Is There a Price Puzzle in Brazil? An Application of Bias-Corrected Bootstrap

Rubens Penha Cysne

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Is There a Price Puzzle in Brazil? An Application of Bias-Corrected Bootstrap*  

Rubens Penha Cysne†  

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Abstract  
Using data from the United States, Japan, Germany, United Kingdom and France, Sims (1992) found that positive innovations to short-term interest rates led to sharp, persistent increases in the price level. The result was confirmed by other authors and, as a consequence of its non-expectable nature, was given the name "price puzzle" by Eichenbaum (1992). In this paper I investigate the existence of a price puzzle in Brazil using the same type of estimation and benchmark identification scheme employed by Christiano et al. (2000). In a methodological improvement over these studies, I qualify the results with the construction of bias-corrected bootstrap confidence intervals. Even though the data does show the existence of a statistically significant price puzzle in Brazil, it lasts for only one quarter and is quantitatively immaterial.  

1 Introduction  

The term "price puzzle" has been introduced in the Vector-Auto-Regression (VAR) literature by Eichenbaum’s (1992) comment to a paper by Sims (1992).  

*I thank the participants of workshops at the Department of Economics of the University of Chicago and at the Graduate School of Economics of the Getulio Vargas Foundation for their comments. I am also thankful to Lutz Kilian, for making available some of the computer codes used in this work. The usual disclaimer applies. Key Words: Price Puzzle, Bootstrap, Bootstrap-After-Bootstrap, VAR, Vector-Auto-Regression, Bias-Corrected Bootstrap. JEL: J30, I30, D33.  

†Professor at the Getulio Vargas Foundation Graduate School of Economics (EPGE/FGV) and, in 2004, a Visiting Scholar at the Department of Economics of the University of Chicago. E mail: rpcysne@uchicago.edu.
In this paper, Sims used the VAR methodology and found out that positive innovations to short-term interest rates led to sharp, persistent increases in the price level in Japan, France, United States, Germany and United Kingdom. The data were monthly and included, for each country, a short-term interest rate, a monetary aggregate, a consumer-price index, an industrial-production index, an index of the foreign exchange value of domestic currency and a commodity price index.

The positive response of prices to the interest rate was found by Sims (1992) to be particularly strong and persistent in France and Japan. In the other countries the responses were weaker and would eventually (in a 48-month horizon) become negative. Running the VAR without the commodity price index and the exchange rate did not substantially modify the results.

Even though there are some models\footnote{Christiano et al. (2000) mentions, e.g., Beaudry and Devereaux (1995), Fuerst (1992) and Christiano, Eichenbaum and Evans (1997).} capable of delivering periods of inflation after a monetary contraction, the common view points out in the opposite direction, thereby the term "price puzzle".

Sims’s explanation to the puzzle is primarily based on the bias in the estimation of the coefficients, on account of possibly-missing variables. He argues that the monetary authority often has information regarding inflationary pressures not captured in the history of the variables included in the VAR. For instance, by knowing of a supply shock, and that inflationary pressure is about to arrive, the authorities can take the preemptive action of raising interest rates, thereby generating a positive (though spurious) correlation between prices and interest rates.

One theoretical consequence of this "missing-variable" explanation is that the same bias (of the estimated parameters of the model) should affect all other responses, not only the one related to the price index. Under such circumstances, the fact that the response of the other variables to an increase of interest rates behaved accordingly to the common sense would lose part of its appeal.

The "missing variable" also implies a subsequent search for one or more variables to be included in the VAR. This has been made, with relative success, by Sims and Zha (1999) and by Christiano et al. (1997 and 2000). These works show that the inclusion of one single leading variable (usually some specific commodity price index) can do the job: the so called "price puzzle" often disappears.

A first purpose of this paper is to investigate if the same type of phenomenon (prices reacting positively to an increase of the interest rate) happens with quarterly Brazilian data ranging from 1980/1 to 2004/2. Some previous
assessments of this paper using Brazilian data are duly referred to in section 3.1.

