On Common Agency with Informed Principals
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Tese submetida à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas como requisito para obtenção do Título de Doutor em Economia

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Rio de Janeiro
2008
LAUDO SOBRE TESE DE DOUTORADO

TERMO DE APROVAÇÃO

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ON COMMON AGENCY WITH INFORMED PRINCIPALS

Tese aprovada como requisito parcial para obtenção do grau de Doutor em Economia no curso de Doutorado em Economia desta Escola de Pós-Graduação em Economia (EPGE) da Fundação Getulio Vargas (FGV), pela banca examinadora composta pelos professores a seguir:

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Rio de Janeiro, 19 de novembro de 2008.
I’d like to thank my colleagues at EPGE\FGV and Toulouse School of Economics for the enriching discussions, specially Romero Barreto da Rocha, João Barroso and Klênio Barbosa. I also thank the Toulouse School of Economics for its hospitality and gratefully acknowledge the financial support of Capes, CNPq and EPGE\FGV. But mainly, I’d like to thank all my family for motivation and support, specially my wife, Amélia Paes, who made this period happier and tenderer.
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Abstract

This thesis consists of three chapters that have as unifying subject the framework of common agency with informed principals. The first two chapters analyze the economic effects of privately informed lobbying applied to tariff protection (Chapter 1) and to customs unions agreements (Chapter 2). The third chapter investigates the choice of retailing structures when principals (the producers) are privately informed about their production costs.

Chapter 1 analyzes how lobbying affects economic policy when the interest groups have private information. I assume that the competitiveness of producers are lobbies’ private information in a Grossman and Helpman (1994) lobby game. This allows us to analyze the effects of information transmission within their model. I show that the information transmission generates two informational asymmetry problems in the political game. One refers to the cost of signaling the lobby’s competitiveness to the policy maker and the other to the cost of screening the rival lobby’s competitiveness from the policy maker. As an important consequence information transmission may improve welfare through the reduction of harmful lobbying activity.

Chapter 2 uses the framework of chapter 1 to study a customs union agreement when governments are subject to the pressure of special interest groups that have better information about the competitiveness of the industries they represent. I focus on the agreement’s effect on the structure of political influence. When join a customs union, the structure of political pressure changes and with privately informed lobbies, a new effect emerges: the governments can use the information they learn from the lobby of one country to extract rents from the lobbies of the other country. I call this the “information transmission effect”.

This effect enhances the governments’ bargaining power in a customs union and makes lobbies demand less protection. Thus, I find that information transmission increases the welfare of the agreement and decreases tariffs towards non-members. I also investigate the incentives for the creation of a customs union and find that information transmission makes such agreement more likely to be politically sustainable.

Chapter 3 investigates the choice of retailing structure when the manufacturers are privately informed about their production costs. Two retailing structures are analyzed, one where each manufacturer chooses her own retailer (exclusive dealing) and another where the manufacturers choose the same retailer (common agency).
It is shown that common agency mitigates downstream competition but gives the retailer bargaining power to extract informational rents from the manufacturers, while in exclusive dealing there is no downstream coordination but also there are no incentives problem in the contract between manufacture and retailer. A preliminary characterization of the choice of the retailing structure for the case of substitute goods shows that when the uncertainty about the cost increases relatively to the size of the market, exclusive dealing tends to be the chosen retailing structure. On the other hand, when the market is big relatively to the costs, common agency emerges as the retailing structure.

This thesis has greatly benefited from the contribution of Professors Humberto Moreira and Thierry Verdier. It also benefited from the stimulating environment of the Toulouse School of Economics, where part of this work was developed during the year of 2007.

Resumo

Esta tese consiste de três artigos que tem como elemento unificador o modelo de agência comum com principais informados. Os dois primeiros capítulos investigam os efeitos econômicos da influência de grupos de pressão (lobbies) sobre a escolha da tarifas de importação (Capítulo 1) e sobre acordos de comércio internacionais (Capítulo 2). O capítulo 3 investiga a escolha da estrutura de revenda quando os produtores possuem informação privada sobre os seus custos.

O capítulo 1 analisa como a atividade de lobby afeta a política econômica quando grupos de interesses possuem mais informação que o governo. Modifica-se o modelo de Grossman e Helpman (1994), assumindo que a competitividade dos produtores é informação privada dos lobbies. Isto permite investigar quais os efeitos de transmissão de informação neste modelo. Esta assimetria de informação gera dois efeitos no jogo político, um associado ao problema de sinalização da competitividade do lobby para o governo e outro associado ao custo de um lobby fazer um screening da competitividade do lobby rival junto ao governo. O principal resultado deste modelo é que a transmissão de informação reduz a capacidade de influência dos lobbies, o que aumenta o bem-estar da sociedade.

O capítulo 2 aplica o modelo do Capítulo 1 para entender os efeitos de transmissão de informação que surgem em uniões aduaneiras quando os lobbies possuem mais informação que o governo. O foco é dado nos efeitos políticos que surgem nestes acordos. Quando os países formam uma união aduaneira o equilíbrio de forças político e um novo efeito surge: os governos usam as informações privadas do lobby de um país para extrair renda dos lobbies dos outros países.

Este efeito aumenta o poder de barganha dos governos dentro de uma união aduaneira e reduzam a capacidade de influência dos lobbies. Desta forma, a transmissão de informação aumenta os benefícios de uma união aduaneira e reduz a tarifas de importação para os países fora do acordo. Além disso, é investigado o
papel da transmissão de informação a criação das uniões aduaneiras e o resultado encontrado é que esta aumenta as chances destes acordos serem implementados.

O capítulo 3 investiga a escolha da estrutura de revenda quando os produtores possuem informação privada sobre seus custos de produção. Duas estruturas de revenda são analisadas, uma onde cada produtor escolhe um revendedor exclusivo (exclusive dealing) e outra onde ambos os produtores escolhem o mesmo revendedor (agência comum). Agência comum reduz a competição no mercado final, mas dá ao revendedor a capacidade de extrair lucro dos produtores utilizando a informação de um contra o outro. Enquanto que exclusive dealing aumenta a competição entre produtores, mas não cria problemas informacionais entre produtor e revendedor. Uma caracterização preliminar da escolha da estrutura de revenda para o caso de bens substitutos mostra que quando a incerteza quanto sobre o custo aumenta relativamente ao tamanho do mercado, exclusive dealing tende a ser a estrutura de revenda escolhida, enquanto que quando o tamanho do mercado é grande em relação aos custos, agência comum tende a ser a estrutura escolhida.

Esta tese se beneficiou enormemente da contribuição dos professores Humberto Moreira e Thierry Verdier. Também se beneficiou o estimulante ambiente acadêmico da Toulouse School of Economis, onde parte dela foi desenvolvida durante o ano de 2007.
Part 1

Lobbying with private information
CHAPTER 1

Information transmission and inefficient lobbying

1. Introduction

Lobbying is a central element in the study of the policy-making process in many fields of economic literature such as trade, taxation and regulation. Yet, there is no consensus about the role of lobbies on the political process. A branch of the literature treats lobbies as groups that have privileged access to information that is relevant to the decision making process and make strategic use of this information. In this context money buys access to the politicians. Another branch views lobbies as groups that exercise influence by giving money contributions to swing the decision of an influenceable policy maker in their favor.

Among the papers that treat the influence aspect, Grossman and Helpman (1994) is one of the most important to capture the effect of lobbies groups. In their model, a political game takes place in a small economy and lobbies represent productive sectors that offer money contributions to an influenceable policy maker in order to receive tariff protection. In their model a fraction of individuals of the economy is not represented by lobbies and do not participate in the political game. Therefore, the country’s trade policy favors the sectors that lobby while the welfare cost of the tariff is bared by individuals that do not lobby. Their analysis assumes that there is perfect information in the political game.

Potters and Van Winden (1992), Austen-Smith (1995) and Krishna and Morgan (2001), to name a few, investigated situations where lobbies are better informed than policy makers. In these papers lobby’s preferences are not aligned with the policy maker’s and she\(^1\) can make strategic use of her private information to influence the policy maker’s choice in her favor.

Our work stands in between these two branches. We assume that the competitiveness of productive sectors in Grossman and Helpman (1994) - GH henceforth - is the lobbies’ private information. To be more precise, each lobby knows its own sector’s competitiveness but does not know the other sectors’. The policy maker has no private information and does not observe the sectors’ competitiveness. In this framework, we are able to analyze the effects of information transmission within a GH model of lobbying for influence.

Under this informational structure two asymmetric information problems arise. The first one hinges on the fact that, facing the same tariff, more competitive
lobbies ("high types") substitute more imports than the less competitive ones ("low types"). Import substitution due to tariff generates inefficiency for the economy because home goods are produced with marginal costs above international prices. Since high type sectors substitute more, they cause higher welfare loss than low type sectors for a given tariff. This informational problem arises because the policy maker does not know the lobbies’ true types. Then, high type lobbies may pretend they are low types in order to contribute less for the protection they receive from the policy maker. If both types offer the same contribution, the policy maker cannot learn their types and can only offer an average compensation for protection. On the other hand, low types do not want to be mixed up with high types because separating themselves they can only pay the true cost of her protection (which is lower than the average cost). Separation allows the policy maker to update correctly through the received contributions, although this is costly for low types. We refer to these distortions as the signaling effect.

The second asymmetric information problem comes from the fact that, in our model, goods are substitutes and each lobby does not know the rival’s type. When the lobby representing good 1 producers asks for more protection, the demand of the substitute good 2 shifts upward because the price of good 1 has increased. In turn, the shift in the demand of good 2 gives the policy maker an increase in the tariff revenue in market 2. This increase in revenue is big if the tariff of good 2 is high, and small if the tariff of good 2 is low. In the perfect information context, the lobby of sector 1 can anticipate the tariff in market 2 and deduct the revenue increase from the contribution she gives to the policy maker. However, lobby 1 does not know the protection in market 2 and therefore she cannot deduct the exact amount from the contribution she offers to the policy maker. Although lobby 1 does not know the tariff that will be given to lobby 2, the policy maker learns the lobbies’ types when he receives the contributions (when they are separating). Hence, lobby 1 knows that the policy maker will learn the rival’s type before the implementation of the policies and she is able to make conditional contributions and screen this information from him. Yet screening is costly and generates distortions in the political game. We refer to these distortions as the screening effect.

Therefore, in our model contributions perform three tasks simultaneously: buy influence, signal the lobby’s type to the policy maker and screen the rival’s type from the policy maker. The low type lobby separates from the high type demanding less protection than she would demand under perfect information. On the other hand, screening the information of the rival type from the policy maker makes the low type lobby leave informational rents to the policy maker and also demand less protection. Both informational effects reduce the lobbies’ ability to influence the policy maker. The policy maker is then able to extract informational rents from both lobbies since each lobby cannot observe his contract with the rival. He uses the information of one lobby to bargain with the other. Thus, information transmission is costly for lobbies and diminishes their influence. As a consequence
tariffs decrease, and imports and the welfare of the society increase compared with the perfect information situation.

**Related literature**

Our approach benefits from Martimort and Moreira (2008) who analyze the divisible public good provision problem as a common agency game with privately informed contributors. In their model contributors are privately informed about their preferences and give conditional money transfers to a common agent who produces the public good. Similarly we introduce private information on the lobbies’ preferences and analyze a common agency game with privately informed principals. The screening effect found in our model is then the same of their paper. However, our model is a common value model once the police maker cares also about the social welfare which includes the lobbies’ profits, while their model is only a private value model because the agent is self interested. Maskin and Tirole (1992) showed that informed principal models with common values have informational distortions in the same spirit of signaling games (e.g., Spence, 1973). Thus, the nature of the signaling effect is directly related to the common value aspect of the model.

Lobby games were also modeled as common agency in Le Breton and Salanié (2003), Martimort and Semenov (2008) and Campante and Ferreira (2007). However, the first two papers consider the ideological uncertainty case, i.e., the policy maker’s preference on contributions and welfare is the private information. The first paper allows individuals to form or not a lobby, which we do not consider here (as in GH, lobbies are assumed to exist).

Our results can be better compared with Martimort and Semenov (2008). They found that ideological uncertainty reduces the lobbies’ influence and the outcome of the game is closer to the policy maker’s preferred policy. This result is similar to ours, since we find that policies are closer to the free trade equilibrium, which in the GH model is the policy maker’s “preferred policy”. However, in contrast with them, we do not find any “influence free” equilibria, i.e., at least some types of lobbies may have protection in our equilibrium.

Campante and Ferreira (2007) break down the commitment assumption of the Dixit, Grossman and Helpman (1997). In their model the wealth of the economy can either be used in lobby or production, thus, lobbying crowds out production. Their equilibrium is inefficient and, under some conditions, policies benefit the less productive sectors.

We cannot compare our model with works that view lobbies as information providers (e.g., Austen-Smith, 1993, 1995) since lobbies do not give money contributions to the policy maker. What lobbies can do is to communicate their private information. In our model, on the other hand, lobbies’ contributions give credibility to the information transmission and an equilibrium with information
transmission always exists. Moreover, this literature commonly assumes that private information is the state of the nature, while in our context the state of the economy is the realization of lobbies’ types. Moreover, each lobby only knows a partition of the information which means that none of the lobbies have all the relevant information for the policy maker.

Bennedsen and Feldmann (2006) are closer to our paper. They analyze a common agency game where lobbies search for information about the state of the economy and also make contributions to influence the policy maker’s decision. They find that the ability to offer contributions reduces the lobby’s willingness to search for information and that competition between lobbies favors those who abstain from searching. Their structure differs from ours first because the information gathered does not affect the lobbies’ preferences while in our model information transmission affects preferences directly. But most importantly, our model is closer to the GH model, which allows us to derive sharp results about welfare.

In the next section we present the economy and the political game, we characterize the efficient policies and the equilibrium of the political game under perfect information. In section 3 we present the informed lobby problem. We define and characterize the equilibrium of the political game in section 4. We then compare this equilibrium with the equilibrium of the game under perfect information. Section 5 concludes.

2. The model

The basic model closely follows the GH model with some minor differences. We consider a small, competitive economy that faces fixed international prices \( p_e \). Within this economy a political game takes place. Special interest groups (lobbies) offer money contributions to the government in exchange for tariff protection, which is the only available policy instrument. Lobbies are better informed about the true impact of tariff in this economy.

The economy has a size one population of consumers. These consumers have preferences for three goods \( (x^0, x^1 \text{ and } x^2) \) represented by the following utility function:

\[
u(x^0, x^1, x^2) = x^0 + \sum_n (\alpha - \beta x^n) x^n + \delta x^1 x^2,\]

where the uppercase \( 0 \) means the numeraire and \( n \in \{1, 2\} \) refers to the productive sector \( n \).

The government’s revenue from import tax is given by

\[
TR = \sum_n (p^n - p_e) (x^n - y^n),
\]

where \( p^n - p_e \) is the import tariff of good \( n \) and the international price \( p_e \) is the same for \( x^1 \) and \( x^2 \). The home production of good \( x^n \) is \( y^n \). This revenue is redistributed to the society through lump-sum transfers.
Good $x^0$ is not taxed and its international price is normalized to 1. It is produced only from labor with constant returns of scale with input-output coefficient of 1. We assume that the labor supply is big enough so that wages can be also normalized to 1.

The wages and the government transfers define the consumers’ income which, together with preferences, allow us to find the market demands:

$$x^n = a - bp^n + dp^{-n},$$

where $d$ is a parameter that defines whether goods are substitutes ($d > 0$) or complements ($d < 0$).

**Assumption 1.** *Goods $x^1$ and $x^2$ are substitutes ($d > 0$).*

Assumption 1 is made to simplify the analysis, but most of the results carry on in the case of complementary goods.

Substituting the demands into the utility function we can compute the indirect utility function, which is denoted by $u(p^1, p^2)$ with some abuse of notation that shall not make confusion.

The goods are produced with sector specific inputs. Hence, the owners of these factors receive all the profit from the production. Moreover, we assume that the owners of productive factors correspond to a negligible fraction of the population, thus the factor ownership is highly concentrated. We refer to the owners of the specific factor as producers.

We consider two different possibilities for the production technology of goods $x^1$ and $x^2$ since they will imply different effects on information transmission. The technologies differ on the elasticity of home supply and they are indexed by a competitiveness (efficiency) parameter of the sector, $\theta$, such that the profit function is given by $\theta \pi(p)$.

**Case 1.** The sectors have zero marginal cost and capacity constraint $\theta$:

$$\frac{\partial c}{\partial y} (\theta, y) = \begin{cases} 0 & \text{if } y \leq \theta \\
\infty & \text{if } y > \theta. \end{cases}$$

Each sector produces and sells $\theta$ for the home price $p$, which implies that $\pi(p) = \theta$. The home supply is inelastic, i.e., the inverse of the supply curve is the vertical line $p = \theta$.

**Case 2.** The marginal cost function is linear with coefficient $\theta^{-1}$:

$$\frac{\partial c}{\partial y} (\theta, y) = \frac{y}{\theta}.$$

The firm’s profit maximization problem implies that $\pi(p) = \frac{\theta^2}{2}$. The home supply curve increases linearly with the home price. In particular, the home supply is more sensible to tariff increases the greater is $\theta$.

