Testing Production Functions Used in Empirical Growth Studies

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Testing Production Functions Used in Empirical Growth Studies*

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Abstract

We estimate and test two alternative functional forms representing the aggregate production function for a panel of countries: the extended neoclassical growth model, and a mincerian formulation of schooling-returns to skills. Estimation is performed using instrumental-variable techniques, and both functional forms are confronted using a Box-Cox test, since human capital inputs enter in levels in the mincerian specification and in logs in the extended neoclassical growth model. Our evidence rejects the extended neoclassical growth model in favor of the mincerian specification, with an estimated capital share of about 42%, a marginal return to education of about 7.5% per year, and an estimated productivity growth of about 1.4% per year. Differences in productivity cannot be disregarded as an explanation of

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why output per worker varies so much across countries: a variance
decomposition exercise shows that productivity alone explains 54% of
the variation in output per worker across countries.

1 Introduction

In this paper we estimate and test two alternative functional forms, which
have been used in the growth literature, representing the aggregate produc-
tion function for a panel of countries. The first model we consider has a long
tradition in the growth literature – the extended neoclassical growth model
– and was proposed by Mankiw, Romer and Weil(1992), among others. The
second is a mincerian formulation of schooling-returns to skills, traditionally
used in the labor-economics literature, e.g., Mincer(1974) and Wills(1986),
but recently incorporated into the growth literature as well, e.g., Bils and
Klenow(1996), and Hall and Jones(1999). In the latter case, schooling enters
exponentially in the production function, while in the former it does not,
paralleling the way physical capital is modelled.

Estimation is performed using instrumental-variable techniques, suitable
for panels, after a careful search for appropriate instruments. After estima-
tion, we ask which of these two functional forms best fits the data, using a
variety of specification tests, including tests of the validity of instruments,
tests of fixed vs. random effects, and of the restriction that productivity is the
same across countries. Finally, both functional forms are confronted using a
Box-Cox test. The data are extracted from Summers and Heston(1991, mark
5.6) and Barro and Lee(1996), and include 95 countries in different stages of
economic development, with observations ranging from 1960 to 1985.

Functional forms and parameter values obtained in either calibration or
estimation exercises are routinely used to decompose the growth of output
per-capita into the growth of its respective inputs. Studies that have tried
to explain the differences in output per worker can be roughly divided into
two groups. The first finds that differences in factors of production alone
(e.g., physical and human capital per worker) can explain most of the ob-
served differences in output per worker; see for example, Mankiw, Romer,
The second group finds that, even controlling for physical and human capital
per worker, there is still a large portion of output-per-worker disparity left
unexplained. Hence, total factor productivity (TFP) disparity can be an im-
important factor in explaining the differences of output per worker across countries; see, for example, Hall and Jones (1999), Prescott (1998), and Klenow and Rodriguez-Clare (1997).

The conclusions in these articles are somewhat influenced by their methodological choices, particularly by the choice of the functional form of the aggregate production function and by the choice of the estimation method and/or by the parameter calibration employed. A simple variance decomposition along the lines of Hall and Jones and Klenow and Rodriguez-Clare shows that if the capital share is calibrated as in Chari, Kehoe and McGrattan (2/3), factors of production alone explain 82% of the variance of output per worker. On the other hand, if the capital share is calibrated as in Hall and Jones (1/3), productivity alone explains 61% of output-per-worker variance, an opposite result. Hence, before looking at specific growth accounting exercises, one must be confident on the validity of the assumptions behind it.

Our evidence rejects the extended neoclassical growth model in favor of the mincerian specification, with an estimated capital share of about 42%, a marginal return to education of about 7.5% per year, and an estimated productivity growth of about 1.4% per year. Productivity estimates vary considerably across countries, and testing whether productivity is the same for all countries strongly rejects this hypothesis. Moreover, differences in productivity cannot be disregarded as an explanation of why output per worker varies so much across countries: a variance decomposition exercise shows that productivity alone explains 54% of the variation in output per worker across countries, a result along the lines of Hall and Jones and of Klenow and Rodriguez-Clare.

Bils and Klenow, Hall and Jones, and Klenow and Rodriguez-Clare make the case for the mincerian form based on microeconomic evidence. The present study confirms the appropriateness of this approach from a macroeconomic perspective. It is also comforting to note that our estimate of the marginal return to education is consistent with those obtained in micro studies. These results increase our confidence that the mincerian specification should be used in future studies and also the confidence with which we regard the growth-accounting results based on it.

This paper has four additional sections. In Section 2 we discuss the functional forms used to run production-function panel regressions. In Section 3, the econometric techniques and the specification tests used are discussed and the estimation results are presented. In Section 4 we use the estimation and tests results to perform a number of variance decomposition exercises.
2 Model Specification

The first production function considered here is the so-called “extended neo-classical growth model,” which uses human capital as an additional explanation for output, jointly with physical capital and raw labor. Start with the following homogenous-of-degree-one production function:

$$ Y_{it} = A_{it} F (K_{it}, H_{it}, L_{it} \exp (g \cdot t)) , $$

where $Y_{it}$, $K_{it}$, $H_{it}$, $L_{it}$, and $A_{it}$ are respectively output, physical capital, human capital, raw labor inputs, and productivity for country $i$ in period $t$, where $i = 1, \cdots, N$, and $t = 1, \cdots, T$. It is assumed that there is exogenous technological progress at rate $g$, which is the same across countries. The production function in per-worker terms can be written as:

$$ \frac{Y_{it}}{L_{it}} = y_{it} = A_{it} F (k_{it}, h_{it}, \exp (g \cdot t)) . $$

Assuming Cobb-Douglas technology (or using a first-order log-linear approximation of the above function) gives:

$$ \ln y_{it} = \ln A_{it} + \alpha \ln k_{it} + \beta \ln h_{it} + \gamma g \cdot t + \epsilon_{it} , $$

where $\epsilon_{it}$ is the inherited measurement error for country $i$ in period $t$. Imposing homogeneity explicitly (i.e., $\gamma = (1 - \alpha - \beta)$), we obtain the following:

$$ \ln y_{it} = \ln A_{i} + \alpha \ln k_{it} + \beta \ln h_{it} + (1 - \alpha - \beta)g \cdot t + \eta_{it} , $$

where in (4) $A_{it}$ is decomposed into a time-invariant component $A_{i}$ and a component that varies across $i$ and $t$ - $\nu_{it}$, such that $\eta_{it} = \nu_{it} + \epsilon_{it}$.

