once inflated, this alternative is no longer available and from then on the realization of η^u is no longer relevant. Given s^i and B'^1, ..., B'^I, consumers of country i know $q^{*i}(s^i,\ B'^i)$ and assess the willingness of international bankers to roll over their loans, z^i. Additionally, they anticipate the decisions of national governments regarding g^i and of the central government about ϑ^u. Therefore, at the time of choosing d'^i, they have rational expectations concerning q^{*i}, q^i, g^i, z^i, ϑ^u and $a^i(s^i,\ z^i)$ and know the decisions they will henceforth make about c^i and k'^i.

In the second period, in deciding about c^i and k'^i, consumers already know the price of a common-currency bond, q^i, and also the price paid by international bankers for dollar bonds, q^{*i}. In addition, they have already discovered z^i, ϑ^u and g^i. Hence, in choosing c^i and k'^i, the state of a representative consumer of country i is (k^i, d^i, s^i, B'^1, ..., B'^I, D'^i, q^{*1}, ..., q^{*I}, g^i, z^i, ϑ^u) and his value function corresponds to

$$\begin{aligned} & V_{c^i}(k^i, d^i, s^i, B'^1, \ldots, B'^I, D'^i, q^{*1}, \ldots, q^{*I}, g^i, z^i, \vartheta^u) \qquad (2) \\ = & \max_{c^i, k'^i, d'^i} \{\varrho^i c^i + v(g^i) + \\ & \beta\ EV_{c^i}[k'^i, d'^i, s'^i, B'^1(s'^1), \ldots, B'^I(s'^I), D'^i(s'^i), q^{*\prime 1}, \ldots, q^{*I}, g'^i, z'^i, \vartheta'^u]\} \end{aligned}$$

s.t.

$$\begin{aligned} c^i &\leq (1-\theta^i)[a^i\ f(k^i) - \delta^i k^i] + \vartheta^u d^i - q^i d'^i + k^i - k'^i \\ c^i, k'^i &\geq 0 \\ d'^i &\geq -\Delta \end{aligned}$$

$$\begin{aligned} s'^i &= \left(K'^i\left(s^i, B'^1, \ldots, B'^I, D'^i, q^{*i}, g^i, z^i, \vartheta^u\right), B'^1, \ldots, B'^I, D'^i, a^i\left(s^i, z^i\right), \vartheta^u, \zeta'^i, \eta'^i, \eta'^u\right) \\ g'^i &= g(s'^i, B'^i\left(s'^i\right), q^*\left(s'^i, B'^i\left(s'^i\right)\right), D'^i\left(s'^i\right), q\left(s'^i, B'^1\left(s'^1\right), \ldots, B'^I\left(s'^I\right)\right), \vartheta'^u) \\ z'^i &= z(s'^i, B'^i\left(s'^i\right), q^*\left(s'^i, B'^i\left(s'^i\right)\right), D'^i\left(s'^i\right), q\left(s'^i, B'^1\left(s'^1\right), \ldots, B'^I\left(s'^I\right)\right), \vartheta'^u) \\ \vartheta'^u &= \left(s'^1, B'^1\left(s'^1\right), q^{*1}\left(s'^1, B'^1\left(s'^1\right)\right), D'^1\left(s'^1\right), q\left(s'^i, B'^1\left(s'^1\right), \ldots, B'^I\left(s'^I\right)\right), \ldots, \right. \\ &\quad \left. s'^I, B'^I\left(s'^I\right), q^{*I}\left(s'^I, B'^I\left(s'^I\right)\right), D'^I\left(s'^I\right), q\left(s'^i, B'^1\left(s'^1\right), \ldots, B'^I\left(s'^I\right)\right)\right) \end{aligned}$$

with Δ a positive constant.

In turn, in choosing b'^i, international bankers know b^i, s^i, B'^i and D'^i for all the countries of the monetary union. Besides this, they take as given q^i, q^{*i} and ϑ^u , since these depend on s^i, $B'^i$and D'^i, which are known for all i. Hence, the state of a representative banker corresponds to (b^1, ..., b^I, s^1, ..., s^I, B'^1, ..., B'^I, D'^1, ..., D'^I) and his value function is:

$$V_b(b^1, \ldots, b^I, s^1, \ldots, s^I, B'^1, \ldots, B'^I, D'^1, \ldots, D'^I) = \tag{3}$$

$$\max_{x, b'^1, \ldots, b'^I} \quad x + \beta\ EV_b(b'^1, \ldots, b'^I, s'^1 \ldots, s'^I, B'^1\left(s'^1\right), \ldots, B'^I\left(s'^I\right), D'^1\left(s'^1\right), \ldots, D'^I\left(s'^I\right))$$

$s.t.$

$$x + \sum_{i=1}^{I} q^*(s^i, B'^i) b'^i \leq \overline{x} + \sum_{i=1}^{I} z^i\left(s^i, B'^i, q^*\left(s^i, B'^i\right), D'^i, q\left(s^i, B'^1, \ldots, B'^I\right), \vartheta^u\right) b^i$$

$$\begin{aligned} \vartheta^u &= \vartheta^u\left(s^1, B'^1, q^{*1}\left(s^1, B'^1\right), D'^1, q^1\left(s^1, B'^1, \ldots, B'^I\right), \ldots, \right. \\ &\quad \left. s^I, B'^I, q^{*I}\left(s^I, B'^I\right), D'^I, q^I\left(s^I, B'^1, \ldots, B'^I\right)\right) \end{aligned}$$

$$x \geq 0$$

$$b'^1, \ldots, b'^I \geq -\Lambda$$

with Λ a positive constant.

The government of country i also makes decisions at two moments in time. At first, when choosing B'^i and D'^i, the government knows s^i, $q^{*i}(s^i,\ B'^i)$ and $q^i(s^i,\ B'^1, \ldots, B'^I)$ for itself and all other member countries. Thereupon, it chooses g^i and z^i. In the Cole-Kehoe model with local currency, the decisions of country i for g^i, z^i and ϑ^i depend only on s^i and its own choices regarding B'^i and D'^i. In our monetary union model, the choice of g^i and z^i also depends on ϑ^u.

In accordance with the timing, the government of country i knows the decision of the central government for ϑ^u on choosing $g^i(s^i,\ B'^i,\ q^{*i}(s^i, B'^i),\ D'^i,\ q^i(s^i, B'^1, \ldots, B'^I),\ \vartheta^u)$ and $z^i(s^i,\ B'^i,\ q^{*i}(s^i,\ B'^i),\ D'^i,\ q^i(s^i, B'^1, \ldots, B'^I),\ \vartheta^u)$, and recognizes that through its actions it can affect the prices that external creditors are willing to pay for its dollar debt, $q^{*i}(s^i,\ B'^i)$ and production parameter $a^i(s^i,\ z^i)$. In addition, it perceives that the choices of national governments regarding B'^i for all i affect ϑ^u, $q^i(s^i,\ B'^1,\ \ldots,\ B'^I)$ and the optimal decisions of consumers $c^i(k^i,\ d^i,\ s^i,\ B'^1,\ \ldots,\ B'^I,\ D'^i,\ q^{*1}, \ldots,\ q^{*I},\ g^i,\ z^i,\ \vartheta^u)$, $k'^i(k^i,\ d^i,\ s^i,\ B'^1,\ \ldots,\ B'^I,\ D'^i,\ q^{*1}, \ldots,\ q^{*I},\ g^i,\ z^i,\ \vartheta^u)$ and $d'(k^i,\ d^i,\ s^i,\ B'^1, \ldots,\ B'^I,\ D'^i)$. Thus, when the national government chooses B'^i and D'^i, it takes as given q^{*i}, q^i, g^i, z^i, ϑ^u, c^i and k'^i, and its value function corresponds to:

$$V_{g^i}(s^i) = \tag{4}$$

$$\max_{B'^i, D'^i} c^i(s^i, B'^1, \ldots, B'^I, D'^i, q^{*1}, \ldots, q^{*I}, g^i, z^i, \vartheta^u) + v(g^i) + \beta E V_{g^i}(s'^i)$$

$s.t.$

$$g^i = g^i\left(s^i, B'^i, q^{*i}\left(s^i, B'^i\right), D'^i, q^i\left(s^i, B'^1, \ldots, B'^I\right), \vartheta^u\right)$$

$$z^i = z^i(s^i, B'^i, q^{*i}\left(s^i, B'^i\right), D'^i, q^i\left(s^i, B'^1, \ldots, B'^I\right), \vartheta^u)$$

$$\begin{aligned} \vartheta^u &= \vartheta^u\left(s^1, B'^1, q^{*1}\left(s^1, B'^1\right), D'^1, q^1\left(s^1, B'^1, \ldots, B'^I\right)\right. \\ &\quad \left., \ldots, s^I, B'^I, q^{*I}\left(s^I, B'^I\right), D'^I, q^I\left(s^I, B'^1, \ldots, B'^I\right)\right) \end{aligned}$$

$$s'^i = \left\{B'^i, D'^i, K'^i\left(s^i, B'^1, \ldots, B'^I, D'^i, q^{*i}, g^i, z^i, \vartheta^u\right), a^i\left(s^i, z^i\right), \vartheta^u, \zeta'^i, \eta'^i, \eta'^u\right\}$$

After national and international creditors have decided about rolling over their loans, the government of country i decides whether to pay its dollar debt, comparing the relative levels of welfare for $z^i \in \{0,\ 1\}$, given $\vartheta^u \in \{\phi^u,\ 1\}$. The choice of z^i determines the level of productivity, $a^i(s^i,\ z^i)$, and of current government spending, g^i, which must be positive. The policy functions $z^i(s^i,\ B'^i,\ q^{*i},\ D'^i,\ q^i,\ \vartheta^u)$ and $g^i(s^i,\ B'^i,\ q^{*i},\ D'^i,\ q^i,\ \vartheta^u)$ are solved for:

$$\max_{g^i, z^i} \quad c^i(s^i, B'^1, \ldots, B'^I, D'^i, q^{*1}, \ldots, q^{*I}, g^i, z^i, \vartheta^u) + v(g^i) + \beta E V_{g^i}(s'^i) \qquad (5)$$

$s.t.$

$$g^i + z^i B^i + \vartheta^u D^i \leq \theta^i [a^i \left(s^i, z^i\right) f(K^i) - \delta^i K^i] + q^{*i}\left(s^i, B'^1, \ldots, B'^I\right) B'^i + q^i \left(s^i, B'^i\right) D'^i$$

$$\begin{aligned} g^i &\geq 0 \\ z^i &= 0 \text{ and } \vartheta^u = 1 \text{ or} \\ z^i &= 0 \text{ and } \vartheta^u = \phi^u \text{ or} \\ z^i &= 1 \text{ and } \vartheta^u = 1 \text{ or} \\ z^i &= 1 \text{ and } \vartheta^u = \phi^u \end{aligned}$$

$$s'^i = \{B'^i, D'^i, K^i\left(s^i, B'^i, D'^i, q^{*i}, g^i, z^i, \vartheta^u\right), a^i\left(s^i, z^i\right), \vartheta^u, \zeta'^i, \eta'^i, \eta'^u\}$$

When the central government chooses ϑ^u, it knows s^i, $B'^i(s^i)$ and $D'^i(s^i)$, for each country i, and also knows how its action will affect the decisions of national governments regarding z^i and g^i, and of consumers regarding c^i and k'^i. Thus, having information about s^i for each country i, the central government is able to assess the preference of each one concerning ϑ^u, $\vartheta^u = 1$ or ϕ^u. If more than two-thirds of the total votes are from countries that prefer an inflation tax, then the central government chooses ϑ^u equal to ϕ^u.

The choice of ϑ^u may be the result of the following procedure. Given the initial state s^i, B'^i and D'^i for all i, the central government envisions what would be the choice of each member country for the debt abatement factor ϑ^i in case each country had its own currency and independent monetary policy. Considering these individual choices, the central government estimates the expected welfare of each country i and the weighted sum of these levels, W, using the weights attributed in the voting system. The central government chooses ϑ^u by equating W to the weighted sum of the welfare of all members of the union, when it supposes that ϑ^u is chosen by all of them instead of ϑ^i. This produces the result

$$W = \sum_{i=1}^{I} \lambda_i V_{g^i}(s^i) \equiv V_{g^u}\left(s^1, \ldots, s^I\right)$$

where V_{g^u} is the value function of the central government.

3.2.3 Definition of an equilibrium

An equilibrium corresponds to a list of value functions V_{c^i} for the representative consumer of country i, V_b for the representative international banker, V_{g^i} for the national government of country i and V_{g^u} for the central government; policy functions c^i, k'^i and d'^i for the consumer of country i, x, $b'^1, \ldots, b'^I$ for the international banker, B'^i, D'^i, g^i and z^i for the national government of country i and ϑ^u for the central government; price functions for the public debt in dollars of country i, q^{*i}, and for the debt in common currency of country i, q^i, and an equation of motion for the aggregate capital stock, K'^i, for all $i = 1, \ldots, I$, such that:

(i) given B'^1, ..., B'^I, D'^i, q^{*i}, g^i, z^i and ϑ^u, V_{c^i} is the value function to solve the problem of the representative consumer of country i, problem (2), and c^i, k'^i and d'^i are the optimal choices of this consumer;

(ii) given B'^1, ..., B'^I, D'^1, ..., D'^I, q^{*1}, ..., q^{*I}, q^1, ..., q^I, z^1, ..., z^I and ϑ^u, V_b is the value function to solve the problem of the representative international banker, problem(3), and x and b'^1, ..., b'^I are the optimal choices;

(iii) given q^{*i}, q^i, g^i, z^i, ϑ^u, c^i and K'^i , V_{g^i} is the value function to solve the problem of the national government of country i , problem (4), and B'^i and D'^i are its optimal choices. Furthermore, given c^i, K'^i, V_{g^i}, B'^i, D'^i and ϑ^u, the functions g^i and z^i are the solutions that maximize problem (5);

(iv) given B'^i, D'^i and ϑ^i for all i, along with the weights λ_i, the central government chooses ϑ^u;

(v) $B'^i(s^i) \in b'^i(B^i, s^i, B'^i, D'^i)$;

(vi) $D'^i(s^i) = d'^i(D^i, s^i, B'^1, \ldots, B'^I, D'^i)$;

(vii) $K'^i(s^i,\, B'^1,\, \ldots,\, B'^I,\, D'^i,\, q^{*1},\, \ldots,\, q^{*I},\, g^i,\, z^i,\, \vartheta^u) = k'^i(K^i,\, D^i,\, s^i,\, B'^1,\, \ldots,\, B'^I,\, D'^i,\, q^{*1},\, \ldots,\, q^{*I},\, g^i,\, z^i,\, \vartheta^u)$.

3.3 Characterization of the equilibrium

Depending on the realization of the sunspot variables ζ^i, η^i and η^u, dollar debt crises for country i can occur with positive probability. In constructing an equilibrium, we assume an uniform distribution in the interval $[0, 1]$ for ζ^i, η^i and η^u. Nevertheless, these three variables are not independent.[5] We suppose this property applies only to ζ^i and η^u and ζ^i and η^i. In this fashion, the probability that external lenders lose confidence in the government of country i and that the central government is willing to resort to inflation to avoid a drop in output corresponds to the product $P(\zeta^i \leq \pi^i) \cdot P(\eta^u \leq \xi^u)$, which is equal to $\pi^i \xi^u$.

The crisis zone of probability $\pi^i(1 - \xi^u)$ for the dollar debt of country i is denoted by the interval $(\overline{b^i}(k^{ni},\, D^i),\, \overline{B^i}(k^{\pi^i\xi^u},\, D^i,\, \pi^i,\, \xi^u)]$.[6] Now suppose that a group of member countries, the sum of whose votes is equal to at least two-thirds of the total, have dollar debts in their respective crisis zones. Then, there is a positive probability ξ^u that the central government will partial default the common-currency debt. For any country i, given the initial state $s^i = (k^{\pi^i\xi^u},\, B^i,\, D^i,\, a^i_{-1},\, \vartheta^u_{-1},\, \zeta^i,\, \eta^i,\, \eta^u)$, with $a^i_{-1} = 1$ and $\vartheta^u_{-1} = 1$, there are eight possible outcomes, depending on the realization of the sunspot variables. These possible situations can be described by a tree diagram that indicates a partition of the sample space Ω, where $\Omega = \{(\zeta^i,\, \eta^i,\, \eta^u)\colon 0 \leq \zeta^i \leq 1,\, 0 \leq \eta^i \leq 1,\, 0 \leq \eta^u \leq 1\}$. We assume π^i, ξ^i and ξ^u are real numbers

[5] In the model with local currency, we suppose that ζ and η are independent and identically distributed with an uniform distribution function in the interval $[0, 1]$.