The supply function of good $n$ is denoted by $y^n(\theta^n, p^n)$. Notice that, by the envelope theorem, $y^n(\theta^n, p^n) = \theta^n \pi'(p^n)$. 
Assumption 2. \( a \) is large enough so that \( x^n (p^n, p_{-n}) > y^n (\theta^n, p^n) \).

Assumption 2 means that we are only going to consider the case of sectors that lobby to protect their market against foreign competition.

The welfare is the sum of the government’s revenues, the consumers’ and producers’ surpluses in all markets, that is,

\[
W (\theta^1, p^1, \theta^2, p^2) = u (p^1, p^2) + \sum_n (p^n - p^e) \left( x^n (p^n, p_{-n}) - \theta^n \pi' (p^n) \right) + \sum_n \theta^n \pi (p^n).
\]

Figure 1 presents the welfare effect of a tariff in market 1.

**Figure 1 - The impact of the tariff**

In market 1 the home price \( p^1 \) is above the international price \( p^e \) given the tariff. The downward sloped line is the home market demand and the upward sloped line is the home supply of good 1. The triangle A below the demand curve and above the home price is the consumers’ surplus; B and F are the producers’ surpluses; D is the tariff revenue; C and E are the deadweight loss of the tariff. The rectangle G in market 2 is an extra revenue due to substitutability and the increase of protection in market 1.

Throughout we will use the following expression of the derivative of the welfare function:

\[
\frac{\partial W}{\partial p^n} (\theta^1, p^1, \theta^2, p^2) = - (b + \theta^n \pi'' (p^n)) (p^n - p^e) + d (p^{-n} - p^e).
\] (2.1)

Notice that the area of triangle C in Figure 1 is \( b (p^1 - p^e) \) in (2.1) and represents the decrease in home consumption. The area of triangle D in Figure 1 is \( \theta^1 \pi'' (p^1) (p^1 - p^e) \) in (2.1) and represents the welfare loss due to import substitution. The area of rectangle G is the last term on the right side of (2.1).

The government’s revenue, the market demands, the home supplies and the international prices define the economy in our model.

**Asymmetric information.** The competitiveness parameter \( \theta \) can only take two values: \( \theta_h \) or \( \theta_l \), where \( \theta_h > \theta_l \). Its realization is private information of the
lobby only. \( \theta \)'s distribution is common knowledge, i.i.d. and \( z \) is the probability of \( \theta = \theta_h \). Therefore, each lobby knows her type but does not know the rival’s type, while the policy maker does not know their types.

In each case we make assumptions about the parameters to assure interior solutions of the lobbies’ problems.\(^2\)

**Assumption 3.** (i) In case 1

\[
(1 - z)b - d > 0 \\
2\theta_i \geq \theta_h.
\]

(ii) In case 2

\[
(1 - z)(\lambda (b + \theta_i) - \theta_i) - \lambda d > 0.
\]

**Political game**

There are three actors: two lobbies and one policy maker. Lobbies offer contributions, \( C \in \mathbb{R}_+ \), to the policy maker. Thus, they are the principals of the common agency game. We assume that consumers cannot lobby.

Each lobby represents the producers of one sector. They care about the profit of the sector they represent and dislike giving money contributions to the policy maker. Their utility function is

\[
V(\theta, p, C) = \theta \pi(p) - C.
\]

The policy maker is the common agent who chooses the import tariffs\(^3\) \( p \in \mathbb{R}_+ \) of the economy. He cares about the social welfare \( W \) but also likes money contributions. Therefore, he is willing to trade economic welfare for money. His utility function is represented by

\[
U(\theta^1, p^1, C^1, \theta^2, p^2, C^2) = \sum_n C^n + \lambda W(\theta^1, p^1, \theta^2, p^2),
\]

where \( \lambda \) is the relative preference between money and welfare.

**Strategy space**

We assume, for sake of simplicity, that lobbies can only demand protection for their own good. Therefore, the contribution schedule of lobby \( n \), \( C^n(\theta^n, p^n) \), specifies the level of contribution \( C^n \) for each policy \( p^n \) and type \( \theta^n \). Moreover, we assume that contributions must be non-negative.

Once the contribution is accepted, the policies are implemented and payments are made accordingly (we are assuming commitment for the political game).

\(^2\)These assumptions are not necessary for interior solutions, however they greatly simplify our analysis since they naturally rule out negative prices. Essentially, they ensure interior solution for the virtual utility maximization problem.

\(^3\)Defining the import tariffs, the policy maker is in fact choosing the home prices \( p \), thus we assume he chooses the home prices directly.
Since the political game is symmetric, we drop the uppercase index, whenever it makes no confusion.

The preference of the lobbies and the policy maker, the information structure and the strategy space define the “political game”.

**Timing**

(0) nature draws the lobbies’ types and each lobby learns her type;
(1) each lobby offers non-cooperatively contribution schedules to the policy maker;
(2) policy maker either accepts or rejects the contracts;
(3) policies are chosen and, when contributions are accepted, payments are made accordingly.

We have two different concepts to evaluate the impact of the information transmission in this model. They differ on the set of individuals whose welfare is taken into account. The first one considers the welfare of the economy, that is, the surplus of consumers and producers plus the tariff revenue. The second concept considers only the welfare of the players of the political game, that is, the payoff of lobbies and the policy maker.

We begin presenting the first concept.

**Free trade**

When the policy maker rejects the contributions, he chooses the import tariffs that maximize the society’s welfare:

**Definition 1.** The free trade prices \( \{\hat{p}_{ik}, \hat{p}_{ki}\} \) are such that

\[
\{\hat{p}_{ik}, \hat{p}_{ki}\} \in \arg \max_{p_{ik}, p_{ki}} W(\theta_i, p_{ik}, \theta_k, p_{ki}),
\]

where the first lowercase index refers to the lobby’s own type and the second index refers to the rival’s type, where \( i, k \in \{h, l\} \).

The first-order conditions of this problem are given by

\[
\frac{\partial W}{\partial \theta_i}(\theta_i, \hat{p}_{ik}, \theta_k, \hat{p}_{ki}) = 0 \tag{2.2}
\]

and symmetric first-order conditions for \( p_{ki} \).

Following GH, the free trade equilibrium is the welfare maximum for the society \( (\hat{p}_{ik}, \hat{p}_{ki} = \tilde{p}^e) \). Therefore, any deviation from these tariffs reduces the welfare and, in particular, those coming from lobby contributions.

This is the threshold concept to evaluate how information transmission in the political game affects the welfare of the society.

---

4In this framework, the second-order condition implies that free trade is the welfare maximum whenever \( b > d \), which trivially holds under assumption 3.
Truthful equilibrium

The second concept consists in the equilibrium of the political game under perfect information. We know that the solution of this game is the same as the solution of a centralized problem that maximizes the surplus of the players. Hence, we have:

**Definition 2.** The prices \( \{\tilde{p}_{ik}, \tilde{p}_{ki}\} \) on the truthful equilibrium are such that

\[
\{\tilde{p}_{ik}, \tilde{p}_{ki}\} \in \arg \max_{p_{ik}, p_{ki}} \theta_i \pi (p_{ik}) + \theta_k \pi (p_{ki}) + \lambda W \left( \theta_i, p_{ik}, \theta_k, p_{ki} \right).
\]

We refer to these prices as truthful equilibrium because, from Bernheim and Whinston (1986b), they maximize the surplus of the players and are coalition proof, which explains why they are focal in common agency games.

The truthful equilibrium will be our threshold to evaluate distortions due to information transmission in the political game. The first-order conditions resulting from the truthful contribution schedules are

\[
\theta_i \pi' (\tilde{p}_{ik}) - \lambda \frac{\partial W}{\partial p} (\theta_i, \tilde{p}_{ik}, \theta_k, \tilde{p}_{ki}) = 0.
\]

They balance the marginal benefit of the lobbies and the marginal welfare cost of the society. Compared to free trade, condition (2.3) gives an extra weight to lobbies and, therefore, policies increase for lobbies and the welfare cost is bared by the rest of the society.

In case 1 policies implemented by the truthful equilibrium are given by

\[
p_{ik} = \frac{\theta_i b + \theta_k d}{\lambda (b^2 - d^2)} + p^e
\]

and in case 2 policies implemented by the truthful equilibrium are given by

\[
p_{ik} = \frac{\left[\theta_i (\lambda b + \theta_k) - \theta_k \right]}{\lambda (b + \theta_k)} \left( \frac{\lambda d \theta_k + p^e}{\lambda \theta_k} \right) + p^e.
\]

3. The political game

In this section we present the lobby’s problem which we refer as the informed lobby problem.

This game is complex and we need some conditions to be able to characterize the equilibria. We will focus on symmetric Perfect Bayesian Equilibria (equilibria, in short) of the political game. We focus on equilibria with separating contribution schedules, that is, different types of lobby offer different contribution schedules.

---

5In this game, a truthful contribution schedule is such that

\[
\frac{\partial C}{\partial p} (\theta, p) = \theta \pi' (p).
\]
More strongly, we focus on policies that are increasing in the lobby’s own type, that is, \( p_{ik} > p_{ik} \).

To find the optimal contribution schedules, we proceed as follows: we take as given the rival’s offer \( C^{-n}(\theta^{-n}, p^{-n}) \) and assume that it is increasing in the rival’s type. Then, we describe the lobby’s contribution choice as an informed principal-agent problem.

We will present the informed lobby problem, that is, the lobby’s utility maximization problem subject to the information transmission constraints. The rival’s variables are presented in bold and, to simplify the notation, we denote \( C(\theta, p_{ik}) \) as \( C_{ik} \).

As we focus on separating equilibrium and the lobby is privately informed, we must impose incentive compatibility constraints on the lobby. If not, some type of lobby may wish to pretend she is a different type. When binding, these constraints will generate distortions in the spirit of signaling games. The distortions coming from these constraints lead to the signaling effect.

Once the rival’s offer is separating, the policy maker will learn the rival’s type when he receives her offer before implementing the policy \( p^n \). Therefore, the policy maker will learn the rival’s type and, thus, the lobby has to screen this information from him. Screening requires incentive compatibility constraints for the policy maker in the informed lobby problem. The distortions coming from these constraints constitute the screening effect.

The policy maker’s incentive compatibility constraint states that type-\(-i\) lobby does not want to offer the contribution schedule of type-\(i\) lobby. This constraint ensures that the policy maker can update correctly the lobby’s type from the contribution schedule. It is given by

\[
E[\theta_{-i}\pi(p_{-i}) - C_{-ik}] \geq E[\theta_{-i}\pi(p_i) - C_{ik}], \tag{IC_{-i}}
\]

where the \(-i \neq i\) is the opposite type.

The policy maker’s individual rationality constraint is given by

\[
E[\theta_i\pi(p_i) - C_{ik}] \geq \theta_i\pi(p^e), \tag{IR_i}
\]

The policy maker’s incentive compatibility constraints ensure that the policy maker chooses the level of protection according to the true type of the rival. In other words, they ensure that the policy maker chooses the contribution associated with his true marginal cost of the tariff:

\[
C_{ik} + C_{ki} + \lambda W(\theta_i, p_{ik}, \theta_k, p_{ki}) \geq C_{i(-k)} + C_{ki} + \lambda W(\theta_i, p_{i(-k)}, \theta_k, p_{ki}), \tag{ICP_{ik}}
\]

where \(-k \neq k\) is the rival’s false type.

Notice that constraint \(ICP_{ik}\) leads to two different constraints in the type-\(i\) informed lobby problem below (one for each possible \(k\)).

The policy maker’s individual rationality constraint is given by

\[
C_{ik} + C_{ki} + \lambda W(\theta_i, p_{ik}, \theta_k, p_{ki}) \geq \lambda W(\theta_i, p^e, \theta_k, p^e). \tag{IRP_{ik}}
\]
Next we present the informed lobby problem.

**The type-\(i\) informed lobby problem**

\[
\max_{p_{ik}, C_{ik}} E \left[ \theta_i \pi (p_i) - C_{ik} \right]
\]

subject to \( (IC_{-i}) \), \( (IRP_{ik}) \), \( (IR_i) \), \( (ICP_{ik}) \) and \( C_{ik} \geq 0 \) for all \(-i\) and \(k\).

We have to identify which of these constraints will bind at the optimal contribution schedule, this requires:

**CONDITION 1.** The rival’s offer is monotone schedule in the lobby’s type.

When goods are substitutes, the lobby’s marginal cost of protection decreases with the protection of the other market, which means that the lobbies’ policies are strategic complements. In other words, a lobby prefers to face a high type opponent because the marginal welfare cost of protection is smaller.

**LEMMA 1.** Under Condition 1, the lobby’s offer is increasing, i.e.,

\[
p_{ih} \geq p_{il}.
\]

Lemma 1 suggests the direction of the policy maker’s incentive compatibility constraints that may bind. The policy maker will be prompt to lie and choose prices as if the rival is the low type (high welfare cost) when he is truly the high type in the absence of proper incentives. Hence, constraints \( (ICP_{ih}) \) and \( (IRP_{il}) \) should bind in the type-\(i\) informed lobby problem. However, we cannot say whether the constraint \( (IC_{-i}) \) will bind. Moreover we assume that \( (IR_i) \) and check ex-post if this is true.

Different combinations of binding constraints for the policy maker can emerge in equilibrium depending on the surplus distribution between lobbies. This can generate countervailing incentives in the screening problem. We avoid this possibility imposing the following condition on the conjecture about the rival’s offer:

**CONDITION 2.** The rival’s offer is such that the policy maker’s utility is increasing in her type, i.e.,

\[
U (\theta_i, p_i, C, \theta_h, p_{hi}, C_{hi}) \geq U (\theta_i, p_i, C, \theta_l, p_{li}, C_{li}).
\]

This condition implies that the difference between the utilities of high and low type rivals is not greater than the increase in surplus of the game in the two states. We provide a discussion of equilibria with countervailing incentives in the Appendix.

Conditions 1 and 2 restrict the set of equilibria of the game. Particularly, Conditions 1 and 2 assure that constraints \( (ICP_{ih}) \) and \( (IRP_{il}) \) are binding and thus, we can eliminate the contributions from (3.1) and optimize only on the policies \( p_{ik} \).
Lemma 2. The first-order conditions of the informed lobby problem are given by
\[ \theta_i \pi'(p_{ik}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{ik}, \theta_k, p_{ki}) + \frac{\mu_{-i}}{1 - \mu_{-i}} \Delta \theta_i \pi'(p_{ik}) + \lambda I(k) \frac{z}{1 - z} d(p_l - p_h) = 0, \]  
where \( \mu_{-i} \) is the Lagrangian multiplier of \( (IC_i) \), \( \Delta \theta_i = \theta_i - \theta_{-i} \), and \( I(k) \) is an indicator function where \( I(l) = 1 \) and \( I(h) = 0 \).

The first terms of (3.4) are the lobby’s marginal benefit and social cost of protection, which are the driving forces of the truthful equilibrium characterization. The second term is related to the cost of separation; \( \mu_{-i} \) is the shadow price of a marginal increase in the difference between type-(\( -i \)) lobby telling the truth and lying. It captures the distortion necessary to ensure separation. Notice that if \( i = l \), the sign of \( \Delta \theta_i \) is negative, which means that if the high type wishes to pretend she is the low type, then the low type has to demand less protection to separate herself.

The last term is due to the informational rent the lobby has to leave to the policy maker in order to induce him to tell the truth. In order to save on the informational rent she demands less when facing a low type rival, but that decreases her utility. This is the usual trade-off between allocative efficiency and rent extraction.

Manipulating condition (3.4) we can retrieve the best-response correspondences of the informed lobby problems.

4. Information transmission and tariff protection

In this section we define and characterize the equilibrium of the political game and discuss the effects of information transmission on the pattern of protection given to the lobbies. We divide the section in two parts: one for each case (1 and 2), since the informational effects are different for these different technologies (in particular, the signaling effect only appears in case 2).

We begin by defining the equilibrium concept.

Definition 3. A symmetric PBE of the political game are pairs of contribution schedules (one for each lobby) that simultaneously solve (3.1), for every type \( i \).

Therefore, the equilibrium is a fixed point of the best responses of the informed lobby problem for all possible realization of types.

4.1. Capacity constraints (case 1). When lobbies have capacity constraint \( \theta \), the home country supply curve is perfectly inelastic. Figure 1 shows that if the supply curve is vertical, the triangle C does not exist. This means that different types of sectors generate the same welfare cost of protection for a given tariff. Hence, the welfare cost comes solely from the loss in the consumer surplus (triangle E in Figure 1) which the policy maker already knows. Then, we have:
4. INFORMATION TRANSMISSION AND TARIFF PROTECTION

**Lemma 3.** The constraint \((IC_i)\) is never binding.

Lemma 3 states that, with capacity constraints, there is no signaling effect in the political game. The screening effect exists because a lobby does not know the type of her rival and goods \(x^1\) and \(x^2\) are substitutes. When goods are substitutes, the tariff in one market affects the demand in the other market. Thus, if a lobby does not know the type of her rival, she does not know the true welfare cost of protection. On the other hand, the policy maker receives contributions that reveal the lobbies’ types before the implementation of the policy. Thus the private information of the rival lobby becomes “private information” of the policy maker. The best the lobby can do is to screen the rival’s information through the policy maker.