The second specification differs from the above in the way human capital is modelled. It uses a mincerian (e.g., Mincer(1974) and Wills(1986)) formulation of schooling returns to skills to model human capital. There is only one type of labor in the economy, which has skill level $\lambda$, determined by educational attainment. It is assumed that the skill level of a worker with $h$ years of schooling is $\exp (\phi h)$ greater than that of a worker with no education at all, leading to the following homogenous-of-degree-one production function:

$$ Y_{it} = A_{it} F (K_{it}, \lambda_{it}, L_{it} \exp (g \cdot t)) . $$
The parameter $\phi$ in $\lambda_{it} = \exp(\phi h_{it})$ gives the skill return of one extra year of education, i.e., $\phi$ can be interpreted as a measure of the percentage increase in income of an additional year of schooling. In per-worker terms, the equation above reduces to:

$$\frac{Y_{it}}{L_{it}} = y_{it} = A_{it} F(k_{it}, \lambda_{it} \exp(g \cdot t)).$$  \hfill (6)

Again, with Cobb-Douglas technology (or with a first order log-linear approximation of the production function) we obtain:

$$\ln y_{it} = \ln A_{it} + \alpha \ln k_{it} + \beta \ln (\lambda_{it} \exp(g \cdot t)) + \varepsilon_{it},$$  \hfill (7)

where $\varepsilon_{it}$ is the inherited measurement error for country $i$ in period $t$. Finally, using $\lambda_{it} = \exp(\phi h_{it})$ and imposing homogeneity explicitly (i.e., $\beta = 1 - \alpha$), we obtain:

$$\ln y_{it} = \ln A_{i} + \alpha \ln k_{it} + (1 - \alpha) (\phi h_{it} + g \cdot t) + \eta_{it},$$  \hfill (8)

where, again, in (8) $A_{it}$ is decomposed into a time-invariant component $A_{i}$ and a component that varies across $i$ and $t - \nu_{it}$, such that $\eta_{it} = \nu_{it} + \varepsilon_{it}$.

Econometrically, the basic difference between equations (4) and (8) is whether human capital enters the (log of the) production function in levels or in logs. If human capital enters in logs — (4), there is a fixed human-capital elasticity in production for all countries. If it enters in levels — (8), human-capital elasticity in production will change across countries (and across time as well).

3 Econometric Estimation, Testing, and Results

3.1 Estimation and Testing

The central focus of this paper is to test whether or not human-capital measures should enter the production function in levels or in logs. As discussed in the previous section, this is the basic difference between the mincerian and the neoclassical growth model specification for the production function.
When choosing between these two types of specifications in regression models, it is customary to resort to the so called “Box and Cox (1964) transformation.” Consider the generic regression equation, using the Box and Cox transformation for the regressor:

\[ y_t = \left( \frac{x_t^\theta - 1}{\theta} \right) \beta + \varepsilon_t. \]  

(9)

Notice that:

\[ \lim_{\theta \to 0} \left( \frac{x_t^\theta - 1}{\theta} \right) = \ln(x_t), \quad \text{and} \]

\[ \lim_{\theta \to 1} \left( \frac{x_t^\theta - 1}{\theta} \right) = x_t - 1, \]

(10)

(11)

where it is clear that for a logarithmic transformation to be valid we must have \( \theta = 0 \), and for the series \( x_t \) to enter the regression in levels we must have \( \theta = 1 \).

These two hypotheses can be tested by means of a Wald test, using a Box-Cox transformation for the human-capital measure in the production function. This is exactly how we proceed here. However, since we impose constant returns to scale in both production functions (4) and (8), there is not a general regression model which nests them. Therefore, we proceed in testing these two functional forms before imposing constant returns to scale constraints in production. We next discuss other modelling choices made here.

Each one of the sets of equations in (4) and (8) constitutes a structural system of equations for a set of countries \( i = 1, 2, \cdots N \) and a set of time periods \( t = 1, 2, \cdots T \). As is usual with such a panel, panel-data techniques can be employed to estimate the structural parameters \( \ln A_i, \alpha, \beta, g, \text{and } \phi \).

If one disregards the panel-data structure in either (4) or (8), exploiting only the cross-sectional dimension of the data set, one cannot estimate either the technological-progress trend coefficient \( g \), or the country-specific productivity level \( \ln A_i \). Trying to do so would inevitably exhaust all available degrees of freedom. This is the main criticism of Islam (1995) of the estimation procedure in Mankiw, Romer and Weil (1992). Because we do not want to rule out \textit{a priori} that \( \ln A_i \) can be an important factor in explaining the observed disparity in output per worker across nations, we choose to consider the panel-data structure of the structural equations in (4) or (8). However,
rather than imposing, we test the hypothesis that productivity varies across countries, which has not been done previously.

In considering the techniques to be employed in estimating either (4) or (8), the following have to be taken into account: (i) in general, ln $k_{it}$, ln $h_{it}$, and $h_{it}$ are correlated with $\eta_{it}$. This occurs for several reasons. In a short list, ln $k_{it}$, ln $h_{it}$, and $h_{it}$ are measured with error, generating an error-in-variables problem in estimation. Second, there is a portion of $\eta_{it}$ that comes from productivity, hence being correlated with ln $k_{it}$, ln $h_{it}$, and $h_{it}$. Because regressors and errors are correlated, if one hopes to get consistent estimates of structural parameters, a list of instrumental variables has to be obtained. These must be correlated with ln $k_{it}$, ln $h_{it}$, and $h_{it}$, but not with $\eta_{it}$; (ii) a choice must be made regarding how to model ln $A_i$. There are two natural candidates in the panel-data literature: modelling ln $A_i$ as a fixed effect or as a random effect.

