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On the Growth Effects of Barriers to Trade

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Abstract
We study the macroeconomic effects of international trade policy by integrating a Heckscher-Ohlin trade model into an optimal growth framework. The model predicts that an open economy will have higher factor productivity and faster growth. Also, under protectionist policies there may be “development traps,” or additional steady states with low income. In the last case, higher tariffs imply lower incomes, so that the large cross-country differences in barriers to trade may explain part of the huge dispersion of per capita income observed across countries. The model simulation shows that the link between trade and macroeconomic performance may be quantitatively important.

1 Introduction
There is a large disparity across countries in income, growth rates, and other key macroeconomic variables. Abundant empirical research has studied whether those cross-country variations are related to differences in international trade policy, finding that open economies have higher levels of capital

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and total factor productivity, achieve faster growth, and enjoy better chances
of convergence in per-capita output. This paper addresses whether the sim-
plest comparative advantage arguments, when framed in a macroeconomic
model, can explain the link between trade policy and macroeconomic per-
formance, and account for a relevant portion of the observed income gap
between rich and poor nations.

Evidence regarding the link between trade liberalization and output lev-
els is provided by Frankel and Romer (1996) and by Hall and Jones (1999).
They elaborate that openness (namely, low tariffs and low incidence of quo-
tas and other non-tariff trade barriers) increases income per-capita both by
enhancing capital accumulation and, notably, by expanding productivity.
Evidence regarding the positive relationship between open trade policy and
growth rates, which is discussed in Krueger (1997), has been documented
recently by Edwards (1997), Frankel, Romer and Cyrus (1996), Harrison
(1995), Lee (1996) and Taylor (1996), using various types of data samples
and techniques. Previous empirical work on the same topic is also surveyed in
Edwards (1993). Finally, evidence regarding the relationship between trade
policy and convergence is provided in Sachs and Warner (1995), who show
that among the countries that they consider open to trade, one cannot statis-
tically reject convergence, while the same is not true in a broader sample of
countries. To us, this finding suggests that very poor countries that choose
isolationist trade policies set for themselves “development traps”, and that
the pervasiveness of protectionism among LDC’s may be responsible for the

These empirical results have been derived from data panels that include
a majority of poor countries. For those nations, differences in factor endow-
ments with respect to their wealthier main trading partners are a strong a
motive for trade, probably as important as any other. Then, without detri-
ment to other possible channels for the macroeconomic effects of trade policy,
it seems natural to inquire whether the links mentioned above between trade
liberalization and macroeconomic performance can be accounted for in a com-
parative advantage framework in the vein of the Heckscher-Ohlin model. Yet,

1Most previous efforts to explain the link between trade and growth emphasize other
motives for trade, especially increasing returns—as in Romer and Rivera-Batiz (1991) or
by doing—Young (1992)—, or the absorption of foreign technology—Holmes and Schmitz
(1995). Two exceptions that use a comparative advantage model are Corden (1971) and
Ventura (1997).
this is not a channel that many believe would be fruitful. As Stiglitz (1998) points out, we understand well why exploiting comparative advantage generates static welfare gains, yet the dynamic improvement in aggregate output that the data suggests is still largely unexplained.\textsuperscript{2}

We merge a factor-endowment trade structure into the standard neoclassical growth model. Like in Corden (1971), Trejos (1992) or Ventura (1997) in order to do this we assume two tradable and non-storable intermediate goods, used in the production of a non-tradable final good. This has the advantage that the potentially complex dynamic trade problem can be solved sequentially. First one solves the static trade and factor allocation problem; the solution takes the form of an endogenous mapping between factor endowments and aggregate output. Second, one solves the optimal growth problem, treating that mapping as an exogenously given production function. We focus on a small, price-taking economy, which trades with a larger “world economy”. We interpret the latter to be a developed country (or group of countries) that has already converged to a balanced growth path and for which, due to its size, trade is relatively unimportant.

The equilibrium mapping that shows the relationship between factors and output, and plays the role of the small economy’s implicit production function, is affected by tariffs and by international prices. Specifically, an increase in trade barriers, or a worsening in terms of trade, have an effect quite similar to a decrease in productivity. In that sense, the model can be useful in understanding the relationship between openness and measured TFP. Furthermore, it is the case that this mapping, under no trade, takes the form of a standard, constant returns to scale Cobb-Douglas function, while for the small, trading economy the mapping is qualitatively different, and not even necessarily concave. As a consequence, we find that a trading economy may have development traps, in the sense that there are not one but two locally asymptotically stable balanced growth paths.\textsuperscript{3} The higher

\textsuperscript{2}Specifically, he writes “Interestingly, the process by which trade liberalization leads to enhanced productivity is not fully understood. The standard Heckscher-Ohlin theory predicts that countries will shift intersectorally, moving along their production possibility frontier, producing more of what they are better at and trading for what they are worse at. In reality, the main gains from trade seem to come intertemporally, from an outward shift in the production possibility frontier as a result of increased efficiency, with little sectoral shift. Understanding the causes of this improvement in efficiency requires an understanding of the links between trade, competition, and liberalization. This is an area that needs to be pursued further.”