A second purpose is to illustrate the use of bias-corrected bootstrap in the construction of confidence intervals for impulse-response functions.

Regarding the technicalities concerning the estimation of the VAR (the type of orthogonalization, the assumption of fixed coefficients over the whole sample range\(^2\), which variables to include, the type of estimation (if in levels of first differences)) I will strictly follow Christiano et al. (2000) and will not consider alternative estimations procedures. Therefore, my answer to the problem is conditional on using Christiano et al.’s (2000) specifications. The only degree of freedom I will explicitly consider regards the number of lags to include in the VAR.

The reason for following Christiano et al.’s (2000) specifications is that these authors were able to generate the puzzle, as well as to solve it (by adding an extra variable), with different sets of data. Using their specifications, therefore, allows for the establishment of a well known benchmark for the purpose of comparisons.

One important improvement of the analysis I develop here, relatively to the ones by Sims (1992), by Eichenbaum (1992), and by Christiano et al. (2000), is the inclusion of confidence bands generated by the method of bias-corrected bootstrap. Sims (1992) does not provide confidence intervals. Eichenbaum uses the parametric Monte Carlo integration procedure of Doan (1990), whereas Christiano et al. (2000) employ the standard bootstrap technique. It happens, though, that these procedures do not perform well in small samples, due to the bias of the estimators of the A.R. processes. Here I use the bias-corrected methodology proposed by Kilian (1998).

2 The Bias-Corrected Confidence Bands

As shown by Tjostheim and Paulsen (1983), Nicholls and Pope (1988) and Pope (1990), among others, the application of the standard bootstrap procedures to auto-regressive models generates replicates which are necessarily biased, on account of the small-sample bias of the estimators of the parameters. In order to deal with this problem, specifically regarding the confidence intervals for impulse-response functions constructed from VAR estimates, Kilian (1998) proposed correcting the bias prior to bootstrapping the estimate.

\(^2\)See Fiorencio and Moreira (1999) for a VAR analysis in which the coefficients are allowed to vary as the economy adapts to a changing environment.
This correction is important. When based on the standard techniques, the bias and skewness of the impulse-response estimators in small samples can lead to very inaccurate intervals. Exemplifying with a previous paper by Bernanke and Gertler (1995), Kilian (1998) has shown that some key stylized facts of the VAR literature can fail to be supported by the data, when bias-corrected bootstrap bands are used\(^3\),\(^4\).

In general, the bias correction can be accomplished by a nested-bootstrap procedure, in which the inner bootstrap is used to correct for the bias of the coefficients, and the outer bootstrap to generate the confidence bands of the impulse-response functions. Kilian (1998) called this procedure bootstrap-after-bootstrap.

Kilian also points out that the employment of a closed-form solution to the bias proposed by Pope (1990) can simplify the procedure, by making the inner bootstrap unnecessary. Pope’s solution is available for the case of demeaned VARs run in levels, with a nonzero intercept, and without trend. The main result reads:

**Theorem 1 (Pope (1990)):** Let \( \hat{A}_n \) be the least-squares estimator of \( A \) in the \( m \)-dimensional AR(1): 

\[
X_t = AX_{t-1} + Z_t, \quad \text{in which } X_t \text{ and } Z_t \text{ are } mx1 \text{ and } A \text{ is } mxm, \text{ based on a sample of size } n. \quad \text{Make:}
\]

\[
C_n(s) = \frac{1}{n-1} \sum_{i=1}^{n-1} U_{i-s}U_i^T
\]

where \( U_t = X_t - \bar{X}_n, \bar{X}_n \) the sample mean. Suppose that, for some \( \varepsilon > 0, E \| C_n(0)^{-1} \|^{1+\varepsilon} \| . \| \) (\( \| \) states for the operator norm) is bounded as \( n \to \infty \) and that the innovations \( Z_t \) are a martingale difference sequence such that all moments of \( Z_t \) up to and including the sixth, conditional on the past, are finite and have values independent of \( t \). Let \( G \) denote the conditional covariance of \( Z_t \), and suppose that \( \| A \| < 1. \) Then, as \( n \to \infty \), the bias \( B_n = E\hat{A}_n - A \) is of the form:

\[
B_n = -\frac{b}{n} + O(n^{-3/2})
\]  \hspace{1cm} (1)