Simultaneous-equation coefficients, such as the ones in either (4) or (8) above, can be consistently estimated by instrumental variable methods; see Hsiao (1986, pp. 115-127). Considering the structure of correlation among errors for different countries is a first step in choosing the estimation method. A reasonable assumption about errors is that their variance is not identical across countries, maybe due to the fact that shocks to specific countries are different. Errors for different countries should also have a non-zero contemporaneous correlation, because of common international shocks that simultaneously affect all countries. Exploiting this leads to efficiency gains in estimation, i.e., more precise parameter estimates.

In our case we have 26 years of data and 95 countries. If the estimate of ln $A_i$ is to vary across countries as a fixed effect, for example, is is not feasible to fully exploit these efficiency gains, because the estimate of the variance-covariance matrix of the errors will be singular. Moreover, full information methods (such as full information maximum likelihood or 3SLS) also have problems of their own, in which mis-specification of one given equation in the system carries over to other system equations, leading to inconsistent estimates for the whole system. The larger the system, the higher the chance of mis-specification. For these reasons, we choose to estimate (4) and (8) in a limited information setting. However, heteroskedasticity of errors in different countries will be considered in estimation. This is done by weighting every equation in the system differently, using the reciprocals of the standard deviations of the country-specific errors as weights.

The second step in instrumental-variable estimation is to obtain valid
instruments. We discuss here the case of the (log of) the capital stock, but a similar argument applies to the measures of human capital as well (ln $h_{it}$ or $h_{it}$). Consider first $\ln k_{jt-1}, i \neq j$, i.e., the lagged (log level of the) capital stock of country $j$, as an instrument for $\ln k_{it}$\(^1\). Even if the capital stock is measured with error, creating an error-in-variable problem, as long as the measurement errors are idiosyncratic in nature (country specific), $\ln k_{jt-1}$ will not be correlated with $\eta_{it}$. A possible problem is that it may not be correlated with $\ln k_{it}$ either. For example, there is no guarantee that the lagged (log of the) capital stock of Fiji will be correlated with the current (log of the) capital stock of Romania. However, if we choose a group of countries $j$ satisfying some geographical (or cultural) criterion, we can increase the chance of $\ln k_{jt-1}$ and $\ln k_{it}$ being correlated. In particular, we propose using for each country $i$ the following instrument for $\ln k_{it}$:

$$\frac{1}{N^i} \sum_{j \in \{N^i\}} \ln k_{jt-1},$$

(12)

where $N^i$ represents the number of additional countries in the same continent that country $i$ is in, and $\{N^i\}$ represents the set of countries in that continent that are not country $i$, i.e., (12) represents rest-of-the-continent average lagged (log of the) capital stock.

Lagged average rest-of-continent capital stock looks promising as an instrument. Countries in the same continent usually trade more with each other than with countries outside that continent. They also tend to have similar macroeconomic policies. These factors contribute to deliver a non-zero correlation between $\frac{1}{N^i} \sum_{j \in \{N^i\}} \ln k_{jt-1}$ and $\ln k_{it}$. On the other hand, one can argue that some component of $\frac{1}{N^i} \sum_{j \in \{N^i\}} \ln k_{jt-1}$ may be correlated with $\eta_{it}$. Although this is always possible, there is a way that the orthogonality between errors and instruments can be tested for over-identified models; basic references are Sargan(1958) and Basmann(1960).

Testing orthogonality for a specific over-identified regression equation in a system requires first using instrumental-variable residuals in running auxiliary regressions, and second constructing a $T \times R^2$ statistic with the output of this auxiliary regression; see Davidson and MacKinnon(1993, Section 7.8). Although the procedure is suitable for testing “orthogonality” for

\(^1\)We choose the lagged capital stock for country $j$ to reduce further the chance of correlation between error and instrument.
single-equations in a system, it is a joint test of orthogonality and of correct specification of the model. Hence, rejection of the null can be due to incorrect specification and not to lack of orthogonality. This requires avoiding imposing any restrictions on panel-regression estimates when testing for orthogonality.

Finally, whether we should use fixed or random effects in modelling individual productivity factors $\ln A_i$ can be investigated by means of a Hausman (1978) test. Estimates using fixed and random effects are compared by searching for departures of the latter from the former, which happens when the random component is correlated with regressors. A large difference is a sign of a non-zero correlation, which violates consistency for the parameters of the random effect model.

### 3.2 The Data

The panel data set used ranges from 1960 to 1985, and combines macroeconomic data for 95 countries in the mark 5.6 of the Summers and Heston (SH, from now on) data set with human-capital measures extracted from Barro and Lee (1996). Since the latter are only available at five-year intervals, we first considered using a database with that frequency. However, this presents a problem, since production-function data will use non-contiguous observations and several years of data would be left unused. Alternatively, we decided to interpolate human-capital measures to fit annual frequency. Although this induces measurement error in human capital, the problem is relatively small, since human capital changes with a highly predictable pattern and the estimation technique used allows for regressors that are measured with error.\(^2\)

The time span was restricted from 1960 to 1985. Before the 1960’s, and after 1985, macroeconomic data is missing for several developing countries. Limiting the final year to 1985 also has the advantage of including in our sample the previously socialist countries. Most of them made their transition into capitalism at the end of the 1980’s, making 1985 the last possible year to include in the sample.

The specific series used are the following: $y_{it}$ is the ratio of real GDP (at 1985 international prices) and the number of workers in the labor force, extracted from SH; $k_{it}$ is the physical capital series per worker. The physical

\(^2\)To check the robustness of estimation results, we compared those using data with five-year intervals and those using yearly data. They were very close indeed, as discussed below.
The physical-capital series were constructed using real investment data from SH (at 1985 international prices); $h_{t}$ is Barro and Lee’s (1996) series of average years of completed education of the labor force, interpolated (in levels) to fit annual frequency.

