Public Debt Sustainability and Endogenous Seigniorage in Brazil: Time-Series Evidence From 1947-92 (Revised Version)

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Public Debt Sustainability and Endogenous Seigniorage in Brazil: Time-Series Evidence from 1947-92

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Abstract
Using national accounts data for the revenue-GDP and expenditure-GDP ratios from 1947 to 1992, we examine three central issues in public finance. First, was the path of public debt sustainable during this period? Second, if debt is sustainable, how has the government historically balanced...
the budget after shocks to either revenues or expenditures? Third, are expenditures exogenous? The results show that (i) public deficit is stationary (bounded asymptotic variance), with the budget in Brazil being balanced almost entirely through changes in taxes, regardless of the cause of the initial imbalance. Expenditures are weakly exogenous, but tax revenues are not; (ii) the behavior of a rational Brazilian consumer may be consistent with Ricardian Equivalence; (iii) seigniorage revenues are critical to restore intertemporal budget equilibrium, since, when we exclude them from total revenues, debt is not sustainable in econometric tests.

1. Introduction

At least since the end of WWII the Brazilian economy has been plagued with chronic public deficits and relatively high inflation - a typical example of seigniorage-financed deficits. However, there have been very few episodes when a sharp increase in public debt was observed. Using national accounts data for the revenue-GDP and the expenditure-GDP ratios from 1947 to 1992, this paper studies three central issues in public finance. First, was the path of public debt sustainable during this period? Second, if debt is sustainable, how has the government historically balanced the budget after shocks to either revenues or expenditures were observed? For example, given an unpredictable increase in expenditures, there are two polar forms to balance the budget. One is to increase the present value of taxes and the other is to decrease the present value of expenditures. From the point of view of a rational Brazilian taxpayer, it is important to learn to what extent these two forms of balancing the budget have occurred. Third, what are the results of exogeneity tests for government expenditures and tax revenues? This last issue is motivated by the fact that models of seigniorage-financed deficits such as Bruno and Fischer (1990), and the extension in Ruge-Murcia (1995), assume respectively that the fiscal deficit and that government expenditures are exogenous. Moreover, in a more flexible framework than Barro (1974), Bohn (1992) shows that the exogeneity of expenditures is a necessary condition for Ricardian Equivalence. Given that post-war Brazil seems to t the seigniorage-financed deficit model, and that it is desirable to discuss Ricardian Equivalence here, we nd it useful to apply exogeneity tests to Brazilian fiscal figures. As far as we know, this paper is the rst work to do it in this context.

Following Hamilton and Flavin (1986) and Bohn (1991), the rst two issues
are investigated using unit-root tests, cointegration tests, and calculating unconventional impulse-response functions based on Vector Error-Correction Models (VECM's) where a balanced-budget restriction is imposed. The search for a sensible test of exogeneity has led us to use the definitions of weak, strong and super exogeneity in Engle, Hendry and Richard (1983). They propose likelihood-based definitions of exogeneity which could be tested, showing why the previous concepts (pre-determinedness and strict exogeneity) were incomplete or misleading. Exogeneity tests were conducted using the framework proposed in Johansen (1992, 1995).

This paper has three main findings. First, debt is sustainable, with the budget in Brazil being balanced almost entirely through changes in taxes, regardless of how the initial imbalance was generated. Expenditures are weakly exogenous in the sense of Engle, Hendry and Richard (1983). Second, the behavior of a rational Brazilian consumer, with Ricardian preferences, is consistent with the Ricardian Equivalence result (Barro (1974)). Given, for example, a tax break today, since his/her best forecast for restoring scalar balance is that the current imbalance will be fully reversed by future tax increases, consumption and welfare will be unchanged\(^1\). Third, we show that seigniorage revenues are critical to restore intertemporal budget equilibrium, since, when we exclude them from total revenues, debt is not sustainable in econometric tests. These results match the conventional wisdom about Brazilian public behavior and are broadly consistent with the theoretical model of optimal seigniorage of Bruno and Fischer (1990), its extension in Ruge-Murcia (1995), and with the empirical findings of Pastore (1995), Ruge-Murcia, and Rocha (1997).

The paper is organized as follows: in Section 2 the methodology is presented; in Section 3 the data set is discussed; in Section 4 the empirical results are presented and in Section 5 we conclude.

2. Methodology

The econometric techniques used here to test whether or not debt is sustainable follow Hamilton and Flavin (1986), and Bohn (1991). The calculations of the "unconventional impulse-response function" follow Bohn (1991). We discuss a slight

\(^1\)As pointed out by Bohn (1992, footnote 7), this argument relies on our estimates not being subject to the Lucas (1976) critique. We return to this issue on the discussion of exogeneity tests.
caveat to his approach. Exogeneity tests are performed following the typology in Engle, Hendry and Richard (1983); see also the test implementation in Johansen (1992, 1995). A brief discussion of these techniques is presented here for the sake of completeness.

The government budget constraint can be written in the following form:

$$B_{t+1} = G_t + T_t + (1 + r)B_t + ²_{t+1};$$

(2.1)

where $T_t$ represents \textsuperscript{-}scal revenues including seigniorage, $G_t$ represents \textsuperscript{-}scal expenditures excluding debt-service payments, $r$ is the \textsuperscript{-}real-interest rate\textsuperscript{2} on debt (assumed \textsuperscript{-}xed), $B_t$ is beginning of period public debt, and $²_{t+1}$ is a stationary measurement error inherited from assuming that $r_t = r$ for all $t$. All time series are measured as a proportion of GDP.

