FINANCIAL DEEPENING IN BRAZIL

CAPÍTULO II

MONETARY CORRECTION AND ACCOUNTING WITH REAL INTEREST

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MONETARY CORRECTION AND REAL INTEREST ACCOUNTING

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2.1) Introduction

Monetary correction is a method used for correcting monetary values when the currency in which claims are denominated has its purchasing power (of a set of goods) changing over time. It is a tool used to reduce distortions from inflation. Ultimately, it represents an implicit institutional acceptance of the need to disassociate two uses of money: as a means of payment, and as a means of accounting. Monetary correction is also generally used as a synonym for the difference between nominal and real interest paid on a security. These amounts are obviously not the same, though, when the inflation index differ from the monetary correction index. To differentiate among both, we shall use the distinct terms "inflationary correction", which takes into consideration the inflation index, and "monetary correction", based on the monetary correction index. By definition, the difference between nominal and real interest is given by the "inflationary correction".

In Brazil, the development of monetary correction techniques and legislation began in 1964. The complicated set of rules developed from this date onward reflect a second best alternative chosen by the country to minimize the costs of inflation. This was done instead of eliminating inflation.

In a certain way, the costly allocation of the nation's scarce resources to create, develop and apply these rules, can be viewed as the price paid for using a fiscal system based more on disguised indirect taxation (inflationary tax) instead of explicit direct taxation. As we show in section 2.3.4, in some years the inflationary tax exceeded five percent of GDP. As a counterpart, two groups have benefitted from this perverse method of government financing an excess of expenditures over receipts:
the commercial banks, which earn high rates of real interest on the free part of demand deposits, and the accountants and economists, who are paid to create, understand, translate and apply this apparatus.

Monetary correction indexes are set each month at the discretion of the government. As table 2.4 shows, they generally have been set below inflation at the present time. The face value of Indexed Treasury Bonds (OTNs) is the indicator of the monetary correction index.

After a heuristic introduction to the necessity of an institutional apparatus related to monetary correction in countries with inflation, this chapter solely concentrates on the differentiation between nominal and real interest accounting.

This problem extends to many important macroeconomic statistics. The first one, studied in sections 2.3.1, 2.3.2, 2.3.3 and 2.3.5, is the controversy about the estimates of public deficits. Since 1982, the operational concept of deficit, which deals with real interest, has been systematically used in Brazil. Many economists, however, still nowadays doubt that this should be the appropriate figure to use to estimate the disequilibrium of public finance. In these sections, besides presenting some econometric results to serve as empirical background for the question, we try to explain the main lines of the methodology used in these calculations. Some simplified ways to evaluate the difference between nominal and real interest paid on financial assets are also presented.

Section 2.3.4 presents the total amounts of inflationary tax and inflationary transfers of the commercial banks in Brazil since 1947. They are defined as the negative real
interest paid by the Monetary Authorities and the commercial banks, respectively, on the stock of High Powered Money and on the excess of demand deposits over reserve requirements of the commercial banks. Added up, they reach, in some years, the amazing figures of almost 9% of GDP.

Section 2.3.6 extends the methodology of real interest accounting to social accounting. The differences are concentrated on the income side. There are two possible distributions of the Gross Domestic Income among the government, households, firms and external sector. One with nominal interest, and the other with real interest accounting. The operational methodology is also introduced. This represents a half-way between the nominal and real interest methodology.

Finally, section 2.3.7 shows that the use of real interest to calculate the budget deficit of the government or in social accounting can be seen as a simple extension of the profits correction mechanism introduced in Brazil in 1964. In this way, it represents nothing new.
2.2) The Demand For Monetary Correction

In an economy facing high rates of inflation, the continuous loss of real value of the accounting unit leads to the necessity of compensatory mechanisms, which allow for the dissociation between the real and nominal values of the transactions. It is obviously a second best solution, since it would always be better to live without inflation.

A trivial problem is the one deflating time series for the purpose of comparisons of values situated at different points in time. The method transforms monetary values denominated in currency of (purchasing power of) period $j$ ($V_j$) into monetary values denominated in currency of period $T$ ($V_T$) by means of the formula:

$$V_T = V_j \left( \frac{P_T}{P_j} \right)$$

$P_j$ and $P_T$ standing for the price level, respectively, at time $j$ and $T$. Another crucial problem is to distinguish, in the same period of time (let us say, one year), the nominal interest payment from the real. In periods of inflation, the creditors real income is less than the nominal one, the opposite happening to the debtors. This distinction is necessary to delineate the appropriate taxation, profits distribution, budgetary forecasts and analysis.

Among the practical problems that arise in the day-to-day life of economic agents, we shall analyze three main categories: a) those related to the necessary correction of nominal values over time; b) those concerning the demand and supply of credit and; c) those concerning the accounting procedures. These last two categories rely on the need for a clear distinction, in the same period of time, between nominal interest and real interest.
a) Expression (1) tells us that without price stability, cruzados received at time $T$ are different (in terms of purchasing power), from cruzados received (or paid) at time $j$. Transactions among economic agents are fundamentally based on purchasing power, rather than on nominal values. Consequently, it would be reasonable to expect that automatic corrections based on equation (1) were endogenously incorporated in contracts and agreements concluded between economic agents. This is the popular escalator clause mechanism, which proves its usefulness whenever the same transaction has its money payment specified in at least two different points in time. Among the most known examples are the penalties incurred by debtor in the case of delaying a previously agreed payment and the setting of different types of income (such as wages, rents) to be received at different points in time.

Although it may seem trivial, many years of inflation-induced arbitrary transfers from creditors to debtors occurred before this institutional apparatus of monetary correction was legally established in Brazil. For example, with respect to tax liabilities, only in 1964 was debt correction introduced. The very high rates of inflation existing in those years (51.2% in 1962, 81.3% in 1963 and 91.9% in 1964) made it highly profitable to delay payments to the government. Law 4357 stipulated that fiscal debts should be corrected according to indexes to be established by the Planning Ministry. Moreover, any fines and penalties associated with delayed payments were to be calculated over the corrected values.

Before 1964, wages generally were corrected on a yearly basis, informally, taking into consideration past inflation. The methodology introduced by Laws 54018 (of July 14, 1964)
and 54228 (of September 1, 1964) established the first official rules for fixing wages in Brazil. The details of this will be studied in the next chapter.

As concerns rents, the situation until 1964 was really a dramatic one. Between 1950 and 1964, rents were systematically corrected below inflation. This led to a decrease of the real income of real estate owners, a sharp increase of the initial prices of the new rents, and a decrease in investments in the sector, aggravating the housing shortage. The following table contrasts movement in the rent cost index with the General Price Index during the period 1953-1968:

Table 2.1

<table>
<thead>
<tr>
<th>YEAR</th>
<th>RENT INDEX (A)</th>
<th>COST OF LIVING INDEX (B)</th>
<th>A/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>100</td>
<td>100</td>
<td>1,000</td>
</tr>
<tr>
<td>1954</td>
<td>120</td>
<td>122</td>
<td>0,984</td>
</tr>
<tr>
<td>1955</td>
<td>137</td>
<td>151</td>
<td>0,907</td>
</tr>
<tr>
<td>1956</td>
<td>160</td>
<td>182</td>
<td>0,879</td>
</tr>
<tr>
<td>1957</td>
<td>191</td>
<td>212</td>
<td>0,901</td>
</tr>
<tr>
<td>1958</td>
<td>223</td>
<td>243</td>
<td>0,918</td>
</tr>
<tr>
<td>1959</td>
<td>267</td>
<td>338</td>
<td>0,790</td>
</tr>
<tr>
<td>1960</td>
<td>317</td>
<td>437</td>
<td>0,725</td>
</tr>
<tr>
<td>1961</td>
<td>372</td>
<td>583</td>
<td>0,638</td>
</tr>
<tr>
<td>1962</td>
<td>509</td>
<td>884</td>
<td>0,576</td>
</tr>
<tr>
<td>1963</td>
<td>774</td>
<td>1507</td>
<td>0,514</td>
</tr>
<tr>
<td>1964</td>
<td>1169</td>
<td>2889</td>
<td>0,405</td>
</tr>
<tr>
<td>1965</td>
<td>2313</td>
<td>4787</td>
<td>0,483</td>
</tr>
<tr>
<td>1966</td>
<td>4050</td>
<td>6764</td>
<td>0,599</td>
</tr>
<tr>
<td>1967</td>
<td>6213</td>
<td>8824</td>
<td>0,704</td>
</tr>
<tr>
<td>1968</td>
<td>8426</td>
<td>10766</td>
<td>0,783</td>
</tr>
</tbody>
</table>

Source: Chacel, Simonsen and Wald (1976).

Original Source: Getulio Vargas Foundation.
One can observe the sharp deterioration of the terms of trade against the landlords until 1964, and the inversion of this trend as of this date, when escalator clauses were introduced and became generalized.

The classical stabilization mechanism, by means of which high rates of inflation lead to a migration of tax payers from lower levels to the higher levels of income tax brackets was not important in Brazil. First, with the high rates of inflation after 1961, these brackets of income were fixed in terms of minimum wages, which were periodically revised (otherwise, the taxation would become unbearable for those situated in the lower income brackets). Second, indirect taxes always represented the largest share of taxation, and the direct taxes on firms were more important than the same taxes on individuals. Beginning with Law 4506 (1965) income brackets became indexed. Nowadays, they are adjusted annually according to a coefficient established at the discretion of the Ministry of Finance, which is generally a measure of the inflation that occurred in the period.

b) We turn our attention to the demand and supply of credit.

A natural solution to unanticipated devaluation of the figures set in loan agreements due to inflation would be a floating interest rate. This interest rate would reflect the costs of attracting money to financial institutions and should vary in proportion to inflation. As is well known though, this mechanism does not work in high inflation environments, because of one simple reason: if the interest is not accrued to the principal, the higher the inflation, the higher will be the fall
of the real value of the debt or credit. This subject is discussed in section 2.3.1. It can lead to acute problems of insolvency. If we take, for instance, a 100% per year inflation, the complete payment of the nominal interest accruing on the principal will reduce the real value of the debt to one half. In other words, in one year, the debtor would have been obliged to pay off half of the real value of the debt.

A feasible alternative would be the floating of the real rate of interest. The nominal periodical payments would be corrected by some price index, and a real and variable interest rate would accrue over this amount.