The way the physical-capital series were constructed deserves comment. We started with the investment series and applied the Perpetual Inventory Method to get measures of capital. This method requires an initial capital level and a depreciation rate for physical capital. Since it is not obvious which is a reasonable depreciation rate to apply for all countries, we chose to use five different rates (3%, 6%, 9%, 12%, and 15%), checking whether the capital series were similar across depreciation rates. As for the initial capital stock, we followed Young (1995) and Hall and Jones (1999) and approximated it by $K_{0} = I_{0} / (g_{I} + \delta)$, where $K_{0}$ is the initial capital stock, $I_{0}$ is the initial investment expenditure, $g_{I}$ is the growth rate of investment, and $\delta$ is the depreciation rate of the capital stock. Since $g_{I}$ is the (population) mean growth rate of investment, the closest we can get to it is by computing sample long-term time averages. That is exactly how we proceed here.

### 3.3 Model Estimation Results

Instrumental-variable estimates of the mincerian model (8), using a limited-information setting for a variety of depreciation rates for the capital stock, are presented in Table 1. It also includes several test results – Box-Cox, Hausman, Sargan, etc. Instruments are country-specific, comprising $\frac{1}{N} \sum_{j \in \{N_{i} \}} \ln k_{jt-1}$, $\frac{1}{N} \sum_{j \in \{N_{i} \}} h_{jt-1}$, and $t$, and productivity ($\ln A_{i}$) is modelled as a fixed-effect. The latter is the result of a Hausman (1978) test for choosing how to model $\ln A_{i}$ (random versus fixed effects); see the p-values for it in Table 1.

< include Table 1 here>

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3 Notice that Young (1995) uses the mean growth of investment in the first five years of his sample as a proxy for $g_{I}$. As argued in the text, it is preferable to use long-term time averages because $1/T \sum_{t=1} g_{I}$ converges in probability to $g_{I}$, and for large enough $T$, these sample averages are close to $g_{I}$. Moreover, for the period of time in question, had we adopted Young’s procedure, we would have biased our estimate of $K_{0}$ downward, since the the early 1960’s were a relatively high growth-rate period for all macroeconomic aggregates across our sample of countries.
The reported estimates for $\alpha$, $\phi$, and $g$ do not change much as we vary depreciation rates. For reasonable values of $\delta$ (6%-12% interval), the estimate of $\alpha$ is about 0.41, of $\phi$ about 0.08, and of $g$ about 0.014; all are statistically significant at the usual levels.

These numbers are close to what could be expected 

**a priori:** several calibrated studies use a capital elasticity $\alpha = 1/3$ (see Cooley and Prescott(1995)). Estimates in Gollin(1997) are also close to 0.40 for a variety of countries, and those in Islam(1995) are 0.52 for non-oil countries – smaller for intermediate and OECD countries. As discussed above, $\phi$ can be interpreted as a measure of the percentage increase in income of an additional year of schooling. Mincerian regressions usually find $\phi \approx 10\%$ (Mincer(1974)). Moreover, Psacharopolos(1994), who surveys schooling-return evidence using a large set of countries, finds an average of 6.8% for OECD countries and 10.1% for the world as a whole. It is therefore comforting to be able to reconcile our growth-regression results, using a macroeconomic model, with the microeconomic evidence found elsewhere on returns to schooling. Regarding the estimate of $g$, an average growth rate of productivity of 1.4% a year is in line with the evidence on long-run growth presented by Maddison(1995).

Instrumental-variable estimates of the extended neoclassical growth model (4) are presented in Table 2. The same country-specific instruments were used. Due to a Hausman-test result, productivity ($\ln(A_i)$) is modelled as a fixed effect. For reasonable depreciation rates (6%-12% interval), the estimate of $\alpha$ is about 0.43, close to that in the mincerian case. The estimate of the growth rate of productivity $g$ is about 1.9% a year, maybe closer to the conventional wisdom than the mincerian estimate. Human capital elasticity estimates $\beta$ are relatively small: about 0.025 and, for some values of $\delta$, not significantly different from zero; see Islam(1995) for a similar result.

The results in Table 2 are completely different from those obtained by Mankiw, Romer and Weil(1992), in which their estimated sum of the shares of capital and labor are much higher: ours are at most 0.45, while theirs is about 0.60. In order to reconcile these conflicting results we re-estimated the extended neoclassical model under the restriction that productivity is the same across countries, i.e., that $\ln(A_i) = \ln(A), \forall i$; see the discussion in Islam. Estimates of $\alpha$ and $\beta$ change completely: the first jumps from 0.43
to 0.60, the second jumps from 0.02 to 0.11 – about five times higher, and their sum increases from about 0.45 to about 0.71. Hence, if productivity is restricted such that $\ln(A_i) = \ln(A)$, $\forall i$, estimates closer to Mankiw, Romer and Weil’s are produced. We perform a Wald test for $\ln(A_i) = \ln(A)$, $\forall i$. Results of these tests for the mincerian and the extended neoclassical model are presented respectively in the last lines of Tables 1 and 2. Test results overwhelmingly reject these restrictions, showing that the fixed-effect specification is appropriate, and that productivity indeed varies across countries.

As discussed above, we can choose which of the two models ((8) or (4)) best fits the data by using a Box-Cox test for the human-capital measure. Panel data estimates using the Box-Cox transformation are presented in Table 3. As noted above, in running these regressions, we did not impose constant-return-to-scale restrictions. Estimates of $\theta$ are indeed very close to unity, favoring the mincerian specification for the production function. Wald test results for $\theta = 1$ and $\theta = 0$ did not reject the former although strongly rejected the latter.

A final check for appropriateness of the mincerian model is also done when constant-return-to-scale restrictions are imposed in estimation; see the results in Table 1. Again, we used a Box-Cox transformation for the human-capital measure, testing whether $\theta = 1$ and $\theta = 0$. As before, we did not reject the former although we strongly rejected the latter. Hence, we find evidence that the mincerian specification is preferable to that used by the extended neoclassical model.

This is a relevant result, since recently a group of papers in the growth literature have made the case for the mincerian form of the production function; see Hall and Jones(1999), Klenow, Rodriguez-Clare(1997) and Bils and Klenow (1999), among others. Their argument is entirely based on microeconomic evidence, such as that presented in Psacharopoulos (1994). The evidence here, using the Box-Cox transformation, confirms the appropriateness of their approach from a formal econometric point of view using a macroeconomic model. At the same time, our evidence points toward the rejection of a competing alternative model also used extensively in the growth literature; see Mankiw, Romer and Weil(1992) and Islam(1995), among others.