Without loss of generality, we work with the following version of equation (2.1):

$$B_{t+1} = G_t + T_t + B_t + ²_{t+1};$$

(2.2)

where $G_t = G_t + rB_t$ is a broader version of government expenditures, including interest paid on debt. Disregarding measurement error, and rearranging (2.2) we get:

$$B_{t+1} - B_t = G_t + T_t = \text{Def}_t;$$

(2.3)

where $\text{Def}_t$ is the public de\textsuperscript{-}cit in period $t$. Equation (2.3) is the basis for the debt sustainability test of Hamilton and Flavin (1986). It shows that whenever $\text{Def}_t$ is not an integrated series, $B_t$ is difference-stationary. Thus, debt sustainability is tested via a unit-root test on $B_t$.

A similar argument can be made from an intertemporal perspective. The government intertemporal budget constraint is:

$$B_t = \sum_{j=0}^{h} \frac{1}{(1+r)^j} E_t \left( T_{t+j} + G_t^{x} + ²_{t+j+1} \right);$$

(2.4)

where $\frac{1}{2} = \frac{1}{(1+r)^j}$ is the one-period discount rate for future taxes and expenditures. Trehan and Walsh (1988) showed that (2.4) holds whenever public debt is

\textsuperscript{2}Since \textsuperscript{-}scal data are measured as a proportion of GDP, to a logarithmic approximation, $r$ should be interpreted as the difference between the real-interest rate on debt and the growth rate of GDP.
di@erence-stationary. From (2.3), this last condition implies that $G_t^\nu$ and $T_t$ coin-
tegrate with coe@cient $(1; i 1)$. This is the test proposed in Bohn(1991) to check
whether or not debt is sustainable. Under (restricted) cointegration, the system
on $X_t = (G_t^\nu; T_t)^0$, in error-correction form is$^3$ (Engle and Granger(1987)):

$$A(L)\hat{\pi}_X t = i \odot^{-1} X_{t-1} + 1 = i \odot D ef_{t-1} 1 + 1_t;$$  \hspace{1cm} (2.5)

where $\hat{\pi} = (1; i 1)^0$ is the cointegrating vector, $\odot$ is the adjustment coe@cient
vector of the error-correction term, and $1_t$ is a multivariate white-noise process.
To simplify rational-expectation computations of \'scal variables, we can rewrite
(2.5) as a \'rst-order system of equations as follows:

$$
\begin{bmatrix}
\hat{\pi} X_t \\
\hat{\pi} X_{t-1} \\
\vdots \\
\hat{\pi} X_{t-k+1} \\
D ef_{t-k} \\
\end{bmatrix} =
\begin{bmatrix}
A_1^\nu & A_2^\nu & \cdots & A_k^\nu & \odot & 1 & 0 & 1 \\
1 & 0 & \cdots & 0 & \odot & \hat{\pi} X_{t-2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 1 & \odot & \hat{\pi} X_{t-k} \\
0 & \cdots & 0 & 1 & \odot & \hat{\pi} X_{t-1} \\
\end{bmatrix}
$$

or, compactly as:

$$X_t^\nu = A^\nu X_{t-1}^\nu + 1_t.$$  \hspace{1cm} (2.6)

where $X_t^\nu = \hat{\pi} X_t, \hat{\pi} X_{t-1}, \cdots, \hat{\pi} X_{t-k+1}, D ef_{t-k}$ and $1_t = 1_t^0; 0; \cdots; 0$ are
nk + 1 vectors, and $A^\nu$ is the $[nk + 1]$ by $[nk + 1]$ loading matrix of $X_{t-1}^\nu$.

Using (2.7) and (2.4), we can analyze the e@ects of unpredictable changes of $G_t^\nu$
and $T_t$ on their respective present discounted values. De@ne the present discounted
value of a generic variable $z$, $P V (z)_t = \frac{1}{2} Z_{t+j}$, an innovation in $z$, $Z_t^i = Z_t i$
$E_{t-1} z_t$, an innovation on the present discounted value of $z$, $PV (z)_t = E_t P V (z)_t i$

$^3$Although we are basically using $X_t = (G_t^\nu; T_t)^0$ as the vector of \'scal variables, Bohn suggests
augmenting it to include series that may increase the information set of the econometrician.
Examples are GNP, hours worked, etc.
$E_t [P V (z)_t \mid E_t] = E_t [P V (2)_t]$, and the inherited measurement error term, $-t = E_t [P V (2)_t] \mid E_{t-1}]$. Consider Campbell's (1987) identity $(1 \frac{1}{2}[z_t + P V (z)_t] = z_t + P V (c z)_t$, and the fact that $c \Phi_t = \Phi_t$, to obtain:

$$
\Phi_t + P V (\Phi T)_t = c G_t^X + P V (\Phi G)_t + r - t. \quad (2.8)
$$

From (2.8), for an unchanged debt value, and disregarding the error term, any increase in expenditures (not accompanied by an increase in taxes) would, in the future, either require a decrease in expenditures or an increase in taxes. In present-value terms, these changes should offset exactly the initial change in expenditures (a similar result applies for a change in taxes), i.e., $\Phi G_t^X = P V (\Phi T)_t \mid P V (\Phi G)_t$ must hold. Notice that when we consider the measurement error term, this equation will not hold exactly. The absolute difference between the left- and the right-hand side is an increasing function of $r$. Of course, it holds exactly when $r = 0$ (i.e., $\frac{1}{2} = 1$).