In Brazil, there was no room for these procedures up to 1964. Law 23501, enacted in November 27, 1933, prohibited the use of any means of account other than cruzeiros. This made it illegal to monetarily correct contracted or agreed values (taking into consideration the exchange rate devaluation or the variation of the domestic price index in the period). At the same time, Law 22626, of April 07, 1933, prohibited the charging of nominal interest rates above twelve percent a year (a similar measure has just been introduced in the new Constitution, but, at this time, concerning the real rate).

Taking these two Laws into consideration interest rates in Brazil had a ceiling of 12% a year. Given the escalation of inflation from 1957 onward, the natural consequence was a decline of the real interest rate and an excess demand for credit. Suddenly it became very cheap to borrow; good for borrowers, but bad for lenders.

Three artifices to increase the effective price of loans were in general use by lenders at this time. The first
was the widespread use of banking commissions. The second was constituted by off-the-record payments. The third, and more important, was the requirement of maintaining a minimum stipulated cash deposit relative to the loan taken.

In practice, this means that the financial institution actually lends only a fraction of what is stipulated in the contract. By this last trick, a 12% a year loan of ten cruzeiros followed by the requirement of five cruzeiros as average balance in cash turns out to be equivalent to a 24% a year cost of the loan. Yet, these artifices were not enough to deal with the interest ceiling problem. Graph 2.1 helps to understand what happens to the credit market on such an occasion. We admit that both borrowers and lenders have no money illusion and that, consequently, both the supply of funds (S) and the demand for credit (D) are functions of the real expected interest rate (\( \tilde{r} - \tilde{\pi}^e \)). \( \tilde{\pi}^e \) stands for the logarithm(1) expected rate of inflation and \( \tilde{r} \) for the logarithm nominal interest rate. For the sake of simplicity, we admit that expected inflation is equal to the actual rate of inflation (\( \tilde{\pi} = \tilde{\pi}^e \)).

**Graph 2.1**

Credit Market

\[
\begin{align*}
S & : 12\% \\
D & : 24\%
\end{align*}
\]

(1) We work with logarithmic rates of inflation and interest defined as \( \tilde{\pi} = \ln(1+\pi) \) and \( \tilde{r} = \ln(1+r) \) where \( \pi \) and \( r \) are the popular rates. This artifice transforms operations with composed interest into operations with simple (logarithmic) interest. By this procedure, \( \tilde{r} - \tilde{\pi}^e \) is not an approximation, but an exact value of the real expected rate of inflation.
We depart from an initial short run equilibrium, shown in Graph 1. With the rise of inflation (and consequently, of the expected inflation), the two curves shift upwards (since both S and D are functions of $\bar{\pi} - \pi^e$) leading to an excess demand for credit given by $D' - S'$:

**Graph 2.2**
Credit Market Rationing

The natural consequence of the interest ceiling is an arbitrary income transfer against those with limited access to the market. Since the minimum between demand and supply must prevail, the statistics of the supplied credit must show a sharp decline (from $S^e$ to $S'$). In fact, they do. Table 2.2 shows that the time deposit share in $M_2$ ($M_1$ plus time deposits) declined from 24% in 1951 to 4.0% in 1965. As of this date, a small recovery can be observed due to the introduction of monetary correction.
### Table 2.2
Commercial Banks Time and Demand Deposits
Balances in Cr$ millions - End of Period

<table>
<thead>
<tr>
<th>YEAR</th>
<th>DEMAND DEPOSITS (1)</th>
<th>TIME DEPOSITS (2)</th>
<th>TOTAL (3)</th>
<th>PERCENTAGE (2)in (3)</th>
<th>GENERAL WHOLESALE PRICE INDEX (1953=100)</th>
<th>DEPOSITS IN CONSTANT CRUZEIROS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DEPOSITS IN CRUZEIROS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DEMAND DEPOSITS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td>52,2</td>
<td>16,9</td>
<td>69,1</td>
<td>24,4</td>
<td>82,5</td>
<td>63,3</td>
</tr>
<tr>
<td>1952</td>
<td>60,3</td>
<td>16,9</td>
<td>72,2</td>
<td>23,4</td>
<td>90,4</td>
<td>66,7</td>
</tr>
<tr>
<td>1953</td>
<td>71,2</td>
<td>17,7</td>
<td>88,9</td>
<td>19,9</td>
<td>113,2</td>
<td>62,9</td>
</tr>
<tr>
<td>1954</td>
<td>84,9</td>
<td>20,4</td>
<td>105,3</td>
<td>13,3</td>
<td>140,3</td>
<td>60,5</td>
</tr>
<tr>
<td>1955</td>
<td>102,4</td>
<td>19,8</td>
<td>122,2</td>
<td>16,2</td>
<td>153,5</td>
<td>66,7</td>
</tr>
<tr>
<td>1956</td>
<td>126,6</td>
<td>21,1</td>
<td>147,7</td>
<td>14,3</td>
<td>192,9</td>
<td>65,6</td>
</tr>
<tr>
<td>1957</td>
<td>176,1</td>
<td>24,3</td>
<td>200,4</td>
<td>12,1</td>
<td>199,4</td>
<td>88,3</td>
</tr>
<tr>
<td>1958</td>
<td>216,1</td>
<td>25,8</td>
<td>241,9</td>
<td>10,6</td>
<td>255,0</td>
<td>84,7</td>
</tr>
<tr>
<td>1959</td>
<td>321,6</td>
<td>30,8</td>
<td>352,4</td>
<td>8,7</td>
<td>347,1</td>
<td>92,7</td>
</tr>
<tr>
<td>1960</td>
<td>438,2</td>
<td>47,4</td>
<td>485,6</td>
<td>9,8</td>
<td>460,8</td>
<td>95,1</td>
</tr>
<tr>
<td>1961</td>
<td>610,8</td>
<td>55,2</td>
<td>666,0</td>
<td>8,3</td>
<td>691,6</td>
<td>88,3</td>
</tr>
<tr>
<td>1962</td>
<td>1,037,7</td>
<td>56,3</td>
<td>1,094,0</td>
<td>5,1</td>
<td>1,037,0</td>
<td>100,0</td>
</tr>
<tr>
<td>1963</td>
<td>1,703,9</td>
<td>89,4</td>
<td>1,793,3</td>
<td>5,0</td>
<td>1,886,0</td>
<td>90,3</td>
</tr>
<tr>
<td>1964</td>
<td>3,069,6</td>
<td>148,3</td>
<td>3,217,9</td>
<td>4,6</td>
<td>3,645,0</td>
<td>84,2</td>
</tr>
<tr>
<td>1965</td>
<td>5,799,7</td>
<td>241,7</td>
<td>6,041,4</td>
<td>4,0</td>
<td>4,676,0</td>
<td>124,0</td>
</tr>
<tr>
<td>1966</td>
<td>6,390,0</td>
<td>696,0</td>
<td>7,086,0</td>
<td>9,8</td>
<td>6,413,0</td>
<td>99,6</td>
</tr>
<tr>
<td>1967</td>
<td>9,622,0</td>
<td>533,5</td>
<td>10,155,5</td>
<td>5,3</td>
<td>7,881,0</td>
<td>122,1</td>
</tr>
<tr>
<td>1968</td>
<td>13,483,8</td>
<td>918,5</td>
<td>14,402,3</td>
<td>6,4</td>
<td>9,854,0</td>
<td>136,9</td>
</tr>
</tbody>
</table>

Source: Chace1, Simonsen and Wald (1976).
Original Source: Central Bank of Brazil.

Concerning debentures, the situation was the same. Despite the premium and the short maturity at which they were sold, their proportion to GDP fell steadily between 1956 and 1964. behind all these hindrances to the efficient working of the financial market and, consequently, to capital formation and economic development, was the law making it illegal to transform cruzeiros of one period into cruzeiros of another. In other words, it was impossible to use escalator clauses in contracts and agreements. These obstacles were finally removed after 1964. On July 17, 1964, Law 4357 allowed the government to issue bonds...
with escalator clauses. This was a first step into a generalized indexation of other assets, including saving accounts, mortgages, labor funds (FGTS), etc.

c) Finally let us now turn our attention to the accounting problems which arise due to inflation. We focus on them for the remainder of this chapter. With respect to the enterprise sector, they are twofold. First, the net financial assets generate a flow of income (nominal interest) whose main part (the inflationary correction) is only to keep constant its real value over time. If this income is incorporated to the profit calculations, there will be an overestimation of this figure, leading the firm to pay more taxes and distribute more dividends to shareholders than it should. As a consequence, other things constant, the real value of the firm's assets will decrease at the end of the period.

Data available for the period 1958-64 show that these illusory profits were responsible for over fifty percent of the profits of corporations. These figures will be presented in section 2.3.7. To avoid such problems the correct procedure would be to distinguish the nominal from the real interest, and only incorporate the latter in profits. In Brazil, the figures calculated under this methodology are called "Lucros Corrigidos" (Corrected Profits). We will get back to this matter in section 2.3.7.

The necessity to distinguish between nominal and real profits is the same when a firm or an individual sells a capital good. Should the difference between the selling and buying
price be interpreted as a profit, the taxation would be higher than it should, since part of this difference is only due to the monetary correction of the physical asset price.

Another problem for business firms which could happen when there is inflation (and this problem is not recognized by the Laws) is the overpayment of taxes due to the underestimation of the allowed fiscal depreciation of physical assets. The point is that with prices steadily rising, the replacement values of the physical assets are higher than the historical ones. If this fact is not legally recognized, the fiscal depreciation of physical assets, calculated in terms of the historical values, will be systematically underestimated.

To illustrate this, let us take a firm which has acquired a Cz$ 100,00 capital good, whose useful life is supposed to be four years. If we work with a hundred percent per year inflation rate, the corrected depreciation and the depreciation based on the historical value of the capital good would be, in current cruzeiros:

Table 2.3

<table>
<thead>
<tr>
<th>Year</th>
<th>Approximate Corrected Value of the Capital Good at Average Prices of the Year</th>
<th>Corrected Depreciation</th>
<th>Allowed Depreciation</th>
<th>Excess Taxed Income (Corrected - Allowed Depreciation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>37,5</td>
<td>25</td>
<td>12,5</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>75</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>150</td>
<td>25</td>
<td>125</td>
</tr>
<tr>
<td>4</td>
<td>1,200</td>
<td>300</td>
<td>25</td>
<td>275</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>562,5</td>
<td>100</td>
<td>462,5</td>
</tr>
</tbody>
</table>
As can be seen from the above table, the underestimation of the amount to be subtracted from the gross income (fiscal allowed depreciation) to obtain the taxable income systematically leads to overtaxation. This problem was first recognized by Law 4357, of July 01, 1964, which allowed for the use of monetarily corrected depreciation values.