[6] The capital stocks k^{ni} and $k^{\pi^i\xi^u}$ are chosen when there is a probability of a dollar-debt crisis of, respectively, zero and $\pi^i\,(1 - \xi^u)$ in the following period.

in the interval [0, 1] and use the notation H for each of the intervals $0 \leq \zeta^i \leq \pi^i$, $0 \leq \eta^i \leq \xi^i$ and $0 \leq \eta^u \leq \xi^u$ and T, for the intervals $\pi^i < \zeta^i \leq 1$, $\xi^i < \eta^i \leq 1$ and $\xi^u < \eta^u \leq 1$. Hence, event B, denoted by (H, H, H), corresponds to the subset of Ω given by $\{(\zeta^i, \eta^i, \eta^u)$: $0 < \zeta^i \leq \pi^i$, $0 < \eta^i \leq \xi^i$, $0 < \eta^u \leq \xi^u)\}$; event C, denoted by (H, T, H) corresponds to $\{(\zeta^i, \eta^i, \eta^u)$: $0 \leq \zeta^i \leq \pi^i$, $\xi^i < \eta^i < 1$, $0 \leq \eta^u \leq \xi^u)\}$; and so on through event I (Figure 1).

Figure 1 is a tree diagram for any country i and initial aggregate state s^i (represented by point A), known at the beginning of period t. The branches of the tree indicate the probabilities that the market participants face before realization of the sunspot variables at time t. The probability of event B is specified by dsi. If this outcome occurs, the national government of country i does not default on the dollar debt and the central monetary authority declares a partial moratorium on the common–currency debt because the central government is revealed to be in favor of stabilizing output, being of the same type as the national government. The probability of outcome C is given by dai. Outcome C is similar to B, except that the shocks that reveal the central and national government types are asymmetric. The shocks are symmetric for events B, E, F and I. The remaining outcomes of Figure 1 correspond to asymmetric shocks.

All the events of Figure 1 are mutually exclusive, and given the assumption of independence between the sunspot variables ζ^i and η^u, the summed probabilities of outcomes B and C correspond to the probability $\pi^i\xi^u$; the sum of the probabilities of outcomes D and E to $\pi^i(1 - \xi^u)$; that of events F and G to $(1 - \pi^i)\xi^u$; and finally, the sum of the probabilities of the final two events equals $(1 - \pi^i)(1 - \xi^u)$. With perfect correlation between the shocks on the types of central and national governments, then the central government is willing to inflate the money anytime the national government is also willing to do so. This is the case of perfect symmetry between the shocks analyzed in this article.

By assumption, once the central government has inflated the common currency, this recourse is thereafter no longer available. In this fashion, if the events B, C,

F or G occur and the dollar debt is in the crisis zone, then there is a probability π^i that a dollar debt crisis will occur in the following period. The crisis zone in this case is denoted by $(\bar{b}(k^{ni}, D^i), \overline{B}(k^{\pi^i}, D^i, \pi^i, \phi^u)]$. The upper bound of this interval indicates that there was a partial moratorium on common-currency debt, given by the term ϕ^u.

In the next step to construct an equilibrium, we describe the optimal behavior of the market participants, supposing that: (i) at least two-thirds of the votes are from member countries with dollar debts in their respective crisis zones; and (ii) there has not been any external debt crisis in any of the countries of the monetary union, nor partial moratoria of common-currency debt, up to the initial state (i.e., $a^i_{-1} = 1$ for all i and $\vartheta^u_{-1} = 1$).

Consumers and International Bankers

The behavior of consumers of country i and of international bankers depends on their expectations regarding whether the national government will default on the dollar debt and the central government will inflate the common currency in the following period.

Consumers of Country i

We assume that consumers know ϑ^u_t, g^i_t and z^i_t when making their decisions as to c^i_t and k^i_{t+1}. Additionally, we suppose that they take these variables as given in deciding on d^i_{t+1}. Given q^{*i}_t, B^i_{t+1} and D^i_{t+1}, g^i_t depends only on the consumer's decision about d^i_{t+1}, which is done at period t and with the choice of q^i_t. In the next period, given B_{t+2}, D_{t+2} and q^*_{t+1}, there are four possible choices for k^i_{t+2} and three for q^i_{t+1}, according to the realization of the sunspot variables ζ^i and η^u and also on the levels of B^i_{t+2}, for $i = 1, \ldots, I$. Hence, fixing k_{t+2} and q^i_{t+1}, the optimization problem at time t corresponds to:

$$\max_{c^i_t, k^i_{t+1}, d^i_{t+1}} \quad c^i_t + \beta E c^i_{t+1}$$

s.t.

$$c_t^i + k_{t+1}^i - k_t^i + q_t^i d_{t+1}^i = \left(1-\theta^i\right)\left[a_t^i\, f\left(k_t^i\right) - \delta^i k_t^i\right] + \vartheta_t^u d_t^i$$

$$c_{t+1}^i + k_{t+2}^i - k_{t+1}^i + q_{t+1}^i d_{t+2}^i = \left(1-\theta^i\right)\left[a_{t+1}^i\, f\left(k_{t+1}^i\right) - \delta^i k_{t+1}^i\right] + \vartheta_{t+1}^u d_{t+1}^i$$

$$c_t^i, c_{t+1}^i, k_{t+1}^i, d_{t+1}^i \geq 0$$

The first-order condition for optimal individual capital accumulation, k_{t+1}^i, is the same as in the original Cole-Kehoe model. The expression for the Cobb-Douglas output function, $f(k) = Ak^{\nu}$, is given by:

$$k_{t+1}^i = \left\{\left[\left(\frac{1}{\beta} - 1\right)\frac{1}{1-\theta^i} + \delta^i\right]\frac{1}{E_t\left[a_{t+1}^i\right]A^i\nu^i}\right\}^{\dfrac{1}{\nu^i - 1}} \tag{6}$$

The optimal capital, k_{t+1}^i, depends on consumers' expectations regarding the productivity of the economy in the following period. The first-order condition with relation to d_{t+1}^i results in the following expression:

$$q_t^i = \beta E_t\left[\vartheta_{t+1}^u\right]$$

The possible values for capital accumulation are: $k^{\pi^i\xi^u}$, k^{π^i}, k^{di} and k^{ni}. If consumers expect a dollar debt crisis to occur with probability $\pi^i(1-\xi^u)$, they are at point A of the tree diagram in Figure 1. They choose capital $k^{\pi^i\xi^u}$, which is obtained by substituting the expression $E_t\left[a_{t+1}\right] = 1 - \pi^i(1-\alpha^i)(1-\xi^u)$ in the first-order condition (6). In addition, national creditors pay $\beta(1-\xi^u+\phi^u\xi^u)$ per common-currency bond. Even with an expectation of symmetric or asymmetric shocks, consumers choose k_{t+1}^i equal to $k^{\pi^i\xi^u}$ and $q_t^i = \beta(1-\xi^u+\phi^u\xi^u)$, because the private sector believes the probability is ξ^u that the central government will inflate the common money in the following period, regardless of whether the shocks on the types of governments are correlated or not. Consumers choose k_{t+1}^i equal to k^{π^i} if the government undertakes a partial default on the common-currency and if they believe that the national government will suspend payment on dollar debts with probability π^i in the following period. In this case, they are in one of the states B, C, F or G of the tree diagram. They choose $k_{t+1}^i = k^{\pi^i}$, given by $E_t\left[a_{t+1}\right] = 1 - \pi^i(1-\alpha^i)$ and pay $\beta\phi^u$ for common-currency bonds no matter what the new dollar debt is. From each of the outcomes B, C, F or G, there are two new branches, representing two possible events: to default or not on dollar debt. Now symmetry of shocks between the national and central governments is no longer important, because there is no longer any chance for the central government to abate the common-currency debt. The third capital stock k^{di}, given by $E_t\left[a_{t+1}\right] = \alpha^i$, is chosen if the government defaults on dollar debt. Besides this, the consumers pay $\beta\phi^u$ per common-currency bond if there was a partial moratorium previously. Otherwise (represented by events D or E), they pay $\beta(1 - \xi^u + \phi^u\xi^u)$. In the period following outcomes D or E, four other events are possible, depending on the realization of the sunspot variables η^i and η^u: inflation of the common money and symmetry of shocks, with probability si; inflation of the common currency and asymmetry of shocks, with probability ai; no inflation and symmetry, sc; and no inflation and asymmetry, ac. The sum of the probabilities si and ai corresponds to ξ^u, and of sc and ac to $(1 - \xi^u)$.

Finally, if consumers are certain that the government will not default on dollar debt in the next period, then they choose k^i_{t+1} equal to k^{ni}, corresponding to $E_t\left[a_{t+1}\right] = 1$, and pay either $\beta\phi^u$ or $\beta(1 - \xi^u + \phi\xi^u)$, depending on whether or not, respectively, the central government has resorted to inflating the common currency.

International Bankers

The optimization problem of international bankers at time t corresponds to:

$$\max_{x_t, b^1_{t+1}, \ldots, b^I_{t+1}} \quad x_t + \beta E_t\left[x_{t+1}\right]$$

s.t.

$$x_t + q^{*1}_t b^1_{t+1} + \ldots + q^{*I}_t b^I_{t+1} = \overline{x} + z^1_t b^1_t + \ldots + z^I_t b^I_t$$

and the first-order condition for b^i_{t+1} is:

$$q^{*i}_t = \beta E_t\left[z^i_{t+1}\right]$$

If international bankers believe that a dollar debt crisis will occur with probability $\pi^i\left(1 - \xi^u\right)$ in the following period, they pay $\beta(1 - \pi^i + \pi^i\xi^u)$ for dollar bonds issued by country i. If the central government has undertaken a partial default in common-currency debt in the current period and external creditors believe that the national government will default the dollar debt with probability π^i in the following period, they pay $\beta\,(1-\pi^i)$ for dollar debt. However, if the government currently defaults on the dollar debt, then international bankers only acquire the bonds of country i if the price is zero. Finally, if external creditors are sure that the government will not resort to a dollar debt moratorium in the following period, they choose q_t^{*i} equal to β.

3.4 The crisis zone

Before describing the behavior of national governments, we define the crisis zone of country i when the common-currency debt is fixed at D^i and the inflation the central government can impose is given by the abatement factor for common-currency debt, ϕ^u.

Crisis zone of probability $\pi^i\,(1-\xi^u)$

The lower bound $\overline{b^i}\,(k^{ni}, D^i, \xi^u)$ of the crisis zone of probability $\pi^i\,(1-\xi^u)$ is the highest dollar debt level, B^i, given k^{ni}, D^i and ξ^u, for which the following restriction is satisfied in equilibrium:

$$V^{n^i\xi^u}\left(s^i, 0, 0, D^i, q^i\right) \geq V^{di}\left(s^i, 0, 0, D^i, q^i\right) \tag{7}$$

where $s^i = (k^{ni},\ B^i,\ D^i,\ a^i_{-1},\ \vartheta^u_{-1},\ \zeta^i,\ \eta^i,\ \eta^u)$ is an initial state in which the national government did not resort to a moratorium on dollar debt ($a^i_{-1}=1$), the central government did not undertake a partial moratorium on common-currency debt ($\vartheta^u_{-1}=1$), and the realization of the sunspot variable η^u matters. The welfare levels $V^{n^i\xi^u}\,(s^i, 0, 0, D^i, q^i)$ and $V^{di}\,(s^i, 0, 0, D^i, q^i)$ refer to the government decision, respectively, not to default and to default on the dollar debt, even if it does not sell new dollar bonds at a positive price at the current period. The second and third positions of the argument of the welfare functions mean that B and q^{*i} are zero. New debt in common currency is sold for q^i equal to $\beta(1-\xi^u+\phi^u\xi^u)$, as long as there is an expectation of inflating the common currency in the next period. Otherwise, it is sold for β.

The upper bound of the crisis zone, $\overline{B^i}(k^{\pi^i \xi^u},\ D^i,\ \pi^i,\ \xi^u)$, is the highest level of dollar debt for which international bankers extend loans to country i, given probability $\pi^i (1 - \xi^u)$ of a dollar debt crisis occurring in the following period. It is calculated as the highest level of dollar debt such that the following restrictions are simultaneously satisfied in equilibrium:

$$V^{\pi^i \xi^u} \left(s^i, B'^i, q^{*i}, D^i, q^i \right) \geq V^{di} \left(s^i, B'^i, q^{*i}, D^i, q^i \right) \tag{8}$$

and

$$V^{\pi^i} \left(s^i, B'^i, q^{*i}, D^i, \beta\phi^u \right) \geq V^{di} \left(s^i, B'^i, q^{*i}, D^i, \beta\phi^u \right) \tag{9}$$

where the initial aggregate state of the condition (8) is now defined by $s^i = (k^{\pi^i \xi^u},\ B^i,\ D^i,\ 1,\ 1,\ \zeta^i,\ \eta^i,\ \eta^u)$, with $a^i_{-1} = 1$, $\vartheta^u_{-1} = 1$, $\zeta^i > \pi^i$, any η^i and $\eta^u > \xi^u$.[7] The welfare levels $V^{\pi^i \xi^u}(s^i,\ B'^i,\ q^{*i},\ D^i,\ q^i)$ and $V^{di}(s^i,\ B'^i,\ q^{*i},\ D^i,\ q^i)$ correspond to the decision of the national government, respectively, not to declare ($z^i = 1$) and to declare ($z^i = 0$) a moratorium on dollar debt, and of the central government not to inflate the common currency ($\vartheta^u = 1$). This condition guarantees that for a given initial aggregate state $s^i = (k^{\pi^i \xi^u},\ B^i,\ D^i,\ 1,\ 1,\ \zeta^i,\ \eta^i,\ \eta^u)$, with $\zeta^i > \pi^i$, any η^i and $\eta^u > \xi^u$, the government of country i repays its dollar debt as long as it manages to sell new dollar debt at price q^{*i} and new common-currency debt at price q^i, accumulating a capital stock of K'^i.

Analogously, restriction (9) determines that the government of country i prefers to honor its dollar obligations rather than resort to a moratorium, given the state $s^i = (k^{\pi^i \xi^u},\ B^i,\ D^i,\ 1,\ 1,\ \zeta^i,\ \eta^i,\ \eta^u)$, with $\zeta^i \leq \pi^i$ and $\eta^u \leq \xi^u$, as long as it is able to sell new dollar debt at q^{*i} and new common-currency debt at $\beta\phi^u$. The welfare levels $V^{\pi^i}(s^i,\ B'^i,\ q^{*i},\ D^i,\ \beta\phi^u)$ and $V^{di}(s^i,\ B'^i,\ q^{*i},\ D^i,\ \beta\phi^u)$ result, respectively, from a decision of the national government not to default and to default on dollar debt

[7] The realization of the sunspot variable η^i may take any value, because we are not making assumptions about the correlation between η^i and η^u. This correlation is irrelevant to national governments and international bankers, when they are taking decisions about the level of dollar debt.

after the central government undertakes a partial moratorium on common-currency debt ($\vartheta^u = \phi^u$).

There is a third condition that must be satisfied to obtain the upper bound of the crisis zone. It is identical to condition (9), except that the realization of the sunspot variables are $\zeta^i > \pi^i$ and $\eta^u \leq \xi^u$. Given these outcomes, the expectation of national and international creditors is that the national government will not default on the dollar debt and the central government will inflate the common currency ($z^i = 1$ and $\vartheta^u = \phi^u$). Based on this belief, they buy dollar debt for $\beta(1 - \pi^i)$ and common-currency debt for $\beta\phi^u$. The optimal new dollar debt also has to satisfy the condition that the welfare of not defaulting is greater than defaulting. This is the cost for country i to join a monetary union. Unfavorable realizations of the sunspot variables ζ^i for the majority of countries can lead the central government to resort to a partial moratorium of common-currency debt even if the external sector of country i is not in any difficulty.