Because we want to check whether or not suitable instruments were used in estimating the structural models, we performed a series of Sargan(1958) tests (orthogonality between instruments and errors, equation-by-equation). The first step is to design an over-identified model, since the ones in Tables 1 and 2 are just-identified. We used three lags of our instrument list above, and
as well, in getting over-identified equations. Since each equation estimates three coefficients, and we are using seven instruments, we should compare the test statistic with a $\chi^2$. Results for the mincerian model are presented in Table 1. If we take the significance level to be 5%, from a total of 95 country regressions, between 14 and 21 countries rejected the null in this “instrument-validity” test. This is about 16%-22% of the sample of countries, a relatively low number. For the extended neoclassical model the results are similar; see Table 2.

Although in terms of number countries these rejections are relatively small, since the data for each country are weighted by the variance of its error term in computing instrumental-variable estimates, it could happen that including these countries makes a big difference in terms of parameter estimates. To check if this was a potential problem, we ran mincerian regressions excluding from our sample of countries those for which we rejected orthogonality at the 5% level in the Sargan test. For all values of $\delta$ used, the results of this exercise showed overwhelmingly that estimates changed very little when these countries were excluded. To illustrate these differences, we report here the case of $\delta = 9\%$. For the restricted sample of countries, parameter estimates are the following: $\hat{\alpha} = 0.4127$, $\hat{\phi} = 0.0798$, and $\hat{g} = 0.0135$, whereas for the unrestricted sample they are: $\hat{\alpha} = 0.420$, $\hat{\phi} = 0.075$, and $\hat{g} = 0.014$, i.e., virtually the same results.

We now turn to a series of robustness checks of our final estimated model. First, we discarded the first five years of our sample in estimation. As we used the perpetual inventory method to construct the capital stock series, it could be the case that the choice of initial value for it drove our results. Discarding some of the early observations avoids this problem. When we only used data from 1965 to 1985, results did not change much: estimates were $\hat{\alpha} = 0.480$, $\hat{\phi} = 0.045$, and $\hat{g} = 0.012$, all significant at 5%. Second, we replaced the linear time trend with a full set of time dummies to allow

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4Since we want to isolate orthogonality as much as possible, the only functional form restriction we imposed was linearity. Hence, we did not impose the restrictions that coefficients are the same across countries, nor did we impose the homogeneity restrictions that $\gamma = (1 - \alpha - \beta)$ for the extended neoclassical model or that $\beta = (1 - \alpha)$ for the mincerian model.

5When using the 9% depreciation rate of capital stock, the 15 countries for which the instrument list is not valid (at 5%) are the following: Swaziland, Canada, Argentina, Colombia, Guyana, Peru, Venezuela, Israel, Jordan, Finland, the Netherlands, Portugal, Switzerland, Fiji, and Czechoslovakia. If we vary the depreciation rate used in constructing capital, this list changes slightly.
for a time-varying trend. Again, estimates were very close to those obtained with our original specification, $\hat{\alpha} = 0.420$ and $\hat{\phi} = 0.060$, both significant at 5%. Third, we re-estimated the mincerian model with data on five-year intervals, e.g., Islam (1995), to prevent the influence of measurement error in our interpolated human-capital measure, of cyclical fluctuations of the data, and of possible serial correlation in the error terms on final estimates. Again, original results changed very little: $\hat{\alpha} = 0.475$, $\hat{\phi} = 0.057$, and $\hat{g} = 0.012$, all significant at 5%. Hence the estimates in Table 1 seem robust to a variety of departures from the original model.

It is useful at this point to summarize our evidence regarding production-function estimates using panel data. First, based on the evidence of the Box-Cox test, we chose the mincerian growth model over the extended neoclassical model. Estimates of the marginal return to education are consistent with those obtained in micro studies. Second, for both production functions considered, it is clear that productivity is better modelled as a fixed effect vis-a-vis a random effect. Finally, if we test whether or not productivity is the same across countries, i.e., that $\ln (A_i) = \ln (A)$, $\forall i$, regardless of the production function and the depreciation rate considered, the results show unequivocally that it is not.

The next step is to get production-function estimates to investigate the nature of income inequality across nations. To do so, however, we first have to choose a depreciation rate. As the results of Table 1 show, it makes little practical difference in terms of parameter estimates which depreciation rate $\delta$ is used in constructing the physical-capital series; see also Benhabib and Spiegel (1994). Here we decided to use $\delta = 9\%$. The results of the mincerian growth model, with a 9% depreciation rate, will be used as the benchmark in examining the nature of income inequality across nations; $\hat{\alpha} = 0.420$, $\hat{\phi} = 0.075$, and $\hat{g} = 0.014$. To get some idea of how much productivity varies across countries for that specification, we computed a kernel-density estimate of $\ln A_i$, displayed in Figure 1. The features of productivity are: (i) as in Islam, it can vary substantially across countries, with the ratio $\exp (\ln A_{\text{max}}) / \exp (\ln A_{\text{min}}) = 8.66$; (ii) it is slightly skewed; and, (iii) the near-Normal shape of the density in Figure 1.

< include Figure 1 here>
4 On The Nature of Output-per-worker Inequality

4.1 Variance Decomposition of Output per Worker

To understand the relative contribution of inputs and productivity to the variance of output per-worker, two variance-decomposition exercises are performed. Initially, we take 1985 variables and disregard the uncertainty in parameter estimates, using $\alpha = 0.420$, $\phi = 0.075$, and $g = 0.014$. The first exercise is a naive decomposition, because in it we disregard the fact that part of the variation of the capital-labor ratio is due to productivity variation across countries. The second exercise follows Hall and Jones (1999), among others, rewriting the production function per-worker in terms of the capital-output ratio. This is a way of coping with the endogeneity problem. We show that the decomposition performed by Hall and Jones has a natural interpretation in terms of distortions to capital accumulation.