Another reason for careful discrimination between nominal and real interest arises in the construction of the flow of funds and social accounting statistics. If there is no money illusion, the figures of the budget deficit calculated with real interest can eventually be more appropriate than the other ones. The same thing happens in social accounting. This will be the main subject of sections 2.3.1 and 2.3.6.
2.3) The Real Interest Accounting Methodology

2.3.1) Controversy Over Different Concepts of the Public Deficit

A striking problem emerged during the negotiations initiated between Brazil and The International Monetary Fund (IMF) representatives at the end of 1982. The IMF maintained that Brazilian negotiators should calculate the budget figures in accordance with the agreed forecast of inflation. The problem is that these forecasts were systematically underestimated. The indexation rules existing at that time made the economy much less sensitive to aggregate demand control than normally would be expected. Consequently, interest paid on the public debt was systematically underestimated. In sum, every time the inflation rate was higher than predicted, interest expenditures of the government paid on public debt also were higher, and, as a result, the budget deficit.

This problem partially explains why the IMF, which generally negotiates one "Letter of Commitment" per year, with countries which seek its financial aid, found it necessary to analyze four proposed agreements in 1983 and three in 1984, during negotiations with the Brazilian authorities.

At this time the Brazilian representative at the IMF suggested that the budget forecasts should deal only with real interest paid on the debt, since the part of interest payments due to inflation would always be beyond the control of the government.

The suggestion was not easily or promptly accepted by the Fund technicians (or even by the majority of Brazilian economists). They argued that nominal interest expenses were to be matched exactly like all other expenses: by tax collection,
money printing, or expansion of the public net debt. In this way, it would be nonsense to try to construct a deficit figure leaving aside these amounts.

Six years later (1988) the real interest approach to the calculus of budget figures seems to be accepted as common sense. In Brazil, the "operational deficit" (which will be defined later) is the statistic connected to fiscal policy management divulged by the government. In the Mexican stabilization plan of 1988 aggregate demand monitoring was based on what is termed "primary deficit", which goes beyond brazilian methodology, also excluding the real interest payments from recorded government expenditures. As a general rule, the same kind of proceeding has been used to describe the fiscal accounts of many countries facing high rates of inflation.

Our main purpose here will be to analyze the possible theoretical bases behind these (previously unusual) procedures. We begin remembering a trivial fact already mentioned in section 2.2: in inflationary environments, if the interest on net debts are not accrued to the principal, the higher the inflation rate, the more quickly the real value of the debt will decline. As a general rule, if the nominal deficit is equal to zero and inflation to \( \pi \), the real amortization of the principal
will be equal to a fraction $\pi/(1+\pi)$\textsuperscript{(1)} of the initial real debt. As can easily be seen, in this case the amortization of the real value of the debt is an increasing function of inflation.

Put another way, this reasoning means that high nominal government deficits in high inflation countries should be seen as a natural consequence of the fact that debtors cannot clear up their real debts given such high rates\textsuperscript{(2)}. And even if they could, there is no economic reason why amortization should be kept proportional to inflation, as in the case of a zero (or any other fixed number) nominal deficit target.

This simple arithmetic shows that, in countries where inflation is high and unstable, it becomes very problematic trying to monitor aggregate demand by forecasting nominal budget deficit figures. The two possible alternatives are a) to fully eliminate any interest receipts or payments from the budget calculations; and b) to work only with real interest rates.

Another point to be noted is that, contrarily to the arguments presented by FMI members, interest payments are not exactly equivalent to wage, dividends or rent payments. The reason is an accounting one: only financial assets systematically lose their value in inflationary environments. This does not mean that wages, for example, are not affected by inflation. But that the possible losses or gains arising from inflation are not systematic, since they are continuously readjusted over time. In other words, it makes no sense, for instance, for a firm, to compare total wage payments with total interest payments, in order

\textsuperscript{(1)} If $F_t$ stands for the nominal debt the real amortization between period zero and one at prices of period zero will be equal to $F_0 - F_0 / (1+\pi) = F_0 \pi / (1+\pi)$.

\textsuperscript{(2)} With an inflation rate of more than two hundred percent a year, for example, as presented by Brazil in the period 1983-85, real amortization would be over two thirds of the debt in each year.
to evaluate the relative shares in production costs. Indeed, included in the nominal interest payments is the real amortization of the firms liabilities, which has nothing to do with production costs.

If inflation is hundred percent a year, and real interest rate zero, a Cz$ 100,00 debt will be responsible for Cz$ 100,00 interest payments, but the real value of the debt will be reduced to a half at the end of the period. Should one want to compare relative shares in production costs, real interest should replace nominal interest. This is equivalent to work with nominal interest and crediting the current account of the economic agent with the real amortization of the respective debt. This distinction between interest and other remunerations originated the differentiated treatment given to interest in the corporate profits evaluation. The operational deficit concept can be seen as a simple extension of this accounting asymmetry.

Of course, the construction of any statistic will be more or less appropriate depending on its intended uses. Let us assume that the main purpose of the budget deficit calculations is to relate it to the control of aggregate demand. As we know, interest receipts and payments affect aggregate demand by means of the changes they impose on the net income of the private sector. Consequently, what we have to ask is: what kind of net income of the private sector should be considered? The one calculated with nominal interest or real interest? In a first approach to this question, wholly based on the possibility of there existing money illusion, the answer
would be the first, if economic agents should decide about their consumption levels taking into consideration the whole amount of interest they get from holding public debt; and the second, if they recognized that a part of these earnings is only a nominal restitution of the real value of the debt that was missed due to inflation. In other words, if there existed money illusion, the nominal deficit would also be the important one to determine private consumption; the other way around, if the household sector of the economy were free of confusion between nominal and real values, they would consider as income only the real interest received (or paid) on the net government debt. This would be the case for real interest accounting.

This last is equivalent to the supposition that the marginal propensity to consume over inflationary correction is equal to zero. Other things constant, at the maturity date of the government securities the roll-over of the debt would not call for any increase of the real interest rate(1). In a two hundred per cent inflation environment, for example, people would recognize that the first two cruzados interest paid on each cruzado invested in Treasury Securities, at the beginning of the period, should not be taken as real income, and in this way, should not affect their consumption levels.

According to some analysts, the possibility of there existing money illusion tells only one part of the story. Following this alternative approach to the question, even if there is no confusion between nominal and real interest, aggregate demand can be correlated with the nominal deficit due to the uncertainty about the monetary correction index. When inflation rate increases and,
consequently, the nominal deficit, monetary correction indexes fail to follow the price jumps due to lags in their calculation. Therefore, there is a flight from indexed bonds to non-indexed bonds and, particularly, to durable consumption goods, fostering aggregate demand.

Table 2.4 shows that the underestimation of monetary corrections indexes, at least relatively to the IGP and IPA indexes, has really been a constant in the last fifteen years.
Table 2.4
MONETARY CORRECTION AND INFLATION INDEXES
(End of Period)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M)netary Correction (M)</td>
<td>15.3</td>
<td>12.8</td>
<td>33.3</td>
<td>24.2</td>
<td>37.2</td>
<td>30.1</td>
<td>36.2</td>
<td>47.2</td>
<td>50.8</td>
<td>95.6</td>
<td>97.8</td>
<td>156.6</td>
<td>215.3</td>
<td>219.4</td>
<td>50.7</td>
<td>391.5</td>
</tr>
<tr>
<td>Inflation $\pi_1$ (IGP-DI)</td>
<td>15.7</td>
<td>15.5</td>
<td>34.5</td>
<td>29.4</td>
<td>46.3</td>
<td>38.8</td>
<td>40.8</td>
<td>77.2</td>
<td>110.2</td>
<td>95.2</td>
<td>99.7</td>
<td>211.0</td>
<td>223.8</td>
<td>235.1</td>
<td>65.0</td>
<td>415.8</td>
</tr>
<tr>
<td>Inflation $\pi_2$ (IPA-DI)</td>
<td>15.9</td>
<td>15.3</td>
<td>29.1</td>
<td>29.3</td>
<td>44.9</td>
<td>35.5</td>
<td>43.0</td>
<td>80.1</td>
<td>121.3</td>
<td>94.3</td>
<td>97.7</td>
<td>234.0</td>
<td>223.6</td>
<td>225.7</td>
<td>62.6</td>
<td>407.2</td>
</tr>
<tr>
<td>$\alpha_1 = \frac{M}{\pi_1}$</td>
<td>0.97</td>
<td>0.83</td>
<td>0.96</td>
<td>0.82</td>
<td>0.80</td>
<td>0.77</td>
<td>0.89</td>
<td>0.61</td>
<td>0.46</td>
<td>1.00</td>
<td>0.98</td>
<td>0.74</td>
<td>0.96</td>
<td>0.93</td>
<td>0.78</td>
<td>0.94</td>
</tr>
<tr>
<td>$\alpha_2 = \frac{M}{\pi_2}$</td>
<td>0.96</td>
<td>0.84</td>
<td>1.14</td>
<td>0.82</td>
<td>0.83</td>
<td>0.85</td>
<td>0.84</td>
<td>0.59</td>
<td>0.42</td>
<td>1.01</td>
<td>1.00</td>
<td>0.66</td>
<td>0.96</td>
<td>0.95</td>
<td>0.81</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Since March, 1987, though, monetary correction indexes have exactly followed the rate of change of the IPG (Índice de Preços ao Consumidor), monthly published by IPGE. With this procedure, inflation is in average one month ahead of monetary correction indexes. This is to mean that, when inflation is increasing, the face value of indexed bonds, relatively to a basket of goods, really declines.

However, this is not an argument for a correlation between nominal interest payments and aggregate demand\(^1\). Indeed, the possible flights from indexed bonds to non indexed bonds and, particularly, to durable consumption goods, would be correlated to the increase of inflation (which would lead to underestimates of the monetary correction index), and not to inflation itself\(^2\). If savers should, coeteris paribus, demand higher real rates of interest to keep the same real value of governments debt in their portfolios, this would be positively correlated to the increase of nominal interest payments, and not to their actual levels.

Consequently, in the absence of money illusion, if the real public deficit is to be complemented by an alternative variable to evaluate the conduction of fiscal policy, this one should be the monthly increase of the nominal deficit, rather than the nominal deficit itself. The effective magnitude of this second order factor in affecting aggregate demand is something to deserve further investigations.