Crisis zone of probability π^i

It is possible to define a crisis zone of probability π^i, denoted by $(\overline{b^i}(k^{ni}, D^i, \phi^u)$, $\overline{B^i}(k^{\pi^{\text{i}}}, D^i, \pi^i, \phi^u)]$. The lower bound is similar to that calculated by condition (7), assuming there was abatement of common-currency debt. Calculation of the upper bound considers, besides this assumption, that the national government prefers not to default on dollar debt, given that it sells B'^i at positive price q^{*i} and D^i at price $\beta\phi^u$. We thus obtain $\overline{B^i}(k^{\pi^{\text{i}}}, D^i, \pi^i, \phi^u)$ as the highest level of B^i, given the initial aggregate state $s^i = (k^{\pi^{\text{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot)$, with $a^i_{-1} = 1$, $\vartheta^u_{-1} = \phi^u$, $\zeta^i > \pi^i$, and any η^i and η^u, that satisfy the following restriction:

$$V^{\pi^{\text{i}}}\left(s^i, B'^i, q^{*i}, D^i, \beta\phi^u\right) \geq V^d\left(s^i, B'^i, q^{*i}, D^i, \beta\phi^u\right) \tag{10}$$

where $V^{\pi^{\text{i}}}(s^i, B'^i, q^{*i}, D^i, \beta\phi^u)$ and $V^d(s^i, B'^i, q^{*i}, D^i, \beta\phi^u)$ refer to the welfare levels of national government i when it decides, respectively, not to default and to default on dollar debt.

A dollar-debt crisis can also occur with probability π^i in country i, given the initial state $s^i = (k^{\pi^i}, B^i, D^i, 1, 1, \zeta^i, \eta^i, \eta^u)$, as long as creditors are sure the central government will not inflate the common currency in the following period. In this case, $P(\eta^u \leq \xi^u) = 0$ and it turns out that ξ^u is zero. The probability that the central government will inflate the common currency is zero when more than two-thirds of the votes come from countries that have no dollar debts in the crisis zone. In this case, the crisis zone of probability π^i corresponds to the interval $(\overline{b^i}(k^{ni}, D^i), \overline{B^i}(k^{\pi^i}, D^i, \pi^i)]$ and the procedure to obtain its extreme values is the same as above.

3.5 Optimal decisions of national government i

Following the same procedure as Cole and Kehoe [6], we obtain the optimal behavior of the national government when its dollar debt, B_t^i, is in the no-crisis zone and in the crisis zone. We suppose that D^i is fixed and ϕ^u is known.

Dollar debt in the no-crisis zone

To characterize the optimal government policy in the no-crisis zone, we solve the following problem for the national government of country i. In this problem, the government chooses g_t^i, g_{t+1}^i and B_{t+1}^i, in order to be sure not to have a dollar-debt crisis in the following period, given $B_t^i \leq \overline{b^i}(k^{ni}, D^i, \xi^u)$, $K_t^i = k^{ni}$, K_{t+1}^i and B_{t+2}^i.

$$\max_{B_{t+1}^i} \beta^t v\left(g_t^i\right) + \beta^{t+1} E v\left(g_{t+1}^i\right)$$

s.t.

$$g_t^i = \theta^i y^{ni} + \beta B_{t+1}^i - B_t^i - \left[1 - \beta\left(1 - \xi^u + \phi^u \xi^u\right)\right] D^i$$

$$g_{t+1}^i = \theta^i y^{ni} + \beta B_{t+2}^i - B_{t+1}^i - \left(\vartheta_{t+1}^u - q_{t+1}^i\right) D^i$$

$$g_t^i, g_{t+1}^i > 0$$

$$y^{ni} = f(k^{ni}) - \delta^i k^{ni}$$

The expectation refers to the possibility of a partial default on common-currency debt in the following period. In a monetary union, although dollar debt of $B_t^i \leq$

$\overline{b^i}(k^{ni},\ D^i,\ \xi^u)$ does not cause external creditors to lose confidence regarding its repayment, there is a probability of ξ^u that the central government will inflate the common currency if this alternative has not already been used.

If $\xi^u = 0$ in all periods, the solution is equal to that of the model with local currency. In this case, for dollar debt in the crisis zone, $q^{*i} = \beta$, $q^i = \beta$, $z = 1$, $K'^i = k^{ni}$ and $\vartheta^u = 1$, in every period. Also, the first-order condition for the government's problem regarding B^i_{t+1} results in $v'(g^i_t) = v'(g^i_{t+1})$ and the optimal behavior of national government i consists of holding its current consumption steady, $g^i_t = g^i_{t+1}$. Hence, if at the start the dollar debt is $B^i_0 \leq \overline{b^i}(k^{ni},\ D^i,\ \xi^u)$, then the optimal new dollar debt is to maintain the same level B^i_0.

On the other hand, if $\xi^u > 0$, the first-order condition is equal to:

$$v'\left(g^i_t\right) = (1-\xi^u)\, v'\left(g^{ni}_{t+1} \mid \eta^u > \xi^u\right) + \xi^u v'\left(g^{dpi}_{t+1} \mid \eta^u \leq \xi^u\right)$$

$$g^i_t = \theta^i y^{ni} - B^i_t + \beta B^i_{t+1} - D^i + \beta\left(1-\xi^u + \phi^u\xi^u\right) D^i$$

$$g^{ni}_{t+1} = \theta^i y^{ni} - B^i_{t+1} + \beta B^i_{t+2} - D^i + \beta\left(1-\xi^u + \phi^u\xi^u\right) D^i$$

$$g^{dpi}_{t+1} = \theta^i y^{ni} - B^i_{t+1} + \beta B^i_{t+2} - \phi^u\left(1-\beta\right) D^i$$

where g^{ni}_{t+1} e g^{dpi}_{t+1} are the national government consumption levels, when the central government decides, respectively, not to partial default ($\vartheta^u_{t+1} = 1$) and to partial default ($\vartheta^u_{t+1} = \phi^u$), the common-currency debt. Now the previous optimal decision to hold government consumption constant in all periods may no longer be the best decision.

Dollar debt in the crisis zone

If the dollar debt is in the crisis zone, the decision regarding B^i_{t+1} is that which provides the highest welfare from the perspective of the national government among the following options: resorting to a moratorium; lowering the debt to $\overline{b}(k^{ni},\ D^i,\ \xi^u)$ in T periods if no crisis occurs; or never lowering it. To characterize this optimal policy, suppose that (8) and (9) are not active and that there is no default, neither on the dollar debt, nor on the common-currency debt currently ($z_t = 1$ and $\vartheta_t =$

1). Moreover, assume that there is probability $\pi^i(1-\xi^u)$ of a default on the dollar debt in the following period and in all other periods in which there was no default on both debts. Under these hypothesis, the first-order condition for the national government problem is

$$v'\left(g_t^i\right)\left(1-\pi^i+\pi^i\xi^u\right)=\left(1-\pi^i\right)\left(1-\xi^u\right)v'\left(g_{t+1}^{nni}\right)+\xi^u v'\left(g_{t+1}^{ndpi}\right)$$

$$g_t^i=\theta^i y^{\pi^i\xi^u}-B_t^i+\beta\left(1-\pi^i+\pi^i\xi^u\right)B_{t+1}^i-D^i+\beta\left(1-\xi^u+\phi^u\xi^u\right)D^i$$

$$g_{t+1}^{nni}=\theta^i y^{\pi^i\xi^u}-B_{t+1}^i+\beta\left(1-\pi^i+\pi^i\xi^u\right)B_{t+2}-D^i+\beta\left(1-\xi^u+\phi^u\xi^u\right)D^i$$

$$g_{t+1}^{ndpi}=\theta^i y^{\pi^i\xi^u}-B_{t+1}^i+\beta\left(1-\pi^i\right)B_{t+2}-\phi^u\left(1-\beta\right)D^i$$

$$y^{\pi^i\xi^u}=f\left(k^{\pi^i\xi^i}\right)-\delta^i k^{\pi^i\xi^u}$$

where g_{t+1}^{nni} and g_{t+1}^{ndpi} are the government consumption levels, when the central government decides, respectively, not to partial default and to partial default on the common-currency debt, given that the national government i did not default on the dollar debt.

This condition does not result in constant government consumption. The analytic expression for the expected welfare is more complex than that obtained in the original Cole-Kehoe model, since we are considering the possibility of a partial default on the common-currency debt. These same conclusions apply to the model with local currency when the dollar debt is in the crisis zone. The optimal solution for new dollar debt, given its current level, is obtained in numerical form as shown in Figure 10.

3.6 Welfare for the national government

The main objective of our work is to describe the expected welfare of the government of country i belonging to a monetary union and subject to a possible crisis of its dollar debt. We compare this result with the expected welfare given by the original Cole-Kehoe model and also to a model with local currency. In assessing the welfare of a country in a monetary union, we consider the possibility of a union central bank subject to political influences and also the presence of imperfect correlation between the national and central government types.

3.6.1 Welfare when union central bank is free of political influences

- *Dollar debt in the no-crisis zone and possibility of inflation*

For dollar debt levels in the no-crisis zone, external creditors know that the national government always prefers to pay back its debts, no matter what the realization of the sunspot variables ζ^i, η^i and η^u are. Given the initial state $s^i = (k^{ni}, B^i, D^i, a^i_{-1}, \vartheta^u_{-1}, \zeta^i, \eta^i, \eta^u)$, with $B^i \leq b(k^{ni}, D^i, \xi^u)$, D^i fixed, $a^i_{-1} = 1$ and $\vartheta^u_{-1} = 1$, the expected welfare for country i before realization of the sunspots depends on the assumption we make regarding the symmetry of shocks between the types of national and central governments. The specification of this welfare also depends on the optimal choices of the government for new dollar debt. As seen in the previous section, there is no confirmation that stationary dollar debt is the optimal choice when there is a positive probability that the central government will inflate the common currency. Nevertheless, we suppose that the government follows a stationary dollar debt policy and we describe the welfare thus provided, in accordance with the beliefs of the private sector regarding the correlation of shocks between the national and central government types, in the following manner[8]

(i) *perfect correlation*

$$V^{n^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^i\right) = \tag{11}$$

$$\frac{1}{1-\beta\left(1-\xi^u\right)}\left[\left(1-\xi^u\right)\cdot u^{nndpB}\left(k^{ni},\xi^u\right)+\frac{\xi^u}{\left(1-\beta\right)}u^{ndpB}\left(k^{ni},\phi^u\right)\right]$$

$$u^{ndpB}\left(k^{ni},\phi^u\right) = \tag{12}$$

$$\varrho^i\left[\left(1-\theta^i\right)y^{ni}+\phi^u\left(1-\beta\right)D^i\right]+$$

$$v\left[\theta^i y^{ni}-\left(1-\beta\right)B^i-\phi^u\left(1-\beta\right)D^i\right]$$

[8]Stationary debt policy means that the government dollar debt stays constant at the initial level. We do not maintain the assumption of stationarity in the numerical exercisies. We procced in this way only here to simplify the analytical expressions.

$$u^{nndpB}\left(k^{ni},\xi^{u}\right) = \tag{13}$$

$$\varrho^{i}\left\{\left(1-\theta^{i}\right)y^{ni}+\left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right]D^{i}\right\}+$$

$$v\left\{\theta^{i}y^{ni}-\left(1-\beta\right)B^{i}-\left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right]D^{i}\right\}$$

where $u^{ndpB}\left(k^{ni},\phi^{u}\right)$ and $u^{nndpB}\left(k^{ni},\xi^{u}\right)$ are the instantaneous utility when the central government decides, respectively, not to partial default and to partial default the common-currency debt, and the national government does not default on the dollar debt.

(ii) *imperfect correlation, with η^{i} and η^{u} symmetric at the initial period*

Now welfare depends on the joint probabilities of η^{i} and η^{u}, specified in Table 4. Given an expectation for the symmetry of these shocks at the initial period, the expected welfare equals

$$Vsy^{n^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^{i}\right) =$$

$$sc\cdot\left[u^{nndpB}\left(k^{ni},\xi^{u}\right)+\beta\cdot V^{n^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^{i}\right)\right]+$$

$$si\cdot\frac{1}{\left(1-\beta\right)}u^{ndpB}\left(k^{ni},\phi^{u}\right)$$

(iii) *imperfect correlation, with η^{i} and η^{u} symmetric at all times*

Supposing the expectation is that at every moment the realization of the sunspot variables indicate that the national and the central government are of the same type, the expected welfare is

$$Vsy^{sc}\left(s^{i}\right) = \tag{14}$$

$$\frac{1}{1-\beta\cdot sc}\left[sc\cdot u^{nndpB}\left(k^{ni},\xi^{u}\right)+\frac{si}{\left(1-\beta\right)}u^{ndpB}\left(k^{ni},\phi^{u}\right)\right]$$

In the case of asymmetric shocks, the expected welfare is analogous, except that the probabilities of the events sc and si are substituted by ac and ai.

- *Dollar debt in the crisis zone*

When dollar debt is in the crisis zone of probability $\pi^i(1-\xi^u)$, realization of the sunspot variable ζ^i, along with η^i and η^u, has bearing. The joint probabilities of shocks on the types of government and on the confidence of external lenders are shown in Table 5. Given initial state $s^i = (k^{\pi^i\xi^u},\ B^i,\ D^i,\ a^i_{-1},\ \vartheta^u_{-1},\ \zeta^i,\ \eta^i,\ \eta^u)$, with $B^i > \overline{b}(k^{ni},\ D^i,\ \xi^u)$, D^i fixed, $a^i_{-1}=1$ and $\vartheta^u_{-1}=1$, the expected welfare for country i, according to the correlation of shocks regarding the type of national and central governments, is given by:

(i) *perfect correlation*

$$V^{\pi^i\xi^u}\left(s^i\right) = \tag{15}$$

$$\left(1-\pi^i\right)\left(1-\xi^u\right)V^{\pi^i\xi^u}\left(s^i, B^i, q^{*i}, D^i, q^i\right) +$$
$$\xi^u\left[u^{\pi^i}\left(k^{\pi^i\xi^u}, \pi^i, \phi^u\right) + \beta V^{\pi^i}\left(k^{\pi^i}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot\right)\right] +$$
$$\pi^i\left(1-\xi^u\right)\left[u^{di}\left(k^{\pi^i\xi^u}, \xi^u\right) + \beta V^{di}\left(k^{di}, \cdot, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)\right]$$

where $V^{\pi^i\xi^u}(s^i,\ B^i,\ q^{*i},\ D^i,\ q^i)$, $u^{\pi^i}(k^{\pi^i\xi^u}, \pi^i, \phi^u)$, $V^{\pi^i}(k^{\pi^i},\ B^i,\ D^i,\ 1,\ \phi^u,\ \zeta^i,\ \cdot,\ \cdot)$, $u^{di}(k^{\pi^i\xi^u},\ \phi^u)$ and $V^{di}(k^{di},\ \cdot,\ D^i,\ \alpha^i,\ 1,\ \cdot,\ \cdot,\ \eta^u)$ are specified in Appendix B, respectively, by the expressions (30), (23), (24), (31) and (22).