\(^3\)In the Monte Carlo studies carried out by Kilian (1998, p.222), the relative frequency at which the 95% confidence interval covers the true impulse response (in a model like the one used here) can be as low as 50% when standard bootstrap methods are employed. The bias-corrected bootstrap, on the other hand, presents a frequency around 90-95%. In general, the amount by which the bias-corrected bootstrap outperforms his competitors depends on the persistence of the process, on the sample size and on the inclusion or not of a time trend.

\(^4\)The bias-corrected bootstrap detailed here has been primarily designed for stationary VARs. However, Kilian (1998, section VI) demonstrates that this technique may still be satisfactory, in practice, for nonstationary processes estimated in levels.
where $b$ is given by:

$$b = G \left[ (I - A^T)^{-1} + A^T(I - (A^T)^2)^{-1} + \sum \lambda(I - \lambda A^T)^{-1} \right] \Gamma(0)^{-1}$$

(2)

The sum is over eigenvalues of $A$, weighted by their multiplicities, and $\Gamma(j) = EX_tX_t^T$.


Since the type of VAR specification assumed by Pope (1990) is the same I shall use here, I perform the bias correction through the employment of (1) and (2). The key point to note is that the bias for the bootstrap is re-estimated, and new coefficients are generated, in each re-sample of the bootstrap.

3 Empirical Results

3.1 Previous Evidences Using Brazilian Data

In an analysis the main purpose of which was comparing the effectiveness of monetary policy in three different periods of the Brazilian economy ((1975-1985), (1985-1994) and (1994-2000)), Minella (2001, section 3.3, 2003) provides estimates that can add to our interests here. His estimates were based on monthly data and did not include the data on bank reserves. His impulse-response bands were constructed using Doan’s (1990) Monte Carlo procedure (standard in the software RATS). Most of the time, Minella works with inflation, rather than with the price level. For our purposes here, only the regressions ran in levels are of direct interest. In such cases, which only cover the third subsample (Figures 5 and 7 in the original) no "price puzzles" are found. The same conclusion is obtained by Fiorencio and Moreira (1999, Tables 3A and 3B, in this case using constant parameters) concerning both periods before and after the Real Plan.

5 Regarding the regressions run in growth rates, Minella (2001, 2003) found no statistically significant response of inflation to interest rates in the first subsample. With respect to the second sample, Minella resports an "inflation puzzle" which disappears (figure 9 in the original) when centered inflation is used to correct for the fact that inflation at this time was increasing at a fast pace. For the third subsample (1994-2000), there is a second "inflation puzzle", solved by the addition of another variable to the regressions. For clarification, "centered inflation" is calculated by comparing the average of price levels at the end of month $t$ to that at the end of month $t - 1$. 

5
3.2 New Measurements

Let $y, p, r, res$ and $m$ denote, respectively, the log of real GDP, the logarithm of the price index, the short term interest rate, the logarithm of total (bank) reserves and the logarithm of $M_1$. The primitive data on $p, m$ and $r$ are from IBRE-FGV’s\(^6\) data bank. The quarterly GDP index and bank reserves are from IPEA\(^7\).

Since the purpose here is the assessment of the existence of a price puzzle, in all econometric evaluations the policy instrument is the short-term interest rate. As mentioned before, the benchmark specification is the same as in Christiano et al. (2000). Particularly regarding the ordering of the variables in the VAR, I use\(^8\): $y, p, r, res, m$.