Figure 2 shows the effect of a tariff increase in its own market and on the other market. For simplicity we set \(p^e = 0\).

![Figure 2 - The screening effect](image)

Figure 2 shows a price increase from \(p^1\) to \(p^1 + \Delta\) in market 1 and the welfare losses in market 1 for the two prices (the darker triangle is the welfare loss of \(p^1\) and the big grey triangle is the welfare loss of \(p^1 + \Delta\)). A price increase in market 1 also shifts positively the demand in market 2, which gives the policy maker an additional tariff revenue given by the grey rectangles. The bigger (and darker) rectangle corresponds to the additional revenue when protection is high in market 2 and the smaller rectangle corresponds to the additional revenue when protection is small.

In order to have protection, a lobby has to compensate the policy maker for the welfare loss caused by the tariff increase. Therefore, the lobby could deduct the revenue increase in the other market from the contributions she gives to the policy maker. The problem is that the lobby does not know the price in the rival market because she does not know the rival lobby’s type. Since the lobby will
hold the rival’s information, she can only screen this information from the policy maker.

This informational problem exists because the lobby cannot observe the contribution of the rival lobby given that offers are simultaneous. As the policy maker learns information that lobbies do not have, he bargains with the lobbies. Thus, the lobby’s inability to observe the contributions of the rival gives the policy maker the power to extract informational rents.

In case 1, the first-order conditions of the type-\(i\) informed lobby are given by

\[
\theta_i - \lambda \left[ b(p_{ih} - p^F) - d(p_{hi} - p^F) \right] + \frac{z}{1 - z} \lambda d(p_{hi} - p_{hi}) = 0 \quad (4.1)
\]

\[
\theta_i - \lambda \left[ b(p_{ih} - p^F) - d(p_{hi} - p^F) \right] = 0. \quad (4.2)
\]

Notice that the lobby distorts her demand for protection whenever she faces a low type opponent in order to save informational rent she has to give to the policy maker in order to make him choose his true welfare cost of protection. This explains the following:

**Theorem 1.** There exists a unique symmetric separating pure strategy PBE of the political game with informed lobbies. Moreover, the equilibrium prices are such that:

\[
p_{hh}^s = \tilde{p}_{hh}
\]

\[
p_{hl}^s = \tilde{p}_{hl} - \frac{z b^2 d (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}
\]

\[
p_{lh}^s = \tilde{p}_{lh} - \frac{z b^2 d (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}
\]

\[
p_{ll}^s = \tilde{p}_{ll} - \frac{z b^2 d (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}
\]

where the uppercase \(s\) refers to the screening equilibrium policies. The equilibrium contributions are computed from the binding constraints (ICP\(_{ih}\)) and (IRP\(_{il}\)).

Notice that prices decrease, except when both lobbies have high types. The screening effect makes lobbies demand less protection than they do under perfect information.

One important question is how the screening equilibrium compares with the truthful equilibrium. First, the screening effect gives power to the policy maker so that lobbies have to pay informational rents. This informational rent makes lobbies distort their demands downward for protection compared to the truthful equilibrium. Since protection is above the free-trade level in the truthful equilibrium, decreasing protection is welfare enhancing. Thus, information transmission increases welfare compared to the perfect information situation.
4. INFORMATION TRANSMISSION AND TARIFF PROTECTION

Corollary 1. The screening equilibrium of the political game with informed lobbies Pareto dominates the truthful equilibrium of the political game under perfect information.

4.2. Linear marginal cost (case 2). The profit function is \( \pi(p) = \frac{p^2}{2} \) and the home supply is given by \( \theta p \), which is more elastic the higher is \( \theta \). Different elasticities of supply imply in different welfare costs of protection as shown in Figure 3.

![Figure 3 - The signaling effect](image)

Figure 3 shows that more competitive sectors \( (\theta_h) \) generate a higher welfare cost than less competitive sectors. The policy maker, however, does not know the true value of \( \theta \). Therefore, high type lobbies may wish to pretend they are low types in order to give small contributions for the tariff increase. If both types of lobby offer the same contribution, the policy maker cannot update the lobbies’ types and has to ask for a compensation for protection that is equal to the average welfare cost. This leads to a signaling problem: the low type lobby has to separate herself to allow the policy maker to learn her type. Then, he has to pay for the true cost of her protection, which is smaller than the average cost.

The more competitive are the sectors, the higher is welfare cost for the same tariff protection because sectors substitute more the imports for a given tariff. Import substitution is harmful because home consumers buy a good produced at a higher marginal cost than the international price. Since the high type causes higher welfare costs, we have the following:

**Lemma 4.** The constraint \( (IC_l) \) is never binding.

Lemma 3 implies that only low type lobbies may have to bear the cost of separation in equilibrium. The first-order conditions of the low type informed
lobby problem are then given by

\[ \theta_l p_{lh} - \lambda \frac{\partial W}{\partial p_{lh}} (\theta_l, p_{lh}, \theta_h, p_{hl}) - \frac{\mu}{1 - \mu} \frac{(\theta_h - \theta_l)}{1 - \mu} p_{lh} = 0 \quad (4.3) \]

\[ \theta_l p_{ll} - \lambda \frac{\partial W}{\partial p_{ll}} (\theta_l, p_{lh}, \theta_l, p_{hl}) - \frac{\mu}{1 - \mu} \frac{(\theta_h - \theta_l)}{1 - \mu} p_{ll} + \lambda d \frac{z}{1 - z} (p_{hl} - p_{hl}) = 0 \quad (4.4) \]

and the first-order conditions for the high type are similar but with no signaling term. Notice also that \( \mu = \mu_h \) since only constraint \((IC_h)\) can be binding.

The low type lobby may be forced to separate herself from the high type and she does so by offering a contribution that demands less protection from the policy maker. We then have:

**Theorem 2.** There exists a symmetric separating pure strategy PBE of the political game with informed lobbies. Moreover, if constraint \((IC_h)\) is binding, the equilibrium policies are such that:

\[ p_{hh}^* = \tilde{p}_{hh} \]

\[ p_{hl}^* < \tilde{p}_{hl} \]

\[ p_{lh}^* < \tilde{p}_{lh} \]

\[ p_{ll}^* < \tilde{p}_{ll} \]

where the uppercase * indicates the equilibrium policy.

Theorem 2 shows that the signaling effect reinforces the screening effect by reducing the protection for low type lobbies; \( p_{hl}^* \) decreases due to strategic complementarity and \( p_{hh}^* \) remains the same. The signaling effect on the equilibrium policies can be better understood for the case where \( d = 0 \) (no screening effect case):

\[ p_{hh}^* = \frac{\theta_h p^e}{\lambda (b + \theta_h) - \theta_h} + p^e = \tilde{p}_{hi} \]

\[ p_{hl}^* = \frac{\left( \theta_l - \frac{\mu (\theta_h - \theta_l)}{1 - \mu} \right) p^e}{\lambda (b + \theta_l) - \theta_l + \frac{\mu (\theta_h - \theta_l)}{1 - \mu}} < \tilde{p}_{li}. \]

Notice that high types do not distort their protection while low type protection falls to achieve separation.

**Corollary 2.** The equilibrium of the political game with informed lobbies Pareto dominates the equilibrium of the political game under perfect information.

Therefore, the existence of private information within the lobby groups generates two informational problems on the political game that reduce the lobbies’ influence on the policy maker. Thus, tariffs decrease, imports increase and the welfare of the society increases.
5. Conclusion

We modified Grossman and Helpman (1994) assuming that competitiveness of productive sectors is the private information of the lobbies. This new element introduces private information on the lobbies preferences in the political game which allows us to analyze the effects of information transmission in the political game for influence.

The information transmission causes two asymmetric information problems. The first one is the screening problem that is a consequence of the fact that the market demands are interdependent. The marginal cost of protection in one market depends on the price (tariff) of the other market, but a lobby does not know which will be the price in the other market. On the other hand, the policy maker knows the prices of both markets after receiving contributions in a separating equilibrium. Thus, the price in the rival market becomes the policy maker’s private information that the other lobby has to screen out. Screening makes lobbies leave informational rents to the policy maker and also to ask for less protection.

The second informational problem is the signaling problem. The policy maker does not know the competitiveness of the lobby and then he does not know the cost of providing protection for this sector. This gives the opportunity for sectors that cause high welfare costs (the more competitive sectors) to pretend they are less competitive in order to give less contribution to the policy maker. Thus, the less competitive sectors have to separate their contributions in order to allow the policy maker to update the true types from the contributions he receives. Separation makes low type lobbies ask for less protection.

We find that lobbies’ best responses in the information transmission problems demand less protection when compared with the perfect information game. Hence, information transmission problems cause distortions on the political game that reduce the lobbies’ ability to influence the policy maker. Since the lobbying activity is inefficient, the welfare of the society increases with their influence reduction.

The results of this paper raise some questions. The first is the question of transparency. It is commonly argued that transparency in the relationship between governments and lobbies is good for the society, that sharply contrasts with the results found here. The arguments in favor of transparency are traditionally based on the accountability of politician on elections (see Coate and Moris, 1995), something that we do not consider in this paper. Nonetheless, we have shown that the absence of information asymmetry would harm the society, what points to a trade-off between better accountability versus less information transmission in political games.

A second question concerns the role of information transmission when policy makers use different policy instruments, such as non tariff barriers. With such instruments, the government does not have the tariff revenue, thus the screening effect may be different. One important issue is that with such instruments, the size of the imports matters, not only the market elasticities, as pointed out by
Maggi and Rodrigues-Clare (2000). Therefore, different instruments can generate different effects given the information transmission problem.

Appendix

Countervailing

The political game, as most of common agency games, has some degree of freedom in the determinacy of the division of the surpluses between lobbies. Up to some level we avoid this indeterminacy by looking for symmetric equilibria. However, symmetry does not account for the surplus division in non-symmetric states of nature (high versus low types).

This indeterminacy raises some questions in this game. Particularly, this can generate equilibria with countervailing incentives. If we look at individual rationality constraints, from the point of view of type-$i$ lobby that takes as given the rival’s offer, we have

\[
C_{ih} + \lambda W(\theta_i, p_{ih}, \theta_h, p_{hi}) \geq \lambda W(\theta_i, p^e, \theta_h, p^e) - C_{hi}
\]
\[
C_{il} + \lambda W(\theta_i, p_{il}, \theta_l, p_{li}) \geq \lambda W(\theta_i, p^e, \theta_l, p^e) - C_{li}.
\]

Notice that the reserve utilities depend on the contribution offered by the rival. Let type-$i$ lobby conjecture that his high type rival will offer a small contribution. This implies that the policy maker’s reserve utility in this state has increased and may be above the reserve utility of the policy maker when the rival is the low type. In this situation, possibly the binding constraints are no longer the ones we have assumed, i.e., there may be countervailing incentives in the type-$i$ problem.

Countervailing incentives change the binding constraints in the informed lobby problem. If, for example, we conjecture that the binding constraints in the type-$i$ informed principal problem are $(ICP_{il})$ and $(IRP_{ih})$ (which are the opposite case we considered in the text). Then, the best-response policies and contributions are such that this lobby makes the same set of constraints bind in the rival’s problem. As a consequence, distortions in the equilibrium policies due to screening are the opposite ones. When the lobby faces a high type rival she demands more protection than in the truthful equilibrium, while when she faces a low type opponent, she demands the same as in the truthful equilibrium. Therefore, the welfare ranking of Corollary 1 is reversed and the welfare ranking of Corollary 2 is ambiguous. As an example, we present the policies of an equilibrium with countervailing incentives for case 1:
Notice that the distortions change. Now policies are upward distorted because lobbies demand more protection in the efficient states (high type rival) to prevent the policy maker saying that the low type rival is the high type.

One way to rule out countervailing is to impose more structure on the conjecture about the rival’s offer. If we assume that the rival’s contribution schedule is such that condition (3.3) holds, that is, the policy maker’s utility is non-decreasing with the rival’s type, there is no countervailing incentives on the type-informed lobby problem. Intuitively, this condition implies that the difference in the utility between high and the low type rivals is not greater than the surplus increase of the political game across the two states.

**Proof of Lemma 1.** The inequality in Lemma 1 comes from the incentive compatibility constraints of the policy maker. The constraint \((ICP_{ih})\) is given by

\[ C_{ih} + C_{hi} + \lambda W(\theta_i, p_{ih}, \theta_h, p_{hi}) \geq C_{il} + C_{li} + \lambda W(\theta_i, p_{il}, \theta_h, p_{hi}) \]

which can be rewritten as

\[ C_{ih} - C_{il} \geq \lambda W(\theta_i, p_{il}, \theta_h, p_{hi}) - \lambda W(\theta_i, p_{ih}, \theta_h, p_{hi}) . \]  

(5.1)

The constraint \((ICP_{il})\) is given by

\[ C_{il} + C_{li} + \lambda W(\theta_i, p_{il}, \theta_l, p_{li}) \geq C_{ih} + C_{li} + \lambda W(\theta_i, p_{ih}, \theta_l, p_{li}) \]

which can be rewritten as

\[ C_{ih} - C_{il} \leq \lambda W(\theta_i, p_{il}, \theta_l, p_{li}) - \lambda W(\theta_i, p_{ih}, \theta_l, p_{li}) . \]  

(5.2)

Combining (5.1) and (5.2) we get

\[ W(\theta_i, p_{il}, \theta_l, p_{li}) - W(\theta_i, p_{ih}, \theta_l, p_{li}) \geq W(\theta_i, p_{il}, \theta_h, p_{hi}) - W(\theta_i, p_{ih}, \theta_h, p_{hi}) . \]

This inequality can be written as

\[ \int_{p_{ih}}^{p_{il}} \frac{\partial W}{\partial p^1}(\theta_i, s, \theta_l, p_{li}) \, ds - \int_{p_{ih}}^{p_{il}} \frac{\partial W}{\partial p^1}(\theta_i, s, \theta_h, p_{hi}) \, ds \geq 0 \]

or as

\[ \int_{p_{ih}}^{p_{il}} \left[ \int_{p_{hi}}^{p_{li}} \frac{\partial^2 W}{\partial p^1 \partial p^2}(\theta_i, s, \theta_l, p) \, d\theta \right] ds \geq 0 . \]
Since 
\[ \frac{\partial W}{\partial p_i} = - (b + \theta \pi''(p^1))(p^1 - p^e) + d(p^2 - p^e) \]
we have that
\[ \frac{\partial^2 W}{\partial \theta^2 \partial p_i}(\theta^1, p^1, \theta^2, p^2) = 0 \]
\[ \frac{\partial^2 W}{\partial p^1 \partial p^2}(\theta^1, p^1, \theta^2, p^2) = d. \]

Hence, since \( p_{hi} \geq p_{li} \), \( \int_{p_{hi}}^{p_{li}} \left[ \int_{p_{li}}^{p_{hi}} \frac{\partial^2 W}{\partial \theta \partial p}(\theta, s, \theta, \tilde{p}) d\tilde{p} \right] ds \geq 0 \) as long as \( p_{ih} \geq p_{il} \), which proves the lemma. \( \square \)

**Proof of Lemma 2.** First we must show what are the binding constraints. Since the lobby does not like to give contributions, we know that at least one of the individual rationality constraints is binding. If not, the lobby could reduce both contributions at once without violating (5.1) and (5.2).

By assumption 4, the policy maker’s preference is increasing in the rival’s type and we have that
\[ C_{hi} - C_{li} + \lambda W(\theta, p, \theta, p_{hi}) - \lambda W(\theta, p^e, \theta, p^e) - \lambda W(\theta, p^e, \theta, p^e) \geq 0. \]

Replacing \( p \) by \( p_{il} \) and adding \( C_{il} \) on both sides of the previous inequality yield
\[ C_{il} + C_{hi} + \lambda W(\theta, p, \theta, p_{hi}) - \lambda W(\theta, p^e, \theta, p^e) \geq C_{il} + C_{li} + \lambda W(\theta, p, \theta, p_{li}) + \lambda W(\theta, p^e, \theta, p^e). \]

From constraint \( (ICP_{ih}) \) we have that
\[ C_{ih} + C_{hi} + \lambda W(\theta, p_{ih}, \theta, p_{hi}) - \lambda W(\theta, p^e, \theta, p^e) \geq C_{il} + C_{hi} + \lambda W(\theta, p_{il}, \theta, p_{li}) - \lambda W(\theta, p^e, \theta, p^e) \]
and from constraint \( (IRP_{il}) \) we have that
\[ C_{il} + C_{li} + \lambda W(\theta, p_{il}, \theta, p_{li}) - \lambda W(\theta, p^e, \theta, p^e) \geq 0, \]
which imply that
\[ C_{ih} + C_{hi} + \lambda W(\theta, p_{ih}, \theta, p_{hi}) - \lambda W(\theta, p^e, \theta, p^e) \geq C_{il} + C_{hi} + \lambda W(\theta, p_{il}, \theta, p_{li}) - \lambda W(\theta, p^e, \theta, p^e) \geq 0. \]

Therefore, constraint \( (ICP_{h}) \) assures that \( (IRP_{ih}) \) is not violated. Hence, we can ignore \( (IRP_{ih}) \) while \( (IRP_{il}) \) is binding.