In the “naive” decomposition, given the structural model in 1985 with its error term $\eta_i$ replaced by its unconditional expectation (zero), we have:

\[
\ln y_i = \ln A_i + \alpha \ln k_i + (1 - \alpha)(\phi h_i + g \cdot 1985). \tag{13}
\]

We decompose the variance of (the log of) output per worker in 1985\(^6\) ($\ln y_i$) in terms of (the log of) productivity ($\ln A_i$), (the log of) capital per worker ($\ln k_i$), and (the level of) human-capital per-worker ($h_i$).

This exercise is naive because it treats each factor as exogenous in calculating the variance decomposition. This is particularly troublesome for physical capital, since part of its variation may be induced by productivity variation; see the discussion in Hall and Jones (1999, p. 88). Indeed, for a given investment rate, an exogenous increase in productivity will increase the incentive to accumulate capital in the long run, raising the capital per-worker ratio. Hence, part of the impact of physical capital on output is induced by productivity, and this is not taken into account in performing the first exercise.

To cope with this problem, Hall and Jones proposed performing the decomposition in terms of the capital-output ratio. The production function is

\(^6\)We used 1985 because it is the last year in our sample. The same exercise was actually performed on other years as well, with results being virtually the same, as reported below.
rewritten as:
\[
\frac{Y_i}{L_i} = A_i^{1-\alpha} H_i \left( \frac{K_i}{Y_i} \right)^{\alpha/(1-\alpha)},
\]
where, in this case, \( H_i = L_i \exp (\phi h_i) \). Taking logs of (14):
\[
\ln y_i = \frac{1}{1-\alpha} \ln A_i + \phi (h_i + g \cdot 1985) + \frac{\alpha}{1-\alpha} \ln \left( \frac{K_i}{Y_i} \right).
\] (15)

This formulation allows decomposing the variation of output per-worker into variations of productivity, human capital, and the capital-output ratio. Moreover, the effect of productivity on capital cancels out with that on output. Hence, variations in the capital-output ratio are free from the effect of productivity on the capital measure, answering the endogeneity problem raised above.

Hall and Jones argue that in the balanced-growth path the capital-output ratio is proportional to the investment rate, which suggests a natural interpretation for the decomposition based on (15). It turns out that we can also interpret it in terms of the distortions to capital accumulation present in each country. First, assume that the net return to capital is the same across countries \( (r) \). Implicitly, this relies solely on free capital mobility. We can find, for each country, its (dynamic) distortion to capital accumulation \( (\tau_i) \) by solving the following equation:
\[
\alpha(1-\tau_i)A_i k_i^{\alpha-1} \exp \left[ (1-\alpha)\phi h_i \right] = \delta + r, \quad \text{or,}
\]
\[
(1-\tau_i) = \left( \frac{K_i}{Y_i} \right) \cdot \frac{\delta + r}{\alpha}.
\] (16) (17)

where \( \delta \) is the depreciation rate of physical capital.

Equation (17) implies that:
\[
\ln(1-\tau_i) = \ln \left( \frac{K_i}{Y_i} \right) + \ln \left( \frac{\delta + r}{\alpha} \right).
\] (18)

Therefore, any cross moments involving \( \ln \left( \frac{K_i}{Y_i} \right) \) will be identical to their respective counterparts using \( \ln(1-\tau_i) \), and the results of the variance decomposition for \( \ln \left( \frac{K_i}{Y_i} \right) \) based on (15) will be numerically identical to those based on \( \ln (1-\tau_i) \). Indeed, because of (18), variance decomposition for
\( \ln \left( K_i \right) \) based on (15) can be interpreted as the relative importance of distortions to capital accumulation\(^7\).

As in the case of the decomposition based on (15), performing it using \( \ln(1 - \tau_i) \) solves the endogeneity problem of physical capital. *Ceteris paribus*, the higher \( \tau_i \) is, the smaller is the incentive for capital accumulation, and hence, the smaller is the capital per-worker ratio in the long run. Therefore, part of the variation of \( \ln k_i \) is induced by that of \( \ln(1 - \tau_i) \), and performing the analysis based on \( \ln(1 - \tau_i) \) isolates a primary factor of \( \ln k_i \) variation. Implicitly, the decomposition based on \( \ln(1 - \tau_i) \) implies that there is no (negative) relationship between capital per worker and market returns – as in the standard neoclassical model – because \( \tau_i \) equates returns across economies.

The approach based on \( \ln(1 - \tau_i) \) is interesting in its own right, since one rarely sees in the growth literature accounting exercises in terms of distortions. Computation of \( \tau_i \) for different countries allows: (i) measuring which of those implicitly “tax” capital accumulation, (ii) classifying countries according to distortions, and, (iii) performing counter-factual exercises in long-run growth, such as the ones discussed below.

Since we used panel-data techniques to estimate structural parameters, one can argue that performing the variance decomposition exercise using data on 1985 alone may “throw away” relevant information on other years. One way of taking all possible years into account is to time aggregate the basic equations used in variance decompositions, performing the latter in terms of time averages. Taking equation (15), for example, if we disregard irrelevant constants, a variance decomposition exercise can be based on:

\[
\frac{1}{T} \sum_{t=1}^{T} \ln y_{it} = \frac{1}{1 - \alpha} \ln A_i + \phi \frac{1}{T} \sum_{t=1}^{T} h_{it} + \frac{\alpha}{1 - \alpha} \frac{1}{T} \sum_{t=1}^{T} \ln \left( \frac{K_{it}}{Y_{it}} \right), \tag{19}
\]

with a corresponding counterpart using (13). Notice that these decomposition exercises take all years into account, being immune to cyclical fluctuations and other effects that may change the cross-sectional distribution of relevant variables.

\(^7\)A caveat to this approach is that we could think of distortions as simply how much the implied marginal product of capital in each country deviates from the world interest rate. In a world with limited capital mobility (see Feldstein and Horioka(1990)) this could reflect differences in savings rates and population growth rates due to demographics, preferences, among other factors, and not only taxation or distortions to capital accumulation.
4.2 Variance Decomposition Results

Table 4 presents the results of the variance-decomposition exercises using 1985 data. In the “naive” decomposition, the variance of productivity, physical capital, and human capital account respectively for 21%, 49% and 2% of the variance of output per-worker. The remaining 28% is accounted for by the covariances between these factors. With all caveats in mind, physical capital variation can be an important factor explaining output-per-worker variation. Also, the relative importance of productivity is undeniable.