Under this ordering of the variables, the information set assumed to be available to the government at time $t$, when choosing the interest rate to target, includes the current and lagged values of $y$ and $p$, as well as lagged values of $r, res$ and $m$. As a consequence of the ordering of the variables in the VAR and of the Cholesky-orthogonalization assumption, neither the price level at time $t$ nor the GDP at time $t$ are supposed to change as a consequence of a contemporaneous interest-rate shock. This can be noticed in Figure 1, in Figure 2, and in Table 1 of this work.

3.3 The Overall Dynamics of the Model

I start by running the VARs with four, eight and twelve lags. Each equation in the system includes the lags of the variables and an intercept. In particular, this time frame is supposed to capture possible seasonalities of the data. The sample period has been mentioned before. Figure 1 plots the most important response, for our concerns, of a one-standard-deviation shock to the interest rate. In Figures 1 and 2 (but not in Table 1), the response of the variables in logarithm have been multiplied by 100. The total horizon is equal to 16 quarters\(^9\).

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\(^6\)Instituto Brasileiro de Economia da Fundação Getulio Vargas.

\(^7\)Instituto de Pesquisa Econômica Aplicada.

\(^8\)Christiano et al. (2000) also includes the logarithm of the non-borrowed reserves (NBR) between $res$ and $m$. I do not include this variable here.

\(^9\)The maximum estimated absolute value of the eigenvalues of the A.R. matrix for 4, 8 and 12 lags are, respectively, 0.9815, 0.9829 and 0.9930. Reestimations of the VAR over the same sample period lead to values of the Akaike Information Criterion, respectively, for the model with 4, 8 and 12 lags, of 1.0969, 1.1392 and −0.7527. This suggests the use of the model with 12 lags. Even though we are dealing with a small sample, I also performed a log-likelihood test under a null of 12 lags and an alternative of 8 lags. Asymptotically, $c = 2 * (L_{12} - L_8)$ has a chi-square distribution with 100 ($5^2 - 4$) degrees of freedom.
As one can notice from Figure 1, the response of prices and reserves to a one-standard-deviation shock of interest rates look very much alike, either for 4, 8 or 12 lags. Regarding GDP, the inclusion of 12 lags leads to a recovery of real GDP before the end of the first year, the same pattern not being observable in the case of 4 and 8 lags. As one would expect, the effect of raising interest rates on GDP is negative in all cases. The response of total reserves to the interest rate is positive till the end of the first year, and negative afterwards.

Particularly regarding the effect on prices, our main concern here, all alternatives lead to a very close pattern. By the eye, the first impression from the upper-right figure above is that there is indeed a very small price puzzle in Brazil, for a period of around three months\textsuperscript{10}, but of a very small value found was 367.2, confirming the Akaike-Criterion option for 12 lags. Sims’ (1980) bias-corrected version of this test calls for multiplying this statistics by \((n_1 - n_2)/n_1\), where \(n_1\) is the number of periods effectively used in the regressions and \(n_2\) is the number of parameters estimated in the equation with the highest number of parameters. This leads to a new statistics \(s = 367.2 \times (86 - 61)/86 = 106.74\). The new p-value of 0.3049 does not point out so definitely in favor of 12 lags.

\textsuperscript{10}In Sims (1992), the positive response of prices to an interest-rate shock can last from around 10 (USA) to 46 months (France).
magnitude. The proper statistical answer to this question, though, requires the construction of adequate confidence bands for the response functions.

3.4 Is There a Price Puzzle in Brazil?

In this subsection I concentrate on the main point in this paper: checking if the response of prices to interest rates happens to be positive for some time. With this purpose, Figure 2 displays the estimated impulse-response functions, as well as the 90% nominal coverage bias-corrected bootstrap bands for a number of lags equal to 4, 8 and 12, and a horizon of 48 months. The dotted lines depict, in the obvious way, the lower and upper confidence intervals. The confidence intervals correspond to the 5th and 95th percentiles of 1000 bias-corrected bootstrap replications of the I.R. function. The solid line depicts the response of prices (p). Note that we are dealing here with a relatively small sample. The overall sample has 98 data points of each variable. In the worst-case scenario of 12 lags, for instance, only 86 points are effectively used in the regressions. Therefore, the OLS estimation of each equation in this case has only $86 - 5 \times 12 - 1 = 25$ degrees of freedom. For this reason, confidence bands based on limiting distributions would not be adequate. This makes a point for the use of alternative methods, the bias-corrected bootstrap in particular.