\textsuperscript{3}In this model trade policy affects not only the set of steady states, but also the speed
balanced growth path is largely invariant to tariffs, while the lower one is sensitive to trade policy so that, the higher the barriers to trade, the lower the levels of inputs, productivity and output.\footnote{Following Parente and Prescott (1994), researchers have attempted to explain the cross-country income disparities as the steady state income disparities emerging from measurable differences in parameters across countries. If one can interpret rich countries as being in the high steady state, and at least some poor countries as being in the low one, in a model that by explicitly modelling trade and trade barriers allows multiple stable steady states, the income disparities that can be explained with a given difference in fundamentals can be much larger.}

We calibrate the model to see what is the order of magnitude of the effects that it predicts for trade policy. The calibration is conservative, in the sense that the factor shares are not allowed to differ much across tradeable goods, thus limiting the size of the potential gains from trade. In fact, under the chosen parameters the static effects from trade are essentially negligible for the world’s rich countries. Still, even under those parameters gains from trade can be important for countries that are relatively poor. For instance, for a country with one-fifth of US output the static difference between having 10\% tariffs and 100\% tariffs can be 7.5\% of total factor productivity. One can also calculate the sensitivity of the output level of the development trap (the lower steady state), as one changes the tariff rate. The effects are, again, large. For our baseline parameters, at very low tariffs the trap’s output per capita is nine-tenths of the output of the world’s richest economy. Meanwhile, at 40\% tariffs, the economy in a trap has only one-third of the richest country’s income; at 100\% tariffs, one-fifth. In summary, a closed economy will be less productive in the short run, grow more slowly towards steady state, and perhaps even converge to a lower steady state level of output, than an open economy.

Section 2 presents the model, and Section 3 derives the theoretical results. The calibration and quantification of the results is performed in Section 4. Section 5 concludes.

\footnote{Trejos (1992) compares the transitional dynamics under free trade to those under no trade. The main result is that a very poor economy initially grows faster under free trade than in autarky.}
2 The economy

Time is discrete and unbounded. Our representative country is populated by a continuum of identical, infinitely-lived individuals. There are three goods produced in this economy. Two of those goods, called A and B, are non-storable intermediate products. They are only used to make the other good, called Y, a final product that can be consumed or invested. There are also two factors of production in this economy: labor L and physical capital K.

The endowment of labor, measured in efficiency units, grows at an exogenous rate \( \alpha \), due to both demographic expansion and technical progress.

The technology is as follows: physical capital and labor can be used in the production of the intermediate goods A and B, with constant returns Cobb-Douglas technologies:

\[
A = K_A^{a_A} L_A^{1 - a_A} \\
B = K_B^{a_B} L_B^{1 - a_B}.
\]

Without loss of generality, we assume that A is the labor-intensive good, so \( a_A < a_B \). The production of the final good Y uses the intermediate goods:

\[
Y = \ell^{a_a b_{\ell}^{1 - a_a}}.
\]

Final goods can be used in either consumption or investment. The law of motion for capital is

\[
K^0 = (1 - \delta) K + Y + C
\]

The representative consumer owns the capital, and faces the standard intertemporal maximization problem, with instantaneous log utility and discount rate denoted \( \gamma \). All markets are perfectly competitive. The ordinary market clearing conditions for both factors are

\[
K, \quad K_a + K_b \\
L, \quad L_a + L_b
\]

The economy is only one of many others in the world, and is small, at least compared to a certain very large country, which we just call the world.

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5 The results are not changed significantly if we assume that factors are used also in the production of final goods, and rewrite (??) as \( Y = \ell^{a_a b_{\ell}^{1 - a_a}} K_a^{a_a} L_a^{1 - a_a} + Y \).
economy. The relative sizes of the two economies mean that, if they trade, the small economy will be a price-taker, as the prices at which they trade are the same as the world-economy’s autarkic prices. We assume that physical and legal characteristics imply that only the intermediate goods A and B can be exchanged internationally; our small economy’s government imposes a flat, ad-valorem tariff \( \xi \); and the proceeds from tariffs are transferred back to households. Factors, final output or any form of financial obligation cannot be traded. This also implies that there will always be trade balance in intermediate goods. All markets that do exist are assumed to be perfectly competitive. In the baseline scenario, all parameters, preferences and technologies are assumed to be common across the two countries.

Since it can be shown that under no trade this model has a unique, globally stable balanced growth path with growth rate \( \dot{Y} \) (a steady state if we measure capital and output in per-labor units) we can assume that the world economy has already converged to said steady state, with a capital-labor ratio denoted \( k^* \). That means that the international relative price of A in terms of B, denoted \( p \), is constant, as it is the no-trade equilibrium price in the world economy, pinned down by \( k^* \). In our small economy, the domestic relative price of A in terms of B (which may differ from \( p \) due to tariffs) is denoted \( q \). The domestic price of final output \( Y \) (again in terms of B) is denoted \( \frac{\pi}{4} \).

3 Equilibrium

The procedure used to find equilibria for the model is the following. First, we solve for the allocation of capital \( K \) and labor \( L \) among the production of A and B, the quantities \( a \) and \( b \) of intermediate goods used domestically, and the amount of final output \( Y \) that is produced. This is a static problem, which yields an equilibrium mapping

\[
Y = F(K; L; \xi; p)
\]

that relates final output with factor endowments. Second, we use that equilibrium mapping \( F \) as if it was an exogenously given technology, and solve the standard dynamic problem that emerges as a result.