(ii) *imperfect correlation, with η^i and η^u possibly symmetric at the initial period*

The expected welfare depends on the joint probabilities of the sunspot variables, shown in Table 5, in Appendix B.

$$Vsy^{\pi^i\xi^u}\left(s^i\right) =$$
$$nsc \cdot V^{\pi^i\xi^u}\left(s^i, B^i, q^{*i}, D^i, q^i\right) +$$
$$(nsi + dsi)\left[u^{\pi^i}\left(k^{\pi^i\xi^u}, \pi^i, \phi^u\right) + \beta V^{\pi^i}\left(k^{\pi^i}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot\right)\right]$$
$$dsc \cdot \left[u^{di}\left(k^{\pi^i\xi^u}, \xi^u\right) + \beta V^{di}\left(k^{di}, \cdot, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)\right]$$

(iii) *imperfect correlation, with η^i and η^u symmetric at all times*

For η^i and η^u symmetric at all times, we calculate $Vsy^{nsc}(s^i)$ instead of $Vsy^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}(s^i)$.

$$Vsy^{nsc}\left(s^i\right) = \tag{16}$$
$$\frac{1}{1-\beta\cdot nsc}\left\{nsc\cdot u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}},\pi^i,\xi^u\right)+\right.$$
$$(dsi+nsi)\left[u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}},\pi^i,\phi^u\right)+\beta V^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}},B^i,D^i,1,\phi^u,\zeta^i,\cdot,\cdot\right)\right]+$$
$$\left.dsc\cdot\left[u^{di}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}},\xi^u\right)+\beta V^{di}\left(k^{di},\cdot,D^i,\alpha^i,1,\cdot,\cdot,\eta^u\right)\right]\right\}$$

In the case of asymmetric shocks, we replace nsc, nsi, dsi and dsc with nac, nai, dai and dac, respectively.

3.6.2 Welfare when union central bank is subject to political pressures

In the model with local currency, the government can exert political pressure on its central bank in order to obtain additional budgetary resources. This possibility is not considered by the private sector in its optimization problem. Hence, national consumers and international bankers are surprised by a decision of the central bank to reduce the real value of its local-currency debt without an external crisis. Nevertheless, for a given initial aggregate state and before realization of the sunspot variables, the private sector attributes a parameter ψ^i to describe its beliefs regarding the independence of the central bank, with ψ^i varying in the interval $[0,\ 1/\xi^i]$. If $\psi^i = 0$, the central bank is free of political pressures (strong), and at the other extreme, if $\psi^i = 1/\xi^i$, the central bank is highly subject to political interference (weak).

In the monetary union model we also allow a situation like the one described above, in which the central government can inflate the common currency even if the majority of votes are not from countries facing external debt crises. Instead, this decision results from pressure by some national governments for the purpose of raising extra revenue through an inflation tax. In particular, suppose that all the union's countries have dollar debt levels in their respective crisis zones of probability $\pi^i(1 - \xi^u)$. Suppose further that more than two-thirds of the votes come from countries whose sunspot variable realization corresponds to $\zeta^i > \pi^i$. If the union central bank is strong, its optimal choice is $\vartheta^u = 1$. If it decides on $\vartheta^u = \phi^u$, then it is beholden to the interests of member countries that want inflation despite the absence of any external crisis.

The private sector and the national government of each country are not aware of when the central bank will act in this manner, but attribute a parameter ψ^u, analogous to ψ^i, varying in the interval $[0,\ 1/\xi^u]$. If they believe the central bank is independent of pressures by national governments, then $\psi^u = 0$. Otherwise, $0 < \psi^u \leq 1/\xi^u$.

The member countries will be affected by an arbitrary decision of a weak central bank, whose effect on their welfare depends on the initial state of their economies. In the monetary union model, surprise inflation can occur even if country i is under a speculative attack, unlike in the local currency model, in which a weak central bank only inflates the local currency when the country is not in a crisis. Therefore, we consider the following cases:

Dollar debt in the no-crisis zone

Given the initial state $s^i = (k^{n^i}, B^i, D^i, a^i_{-1}, \vartheta^u_{-1}, \cdot, \eta^i, \eta^u)$, with $a^i_{-1} = 1$, $\vartheta^u_{-1} = 1$, any ζ^i and $\eta^u > \xi^u$. In accordance with the realization of the sunspot variables, domestic creditors do not believe that the government will abate its debt in common currency and thus pay $\beta(1 - \xi^u + \xi^u\phi^u)$ for this debt at the start. However, the central government decides on $\vartheta^u = \phi^u$, surprising consumers. Taking into account a value of ϖ^i not equal to 1, the utility of a member country i at the moment of this surprise decision by the central government is given by:

$$u\left(k^{ni}, \psi^u\right) =$$

$$\varrho^i \left\{\left(1-\theta^i\right) y^{ni} - \left(1-\phi^u\right) D^i + \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} +$$
$$v\left\{\theta^i y^{ni} - \left(1-\beta\right) B^i + \varpi^i \left(1-\phi^u\right) D^i - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\}$$

After a decision to inflate the common currency and given that there has been no dollar-debt moratorium, the state of the economy is $s^i = (k^{ni},\ B^i,\ D^i,\ 1,\ \phi^u,\ \cdot,\ \cdot,\ \cdot)$ and welfare from then on corresponds to:

$$\frac{1}{1-\beta} u^{ndpB}\left(k^{ni}, \phi^u\right)$$

where $u^{ndpB}(k^{ni}, \phi^u)$ is given by expression (12). Therefore, given initial state $s^i = (k^{n^i},\ B^i,\ D^i,\ 1,\ 1,\ \cdot,\ \eta^i,\ \eta^u)$ and a weak union central bank, the expected welfare for country i, according to the correlation of shocks regarding the type of national and central governments, is given by:

(i) *perfect correlation*

$$\begin{gathered} V^{n^i\xi^u}\left(s^i, \psi^u\right) = \\ \frac{\xi^u}{1-\beta} u^{ndpB}\left(k^{ni}, \phi^u\right) + \\ \left(1-\xi^u\right) \cdot \left(1-\psi^u \xi^u\right)\left[u^{nndpB}\left(k^{ni}, \xi^u\right) + \beta V s y^{sc}\left(s^i\right)\right] + \\ \left(1-\xi^u\right) \cdot \psi^u \xi^u\left[u\left(k^{ni}, \psi^u\right) + \frac{\beta}{1-\beta} u^{ndpB}\left(k^{ni}, \phi^u\right)\right] \end{gathered}$$

(ii) *imperfect correlation, with η^i and η^u possibly symmetric at the initial period*

$$\begin{gathered} V s y^{n^i\xi^u}\left(s^i, \psi^u\right) = \\ \frac{si}{1-\beta} u^{ndpB}\left(k^{ni}, \phi^u\right) + \\ sc \cdot \left(1-\psi^u \xi^u\right)\left[u^{nndpB}\left(k^{ni}, \xi^u\right) + \beta V^{n^i\xi^u}\left(s^i\right)\right] + \\ sc \cdot \psi^u \xi^u\left[u\left(k^{ni}, \psi^u\right) + \frac{\beta}{1-\beta} u^{ndpB}\left(k^{ni}, \phi^u\right)\right] \end{gathered}$$

where $u^{ndpB}(k^{ni},\ \phi^u)$, $u^{nndpB}(k^{ni},\ \xi^u)$ and $V^{n^i\xi^u}\left(s^i\right)$ correspond to expression (12), (13) and (11).

(iii) *imperfect correlation, with η^i and η^u symmetric at all times*

$$V s y^{n^i\xi^u}\left(s^i, \psi^u\right) =$$

$$\frac{si}{1-\beta}u^{ndpB}\left(k^{ni},\phi^{u}\right)+$$
$$sc\cdot\left(1-\psi^{u}\xi^{u}\right)\left[u^{nndpB}\left(k^{ni},\xi^{u}\right)+\beta Vsy^{sc}\left(s^{i}\right)\right]+$$
$$sc\cdot\psi^{u}\xi^{u}\left[u\left(k^{ni},\psi^{u}\right)+\frac{\beta}{1-\beta}u^{ndpB}\left(k^{ni},\phi^{u}\right)\right]$$

where $Vsy^{sc}\left(s^{i}\right)$ corresponds to expression (14).

If there are asymmetric shocks, we replace sc and si with ac and ai.

Dollar debt in the crisis zone

1. Initial state $s^{i}=(k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}},\ B^{i},\ D^{i},\ a_{-1}^{i},\ \vartheta_{-1}^{u},\ \zeta^{i},\ \eta^{i},\ \eta^{u})$, with $a_{-1}^{i}=1$, $\vartheta_{-1}^{u}=1$, $\zeta^{i}\leq\pi^{i}$ and $\eta^{u}>\xi^{u}$

Given the realization of the sunspot variables, international bankers pay zero for the dollar bonds of country i and consumers pay $\beta(1-\xi^{u}+\xi^{u}\phi^{u})$ for common currency debt. However, after these choices, the union central bank decides to inflate the common currency ($\vartheta^{u}=\phi^{u}$). Each government receives revenue of $\varpi^{i}\left(1-\phi^{u}\right)D^{i}$ and since external creditors refuse to extend new loans, each government has two choices: (i) to use the inflation tax to repay international bankers and avoid a default on the dollar debt; and (ii) not to do so. The optimal choice is that providing the greatest welfare.

In the first case, consumers choose capital $k_{t+1}=k^{ni}$, because international bankers do not buy dollar debt ($B^{i}=0$). Hence, the government utility at the start is given by:

$$u^{n}\left(k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}},\psi^{u}\right)=$$
$$\varrho^{i}\left\{\left(1-\theta^{i}\right)y^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}-k^{ni}+k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}-\left(1-\phi^{u}\right)D^{i}+\left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right]D^{i}\right\}+$$
$$v\left\{\theta^{i}y^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}-B^{i}+\varpi^{i}\left(1-\phi^{u}\right)D^{i}-\left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right]D^{i}\right\}$$

The aggregate state in the next period is $s^{i}=(k^{ni},\ 0,\ D^{i},\ 1,\ \phi^{u},\ \cdot,\ \cdot,\ \cdot)$, with $B^{i}=0$, $a_{-1}^{i}=1$, $\vartheta_{-1}^{u}=\phi^{u}$ and any ζ^{i}, η^{i} and η^{u}. The optimal choice of the government is to follow a stationary debt policy, with $B^{i\prime}=B^{i}=0$ and consumers pay $\beta\phi^{u}$ for common-currency debt and choose k^{ni}. Accordingly, the welfare from this instant on is

$$\frac{1}{1-\beta} u^{ndp}\left(k^{ni}, \phi^{u}\right)$$

where $u^{ndp}\left(k^{ni}, \phi^{u}\right)$ is given by expression (18) in Appendix A.

In the second case, the national government chooses $z^{i} = 0$ and its utility at the start is:

$$\begin{gathered} u^{d}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \psi^{u}\right) = \\ \varrho^{i}\left\{\left(1-\theta^{i}\right)\left[\alpha^{i} f\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right) - \delta^{i} k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right] - k^{di} + k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right. \\ \left. - \left(1-\phi^{u}\right) D^{i} + \left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right] D^{i}\right\} + \\ v\left\{\theta^{i}\left[\alpha^{i} f\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right) - \delta^{i} k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right]\right. \\ \left. + \varpi^{i}\left(1-\phi^{u}\right) D^{i} - \left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right] D^{i}\right\} \end{gathered}$$

The initial state in the next period is $s^{i} = (k^{di}, 0, D^{i}, \alpha^{i}, \phi^{u}, \cdot, \cdot, \cdot)$, with $B^{i} = 0$, $a^{i}_{-1} = \alpha^{i}$, $\vartheta^{u}_{-1} = \phi^{u}$ and any ζ^{i}, η^{i} and η^{u}. The expected welfare is

$$\frac{1}{1-\beta} u^{ddp}\left(k^{di}, \phi^{u}\right)$$

where $u^{ddp}\left(k^{ni}, \phi^{u}\right)$ is given by expression (21) in Appendix A.

2. Initial state $s^{i} = (k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, B^{i}, D^{i}, a^{i}_{-1}, \vartheta^{u}_{-1}, \zeta^{i}, \eta^{i}, \eta^{u})$, with $a^{i}_{-1} = 1$, $\vartheta^{u}_{-1} = 1$, $\zeta^{i} > \pi^{i}$ and $\eta^{u} > \xi^{u}$:

The realizations of ζ^{i} and η^{u} induce international bankers to pay $\beta(1 - \pi^{i} + \pi^{i}\xi^{u})$ for the dollar debt, and national consumers to buy common-currency debt for $\beta(1 - \xi^{u} + \xi^{u}\phi^{u})$. After these decisions, however, the union central bank declares a partial moratorium on debt in the hands of domestic creditors, leading them to choose k' equal to $k^{\pi^{\mathrm{i}}}$. Hence, at the start

$$\begin{gathered} u\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \psi^{u}\right) = \\ \varrho^{i}\left\{\left(1-\theta^{i}\right) y^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}} - k^{\pi^{\mathrm{i}}} + k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}} - \left(1-\phi^{u}\right) D^{i}\right. \\ \left. + \left[1-\beta\left(1-\xi^{u}+\xi^{u}\phi^{u}\right)\right] D^{i}\right\} + \end{gathered}$$

$$v\left\{\theta^i y^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}} - \left[1-\beta\left(1-\pi^i+\pi^i\xi^u\right)\right]B^i + \right.$$
$$\left.\varpi\left(1-\phi^u\right)D^i - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right]D^i\right\}$$

The initial state in the next period is $s^i = (k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot)$ and the expected welfare based thereon is given by $V^{\pi^{\mathrm{i}}}(k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot)$, in accordance with expression (24).

Finally, we describe the expected welfare of country i when the union central bank is weak and the dollar debt is in the crisis zone. We suppose that if $\zeta^i \leq \pi^i$ in the initial state, the national government prefers to repay its external debt upon receiving its share of inflation tax. Hence, given initial state $s^i = (k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, B^i, D^i, a^i_{-1}, \vartheta^u_{-1}, \zeta^i, \eta^i, \eta^u)$, with $a^i_{-1} = 1$, $\vartheta^u_{-1} = 1$, before realization of the sunspot variables and revelation of the type of union central bank, the expected welfare of the country is as below, depending on the correlation of shocks between the government types (national versus central):

(i) *perfectly correlated*

$$V^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^i, \psi^u\right) =$$
$$\pi^i\left(1-\xi^u\right)\left(1-\psi^u\xi^u\right)\left[u^{di}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \xi^u\right) + \beta V^{di}\left(k^{di}, 0, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)\right] +$$
$$\pi^i\left(1-\xi^u\right)\psi^u\xi^u\left[u^n\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \psi^u\right) + \frac{\beta}{1-\beta}u^{ndp}\left(k^{ni}, \phi^u\right)\right] +$$
$$\xi^u\left[u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \pi^i, \phi^u\right) + \beta V^{\pi^{\mathrm{i}}}(k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot)\right] +$$
$$\left(1-\pi^i\right)\left(1-\xi^u\right)\psi^u\xi^u\left[u\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \psi^u\right) + \beta V^{\pi^{\mathrm{i}}}(k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot)\right]$$
$$\left(1-\pi^i\right)\left(1-\xi^u\right)\left(1-\psi^u\xi^u\right)V^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^i, B^i, q^{*i}, D^i, q^i\right)$$

(ii) *imperfectly correlated, with η^i and η^u possibly symmetric at the start*

$$Vsy^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\left(s^i, \psi^u\right) = \tag{17}$$
$$dsc \cdot \left(1-\psi^u\xi^u\right)\left[u^{di}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \xi^u\right) + \beta V^{di}\left(k^{di}, 0, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)\right] +$$

$$dsc \cdot \psi^u \xi^u \left[u^n \left(k^{\pi^{\mathsf{i}} \xi^{\mathsf{u}}}, \psi^u \right) + \frac{\beta}{1-\beta} u^{ndp} \left(k^{ni}, \phi^u \right) \right] +$$

$$(nsi + dsi) \left[u^{\pi^{\mathsf{i}}} \left(k^{\pi^{\mathsf{i}} \xi^{\mathsf{u}}}, \pi^i, \phi^u \right) + \beta V^{\pi^{\mathsf{i}}} \left(k^{\pi^{\mathsf{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot \right) \right] +$$

$$nsc \cdot \psi^u \xi^u \left[u \left(k^{\pi^{\mathsf{i}} \xi^{\mathsf{u}}}, \psi^u \right) + \beta V^{\pi^{\mathsf{i}}} (k^{\pi^{\mathsf{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot) \right] +$$

$$nsc \cdot (1 - \psi^u \xi^u) V^{\pi^{\mathsf{i}} \xi^{\mathsf{u}}} \left(s^i \right)$$

(iii) *imperfectly correlated, with η^i and η^u possibly symmetric at all times*

The expected welfare is the same as (17), except that $V^{\pi^{\mathsf{i}} \xi^{\mathsf{u}}} (s^i)$ is changed to $Vsy^{nsc} (s^i)$, which is given by (16).