Now, assume that \( (ICP_{il}) \) is not binding in the solution of the informed lobby problem. Then, since \( (ICP_{il}) \) and \( (IRP_{ih}) \) are not binding, we could choose a smaller contribution \( C_{ih} \) without violating \( (ICP_{il}) \) as we can see from (5.2). Since this new contribution schedule gives the lobby the same policies with less...
contribution, she is strictly better-off. Therefore, the constraint \((ICP_{ih})\) must be binding in the solution of the informed lobby problem.

Provided that constraints \((ICP_{ih})\) and \((IRP_{il})\) are binding, the contributions are given by

\[
C_{il} = -C_{li} - \lambda W (\theta_i, p_{il}, \theta_l, p_{hi}) + \lambda W (\theta_i, p^e, \theta_l, p^e)
\]

and

\[
C_{ih} = C_{il} - \lambda W (\theta_i, p_{ih}, \theta_h, p_{hi}) + \lambda W (\theta_i, p_{il}, \theta_h, p_{hi}).
\]

We plug these contributions into the lobby’s utility function in (3.1) to get

\[
\max_{p_{ih}, p_{il}} z \left[ \theta_i \pi (p_{ih}) + \lambda W (\theta_i, p_{ih}, \theta_h, p_{hi}) - \lambda W (\theta_i, p_{il}, \theta_h, p_{hi}) \right. \\
+ \lambda W (\theta_i, p_{il}, \theta_l, p_{li}) - \lambda W (\theta_i, p^e, \theta_l, p^e)
\]

\[(1 - z) \left[ \theta_i \pi (p_{il}) + \lambda W (\theta_i, p_{il}, \theta_l, p_{li}) - \lambda W (\theta_i, p^e, \theta_l, p^e) \right]
\]

s.t. \((IC_{-i})\).

The first-order conditions of this problem are

\[
z \left[ \theta_i \pi' (p_{ih}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{ih}, \theta_h, p_{hi}) \right] - \mu_{-i} z \left[ \theta_i \pi' (p_{ih}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{ih}, \theta_h, p_{hi}) \right] = 0
\]

\[(1 - z) \left[ \theta_i \pi' (p_{il}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{il}, \theta_l, p_{li}) \right] + z \left[ \lambda d (p_{li} - p^e) - \lambda d (p_{ih} - p^e) \right]
\]

\[- (1 - z) \mu_{-i} \left[ \theta_i \pi' (p_{il}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{il}, \theta_l, p_{li}) \right] - \mu_{-i} z \left[ \lambda d (p_{li} - p^e) - \lambda d (p_{ih} - p^e) \right] = 0.
\]

We add and subtract the term \(\mu_{-i} z \theta_i \pi' (p_{ih})\) in the first-order condition of \(p_{ih}\), add and subtract the term \((1 - z) \mu_{-i} \theta_i \pi' (p_{il})\) in the first-order condition of \(p_{il}\), and then divide both by \((1 - \mu_{-i})\) to get

\[
z \left[ \theta_i \pi' (p_{ih}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{ih}, \theta_h, p_{hi}) \right] + \frac{\mu_{-i}}{1 - \mu_{-i}} \Delta \theta_i \pi' (p_{ih}) = 0
\]

and

\[
(1 - z) \left[ \theta_i \pi' (p_{il}) - \lambda \frac{\partial W}{\partial p^i} (\theta_i, p_{il}, \theta_l, p_{li}) \right] + \frac{\mu_{-i}}{1 - \mu_{-i}} \Delta \theta_i \pi' (p_{il})
\]

\[+ z \left[ \lambda d (p_{li} - p^e) - \lambda d (p_{ih} - p^e) \right] = 0.
\]

Dividing these first-order conditions, respectively, by \(z\) and \((1 - z)\) gives condition (3.4).

Proof of Lemma 3. The constraints \((IC_{-i})\) are not binding if the utility of the type-\(i\) lobby does not wish to pretend she is type \(-i\). This implies that

\[
E[V (\theta_i, p (\theta_i, .), C (p (\theta_i, .), \theta_i))] \geq E[V \theta_i, p (\theta_{-i}, .), C (\theta_{-i}, p (\theta_{-i}, .))].
\]
This inequality can be artificially written as
\[ E \left[ \int_{\theta_{-i}}^{\theta_i} \frac{\partial V}{\partial \theta} \left( \theta_i, p \left( \tilde{\theta}_{-i} \right), C \left( \tilde{\theta}_i, p \left( \tilde{\theta}_{-i} \right) \right) \right) \, d\tilde{\theta} \right] \geq 0 \] 
(5.3)

where the expectation is taken with respect to the rival’s type.

In turn, we have
\[ \frac{\partial V}{\partial \theta_i} \left( \theta_i, p_{ih}, C \left( \tilde{\theta}_i, p_{hi}, p_{il} \right) \right) = \theta_i - \tilde{\theta}_i \frac{\partial p_{ih}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} \frac{\partial p_{il}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} - \frac{\partial p_{ih}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} \frac{\partial p_{il}}{\partial \theta_i} \]

Using that constraints (ICP_{ih}) and (IRP_{il}) are binding, we have
\[ C_{il} = -C_{li} - \lambda W \left( \tilde{\theta}_i, p_{il}, \theta_i, p_i \right) + \lambda W \left( \tilde{\theta}_i, p_f, \theta_i, p^e \right) \] 
(5.4)

and
\[ C_{ih} = C_{il} - \lambda W \left( \tilde{\theta}_i, p_{ih}, \theta_h, p_{hi} \right) + \lambda W \left( \tilde{\theta}_i, p_{il}, \theta_h, p_{hi} \right). \] 
(5.5)

Thus, we can apply the envelope theorem to get \( \tilde{\theta} = \frac{\partial C_{ih}}{\partial p_{ih}} + \frac{\partial C_{ih}}{\partial p_{il}} \).

Hence, the derivatives of the lobby’s utility simplify to
\[ \frac{\partial V}{\partial \theta_i} \left( \theta_i, p_{ih}, C \left( \tilde{\theta}_i, p_{hi}, p_{il} \right) \right) = \left( \theta_i - \tilde{\theta}_i \right) \frac{\partial p_{ih}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} - \frac{\partial C_{ih}}{\partial \theta_i} \frac{\partial p_{il}}{\partial \theta_i} \]
\[ \frac{\partial V}{\partial \theta_i} \left( \theta_i, p_{il}, C \left( \tilde{\theta}_i, p_{il} \right) \right) = \left( \theta_i - \tilde{\theta}_i \right) \frac{\partial p_{il}}{\partial \theta_i} - \frac{\partial C_{il}}{\partial \theta_i} + \frac{z}{1 - z} \frac{\partial C_{ih}}{\partial \theta_i} \frac{\partial p_{il}}{\partial \theta_i} \]

Substituting them back into (5.3) gives
\[ E \left[ \int_{\theta_{-i}}^{\theta_i} \left( \theta_i - \tilde{\theta}_i \right) \frac{\partial p}{\partial \theta} \left( \tilde{\theta} \right) - \frac{\partial C}{\partial \theta} \, d\tilde{\theta} \right] \geq 0. \] 
(5.6)

Again, from the constraints (ICP_{ih}) and (IRP_{il}), we have
\[ \frac{\partial C_{il}}{\partial \theta_i} = \lambda \frac{\partial W}{\partial \theta_i} \left( \tilde{\theta}_i, p_f, \theta_i, p_i \right) - \lambda \frac{\partial W}{\partial \theta_i} \left( \tilde{\theta}_i, p_{il}, \theta_i, p_i \right) \]
\[ \frac{\partial C_{il}}{\partial \theta_i} = \lambda \frac{\partial C_{il}}{\partial \theta_i} + \lambda \frac{\partial W}{\partial \theta_i} \left( \tilde{\theta}_i, p_f, \theta_i, p_i \right) - \lambda \frac{\partial W}{\partial \theta_i} \left( \tilde{\theta}_i, p_{il}, \theta_i, p_i \right) \]

However, we know that
\[ \frac{\partial W_{ik}}{\partial \theta_i} = p^e \]
which, in turn, implies that \( \frac{\partial C_{il}}{\partial \theta_i}, \frac{\partial C_{ih}}{\partial \theta_i} = 0. \)

Therefore, (5.6) becomes
\[ E \left[ \int_{\theta_{-i}}^{\theta_i} \left( \theta_i - \tilde{\theta}_i \right) \frac{\partial p}{\partial \theta} \left( \tilde{\theta} \right) \, d\tilde{\theta} \right] \geq 0. \]
Since this is a discrete type model we do not have intermediate values $\tilde{\theta}$. However, let $p(\tilde{\theta})$ be the linear function that links the values $p(\theta_i)$ and $p(\theta_{-i})$. In particular, $\frac{\partial p}{\partial \tilde{\theta}}(\tilde{\theta}) \geq 0$ since prices are increasing in the lobby’s type. Hence, the above inequality holds because $\theta_i > \theta_{-i}$ if and only if $(\theta_i - \tilde{\theta}) \frac{\partial p}{\partial \tilde{\theta}}(\tilde{\theta}) \geq 0$, for all $\tilde{\theta}$ between $\theta_i$ and $\theta_{-i}$. Therefore, the lobby is always better-off telling the truth and the constraints $(IC_{-i})$ are not binding in case 1.

**Proof of Theorem 1.** The first-order conditions (4.1) and (4.2) for lobby 1 and 2 problems constitute a system of linear equations that can be written in a matrix form. For instance, when both lobbies are high type (state $f_hh$), we have that

$$
\begin{bmatrix}
-\lambda b & \lambda d \\
\lambda d & -\lambda b
\end{bmatrix}
\begin{bmatrix}
p^1_{hh} - p^e \\
p^2_{hh} - p^e
\end{bmatrix}
= \begin{bmatrix}
-\theta_h \\
-\theta_h
\end{bmatrix},
$$

where a solution exists since the determinant of the coefficient matrix is $\lambda b^2 - (\lambda d)^2 > 0$ since, by assumption 1, $b(1 - z) > d$.

Given the solution in this state, we have a similar system of the first-order conditions for the state $f_hl$ given by

$$
\begin{bmatrix}
-\lambda b (1 - z) & \lambda d \\
\lambda d & -\lambda b (1 - z)
\end{bmatrix}
\begin{bmatrix}
p^1_{hl} - p^e \\
p^2_{hl} - p^e
\end{bmatrix}
= \begin{bmatrix}
-\theta_h (1 - z) + z\lambda d(p^e_{hh} - p^e) \\
-\theta_l (1 - z) + z(p^e_{hh} - p^e)
\end{bmatrix},
$$

which has a positive determinant for the same reason. We also have a symmetric system for state $f_lh$.

Given the solution of these systems, we have the system of first-order conditions for state $f_{ll}$, given by

$$
\begin{bmatrix}
-\lambda b (1 - z) & \lambda d \\
\lambda d & -\lambda b (1 - z)
\end{bmatrix}
\begin{bmatrix}
p^1_{ll} - p^e \\
p^2_{ll} - p^e
\end{bmatrix}
= \begin{bmatrix}
-\theta_l (1 - z) + z(p^e_{ll} - p^e) \\
-\theta_l (1 - z) + z(p^e_{ll} - p^e)
\end{bmatrix}.
$$

The determinant of the coefficient matrix is given by $(\lambda (1 - z) b)^2 - (\lambda d)^2 > 0$, since $(1 - z) b > d$ by assumption A1. Thus, there exists a solution for this system.

To find the policies we first compute the best-response functions:

$$
p^1_{hh} = f_{hh}(p^2_{hh}) = p^e + \frac{\theta_h + \lambda d(p^2_{hh} - p^e)}{\lambda b}
$$

$$
p^2_{hh} = f_{hh}(p^1_{hh}) = p^e + \frac{\theta_h + \lambda d(p^1_{hh} - p^e)}{\lambda b}.
$$

Then, we substitute one function into the other to get:

$$
p^*_{hh} = f_{hh}(f_{hh}(p^*_{hh})) = p^e + \frac{\theta_h + \lambda d(p^e_{hh} - p^e)}{\lambda b}
$$

which can be rearranged as
$$p_{hh}^s - p^e = \frac{\lambda b \theta_h + \lambda d \theta_h + (\lambda d)^2 (p_{hh}^s - p^e)}{(\lambda b)^2}$$

and we get

$$(p_{hh}^s - p^e) \lambda^2 (b^2 - d^2) = \lambda (b \theta_h + d \theta_h) \iff p_{hh}^s - p^e = \frac{b \theta_h + d \theta_h}{\lambda (b^2 - d^2)}.$$

Finally, we have that

$$p_{hh}^s = \frac{b \theta_h + d \theta_h}{\lambda (b^2 - d^2)} + p^e = \tilde{p}_{hh}.$$ Prices in the other states can be found in the same way.

The equilibrium contributions are given by (5.4) and (5.5) for the equilibrium policies.

We now have to check whether $(IR_i)$ and Conditions 1 and 2 are satisfied in this equilibrium.

The equilibrium policies are given by

$$p_{hl}^s = \frac{b \theta_h + d \theta_l}{\lambda (b^2 - d^2)} - \frac{zb^2d (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}$$

$$p_{lh}^s = \frac{b \theta_l + d \theta_h}{\lambda (b^2 - d^2)} - \frac{zbd^2 (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}$$

$$p_{ll}^s = \frac{b \theta_l + d \theta_l}{\lambda (b^2 - d^2)} - \frac{zb (b + d) (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}.$$ Notice that the equilibrium policies are increasing in the lobby’s own type since

$$p_{hh}^s - p_{lh}^s = \frac{b (\theta_h - \theta_l)}{\lambda (b^2 - d^2)} + \frac{zb^2d (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)} > 0$$

$$p_{hl}^s - p_{ll}^s = \frac{b (\theta_h - \theta_l)}{\lambda (b^2 - d^2)} + \frac{zb (b + d) (\theta_h - \theta_l)}{\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)} > 0.$$ Therefore, Condition 1 is satisfied in this equilibrium.

The set of binding constraints assure that Condition 2 holds as well. Indeed, in the state constraint $(IR_{P_l})$ is binding, the policy maker receives his reserve utility. Since constraint $(IR_{P_h})$ is binding, then the policy maker also receives his reserve utility in states $\{\theta_h, \theta_l\}$ and $\{\theta_l, \theta_h\}$. Constraint $(IR_{P_{hh}})$ is automatically satisfied since constraint $(IC_{P_{hh}})$ holds and, then it is not binding and the policy maker receives positive rent when both lobbies have high types. Therefore, the policy maker’s utility is increasing in the lobbies’ types and Condition 2 is also satisfied in equilibrium.