Quantitative results change considerably once physical capital is treated endogenously as in the decomposition used by Hall and Jones (1999), i.e., equation (15). The second line of Table 4 shows that productivity alone explains 54% of the variance of ln $y_i$. Human capital explains 5%, and the capital-output ratio explains 21%. These numbers are very different from those of the previous exercise, showing that, when the indirect effect of productivity on capital is accounted for, productivity explains about one-half of the variance of ln $y_i$, rather than one-fifth. The last row of Table 4 presents the results of the variance-decomposition exercise, based on equation (15), when we restricted the number of countries to include only the 5 richest and 5 poorest in our sample. Productivity differences are still the main reason for income dispersion across countries.

We perform robustness checks on the results in Table 4 first by running variance decompositions of output per worker for all years we have actual data on human capital (five year intervals, starting in 1965). Results based on 1985 data were virtually unchanged. Most year-to-year changes were of about one percentage point, and, with a few exceptions, not larger than six percentage points. Next, we decomposed the variance of $\frac{1}{T} \sum_{t=1}^{T} \ln y_{it}$ using equation (19). Again, ln $A_i$ accounts for most of the variation of $\frac{1}{T} \sum_{t=1}^{T} \ln y_{it}$ – 56%, followed by $\frac{1}{T} \sum_{t=1}^{T} \ln (K/Y)_{it} – 24\%$ and then $\frac{1}{T} \sum_{t=1}^{T} h_{ti} – 5\%$. The naive decomposition had its results virtually unchanged and so did the decomposition based on the 5 poorest and richest nations. Therefore, we conclude that, once the endogeneity of the capital measure is taken into account, productivity is the most important factor in explaining the variation of output per worker across nations.
Our results are similar to those in Hall and Jones (1999) and Klenow and Rodriguez-Clare (1997). This can be expected _a priori_ for two reasons. First, these authors use a mincerian specification for the production function. As we showed here after extensive econometric testing, this specification should be the one used in growth-accounting exercises. Second, our estimated parameter values are very close to those used by these authors in their calibrated exercises. Again, our econometric results show that their choice of parameter values in calibration is sensible. Comparing our results to theirs shows the following: the variance decomposition of model BK4 in Table 2 of Klenow and Rodriguez-Clare - their preferred model - found productivity explaining 66% of output per worker variation vis-a-vis only 34% for inputs. Given the zero covariance restriction in their exercise, if we impose it in ours (Table 4), we find almost the same values, 67.5% and 32.5%, for productivity and inputs respectively. Hall and Jones, compare the 5 richest to the 5 poorest countries. They find that productivity alone explains 67% of income variation. Again ignoring covariances, our results in Table 4 show that productivity explains 62.3% of the income difference of these two groups.

### 4.3 Productivity, Distortions, and Counter-Factual Exercises on Long-Run Growth

Before our final exercise on “the nature of output-per-worker inequality across nations” is performed, we present basic statistics for a select group of countries on estimated total factor productivity \( \ln A_i \), distortions to capital accumulation \( \tau_i \) and human capital measures \( h_i \), relative to their U.S. counterparts. Results are reported in Table 5.

< include Table 5 here>

Productivity levels of rich countries – particularly those in Europe – are above average and _vice-versa_ – African countries, for example. Only six economies are more productive than the U.S. economy, five of which are oil producers. This happens because our measure of capital does not include the endowment of mineral and/or natural resources; see Hall and Jones(1999). Additionally, there are two groups of countries that have low productivity levels. The first is the group of ex-communist countries. For example, Romania has the second smallest productivity overall, which is more than four
times smaller than that of the U.S. economy. Moreover, the U.S.S.R. and Czechoslovakia are respectively only 43% and 37% as productive as the U.S. Also, the productivity of the U.S.S.R. is the same as Ghana’s. The second group is composed of some Asian countries: Japan, Taiwan, and South Korea all have below-average productivity by world standards.

Rich (and more educated) nations distort capital accumulation less than poor (and under-educated) nations do. For instance, while the average per-capita income of the group of 20 countries with the highest estimated distortion is only 7.1% of the USA, that of the 20 least distortive countries is 59.2% income in the U.S. However, since the correlation between $\ln (1 - \tau_i)$ and $\ln A_i$ is close to zero (actually 0.08), economies that are very good at combining inputs (i.e., are highly productive) do not necessarily have the right incentives to boost capital accumulation. Ex-communist countries and some Asian countries have small distortions and are relatively unproductive; e.g., Japan, Korea, and Taiwan. For the latter group, the case of Japan is very interesting: its productivity is below world average but its distortion is the third lowest amongst all nations (it is negative – a subsidy – as a matter of fact). These findings are consistent with Young’s(1995) result that the good growth performance of some Asian countries in the recent past was mostly due to factor accumulation, not productivity.

Table 6 displays a counter-factual exercise on long-run growth, which might help in understanding the nature of income inequality across nations. The second column displays 1985 output per worker (relative to the U.S.) – $Y_i/Y_{US}$. The third column shows relative income corrected for $\tau_i$, i.e., where country $i$ is given the same $\tau$ as the U.S. economy. The fourth column corrects for human capital and $\tau$, i.e., where country $i$ is given the same $\tau$ and human capital as that of the U.S. economy.

Most of the time, relative output increases when we allow a country to have the U.S. $\tau$ and $h$ measures; see the case of Argentina, Mexico, and particularly Mozambique, where output per worker increases by almost ten times. However, there are exceptions: for Japan and the ex-communist countries, output decreases when we allow them to have the U.S. distortion. Given

\[ < \text{include Table 6 here}> \]

\[ \text{Note: A different way to look at the fourth column of Table 6 is to regard it as the relative output of a country which is identical to the U.S. in everything but productivity. A table with the entire set of countries is available upon request.} \]
that the education level observed in these countries is similar to the U.S. level, the fourth column shows that, if it were not for capital accumulation, output per worker in these countries would be almost half of the actual difference.