In the case of asymmetric shocks, we replace nsc, nsi, dsi and dsc by, respectively, nac, nai, dai, and dac.

4 A Numerical Exercise

Using the extension of the Cole-Kehoe model for monetary union, we carry out simulations for the recent Brazilian economy, as if Brazil were a member of such a union. Besides this, we compare these results with those of a simulation using the original Cole-Kehoe model (dollarization) and one based on purely local currency. The parameters used in the simulations are supported by suppositions very similar to those of Cole and Kehoe [4].

4.1 The parameters for the Brazilian economy

We have chosen the parameters so that the initial period reproduces the situation of the Brazilian economy between June 1999 and May 2001. This two-year interval equals the average maturity period of Brazilian government debt. We assume that the average maturity of local-currency debt follows the average for debt indexed by the SELIC rate (the basic rate set by the Brazilian Central Bank), while that for dollar debt is the same as for dollar-indexed bonds. Both average maturities are approximately 24 months for the period under study.

We designate values equal to those used by Cole-Kehoe for Mexico for the production function and the coefficient of relative risk-aversion, $risk$. The production function is specified by $f(k) = Ak^{\nu}$, with ν being the capital share, equal to 0.5. The $risk$ parameter corresponds to 1, implying a logarithmic function for the utility of current government consumption. Additionally, Cole and Kehoe supposed that if the government did not pay its debt, Mexican production would fall to 95% of its former level for the post-crisis period. Hence, the coefficient α equals 0.95. We use the same hypothesis for Brazil. The tax rate on net income, θ, corresponds to the total tax burden as a percentage of GDP. We set this parameter at 0.30 for the two-year period, very near official figures of 29.33% for 1998 and 31.67% for 1999.

According to Cole and Kehoe, the probability of default on the dollar debt, π, is based on the yearly yield on short-term, dollar-indexed, Mexican government bonds (*tesobonos*). The expression for this probability corresponds to:

$$\pi = 1 - \left[\left(\frac{1+r^*}{1+r} \right)^{\frac{1}{n}} \right]$$

where r is the yearly average yield of the government dollar-denominated or dollar-indexed bonds of the country that can suspend its payments; r^*, is the yearly average yield of U.S. Treasury bonds; and n, the number of periods in the simulation making up a year. For Brazil, r is the net yield of Brazilian federal government bonds, with values updated (monetarily restated) by the dollar and maturing in two years, sold at public auctions by the Brazilian Central Bank. Using these data for Brazilian and U.S. government bonds with similar characteristics, we estimate π at between 4% and 8% during the year 2000 and the first half of 2001.

Employing the same procedure adopted by Cole-Kehoe, we calculate the discount factor, β, as the yield to maturity of Treasury bonds with two-year maturities, deflated by the inflation expectation as published in *The Economist* for the U.S. Consumer Price Index. Based on these figures, we obtain four estimates for β during the period under analysis, varying between 0.93 and 0.96. The depreciation factor,

δ, depends on the value given to β, under the condition that the capital-output ratio is 3.0. Total factor productivity, A, is considered a scale parameter in the simulation, as is the weight of the utility of private consumption relative to the utility of government consumption, represented by ϱ.

In order to estimate welfare, we specify the government debt in local (or common) currency. Public-sector debt in local (or common) currency, D, relative to GDP was 30% during the months of June 1999 to May 2001. Also, as reference, the government dollar debt, B, for the same period, is equal to 0.20 relative to GDP and corresponds to the public-sector debt held by foreign lenders (less international reserves) plus dollar-indexed debt.

The simulation requires three other parameters. The first is the share of real return of debt denominated in common currency effectively paid, ϑ^u, which is defined as the inverse of the inflation factor. We use the official inflation measured for the Brazilian economy, which totaled 14% for the period May 1999 to June 2001. Hence, ϑ^u equals 0.88.

This factor ϑ^u and its parameterization can be better explained in the following manner. Suppose, as is the case for the numerical exercise, that total public debt equals 50% of GDP, with 20% corresponding to dollar-based debt and the rest to the (local or common) currency debt. If international bankers are unwilling to extend further loans, then to avoid a default on the dollar debt, the national government carries out a fiscal adjustment amounting to approximately 5% of GDP, and the central government decides to pay only a fraction, ϑ^u of the common-currency debt. For the national government to have 15% of GDP available to pay its external debt, what should the factor ϑ^u be?

If the central government pays half of the real value of the common-currency debt ($\vartheta^u = 0.5$), then this analogous to 100% inflation. In particular, suppose that initially the nominal price of each common-currency bond is $\beta \cdot \$1$ and the price of one unit of the good is \$1. The expense incurred by a national creditor is $10 \cdot \beta \cdot \$1$ to acquire 10 common-currency bonds. In the following period, this investment yields the creditor \$10, and if there were no inflation, each consumer could then purchase 10 units of the reference good. However, with inflation of 100%, the nominal result of the investment remains \$10, but the creditor can acquire only five rather than ten units of this good. The central government thus retains the other five units, using them to pay off its foreign debt. Hence, revenue from inflation, in units of the good, corresponds to $(1 - p_{t-1}/p_t) \cdot D$. Given $\vartheta^u = 0.88$, this expression equals $(1 - 0.88) \cdot 0.30$, meaning an inflation tax equivalent to 3.6% of GDP in the period. To obtain receipts equal to 15% of GDP, then, the reduction in common-currency debt should be 50% ($\vartheta^u = 0.5$), corresponding to inflation of 100%.

To proceed with the numerical exercise, the other three parameters needed are ξ, ψ and ϖ. In the case of a weak central bank, ψ chosen is that of the extreme case, $1/\xi$. In the simulations, ξ and ϖ take on arbitrary values.

4.2 Preliminary results

Experimenting with these parameters allowed us to select a set of them to use in the simulations of the three models. In particular, we chose the joint probabilities in such a way that in the monetary union model there is strong symmetry among the national and central government types. For such symmetry, we use *dai*, *dac*, *nai* and *nac* very near zero. Besides this, we assume ξ^i and ξ^u are identical, with both denoted by ξ alone in the simulations, and we suppose that the welfare for imperfect correlation between η^i and η^u is characterized by the expression for possible symmetry at the initial period. Given π and ξ, we arbitrarily calculate the probabilities of ζ^i, η^i and η^u occurring, subject only to the values of the marginal densities. The levels of welfare are estimated, supposing different values for ξ in the set $\{0.1; 0.2; ... ; 0.9\}$. At the same time, we parameterize the influence of the member country on a weak union's central bank, with ϖ both zero and 10. Making ϖ equal to 10 means that the national government of a monetary union collects ten times the revenue extracted through the inflation tax when the central bank imposes surprise inflation. This is doubtless a strong hypothesis, but it allows a better depiction of the effect on welfare.

According to the model, the higher ξ is, the greater the difference between the levels of welfare for strong (independent) and weak (subject to political pressure) central banks in the model with local currency. The expected welfare is greater the higher the chance is of a strong central bank inflating the local money, because the prices of local currency bonds are lower, and if there is no speculative attack on dollar debt, then the financial gains of local consumers are greater. On the other hand, given these lower prices for bonds in local currency, surprise inflation extracts a portion of domestic creditors' gains and also makes it impossible to them realize those high financial gains in the future, since there is no longer the chance to inflate.

In the model with common currency, as ξ rises, so does the weighting on expected welfare referring to the welfare with decision to inflate, for both weak and strong union central banks. Therefore, the effect of ξ is different in the two models (local and common currency) in respect to the expected welfare for distinct types of central banks. These results are obtained by considering that $\varpi = 1$. When ϖ is greater than 1, the effect of surprise inflation makes the expected welfare greater for a member country with strong influence over the union's central bank, and hence there is a distancing between the expected welfare with a strong versus a weak union central bank.

We carry out numerical exercises with different values of the parameters ϖ and ξ and for the joint probabilities. The parameters for the baseline model and the probabilities of the sunspot variables are shown in Tables 1 and 2, in Appendix B.

We modify the function v in relation to the Cole-Kehoe model. In the original model, v is a logarithmic function of the type $a\ln(g) + b$, with a and b equal to 1 and zero respectively. This change is necessary because the combination of the parameters and ϖ equal to 1 generates greater welfare when the central bank is weak than when it is strong. This result arises from the chosen parameterization, which is responsible for producing disutility of public consumption for consumers when the level of public consumption is low. With surprise inflation and $\varpi = 1$, private consumption is subtracted from the inflation tax that brings one-time revenues for the government. At the same time, the lower disutility, caused by increased public spending, more than offsets the effect of the drop in private consumption at the instant the weak central bank carries out surprise inflation. This effect is so intense that it offsets the loss of the alternative of inflating the common currency in subsequent periods. This result is undesirable. Therefore, we parameterize a and b, respectively, at $1/10$ and 1. Besides this, to reduce the weight of private consumption, c, in consumer utility it is necessary to reduce ϱ.

The first exercise refers to a simulation with ξ equal to 0.9 and ϖ zero. In this case, there is a 90% chance that the government will not pay 12% of the real return on its bonds in local or common currency. Only 0.88 of the unit value of the good will be redeemed at maturity, not the full value as would be the case in dollar-denominated bonds. In a monetary union, $\varpi = 0$ means that there are no transfers to the national government of the tax so collected. In an economy with local currency, ϖ is assumed always to equal 1, because the national government destines all the tax for its own use. We compare, for a given ξ, the situation with ϖ zero and also equal to 10 in order to characterize, respectively, a member country whose entire inflation tax revenue is destined to another country and a country that receives transfers of 10 times the inflation tax.

In the figures we present the levels of welfare under uncertainty for the three monetary regimes: dollarization, monetary union and local currency. For the local-currency regime, we conduct variations regarding the type of central bank, strong or weak. For monetary union, besides these variations, we conduct exercises regarding the symmetry between shocks that determines the types of central and national governments, which can be either perfectly correlated or symmetric. The results show that the three levels of welfare are decreasing with the total amount of public debt. Welfare with common currency (denoted by COM) and with local currency (LOCAL) vary only for values of total debt relative to GDP above 0.30, because both models include debt in the money of the country or of the union, which are fixed at this level.

In Figure 2, the greatest welfare corresponds to the local currency regime with a strong central bank (LOCAL,STR), depicted in the graphs by dashed lines. In comparing Graphs I and II, there is no significant difference between welfare levels for monetary union with a strong central bank (COM,STR) and poor central bank (COM,WEA), since the probability ξ is high. Furthermore, Graph I shows that for debt relative to GDP greater than 30%, the country prefers monetary union over dollarization, given the hypothesis of symmetry with ξ equal to 0.9, denoted by $sy1$. In the case of perfectly correlated shocks, welfare referring to common currency, weak central bank and perfect correlation (COM,WEA,P.CO) is much nearer that with local currency (LOCAL,STR) than to the welfare referring to symmetric shocks (COM,WEA,SYM), as can be seen by comparing the distances between the solid and dashed lines in Graphs II and III. Hence, more symmetric shocks bring monetary union closer to an independent monetary regime. We make a final observation in the case of $\xi = 0.9$ and $\varpi = 0$, shown in Graphs III and IV, respectively, of Figure 2, finding that the behavior of the welfare curve for an economy with local currency with a weak central bank slopes downward relative to that for a strong central bank.

Carrying out the simulation with $\xi = 0.9$ and $\varpi = 10$, we obtain the four graphs shown in Figure 3, which are very similar to those of Figure 2. The change in the parameter ϖ from zero to 10 causes little variation in the levels of welfare. However, one can see that the welfare with common currency and perfectly correlated shocks (COM,WEA,P.CO) is slightly greater than that with local currency (LOCAL,WEA) when the central banks are subject to political pressures. This result can be seen in Graph IV of Figure 3. The difference between the two levels of welfare is never greater than 0.13% of the welfare with local currency. Under a regime of monetary union with a weak central bank and $\varpi = 10$, consumers suffer from surprise inflation and the government receives 10 times this value in exchange, while with local currency the extra government revenue is equal to the transfer. This effect is even more evident when the exercise is conducted with $\xi = 0.1$, as shown below.

By altering ξ to 0.1, the expected welfare with dollarization remains unchanged. However, with low ξ the expected welfare with local currency for strong (LOCAL,STR) and weak (LOCAL,WEA) central banks is very near. This effect can be seen by comparing the dashed-line curves in Graphs III and IV of Figure 4, which are practically the same relative to the solid-line curves. Furthermore, with $\xi = 0.1$, one can see in more accentuated form the fall in welfare of a government under the three monetary regimes, for dollar-denominated debt levels in the crisis zone. There is a slight but sudden drop in the welfare at the total debt mark beyond which there is uncertainty regarding repayment of dollar debt. This mark refers to a dollar debt/GDP ratio of 0.4 or to total debt/GDP of 0.7. The case of $\xi = 0.1$, also shows that greater symmetry of shocks also raises the welfare of the economy nearer that of the economy with local currency and a strong central bank.

When we suppose that $\varpi = 10$, the welfare under a regime with common currency and symmetry of shocks moves upward. This movement can be observed by comparing the two solid-line curves in Graphs II of Figures 4 and 5. In the same fashion, one can see by comparing Graphs III of these two figures that the change in ϖ from 0 to 10 causes an upward shift in the expected welfare of a government with common currency and perfectly correlated shocks (COM,WEA,P.CO), which surpasses the expected welfare with local currency and strong central bank (LOCAL,STR). This behavior of the graphs results exclusively from the surprise inflation and its effect on the instantaneous utility of individuals at the initial moment.

In the exercises that follow, we modify the values given to the joint probabilities to characterize the shocks on the central and national types of government with less symmetry than that specified previously above. The new values for the joint probabilities of the sunspot variables are depicted in Table 3 and they are denoted as *sy2*.

In the next four figures (6-9), besides making the shocks slightly less symmetric and ϖ equal to 10, we calculate the expected welfare levels for five different values of ξ. With this new specification for symmetry of shocks, the welfare with common currency and strong central bank is below that for a dollarized regime for any level of total public debt that external creditors are willing to acquire under these regimes, as shown in Figure 6. This result holds for different values of ξ. Moreover, welfare of a government belonging to a monetary union with a strong central bank varies very little with ξ. In this grouping of curves, the uppermost one refers to $\xi = 0.9$, while the lowest one corresponds to $\xi = 0.1$.

A country with local currency and strong central bank shows greater expected welfare than does one with dollarization, as can be seen in Figure 7. There is more significant variation in the expected welfare with local currency for different values of ξ than with common currency. The highest curve refers to $\xi = 0.9$. As ξ gets higher, the curve drops off less drastically. The high probability of the government resorting to an inflation tax to meet foreign debt commitments reduces the uncertainty of a debt crisis and hence the dropoff is lower. Therefore, the lower the probability of the national or central government to inflate, respectively, the local and common currencies, the less attractive these regimes become in terms of expected welfare, given that the respective central banks are strong (immune to political pressures).

Figures 8 and 9 show the results supposing the central bank is weak. In Figure 8, the expected welfare with common currency is not always lower than that for a dollarized economy for all levels of debt under this regime. This result depends on ξ. The curve for $\xi = 0.1$ is the uppermost one and, the lowest, corresponds to $\xi = 0.9$. The lower ξ is, the higher the price consumers will pay for bonds in the common currency, $\beta[1 - \xi(1 - \phi^u)]$, because the private sector sees little risk of devaluing that currency. However, if the central bank surprises domestic creditors by inflating the common currency, the losses for domestic creditors are greater when they believe the risk of devaluation is low (low ξ). This loss for domestic bondholders translates into gains for the government that are leveraged tenfold because $\varpi = 10$. Therefore, the welfare is greater for $\xi = 0.1$ than for $\xi = 0.9$, when the central bank is weak. Figure 8 seeks to show the situations in which the economic parameters can indicate that joining an economic union can be a better solution than dollarization and vice-versa. On the other hand, this conclusion does not extend to the local currency regime.

In Figure 9, even with a weak central bank, the expected welfare of a government with local currency and independent monetary policy surpasses that for a dollarized regime for different values of ξ. The effect of ξ on expected welfare is very slight. One might expect that when the central bank is easily influenced by political pressures, the expected welfare of a dollarized regime would be more attractive, but the result does not show this.