Calculating the marginal impact of current innovations to taxes and expenditures on the innovations of the present discounted values can be easily done using (2.7). It is straightforward to show that:

$$
P V (\Phi X)_t = \left. \frac{\partial P V (\Phi X)_t}{\partial \Phi X} \right|_{\Phi X} = X \left( \frac{1}{2}A^X \right)^k 1^X. \quad (2.9)
$$

The partial derivative of $P V (\Phi X)_t$ with respect to $1^X$ is simply:

$$
\frac{\partial P V (\Phi X)_t}{\partial 1^X} = X \left( \frac{1}{2}A^X \right)^k 1^X = \frac{1}{2}A^X (I \mid \frac{1}{2}A^X)^i 1. \quad (2.10)
$$

Equation (2.10) is what we have labelled the unconventional impulse-response function. It depends only on $\frac{1}{2}$ and $A^X$. The latter can be consistently estimated and the former can be either estimated or calibrated. Equation (2.10) is "unconventional" since it calculates the innovation present-value response to shocks to the system. It is tempting to associate the $l$-th element of the vector $h_l = \frac{1}{2}A^X (I \mid \frac{1}{2}A^X)^i 1$, where $h_l$ is a selection vector, with the response to a unit impulse of the $l$-th element of $1_t$. Although Bohn (1991) claims that this impulse-response function requires no orthogonalization of the shocks, the interpretation

---

$^4$There is no contradiction between the definition of $b_t = z_t \mid E_{t-1} z_t$, and $P V (z)_t = E_t P V (z)_t \mid E_{t-1} z_t$. The former can written as $b_t = E_t z_t \mid E_{t-1} z_t$, due to the fact that $z_t$ is in the information set when the conditional expectation is taken.
above embeds the assumption that no other element of \( x_t \) changed when its \( l \)-th element did. Of course, this can only be true when the covariance matrix of the shocks is diagonal.

The exogeneity tests conducted here follow the typology in Engle, Hendry and Richard (1983); see Maddala (1992) for an introduction, Ericsson and Irons (1994) for a more complete review, and Ericsson and Irons and Johansen (1992, 1995) for testing procedures. There are three definitions of exogeneity: weak, strong, and super exogeneity. We only discuss the first two here\(^5\). They are relevant respectively to conduct conditional inference and to perform conditional forecasting.

Engle, Hendry and Richard proposed definitions of exogeneity that took into account the parameters of interest, something missing from the definitions of predeterminedness and strict exogeneity; see Engle, Hendry and Richard (p. 280) and Maddala (1992, p. 391). It does so by decomposing the joint density of \((y_t; x_t)^0\) into the product of the conditional and marginal densities: \( f(y_t|x_t) \cdot f(x_t) \). Heuristically, \( x_t \) is said to be weakly exogenous for \( \theta \) (a function of the parameters of the conditional density) if \( f(x_t) \) contains only nuisance parameters which are irrelevant for conducting inference on \( \theta \). Thus, to do inference on \( \theta \), we can "discard" \( f(x_t) \) and maximize the likelihood using only the terms arising from \( f(y_t|x_t) \). Weak exogeneity is a necessary condition to have strong and super exogeneity. In addition, each of them require an extra condition. In particular, strong exogeneity requires that \( y \) does not Granger-cause \( x \).

A key ingredient of the weak exogeneity is the "separation" property for the parameters associated with the conditional and marginal distributions. This is relevant for showing that innovations to the present discounted value of \( x \) are not affected by shocks to the conditional mean of \( y \). Consider the simple example in Ericsson (1994, pp. 22-24) where \( X_t = (y_t; x_t)^0 \) follows a Gaussian Vector Autoregression of order one, conveniently reparameterized as:

\[
\begin{align*}
\phi y_t &= \frac{1}{41}y_{t-1} + \frac{1}{42}x_{t-1} + \mu_1 \\
\phi x_t &= \frac{1}{21}y_{t-1} + \frac{1}{22}x_{t-1} + \mu_2,
\end{align*}
\]

where \((\mu_1; \mu_2)^0 \sim N(0; \Sigma)\). \( \mu \) reads independent and normally distributed random vector, where \( \Sigma \) is a non-diagonal positive-definite matrix with

\(^5\)Super exogeneity may be relevant to discuss Ricardian Equivalence, since this is the concept of exogeneity that allows for counterfactual (policy) analysis using econometric estimates. However, including it in the text would have taken us too far from the scope of this paper. Thus, it is left for future research.
If there is one cointegration vector \( \bar{\theta}^\top = (1; \pm) \), where \( \pm = i \frac{1}{\bar{\theta}^2} = \frac{1}{\bar{\theta}^2} \), \( \bar{\theta}^\top = (y_{t,1} \ i \ \bar{x}_{t,1}) \), \( \bar{\theta}_1 = \frac{1}{\bar{\theta}^2} \), and \( \bar{\theta}_2 = \frac{1}{\bar{\theta}^2} \), the error-correction form is:

\[
\begin{align*}
\zeta_1 y_t &= \bar{\theta}_1 (y_{t,1} \ i \ \bar{x}_{t,1}) + \eta_1 t \\
\zeta_2 x_t &= \bar{\theta}_2 (y_{t,1} \ i \ \bar{x}_{t,1}) + \eta_2 t \quad \text{(2.11)}
\end{align*}
\]