**Proof of Corollary 1.** Since preferences are quasi-linear, in order to compare the welfare of the political game with and without private information we do not have to look for contributions, we only need to compare prices.
The welfare is the same with truthful prices for the state \( \{ \theta_h, \theta_h \} \) since prices are the same. Prices in the other states can be written as

\[
\begin{align*}
p_{hl}^* &= \frac{\theta_h b \left( \frac{(1-z)b-d}{b-d} \right) + \theta_l d}{\lambda ((1-z) b^2 - d^2)} + p^c < \bar{p}_{hl} \\
p_{lh}^* &= \frac{\theta_l b (1-z) + \theta_h d \left( \frac{(1-z)b-d}{b-d} \right)}{\lambda ((1-z) b^2 - d^2)} + p^e < \bar{p}_{lh} \\
p_{ll}^* &= \frac{\theta_l ((1-z) b + d) - \theta_h d \left( \frac{z b}{b-d} \right)}{\lambda ((1-z) b^2 - d^2)} + p^f < \bar{p}_{ll}. 
\end{align*}
\]

The maximum welfare for the society is given by free trade. Thus, when lobbies receive positive protection, welfare decreases. This means that \( \frac{\partial W}{\partial p_n} < 0 \) for \( p > p^c \). In states \( \{ \theta_h, \theta_l \} \) and \( \{ \theta_l, \theta_h \} \) prices are below the truthful prices and above international prices (since \( (1-z) b > d \)). Therefore, the welfare is higher with truthful prices. In the state \( \{ \theta_l, \theta_l \} \) prices are also below the truthful prices, but they can fall below international prices as well. Thus, we have to compare explicitly the welfare with different prices. That is, if

\[
W (\theta_l, \bar{p}_{ll}, \theta_l, \bar{p}_{ll}) \leq W (\theta_l, p_{ll}^*, \theta_l, p_{ll}^*),
\]

then welfare is lower in this state with truthful prices. Given our assumptions, this inequality is equivalent to

\[
(\bar{p}_{ll} - p^c)^2 (b - d) - (p_{ll}^* - p^c)^2 (b - d) > 0.
\]

Since \( p_{ll}^* < \bar{p}_{ll} \), we must have

\[
(\bar{p}_{ll} - p^c)^2 > (p_{ll}^* - p^c)^2
\]

which simplifies to

\[
\bar{p}_{ll} - p^c > p^c - p_{ll}^*.
\]

Substituting the previous formulas for these prices, we get:

\[
\theta_l (b + d) > -\frac{\theta_l (b + d)}{\lambda (b^2 - d^2)} + \frac{zbd (\theta_h - \theta_l) (b + d)}{\lambda ((1-z) b^2 - d^2) (b^2 - d^2)},
\]

which we can rewrite as

\[
\frac{2\theta_l (b + d)}{\lambda (b^2 - d^2)} > \frac{zbd (\theta_h - \theta_l) (b + d)}{\lambda ((1-z) b^2 - d^2) (b^2 - d^2)}.
\]

After some algebra, the last expression simplifies to

\[
\frac{2 ((1-z) b^2 - d^2)}{zbd} > \frac{\theta_h - \theta_l}{\theta_l}.
\]

By assumption 1, the previous inequality holds. Therefore, the welfare in this state is greater than in the perfect information case. \( \square \)
Proof of Lemma 4. We will use the proof of Lemma 3 for the case where \( i = l \). However, we are in case 2 and the derivative of the lobby’s utility is given by

\[
\frac{\partial V}{\partial \tilde{\theta}} \left( \theta_i, p_i^h, C \left( \tilde{\theta}, p_i^h, p_i^l \right) \right) = \theta_i p_i^h \frac{\partial p_i^h}{\partial \tilde{\theta}} - \frac{\partial C_i}{\partial \tilde{\theta}} \frac{\partial p_i^h}{\partial \tilde{\theta}} - \frac{\partial C_i}{\partial \tilde{\theta}} \frac{\partial p_i^l}{\partial \tilde{\theta}}.
\]

The derivatives of the welfare function are given by

\[
\frac{\partial W_{lk}}{\partial \theta_i} (\theta_i, p_{ik}, \theta_k, p_{ik}) = -p_{ik} \left( \frac{p_{ik}}{2} - p^e \right)
\]

and also

\[
\frac{\partial W_{lk}}{\partial \theta_i} (\theta_i, p^e, \theta_k, p^e) = \left( \frac{p^e}{2} \right).
\]

Therefore, we have

\[
\frac{\partial C_{il}}{\partial \tilde{\theta}} = \frac{1}{2} (p_{ik} - p^e)^2
\]

and, the inequality (5.3) for low type lobbies becomes

\[
E \left[ \int_{\theta_l}^{\theta_h} \left( \theta_i - \tilde{\theta} \right) p \left( \tilde{\theta} \right) \frac{\partial p}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) - \frac{1}{2} (p_{ik} - p^e)^2 \right] d\tilde{\theta} \geq 0
\]

i.e.,

\[
E \left[ \int_{\theta_l}^{\theta_h} \left( \tilde{\theta} - \theta_l \right) p \left( \tilde{\theta} \right) \frac{\partial p}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) + \frac{1}{2} (p_{ik} - p^e)^2 \right] d\tilde{\theta} \geq 0.
\]

Since we can choose a linear function \( p \left( \tilde{\theta} \right) \) linking \( p \left( \theta_l \right) \) to \( p \left( \theta_l \right) \), \( \frac{\partial p}{\partial \theta} \left( \tilde{\theta} \right) \) is positive. Hence, the above inequality holds and the low type prefers to tell the truth, i.e., the constraint \((IC_1)\) is not binding. Notice that for the high type

\[
E \left[ \int_{\theta_l}^{\theta_h} \left( \theta_i - \tilde{\theta} \right) p \left( \tilde{\theta} \right) \frac{\partial p}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) - \frac{1}{2} (p_{ik} - p^e)^2 \right] d\tilde{\theta}
\]

clearly may not be positive since \( p_{ik} - p^e > 0 \) and \( \frac{\partial p}{\partial \theta} \left( \tilde{\theta} \right) \geq 0 \).

Proof of Theorem 2. The first-order conditions (4.3) and (4.4) for lobbies 1 and 2 constitute a system of linear equations that can be written in a matrix form as in the proof of Theorem 1. For state \( \{\theta_h, \theta_h\} \) this system is given by

\[
\begin{bmatrix}
\theta_h - \lambda (b + \theta_h) & \lambda d \\
\lambda d & \theta_h - \lambda (b + \theta_h)
\end{bmatrix}
\begin{bmatrix}
p_{lh}^1 - p^e \\
p_{lh}^2 - p^e
\end{bmatrix} =
\begin{bmatrix}
-p^e \theta_h \\
-p^e \theta_l
\end{bmatrix}.
\]

It has a solution because the coefficient matrix has a positive determinant since \((1 - z) (\lambda (b + \theta_h) - \theta_h) > \lambda d\).
With policy $p^e_{ih}$, we have the following system of first-order conditions for the state \{\theta_h, \theta_i\}:

$$
\begin{bmatrix}
(1-z) (\theta_h - \lambda (b + \theta_h)) \\
\lambda d
\end{bmatrix}
\begin{pmatrix}
\lambda d \\
\theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} - \lambda (b + \theta_i)
\end{pmatrix}
\begin{bmatrix}
p^1_{ih} - p^e \\
p^2_{ih} - p^e
\end{bmatrix}
= 
\begin{bmatrix}
-p^e (1-z) \theta_h + z (p^e_{ih} - p^e) \\
-p^e (1-z) \theta_i + z (p^e_{ih} - p^e)
\end{bmatrix}
$$

and a symmetric system for state \{\theta_i, \theta_h\}. This system has a determinant given by

$$(1-z) (\theta_h - \lambda (b + \theta_h)) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} - \lambda b (b + \theta_i) \right) - (\lambda d)^2 > 0$$

since $(1-z) (\lambda (b + \theta_i) - \theta_i) > \lambda d$ and $\mu \in [0, 1)$.

In turn, given the policies of these states the following system of best-responses for state \{\theta_i, \theta_i\}:

$$
\begin{bmatrix}
(1-z) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} - \lambda (b + \theta_i) \right) \\
\lambda d
\end{bmatrix}
\begin{pmatrix}
\lambda d \\
(1-z) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} - \lambda b (b + \theta_i) \right)
\end{pmatrix}
\begin{bmatrix}
p^1_{ii} - p^e \\
p^2_{ii} - p^e
\end{bmatrix}
= 
\begin{bmatrix}
-p^e (1-z) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right) + z (p^e_{ih} - p^e) \\
-p^e (1-z) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right) + z (p^e_{ih} - p^e)
\end{bmatrix}
$$

The determinant of the coefficient matrix

$$
\left[ (1-z) \left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} - \lambda b (b + \theta_i) \right) \right]^2 - (\lambda d)^2 > 0
$$

since $(1-z) (\lambda (b + \theta_i) - \theta_i) > \lambda d$ and $\mu \in [0, 1)$. Therefore, this system has a unique solution for each given $\mu \in [0, 1)$.

Now, we show there exists $\mu \in [0, 1)$ which ensures that constraint (ICa) holds. If the constraint (ICa) holds for $\mu = 0$, then there is no signaling effect and this is the equilibrium of the game (as in case 1). If, however, the constraint (ICa) does not hold for $\mu = 0$, then $\mu > 0$. In this case policies for the low type are given by

$$
p^e_{ih} - p^e = \frac{\left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right) p^e (1-z) (\lambda (b + \theta_h) - \theta_h) + \lambda d \theta_i p^e (1-z) (\frac{(1-z)(\lambda (b + \theta_h) - \theta_h) - \lambda d}{\lambda (b + \theta_h) - \theta_h - \theta_i + \frac{\mu(\theta_h - \theta_i)}{(1-\mu)}})}{(1-z) \left( \lambda (b + \theta_h) - \theta_h \right) \left( \lambda (b + \theta_i) - \theta_i + \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right)} - (\lambda d)^2,
$$

$$
p^e_{ii} - p^e = \frac{\left( \theta_i - \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right) p^e (1-z) - z \lambda d (p^e_{ih} - p^e)}{(1-z) \left( \lambda (b + \theta_i) - \theta_i + \frac{\mu(\theta_h - \theta_i)}{(1-\mu)} \right) - \lambda d}.
$$
Notice that both prices are continuous functions of $\mu \in (0, 1)$ and also that $\frac{\partial p_h}{\partial \mu}, \frac{\partial p_l}{\partial \mu} < 0$. Moreover, we have that if $\mu \to 1$, then $p_h^*, p_l^* \to 0$. Thus, by continuity, there exists $\tilde{\mu} \in (0, 1)$ such that
\[ E [\pi (p_h^*) - \pi (p^e)] = 0. \tag{5.7} \]
Since the expected profit is equal to free trade, then it must be that $E [C_h^*] = 0$. Therefore, the low type lobby is indifferent between playing or not this game. On the other hand, multiplying (5.7) by $\theta_h$ shows that the high type lobby gets $E [\theta_h \pi (p_h^*)] = \theta_h \pi (p^e)$ when she lies to the policy maker. However, by constraint ($IR_i$), we have that
\[ E [\theta_h \pi (p_h^*) - C_h^*] \geq E [\theta_h \pi (p^e)] = E [\theta_h \pi (p_l^*)]. \tag{5.8} \]
Therefore, when $\mu = \tilde{\mu}$ the high type lobby has no incentive to lie and the constraint ($IC_h$) holds.

We also have that for $\mu = 0$
\[ E [\theta_h \pi (p_h^*) - C_h^*] < E [\theta_h \pi (p_l^*) - C_l^*], \tag{5.9} \]
because we are in the case where signaling is costly. Therefore, given (5.8) and (5.9), there exists $\mu^* \in (0, \tilde{\mu}]$ such that $E [\theta_h \pi (p_h^*) - C_h^*] = E [\theta_h \pi (p_l^*) - C_l^*]$, and the constraint ($IC_h$) is binding. In fact, there may be more than one $\mu^*$ such that ($IC_h$) is binding. Pick the smallest one (which generates the least distortion). Given this $\mu^*$, there exists an equilibrium for the political game in case 2.

The equilibrium contributions are given by (5.4) and (5.5) for the equilibrium policies $p_{ik}^*$ for all $i$ and $k$.

A similar argument to the proof of Theorem 1 shows that Conditions 1 and 2 are satisfied in equilibrium.

**PROOF OF COROLLARY 2.** Analogous to the proof of Corollary 1. \qed
Bibliography


CHAPTER 2

Lobbying and information transmission in customs unions

1. Introduction

Motivated by the vast increase in the number of regional trade agreements after the creation of the World Trade Organization, the recent literature on preferential trade agreements (PTAs) has advanced in characterizing the kinds of PTAs that are actually implementable, how they affect the countries’ protection towards non-members and whether they influence the countries’ willingness to pursue further trade liberalization.

This new literature recognizes politics as a central element to the decision making in trade policy. The agreements and the tariffs are endogenously chosen by governments subject to the pressure of special interest groups.

In this paper we extend this literature evaluating the costs and benefits of customs union agreements when governments are subject to the pressure of special interest groups that have better information than governments. We focus on the political effects of the agreement and therefore, our setup is suitably simple so that there are no terms of trade effects. Without the agreement, each lobby demands protection for the government of her own country, while in a customs union the lobbies from both counties compete to get protection and have to “please” the governments in the union.

Three effects arise in a customs union. The first one is the dilution of the lobbies’ influence: with the agreement the decision making takes into consideration the welfare of all partner countries, thus the benefit of a particular group becomes relatively small and the lobbies’ ability to influence decreases. The second effect is the free riding. Politically weak groups benefit from a customs union since they free-ride on the lobbying efforts of groups from the other countries.

The third and new effect is due to asymmetric information: since goods are substitutes, when a lobby offers contributions she should know the price of the other goods in order to learn the true marginal cost of her protection. However, prices of other goods are determined by the protection demanded by the lobbies of the other countries which, in turn, depends on the other lobbies’ capacities. Since lobbies do not know the capacity of the rival lobbies, they do not know the true marginal cost of their own protection.

\footnote{1We refer to lobbies with feminine pronouns and to governments with masculine pronouns.}
On the other hand, when a lobby negotiates protection with the governments, she conveys her information through contributions and, in our model, the outcome of this negotiation reveals the competitiveness of the lobbies to the policy maker. This gives the governments an advantage to use the information of the lobbies from one country to extract rents from the lobbies of the other country in a customs union. Thus, from the point of view of a given lobby, the private information of foreign lobbies becomes private information of the coalition of governments that form the customs union. Lobbies can only screen the rival’s information from the policy maker, yet, screening generates distortions on the demands for protection. We refer to this distortions as the “information transmission effect”.

Information transmission allows the governments to extract informational rents and makes the lobbies demand less protection to save on rents they pay. Therefore, this effect decreases the tariffs imposed to non-members in a customs union, which increases the welfare of societies of the member countries.

We then consider the incentives for the creation of customs unions. We find that lobbies have no interest in participating of a customs union. Thus, under perfect information, customs unions are always blocked by lobbies. However, with informed lobbies, if the information transmission effect is relatively large, the welfare increase of the agreement becomes too big that lobbies are no longer able to block it. Therefore, information transmission can make a customs union politically sustainable.

Related literature

The first effect of a customs union was called the “preference-dilution effect” by de Melo, Panagariya and Rodrik (1993). In a setting quite different from ours, they found that a trade agreement (not only a customs union) reduces the relative weight of lobbies in the eyes of the decision makers when countries decide policies taking into consideration its effects on partner countries.

Richardson (1993) has compared free trade areas (FTAs) and customs unions, and found that the second is welfare superior because tariffs are public good for lobbies on the same sector from different countries. Thus, they free ride on the contributions of each other and the overall protection falls. In a FTA each lobby demands protection for the policy maker of their country. In the perfect information version of our framework, the welfare effects of a customs union are similar to those analyzed by these authors.

Grossman and Helpman (1995) and Krishna (1998) considered the role of politics on the incentives to sign PTAs. In a context where tariffs are endogenously defined by lobbying, they found that trade diverting FTAs were more likely to be supported. Krishna (1998) also finds that the incentives for engaging in multilateral liberalization decrease after joining a FTA. However, the role of politics in theses papers is just partially accounted for since they assume that the countries’
towards the rest of the world are the same before and after the agreement is signed.

In their setup, lobbies cannot influence the external tariffs after the agreements were signed, thus, they do not take into account the effect of the agreement on the political game, which is the focus of this paper. We, instead, compare the pattern of politically motivated protection outside and inside a customs union. Thus, in our framework, protection changes because the agreement changes the structure of the political game.

Ornelas (2005) and Maggi and Rodrígues-Clare (2007) have already considered the role for lobbying after an agreement is signed. The first paper shows that the rent that lobbies can obtain decreases in a FTA, which makes welfare decreasing agreements less likely to be implemented. The second paper considers the role of trade agreements as a commitment against future lobbying and also finds that trade agreements result in deeper liberalization when countries are more politically motivated.

The trade literature, to our knowledge, has always considered the political pressure game as one with perfect information. However, there is a vast literature on political economy that investigates situations where lobbies have more information than the decision makers, like Austen-Smith (1995) and Potters and Wan Winden (1991) to name a few. We link these two literatures and we acknowledge the role of information asymmetries in political pressure for trade protection.

Our work benefits from Maskin and Tirole (1990, 1992) and from Costa Lima and Moreira (2008). The first authors have studied informed principal problems which represents the structure of political pressure in our model when the governments do not reach the agreement. The second analyzed a lobbying game like Grossman and Helpman (1994) with informed lobbies and the informational effects we find in a customs union is similar to theirs.

The next section presents the two countries’ economies and the political game. In sections 3 and 4 we find the trade protection when the agreement is not signed and in a customs union, respectively. Section 5 compares the welfare under each political structure. In section 6 we investigate the incentives for the creation of a customs union. Section 7 concludes.

2. Analytical framework

We consider two small countries (A and B) that produce and trade goods with each other and the rest of the world. Within each country a political game takes place. Special interest groups that represent productive sectors offer money contributions to governments in order to receive tariff protection. Since governments are influenceable, lobbying shifts rents from consumers to producers.

We compare two different regimes for the political game, one where the governments choose their import tariffs unilaterally (the unilateral regime), and another where the countries sign a customs union agreement to remove all tariffs between
them and set the same import tariff towards the rest of the world (customs union regime).

The economies presented are similar to that described in Grossman and Helpman (1994) with particular functional forms. We begin presenting the economies, and then we present the political game under each regime.

**The economy**

We assume that countries are symmetric, thus we only present the economy of country $A$. The country produces and trades three goods, $x_0^A$, $x_1^A$, and $x_2^A$. It has a size one population with preferences given by

$$u(x_0^A, x_1^A, x_2^A) = x_0^A + \sum_n (\alpha - \beta x_n^A) x_n^A - \delta x_1^A x_2^A,$$

where the uppercase index $n \in \{1, 2\}$ refers to the market and the lowercase index refers to the country and we assume that $\alpha, \beta > 0$.