Finally, Figure 10 represents an extension of the Cole-Kehoe exercise to describe the optimal path of dollar debt for a country wishing to leave the crisis zone. At each instant, given a level of current debt in dollars (on the horizontal axis), the curves determine the optimal choice for new dollar debt (on the vertical axis). Each of the graphs in this figure represents one of the monetary regimes under analysis. Dollarization produces a result analogous to that of Cole-Kehoe, only with parameters for the Brazilian economy. In the first and second graphs, corresponding respectively to local currency and dollarization, the optimal government choice is to maintain debt at a constant level for all current dollar debt up to the lower limit of the crisis zone. This lower bound, $\overline{b}(k^n,\ D)$, for the model with local currency equals 0.38 relative to GDP, and for dollarization, $\overline{b}(k^n)$ is 0.40. In this fashion, in an interval of current dollar debt $[0, \overline{b}\,(k^n, D)]$ or $[0, \overline{b}\,(k^n)]$, the curve for the optimal government choice coincides with a 45-degree line in the two monetary regimes. In the lower graph, referring to monetary union, this coincidence does not occur. The greater the probability that the central government will inflate the common currency, the further the optimal-choice curve deviates from the 45-degree line. Consequently, the optimal path does not consist of maintaining the debt constant when the current dollar debt is in the no-crisis zone, where the lower bound, $\overline{b}(k^n,\ D,\ \xi)$, corresponds to 0.35 relative to GDP, given $\xi = 0.9$. For all initial debt in the interval $[0, \overline{b}(k^n,\ D,\ \xi)]$, at each point in time, the optimal choice of the national government is to increase its dollar indebtedness up to $\bar{b}(k^n,\ D,\ \xi)$ and henceforth to maintain this level. Thus, in a monetary union the chance for the central government to inflate the common money encourages incurring dollar debt in the countries without credit risk.

For current dollar debt in the crisis zone, the optimal path consists of choosing a lower debt level than the initial one in each period until the lower bound of the crisis zone is reached. This can be seen for the three monetary regimes. Furthermore, under the local currency regime, the greater the probability of the government's creating inflation tax, the higher the level of dollar debt that external lenders are willing to bear. This result is due to the lower probability of an external debt crisis insofar as the government is more willing to use resources from abating real debt in local currency to honor its international payments. The probability of an external debt crisis is $\pi(1-\xi)$ in the model with local currency, and thus is less for ξ equal to 0.9 than 0.1. As the probability of a debt crisis falls, the optimal-choice curve approaches the 45-degree line, indicating less uncertainty concerning payment of dollar debt.

5 Conclusion

This paper brings into discussion one aspect of the debate about the monetary regimes of dollarization, monetary union and local currency with independent monetary policy. In particular, it develops a framework more suited for economies that are heavily dependent on international lending, like the Latin-American and emerging market economies from Southeast Asia, in their evaluation about the different options of monetary regimes. We have done this with a macroeconomic model that incorporates microfundamentals, rational expectations and dynamic optimization, using as a reference the model developed by Cole and Kehoe.

Cole-Kehoe's procedure to obtain the welfare for an economy subject to speculative attacks on its external debt has served as the starting point to describe an economy under the local currency regime and monetary union. The main ingredients of these new versions are the incorporation of debt in local and common currency, thus allowing a national government or the central government of a monetary union, respectively, to resort to lowering the real return on this debt (owned by domestic consumers), using the revenue so extracted to avoid an external debt crisis whose consequences could be even worse in terms of welfare. Besides these modifications, we also have included a parameter that seeks to distinguish strong and weak central banks for both these regimes. We went a bit further by describing the symmetry of shocks between the national and central types of governments for the monetary union model.

The numerical exercise for Brazil consisted of a test of the proposed models and is not intended to provide a definitive answer to the decision a country should take as to its monetary regime. Nevertheless, the model contributes to a better understanding of some questions involved in the issue of whether or not a country should keep its own currency, given that it is heavily dependent on foreign capital.

The preliminary results indicate, for example, that the regime of independent monetary policy with local currency and a strong central bank dominates, in terms of expected welfare, all the other models except that in which a country exercises strong bargaining power over the central bank of a monetary union and is able to obtain a greater portion of the inflation tax revenue than that extracted from its own consumers. In the absence of such exceptional bargaining power, the simulations reveal that the expected welfare with common currency more closely approaches a situation with local currency as the symmetry increases between the types of national and central governments in their respective decisions regarding debt devaluation. Other results show that for strong national and union central banks (i.e., that only resort to an inflation tax in case of an external crisis), the expected welfare rises as the probability increases that the government will take this alternative to avoid an external debt crisis. This happens because of the reduced uncertainty of a default on the dollar debt. On the other hand, in the case of weak central banks, the less the belief by the private sector that the government will devalue the currency, the greater the expected welfare. Finally, the simulations also indicate that for a given initial debt in dollars in the crisis zone, international bankers extend larger loans to economies with local currency and strong central banks the greater the willingness to inflate the money to avoid a default. This effect is not so evident with a common currency. In this case, what calls attention is the greater incentive governments of the member countries have to increase their new dollar debts, for a given current dollar debt in the no-crisis zone, as stronger the beliefs of the private sector get that the central government is willing to devalue the common currency

The framework we develop in this paper, which constitutes of the original Cole-Kehoe model and the two extensions, should be placed among other models of modern macroeconomics that discuss the issue of whether or not a country should keep its own currency. In this group includes the works of Cooley and Quadrini [7], Schmitt-Grohé and Uribe [21] and Mendoza [16]. They mainly focus on the debate over dollarization and use an approach very similar to the one developed in our paper, which is made up of two parts: a theoretical model and numerical simulations. Future extensions of our work will be aimed at carrying out simulations in which debt in local or common currency is not fixed but instead results from an optimization exercise as is the case for dollar debt. Besides this, from a theoretical standpoint, the welfare under a local currency regime needs to be refined so as to react more strongly to changes in the type of central bank. It would be more representative if dollarization indeed posed more effective competition to the local currency regime with a weak central bank.

References

[1] Alesina, Alberto; and Grilli, Vittorio. "The European Central Bank: Reshaping Monetary Politics in Europe". In: Matthew B. Canzoneri, Vittorio Grilli and Paul Masson (eds.), *Establishing a Central Bank: Issues in Europe and Lessons from the U.S.* Cambridge, England: Cambridge University Press, 1992, p. 42-69.

[2] Araujo, Aloisio P.; Pascoa, Mario R.; and Torres, Juan P. "Collateral

Avoids Ponzi Schemes in Incomplete Markets". *Ensaios Econômicos*, EPGE, n. 419. Rio de Janeiro: Fundação Getúlio Vargas, mai. 2001 (to appear in *Econometrica*)

[3] Chang, Roberto. "Dollarization: A Scorecard". *Economic Review*, Federal Reserve Bank of Atlanta, Third Quarter, 2000. Available at: <http://www.frbatlanta.org/publica/eco-rev/rev_abs/00er/q3/chang.pdf>. Accessed in: Nov. 2001.

[4] Cole, Harold L.; and Kehoe, Timothy J. "A self-fulfilling model of Mexico's 1994-1995 debt crisis". *Journal of International Economics*, 41:309-330, 1996.

[5] ———. "Self-Fulfilling Debt Crises". Minneapolis, Federal Reserve Bank, Research Department, *Staff Report* 211, Jul. 1998.

[6] ———. "Self-fulfilling Debt Crises". *Review of Economic Studies*, London: Blackwell, v. 67, n. 1, p. 91-116, Jan. 2000.

[7] Cooley, Thomas F.; and Quadrini, Vicenzo. "The Costs of Losing Monetary Independence: The Case of Mexico". May 2000. (Paper prepared for the conference *Optimal Monetary Institutions for Mexico*, 1999, Mexico D.F, Instituto Tecnológico Autónomo de México). Available at: <http://www.stern.nyu.edu/~tcooley/WorkingPapers/capmob.pdf >. Accessed in: Apr. 2002.

[8] Cooper, Russel; and Kempf, Hubert. "Dollarization and the Conquest of Hyperinflation in Divided Societies". *FRB of Minneapolis Quarterly Review*, v. 25, n. 3, Summer 2001, p. 3-12.

[9] Dubey, Pr.; Geanakoplos, J.; and Zame, W.R. "Default, Collateral and Derivatives". Yale University, 1995. (mimeo)

[10] Fratzscher, Marcel. "On Currency Crises and Contagion". Institute of International Economics, Working Paper Number 00-9. Washington, Dec. 2000. Available at: <http://www.iie.com/catalog/WP/Wpauthor.htm#fratzscher>

[11] Goldfajn, Ilan; and Olivares, Gino. "Is Adopting Full Dollarization a Solution? Looking at the Evidence". Working text, n. 416, Departamento de Economia, PUC-RIO, Rio de Janeiro, fev. 2000.

[12] Hanke, Steve H.; and Schuler, Kurt. "A Dollarization Blueprint for Argentina". Mar. 1999. Available at: <http://www.cato.org/pubs/fpbriefs/fpb52.pdf>. Accessed in: Nov. 2001

[13] Hernández, Leonardo F.; and Valdés, Rodrigo O. "What Drives Contagion: Trade, Neighborhood, or Financial Links?" *IMF Working Paper* WP/01/29. International Monetary Fund, Mar. 2001.

[14] Kenen, Peter A. "The Theory of Optimum Currency Areas: An Eclectic View". In: R. A. Mundell and A. K. Swoboda (eds.), *Monetary Problems of the International Economy*. Chicago: University of Chicago Press, 1969.

[15] McKinnon, Ronald A. "Optimum Currency Areas". *American Economic Review*, v. 53, n. 4, p. 717-725, Sep. 1963.

[16] Mendoza, Enrique G. "On the Benefits of Dollarization When Stabilization Policy Is Not Credible and Financial Markets are Imperfect". *NBER Working Paper Series*, Working Paper 7824. Cambridge, MA: National Bureau of Economic Research, Aug. 2000.

[17] Morandé, Felipe; and Schmidt-Hebbel, Klaus. "Chile's Peso: Better Than (Just) Living with the Dollar?" Working Paper n. 68, Banco Central de Chile, Mar. 2000.

[18] Mundell, Robert A. "The Theory of Optimum Currency Areas". *American Economic Review*, v. 51, n. 4, p. 657-665, Sep. 1961.

[19] Radelet, Steven; and Sachs, Jeffrey A. “The Onset of the East Asian Financial Crisis”. *NBER Working Paper Series*. Cambridge, MA: National Bureau of Economic Research, n. 6680, Aug. 1998.

[20] Sargent, Thomas J. Comment on “Fiscal Consequences for Mexico of Adopting the Dollar”, Jun. 2000. Available at: <http://www.stanford.edu/~sargent/>.

[21] Schmitt-Grohé, Stephanie; and Uribe, Martín. “Stabilization Policy and the Costs of Dollarization”. (Paper prepared for the conference *Optimal Monetary Institutions for Mexico*, 1999, Mexico D.F, Instituto Tecnológico Autónomo de México). Available at: <http://econweb.rutgers.edu/grohe/research/research.htm>. Accessed in: Dec. 2001. (Published at *Journal of Money, Credit, and Banking*, v. 33, p. 482-509, May 2001.)

[22] Sims, Christopher A. “Fiscal Consequences for Mexico of Adopting the Dollar”. (Paper presented at the conference *Optimal Monetary Institutions for Mexico*, 1999, Mexico D.F, Instituto Tecnológico Autónomo de México). Available at: http://www.princeton.edu/~sims/#Papers. Accessed in: Dec. 2001.

[23] Van Rijckeghem, Caroline; and Weder, Beatrice. “Spillovers Through Banking Centers: A Panel Data Analysis”. *IMF Working Paper* WP/00/88, May 2000.

A Welfare Levels to Construct the Crisis Zone

The description of the levels of welfare to construct the crisis zone of probability $\pi^i(1-\xi^u)$ is carried out supposing that the indebtedness in common currency, D^i, is fixed and that the abatement factor for this debt, ϕ^u, is a known fraction.

For the initial state $s^i = (k^{ni}, B^i, D^i, a^i_{-1}, \vartheta^u_{-1}, \zeta^i, \eta^i, \eta^u)$, with $B^i \leq b(k^{ni}, D^i, \xi^u)$, $a^i_{-1} = 1$, $\vartheta^u_{-1} = 1$, any ζ^i and η^i and $\eta^u > \xi^u$, the levels of welfare on the left and right sides of the condition (7) are, respectively, equal to:

$$V^{n^i\xi^u}\left(s^i, 0, 0, D^i, \beta\left(1-\xi^u+\phi\xi^u\right)\right) =$$

$$\varrho^i \left\{\left(1-\theta^i\right) y^{ni} + \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} +$$
$$v\left\{\theta^i y^{ni} - B^i - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} +$$
$$\frac{\beta}{1-\beta\left(1-\xi^u\right)}\left[\xi^u \frac{1}{1-\beta} u^{ndp}\left(k^{ni},\phi^u\right) + \left(1-\xi^u\right) u^{nndp}\left(k^{ni},\xi^u\right)\right]$$

$$u^{ndp}\left(k^{ni},\phi^u\right) = \varrho^i\left\{\left(1-\theta^i\right) y^{ni} + \phi^u\left(1-\beta\right) D^i\right\} + v\left\{\theta^i y^{ni} - \phi^u\left(1-\beta\right) D^i\right\} \tag{18}$$

$$u^{nndp}\left(k^{ni},\xi^u\right) = \varrho^i\left\{\left(1-\theta^i\right) y^{ni} + \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} + v\left\{\theta^i y^{ni} - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\}$$

where $u^{ndp}\left(k^{ni},\phi^u\right)$ and $u^{nndp}\left(k^{ni},\xi^u\right)$ are equal to expressions (12) and (13) when $B=0$.

$$V^{di\xi^u}\left(s^i,0,0,D^i,\beta\left(1-\xi^u+\phi\xi^u\right)\right) = \tag{19}$$

$$\varrho^i\left\{\left(1-\theta^i\right) y^{di} - k^{di} + k^{ni} + \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} +$$
$$v\left\{\theta^i y^{di} - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} + \beta V^{di}\left(k^{di},0,D^i,\alpha^i,1,\cdot,\cdot,\eta^u\right)$$

$$u^{dndp}\left(k^{di},\xi^u\right) = \tag{20}$$

$$\varrho^i\left\{\left(1-\theta^i\right) y^{di} + \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\} +$$
$$v\left\{\theta^i y^{di} - \left[1-\beta\left(1-\xi^u+\xi^u\phi^u\right)\right] D^i\right\}$$

$$u^{ddp}\left(k^{di},\phi^u\right) = \varrho^i\left[\left(1-\theta^i\right) y^{di} + \phi^u\left(1-\beta\right) D^i\right] + v\left[\theta^i y^{di} - \phi^u\left(1-\beta\right) D^i\right] \tag{21}$$

$$V^{di}\left(k^{di},0,D^i,\alpha^i,1,\cdot,\cdot,\eta^u\right) = \tag{22}$$
$$\frac{1}{1-\beta\left(1-\xi^u\right)}\left[\xi^u \frac{1}{1-\beta} u^{ddp}\left(k^{di},\phi^u\right) + \left(1-\xi^u\right) u^{dndp}\left(k^{di},\xi^u\right)\right]$$

where, $y^{di} = \alpha^i f\left(k^{di}\right) - \delta^i k^{di}$ and $V^{di}(k^{di},\ 0,\ D^i,\ \alpha^i,\ 1,\ \cdot,\ \cdot,\ \eta^u)$ is the expected welfare after a default on the dollar debt and probability ξ^u of a partial default on the common-currency debt.

Since $V^{n^{\mathrm{i}}\xi^{\mathrm{u}}}(s^i,\ 0,\ 0,\ D^i,\ \beta(1-\xi^u+\phi\xi^u))$ is a decreasing function of B^i and $V^{di\xi^{\mathrm{u}}}(s^i,\ 0,\ 0,\ D^i,\ \beta(1-\xi^u+\phi\xi^u))$ does not change with this variable, then the highest dollar debt level, $b(k^{ni}, D^i, \xi^u)$, is obtained when condition (7) is satisfied with equality. Moreover, for $\xi^u = 0$, we have $\overline{b^i}(k^{ni}, D^i, 0) = \overline{b^i}(k^{ni}, D^i)$.