If we factor the joint density of the elements in \( X_t \) (given \( X_{t-1} \)) into the conditional density of \( y_t \) given \( x_t \) (and \( X_{t-1} \)), and the marginal density of \( x_t \) (given \( X_{t-1} \)), we get:

\[
\begin{align*}
\zeta_1 y_t &= \gamma_1 \zeta_1 x_t + \gamma_2 (y_{t,1} \ i \ \bar{x}_{t,1}) + \eta_1 t \\
\zeta_2 x_t &= \gamma_2 (y_{t,1} \ i \ \bar{x}_{t,1}) + \eta_2 t \quad \text{(2.12)}
\end{align*}
\]

where \( \gamma_1 = i 12 \), \( \gamma_2 = \bar{\theta}_1 \ i \ (i 12 \bar{\theta}_2 \) \( \bar{\theta}_2 \) \, and \( \eta_1 t \to N (0; 3/2) \) and \( \eta_2 t \to N (0; i 22) \). First, notice that the two shocks are now independent, which did not happen to shocks in the reduced form (2.11). Second, we can collect the parameters of the conditional and marginal models in \( (\gamma_1; \gamma_2; \pm 3/2) \) and \( (\bar{\theta}_2; \pm i 22) \) respectively. Since \( \pm \) is an element of both, and \( \bar{\theta}_2 \) is in the conditional model through \( \gamma_2 \), the parameters of the conditional and marginal models are not separable in general.

However, if \( \bar{\theta}_2 = 0 \), \( x_t \) is weakly exogenous for \( \bar{\theta}_0 \). In this case, the parameters of the conditional and marginal models are respectively \( (\gamma_1; \gamma_2; \pm 3/2) \) and \( (\bar{\theta}_2; \pm i 22) \); the \( \gamma_1 \)st equation is sufficient to conduct inference on \( \pm \) and thus on \( \bar{\theta}_0 = (1; \pm) \); the second equation has only the nuisance parameter \( i 22 \); and (2.12) simplifies to:

\[
\begin{align*}
\zeta_1 y_t &= \gamma_1 \zeta_1 x_t + \gamma_2 (y_{t,1} \ i \ \bar{x}_{t,1}) + \eta_1 t \\
\zeta_2 x_t &= \eta_2 t \quad \text{(2.13)}
\end{align*}
\]

Since \( \eta_{2t+i} \) is independent of \( \eta_{2t} \), \( \gamma_i \), and \( \bar{\theta} V (\zeta x)_t \) is a function of \( \eta_{2t+i} \), \( \gamma_i > 1 \), it cannot be affected by changes in \( \eta_{2t} \). This makes weak exogeneity relevant for inferring how the government balances the budget given a shock to either tax revenues or expenditures. For example, if we find that expenditures are weakly exogenous for the cointegration vector, as in (2.13), \( \bar{\theta} V (\zeta G)_t \) cannot be affected by a shock to tax revenues (and vice-versa)\(^6\). Hence, after a shock to taxes, intertemporal equilibrium is restored via a change in \( \bar{\theta} V (\zeta T)_t \) alone.

\(^6\)A formal proof is available upon request.
Given the discussion above, a test for weak exogeneity was proposed by Johansen (1992, 1995). It consists of an exclusion test for $\bar{\Phi}_2$ in (2.12). This is exactly the test implemented here.

3. The Database

Since we apply unit-root and cointegration tests to investigate whether public-debt is sustainable, it is desirable to work with data with a long span. Unfortunately, Brazil does not have long-span time-series data on debt. An alternative is to use the fact that, $B_{t+1} - B_t = G_t^x - T_t = Def_t$, relying on data on $G_t^x$ and $T_t$. FIBGE provides annual national-account data on expenditures, which include payments of nominal interest on public debt, and cannot be disaggregated further, and also annual data on revenues (excluding seigniorage). They cover the period from 1947 to 1992. Since $G_t^x$ includes real-interest expenditures, not nominal, there is a mismatch between the data and the theoretical framework.

Seigniorage, approximated by inflation tax, is extracted from Cysne (1995), and then added to national-accounts tax revenues to form a series of total tax revenues. FIBGE provides annual national-account data on expenditures, which include payments of nominal interest on public debt, and cannot be disaggregated further, and also annual data on revenues (excluding seigniorage). They cover the period from 1947 to 1992. Since $G_t^x$ includes real-interest expenditures, not nominal, there is a mismatch between the data and the theoretical framework.

Seigniorage, approximated by inflation tax, is extracted from Cysne (1995), and then added to national-accounts tax revenues to form a series of total tax revenues. The former, and expenditures, as described above, were divided by GDP, and labelled $T_t$ and $G_t^x$ respectively.

According to Ahmed and Rogers (1995), using nominal-interest payments in place of $rB_t$ may bias toward rejection the restricted version of the cointegration test between $G_t^x$, and $T_t$. Indeed, we may get a cointegrating vector different from $(1; -1)^0$, possibly with the coefficient of $G_t^x$ being greater than unity in absolute value. This, however, is not the only problem. Since the nominal-interest rate can be thought of as the sum of the real-interest rate and the ex-ante rate of inflation, we may not get cointegration at all if inflation is an integrated process, which is a real possibility for Brazil. In order to account for that problem with the data, we increased the significance level of the cointegration test from the usual 5% or 10% levels (Johansen and Juselius (1990)), to the 20% level.

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7Seigniorage revenues, i.e., the change in monetary base used exclusively to finance government deficits, is not available for 1947-92, although monetary base is. Since high-powered money could change for reasons other than to finance government deficits, we chose not to use the first-difference in monetary base as a proxy for seigniorage.