\(^1\) In other words, this is not an argument for the use of deficit figures calculated with nominal, instead of real interest.

\(^2\) What is backed up by empirical estimates based in monthly data.
Given the remaining hypothesis of money illusion, the choice between the use of nominal or real interest figures in the calculations of the budget deficit is, therefore, a purely empirical question. In each economy, at each period in time, the task is to determine if the possible confusion between nominal and real interest makes aggregate demand a positive function of monetary correction. Two possible kinds of tests are: a) to obtain estimates of the consumption function of the private sector taking as one of the explanatory variables the monetary correction on the government internal debt, and b) to accompany the roll-over process of the debt, to determine if higher real interest rates are demanded by savers when the nominal value of the debt increases.

In Brazil, no serious attempt has been made up to now to evaluate what definition of public deficit should be relevant to determine control over aggregate demand. After the introduction of the "operational deficit" concept, (which works with real interest), in 1982, this statistic has been systematically used, not only in academic and theoretical discussions, but also in negotiations with international organizations such as the IMF and the World Bank. The nominal deficit figures also are divulged by the Central Bank, but have deserved almost no references in economic discussions.

A simple correlation exercise shows us that the marginal propensity to consume over the difference between nominal and real interest, if different from zero, could not be very high. Indeed, after 1982 inflation rates more than doubled and with this the inflationary correction of the government debt received by the private
sector. Nonetheless, private savings increased as a percent of GNP in the years 1983, 1984 and 1985 (from 13.5% in 1983 to 19.4% in 1984 and 25.9% in 1985).

In order to provide empirical evidence for this question, we present here some econometric exercises, where the consumption function is taken as the endogenous variable and the GDP and the monetary correction on the internal (federal, state and local government) debt as the explanatory variables. We work with data available for the period 1970-1986. Due to lags, variables definitions (using variations) and the use of econometric methods other than Ordinary Least Squares (1), the number of observations range from 13 to 15. To avoid multicollinearity, all variables are used in constant cruzados.

The basic equation to be estimated is given by:

$$ C = a + bY_D + c ICI, $$

where

$C =$ Consumption,

$Y_D =$ Disposable Income of the Private Sector

$ICI =$ Inflationary correction on the internal debt

The ICI variable was obtained taking into consideration the inflation in each period, rather than the correction indexes arbitrarily set by the government. The disposable income of the private sector was calculated subtracting from the Gross National Product (GNP) the government net receipts (taxes minus subsidies and transferences (except interest) and the inflationary tax. The nominal interest on the public debt received by the private sector was not included. Other technical details concerning the calculations are presented in appendix.

(1) Due to the well known problem of simultaneity related to the estimation of the consumption function, we also present some results based on two stage least squares.
Table 2.5 displays the main results. Theoretically, $b$ should be positive, and range between zero and one. Were there money illusion, coefficient $c$ should be strictly positive.

One can observe in all equations that it is statistically impossible to neglect the hypothesis that aggregate consumption does not depend on the monetary correction. This result constitutes an empirical background to the use of real interest in fiscal accounting.(1)

(1) It must be emphasized that this result is very intuitive. Otherwise, how could the government have afforded to domestically finance the major part (67%) of its (nominal) deficit, around 30% of GDP, with almost zero average real interest rates, as it happened in 1987?
### Table 2.5

**ESTIMATES OF THE CONSUMPTION FUNCTION**

<table>
<thead>
<tr>
<th>Equations</th>
<th>Variables</th>
<th>Constant</th>
<th>$Y_t$</th>
<th>$Y_{t-1}$</th>
<th>ICI$_t$</th>
<th>ICI$_{t-1}$</th>
<th>Dummy</th>
<th>Method</th>
<th>I.V.</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>Range</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>9,4.10$^4$</td>
<td>0,85</td>
<td>-</td>
<td>0,093</td>
<td>-</td>
<td>-</td>
<td>O.L.S.Q.</td>
<td>-</td>
<td>0,988</td>
<td>1,48</td>
<td>1971/1985</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0,14)</td>
<td>(23,17)</td>
<td>(0,61)</td>
<td>(0,61)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>6,9.10$^5$</td>
<td>0,802</td>
<td>-</td>
<td>0,015</td>
<td>-</td>
<td>-</td>
<td>O.L.S.Q.</td>
<td>-</td>
<td>0,989</td>
<td>1,94</td>
<td>1971/1985</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0,81)</td>
<td>(14,33)</td>
<td>(0,059)</td>
<td>(0,059)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3,1.10$^6$</td>
<td>-</td>
<td>0,734</td>
<td>0,312</td>
<td>-</td>
<td>-</td>
<td>O.L.S.Q.</td>
<td>-</td>
<td>0,966</td>
<td>1,87</td>
<td>1971/1985</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3,24)</td>
<td>(13,23)</td>
<td>(1,243)</td>
<td>(1,243)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>-5,2.10$^5$</td>
<td>0,882</td>
<td>-</td>
<td>0,022</td>
<td>-</td>
<td>-</td>
<td>I.V.</td>
<td>$Y_{t-1}$</td>
<td>0,983</td>
<td>1,51</td>
<td>1972/1985</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0,56)</td>
<td>(17,65)</td>
<td>(0,116)</td>
<td>(0,116)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>11,3.10$^3$</td>
<td>0,845</td>
<td>-</td>
<td>-0,035</td>
<td>-</td>
<td>-</td>
<td>I.V.</td>
<td>$Y_{t-1}$</td>
<td>0,984</td>
<td>1,81</td>
<td>1972/1985</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9,1.10$^{-3}$)</td>
<td>(11,03)</td>
<td>(-0,166)</td>
<td>(-0,166)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>3,7.10$^6$</td>
<td>0,71</td>
<td>-</td>
<td>0,39</td>
<td>-</td>
<td>-</td>
<td>O.L.S.Q.</td>
<td>-</td>
<td>0,95</td>
<td>2,07</td>
<td>1972/1985</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3,10)</td>
<td>(10,02)</td>
<td>(0,94)</td>
<td>(0,94)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations:
1) Endogenous Variable = $C_t$.
2) I.V. = Instrumental Variables.
3) D.W. = Durbin-Watson Statistic.
4) The number below the estimates of the coefficients stands for the $t$-statistics.
2.3.2) The Methodology Behind the Public Deficit Estimates

The developments here presented derive from Simonsen and Cysne (1987). Our main purpose is to define the real and operational concepts of public deficit under two different assumptions. The first includes capital gains or losses due to real exchange valuations or depreciations. The second does not take it into consideration.

We start with the deficit financing equation:

\[ D_g = I_g + G - GNR = \Delta V \]  

(3)

where \( D_g \) = Public (Government) Deficit

\( I_g \) = Government Investment

\( G \) = Government Consumption (Excluding Interest Payments and Receipts)

\( GNR \) = Government Net Receipts (Direct and Indirect Taxes + Other Current Receipts - Transfers - Subsidies - Other Current Expenditures)

The terms "Other Current Expenditures" and "Other Current Receipts" include the interest paid and received on the external debt, as well as profits and rents. Interest related to the internal debt is included in the item "Transfers".

\( \Delta V \) = Net increase of government liabilities.

We subdivide \( V \) into three different parts: \( V_1 \), bought by the Central Bank; \( V_2 \), bought by the remaining domestic economic agents and \( EK \) acquired by non-residents (we admit, with no loss in generality, that this part of the debt (\( K \)) is entirely dollar denominated). \( E \) stands for the nominal cruzado/dollar exchange rate. The Central Bank balance sheet is supposed to be given by:
Central Bank Simplified Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>International Reserves (Res)</td>
<td>Monetary Base (B)</td>
</tr>
<tr>
<td>Net Loans to the Government ($V_1$)</td>
<td></td>
</tr>
<tr>
<td>Net Loans to the Private Sector ($L$)</td>
<td></td>
</tr>
</tbody>
</table>

As a result we have, from equation (2):

\[ D_g = \Delta V_1 + \Delta V_2 + \Delta EK = \Delta B + \Delta V_2 - \Delta Res - \Delta L + \Delta EK \]

Making \[ F = V_2 - Res - L, \]

\[ D_g = \Delta B + \Delta F + \Delta EK \]  \( (4) \)

This is the classical equation that relates the budget deficit to money and net debt creation. All this development aimed at correctly qualifying the definition of the domestic net debt ($F$) that should be used in it. It must be equal to the net increase of public debt in the hands of the public minus the net assets of the Central Bank against the private and external sector of the economy.

There is nothing new with equation (4). It simply shows that an excess of expenditures over receipts must correspond to an increase of net liabilities. $D_g$ corresponds to the nominal deficit, and the right member to the net nominal increase of the government liabilities.
The adjustments to be made to pass from the nominal to the real interest rate estimate of the public deficit lie entirely on the term GNR. The items "Transfers", "Other Current Receipts" and "Other Current Expenditures" should be properly modified.

The basic operation to be carried out is to transform nominal interest into real by means of the following operation:

Real receipts = Nominal receipts minus nominal interest received plus nominal interest paid minus real interest paid plus real interest received.

The symmetric calculations applies for the expenditures:

Real Expenditures = Nominal Expenditures minus nominal interest paid plus nominal interest received minus real interest received plus real interest paid.

Following these procedures, the term "real transfers" will differ from the nominal one by the exclusion of the monetary correction paid on the net government debt. The real figures of "Other Current Receipts" and "Other Current Expenditures" will include the inflationary tax, which represent the real interest earned by the Central Bank (here, consolidated with the public sector) stock of High Powered Money. All these corrections are automatically transferred to the government net receipts, saving and deficit, originating the real concepts of these derived variables.

Finally, profits of government enterprises should be corrected, excluding the possible monetary correction earnings on their net financial assets (this procedure will be further detailed in section 2.3.7).
To calculate these differences between nominal and real interest rates, we work with the right side of equation (4). This amount (which we denote by NR) will be given, at prices of an arbitrary period $j$, by

$$NR_j = P_j \int_0^1 \frac{B + F + EK}{P} \frac{dP}{dt} \ dt$$

(5)

We decompose this total into three parts, all of them denominated in prices of periods $j$:

$$II_j = P_j \int_0^1 \frac{B}{P} \frac{dP}{dt} \ dt$$

(6)

$$ICI_j = P_j \int_0^1 \frac{F}{P} \frac{dP}{dt} \ dt$$

(7)

$$MCE_j = P_j \int_0^1 \frac{EK}{P} \frac{dP}{dt} \ dt$$

(8)

where $P = \text{Price Index}$ and

$$NR_j = II_j + ICI_j + MCE_j$$

(5')

The first term in the second member of (5'), $II$, is the inflationary tax. The other two, $ICI$ and $MCE$, stand for, respectively, the inflationary correction on the domestic net debt and on the external debt.