There are groups of countries, such as India and Niger, where the increase in relative income brought about by the reduction of $\tau$ and improvement in education is not very large. In this case, most of the difference between them and the U.S. is due to productivity differences. European countries which have output per worker close to that of the U.S., such as the Netherlands, Austria, and France, would not change much either, but for different reasons: their $\tau_i$, $h_i$ and $\ln(A_i)$ are already very close to those of the U.S. economy. However, this pattern is not uniform across Europe: if Spain had the same incentives to capital accumulation and educational level as the U.S., its relative output would jump from 45% to 73% of the latter’s.

It deserves note that even after correcting for factor differences across countries, there still remains a large income disparity left unexplained. On average, output per worker of the 95 nations in our data set is 29% of that of the U.S. After substituting their $\tau_i$ and $h_i$ with the corresponding values of the American economy, the average output per worker increases to only 48% of the U.S. output; the rest corresponds to total factor productivity differences.

## 5 Conclusion

In this paper we estimate and test two alternative functional forms, that have been used in the growth literature, representing the aggregate production function for a panel of countries: the extended neoclassical growth model – proposed by Mankiw Romer and Weil(1992), among others, and a mincerian formulation of schooling-returns to skills – e.g., Mincer(1974) and Wills(1986) – recently incorporated into the growth literature by Bils and Klenow(2000), and Hall and Jones(1999).

Estimation is performed using instrumental-variable techniques, suitable for panels, after a careful search for appropriate instruments. After estimation, we ask which of these two functional forms best fits the data using a variety of econometric specification tests. Finally, both functional forms are confronted using a Box-Cox test, since human capital inputs enter in levels in the mincerian specification and in logs in the extended neoclassical growth model.
Our evidence rejects the extended neoclassical growth model in favor of the mincerian specification, with an estimated capital share of about 42%, a marginal return to education of about 7.5% per year of schooling, and an estimated productivity growth of about 1.4% per year. Previously, several authors have made the case for the mincerian form based on microeconomic evidence; see Hall and Jones, Klenow and Rodríguez-Clare, and Bils and Klenow. The present study confirms the appropriateness of this approach from a macroeconomic perspective, reconciling our estimate of the marginal return to education with those obtained in micro studies. This result increases our confidence that the mincerian specification should be used in future studies and also the confidence with which we regard the growth-accounting results based on it.

In panel-data regressions, productivity was preferably modelled as a fixed effect, varying across countries but not across time. Regression results show that productivity estimates vary considerably across countries. In growth accounting exercises, once the endogeneity of physical capital per worker is taken into account, differences in productivity emerge as a fundamental variable in explaining why output per worker varies so much across countries, a result along the lines of Hall and Jones and of Klenow and Rodriguez-Clare. Thus, the conclusion that inputs alone can explain the variation of output per worker can be called into question. However, our decomposition exercises show that there is not a single cause for poverty, and some productive economies with either poor incentives to capital accumulation or low levels of schooling, or both, can have low levels of per-capita income.

References


6 Figures and Tables

Figure 1: Kernel Density Estimate of (the log of) Productivity
Table 1: Estimates of the Mincerian Growth Model Log-Level Model

Estimated Equation: \( \ln y_{it} = \ln A_i + \alpha \ln k_{it} + (1 - \alpha)(\phi h_{it} + g \cdot t) + \eta_{it} \).

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Table 2: Estimates of the Extended Neoclassical Growth Model - Double-Log Model

Estimated Equation: \( \ln y_{it} = \ln A_i + \alpha \ln k_{it} + \beta \ln h_{it} + (1 - \alpha - \beta) \cdot g \cdot t + \eta_{it} \).

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Table 3: Estimates of the Models in Box-Cox Form

Estimated Equation: \( \ln y_{it} = \ln A_i + \lambda_1 \ln k_{it} + \lambda_2 \left( \frac{k_i^{\frac{1}{\theta}} - 1}{\theta} \right) + \lambda_3 \cdot t + \eta_{it}, \)

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Table 4: Variance Decomposition of Output per Worker (1985) in Terms of Different Factors

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Table 5: Relative Productivity Estimate for Selected Countries (U.S.=1.00)
Factors and Productivity Relative to the U.S

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<td>0.75</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.44</td>
<td>1.35</td>
<td>0.35</td>
</tr>
<tr>
<td>U.S.S.R.</td>
<td>0.43</td>
<td>-0.64</td>
<td>0.83</td>
</tr>
<tr>
<td>Ghana</td>
<td>0.43</td>
<td>2.11</td>
<td>0.31</td>
</tr>
<tr>
<td>India</td>
<td>0.36</td>
<td>1.55</td>
<td>0.32</td>
</tr>
<tr>
<td>Kenya</td>
<td>0.29</td>
<td>1.41</td>
<td>0.29</td>
</tr>
<tr>
<td>Romania</td>
<td>0.23</td>
<td>0.05</td>
<td>0.68</td>
</tr>
<tr>
<td>Malawi</td>
<td>0.20</td>
<td>1.39</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 6: Relative Output of Selected Countries in Counter-Factual Analysis

<table>
<thead>
<tr>
<th>Country/Statistics</th>
<th>( \frac{Y_i}{Y_{US}} ) (Uncorrected)</th>
<th>( \frac{Y_i}{Y_{US}} ) ( (\tau_i = \tau_{US}) )</th>
<th>( \frac{Y_i}{Y_{US}} ) ( (h_i = h_{US}, \text{and}\ \tau_i = \tau_{US}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.32</td>
<td>0.53</td>
<td>0.64</td>
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<tr>
<td>Brazil</td>
<td>0.24</td>
<td>0.34</td>
<td>0.48</td>
</tr>
<tr>
<td>Mozambique</td>
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<td>0.34</td>
<td>0.55</td>
</tr>
<tr>
<td>Niger</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
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<tr>
<td>India</td>
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<td>0.10</td>
<td>0.15</td>
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<td>Japan</td>
<td>0.71</td>
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<td>0.38</td>
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<tr>
<td>U.S.S.R.</td>
<td>0.42</td>
<td>0.21</td>
<td>0.23</td>
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<tr>
<td>Spain</td>
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<tr>
<td>Netherlands</td>
<td>0.71</td>
<td>0.73</td>
<td>0.83</td>
</tr>
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</table>