8We only use the 20% level when testing sequentially for the rank of cointegration: $H_0: r = 0$, and then $H_0: r = 1$. In testing the restriction $H_0: \bar{\gamma} = (1; -1)^0$, conditional on the rank being one, we use the usual levels (5% or 10%).
4. Empirical Results

4.1. Unit-Root and Cointegration Tests

Before presenting the results of unit-root tests it should be mentioned that the fact that our series are ratios, therefore lying in the real interval \([0;1]\), does not rule them out being integrated processes; see Ahmed and Yoo(1989). Figure 1 shows the data set used in this paper. It is clear that all series are smooth, indicating a high level of persistence. The sample average (in percentage) of the ratios \(G_t\) and \(T_t\) are very close, respectively 24.8% and 24.4%. Despite \(G_t\) and \(T_t\) showing a steady increase since the beginning of the sample, \(G_t - T_t\) shows mean reversion, especially after the expenditure and tax increases of the mid-eighties.

Table 1 shows the results of the unit-root tests performed: the Augmented Dickey-Fuller (ADF) and the Phillips-Perron test\(^9\). Regardless of the test type, the results support one unit root for expenditures and taxes, whereas the deficit \((G_t - T_t)\) is stationary even at 1%\(^{10}\).

The next step is to perform the cointegration test between \(G_t\) and \(T_t\). We used the likelihood-based cointegration test of Johansen(1991). It is well known that the results of cointegration tests using this technique depend on the deterministic components included in the VAR and on the chosen lag length. Therefore some pre-testing is done in order to choose these. The lag length was selected using two types of information criteria (Schwarz and Hannan-Quinn). To choose the deterministic components we used the likelihood ratio test discussed in Johansen. The pre-test results are presented in Tables 2 and 3.

The results in Table 2a are based on a VAR estimated with an unrestricted constant term and the results in Table 2b on a VAR estimated without a constant. In both cases, using the Schwarz criterium, the optimal lag length is two, but using the Hannan-Quinn criterium the optimal lag length is three. Given the Monte-Carlo results in Gonzalo(1994), we chose to work with three lags in the VAR\(^{11}\). Testing the deterministic components in the VAR is more subtle, since it requires conditioning on the number of cointegrating relationships to be performed; see

\(^9\)The results of the latter are especially relevant, since the data clearly show signs of heteroskedasticity.

\(^{10}\)Tests for two unit roots reject this hypothesis for all three series.

\(^{11}\)The Monte-Carlo results in Gonzalo(1995) show that the efficiency loss is small for overestimating, while consistency is lost if the lag length chosen is too small. It is also worth mentioning that the VAR with two lags showed signs of autocorrelated residuals
Johansen (1991). Table 3 shows test results when the number of cointegrating vectors is one. When the model with an unrestricted constant is tested against the one with a restricted constant the test statistic is 1.64, distributed as a $\chi^2_1$. Thus, we cannot reject the restricted model. The same result was obtained when we tested the restricted constant against the no-constant model, or the unrestricted constant versus the no-constant model. Thus, we used a VAR without a constant.

The results of the cointegration test are presented in Table 4. As discussed above, we used the 20% significance level for the critical values of the Trace and the $\lambda_{\max}$ statistics. At 20% we reject the null of no cointegrating vectors and cannot reject the null of one cointegrating vector. Although we used the 20% level, it is worth mentioning that the test statistics for the null of zero cointegrating vectors are very close to the critical value at 10%. The point estimate of the cointegrating vector is $(1; -0.94)$ (normalization on expenditures), which matches the prior that using the nominal-interest payments rather than the real-interest payments would bias upwards the absolute value of the expenditures coefficient. It is natural at this point to test the theoretical value of $(1; 1)$ for the cointegrating vector. At usual significance levels, this theoretical vector could not be rejected. Hence, cointegration tests corroborate our prior findings from unit-root tests that the deficit is stationary, and thus debt is sustainable.

4.2. Exogeneity Tests

Table 5 shows the results of the error-correction model estimates. First, it seems that our choice of 3 lags for the VAR was appropriate. For the system as a whole, $c \beta_{\tau}$ is significant, while $c \gamma_{\tau}$ is marginally significant. Table 6 shows the results of the weak exogeneity tests for the cointegrating vector if the cointegrating rank is one. At usual significance levels we found that expenditures are weakly exogenous for the parameters of interest in the conditional model of tax-revenues, but the converse is not true for revenues. The results in Table 7 clearly indicate that expenditures Granger-cause taxes and vice-versa at the 5% significance level. At 1%, however, taxes do not precede expenditures. Therefore, although expenditures are weakly exogenous, they are not strongly exogenous, since they are Granger-caused by tax-revenues.

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12 The fact that the coefficient of $\delta_{\tau}$ on the $c \beta_{\tau}$ equation is not significant led us to test the joint hypotheses that the cointegrating vector and the adjustment coefficient are respectively $(1; 1)$ and $(0; \delta)$. The $\chi^2$ statistic of this test is 1.78, which is not rejected at the 41% significance level.
4.3. Unconventional Impulse-Response Function

As discussed above, impulse-response results are only interpretable when the variance-covariance matrix of the VECM errors is diagonal. The correlation coefficient between the two reduced-form residuals is 0.244. The log-likelihood ratio statistic testing for a diagonal covariance matrix is 2.67, with a $\chi^2_1$ distribution. The critical values are 3.84 and 2.71 at 5% and 10% respectively. Thus, we do not reject the null of orthogonal innovations, and no orthogonalization technique is used for reduced-form residuals.