To characterize the conditions for the upper limit of the crisis zone, first we specify the left side of condition (9). Given the initial state $s^i = (k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}},\ B^i,\ D^i,\ a^i_{-1},\ \vartheta^u_{-1},\ \zeta^i,\ \eta^i,\ \eta^u)$, with B^i stationary and inside the crisis zone of probability $\pi^i(1-\xi^u)$, $a^i_{-1}=1$, $\vartheta^u_{-1}=1$, $\zeta^i \leq \pi^i$, $\eta^u \leq \xi^u$, and any η^i, the left side of condition (9) is characterized by:

$$V^{\pi^{\mathrm{i}}}\left(s^i, B^i, q^{*i}, D^i, \beta\phi^u\right) =$$

$$= u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \pi^i, \phi^u\right) + \beta V^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot\right)$$

where,

$$u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}, \pi^i, \phi^u\right) = \tag{23}$$

$$\varrho^i\left\{\left(1-\theta^i\right)\left[f\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right)-\delta^i k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right]-k^{\pi^{\mathrm{i}}}+k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}+\phi^u(1-\beta)D^i\right\}+ \\ v\left\{\theta^i\left[f\left(k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right)-\delta^i k^{\pi^{\mathrm{i}}\xi^{\mathrm{u}}}\right]-\left[1-\beta\left(1-\pi^i\right)\right]B^i-\phi^u(1-\beta)D^i\right\}$$

$$V^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot\right) = \tag{24}$$

$$= \frac{(1-\pi^i)}{1-\beta(1-\pi^i)}u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}}, \phi^u, \pi^i\right) + \frac{\pi^i}{1-\beta(1-\pi^i)}u^{di}\left(k^{\pi^{\mathrm{i}}}, \phi^u\right) + \\ \frac{\beta\pi^i}{(1-\beta)\left[1-\beta(1-\pi^i)\right]}u^{di}\left(k^{di}, \phi^u\right)$$

$$u^{\pi^{\mathrm{i}}}\left(k^{\pi^{\mathrm{i}}}, \pi^i, \phi^u\right) = \tag{25}$$

$$= \varrho^i\left\{\left(1-\theta^i\right)\left[f\left(k^{\pi^{\mathrm{i}}}\right)-\delta^i k^{\pi^{\mathrm{i}}}\right]+\phi^u(1-\beta)D^i\right\}+ \\ v\left\{\theta^i\left[f\left(k^{\pi^{\mathrm{i}}}\right)-\delta^i k^{\pi^{\mathrm{i}}}\right]-\left[1-\beta\left(1-\pi^i\right)\right]B^i-\phi^u(1-\beta)D^i\right\}$$

$$u^{di}\left(k^{\pi^{\mathrm{i}}}, \phi^u\right) =$$

$$= \varrho^i \left\{ \left(1-\theta^i\right) \left[\alpha^i f\left(k^{\pi^{\text{i}}}\right) - \delta^i k^{\pi^{\text{i}}}\right] - k^{di} + k^{\pi^{\text{i}}} + \phi^u \left(1-\beta\right) D^i \right\} +$$
$$v \left\{ \theta^i \left[\alpha^i f\left(k^{\pi^{\text{i}}}\right) - \delta^i k^{\pi^{\text{i}}}\right] - \phi^u \left(1-\beta\right) D^i \right\}$$

$$u^{di}\left(k^{di}, \phi^u\right) = \tag{26}$$
$$= \varrho^i \left[\left(1-\theta^i\right) y^{di} + \phi^u \left(1-\beta\right) D^i\right] + v \left[\theta^i y^{di} - \phi^u \left(1-\beta\right) D^i\right]$$

where $u^{\pi^{\text{i}}}(k^{\pi^{\text{i}}\xi^{\text{u}}},\ \pi^i,\ \phi^u)$ is the instantaneous utility when the central government decides to make a partial default on the common-currency debt; $V^{\pi^{\text{i}}}(k^{\pi^{\text{i}}},\ B^i,\ D^i,\ 1,\ \phi^u,\ \zeta^i,\ \cdot,\ \cdot)$ is the expected welfare, given initial aggregate state $s^i = (k^{\pi^{\text{i}}},\ B^i,\ D^i,\ 1,\ \phi^u,\ \zeta^i,\ \cdot,\ \cdot)$, after a partial default on the common-currency debt and probability π of a dollar debt crisis occurring next period; $u^{\pi^{\text{i}}}(k^{\pi^{\text{i}}},\ \phi^u,\ \pi^i)$, $u^{di}(k^{\pi^{\text{i}}},\ \phi^u)$ and $u^{di}(k^{di},\ \phi^u)$ are the instantaneous utilities when the national government decides, respectively, not to default, to default given $K^i = k^{\pi^{\text{i}}}$ and to default given $K^i = k^{di}$.

The right side of condition (9), $V^{di}(s^i,\ B^i,\ q^{*i}(B^i),\ D^i,\ \beta\phi^u)$, is obtained by supposing that the government of country i has chosen $z^i = 0$ and the central government $\vartheta^u = \phi^u$. As seen in the previous paragraph, given the realizations of ζ^i and η^u, lenders acquire dollar debt at a positive price $\beta(1 - \pi^i)$ and common-currency debt at $\beta\phi^u$. Even though the external creditors renew their loans, the national government defaults. Thus we have that

$$V^{di}\left(s^i, B^i, q^{*i}, D^i, \beta\phi^u\right) = \tag{27}$$
$$= u^{di}\left(k^{\pi^{\text{i}}\xi^{\text{u}}}, \phi^u\right) + \beta V^{di}\left(k^{di}, B^i, D^i, \alpha^i, \phi^u, \cdot, \cdot\right)$$

where,

$$u^{di}\left(k^{\pi^{\text{i}}\xi^{\text{u}}}, \phi^u\right) = \tag{28}$$
$$\varrho^i \left\{ \left(1-\theta^i\right) \left[\alpha^i f\left(k^{\pi^{\text{i}}\xi^{\text{u}}}\right) - \delta^i k^{\pi^{\text{i}}\xi^{\text{u}}}\right] - k^{di} + k^{\pi^{\text{i}}\xi^{\text{u}}} + \phi^u \left(1-\beta\right) D^i \right\} +$$
$$v \left\{ \theta^i \left[\alpha^i f\left(k^{\pi^{\text{i}}\xi^{\text{u}}}\right) - \delta^i k^{\pi^{\text{i}}\xi^{\text{u}}}\right] + \beta\left(1-\pi^i\right) B^i - \phi^u \left(1-\beta\right) D^i \right\}$$

where $u^{di}\left(k^{\pi^{\text{i}}\xi^{\text{u}}}, \phi^u\right)$ is the instantaneous utility when the national government decides to default on the dollar debt and

$$V^{di}\left(k^{di}, B^i, D^i, \alpha^i, \phi^u, \cdot, \cdot\right) = \frac{1}{1-\beta} u^{di}\left(k^{di}, \phi^u\right) \tag{29}$$

such that at time $t+1$, the state is $s^i = (k^{di},\, B^i,\, D^i,\, \alpha^i,\, 1,\, \cdot,\, \eta^u)$, characterized by a moratorium in dollar debt in the previous period, and $u^{di}\left(k^{di}, \phi^u\right)$ is defined as per expression (26).

In a similar procedure, we characterize condition (8). For an initial state $s^i = (k^{\pi^i\xi^u},\, B^i,\, D^i,\, 1,\, 1,\, \zeta^i,\, \eta^i,\, \eta^u)$, with B^i stationary and in the crisis zone of probability $\pi^i(1-\xi^u)$, $\zeta^i > \pi^i$ and $\eta^u > \xi^u$, the level of welfare on the left side is given by

$$V^{\pi^i\xi^u}\left(s^i, B^i, q^{*i}, D^i, q^i\right) = \tag{30}$$

$$u^{\pi^i\xi^u}\left(k^{\pi^i\xi^u}, \pi^i, \xi^u\right) + \beta\left\{\frac{(1-\pi^i)(1-\xi^u)}{1-\beta(1-\pi^i)(1-\xi^u)} u^{\pi^i\xi^u}\left(k^{\pi^i\xi^u}, \pi^i, \xi^u\right) + \right.$$
$$\frac{\xi^u}{1-\beta(1-\pi^i)(1-\xi^u)}\left[u^{\pi^i}\left(k^{\pi^i\xi^u}, \pi^i, \phi^u\right) + \beta V^{\pi^i}\left(k^{\pi^i}, B^i, D^i, 1, \phi^u, \zeta^i, \cdot, \cdot\right)\right] +$$
$$\left.\frac{\pi^i(1-\xi^u)}{1-\beta(1-\pi^i)(1-\xi^u)} u^{di}\left(k^{\pi^i\xi^u}, \xi^u\right) + \beta V^{di}\left(k^{di}, 0, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)\right\}$$

where,

$$u^{\pi^i\xi^u}\left(k^{\pi^i\xi^u}, \pi^i, \xi^u\right) =$$
$$\varrho^i\left\{(1-\theta^i)\, y^{\pi^i\xi^u} + [1-\beta(1-\xi^u+\xi^u\phi^u)]\, D^i\right\} +$$
$$v\left\{\theta^i y^{\pi^i\xi^u} - \left[1-\beta\left(1-\pi^i+\pi^i\xi^u\right)\right] B^i - [1-\beta(1-\xi^u+\xi^u\phi^u)]\, D^i\right\}$$

$$u^{di}\left(k^{\pi^i\xi^u}, \xi^u\right) = \tag{31}$$
$$\varrho^i\left\{(1-\theta^i)\left[\alpha^i f\left(k^{\pi^i\xi^u}\right) - \delta^i k^{\pi^i\xi^u}\right] - k^{di} + k^{\pi^i\xi^u} + [1-\beta(1-\xi^u+\xi^u\phi^u)]\, D^i\right\} +$$
$$v\left\{\theta^i\left[\alpha^i f\left(k^{\pi^i\xi^u}\right) - \delta^i k^{\pi^i\xi^u}\right] - [1-\beta(1-\xi^u+\xi^u\phi^u)]\, D^i\right\}$$

where $u^{\pi^i}(k^{\pi^i\xi^u}, \pi^i, \phi^u)$, $V^{\pi^i}(k^{\pi^i},\, B^i,\, D^i,\, 1,\, \phi^u,\, \zeta^i,\, \cdot,\, \cdot)$, $u^{di}(k^{\pi^i\xi^u},\, \xi^u)$ and $V^{di}(k^{di},\, 0,\, D^i,\, \alpha^i,\, 1,\, \cdot,\, \cdot,\, \eta^u)$ are defined by the expressions (23), (24) and (22), respectively. Moreover, $u^{\pi^i\xi^u}\left(k^{\pi^i\xi^u}, \pi^i, \xi^u\right)$ and $u^{di}\left(k^{\pi^i\xi^u}, \xi^u\right)$ are the instantaneous utilities when the government decides, respectively, not to default and to default on the dollar debt, given initial aggregate state $s = (k^{\pi^i\xi^u},\, B^i,\, D^i,\, 1,\, 1,\, \zeta^i,\, \eta^i,\, \eta^u)$, with $\zeta^i > \pi^i$and $\eta^u > \xi^u$ in the first case and $\zeta^i \leq \pi^i$and $\eta^u > \xi^u$, in the later one. In both cases, the central government does not partial default on the common-currency debt.

The right side of condition (8), $V^{di}(s^i,\ B^i,\ q^{*i},\ D^i,\ q^i)$, is obtained when we suppose that the national government chooses $z^i = 0$ and the central government $\vartheta^u = 1$. Given the realization of the sunspot variables, lenders buy dollar bonds at a positive price, q^{*i}, and bonds in common currency at q^i. Assuming a stationary debt level and positive probability that the central government will inflate the common currency, then the welfare for the government of country i, on deciding to undertake a moratorium in dollar debt despite having sold new debt at price q^{*i}, corresponds to:

$$V^{di}(s^i, B^i, q^{*i}, D^i, \beta\,(1 - \xi^u + \xi^u \phi^u)) =$$
$$\varrho^i \left\{ \left(1 - \theta^i\right) \left[\alpha^i f\left(k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}\right) - \delta^i k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}\right] - k^{di} + k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}} + \left[1 - \beta\,(1 - \xi^u + \xi^u \phi^u)\right] D^i \right\} +$$
$$v \left\{ \theta^i \left[\alpha^i f\left(k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}\right) - \delta^i k^{\pi^{\mathsf{i}}\xi^{\mathsf{u}}}\right] + q^{*i} - \left[1 - \beta\,(1 - \xi^u + \xi^u \phi^u)\right] D^i \right\} +$$
$$\beta V^{di}\left(k^{di}, B^i, D^i, \alpha^i, 1, \cdot, \cdot, \eta^u\right)$$

where, $V^{di}(k^{di},\ B^i,\ D^i,\ \alpha^i,\ 1,\ \cdot,\ \eta^u)$ is given by the expression (22), with $z = 0$.

B Tables and Figures

β	$risk$	ϱ	α	ν	A	δ	π	θ	ϑ
0.93	1	0.7	0.95	0.5	0.8	0.20	0.04	0.3	0.88

Table 1: Simulation Parameters

$\xi = 0.1$							
dsi	dai	dsc	dac	nsi	nai	nsc	nac
0.004	0.000	0.035	0.001	0.095	0.001	0.86	0.004
$\xi = 0.9$							
dsi	dai	dsc	dac	nsi	nai	nsc	nac
0.035	0.001	0.004	0.000	0.86	0.004	0.095	0.001

Table 2: Parameterization of the Joint Probabilities (sy1)

$\xi = 0.1$							
dsi	dai	dsc	dac	nsi	nai	nsc	nac
0.004	0.000	0.034	0.002	0.094	0.002	0.855	0.009
$\xi = 0.9$							
dsi	dai	dsc	dac	nsi	nai	nsc	nac
0.034	0.002	0.004	0.000	0.855	0.009	0.094	0.002

Table 3: Parameterization of the Joint Probabilities (sy2)

	$\eta^u > \xi^u$	$\eta^u \leq \xi^u$
$\eta^i > \xi^i$	sc	ai
$\eta^i \leq \xi^i$	ac	si

Table 4: Joint Probability for the Shocks about Government Types

	$\zeta^i > \pi^i$ e $\eta^u > \xi^u$	$\zeta^i > \pi^i$ e $\eta^u \leq \xi^u$	$\zeta^i \leq \pi^i$ e $\eta^u \leq \xi^u$	$\zeta^i \leq \pi^i$ e $\eta^u > \xi^u$
$\zeta^i > \pi^i$ e $\eta^i > \xi^i$	nsc	nai	–	–
$\zeta^i > \pi^i$ e $\eta^i \leq \xi^i$	nac	nsi	–	–
$\zeta^i \leq \pi^i$ e $\eta^i \leq \xi^i$	–	–	dsi	dac
$\zeta^i \leq \pi^i$ e $\eta^i > \xi^i$	–	–	dai	dsc