6 This because trade with the small economy is a miniscule fraction of economic activity for the world economy, even if it is large from the perspective of the small economy.
To get $F$ notice that the equilibrium solutions for $f_A; B; a; b; K_i; L_i$ must satisfy, each period, the following properties\(^7\):

1. The allocation of $K$ and $L$ maximizes the value of intermediate-good output at domestic prices, given endowments and technology:

$$A; B = \arg\max_{A; B} qA + B$$

$$\text{s.t.:} \quad A \cdot K_A L_A \leq a_A \quad B \cdot K_B L_B \leq b_B \quad K = K_A + K_B \quad L = L_A + L_B$$

2. Producers of final goods maximize profits, taking domestic prices as given:

$$a; b = \arg\max_{a; b} \frac{1}{2} a^b \cdot q a \cdot b$$

3. With no factor flows and no debt there is current account balance

$$p_a + b = pA + B$$

4. The local prices must satisfy an after-tariff law of one price

$$q = \begin{cases} p(1 + \xi) & \text{if} \quad K = L < k^w \\ p d(1 + \xi) & \text{if} \quad K = L > k^w \end{cases}$$

Based on the requisites 1-4 mentioned above, one can derive the equilibrium relationship $F$. The basic calculations are relegated to the Appendix, while here we stress what is important, as follows

1. If the capital-labor ratio $k = K = L$ is very different from the world's $k^w$, the economy will specialize in the production of only one intermediate good: the one that uses intensively the factor that is relatively abundant locally. Thus, there are critical levels $\bar{R}_A < k^w$ and $\bar{R}_B > k^w$ such that if $k \cdot \bar{R}_A$ then the country only produces $A$, and if $k \cdot \bar{R}_B$ then the country only produces $B$. In those cases, $Y$ is a Cobb-Douglas

\(^7\)For an exposition on the basic Heckscher-Ohlin factor endowments model, see Dixit and Norman (1980).
function of $K$ and $L$, with capital share $\alpha_b$ or $\alpha_a$, depending on which intermediate good gets produced. Furthermore, the critical values $R_A$ and $R_B$ are sensitive to $\xi$. In particular, with higher tariffs the economy is less prone to specialize, so $\partial R_A = \alpha < 0$ and $\partial R_B = \alpha > 0$, with $R_A ! 0$ and $R_B ! 1$ as $\xi ! 1$.

2. If $k$ is close to $k^*$ (close enough that the difference between $p$ and the autarkic price in the small country is less than the tariff rate), the incentives to trade are not enough to overcome the barriers, and so the economy does not trade at all. In other words, there exist $R_1$ and $R_2$, where $R_A < R_1 \cdot k^a$ and $k^a \cdot R_2 < R_B$ such that if $k \in [R_1; R_2]$ then there is no trade, so $a = A, b = B$. Again, in this case $Y$ is a Cobb-Douglas function of $K$ and $L$, with capital share $\alpha = \alpha_a + (1 - \alpha_b) \alpha_b$. Also, the critical values $R_1$ and $R_2$ are sensitive to $\xi$. In particular, $\partial R_1 = \alpha < 0$ and $\partial R_2 = \alpha > 0$: hence, the higher the tariff rate, the broader the interval of capital-labor ratios under which there is no trade. Also, $R_1 ! 0$ and $R_2 ! 1$ as $\xi ! 1$, while $R_1 = R_2$ if $\xi = 0$.

3. If $k$ is neither too close nor too far from $k^*$, the economy will produce both intermediate goods, yet still trade. In those cases holds a result analogous to the Factor Price Equalization Theorem, which states that equilibrium marginal returns of capital and labor are not sensitive to small variations in the factor endowment. What that means is that final output $Y$ is linear in $K$ and $L$ when $k \in [R_A; R_1]$ or when $k \in [R_2; R_B]$.

Hence, the equilibrium relationship from $K$ and $L$ to $Y$ takes the form

$$F(K; L; \xi; p) = \begin{cases} -_1 K \alpha_a L^1 \alpha_b & \text{if } K = L < R_A \\ -_2 K + -_3 L & \text{if } K = L \in [R_A; R_1] \\ -_4 K \alpha_b L^1 \alpha_a & \text{if } K = L \in [R_2; R_B] \\ -_5 K + -_6 L & \text{if } K = L \in [R_2; R_B] \\ -_7 K \alpha_a L^1 \alpha_b & \text{if } K = L > R_B \\ \end{cases}$$

where the values -i are functions of parameters, and are affected by $p$ and $\xi$. The derivation of $F$ is presented in details in the appendix. At this point, note that for a closed economy (be it an economy with a very large tariff rate $\xi$, or be it the price-setting world economy), it is the case that $[R_1; R_2] = \{\}$. Consequently, without trade our model simply collapses to one with the aggregate production function $F^0(K; L; \xi; p) = -_4 K \alpha_b L^1 \alpha_a$. 

8
The function $F$ is decreasing in $\zeta$ (strictly decreasing if $k \geq [R_1; R_2]$), as the gains from trade in this economy manifest in the mix of inputs that goes into a "bi"., and tariffs unambiguously reduce the gains from trade in the Heckscher-Ohlin framework. Not only total output, but marginal output is also affected by tariffs. In particular, $\frac{\partial F}{\partial K} = @ F < 0$ if $K = L$ is very small, and vice versa. Also, output is sensitive to the terms of trade. In particular, $F$ is increasing in $p$ if the economy is an exporter of good A (that is, if $k < R_1$) and decreasing in $p$ if B is the export good (that is, if $k > R_2$). These relationships of $F$ with $\zeta$ and $p$ are continuous. The effects of tariffs on output are illustrated in Figures 1 and 2, for the case where $\zeta = 0$ and $\zeta = 1$ and the case where $\zeta = 0$ and $\zeta = 0.3$, respectively.

Figure 1: Production functions when $\zeta = 0$ and $\zeta = 1$
Figure 2: Production functions when $\xi = 0$ and $\xi = 0.3$

Note that in the first case the curve corresponding to $\xi = 0$ is everywhere (i.e., for any $k$ but $k^*$) above the curve corresponding to $\xi = 1$, while in the second case there is an interval, $[R_1; R_2]$, where the curves coincide.