It is clear from Figure 2 that there is no price puzzle for horizons superior to one year. This allows us to concentrate our attention only in the initial periods. The values of the lower band (lb), upper band (ub) and of prices, till the third quarter, for VARs ran with 4, 8 and 12 lags, are given by the table below:

As the data shows, there does exist a positive response of prices to an increase interest rates. However, it is statistically significant for just one quarter, and has the magnitude no greater than 3.2% in a quarter. In the case with 4 lags, for instance, this corresponds to around 28% (0.03197/0.1158) of one (conditional) standard deviation of (the logarithm of) prices$^{11}$.

$^{11}$The number 0.1158, the one-step-ahead forecast error of prices (a relatively high error for after-Real standards, though reasonable for the 1980/1-2004/2 period considered here), corresponds to the square root of the second number in the diagonal of the variance-covariance matrix of the VAR estimated with 4 lags.
Figure 2: Confidence Bands of the Response of Prices to Interest Rates

\[
\text{lags}=4 = \\
\begin{array}{ccc}
\text{lb} & 0 & 0.05592 \\
\text{ub} & 0 & 0.00354 \\
\end{array} \quad \\
\begin{array}{ccc}
t=1 & t=2 & t=3 \\
0 & 0.03924 & 0.01967 \\
0 & -0.00989 & -0.05995 \\
\end{array}
\]

\[
\text{lags}=8 = \\
\begin{array}{ccc}
\text{lb} & 0 & 0.05521 \\
\text{ub} & 0 & 0.00411 \\
\end{array} \quad \\
\begin{array}{ccc}
t=1 & t=2 & t=3 \\
0 & 0.05440 & 0.05143 \\
0 & 0.00185 & -0.03346 \\
\end{array}
\]

\[
\text{lags}=12 = \\
\begin{array}{ccc}
\text{lb} & 0 & 0.03351 \\
\text{ub} & 0 & 0.00248 \\
\end{array} \quad \\
\begin{array}{ccc}
t=1 & t=2 & t=3 \\
0 & 0.02051 & 0.00053 \\
0 & -0.00536 & -0.03494 \\
\end{array}
\]
4 Shortcomings

The statistical analysis developed here has at least two shortcomings. First, the non-diagonal terms of the covariance matrix are not close to zero, making the results dependent on the ordering of estimation of the VAR. Therefore, the conclusions are contingent to the ordering $y, p, r, res$ and $m$.

Second, during the period covered by the data, the Brazilian economy went through six different stabilization Plans and several institutional changes\textsuperscript{12}. Even though one can argue that some of these stabilization plans could be inferred from the data we use here, it is reasonable to assume that the sigma algebra generated by such stochastic processes is a poor approximation of the information available to economic agents in general.

5 Conclusion

In this paper I have investigated the possibility of the existence of a price puzzle in Brazil, as it happened with United States, Japan, Germany, Britain and France, according to the VARs estimated by Sims (1992), Eichenbaum (1992) and Christiano et al. (2000), among others. I have addressed the problem using the same estimation procedures employed by Christiano et al. (2000). As a methodological improvement over these studies, I have provided bootstrap confidence bands based on bias-corrected estimated coefficients.

The conclusion of the work, contingent on the estimation procedures and on the provisos detailed above, is that even though the data does show that prices increase when interest rates increase, the effect is quantitatively and temporally insignificant. It lasts for only one quarter and has a magnitude no greater than 3.2\%, or around 28\% of the standard deviation of (the logarithm of) prices.

References


\textsuperscript{12}See Cysne (2002).


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