The consumer’s income comes from labor and government transfers. With the income, we can find the market demands for goods 1 and 2, which are given by

$$x_n^A (p_n^A, p_{-n}^A) = a - bp_n^A + dp_{-n}^A,$$

where $-n$ indicates the price of the good in the other market. If $d > 0$, then the goods are substitutes, if $d < 0$, they are complements. The fact that $d \neq 0$ implies that the welfare cost of tariffs depends on the tariffs of other goods. This is important for the political game since it implies that the choice of tariffs of different goods are interrelated. For simplicity, we impose:

**Assumption 1.** Goods $x^1$ and $x^2$ are substitutes ($d > 0$).

Goods are substitutes throughout the paper, although the results carry on when goods are complements as well.

If we substitute these demands into the utility function, we find the indirect utility function, denoted by $u_A (p_1^A, p_2^A)$ with some abuse of notation that shall not make confusion.

Good $x_0^A$ is the numeraire good. It is produced using labor with constant returns to scale with an input-output coefficient of one. We assume that the labor supply is big enough, so that the wages are fixed at one. Goods $x_1^A$ and $x_2^A$ are produced by competitive firms with sector specific inputs. Thus, the owners of these inputs receive all the profit coming from the production of the goods. We assume that the ownership of these inputs is highly concentrated, so that the fraction of the owners in the society is negligible. The country’s industries have a fixed capacity constraint, that is, the industries’ marginal cost ($c$) is given by

$$c (y_{-n}^A) = \begin{cases} 0 & \text{if } y_{-n}^A \leq \theta^n \\ \infty & \text{if } y_{-n}^A > \theta^n \end{cases}$$
where \( y^*_n \) is the home production of good \( n \) and \( \theta^*_A \) is the capacity constraint. With this technology, the industry produces \( \theta^*_A \), sells it for price \( p^*_A \) and earns profit
\[
\theta^*_A \pi (p^*_A) = \theta^*_A p^*_A.
\]

There are no fixed costs.

The government’s revenue comes from import tariffs, redistributed by lump-sum transfers. The revenue is given by
\[
R_A = \sum_n (p^*_A - p^e) (x_A^n (p^*_A, p^-_A) - \theta^*_A)
\]
where \( p^e \) is the international price of goods \( x^1_A \) and \( x^2_A \), \( p^*_A - p^e \) is the protection and \( x_A^n (p^*_A, p^-_A) - \theta^*_A \) is the size of the imports. We assume that \( x^0 \) is untaxed in all countries.

For simplicity, we restrict the analysis to the cases where countries import both goods 1 and 2, that is:

**Assumption 2.** \( a \) is large enough so that \( x^n (p^*_A, p^-_A) > \theta^*_A \).

This implies that lobbies offer contributions to protect their markets from foreign competition. Together with capacity constraints, this assumption eliminates the usual trade diversion and trade creation effects from our analysis because neither country exports goods 1 and 2. Thus, trade can only increase in the good that is already untaxed. Although restrictive, this assumption allows us to clear out the terms of trade effects and focus on the role of politics on tariffs protection between the two regimes.

The consumer’s preferences, the government’s revenues, the technologies of production and the assets ownership define the economy of each country. We now present the political game under each regime.

**The political game**

There is one lobby in each country that represents the producers of one market. In country \( A \) the lobby represents producers of sector 1 and in country \( B \) the lobby represents producers of sector 2. The lobbies can offer contributions to the governments in exchange for tariff protection for their product.

In the unilateral regime, each lobby offers contributions to the government of her country. Hence, the political game is an informed principal game, where the lobby is the principal and the government is the agent. On the other hand, in a customs union, the two lobbies offer contributions to the same policy maker. Thus, the political game is a common agency game with informed principals.

The lobbies care about the profits of the producers from the sector they represent and dislike giving money contributions to governments. Their utility function is given by
\[
V_A (\theta^1_A, p^1_A, C^1_A) = \theta^1_A p^1_A - C^1_A.
\]
The capacity constraint $\theta$ is private information of the lobby. It can assume two values: $\theta_h$ and $\theta_l$, where $\theta_h > \theta_l$, and $\theta_h$’s ex-ante probability ($z$) is common knowledge.

For simplicity, we assume that lobbies can only demand protection for their goods and that contributions must be non-negative. A contribution schedule $C(\theta, p)$ specifies a monetary transfer for each policy $p$ and for each lobby type $\theta$.

The governments are the policy makers that choose the import tariffs of the economies. We assume that import tariffs are the only policy instrument available. They care about the economic welfare but also like money contributions. In the unilateral regime, each government chooses his import tariffs, while in a customs union, governments coordinate to a unique import tariff for the rest of the world and we assume they do so with preferences given by the sum the governments’ preferences. The preferences of government $A$ are given by

$$U_A (\theta_A, p^1_A, p^2_A, C^1_A) = C^1_A + \lambda W_A (\theta_A, p^1_A, p^2_A)$$

and the welfare ($W_A$) is given by the government’s revenues plus the consumer’s and producer’s surpluses

$$W_A (\theta_A, p^1_A, p^2_A) = \sum^n \theta^n_A p^n_A + u (p^1_A, p^2_A) + \sum^n (p^n_A - p^e) (x^n_A - \theta^n_A).$$

Figure 1 gives the welfare of market 1 in country $A$.

**Figure 1: The welfare in market 1**

The downward sloped line is the home market demand and the vertical line is the home supply of good one. The triangle A, below the demand and above the price $p^1$ is the consumer’s surplus. The rectangles B and E are the producers’ surplus, the rectangle C is the tariff revenue and the triangle D is the deadweight loss of the tariff.
3. UNILATERAL TARIFF SETTING

To simplify the notation throughout the paper we will omit the explicit form of the welfare function, but we use of the fact that
\[
\frac{\partial W_A}{\partial p}(\theta_A^1, p_A^1, p_A^2) = -b(p_A^1 - p^e) + d(p_A^2 - p^e) \quad (2.1)
\]
\[
\frac{\partial W_A}{\partial \theta_A}(\theta_A^1, p_A^1, p_A^2) = \theta_A p^e. \quad (2.2)
\]

We want to avoid corner solutions, where either lobbies get the entire surplus from the society or they do not get any protection:

**Assumption 3.** (i) \(b(1 - z) > d\);
(ii) \(\theta_h \leq 2\theta_1\);
(iii) \(\frac{b}{d} > \frac{\theta_h}{\theta_1}\).

The first two items of Assumption 3 are sufficient conditions for interior solution. Item (iii) is useful, although not necessary for our main results.

If governments reject contributions, they maximize their utility without influence, i.e., they maximize economic welfare. In this framework, the welfare maximum for the economies is the free-trade \((p^n = p^e)\), regardless of the regime.

The preferences of the lobbies and the policy makers, the information structure, the regime and the strategy space define the political game.

The timing of this game is given by:
(0) nature draws the lobbies’ types;
(1) each lobby offers a contribution schedule to the policy maker;
(2) policy makers accept or reject the offers;
(3) policies are chosen and, if contributions are accepted, payments are made accordingly.

3. Unilateral tariff setting

In the unilateral regime, governments choose their tariffs separately. In this case the unique informed lobby offers contributions to the policy maker of her own country to get protection. Thus, the political game is an informed principal game where the policy maker has to choose tariffs for the two home markets subject to the pressure of one privately informed lobby. Since markets and countries are symmetric, we drop the country’s and the good’s index and we identify the variables only by the type of the lobby. We begin looking at the perfect information setup.

**Perfect information**

We assume that the lobby has all the bargaining power. Then, a type-\(i\) lobby offers a contribution schedule that solves
\[
\max_{p_i, C(\theta_i, p_i)} \theta_i p_i - C(\theta_i, p_i) \quad \text{(lobby’s problem I)}
\]
subject to
\[ C(\theta_i, p_i) + \lambda W_A(\theta_i, p_i, p_e) \geq \lambda W_A(\theta_i, p^e, p^e), \]  
where the lowercase index \( i \in \{h, l\} \) refers to the lobby’s type and \( p \) is the price in the market that has no lobby.

The lobby has no incentives to leave rent to the policy maker, which implies that the \((IR)\) constraint is binding and we can use it to substitute out the contribution. Then, lobby’s problem I becomes
\[ \max_{p_i} \theta_i p_i + \lambda W_A(\theta_i, p_i, p_e) - \lambda W_A(\theta_i, p^e, p^e). \]

The first-order conditions of this problem are given by
\[ \theta_i - \lambda b (\hat{p}_i - p^e) + \lambda d (\hat{p}_i - p^e) = 0 \]
\[ -b (\hat{p}_i - p^e) + d (\hat{p}_i - p^e) = 0, \]
where \( p_i \) is chosen by the policy maker to accommodate the cost of protection of the other good. We refer to the policies that solve lobby’s problem I with a “hat” for the sectors that have lobby and with an inverted “hat” for the sectors without lobby. These policies are given by
\[ \hat{p}^{1}_{Ai} = \hat{p}^{2}_{Bi} = \hat{p}_i = \frac{\theta_i b}{\lambda (b^2 - d^2)} + p^e \]
\[ \hat{p}^{2}_{A} = \hat{p}^{1}_{B} = \hat{p}_i = \frac{\theta_i d}{\lambda (b^2 - d^2)} + p^e. \]

Notice that by symmetry \( \hat{p}^{1}_{Ai} = \hat{p}^{2}_{Bi} \) and \( \hat{p}^{2}_{A} = \hat{p}^{1}_{B} \).

The tariff is positive for the lobbying industry, and it is increasing in the industry’s capacity \( (\theta_i) \). The unorganized sector receives the tariff that best accommodates the cost of giving protection to the lobby. Since the goods are substitutes, increasing the tariff for the sector that does not lobby reduces the marginal cost of protection for the sector that has a lobby. Therefore, even without exerting pressure, the unorganized sector receives some protection.

**Private information**

In the unilateral regime, the political game is an informed principal problem with common values because the government cares about the lobby’s competitiveness. From Maskin and Tirole (1992), we know that informed principal problems with common values have informational distortions like in signaling games. However, in our particular structure, the capacity constraint does not affect the marginal welfare cost of protection, which implies that informed lobbies reveal their private information without any cost. This is the essence of the following:

**Proposition 1.** There are no distortions on the political game in the unilateral regime.
4. Customs union

A customs union is a trade agreement where two or more countries decide to eliminate all tariffs between them and set the same import tariff regarding the countries outside the agreement.

In our framework, such agreement does not increase trade between the partners since home production is fixed and the countries are importers of goods 1 and 2 (they export good 0, but this good is already untaxed). Therefore, tariffs change between the regimes solely as consequence of the agreement’s effects on the political structure. Obviously, in real negotiations of PTAs, countries are looking to gain access to other markets and to increase exports. Yet, these simplifying assumptions help us to isolate the effects of politics on trade agreements.

In this regime, we assume that governments choose the import tariffs with preferences given by the sum of the government’s preferences, i.e., the policy maker’s

\[ \theta, \theta \]

\[ s(p', p^*) \]

\[ p, p' \]

\[ p'' \]

\[ x \]

**Figure 2: No informational distortion**

Figure 2 shows the welfare cost of protection (the gray triangle) for two different production capacities (which are unknown to the policy maker). Notice that the size of the welfare cost depends only on the size of the tariff and on the shape of the demand, which the policy maker knows. The true type of the lobby of the home sector does not affect the welfare cost of protection. Hence, lobbies do not have incentives to withdraw information from the government.\(^2\)

Since there are no distortions on the political game, the policy outcome under unilateral tariff setting is the same as in the perfect information case.

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\(^2\) Notice also that this result hinges on the specification of the technology of production.
preferences are given by
\[ U(\theta_A^1, p^1, C_A^1, \theta_B^2, p^2, C_B^2) = C_A^1 + C_B^2 + \lambda W_A(\theta_A^1, p^1, p^2) + \lambda W_B(\theta_B^2, p^2, p^1). \]

This means that the governments have no coordination problem in this regime. The lobbies offer contributions to the same policy maker that choose the import tariffs of the two countries and the political game is a common agency game with informed lobbies, similar to Costa Lima and Moreira (2008).

We begin analyzing the perfect information case. The variables of the rival lobby are in bold and, to simplify the notation, we denote \( C(\theta_i, p_{ik}) \) by \( C_{ik} \).

**Perfect information**

With perfect information, a type-\( i \) lobby, facing a type-\( k \) rival lobby offers a contribution schedule that solves

\[
\max_{\theta_i, p_{ik} \in [0,1]} \theta_i p_{ik} - C_{ik} \quad \text{(lobby’s problem II)}
\]

subject to the individual rationality of the policy maker

\[ C_{ik} + C_{ki} + \lambda W_A(\theta_i, p_{ik}, p_{ki}) + \lambda W_B(\theta_k, p_{ki}, p_{ik}) \geq \bar{U}(\theta_i, \theta_k), \quad (IR_{ik}) \]

where the second lowercase index \( k \in \{h,l\} \) refers to the rival lobby’s type and \( \bar{U}(\theta_i, \theta_k) \) is the policy maker’s reserve utility.

Usually \( \bar{U}(\theta_i, \theta_k) \) gives the payoff of the policy maker when he rejects the contribution of only this lobby. However, once we assume that lobbies have the bargaining power, rejecting one lobby does not give any rents to the policy maker. Therefore,

\[
\bar{U}(\theta_i, \theta_k) = \lambda W_A(\theta_i, p^e, p^e) + \lambda W_B(\theta_k, p^e, p^e).
\]

Since the lobby has the bargaining power, the constraint \((IR_{ik})\) is binding and we can substitute out the contribution. Therefore, the lobby’s problem II becomes

\[
\max_{p_{ik}} \theta_i p_{ik} + \lambda \left[ W_A(\theta_i, p_{ik}, p_{ki}) + W_B(\theta_k, p_{ki}, p_{ik}) - W_A(\theta_i, p^e, p^e) - W_B(\theta_k, p^e, p^e) \right].
\]

We follow Bernheim and Whinston (1986) and assume the lobbies’ contributions are truthful, and then the first-order condition of \( \bar{p}_{ik} \) is given by

\[
\theta_i - 2\lambda b (p_{ik} - p^e) + 2\lambda d (p_{ki} - p^e) = 0
\]

which implies that the equilibrium policy under perfect information is given by

\[
\bar{p}_{ik} = \frac{b \theta_i + d \theta_k}{2\lambda (b^2 - d^2)} + p^e. \quad (4.1)
\]

We see that policies increase with the lobby’s own type because high types have higher willingness to pay for protection. They also increase with the protection for the lobby in the other market because goods are substitutes, which means that higher protection for the rival implies lower marginal cost of protection.
Comparing these policies with the prices in the unilateral regime under Assumption 3 gives

\[ \tilde{p}_{ik} < \hat{p}_i \]
\[ \bar{p}_{ik} > \tilde{p}_i. \]

These inequalities show that the lobbies get less protection in a customs union because they have to compensate the policy maker for the welfare loss in two countries. Thus, the relative weight of lobbies decreases compared to welfare in the preference of the policy maker. This is exactly the preference-dilution effect presented in de Melo, Panagariya and Rodrik (1993). On the other hand, the protection increases in sectors that do not lobby because they free-ride in the protection demanded by the lobby in the other country. This effect is similar to the one presented by Richardson (1993).

Notice that prices are equalized in the two countries. However, we cannot anticipate which regime generates higher welfare under perfect information, because protection decreases in markets that have lobby but it increases in markets that do not have lobby. Therefore, in each country, some markets get less while some get more protection. We discuss the welfare properties of each regime in the next section.

We now turn to the case of informed lobbies in a customs union.

Privately informed lobbies

With privately informed lobbies, new elements arise in the political game. In this common agency game with informed lobbies, the lobby cannot access the true marginal cost of her protection because she does not know the price of the substitute good. This is determined by the pressure of the lobby from the other country whose type she does not know.

Obviously, the policy maker knows the policy that is to be implemented in both markets but a lobby does not know the policy to be granted for her rival. Therefore, the policy maker has an informational advantage against the lobbies. From the lobby’s point of view, the policy maker holds the information of the lobby from the partner country. The best the lobby can do is to offer conditional contribution schedules that induce the governments to choose protection according to the true welfare cost. That is, to screen the rival’s information from the policy maker. Yet screening is costly and generates distortions on the political game. These distortions are what we call “the information transmission effect”.

Before clearly presenting these effects, we must discuss some technical issues about our approach.

We restrict the analysis symmetric Perfect Bayesian Equilibrium (equilibrium, in short) of the political game and we focus on separating contribution schedules. To find the optimal contribution schedules, we proceed as follows: we take as given
the rival’s offer \( C^{-n} (\theta^{-n}, p^{-n}) \) and assume that it is separating, increasing in the rival’s type and that

\[
U (\theta_i, p, C, \theta_{hi}, p_{hi}, C_{hi}) \geq U (\theta_i, p, C, \theta_{li}, p_{li}, C_{li}).
\]

This condition implies that the difference between the utility of high and low type rivals is not greater than the surplus increase between the two states of nature.