For all values of $p$ and $\xi$, $F$ is homogeneous of degree one and continuous in $K$ and $L$. Hence, we can rewrite as $y = f(k; p)$, where $y = Y = L$ and $k = K = L$, and $f$ is a continuous function. Generically, $f$ is also locally concave and continuously differentiable, although, if $\xi > 0$, global concavity and continuous differentiability is lost because $f'(k)$ has discrete variations (up or down) at the critical values $R_i$. This is illustrated in Figures 3 and 4: for $k < R_A$; the slope of $f'(k)$ is negative; it jumps down at $R_A$ and for $k > R_A$; $f'(k)$ is horizontal; it jumps up at $R_1$ and for $k > R_1$; $f'(k)$ is negative. After $R_2$, the curve is horizontal and negative again after $R_B$ (not shown in Figure 3).

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8 Notice however that $F$ is, on the other hand, globally concave and $C^1$ if $\xi = 0$ (because $R_1 = R_2$) or if $\xi = 1$. This will be useful later.
Figure 3: Slope of $f(k)$ when $\zeta = 0.1$

Figure 4: Slope of $f(k)$ when $\zeta = 0.3$

Now consider the macroeconomics of an intertemporal model with an aggregate technology given by $F$. A first implication is that higher tariffs will correspond to lower measured total factor productivity in this model, as the gains from trade affect output. This is consistent with the empirical findings mentioned above, about how productivity residuals in cross-country development accounting exercises are correlated with trade liberalization. One should point out that the effect of tariffs on output is not uniform, but rather is stronger the farther away is $k$ from $k^*$; as trade is bigger when the
economy is very different from the world economy in which the price p is determined.

Now, consider the dynamic problem of an economy with technology F. Usual derivations, assuming log utility, lead to the Euler equation \( \ddot{A}(k; k^0, k^\delta) = 0 \); where

\[
\ddot{A}(k; k^0, k^\delta) = [f'(k) + (1 + \delta)k^0(1 + \delta)] - (1 + \delta)[f'(k) + (1 + \delta)k^\delta][f'(k) + 1] \]

which characterizes the law of motion of k. We discuss transitional dynamics in the next section, after providing a quantitative version of the model. For now, we are interested in identifying the set of locally stable balanced growth paths (steady states in k). We know that any balanced growth path will be characterized by a capital-labor ratio \( R \) that satisfies

\[
\frac{1 + \delta}{1 - \delta} + \delta |1 < f'(R) \quad (5)
\]

where \( R_1 \) and \( R_2 \) are the limits as \( k \) approaches \( \infty \) from below and above. Notice that (5) is written in this way rather than a single equation since the global concavity of \( f \) is not guaranteed. In other words, (5) allows for steady states at the critical values \( R \). For the baseline scenario, where we have assumed that our small country is identical in preferences and technology to the world economy, the following proposition characterizes the set of steady states.

**Proposition 1** For all \( \delta > 0 \) there are two locally stable balanced growth paths. One of them is invariant in \( \delta \), and with capital-labor ratio \( k^\ast \). The other one has a lower capital-labor ratio, given by \( R_1 \), which is decreasing in \( \delta \).

**Proof:** First, recall that for all practical purposes the world economy does not trade, and its production function is \( f''(k) = -4kg^\delta \), the same as our small economy's in an interval \([R_1; R_2]\) that contains \( k^\ast \): Therefore, as \( k^\ast \) satisfies (5) for the world economy, it must also satisfy the same condition for the small economy, and thus constitutes a balanced growth path. Furthermore, one can show that

\[
\lim_{k! R_1} f'(k) > \frac{1 + \delta}{1 - \delta} + \delta |1 > \max_{z; g} f'(R) \]
which implies that $\bar{R}_A$ satisfies (??), and is the only balanced growth path in $[\bar{R}_A; k^a]$ [ $(k^a; 1)$]. Finally, as $f'(k)$ is strictly decreasing in $0; \bar{R}_A$; there are no other balanced growth paths.

Proposition ?? establishes that a small country (identical to the world economy) that starts out poor enough will always fall in a low balanced growth path, (or development trap). For low tariff rates, this lower path has a similar $k$ as the world economy, but as tariffs increase this level of $k$ falls. As $F$ itself is also decreasing in $\xi$ everywhere but at $k_0$; output is also decreasing in $\xi$ at the development trap, and invariant in $\xi$ at the high steady state.\footnote{Noticethat figures 3 and 4 display, below the horizontal axes, the line $\bot \ bot + \bot \bot \bot \bot$. See section 4 for parameterization.}

This result is then very consistent with the finding that one cannot statistically reject convergence among countries with low barriers to trade, while countries with high barriers tend to stagnate at much lower levels of capital and output, because even if a low-tariff economy falls in the trap (converges to the lower steady state), it involves similar inputs (and the same productivity) as the world economy. Meanwhile, when a high-tariff country falls in the trap, it is at low levels of inputs and of productivity.

More is learned when one considers also deviations from the baseline scenario, allowing differences in preferences or technology. Consider for instance what happens if there are cross-country differences in depreciation rates, or $\pm \xi \neq \pm \xi$. On the one hand, if $\pm < \pm \xi$, then for low tariff rates there is a unique balanced growth path (with $k > k^a$), while at a high enough tariff there are two balanced growth paths, one with $k$ higher and one with $k$ lower than $k^a$, and with the lower balanced growth paths decreasing in $\xi$. In other words, for low tariffs there is no trap. On the other hand, for $\pm > \pm \xi$, again for low tariff rates there is a unique balanced growth path, with $k < k^a$ and decreasing in $\xi$; at higher tariffs there exist two balanced growth paths.