Table 5: Joint Probability for the Three Sunspot Variables

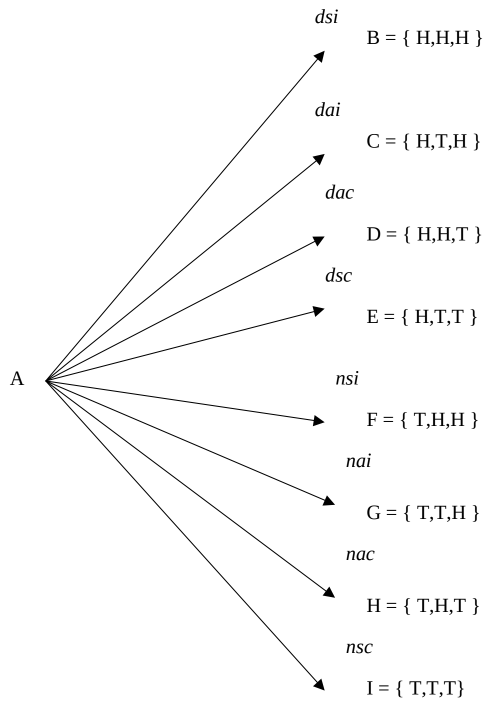

Figure 1: Tree Diagram

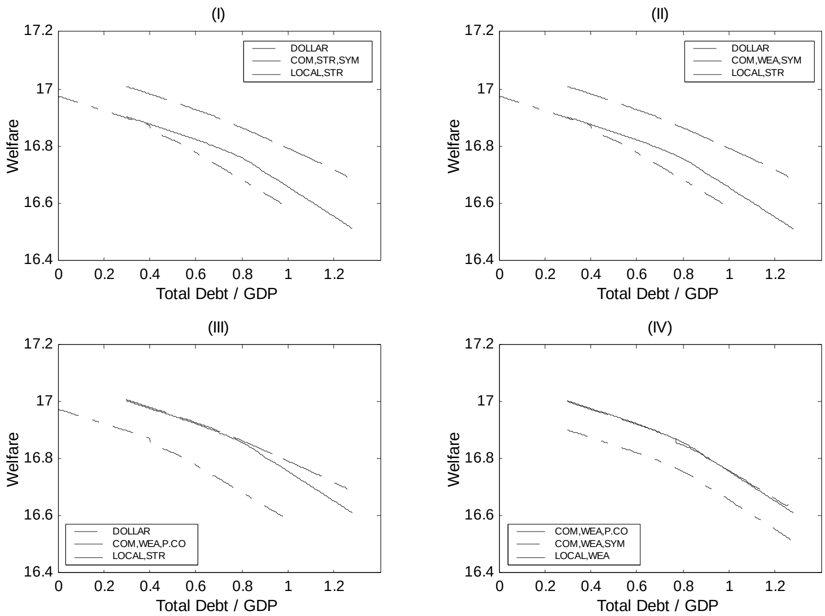

Figure 2: Welfare under Different Monetary Regimes ($\xi = 0.9; \varpi = 0$)

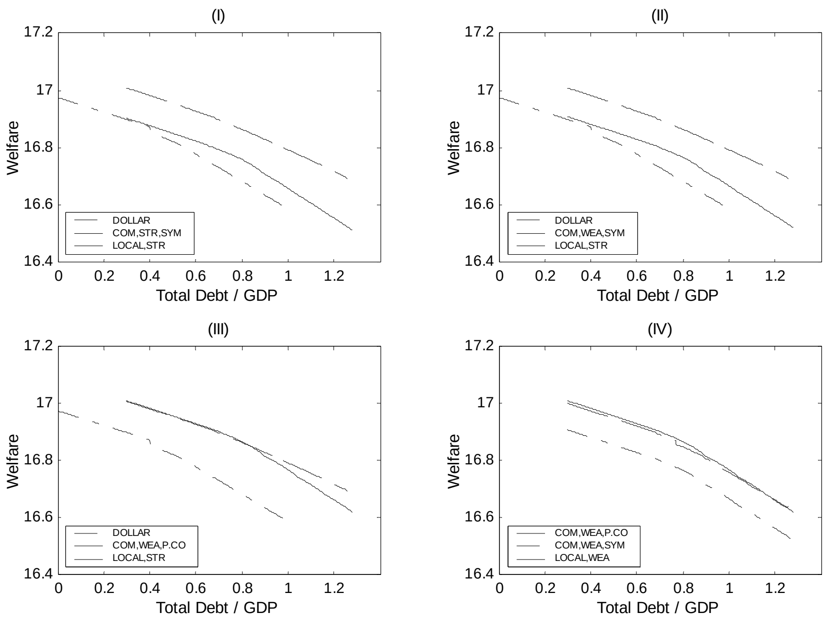

Figure 3: Welfare for Different Monetary Regimes ($\xi = 0.9; \varpi = 10$)

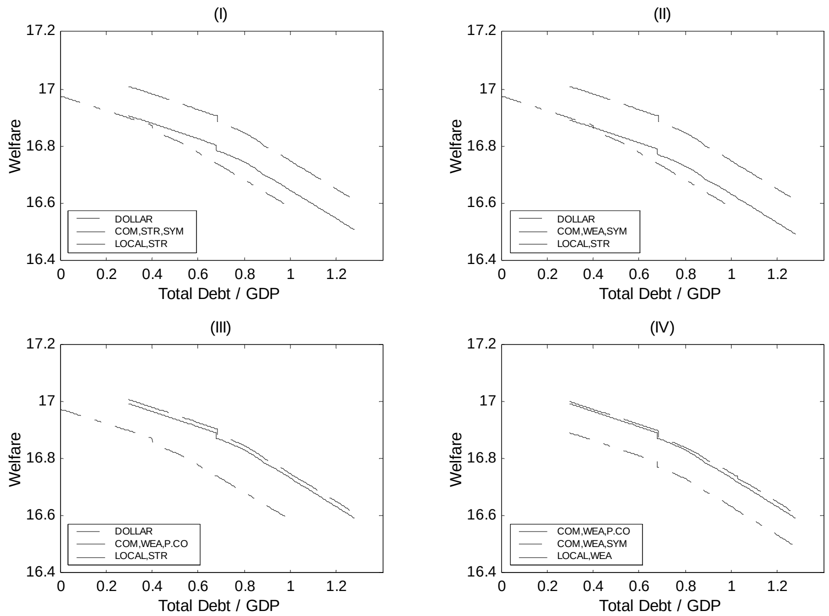

Figure 4: Welfare for Different Monetary Regimes ($\xi = 0.1; \varpi = 0$)

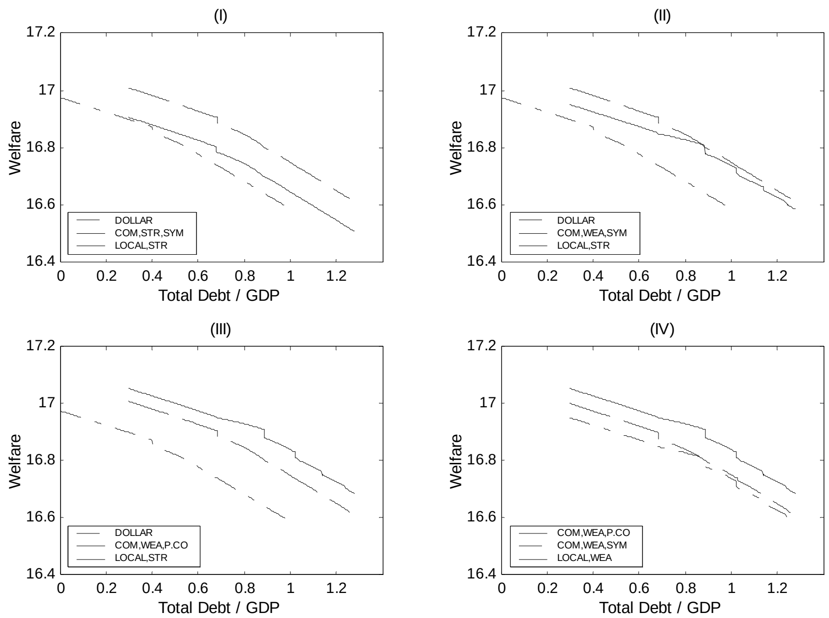

Figure 5: Welfare for Different Monetary Regimes ($\xi = 0.1; \varpi = 10$)

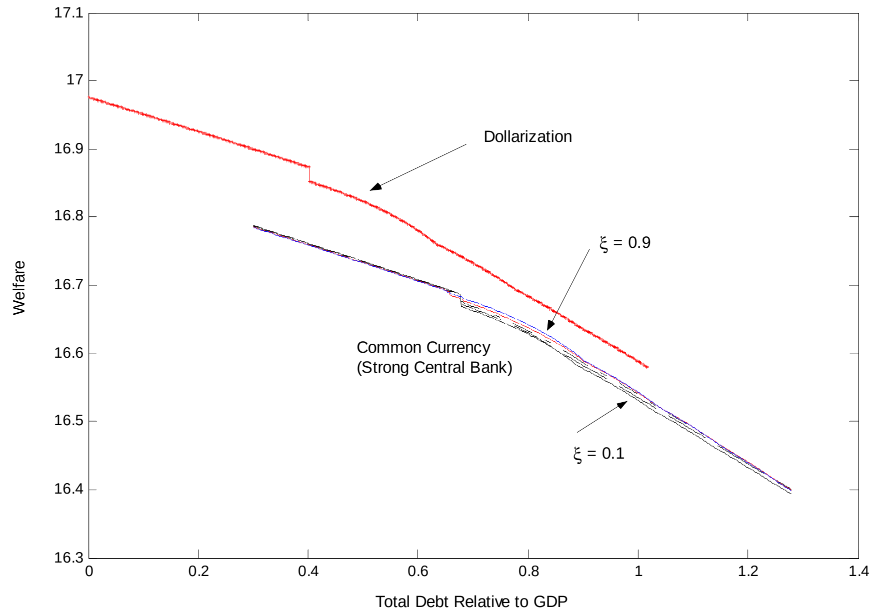

Figure 6: Welfare in a Monetary Union with Strong Central Bank and Various ξ $(\varpi = 10; si2)$

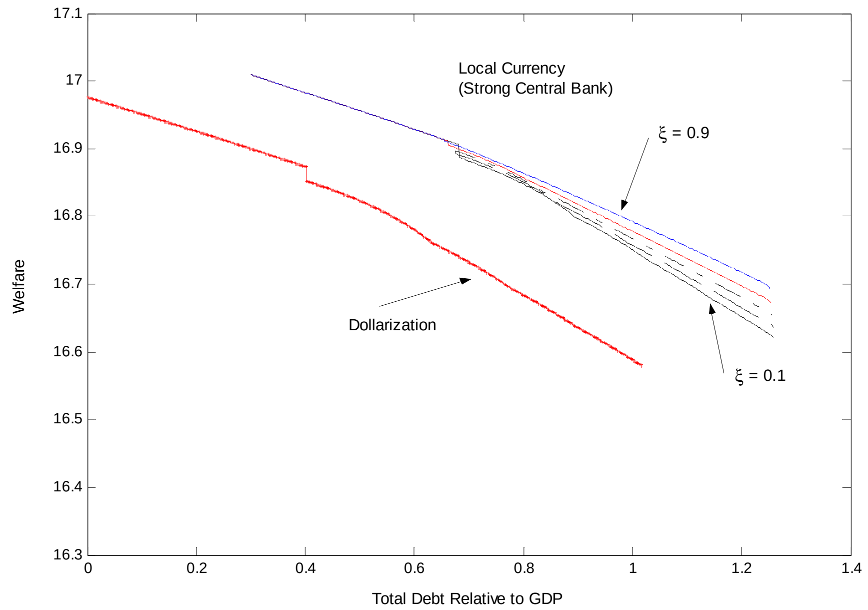

Figure 7: Welfare for Local Currency Regime with Strong Central Bank for Various ξ

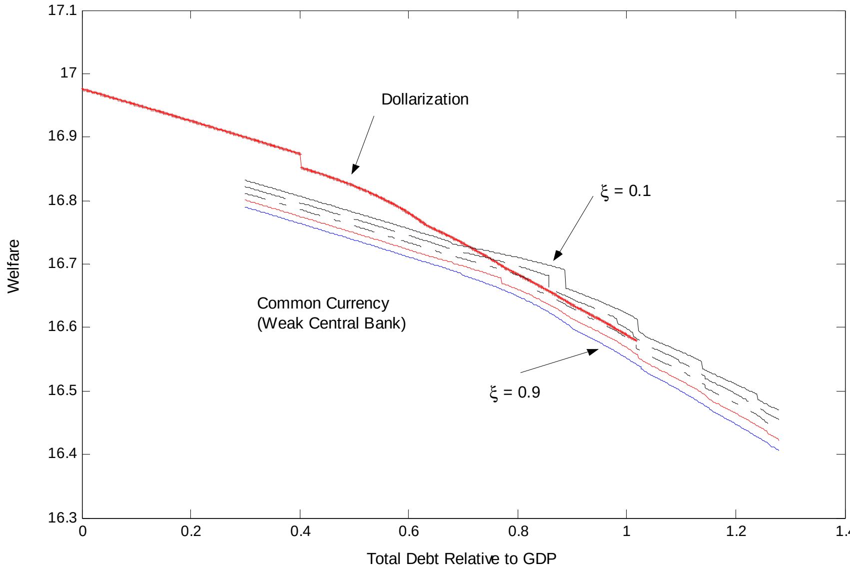

Figure 8: Welfare for a Monetary Union with Weak Central Bank and Various ξ ($\varpi = 10; si2$)

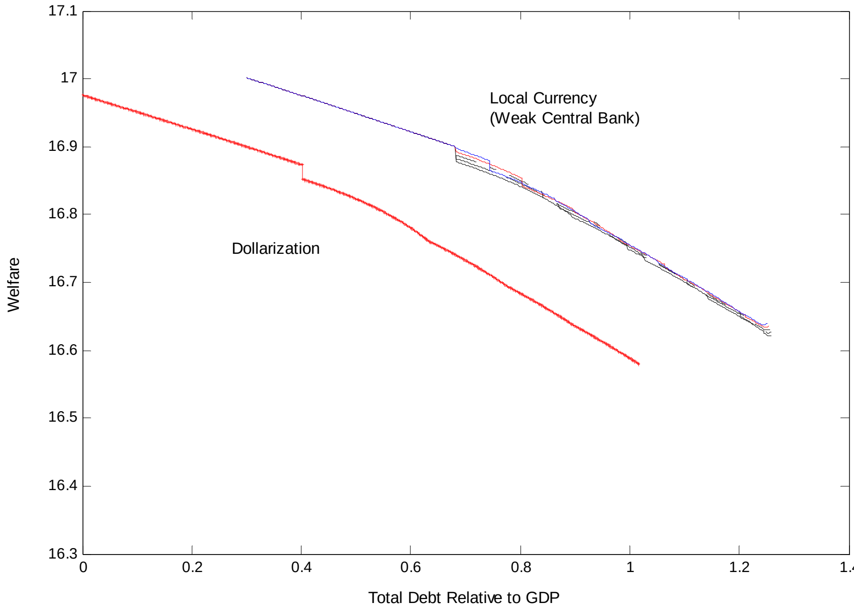

Figure 9: Welfare for a Local Currency Regime with Weak central bank and Various ξ

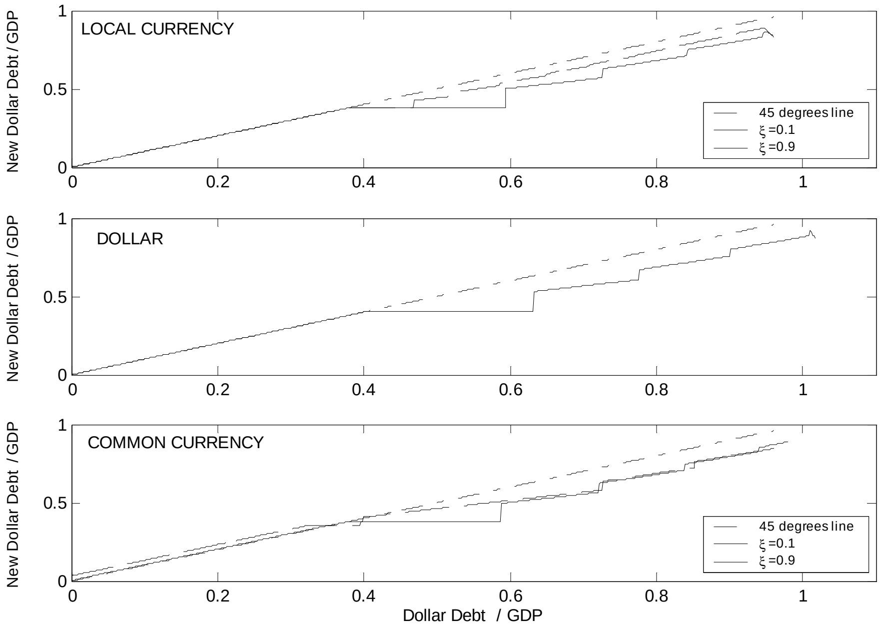

Figure 10: Government Dollar Debt Policy Function