Then, we approach the lobby’s choice of contribution schedules as a principal-agent problem. In turn, we present the lobby’s utility maximization problem subject to the information transmission constraints, which we refer to as “the informed lobby problem”. For further details about this approach see Costa Lima and Moreira (2008).

Once the rival’s offer is separating, the policy maker will learn the rival’s type when he receives her offer, before the implementation of the policy. Therefore, the lobby can screen this information from him. In order to do that, a lobby has to offer a conditional contribution that imposes incentive compatibility on the policy maker.

The incentive compatibility constraints for the policy maker ensures that he will choose the import tariff according to the true marginal of protection, that is, according to the true type of the rival. When the type-\( i \) lobby faces a high type rival, we must have

\[
C_{ih} + C_{hi} + \lambda W_A (\theta_i, p_{ih}, p_{hi}) + \lambda W_B (\theta_h, p_{hi}, p_{ih}) \geq C_{il} + C_{li} + \lambda W_A (\theta_i, p_{il}, p_{hi}) + \lambda W_B (\theta_h, p_{hi}, p_{ih}) \quad (IC_{ih})
\]

and when the type-\( i \) lobby faces a low type opponent we must have

\[
C_{il} + C_{li} + \lambda W_A (\theta_i, p_{il}, p_{li}) + \lambda W_B (\theta_i, p_{li}, p_{ih}) \geq C_{ih} + C_{hi} + \lambda W_A (\theta_i, p_{ih}, p_{hi}) + \lambda W_B (\theta_i, p_{li}, p_{ih}) \quad (IC_{il})
\]

The policy maker’s individual rationality constraints are given by \((IR_{ik})\) for each \( k \).

Now we can present the informed lobby problem.

**The type-\( i \) informed lobby problem**

\[
\max_{p_{ih}, p_{il}} E [\theta_i p_i - C_i] \quad \text{(informed lobby’s problem)}
\]

subject to \((IC_{ik}), (IR_{ik})\) and \( C_{ik} \geq 0, \) for all \( k \).

We need to identify which of these constraints are binding on the informed lobby problem. The lobbies’ goods are substitutes \((d > 0)\), which implies that \( \frac{\partial^2 W}{\partial p_i \partial p_j} = d \). So the marginal cost of protection for lobby 1 decreases with the protection of lobby 2. Therefore, lobby 1 prefers to face a high type rival because high types demand more protection, which reduces the marginal cost of her protection. On the other hand, in the absence of some compensation, the policy maker will be
prompt to lie and choose lobby 1’s policy as if the marginal cost of protection were high (i.e., the rival is low type). Therefore, the binding incentive compatibility constraint must be \((IC_{ih})\) and the binding individual rationality constraint must be \((IR_{il})\). \(^3\)

With these constraints we can eliminate the contributions from the informed lobby’s problem which becomes

\[
\max_{p_{ih}, p_{il}} z [\theta_i p_{ih} + \lambda (W (\theta_i, p_{ih}, \theta_h, p_{hi}) - W (\theta_i, p_{il}, \theta_i, p_{li}) + W (\theta_i, p_{il}, \theta_i, p_{li}))]
\]

\[
+ (1 - z) [\theta_i p_{il} + \lambda W (\theta_i, p_{il}, \theta_i, p_{li})] - \bar{U} (\theta_i, \theta_i) + C_{li}
\]

where \(W (\theta_i, p_{ik}, \theta_k, p_{ki}) = W_A (\theta_i, p_{ik}, p_{ki}) + W_B (\theta_k, p_{ki}, p_{ik})\).

Taking the derivatives with respect to the policies gives the following:

**Lemma 1.** The first-order conditions of the informed lobby problem are given by

\[
\theta_i - 2\lambda b (p_{ih} - p^e) + 2\lambda d (p_{hi} - p^e) = 0 \tag{4.2}
\]

\[
\theta_i - 2\lambda b (p_{il} - p^e) + 2\lambda d (p_{li} - p^e) + 2\lambda d \frac{z}{1 - z} [(p_{li} - p^e) - (p_{hi} - p^e)] = 0. \tag{4.3}
\]

The first two terms of (4.2) and (4.3) refer to the trade-off between the marginal cost of the policy and the lobby’s benefit from protection. The last term of (4.3), inside brackets, comes from the informational rent the lobby has to leave to the policy maker in order to give him the proper incentives. It makes a lobby demand less protection when she faces a low type opponent in order to reduce the policy maker’s payoff of “lying” when the rival is high type. Therefore, the binding informational constraint makes the lobby demand less protection than she would under perfect information.

Figure 3 helps us to explain this information transmission effect (for simplicity, we set \(p^e = 0\)).

\(^3\)If the utility of the policy maker is not increasing in the rival lobby’s type, we can have equilibrium with countervailing incentives as shown in Costa Lima and Moreira (2008).
Figure 3: The screening effect

When lobby 1 increases her demand for protection, say from \( p^1 \) to \( p^1 + \Delta \), by substitutability, the demand for good 2 shifts upwards as shown on the right side of Figure 3. This shift in the demand gives the policy maker an increase in the tariff revenue given by the two grey rectangles in market 2. Notice that the size of this revenue increase depends on the price (protection) in market 2. If this price \( (p^2_h) \) is big, then the revenue increase is big as shown by the darker rectangle in market 2. If the price \( (p^2_l) \) is small, the revenue increase is small as shown by the lighter rectangle in market 2. In a perfect information context, the lobby anticipates the price in market 2 and deducts the revenue increase from her contributions. However, with information asymmetries, the lobby has to screen this information from the policy maker, distorting her demand for protection.

The equilibrium prices are those that simultaneously solve the condition informed lobby’s problem for the two lobbies and for all possible type \( i \). We then have:

**Proposition 2.** There exists a unique symmetric equilibrium of the political game with informed lobbies. Moreover, the equilibrium prices are given by

\[
\begin{align*}
    p^*_{hh} &= \bar{p}_{hh} - \frac{zb^2d(\theta_h - \theta_l)}{2\lambda((1 - z)b^2 - d^2)(b^2 - d^2)} \\
    p^*_{hl} &= \bar{p}_{hl} - \frac{zb^2d(\theta_h - \theta_l)}{2\lambda((1 - z)b^2 - d^2)(b^2 - d^2)} \\
    p^*_{lh} &= \bar{p}_{lh} - \frac{zb^2d(\theta_h - \theta_l)}{2\lambda((1 - z)b^2 - d^2)(b^2 - d^2)} \\
    p^*_{ll} &= \bar{p}_{ll} - \frac{zb^2d(\theta_h - \theta_l)}{2\lambda((1 - z)b^2 - d^2)(b^2 - d^2)}
\end{align*}
\]
where the uppercase * refers to the equilibrium prices in a customs union. The equilibrium contributions are characterized by the binding constraint \((IC_{ih})\) and \((IR_{il})\) for the equilibrium prices.

Notice that prices decrease when compared to the prices in a customs union under perfect information, except when both lobbies have high type. The lobbies distort their demand for protection whenever they face a low type opponent, then \(p_{ll}^* < \hat{p}_{ll}\) and \(p_{hl}^* < \hat{p}_{hl}\); \(p_{lh}^*\) decreases due to strategic complementarity. From Proposition 2 we have the following:

**Corollary 1.** In a customs union, the equilibrium of the political game with informed lobbies is welfare superior to the equilibrium of the political game with perfect information.

This corollary shows that information transmission increases the benefits of a customs union. In this regime, the policy maker has the ability to use the information of one lobby against the other reducing the lobbies’ power to extract rents. Since lobbying is harmful for the society, information transmission is welfare improving.

Now we can compare the prices in a customs union and in the unilateral tariff setting:

\[
\begin{align*}
\hat{p}_h & > p_{hh}^* > \hat{p}_h \\
\hat{p}_l & > p_{lh}^* \\
\hat{p}_h & > p_{hl}^* \\
\hat{p}_l & > p_{ll}^*.
\end{align*}
\]

We can see that when a lobby with high type lobby faces a high type rival, she does not distort her demand for protection and the ordering of the tariffs remains the same as in the perfect information case. The other inequalities show that information transmission makes the lobbies distort downward their protection and the sectors that free ride may receive less protection in a customs union.

5. Welfare

In this section we compare the welfare of the two regimes. In our particular framework the welfare effects are due to the differences between the political game under the two regimes. We begin comparing the two regimes under perfect information.

In the unilateral regime lobbies have more protection than in the customs union because in the first they only have to compensate for the welfare loss of one economy. Therefore, a customs union reduces the relative weight of the lobby’s profits compared to the welfare costs of protection. On the other hand, the sectors that do not have a lobby receive less protection in the unilateral regime than in
the customs union because they free-ride on the protection demanded by the lobby of the other country.

Hence, a customs union can increase protection for politically weak sectors and decrease protection of politically stronger sectors. The proposition below shows that the second effect dominates.

**Proposition 3.** Under perfect information, the welfare of a customs union is greater than the welfare of the unilateral regime.

Proposition 3 shows that the preference-dilution dominates the free-riding effect. This means that, when tariffs are endogenously determined by lobbying and not all countries’ sectors have lobby, a customs union changes the relative forces on the political game and makes countries, on average, more open to international trade than in the unilateral regime.

From Corollary 1, we know that in a customs union the solution of the political game with informed lobbies is welfare superior to the equilibrium under perfect information. From Proposition 1 we also know that in the unilateral regime, the solution of the political game is the same with or without informed lobbies. Therefore, we have the following:

**Corollary 2.** With privately informed lobbies, the welfare of a customs union is greater than the welfare of the unilateral regime.

Corollary 2 shows that information transmission makes customs unions even more beneficial for the two countries decreasing their protection towards non-members even more than in the perfect information case. Therefore, the strategic use of the lobby’s private information is an additional benefit of this trade agreement.

Of course, given the simplicity of our framework, we cannot generalize our results and say that customs unions are always welfare improving. Indeed, this may not be the case for some of these agreements that are implemented. However, what we do want to stress is that the agreement’s effect on the political game is more beneficial for the countries when lobbies have private information.

**6. Blocking the agreement**

In this section we investigate the incentives for the creation of a customs union. We introduce a previous stage in the game that allows the governments to choose the regime before implementing the tariffs and the lobbies to offer contributions to support one of the regimes. The choice of the regime is binary: each government decides whether or not to join the customs union. If both governments decide to join the customs union, it is created. The lobbies offer money contributions to the government of her country to swing their decision to join or not the customs union. Given the chosen regime, lobbies offer contributions to influence the choice of import tariffs.
Since the choice of the regime is binary, the lobbies’ contribution in this previous stage is a positive money transfer ($\Delta$) in favor of the regime she prefers.

The timing of this new game is given by:

0) lobbies offer contributions pro, or against the trade agreement;
1) governments choose to join or not the customs union;
2) nature draws the lobbies’ types;
3) lobbies offer contributions to influence the choice of import tariffs;
4) policy maker(s) accept(s) or reject(s) the contributions;
5) tariffs are chosen and, if contributions are accepted, payments are made accordingly.

For simplicity, we assume that nature only draws the lobbies’ types after the choice of the regime. This assumption eliminates the possibility of information revelation in the first stage of contributions for the regime. If otherwise, lobbies could reveal information in this stage and the game would become quite complex. More importantly, we would not be able to compare the results of this section with the results of the previous sections.

With this timing, we only have to analyze the first stage of the game because, once the regime is chosen, the game unfolds as in the previous sections. The government prefers not to join the customs union if his utility in this regime is smaller than his utility under the unilateral regime. We assume that, once in a customs union, the governments share the benefits equally, since the governments are ex ante symmetric. We have that

$$\Delta^* + E[C^* + \lambda W_A (\theta, p^*, p^*)] \leq \hat{\Delta} + E[\hat{C}_{ik} + \lambda W_A (\theta, \hat{p}, \hat{p})]$$

where the expectation is taken in both type (thus, we omit the indexes). The “hat” refers to the policies of the unilateral regime while the asterisk refers to the policies of the customs union. $\Delta^* (\hat{\Delta})$ is the contribution in favor of (against) the customs union. We can rewrite this inequality as

$$\hat{\Delta} - \Delta^* \geq E[C^* - \hat{C} + \lambda W_A (\theta, p^*, p^*) - \lambda W_A (\theta, \hat{p}, \hat{p})].$$

We know that lobbies have less protection under a customs union. Thus, they are better off when their governments choose not to join this regime and, therefore, $\Delta^* = 0$. A lobby’s willingness to contribute against the customs union is at most the difference in the profits she receives in the two regimes, that is,

$$E[\theta \hat{p} - \hat{C}] - \hat{\Delta} \geq E[\theta p^* - C^*]$$

which we rewrite as

$$\hat{\Delta} \leq E[\theta (\hat{p} - p^*) - (\hat{C} - C^*)].$$
Therefore, the regime will be blocked by lobbies if
\[ E \left[ \theta (\hat{p} - p^*) - \left( \hat{C} - C^* \right) \right] \geq E \left[ C^* - \hat{C} + \lambda W_A (\theta, p^*, \hat{p}) - \lambda W_A (\theta, \hat{p}, \hat{p}) \right] \]

or
\[ E [\theta (\hat{p} - p^*)] \geq \lambda E \left[ W_A (\theta, p^*, \hat{p}) - W_A (\theta, \hat{p}, \hat{p}) \right] \quad (6.1) \]

which simply states that if the loss in the lobby’s surplus due to the agreement is greater (in absolute value) to the welfare increase times \( \lambda \), the lobby blocks the agreement. As we did in the previous sections, we begin analyzing the perfect information case. We have the following:

**Proposition 4.** Under perfect information, a customs union is always blocked by lobbies.

Proposition 4 states that the profit loss the lobbies incur in a customs union is always greater than the contribution necessary to swing the governments decision away from the agreement under perfect information.

When the lobbies are privately informed, information transmission plays a role in this game and in some cases the governments choose to joint the agreement.

**Proposition 5.** Let \( z \) be equal to \( \frac{b - d}{b} \). Then, there exists \( \hat{d} \) such that if \( d > \hat{d} \) then lobbies cannot block a customs union.

The intuition for the Proposition 6 is the following. We set \( z \) to be the maximum value that ensures interior solutions and reduce the difference between \( b \) and \( d \). As \( d \) increases relatively to \( b \), the preference-dilution effect decreases because the protection of the rival has a greater impact in reducing the lobby’s marginal cost of protection. Moreover, the distortions of the information transmission effect are bigger since the substitutability’s relative weight (\( d \)) is bigger. This increases the welfare of a customs union and implies in greater costs to block the agreement. Both effects make the customs union more likely to be politically viable.

The simplicity of our model also does not allow us to draw sharp conclusion about the qualitative properties of customs unions that are actually implemented. The lobbies may have incentive to support agreements that are trade diverting as pointed out by Grossman and Helpman (1995) and Krishna (1998). However, what we point out is the fact that information transmission makes the customs unions more beneficial and, in turn, more likely to be supported. Hence, it reduces the possibility of implementing a welfare decreasing customs union.

7. Conclusion

We analyzed the information transmission effect in a political game of customs unions when lobbies have private information about the competitiveness of the sectors they represent.

The information transmission effect arises in customs unions because the cost of protection for a lobby depends on the size of the protection granted for lobbies
in other markets since goods are substitutes. However, a lobby does not know the competitiveness of the lobbies from the other country. Hence, she cannot anticipate which will be the protection in other sectors. In turn, this implies that she does not know the marginal cost of her own protection.

Although the lobby cannot anticipate the protection of the other markets, she knows that the policy maker learns the foreign lobbies’ information before implementing the policies. Therefore, from the point of view of the lobby, the rival’s private information becomes the policy maker’s “private information”. Hence, the lobby screens the rival’s information from the policy maker. Screening gives informational rents to the governments and distorts the lobbies’ demands for protection downward.

On the other hand, if such agreement is not reached, governments choose tariffs separately and the political game is an informed principal problem. In this case, the information asymmetry does not generate distortions because lobbies reveal their information without any cost. Thus, protection is as predicted by Grossman and Helpman (1994): sectors that lobbies are protected and sectors that do not lobby receive the tariff that accommodates the protection granted and the society bears the welfare cost of protection.

We found that the information transmission effect increases the welfare of a customs union because it reduces the protection of countries towards non-members. We also found that this effect adds up with the free riding effect identified by Richardson (1993) and makes the welfare under a customs union greater than without agreement.

Moreover, we investigated the incentives for the creation of customs unions. We introduced a previous stage in the game where lobbies offer contributions pro or against the agreement; and then the governments choose to join or not the customs union. We found that, under perfect information, the agreement is never politically sustainable, while with the information transmission effect the results can be reversed. Hence, information transmission can make customs unions politically sustainable.

Therefore, the information transmission effect may favor customs union agreements. The effect also points to the fact that, when information asymmetries are important for politics, then the changes on institutions must take into account the informational distortions that may be created or eliminated from the decision making process.