4 Quantiﬁcation

In this section we try to quantify the order of magnitude of the effects of introducing trade into the basic growth model. For that purpose, we make some modifications to the model presented in the last section, and then calibrate the model. With the calibrated model, we can do numerical exercises.
that address three questions regarding the effects of high tariffs. First, we ask about the size of the productivity decrease. Second, we ask about the relative income at the development traps. Third, we ask about the quantitative impact on the transitional dynamics towards steady state. The bottom line of all three exercises is that the macroeconomic effects of increasing tariffs can be important.

Before we address these questions, we modify the model by introducing human capital. It has been argued before (see Mankiw, Romer and Weil (1992) or Chari, Kehoe and McGrattan (1997), for example) that the standard growth models perform much better quantitatively if one allows for this other type of capital; we have done the quantitative analysis both with and without human capital, and confirmed that this is true here as well. The way we will do this is simply to assume that the nal good production process uses human capital (denoted $H$), in the form

$$Y = \ell^3 a^\gamma b^{1-\gamma} H^{1-\gamma}.$$  

(6)

It is easy to show that in equilibrium the mapping from factors to output takes the form

$$Y = G(K; H; L; \ell; p) = F(K; L; \ell; p)^{\theta H^{1-\gamma}}$$

(7)

where $F$ is the same function defined in (??). We give human capital the same law of motion as we gave to physical capital, defined in (??). The theoretical results found in the previous section still hold in this alternative formulation.

The reader may question whether this is the most reasonable formulation. For instance, one may be bothered by the asymmetric treatment of both capitals, as $H$ is only used to make nal goods while $K$ is only used to make intermediate goods. As it turns out, we could assume that all factors are used in nal good production, and still derive (??), (although the formulae for $\ell$ would change, of course). Even the calibration would not change, as we pick the parameters in $\ell$ to match gains from trade, rather than share-per-industry observations. Second, we could introduce $H$ in the intermediate good production, but this does not seem reasonable as it implies all the well-known complications associated with n.m. Heckscher-Ohlin models, which deviate from the focus of this paper, while adding nothing to the macroeconomic analysis. Third, of course, we could just leave human capital out of the model, and calibrate in a way that interprets $K$ as
a broader physical-and-human capital measure. This clearly does not affect the qualitative results, and in fact yields similar quantitative results.

We now calibrate the model. Recall that, in the larger world economy, the equilibrium determination of final good output per unit of labor is \( y = £k^{\frac{\theta}{h}} \). We interpret that economy to be the US (or the US plus other rich countries). Then, we can pick parameters as to mimic the calibration of real business cycles, closed economy models for the US that is common in the literature. That leads us to pick the parameters \( \frac{\theta}{h} = 3 \) that would generate without trade a 2% per-capita growth, 6.1% net return on capital, and a 2.75 physical capital to annual output ratio along the balanced growth path. The shares of human and physical capital are conventionally picked to be \( \frac{\theta}{h} = 1; \frac{\theta}{h} = 1 = 3 \).

For lack of information on the parameter \( \omega \), we opt for symmetry and chose the value \( 1=2 \). The chosen value of \( \omega \) does not affect the results much, provided that one adjusts the other parameters to maintain the calibration of \( \frac{\theta}{h} \) and \( \frac{\theta}{h} \). The value of \( \frac{\theta}{h} \) is selected as to normalize, without loss of generality, to \( k^{\frac{\theta}{h}} = h^{\frac{\theta}{h}} = 1 \); under that normalization and given \( \frac{\theta}{h} = 1=2 \); it follows that \( p = 1 \).

This pins down the average \( \frac{\theta}{h} \) of the two parameters \( \frac{\theta}{h} \) and \( \frac{\theta}{h} \), but leaves freedom of choosing one of them. The choice of these two parameters is important, as the quantitative effects of all trade-related phenomena are bound to be larger the spread \( \frac{\theta}{h} \) is, leaving \( \frac{\theta}{h} \) constant. One way to discipline the choice of parameters is by choosing \( \frac{\theta}{h} \) and \( \frac{\theta}{h} \) conservatively to match the relatively small estimations in the literature about the size of gains from trade in rich countries. Our baseline choice will be \( \frac{\theta}{h} = 0.42 \) and \( \frac{\theta}{h} = 0.58 \). With those values, the model predicts that the total static gains from trade, interpreted as the total difference in final good output associated with comparing \( \omega = 0 \) with \( \omega = 1 \), are less than 1% of GDP for any country with 75% or more of US output per worker (the world’s 18 wealthiest countries). The parameters also imply that total static gains from trade are very small for the US and Canada, and only worth between 2 and 5 percent of GDP for Mexico, in a static, applied general equilibrium model. Considering that NAFTA does not imply a change from autarky to unrestricted free trade, but rather a much smaller change in effective protection, our calibration is conservative even according to their results.

\[ ^{10} \text{For instance, Ventura (1997) demonstrates that one can even get endogenous growth by assuming } \frac{\theta}{h} = 0, \frac{\theta}{h} = 1 \text{. On the other hand, there is no trade at all under any circumstances if } \frac{\theta}{h} = \frac{\theta}{h}. \]

\[ ^{11} \text{As an example, Kehoe and Kehoe (1994) estimate the effect of NAFTA to be very small for the US and Canada, and only worth between 2 and 5 percent of GDP for Mexico, in a static, applied general equilibrium model. Considering that NAFTA does not imply a change from autarky to unrestricted free trade, but rather a much smaller change in effective protection, our calibration is conservative even according to their results.} \]
trade (that is, the whole output difference in going from $\bar{\gamma} = 1$ to $\bar{\gamma} = 0$) are only 4.8\% of GDP for a country with Mexico's income, and only 9\% of GDP for countries, like Brazil or Costa Rica, with roughly a third of US output per worker.