Expression (5) implicitly admits that $B$, $F$ and $EK$ can be expressed as continuous functions of time. Approximations to these calculations can be carried out in two ways. The first is to work with summations, instead of integrals, taking the average monthly values of each considered variable. The second,
a rough simplification of this tedious calculation, can be accomplished by means of some hypotheses concerning the path time of B, F, Q, E, K and the instantaneous inflation rate \((1/P)(dP/dt)\).

Before displaying each one of these possibilities, we shall define:

a) The Real Deficit \(D'_{gr}\) = Nominal Deficit - 
\[(II + ICI + MCE)\]

b) The Operational Deficit \(D'_{go}\) = Nominal Deficit - (ICI + MCE) = Real Deficit + II.

The concept of real deficit extends the discrimination between nominal and real interest to the total amount of the government net liabilities. The operational concept excludes the monetary base from this total. There being no money illusion, the real concept (as against the operational one) is a better statistic to translate the role of the government in determining aggregate demand. Indeed, the higher the inflationary tax, the lower will be the disposable income of the private sector\(^{(1)}\) and, consequently, the private consumption. At the same time, the lower will be the real deficit, in accordance with the fall of aggregate demand. The operational concept, on the other hand, is a useful concept when one desires to determine the disequilibrium of public finance. In fact, this statistic presents the excess of real expenditures over real receipts before the inflationary tax has been added as a current real receipt\(^{(2)}\).

It is interesting to show that the tautology which makes the budget deficit equal to the increase of the net public

\(^{(1)}\) Calculated under the real interest methodology. See section 2.3.7.
\(^{(2)}\) It must be observed that, while in the nominal interest accounting it is totally meaningless to add the inflationary tax (in the way here defined, which, as we shall see, does not include the real seignorage) as a current receipt, in the real interest accounting it is an common proceeding.
debt remains valid even in the real accounting. Indeed, if \( Z \) stands for \( F + B + EK \), the real deficit \( (D_{gr}) \) between time \( t \) and \( t + dt \) will be given by:

\[
dD'_{gr} = dZ - Z(dP/P)
\]

At prices of period \( j \),

\[
dD'_{grj} = P_j \left( \frac{dZ}{P} - \frac{Z}{P} \frac{dP}{P} \right) = P_j D\left( \frac{Z}{P} \right)
\]  

(8')

Integrating between periods 0 and 1,

\[
D'_{grj} = P_j \left( \frac{B_1}{P_1} - \frac{P_0}{P_0} + \frac{F_1}{F_0} - \frac{F_0}{F_0} + \frac{E_1K_1}{P_1} - \frac{E_0K_0}{P_0} \right)
\]  

(9)

Equation (9) tells us that the real deficit can be measured in term of the increase, during the time period considered, of the real values of the monetary base, of the domestic net debt and of the government external debt.

It is sometimes useful to decompose the here presented definition of the real external deficit as the sum of two componentes:

\[
D'_{grj} = D_{grj} + CL_{j}
\]  

(10)

The first, \( D_{grj} \), is the concept of deficit which does not incorporate the capital gains or losses due to real exchange rate changes. To obtain it, the correction from nominal to real interest with respect to the share of government debt denominated in foreign currency must be carried out taking into
consideration the external, rather than the internal inflation. The result must be translated to cruzados at the given exchange rate. Thus, between time \( t \) and \( t + dt \), if \( Q \) stands for the external price level, and making, to simplify, \( B + F = 0 \),

\[
dD_{gr} = E \left( dK - K dQ/Q \right)
\]  

(11)

Since, with respect to the external debt,

\[
dO' = K dE + EdK - (EK) \frac{dp}{P}
\]

we get, from this expression and (11),

\[
dD'_{gr} - dD_{gr} = EK \left( \frac{dE}{E} + \frac{dQ}{Q} - \frac{dp}{P} \right) = EK \frac{d(EQ/P)}{EQ/P}
\]

In currency of period \( j \),

\[
dD'_{grj} - dD_{gr} = P_j EK \frac{d(EQ/P)}{EQ/P}
\]

Integrating between periods 0 and 1, we get expression (10),

\[
D'_{grj} = D_{grj} + CL_j
\]

where

\[
CL_j = \frac{1}{P_j} \int_{0}^{1} \frac{E}{P} \frac{d(EQ/P)}{(EQ/P)} dt
\]

\( CL_j \) translates the increase of the real deficit \( (D'_{grj}) \) due to real exchange rate devaluations.

The value of the narrower concept of real deficit \( D_{gr} \) can be found using expressions (9), (10) and (12). Alternatively, since the distinction between \( D_{gr} \) and \( D'_{gr} \) relies solely on the part of the government debt denominated in foreign currency:
In the above expression, which follows directly from (11) and (9)(1), the first integral in the second member stands for the nominal external deficit, and the second for the difference between nominal and real interest (taking into consideration the foreign inflation). It is sometimes reasonable to admit that the nominal exchange rate depreciations equals internal inflation \((E/P = \text{constant})\), and that the logarithmic external inflation rate is kept unchanged during the period under consideration \(((1/Q)(\frac{\partial Q}{\partial t}) = \pi^*)\).

In this case, we get the useful expression derived from (13) (which we shall use in the next chapter to calculate the real deficit figures prevailing between 1983 and 1987):

\[ D_{grj} = p_j \left( \frac{B_1 - B_0}{P_1 - P_0} + F_1 - F_0 \right) + E_j \left( \frac{K_1 - K_0}{\bar{K}} \right) - E_j \pi^* \]  

\( \bar{K} \) standing for the average value of the foreign debt during the year.

What we have shown is that there are two possible concepts for the real deficit, taking into consideration the treatment to external liabilities. The first, \(D_{gr}^{(1)}\) given by (11), solely due to the real increase of these foreign obligations. The second concept, \((D_{gr}^{'})\), which we shall call "extended deficit", adds to \(D_{gr}\) the capital gains and losses due to real exchange rate appreciations or depreciations (CL).

(1) Remember that we have made in (11), to simplify the notation, \(B+F=0\). Without this assumption, the integration with respect to \(B+F\) follows exactly like in the other case, presented in (9).
The inclusion or not of capital gains and losses in the deficit concept depends upon the use which will be made of this variable. The extended version presents what is called a "consistency property", which means that the deficit figure is always equal to the increase of borrowing requirements. But one must have in mind that any increase in the extended public deficit figures due to real exchange rate changes do not lead to additional nominal borrowing requirements. In other words, it does not generate a lending operation. The increase in the real borrowing requirements is automatic, since the cruzado value of the net government debt is automatically increased after a real exchange depreciation.

Because of this reason, and also because the increase in the real value of the public debt originated from real deprecinations of the exchange rate are not supposed to affect aggregate demand (since the external debt is in the hands of non-residents), the non-extended definition is the one we shall use for macroeconomic purposes. The use of the extended concept in periods when there is a sharp real exchange appreciation can be very misleading. With the capital gains, public deficit figures can remain constant, while aggregate demand is being highly fostered by government expenditures.

Another observation must be done, this time with respect to the inflationary tax definition which we have been using. There is another definition in the literature, given by:

\[ II'_j = p_j \int_0^1 \frac{dB}{Pdt} dt \]
Since $\frac{dB}{P} = \frac{B}{P} \frac{dP}{P} + d\left(\frac{B}{P}\right)$, it is immediate that

$$II'_{j} = II_{j} + P_{j} \left(\frac{B_{1}}{P_{1}} - \frac{B_{0}}{P_{0}}\right)$$  \hspace{1cm} (14)$$

In other words, the alternative definition $II'$ (which presents a semantic disadvantage, since one can have inflationary tax with zero inflation) is equal to ours (II) plus the real seignorage gain occurred at the period.

Given the definition of the operational deficit, it follows from (9) that:

$$D'_{goj} = P_{j} \left(\frac{F_{1}}{P_{1}} - \frac{F_{0}}{P_{0}}\right) + P_{j} \left(\frac{E_{1} K_{1}}{P_{1}} - \frac{E_{0} K_{0}}{P_{0}}\right) + P_{j} \left(\frac{B_{1}}{P_{1}} - \frac{B_{0}}{P_{0}}\right) + II_{j}$$

or, given (13),

$$D'_{goj} = P_{j} \left(\frac{F_{1}}{P_{1}} - \frac{F_{0}}{P_{0}}\right) + P_{j} \left(\frac{E_{1} K_{1}}{P_{1}} - \frac{E_{0} K_{0}}{P_{0}}\right) + II'_{j}$$

The non extended concept of operational deficit can be directly obtained from $D_{grj}$, by adding the inflationary tax:

$$D_{goj} = D_{grj} + II$$

where $D_{grj}$ is given by (13) or, if $E/P$ and $K/Q$ remain constant in the time period considered, (13a).
2.3.3) Some Simplified Expressions For the Evaluation of The Difference Between Nominal and Real Interest (NR)

In this section we develop some analytical expressions that might be of help in obtaining estimates of the variables previously mentioned (II, ICI, and MCE). We shall deal with three different possibilities, all expressions following from (6), (7) and (8). To simplify, variable X stands for B, F or EK.

a) The nominal value of the considered net liability is kept constant over the period \( X = \bar{X} \).

\[
NR_j = \left( \frac{P_j}{P_1} \right) \bar{X}_\pi
\]  
(15)

b) The real value of the considered net liability is kept constant over the time period considered.

\[
NR_j = \tilde{\pi}X_j
\]  
(16)

where \( \tilde{\pi} = \ln(1 + \pi) = \) logarithmic inflation rate.

c) The instantaneous rate of growth of the real value of the net liability (a) and of the inflation rate are \( (\tilde{\pi}_i) \) are kept constant over time. This means

\[
\frac{X}{P} = \frac{X_0}{P_0} e^{at} \quad \text{a = constant}
\]

\[
\frac{1}{P} \frac{dP}{dt} = \tilde{\pi}_i \quad \tilde{\pi}_i = \text{constant}
\]
In this case,

\[ N_{R_j} = \left( \frac{P_j}{P_0} \right) X_0 \bar{\pi} \left( e^a - 1 \right) / a \] (17)

To illustrate the use of these expressions, we shall present some examples:

1) We use expression (15) to calculate the amount of inflationary tax paid by an economic agent who keeps Cz$ 100,00 in cash (currency) during a month when the rate of inflation was 15%:

\[ N_{R_0} = 100 \cdot \frac{\pi}{(1 + \pi)} = \text{Cz$} \ 13,04 \]

or, in currency of the end of the month,

\[ N_{R_1} = 100 \cdot \pi = \text{Cz$} \ 15,00 \]

One can observe that the calculated figures do not depend upon the trajectory of inflation between periods zero and one. The prices can present a uniform growth during the month or rise gradually, and jump at the last day.