Unconventional impulse-response functions were calculated using $(1; -1)$ as the cointegrating vector and the adjustment factor in the form $(0; \beta)^T$. From the VECM with two lags, insignificant regressors were deleted based on their robust t-tests. Based on these restricted VECM estimates, and using (2.10), the marginal impacts of innovations to $G_t$ and $T_t$ on $\Delta V(\xi G^n)_t$ and $\Delta V(\xi T)_t$ were calculated.

Table 8 shows the impulse-response results for the following values of $\gamma$: $0.97$, $0.98$, $0.99$, and $1$. To be able to test hypotheses on the impulse-response parameters, we performed a Monte-Carlo experiment with 1,000 replications for each value of $\gamma$. The sample size used for each replication is the same as that of the original data, and pre-sample observations were drawn to avoid dependence on their values. Standard errors of point-estimates were then computed and are included in Table 8.

Table 8 has two interesting results. First, it is clear that expenditures change very little when we consider innovations in either expenditures or revenues. For $\gamma = 1$, and thus with no measurement error, 89% of expenditure-generated deficits are eliminated via an increase in taxes, versus only 11% of decrease in future expenditures. Notice that the former is not statistically different from 100% and the latter not statistically different from zero. Second, although it is impossible to infer consumer behavior from scalar data, impulse-response results are consistent with Ricardian-Equivalence behavior for Brazilian consumers (Barro (1974, 1989)) with the appropriate preferences. This is true since a change in taxes, not accompanied by a change in expenditures, will be offset 100% by a change in future taxes. Thus, Ricardian consumers using VAR’s to infer the meaning of a tax break (hike) today will not change their optimal consumption allocation, since they expect this tax break (hike) to be offset by an equivalent increase (decrease)

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13 The estimate for $\beta$ is 0.25.
14 Since VAR residuals showed signs of heteroskedasticity, we computed robust standard errors.
in future taxes. This result is a direct consequence of the weak exogeneity of expenditures for the cointegration vector\textsuperscript{15}.

These two basic results are also obtained for \( \frac{1}{2} \leq \theta \leq 1 \). For example, for \( \frac{1}{2} = 0.99 \), point estimates of \( \frac{\partial V(\xi T)}{\partial G_t} \) and \( \frac{\partial V(\xi G^e)}{\partial G_t} \) are 93% and -11% respectively, and are statistically equal to 100% and zero respectively. Moreover, \( \frac{\partial V(\xi T)}{\partial G_t} \) is still statistically equal to -100%. For other values of \( \frac{1}{2} \geq 1 \), impulse-response estimates are not very different from the results above, although not precisely estimated.

It is always appropriate to ask what is the proper value of \( \frac{1}{2} \) to use for the sample period 1947-1992. Since fiscal variables are expressed as a proportion of GDP, \( r \) should be interpreted (to a logarithmic approximation) as the difference between the real-interest rate on debt and the growth rate of GDP. Due to the lack of long-span data on interest rates, we can only conjecture here what is the most appropriate value of \( \frac{1}{2} \) during this period, the relatively high inflation rate, coupled with a relatively high growth rate for GDP, would indicate that \( r = 0 \) is a reasonable value for \( r \). Therefore, we propose using \( \frac{1}{2} = 1 \).

Our last empirical test checks whether or not seigniorage was important for long-term fiscal balance. We perform a cointegration test with the same revenue series as before with seigniorage revenues excluded, using, again, the same significance level (20%). The results are presented in Table 9. We find no cointegration in this case. Notice that the Trace and the \( \lambda_{\max} \) statistic are much smaller than their critical values at 20%.

The evidence for Brazil suggests that debt is sustainable and that expenditures are weakly exogenous. Thus, the country followed much more closely a spend-and-tax than a tax-and-spend policy. Indeed, our impulse-response results suggest that, given a shock to expenditures, the present value of taxes will fully accommodate this imbalance restoring long-run equilibrium. Given that seigniorage is critical for the existence of equilibrium, it was probably through seigniorage revenues that most expenditure increases were financed. In this case Brazilian inflation is simply a consequence of the type of spend-and-tax policy followed in the post-war period.

It is interesting to compare our results to those of others who investigated public-finance issues in Brazil. Pastore(1995) tests debt sustainability using the techniques of Hamilton and Flavin(1986) on a short-span fiscal data set. From

\textsuperscript{15}Although weak exogeneity is not explicitly mentioned in Bohn(1992), he discusses the need of exogeneity for Ricardian Equivalence if the government is allowed to optimize welfare.
unit-root tests, he finds that the first difference of public debt is stationary. Since this series is equal to \( \text{Def}_t \) (see (2.3)), his results and ours are identical. Rocha(1997), also using a short-span data set, concludes too that debt is sustainable. Since all these results are achieved using different techniques, different series, and different samples, they complement each other in confirming the conventional wisdom about Brazilian public finance.

As far as we know, one of the original contributions of this paper is to test for exogeneity of fiscal data, showing that expenditures are exogenous for a high-inflation country such as Brazil. This assumption is implicit in the seminal theoretical paper of Bruno and Fischer(1990) on optimal seigniorage for high-inflation countries\(^{16}\). There, central banks choose optimal seigniorage conditional on a given deficit. Of course, this conditional optimization relies on the exogeneity of expenditures, which was validated here for Brazil under proper econometric tests.