Under those parameters, consider first the static effect of tariffs on measured productivity. Why do tariffs reduce productivity? For two reasons, both completely standard within the Heckscher-Ohlin model of international trade. First, they distort the prices used in the decision of how to allocate the factors $K$ and $L$ between the intermediate goods production, a distortion that reduces the value of national product at international prices. Second, the same price distortion affects the ratio of the two intermediate inputs used by the final good producers, as the relative price they see, $q$, is not the same as the actual opportunity cost posed by the international market, which is the price $p$.

Figure 5 shows proportionately how much higher would productivity be with $\bar{\gamma} = 10\%$ than with $\bar{\gamma} = 40\%$, as a function of the stock of physical capital, given human capital. We can see that, while for rich enough countries the difference is negligible, for the very poor countries these two policies yield a difference in productivity of around 9\%.

![Figure 5: Productivity differences as function of inputs](image)

Figure 6 shows the same thing, but comparing $\bar{\gamma} = 10\%$ with $\bar{\gamma} = 100\%$.
Estimates of the effective rate of protection for many developing countries in the 70's and 80's are around 100%. Now, the effects in productivity are about twice as large. In a closed-economy model, these differences in measured productivity would go unexplained. It is not surprising that empirical studies find a cross-country relationship between trade barriers and measured TFP.

![Figure 6: Productivity differences as function of inputs](image)

We now study the magnitude of the development traps presented in Proposition 1. We focus on the case where there is no difference in parameters between the world economy and our small economy, which means that both have a high steady state with the same input and output levels. What we ask is how big are the differences between the two steady states (between an economy that has fallen in the trap and one that has converged to the higher balanced growth path), and the sensitivity of those differences to trade protection.

For our baseline parameters, we find that for low tariffs the gap is actually fairly small. At $\tilde{\zeta} = 0$, income at the lower steady state is 90% of income at the higher steady state. As $\tilde{\zeta}$ increases, however, the gap widens. When the tariff rate is 50%, income at the trap is only a third of income at the higher steady state; when the tariff rate is 100%, less than a third. This is also illustrated in Figure 7.
Figure 7: Relative output levels (development trap=high steady state) as function of tariffs.

This result is consistent with the empirical work mentioned in the introduction, which found that inputs, and not only productivity, are affected by trade protection. In fact, the cross-country differences in output that one can generate with realistic tariff rates are fairly large, since countries with 100% effective rates of protection are, unfortunately, not rare. The observed income differences across countries can thus be explained as being caused by the dispersion of trade policies in the recent past, as reflected by differences in tariffs but also by their non-tariff equivalents such as quotas and import bans.

Finally, consider now the effects of trade on the transitional dynamics to the balanced growth path. To see this, we solve the dynamic problem posed by (??), using Coleman’s policy function iteration, for the cases where $i = 0$ and $i = 1$.

We then generate simulated transition paths, for various initial conditions, using the policy functions for the two tariff choices. We are interested in comparing the difference in output and the difference in

\[\text{Because for } i = 0 \text{ and } i = 1 \text{ the production function is globally concave and } C^1 \text{ we can use this method to solve the dynamic problem. In the more general case for arbitrary } i, \text{ it is necessary to use value function iteration, since not all local solutions to the Euler equations are going to be real maximands of the one-period maximization problem.}\]
consumption between the open economy path and the closed economy path, at each point in time, assuming the same initial level of \( H \) and \( K=L \).

Due to the static gains mentioned above, even at the initial (and common) levels of \( k \) and \( h \) there is a gap between the open and closed economies. If the initial condition is at a low level of \( h \) and \( k \), we also see the open economy accumulate factors faster, and thus amplify the difference in output over its closed-economy counterpart as time goes by. Over the first few years, the output difference becomes much larger; for many initial conditions it gets to be 3 times or more than the original productivity difference. As time goes by, however, the closed economy begins to catch up and eventually overtake. Figures 8 and 9 illustrate the differences in the transition. It is drawn for a case where the initial stocks of both capitals are at around 6% of their steady state values.

![Figure 8: Output differences during the transition path](image-url)
5 Conclusion

In this paper, we have studied a model that integrates a simple comparative-advantage trade model (the Heckscher-Ohlin model) in an dynamic optimal growth model. The main theoretical results are that barriers to international trade reduce the total factor productivity of an economy and, more importantly, can cause the existence of development traps, or low-income steady states. We then calibrate the model, to get an idea of the possible order of magnitude of the effects of trade policy on macroeconomic performance. We find that the aggregate effects of trade barriers can be large, in productivity, in speed of convergence, and in long run output.

We plan to explore in future work some applications of the model presented here. In particular, one could inquire about the aggregate long-run effects of trends in the terms of trade. This is relevant because the supposed tendency to worsening terms of trade has been used as an argument to justify protectionist policy, at least in Latin America. One could also perform developing accounting exercises, explicitly using our reduced form production function and empirical evidence on trade barriers. This would relate to the question of what fraction of observed cross country variations in output can be attributed to national differences in trade policy, rather than to other sources of productivity variation.
A The production function

In the appendix we present in details the derivation of the production function used in the paper. After presenting general properties of the model we derive the production function for the closed economy case, then for the case of an open economy with no tariffs and finally the most relevant case, the open economy with positive tariffs.