2) Expression (16) is used to evaluate inflationary tax when the economic agent begins the month with Cz$ 100,00, but increases this amount continually in order to keep constant, at each point in time, the purchasing power of his balances:

\[ N_{R_0} = X_0 \bar{\pi} = \text{Cz$} \ 0 \left( 100 \cdot \ln 1,15 \right) = \text{Cz$} \ 13,98 \]
or, in currency at the end of the period,

\[ NR_1 = \text{Cz\$}_1 \ 16,08 \]

As one should expect, the amount of inflationary tax is higher than in example (1), since the average cash balances during the month are also higher.
2.3.4) **Inflationary Tax and Inflationary Transfers to Commercial Banks**

By definition, inflationary tax is the amount of real interest earned by the Monetary Authorities on the stock of High Powered Money. Since this liability of the Monetary Authorities pays zero nominal interest, the real interest earned in each point in time is equal to the monetary base \(B\) times the inflation rate.

An extension of this concept to the commercial banks is the inflationary transfer to commercial banks (IT). In each point in time, if \(M_1\) stands for the means of payment, it is given by \((M_1 - B)\). Since \(M_1 - B\) is also equal to the difference between commercial banks sight deposits and total reserve requirements, IT can be seen as the real interest earned by the banks on its demand deposits minus the real interest on its total reserve requirements which they pay to the Monetary Authorities. In other words, it is equal to the real interest earned by the commercial banks on the net quantity of money they create \((M_1 - B)\).

From the above exposition it follows that by adding the inflationary tax (IT) to the inflationary transfers to commercial banks (IT) one can get the total amount of real interest paid\(^{(1)}\) by the non-banking system to the banking system due to inflation. We shall denominate total inflationary transfers (TIT):

\[(1) \text{ We use the word "payment" despite the fact that the inflationary transfers do not generate a payment in the strict sense of the word. This is a distinction between the inflationary tax and all other taxes: it does not generate a withdrawing operation; income is transferred to the government in real terms, rather than in nominal (monetary) terms. It can be imagined, if one desires, that economic agents receive the necessary amount of money from the government to keep constant the real value of their cash balances and, in a tax payment operation (now in the strict sense of "payment") return this money to the government as a tax (we are obviously consolidating the Central Bank with the government, a common praxis in the analysis of Brazilian economy).} \]
\[
TIT = II + IT = B\pi + (M_1 - B)\pi = M_1\pi
\] (18)

TIT is equal, in each point in time, to \( M_1 \) times the rate of inflation. This equation tells us that the non-banking system, by holding an asset which pays zero nominal interest \( (M_1) \) in an inflationary environment, is actually paying real interest to the banking system. These payments are subdivided in two categories: payments to the Monetary Authorities (in Brazil, they should include Central Bank and Banco do Brasil) and payments to the remainder of the banking system, the commercial banks (we define commercial bank as a financial institution allowed to receive demand deposits).

Table 2.6 presents the value of II, IT and TIT as a percentage of GDP and also at dollars of 1987. The figures range from 1947 up to 1987 (31 years). Other technical details are presented in Appendix.
Table 2.6  

Inflationary Tax (IT), Inflationary Transfers to Commercial Banks (IT) and Total Transfers (TIT = IT + IT)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>INFLATION (%)</th>
<th>IT/GDP (%)</th>
<th>IT/GDP (%)</th>
<th>IT/GDP (%)</th>
<th>IT/GDP (%)</th>
<th>IT/GDP (%)</th>
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Observations:  
1. The GDP figure used for 1987 was a preliminary one.  
2. The last three columns present figures in constant dollars of 1987.  
3. For 1986 and 1987 the new concept of Monetary Based was used.
Some preliminary comments about the raw data must be made, before we go through these results. For the years 1986 and 1987, the new concept of Monetary Base (taking Banco do Brasil as a commercial bank) was used\(^1\). This explains the relative increase of the rate IT/II starting 1986. Because of this change in the definition of B (which became narrower), the numbers in each series II and IT should only be compared between 1947 and 1985. The means of payment definition was also changed as of March, 1986. Demand deposits in the "Caixas Econômicas" and "Banco Nacional de Crédito Cooperativo" were added to the old concept. This constitutes an autonomous motive for increase of the total transfers figures (TIT) we are calculating, as of this date. With this kept in mind, TIT can be compared in the whole series.

The following remarks derive from table 2.6:

a) In constant dollars of 1987, the year 1987 appears as the one in which the total transfers from the non-banking system were the highest (approximately 23,7 billion dollars). This was also the year when inflation achieved its record (415,8%). From this total, around 11 billion dollars (3,53% of GDP) represented transfers to the Central Bank, and 14,9 billion dollars (3,46% of GDP), transfers to the commercial banks.

b) As a percentage of GDP, the highest value of the total transfers (TIT) happened in 1964 (8,62% of GDP). From this total, 5,04% of GDP were transfered to the Monetary Authorities and 3,44% of GDP to the commercial banks.

\(^1\) Data on the old concept of Monetary Base is not available as of March, 1986.
\(^2\) We are only referring to the real interest earned on B (in the case of the Monetary Authorities) and \(M_1 - B\) (in the case of the commercial banks). Other compensatory gains and losses due to inflation or taxation (in the case of the commercial banks) may exist, but are not considered here. In short, we are working with a gross type of income, rather than a net concept.
c) Between 1947 and 1985, the inflationary tax (IT) and transfers to commercial banks (IT) reached a maximum in 1987, in constant dollars, or 1964(II) and 1963(TI), as a percentage of GDP. The high values of IT and II in the years 1963 and 1964, in spite of the relatively low inflation then existing (relative to the levels occurred in the eighties) reflect the increase in the income velocity of money as of 1964. Financial innovations (defined as anything which leads to a decrease in the cost of shifting funds from $M_1$ to interest bearing assets) played an important role in this process of reducing the equilibrium value of real balances as a percentage of GDP.

d) After the fifties the period of lowest inflationary transfers to the banking system as a percentage of GDP occurred in the years 1969-1973 (popularly known as the "Miracle Years", when the real GDP grew at rates above 10% a year). The reason relies on the very low rates of inflation in these years, mostly due to the positive supply shocks, relative monetary stability and real product outburst.

e) During the period 1980-86, the average inflation rate of the three last years (1983-85) was a bit higher than the double of that prevailing during 1980-82. The inflationary tax collected by the government, though, remained almost the same. The total transfers, in the average, fell down with the rise of inflation. These facts indicate a elasticity of money demand superior to one (in absolute value). This was surely corroborated by the financial innovations occurred in the period.

f) The average value of the inflationary tax as a percentage of GDP after 1960 was 2.4%.
2.3.5) **Approximate Estimates of the Difference Between Nominal and Operational Deficit**

We now use expression (17) to evaluate the difference between nominal and real interest paid by the government on its net internal debt. Following the hypotheses considered previously, where this expression was obtained, the real value of the internal debt is admitted to follow the trajectory:

\[
\frac{F(t)}{P(t)} = \frac{P_0}{P_0} e^{bt} \quad 0 \leq t \leq 1
\]

and the price level to be given by:

\[
P(t) = P_0 e^{\gamma t}
\]

We work with the data available at the "Brazil Economic Program", volume 16, published by the Central Bank of Brazil in March, 1988. The necessary data for the calculations is presented below:
### Table 2.7
Data For Calculation of the Difference Between Nominal and Real Interest on the Internal Net Debt

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<th>1986</th>
<th>1987</th>
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<tr>
<td>Net Internal Debt (P) (Cz$ Millions)</td>
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<td>5111950</td>
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<tr>
<td>Price Level (P) (Consumer Price Index March/86 = 100)</td>
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<td>Operational Deficit/GDP</td>
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</tr>
<tr>
<td>GDP (Cz$ Millions)</td>
<td>3681145</td>
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</tr>
</tbody>
</table>

(1) Monetary Unities Expressed in Cz$ Millions
(2) The Internal Debt figure is evaluated at the end of period.

From these data we get

\[ \tilde{\pi} = \ln(5698/1223) = 1.538 \]
\[ b = \ln(8971/7098) = 0.234 \]
\[ \bar{P}_{87} = \text{Average Price Index in 1987} = P(0) \left( \frac{e^{\tilde{\pi}} - 1}{\tilde{\pi}} \right) = 290.9 \]

Using expression (17),

\[ ICI_{87}^{(1)} = \left( \frac{\bar{P}_{87}}{P_{Dec86}} \right) \cdot P_{Dec86} \cdot \tilde{\pi} \cdot \left( \frac{e^b - 1}{b} \right) \]

\[ ICI_{87} = \text{MCz$} \quad 3576106 \quad 87 \]

(1) \( ICI_{87} \) means \( ICI \) at average prices of 1967. \( \text{MCz$} \) means millions of cruzados.
As a percentage of GDP, we have

$$\frac{ICI}{GDP} = 0.29$$

Given the first way we defined the operational budget deficit ($D'_o$ instead of $D_{go}$), the difference between the nominal and operational concepts include not only the inflationary correction on the internal debt, but also on the external one. Therefore, to get the appropriate number related to this difference, we should repeat the same calculations taking into consideration also the external debt. Since the latter corresponds to 56.1% of the total government debt, we would find the approximate estimates of the total difference between nominal and real interest to be about 66% of GDP.