When we consider the fact that debt was only sustainable when seigniorage was included as a government revenue, it becomes clear that the fundamental character of Brazilian public finance was that of endogenous seigniorage used to accommodate expenditure increases. Indeed, this conclusion is a common feature of the work of Pastore(1995), Ruge-Murcia(1995), Rocha(1997), and ours.

5. Conclusions

This paper presents tests on the sustainability of Brazilian public debt for the post-war period - 1947-92. They show that debt is sustainable only if seigniorage is included as a government revenue. In this case, exogeneity tests suggest that expenditures are exogenous. Unconventional impulse-response results show that, regardless of how an initial fiscal imbalance is originated (shocks to expenditures or revenues), it is eliminated through a change in taxes. This last result is consistent with Ricardian-Equivalence behavior for Brazilian consumers with proper preferences. Based on the evidence, we find that Brazil has followed closely a spend-and-tax policy in the post-war era, with seigniorage having a crucial role in balancing the budget. As a consequence, Brazilian inflation was consistently high during this period.

A brief reflection on the status of the Real Plan is appropriate. Since the beginning of the plan in July 1994, seigniorage revenues have decreased sharply.

\(^{16}\)In the extension of Ruge-Murcia(1995) weak exogeneity is explicitly assumed; see also the references therein.
Expenditures have increased considerably, generating a persistent deficit that runs (at the time of writing - June, 1998) at about 7% of GDP. Public debt have increased almost three-fold since then. For exogenous expenditures, as verified in the sample 1947-92, there are two polar forms of restoring long-run equilibrium: (i) increase taxes, excluding seigniorage, or (ii) increase seigniorage revenues. In the first case, Brazilians will be the most heavily taxed citizens in Latin America. However, there are very few services as a counterpart of these taxes - education, infrastructure, legal system, etc. In the second case, inflation will increase again, a price Brazilians may not be willing to pay for fiscal balance. Of course, expenditures could also be cut: let us just hope that they have ceased to be exogenous."
References


Figure 1
Expenditures (Including Nominal Interest Paid on Debt) and Revenues (Including Seigniorage) as a Proportion of GDP

\[ G^*_t = G_t + rB_t \] and \( T_t \)
Table 1

Unit-Root Tests

<table>
<thead>
<tr>
<th>Series</th>
<th>Lags$^1$</th>
<th>ADF Test</th>
<th>Phillips-Perron Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Revenues (T)</td>
<td>3</td>
<td>-3.03</td>
<td>-3.37</td>
</tr>
<tr>
<td>Expenditures (G$^*$)</td>
<td>4</td>
<td>-2.14</td>
<td>-2.54</td>
</tr>
<tr>
<td>Deficit (G$^*$ - T)</td>
<td>4</td>
<td>-2.98**</td>
<td>-3.27**</td>
</tr>
</tbody>
</table>

Notes: (1) The number of lags applies only to the ADF test. The final choice was made based on the t-test of significance of the last lagged first-difference. (2) The lag truncation chosen for the Bartlett kernel was 3. For (G$^*$) and (T) a constant and a time trend were used. Critical values are -3.53 and -4.20 for the 5 and 1% significance levels respectively. For (G$^*$ - T) the no-constant specification was used. Critical values are -1.95 and -2.62 for the 5 and 1% significance levels respectively. The symbols (*) and (**) represent rejection of the null of a unit root at the 5 and 1% significance levels respectively.
## Table 2

### VAR Lag Truncation

<table>
<thead>
<tr>
<th>VAR Order</th>
<th>Constant</th>
<th>Linear Trend</th>
<th>Schwarz Criterium</th>
<th>Hannan-Quinn Criterium</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>unrestricted</td>
<td>no trend</td>
<td>-14.21</td>
<td>-14.36</td>
</tr>
<tr>
<td>2</td>
<td>unrestricted</td>
<td>no trend</td>
<td>-14.25</td>
<td>-14.52</td>
</tr>
<tr>
<td>3</td>
<td>unrestricted</td>
<td>no trend</td>
<td>-14.24</td>
<td>-14.61</td>
</tr>
<tr>
<td>4</td>
<td>unrestricted</td>
<td>no trend</td>
<td>-13.98</td>
<td>-14.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VAR Order</th>
<th>Constant</th>
<th>Linear Trend</th>
<th>Schwarz Criterium</th>
<th>Hannan-Quinn Criterium</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>no constant</td>
<td>no trend</td>
<td>-14.21</td>
<td>-14.31</td>
</tr>
<tr>
<td>2</td>
<td>no constant</td>
<td>no trend</td>
<td>-14.39</td>
<td>-14.60</td>
</tr>
<tr>
<td>3</td>
<td>no constant</td>
<td>no trend</td>
<td>-14.36</td>
<td>-14.68</td>
</tr>
<tr>
<td>4</td>
<td>no constant</td>
<td>no trend</td>
<td>-14.12</td>
<td>-14.54</td>
</tr>
</tbody>
</table>
### Table 3
Deterministic Components Test (VAR(3))

<table>
<thead>
<tr>
<th>Model Order</th>
<th>VAR Order</th>
<th>Constant</th>
<th>Linear Trend</th>
<th>Restricted Vs. Unrestricted Model</th>
<th>Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>unrestricted</td>
<td>no trend</td>
<td>2 versus 1</td>
<td>1.64</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>restricted</td>
<td>no trend</td>
<td>3 versus 2</td>
<td>2.02</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>no constant</td>
<td>no trend</td>
<td>3 versus 1</td>
<td>3.66</td>
</tr>
</tbody>
</table>