The production function for the two intermediate products, a and b, is given by:

\[ A = K_A L_A^{1-a} \quad \text{and} \quad B = K_B L_B^{1-b} \]

Total production of the final good is given by

\[ Y = a^a b^b \]

(8)

where a and b are the total inputs used of the two intermediate products. In the closed economy, these inputs have to be locally produced, so the economy is constrained by \( a = A \); \( b = B \); while in the open economy trade is possible, yet we do not allow external debt, and so the constraint is

\[ p_a + b = p A + B \]

where \( p \) is, of course, the international price of \( A \) relative to \( B \), which we will assume that our economy takes as a parameter. We assume that final goods are not tradable. They are consumable and investable. It is useful to get \( Y \) as a function of \( K \) and \( L \), taking the equilibrium \( A; B; a; b \) implicitly. We know that in the closed economy, the planner maximizes

\[ K_A L_A^{1-a} \quad \text{h} \quad (K_i K)^{(i)} (L_i L_A)^{(i)} \]

First order conditions are given by

\[ \frac{\partial (1_i \circ L_A^{(i)})}{L_A} = (1_i \circ \frac{(1_i \circ L_A^{(i)})}{(L_i L_A)^{(i)}}) \]

\[ \frac{\partial (1_i \circ K_A^{(i)})}{K_A} = (1_i \circ \frac{(1_i \circ K_A^{(i)})}{(K_i K_A)^{(i)}}) \]

which yield

\[ L_A = \frac{\circ 1_i \circ L_A^{(i)} (1_i \circ K_A^{(i)})}{1_i \circ \circ \circ 1_i \circ (1_i \circ K_A^{(i)})} : L \]
\[
\begin{align*}
K_A &= \frac{K}{\beta_a \beta_b + \beta_b i \beta_b^o} \\
L_B &= L(1_i \beta_b) \frac{1_i \beta_b^o}{1_i \beta_b i (1_i \beta_b^o) \beta_b} \\
K_B &= K \beta_b^o \frac{1_i \beta_b}{\beta_a + \beta_b i \beta_b^o}
\end{align*}
\]

So that output under autarky is given by

\[
Y = \beta_a(1_i \beta_b)^{1_i \beta_b^o} K \beta_a^o + \beta_b(1_i \beta_b) L(1_i \beta_a^o + (1_i \beta_b)(1_i \beta_b^o))
\]

where

\[
i = \frac{[\beta_a^o (1_i \beta_b)^{1_i \beta_b^o} \beta_a^o] \beta_b^o \beta_b^o \beta_b (1_i \beta_b^o)}{[1_i \beta_b^o \beta_b i (1_i \beta_b^o) \beta_b^o] \beta_b^o \beta_b^o \beta_b (1_i \beta_a^o + (1_i \beta_b)(1_i \beta_b^o))}
\]

The relative price of A in autarky equilibrium becomes

\[
\frac{\beta_b K_b \beta_b^o}{\beta_a K_a \beta_a^o} = \frac{1_L}{1_L \beta_b^o}
\]

which in equilibrium becomes

\[
\frac{\beta_b (1_i \beta_b)^{1_i \beta_b^o} \beta_a^o}{\beta_a^o (1_i \beta_a^o) \beta_b^o} \frac{K \beta_a^o \beta_b^o \beta_b}{L \beta_a^o \beta_b^o \beta_b} = \frac{1}{p^a}
\]

The capital-labor ratio corresponding to \( p^a \) is \( k^2 \):

For the case where \( 0 < \beta < 1 \) there are several relevant cases. First, the country could produce only A, only B or be diversified; then, when diversified, it could be exporting A, exporting B or not trading and at the closed economy solution written above.

The economy will be in autarky whenever

\[
p > p^a > p=(1+\beta) \quad \text{or} \quad p < p^a < p(1+\beta)
\]

because it loses the comparative advantage it has in good A or B, respectively. In that event the production function is given by the closed economy solution written above (equation [??]).

Consider the case where the country has comparative advantage in A and it trades (that is, \( p^a < p=(1+\beta) \)). In this case (and also in the case when
tau is equal zero), we have to work out in two stages. In the first one the factor allocation and intermediate production of a and b is determined and Q = pa + b is desired: In the second stage, the demand for a and b given Q is solved.

The demands for a and b satisfy

\[ pa + b = Q \quad \text{ and } \quad \frac{p}{1 + \theta} = \frac{b}{a} \]

Solving for these equations and substituting into the final good production function (eq. ??), we get:

\[ Y = \theta^0 (1 - \theta) L_A^1 i_1^{1 - \theta} p_1^1 \theta Q \frac{1 + \theta}{1 + \theta} \]

To obtain Q note that there is a bound \( p_a \) such that if \( p > p_a \), then Q corresponds to the value of potential a output. In this case Q = \( pK_A^{1 - \theta} L_A^{\theta} \) and then:

\[ Y = \theta^0 (1 - \theta) L_A^1 i_1^{1 - \theta} p_1^1 \theta K_A^{1 - \theta} L_A^{\theta} \]

When \( p < p_a \) the economy is diversified in intermediate good production and then \( Q = pa + b \) where a and b are the solutions to the factor allocation problem. In other words, we solve for the maximands of

\[ \frac{p}{1 + \theta} K_A^{1 - \theta} L_A^{\theta} + (K_i K_A)^{\theta} (L_i L_A)^{\theta} \]

Assuming there are no restrictions on the inputs for each sector, unconstrained maximization of the above expression yields:

\[ \frac{K_A}{L_A} = \frac{p^{1 - \theta} \frac{1}{1 + \theta}}{i_1^{1 - \theta} \theta} = s_1 \]

\[ \frac{K_i K_A}{L_i L_A} = \frac{p^{1 - \theta} \frac{1}{1 + \theta}}{i_1^{1 - \theta} \theta} = s_2 \]

Notice that, under the assumption that b is the capital intensive good

\[ \frac{s_2}{s_1} = \frac{\theta}{\frac{1}{i_1^{1 - \theta} \theta}} > 1 \]