If we work with the concept of deficit which does not include capital gains or losses due to real exchange rate changes ($D_{go}$), the calculations with respect to the dollar denominated debt would become totally different. The inflation to be considered would be that taking place overseas, rather than the domestic one. Let us provide an approximation to this total. The amount to be added to the previous estimate would be given by:

$$NR_e = \sum_{j} \int_{0}^{1} \frac{EK}{P} \frac{dQ}{Qdt} \, dt \quad (18)$$

We suppose that the real exchange rate ($\theta = EQ/P$), the real external debt and the instantaneous external inflation are given by

$$\theta(t) = \theta_0 \, e^{ct}$$

$$\frac{K}{Q} = \frac{K_0}{Q_0} \, e^{ct}, \quad c + d \neq 0$$
and \( \frac{d\ln Q}{dt} = r^* \). Under these hypotheses,

\[
NR_{eq} = (P_j / P_0) \left( \frac{E_0 K_0 r^*}{(e^{C+d} - 1)} \right) / (c + d)
\]

The following data (from Brazil Economic Program) were used:

**Table 4A**

<table>
<thead>
<tr>
<th></th>
<th>1986</th>
<th>1987</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Net Government Debt (US$ Millions)</td>
<td>81807</td>
<td>90733</td>
</tr>
<tr>
<td>External Price Index</td>
<td>111</td>
<td>116</td>
</tr>
<tr>
<td>Domestic Price Index</td>
<td>122.3</td>
<td>569.8</td>
</tr>
<tr>
<td>Nominal Exchange Rate (Cz$/US$)</td>
<td>14.6</td>
<td>67.9</td>
</tr>
<tr>
<td>Real Exchange Rate Index</td>
<td>100</td>
<td>104.3</td>
</tr>
</tbody>
</table>

All data reflect end of period (December 31st) positions.

From these, we get:

\[
\begin{align*}
c &= 0.06 \\
r^* &= 0.044 \\
d &= 0.042 \\
P_j / P_0 &= P_{87} / P_0 = 2.38
\end{align*}
\]
From these data, it follows that (1)

\[ NR_e^{87} = MCz$^{87} 131.811 \] or

\[ NR_e/PIB = 0.0107 \]

This figure means that approximately one percent of GDP was paid by the government to its external creditors only to replace the loss of value of the foreign debt due to external inflation.

Central Bank calculations of the government operational deficit, as carried out by its economic department (DEPEC), treats the external part of the debt translating to cruzados, at the average exchange rate of the period, the dollar figures related to interest and other current expenditures paid on the debt. By this methodology, which does not include a correction for external inflation, the differentiation between nominal and real interest is only due to the monetary correction on the domestic (cruzados denominated) debt.

Consequently, one could expect the first number we calculated (29% of GDP) to be close to the difference between the nominal and operational deficit presented in official estimates. Two main reasons prevent this from happening: first, Central Bank's calculations only include the debt service of the part of the government debt which does not belong to Central Bank's portfolio; second, we calculate the difference between nominal and real interest taking into consideration the price index, rather than the monetary correction index stipulated by the government. Since, as we can see from table 2.4, the rate of change of the former is systematically higher than the rate of change of the latter, the difference between inflationary correction

(1) We worked with a log linear trend for domestic prices.
and monetary correction is generally positive.

These facts chiefly explain why our number for the difference between nominal and real interest paid on the debt (9% of GDP in 1986 and 29% of GDP in 1987) are systematically higher than Central Bank's (7.6% of GDP in 1986 and 25.1% of GDP in 1987).

The use of monetary correction indexes to carry out the estimates between nominal and real interest can lead to sharp miscalculations. Inflation indexes are obviously the most appropriate, since the asset purchasing power of interest in this respect is better measured in terms of goods, rather than in OTNs.

Both calculations work with the accrual methodology, taking into consideration not only the part of the debt falling due during the analyzed period, but also the remaining part whose maturity falls due beyond this date.

Disadvantages of the simplified calculations here presented are obviously the arbitrary hypotheses related to the time path of the relevant variables. The more appropriate formula, with summations (which was used with monthly data to evaluate the difference between nominal and real interest necessary to the regressions presented in section 2.3.1) can be found in the appendix. A comparison with the numbers here presented is not possible, since in the former case (of section 2.3.1) we worked with a narrower concept of government debt. Data for the larger concept, used in this section, are not available before 1982.
2.3.6) **Extension to Social Accounting**

Let us take the two basic identities of social accounting:

\[ C + S_p + GNR + RLE = Y = C + I + G + H \]  \hspace{1cm} (20)

where

- \( C \) = Private Consumption
- \( S_p \) = Gross Private Saving (Firms and Households)
- \( GNR \) = Government Net Receipts
- \( RLE \) = Net Transfer of Income to Abroad
- \( Y \) = Gross Domestic Product
- \( I \) = Gross Investment
- \( G \) = Government Consumption
- \( H \) = Net Transfer of Resources to Abroad

The first equation equals the income to GDP, and, the second, the GDP to the ex-post demand. The values of \( C, I, G \) or \( H \) are the same, under the nominal or real interest criteria. The components of demand share no links with the deterioration of the purchasing power of the financial assets. As suggested by the United Nations Operational Surplus Methodology of the Social Accounts, the debt side of the Production Account should include no receipt or payments of interest. Therefore, under the production view (as against the income view), there is no reason to distinguish between nominal and real interest.

With respect to the foreign currency denominated liabilities, the correction between nominal and real interest must be made by using formula (18), rather than (8), since it is
a common practice in social accounting not to include capital gains or losses.

Equation (20) can be written in three different ways. Using the sub-indexes "p" and "r" to denote, respectively, operational and real, and leaving it blank in the case of the nominal concept, we have:

\[ C + S + GNR_p + RLE_p = C + S + RIE_r + RLE_r = C + S + GNR_0 + RLE_0 = Y \]  (20)

This is to say that the total sum of the different parcels of income is the same (and equal to GDP measured at market prices) under any accounting system. What really changes, from one case to the other, is the distribution among the parts. As we previously mentioned, in periods of inflation the real net income of the debtors is higher than the nominal one, the opposite happening to the creditors.

To begin with, let us take a closed economy where the government presents a net positive debt, and restrict ourselves to the nominal and real accounting, leaving the operational one aside for a moment. From (21) we have:

\[ S_p + GNR_p + S_{pr} + GNR_r = Y \]  (21)

which allows us to conclude that

\[ GNR_r - GNR_p = S_p - S_{pr} \]  (23)
This equation tells us that the excess of the real concept of GNR over the nominal one (which is equal to the excess of the nominal government deficit over the real one) is equal to the difference between the nominal and the real concepts of the private gross savings. While one sector experiences a net earnings increase with the change in methodology, the other experiences a decrease.

Subtracting the inflationary tax from both sides of the above equation, we get its operational counterpart:

\[ \text{GNR}_o - \text{GNR} = S_p - S_{po} \]  

(24)

Since the inflationary tax is supposed to be positive, the difference between the operational and nominal concepts of government net receipts (and, consequently of government nominal and operational deficits) is lower than in the real case.

The difference between the operational and real concepts is due only to the inflationary tax (II). At currency of the same period, are

\[ S_{gr} - S_{go} = \text{GNR}_r - \text{GNR}_o = D_{go} - D_{gr} = S_{po} - S_{pr} = II \]  

(25)

\[ S_g = \text{Government Savings} \]

Opening the economy makes it necessary to introduce three different concepts of the net transfer of income to abroad into the discussion. We remember that the change from nominal to real interest must take into consideration the loss of purchasing power of the currency in which the specific asset or
liability is denominated. Consequently, following our simplifying hypothesis introduced in section 2.3.2, which assumed all net liabilities in the hands of non-resident to be dollar denominated, the operational and real values of RLE will coincide (since this assumption implies that all the stock of High Powered Money is in the hands of residents). It can then be written

\[ S_{pr} + GNR_r = S_{po} + GNR_o \]  

(25a)

In the case the external (dollar) inflation may be considered null, it can still be written

\[ S_{pr} + GNR_r = S_p + GNR \]  

(25b)

since the nominal and real values of RLE will be the same.

From equations, (20) and (21) we also conclude that the other generally utilized national accounts identities:
\[ S_g + S_p + S_e = I \]  \hspace{1cm} (26)

\[ D_g = S_p - I_p + S_e \]  \hspace{1cm} (27)

are all valid under the operational and real criteria:

\[ S_{go} + S_{po} + S_{eo} = I \]  \hspace{1cm} (26a)

\[ S_{gr} + S_{pr} + S_{er} = I \]  \hspace{1cm} (26b)

\[ D_{go} = S_{po} - I_p + S_{eo} \]  \hspace{1cm} (27a)

\[ D_{gr} = S_{pr} - I_p + S_{er} \]  \hspace{1cm} (27b)

These extensions provide a useful tool to deal with the social account statistics. As it was argued before, if there is no money illusion, the figures calculated with real interest should be more appropriate. Moreover, if one desires to have an idea of the real values of the social aggregates without extending the distinction between nominal and real interest to the monetary base, the operational concept should be used.
2.3.7) Extension to Correction of Corporate Profits\(^{(1)}\)

In a country where there is inflation, at least two reasons make it necessary to distinguish between the nominal and inflation corrected profits: first, the dividends distribution, and second, taxation. In the absence of this mechanism, a firm with positive net financial assets will show illusory profits, since a part of its interest income only aims at replacing the loss of value of these financial assets due to inflation. If this fact is not recognized in the related tax legislation, the higher the inflation, the more quickly the real value of the firms financial assets will decline, due to an overvaluation of the profits to be taxed and distributed to the shareholders.

In order to make this point clear, let us take a firm whose simplified balance sheet in cruzados is given, at the beginning of the year, by:

\[
\begin{array}{c|c}
\text{t = 0} \\
\hline
\text{Assets} & \text{Liabilities} \\
\text{Financial Assets} & \text{Net Worth} \\
\text{Cz\$0 10} & \text{Cz\$0 10} \\
\end{array}
\]

We suppose, to simplify, that the ad-valorem tax on the gross profits is 35\%, and that there are no retained profits. If the inflation rate is 100\% a year and the real interest rate equal to zero, the firms balance at the end of period, before and after tax and dividends distribution will be:

\[\text{(1)}\text{ We suppose in this section that monetary correction indexes are equal to inflation indexes. But one should keep in mind that the profits correction mechanism assured by the laws is based on the monetary correction indexes.}\]
Before

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Assets 10</td>
<td>Cz$1 10</td>
</tr>
<tr>
<td>Cash 10</td>
<td>Net Worth Cz$20</td>
</tr>
</tbody>
</table>

After

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Assets 10</td>
<td>Cz$1 10</td>
</tr>
</tbody>
</table>

The last balance, written with cruzados of period zero, would be, according to (1):

\[ t = 1 \]

Balance After Tax and Profits Distribution

(in cruzados of period 0)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Assets 5</td>
<td>Net Worth Cz$0 5</td>
</tr>
</tbody>
</table>

Comparing this balance with the first one, one can notice that, should the Cz$0 10 of monetary correction be considered for taxation and profits distribution, the real assets of the firm will have declined to a half at the end of period. This happened because the Cz$3.5 paid to the government and the remaining Cz$6.5 distributed to the shareholders were not real income, but only an amount paid by the debtor to the firm to keep unchanged the real value (purchasing power) of the debt. Taxation in this case actually happens to be a capital levy, rather than an income tax. In the same way, what the firm is distributing to its shareholders is not current profit, but part
of its net patrimony.