**Notes:** Results conditioned on the existence of one cointegrating vector. The critical value for the $\chi^2(1)$ statistic is 3.84 and 6.63 for the 5 and 1% significance levels respectively.
Table 4
Johansen’s Cointegration Test (Seigniorage Included as Revenues)

<table>
<thead>
<tr>
<th>Test Statistic (Critical Value at the 20% Level)</th>
<th>Cointegrating Vector (Expend., Taxes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>Trace</td>
</tr>
<tr>
<td>$9.185^*$</td>
<td>1.638</td>
</tr>
<tr>
<td>(7.58)</td>
<td>(1.82)</td>
</tr>
</tbody>
</table>

Cointegration Restriction Test

Restriction $(1, -1) \chi^2(1) = 1.50$; $p$-value: 22.14%

Notes: The symbol (*) indicates rejection of the null at the 20% significance level.
### Table 5

**Vector Error Correction Model Estimates**

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Est. Coeff.</th>
<th>$t$-stat</th>
<th>$t$-prob</th>
<th>Est. Coeff.</th>
<th>$t$-stat</th>
<th>$t$-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta G_{t-1}^*$</td>
<td>0.17964</td>
<td>0.987</td>
<td>0.3298</td>
<td>0.23177</td>
<td>2.04</td>
<td>0.0474</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td></td>
<td></td>
<td>(0.09)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta G_{t-2}^*$</td>
<td>-0.73603</td>
<td>-3.558</td>
<td>0.0010</td>
<td>-0.1790</td>
<td>-1.39</td>
<td>0.1720</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td></td>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta T_{t-1}$</td>
<td>-0.30270</td>
<td>-1.163</td>
<td>0.2521</td>
<td>-0.3066</td>
<td>-1.89</td>
<td>0.0657</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td></td>
<td></td>
<td>(0.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta T_{t-2}$</td>
<td>0.61572</td>
<td>2.459</td>
<td>0.0186</td>
<td>0.15161</td>
<td>0.97</td>
<td>0.3362</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td></td>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Def$_{t-1}$</td>
<td>0.07624</td>
<td>0.503</td>
<td>0.6179</td>
<td>0.25661</td>
<td>2.72</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td></td>
<td></td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Heteroskedasticity-Consistent standard errors in parentheses.
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test Statistic</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_i$ is weakly exogenous for the parameter of interest of the $G_i$ conditional model</td>
<td>7.36</td>
<td>0.0067**</td>
</tr>
<tr>
<td>$G_i$ is weakly exogenous for the parameter of interest of the $T_i$ conditional model</td>
<td>1.11</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Note: The symbol (**) indicates rejection of the null at the 1% significance level.
Table 7  
Granger-Causality Test

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_t$ does not Granger-cause $G_i^*$</td>
<td>0.023*</td>
</tr>
<tr>
<td>$G_i^*$ does not Granger-cause $T_t$</td>
<td>0.0000**</td>
</tr>
</tbody>
</table>

Notes: The symbols (*) and (**) represent rejection of the null at the 5 and 1% significance levels respectively.
Table 8: Unconventional Impulse-Response Function

| (ρ=0.97) |  
|----------|----------|
| Impulse  | T | T | G* | G* |
| Response | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ |
| Pt-Estimate | -1.22 | -0.07 | 1.12 | 0.05 |
| SE       | (2.13) | (0.47) | (2.13) | (0.51) |

| (ρ=0.98) |  
|----------|----------|
| Impulse  | T | T | G* | G* |
| Response | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ |
| Pt-Estimate | -1.13 | -0.04 | 1.01 | -0.09 |
| SE       | (1.05) | (0.39) | (0.99) | (0.42) |

| (ρ=0.99) |  
|----------|----------|
| Impulse  | T | T | G* | G* |
| Response | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ |
| Pt-Estimate | -1.06 | -0.02 | 0.93 | -0.11 |
| SE       | (0.09) | (0.03) | (0.20) | (0.18) |

| (ρ=1.00) |  
|----------|----------|
| Impulse  | T | T | G* | G* |
| Response | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ | $P\hat{V}(\Delta T)$ | $P\hat{V}(\Delta G^*)$ |
| Pt-Estimate | -1.00 | 0 | 0.89 | -0.11 |
| SE       | (-) | (-) | (0.15) | (0.15) |

Notes: Standard errors calculated using a Monte Carlo, where the Data Generating Process (DGP) was assumed to be the Restricted (cointegration) Vector Error Correction Model. The estimated model in the Monte Carlo corresponds to the DGP. Pre-sample data were also drawn, and not assumed fixed. The number of replications was set to 1,000. When ρ=1, the responses to innovations to $T_t$ are not stochastic.
Table 9
Johansen’s Cointegration Test (*Seigniorage Excluded as Revenues*)

<table>
<thead>
<tr>
<th>Test Statistic (Critical Value at the 20% Level)</th>
<th>Cointegrating Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>Trace</td>
</tr>
<tr>
<td>$K = 0$</td>
<td>$K \leq 1$</td>
</tr>
<tr>
<td>7.305</td>
<td>0.0454</td>
</tr>
<tr>
<td>(10.04)</td>
<td>(1.66)</td>
</tr>
</tbody>
</table>

Notes: The symbol (*) indicates rejection of the null at the 20% significance level. The VAR is modeled with an unrestricted constant and two lags, according to the Schwarz Criterium and to likelihood ratio tests of the deterministic components respectively.