To solve for the factor inputs, then,
\[ K_A = s_2 L_A \]

imply

\[ K_A = s_2 (L_A - L_A) \]

Under these inputs, then, total \( Q \) is given by:

\[ Q = L \frac{ps_1^a s_2}{s_1} \frac{s_2^b s_1}{s_1} + K \frac{s_2^b}{s_2} \frac{ps_1}{s_2} \]

Plugging the above expression in equation (??) we obtain:

\[ Y = \alpha (1_i - \alpha) \cdot p^i \cdot \frac{(1+\xi)}{1+\xi} \cdot L \frac{ps_1^a s_2}{s_1} \frac{s_2^b s_1}{s_1} + K \frac{s_2^b}{s_2} \frac{ps_1}{s_2} \# \]

Finally, \( p_{za} \) comes from the expression of \( s_1 \):

\[ \frac{\mu K}{1} \frac{\frac{\partial q}{\partial a}}{\frac{\partial q}{\partial b}} \frac{1_i - \alpha}{1_i - \alpha} = p_{za} \]

The capital-labor ratio corresponding to \( p_{za} \) is \( \frac{K}{L} \).

For the case where the country exercises comparative advantage in \( b \) (that is, \( p^b > p(1 + \xi) \)) demand for \( a \) and \( b \) must to satisfy:

\[ p_a + b = Q \quad \text{and} \quad \frac{b}{1_i - \alpha} = p (1 + \xi) \]

Solving for these equations and substituting into the final good production function, we get:

\[ Y = \alpha (1_i - \alpha) \cdot p^b \cdot \frac{(1+\xi)}{1+\xi} \cdot K \frac{s_2^b}{s_2} \frac{ps_1}{s_2} \]

Again there are two relevant sub-cases here. There is a \( p_{zb} \) such that if \( p < p_{zb} \) the economy only produces \( b \), and then:

\[ Y = \alpha (1_i - \alpha) \cdot p^b \cdot \frac{(1+\xi)}{1+\xi} \cdot K \frac{s_2^b}{s_2} \frac{ps_1}{s_2} \]

On the other hand, if \( \frac{p^b}{(1+\xi)} > p > p_{zb} \) both goods will be produced and we follow similar steps as in the previous case (solve equation similar to (??) with \( p(1+\xi) \) instead of \( p=(1+\xi) \)) to obtain expressions for \( K_A=L_A = z_1 \) and
\[ (K_i, K_A) = (L_i, L_A) = z_2 \] similar to (13) but, again, with \( p(1 + \zeta) \) instead of \( p = (1 + \zeta) \): These expressions, after some manipulations, will give us:

\[
Y = a^{i_1} (1 + \zeta) i_1 p^j \left( \frac{(1 + \zeta)^{1_1}}{(1 + (1 + \zeta))} \right) L \frac{p_z^{i_1} z_2 i_1}{z_2 i_1 + K} \frac{z_2^{i_1}}{z_1^{i_1} + K} \]

Finally, \( p_{zb} \) comes from the expression of \( z_2 \):

\[
\mu K \frac{L}{z} \frac{L}{z} \frac{L}{z} \frac{1}{z} = p_{zb} \]

The capital-labor ratio corresponding to \( p_{zb} \) is \( \overline{K}_b \).

We have already defined all segments of function \( F(K; L; \zeta; p) \) (eq. [3.1]):

1. For \( K = L < \overline{K}_A \); \( F(K; L; \zeta; p) \) is given by eq. (13) and \( -1 \) corresponds to \( a^{i_1} (1 + \zeta)^i_1 p^j \left( 1 + \zeta \right)^i_1 = (1 + \zeta)^i \):

2. For \( K = L > \overline{K}_B \); it is given by eq. (13) and \( -7 \) corresponds to \( a^{i_1} (1 + \zeta)^i_1 p^j \left( 1 + \zeta \right)^i_1 = (1 + \zeta)^i_1 \):

3. If \( K = L \leq [\overline{K}_A; \overline{K}_B] \) ( \( \overline{K}_A \) being the capital-ratio which corresponds to \( p^n[(1 + \zeta)] \), then \( F(K; L; \zeta; p) \) is given by eq.(13), \( -2 \) corresponding to \( -1[s_2^{i_1} \overline{p}_2^{i_1} \overline{s}_2^{i_1}] = p(s_2 i_1 s_2) \) and \( -3 \) to \(-1[p_s^{i_1} s_2 i_2 \overline{s}_2^{i_1} s_2] = p(s_2 i_1 s_2) \):

4. For the case where \( K = L \leq [\overline{K}_A; \overline{K}_B] \) ( \( \overline{K}_B \) being the capital-ratio which corresponds to \( p^n[(1 + \zeta)] \), the expressions for \( -5 \) and \( -6 \) are symmetric to \( -2 \) and \( -3 \), with \( -7 \) substituting \( -1 \) and \( z_1 \) and \( z_2 \) substituting \( s_1 \) and \( s_2 \) respectively.

5. Finally, when \( K = L \leq [\overline{K}_A; \overline{K}_B] \) the economy is closed and \( F(K; L; \zeta; p) \) corresponds to eq. (13) and \( -4 \) is given by \( a^{i_1} (1 + \zeta)^i_1 \):

For the case where \( \zeta = 0 \); we have only 3 relevant cases, as \( \overline{K}_A \) and \( \overline{K}_B \) collapse to \( k^n \): 1) The economy specializes in good a; 2) The economy specializes in good b; 3) The economy diversify. In the latter case the production function is linear on \( K \) and \( L \) while for the other two it will be Cobb-Douglas with coefficients \( \overline{a}_i \) and \( \overline{a}_b \), respectively. The economy will always trade, except at \( k = k^n \):

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References