What the real interest accounting actually does is to recognize this fact, allowing for the correction, in the profits calculation, of the of earned interest which is only due to the loss of purchasing power of the currency in which the financial assets are denominated. In the previous examples, taxes and profit distribution would be zero, since this was also the real profit of the firm. Of course, had the firm positive financial liabilities, instead of assets, and the opposite would occur (or in the case of positive net assets and deflation). The non-corrected profits would be lower than the corrected figures, leading to an undervaluation of taxation and profits distribution.

This problem was very important in Brazil up to 1964, when Law 4357 (enacted in July 17, 1964) first recognized the necessity of distinguishing between nominal and real interest rates. The next table present some data about it. It becomes evident the importance of the distinction between nominal and corrected profits. Illusory profits ranged between 52.1% and 74% of total profits of the open corporations between 1958 and 1964.
### Table 2.8
Illusory Profits Estimates for Open Corporations
Period 1958-64

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Number of Firms</td>
<td>6.818</td>
<td>7.104</td>
<td>5.587</td>
<td>6.441</td>
<td>6.882</td>
<td>6.998</td>
<td>7.915</td>
</tr>
<tr>
<td>B) Financial Net Assets At the beginning of Period (in million of cruzeiros)</td>
<td>112.876</td>
<td>156.876</td>
<td>163.873</td>
<td>201.279</td>
<td>298.284</td>
<td>404.639</td>
<td>712.357</td>
</tr>
<tr>
<td>C) Inflation rate</td>
<td>27.9</td>
<td>36.1</td>
<td>32.8</td>
<td>50.1</td>
<td>50.3</td>
<td>81.9</td>
<td>93.3</td>
</tr>
<tr>
<td>D) Illusory Profits</td>
<td>31.493</td>
<td>56.474</td>
<td>53.750</td>
<td>100.838</td>
<td>150.037</td>
<td>331.399</td>
<td>664.629</td>
</tr>
<tr>
<td>E) Total Profits (in millions of cruzeiros)</td>
<td>58.399</td>
<td>84.936</td>
<td>102.849</td>
<td>168.287</td>
<td>288.193</td>
<td>447.975</td>
<td>1.140.187</td>
</tr>
<tr>
<td>F) Percentage D/E (%)</td>
<td>53.9</td>
<td>66.5</td>
<td>52.3</td>
<td>59.9</td>
<td>52.1</td>
<td>74.0</td>
<td>58.3</td>
</tr>
</tbody>
</table>

(1) The calculus of item (E) implicitly admits that:
- a) The net financial assets related to item (b) are kept constant in nominal terms during the whole year;
- b) The total profits presented in item (c) are denominated in currency of the end of the period.
Under these circumstances, equation (15) can be used.
To avoid a sizable loss of receipts due to the sudden recognition of the illusory profits, Law 4357, only allowed for a partial correction of balances. Three Decree-Laws about the matter were enacted after 1964: the Decree-Law 62 (1967) Decree-Law 401 (December 30, 1968) and Decree-Law 417 (1969). Each of these provided mechanisms to avoid over taxation on financial earnings.

The profits correction was last regulated by the Decree-Law 1758, enacted in 1976. In order to understand it, let us take the simplified balance sheet of a firm:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A- Physical Assets</td>
<td>W- Net Worth</td>
</tr>
<tr>
<td>B- Net Financial Assets</td>
<td></td>
</tr>
<tr>
<td>Denominated in Domestic</td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td></td>
</tr>
<tr>
<td>C- Net Financial Assets</td>
<td></td>
</tr>
<tr>
<td>Denominated in Foreign</td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td></td>
</tr>
</tbody>
</table>

The so called "Corrected Profits" are obtained from the nominal profits by the following steps:

1) Monetary Correction is added to the physical assets (since their overvaluation due to inflation is not included in the nominal profits);
2) Exchange rate correction is added to the net assets denominated in foreign currency;
3) Monetary correction is subtracted from the net patrimony.
It must be observed that, since \( W = A + B + C \):

a) With respect to the domestic currency denominated financial assets, this procedure is equivalent to transform nominal into real interest (since the net result of these proceedings, with respect to \( B \), is the deduction of monetary correction).

b) With respect to the foreign currency denominated financial assets, the real interests above mentioned incorporate the excess of exchange devaluation over the domestic inflation. This is a capital gain for the firm, which should certainly be considered in terms of taxation and profits distribution.

Item (b) suggests that if we define the real profits taking into consideration, with respect to the external liquid assets, the foreign inflation, the corrected profits should be equal to the real profits plus the capital gains and losses due to real exchange rate appreciations or depreciations \(^{(1)}\). This actually happens, given our previous developments,

\[
\text{Nominal Profits} - \text{Real Profits} = P \int_{0}^{1} \frac{B}{F} \frac{dP}{dt} \, dt + P \int_{0}^{1} \frac{EC}{P \cdot Q} \, dt
\]

On the other hand, following the three steps of profit correction:

\[
\text{Corrected Profits} - \text{Nominal Profits} = P \int_{0}^{1} \frac{C}{F} \frac{dE}{dt} \, dt + P \int_{0}^{1} \frac{A - W}{F^2} \frac{dP}{dt} \, dt
\]

Since \( A - W = -(B + EC) \)

\(^{(1)}\) This result was first disclosed by Simonsen, and is presented in Simonsen and Cysne (1987).
Corrected Profits - Real Profits = \( P_j \int_0^1 \frac{\text{EC}}{P} \left( \frac{1}{E} \frac{dE}{dt} + \frac{1}{Q} \frac{dQ}{dt} - \frac{1}{P} \frac{dP}{dt} \right) dt = \)

\[= P_j \int_0^1 \frac{\text{EC}}{P \Theta} \frac{d\Theta}{dt} dt \]

where \( \Theta = \frac{E_0}{P} \) stands for the real exchange rate, and \( P_j \int_0^1 \frac{\text{EC}}{P \Theta} \frac{d\Theta}{dt} dt \)

represents the capital gain or loss of the firm due to real exchange rate variations.
Appendix

1) Section 2.3.1 - Evaluation of the Difference Between Nominal and Real Interest on the Net Internal Public Debt, at Prices of Period $j(NR_j)$.

Formula Used: \[ NR_j = P_j \sum_{t=1}^{12} \frac{F_t + F_{t-1}}{P_t + P_{t-1}} \frac{P_t + P_{t-1}}{P_{t-1}} \] (1)

$P_j$ = Arithmetic Average Price Level of Year $j$(IGP - DI).

$F_t$ = Total Internal Government (Federal, State and Local) Debt, Relative to Month $t$ (End of Period).

$P_t$ = Price Index Relative to Month $t$(IGP - DI).

\[ \frac{F_t + F_{t-1}}{P_t + P_{t-1}} = \text{Average Debt Balance/Average Price Level, in each month.} \]

\[ \frac{P_t + P_{t-1}}{P_{t-1}} = \text{Monthly Inflation.} \]

This formula presents an approximation of the actual difference between nominal and real interest (which does not depend on the monetary correction index), taking composed interest as simple interest.

2) Section 2.3.4 - Evaluation of the Inflationary Tax ($II_j$) and Inflationary Transfers to Commercial Banks ($IT_j$) at Prices of Period $j$. 
\[ II_j = P_j \sum_{t=1}^{12} \frac{B_t + B_{t-1}}{P_t + P_{t-1}} \frac{P_t + P_{t-1}}{P_t} \]  
\[ IT_j = P_j \sum_{t=1}^{12} \frac{(M_t - B_t) + (M_{t-1} - B_{t-1})}{P_t + P_{t-1}} \frac{P_t - P_{t-1}}{P_t} \]  

\[ P_j = \text{Arithmetic Average Price Level of Year } j (\text{IGP - DI}). \]

\[ B_t = \text{Stock of High Powered Money at the End of Month } t. \]

\[ M_t = \text{Stock of Means of Payment (M}_1 \text{) at the End Month } t. \]

\[ P_t = \text{Price Level (IGP - DI) in Month } t. \]

From the equations which defines the relation between the real interest rate \((r)\) and nominal interest \((i)\):

\[(1 + r)(1 + \pi) = (1 + i) \]  

Since \(M\) and \(B\) pay zero nominal interest, the real interest paid can be calculated putting \(i = 0\) in (4):

\[ r + \pi + r\pi = 0 \rightarrow r = \frac{-\pi}{1 + \pi} \]

The real interest received by the banking system on \(M_1\) or \(B\) is then given by

\[-r = \pi / (1 + \pi) = \frac{P_t - P_{t-1}}{P_t} \]
Thus,

Nominal Interest Paid - Real Interest Paid = Real Interest Received = \( \overline{B} \cdot \frac{(P_t - P_{t-1})}{P_t} \) in the case of the Monetary Authorities, or \( \overline{(M_1 - B)} \cdot \frac{(P_t - P_{t-1})}{P_t} \) in the case of the commercial banks. \( \overline{B} \) and \( \overline{(M_1 - B)} \) represent, respectively, the monthly average balance of High Powered Money, and Demand Deposits in the Commercial Banks minus Total Reserve Requirements. In (2) and (3), they are approximated by \( \frac{(B_t + B_{t-1})}{2} \) and \( \frac{(M_t + M_{t-1})}{2} \). An arithmetic average is also used to approximate the average price level during the month.

Some additional calculations using the monthly averages of the daily balances of \( B \) and \( M-B \) were also used for the period 1982-1987. This option is surely more appropriate (because of the sharp balance variations of both \( B \) and \( M \) that there can be at the last day of the month), but the necessary data is not available for the period 1947-1982. Anyway, the results as of this data did not show substantive variations, when compared with the ones here presented.

Source of Data: Fundação Getulio Vargas - Conjuntura Economica and ARIES Project.

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