Appendix

Proof of Proposition 1. A lobby does not lie about her type if

\[ V(\theta_i, p(\theta_i), C(p(\theta_i), \theta_i)) \geq V(\theta, p(\theta_{-i}), C(p(\theta_{-i}), \theta_{-i})) \]

where \(-i \neq i\).
This inequality can be artificially written as
\[
\int_{\theta_i}^{\theta_{-i}} \frac{\partial V}{\partial \tilde{\theta}} \left( \theta_i, p \left( \tilde{\theta} \right), C \left( p \left( \tilde{\theta} \right), \tilde{\theta} \right) \right) \, d\tilde{\theta} \geq 0. \tag{7.1}
\]

Using that constraints \((IR_i)\) are binding, we have
\[
C_i = -\lambda W_A \left( \tilde{\theta}_i, p_i, p_i \right) + \lambda W_A \left( \tilde{\theta}_i, p^e, p^e \right).
\]

We also use that
\[
\frac{\partial V}{\partial \tilde{\theta}} \left( \theta_i, p_i, C \left( p_i, \tilde{\theta} \right) \right) = \theta_i \frac{\partial p_i}{\partial \tilde{\theta}} - \frac{\partial C_i}{\partial \tilde{\theta}} - \frac{\partial C_i}{\partial p_i} \frac{\partial p_i}{\partial \tilde{\theta}}.
\]

Applying the envelope theorem we have that \(\tilde{\theta} = \frac{\partial C_i}{\partial p_i} \frac{\partial p_i}{\partial \tilde{\theta}}\). Hence, the above derivatives of the lobby’s utility simplify to
\[
\frac{\partial V}{\partial \tilde{\theta}} \left( \theta_i, p_i, C \left( p_i, \tilde{\theta} \right) \right) = \left( \theta_i - \tilde{\theta} \right) \frac{\partial p_i}{\partial \tilde{\theta}} - \frac{\partial C_i}{\partial \tilde{\theta}}.
\]

Therefore, condition (7.1) becomes
\[
\int_{\theta_i}^{\theta_{-i}} \left[ \left( \theta_i - \tilde{\theta} \right) \frac{\partial p_i}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) \right] \, d\tilde{\theta} \geq 0. \tag{7.2}
\]

Again, from the constraint \((IR_{il})\) we have
\[
\frac{\partial C_{il}}{\partial \tilde{\theta}} = \lambda \frac{\partial W_A}{\partial \tilde{\theta}} \left( \tilde{\theta}, p^e, p^e \right) - \lambda \frac{\partial W_A}{\partial \tilde{\theta}} \left( \tilde{\theta}, p_i, p_i \right).
\]

However, using (2.2) gives \(\frac{\partial C_{il}}{\partial \tilde{\theta}} = 0\). Therefore, (7.2) becomes
\[
\int_{\theta_i}^{\theta_{-i}} \left( \theta_i - \tilde{\theta} \right) \frac{\partial p_i}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) \, d\tilde{\theta} \geq 0.
\]

Since this is a discrete type model there is no intermediate values for \(\tilde{\theta}\). However, if we assume that \(p \left( \tilde{\theta} \right)\) is a linear function that links \(p \left( \theta_i \right)\) and \(p \left( \theta_{-i} \right)\), then \(\frac{\partial p_i}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) \geq 0\) since the price is increasing in the lobby’s type. Hence, the above inequality holds because if \(\theta_i > \theta_{-i}\) then \(\left( \theta_i - \tilde{\theta} \right) \frac{\partial p_i}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) \geq 0\), while if \(\theta_{-i} > \theta_i\) then \(\left( \tilde{\theta} - \theta_i \right) \frac{\partial p_i}{\partial \tilde{\theta}} \left( \tilde{\theta} \right) \, d\tilde{\theta} \geq 0\). Therefore, the lobby is always better-off telling the truth. Therefore, she reveals her type without any cost. \(\square\)

**Proof of Lemma 1.** When the constraints \((IC_{il})\) and \((IR_{il})\) are binding we have that
\[
C_{il} = -C_{li} - \lambda W_A \left( \theta_i, p_{il}, p_{il} \right) - \lambda W_B \left( \theta_i, p_{il}, p_{il} \right) + \lambda W_A \left( \theta_i, p^e, p^e \right) + \lambda W_B \left( \theta_i, p^e, p^e \right) \tag{7.3}
\]
and
\[
C_{ih} = C_{il} - \lambda W_A \left( \theta_i, p_{ih}, p_{hi} \right) - \lambda W_B \left( \theta_h, p_{ih}, p_{hi} \right) + \lambda W_A \left( \theta_i, p_{il}, p_{hi} \right) + \lambda W_B \left( \theta_l, p_{ih}, p_{il} \right). 
\]

We plug these contributions into the lobby’s utility function and the problem informed lobby’s problem becomes

\[
\max_{p_{ih}, p_{li}} z \left[ \theta_i p_{ih} + \lambda \left( W_A \left( \theta_i, p_{ih}, p_{hi} \right) + W_B \left( \theta_h, p_{ih}, p_{hi} \right) - W_A \left( \theta_i, p_{il}, p_{hi} \right) - W_B \left( \theta_l, p_{ih}, p_{il} \right) \right) \right. 
+ \lambda \left( W_A \left( \theta_i, p_{il}, p_{hi} \right) + W_B \left( \theta_l, p_{il}, p_{hi} \right) - W_A \left( \theta_i, p^e, p^c \right) - W_B \left( \theta_l, p^e, p^c \right) \right) + C_{li} 
\left. \right] + (1 - z) \left[ \theta_i p_{il} + \lambda \left( W_A \left( \theta_i, p_{il}, p_{hi} \right) + W_B \left( \theta_l, p_{il}, p_{il} \right) - W_A \left( \theta_i, p^e, p^c \right) - W_B \left( \theta_l, p^e, p^c \right) \right) + C_{li} \right].
\]

The first-order conditions of this problem are given by

\[
z \left[ \theta_i p_{ih} - 2 \lambda b \left( p_{ih} - p^e \right) + 2 \lambda d \left( p_{hi} - p^e \right) \right] = 0
\]

\[
(1 - z) \left[ \theta_i p_{il} - 2 \lambda b \left( p_{il} - p^e \right) + 2 \lambda d \left( p_{li} - p^e \right) \right] + z \left[ 2 \lambda d \left( p_{li} - p^e \right) - 2 \lambda d \left( p_{hi} - p^e \right) \right] = 0.
\]

Dividing the first-order conditions, respectively, by \( z \) and \( (1 - z) \) gives (4.2) and (4.3).

**Proof of Proposition 2.** The first-order conditions (4.2) and (4.3) for the problems of lobby 1 and 2 constitute a system of linear equations that can be written in matrix form.

When both lobbies are high type (state \( \{\theta_h, \theta_h\} \)), we have

\[
\begin{bmatrix}
-2 \lambda b \\
2 \lambda d
\end{bmatrix}
\begin{bmatrix}
p_{ih}^1 - p^e \\
p_{hi}^1 - p^e
\end{bmatrix} = \begin{bmatrix}
- \theta_h \\
- \theta_h
\end{bmatrix}
\]

and a solution always exists since, by Assumption 1, the determinant of the coefficient matrix is \( 4 \left( \lambda b \right)^2 - 4 \left( \lambda d \right)^2 > 0 \).

Given the solution of the system in this state, we can find policies for the state \( \{\theta_h, \theta_l\} \):

\[
\begin{bmatrix}
-2 \lambda b \left( 1 - z \right) \\
2 \lambda d
\end{bmatrix}
\begin{bmatrix}
p_{il}^1 - p^e \\
p_{hi}^2 - p^e
\end{bmatrix} = \begin{bmatrix}
- \theta_h \left( 1 - z \right) + 2 z \lambda d \left( p_{ih}^* - p^e \right) \\
- \theta_l \left( 1 - z \right)
\end{bmatrix}
\]

because the coefficient matrix has a positive determinant for the same reason as before. We also have a symmetric system for state \( \{\theta_l, \theta_h\} \).

Given the solution of these systems, we can find the policies for state \( \{\theta_l, \theta_l\} \):

\[
\begin{bmatrix}
-2 \lambda b \left( 1 - z \right) \\
2 \lambda d
\end{bmatrix}
\begin{bmatrix}
p_{il}^2 - p^e \\
p_{hi}^* - p^e
\end{bmatrix} = \begin{bmatrix}
- \theta_l \left( 1 - z \right) + 2 z \lambda d \left( p_{il}^* - p^e \right) \\
- \theta_l \left( 1 - z \right) + 2 z \lambda d \left( p_{hi}^* - p^e \right)
\end{bmatrix},
\]

because the determinant of the coefficients matrix is \( 4 \left( \lambda \left( 1 - z \right) b \right)^2 - 4 \left( \lambda d \right)^2 > 0 \) since \( (1 - z) b > d \) (by Assumption 1).
We find policies for the case where both lobbies have high types. We begin computing the best-response functions, which are given by

\[ p^1_{hh} = f_{hh}(p^2_{hh}) = p^e + \frac{\theta_h + 2\lambda d(p^2_{hh} - p^e)}{2\lambda} \]

\[ p^2_{hh} = f_{hh}(p^1_{hh}) = p^e + \frac{\theta_h + 2\lambda d(p^1_{hh} - p^e)}{2\lambda}. \]

Then we substitute one function into the other to get:

\[ p^*_{hh} = f_{hh}(f_{hh}(p^*_{hh})) = p^e + \frac{\theta_h + 2\lambda d(p^*_{hh} - p^e)}{2\lambda} \]

that can be rearranged to

\[ p^*_{hh} - p^e = \frac{2\lambda b\theta_h + 2\lambda d\theta_h + 4(\lambda d)^2(p^*_{hh} - p^e)}{4(\lambda b)^2} \]

and we get

\[ (p^*_{hh} - p^e) 4\lambda^2 \left(b^2 - d^2\right) = \lambda 2 \left(b\theta_h + d\theta_h\right) \Leftrightarrow p^*_{hh} - p^e = \frac{b\theta_h + d\theta_h}{2\lambda (b^2 - d^2)}. \]

Finally, we have that

\[ p^*_{hh} = \frac{b\theta_h + d\theta_h}{2\lambda (b^2 - d^2)} + p^e = \bar{p}_{hh}. \]

The prices in the other states are found in the same way.

The contributions are obtained by substituting the equilibrium policies into (7.3) and (7.4).

**Proof of Corollary 1.** Since preferences are quasi-linear, in order to compare the welfare of the political game with and without private information, we only need to compare prices.

The welfare of the customs union in the state \( \{\theta_h, \theta_h\} \) is the same for the perfect and private information cases, since prices are the same. The prices in the other states can be written as

\[ p^*_h = \frac{\theta_h b \left((1-z)b-d\right) + \theta_h d}{2\lambda \left((1-z)b^2 - d^2\right)} + p^e < \bar{p}_{hl} \]

\[ p^*_l = \frac{\theta_l \left((1-z)b + d\right) - \theta_h d \left(\frac{b}{b-d}\right)}{2\lambda \left((1-z)b^2 - d^2\right)} + p^e < \bar{p}_{ll} \]

The maximum welfare for the society is the free trade. Thus, when lobbies receive positive protection, the welfare decreases. This means that \( \frac{\partial W}{\partial p^*} (\theta^p, p^*, \theta^-) < 0 \)
0 for \( p^n > p^e \). From the above formulas, we know that \( \bar{p}_{hl} > p^*_h > p^e \) and \( \bar{p}_{lh} > p^*_l > p^e \). Therefore, the welfare with informed lobbies in state \( \{\theta_h, \theta_l\} \) is greater than under perfect information. In state \( \{\theta_l, \theta_l\} \) prices are also below the truthful prices, but they may fall below the international prices and the welfare can decrease compared to the perfect information case. Thus, we must have

\[
W_A(\theta_l, \bar{p}_{hl}, \bar{p}_{lh}) + W_B(\theta_l, \bar{p}_{hl}, \bar{p}_{lh}) < W_A(\theta_l, p^*_h, p^*_l) + W_B(\theta_l, p^*_h, p^*_l).
\]

This last inequality is equivalent to

\[
(\bar{p}_{hl} - p^e)^2 (b - d) - (p^*_l - p^e)^2 (b - d) > 0.
\]

Since \( p^*_h < \bar{p}_{hl} \), we must have

\[
(\bar{p}_{hl} - p^e)^2 > (p^*_l - p^e)^2
\]

which simplifies to

\[
\bar{p}_{hl} - p^e > p^e - p^*_h.
\]

Substituting the formulas for these prices, we get:

\[
\frac{\theta_l (b + d)}{2\lambda (b^2 - d^2)} > \frac{-\theta_l (b + d)}{2\lambda (b^2 - d^2)} + \frac{zbd (\theta_h - \theta_l) (b + d)}{2\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}
\]

which we can rewrite as

\[
\frac{\theta_l (b + d)}{\lambda (b^2 - d^2)} > \frac{zbd (\theta_h - \theta_l) (b + d)}{2\lambda ((1 - z) b^2 - d^2) (b^2 - d^2)}
\]

that, after some algebra, simplifies to

\[
\frac{2 ((1 - z) b^2 - d^2)}{zbd} > \frac{\theta_h - \theta_l}{\theta_l}.
\]

By Assumption 3, the welfare of equilibrium of the political game with informed lobbies is greater than the welfare of equilibrium of the political game with perfect information. \( \square \)

**Proof of Proposition 3.** The economic welfare in one country is given by

\[
W_A(\theta_i, p_{ik}, p_{ki}) = (b - d) \left( \frac{a}{b - d} - p^e \right)^2 - \frac{b}{2} \left[ (p_{ik} - p^e)^2 + (p_{ki} - p^e)^2 \right] + d \left( p_{ik} - p^e \right) \left( p_{ki} - p^e \right) + \left( \theta_i + \bar{\theta} \right) p^e.
\]

Therefore, we have that

\[
W_A(\theta_i, \bar{p}_{ik}, \bar{p}_{ki}) - W_A(\theta_i, \bar{p}_{ik}, \bar{p}_{ki}) = -\frac{b}{2} \left[ (\bar{p}_{ik} - p^e)^2 + (\bar{p}_{ki} - p^e)^2 \right] + d \left( \bar{p}_{ik} - p^e \right) \left( \bar{p}_{ki} - p^e \right) + \frac{b}{2} \left[ (\bar{p}_{ik} - p^e)^2 + (\bar{p}_{ki} - p^e)^2 \right] - d \left( \bar{p}_{ik} - p^e \right) \left( \bar{p}_{ki} - p^e \right)
\]
which, substituting the equilibrium formulas of the policies, simplifies to

\[ W_A(\theta_i, \hat{p}_{ik}, \tilde{p}_{ki}) - W_A(\theta_i, \hat{p}_{ik}, \tilde{p}_{ki}) = \frac{\theta_i}{4\lambda} (2(\hat{p}_i - p^e) - (p_{ik} - p^e)) - \frac{\theta_k}{4\lambda} (p_{ki} - p^e). \]

Taking the expected value both in \( i \) and \( k \), we have

\[ E[W_A(\theta, \hat{p}, \tilde{p}) - W_A(\theta, \hat{p}, \tilde{p})] = E\left[\frac{\theta}{2\lambda} ((\hat{p} - p^e) - (\tilde{p} - p^e))\right] > 0 \tag{7.5} \]

which proves the result.

\[ \square \]

**Proof of Corollary 2.** From Proposition 4 we have that

\[ W_A(\theta_i, \hat{p}_{ik}, \tilde{p}_{ki}) + W_B(\theta_k, \tilde{p}_{ki}, \hat{p}_{ik}) < W_A(\theta_i, \hat{p}_{ik}, \tilde{p}_{ki}) + W_B(\theta_k, \tilde{p}_{ki}, \hat{p}_{ik}) \]

and from Corollary 1 we have that

\[ W_A(\theta_i, \tilde{p}_{ik}, \hat{p}_{ki}) + W_B(\theta_k, \hat{p}_{ki}, \tilde{p}_{ik}) < W_A(\theta_i, \tilde{p}_{ik}, \hat{p}_{ki}) + W_B(\theta_k, \hat{p}_{ki}, \tilde{p}_{ik}). \]

Hence, it follows that

\[ W_A(\theta_i, \tilde{p}_{ik}, \hat{p}_{ki}) + W_B(\theta_k, \hat{p}_{ki}, \tilde{p}_{ik}) < W_A(\theta_i, \tilde{p}_{ik}, \hat{p}_{ki}) + W_B(\theta_k, \tilde{p}_{ki}, \hat{p}_{ik}). \]

\[ \square \]

**Proof of Proposition 4.** Substituting (7.5) into (6.1) gives us

\[ E[\theta ((\hat{p} - p^e) - (\tilde{p} - p^e))] > \lambda E\left[\frac{\theta}{2\lambda} ((\hat{p} - p^e) - (\tilde{p} - p^e))\right] \tag{7.6} \]

which simplifies to

\[ E\left[\frac{\theta}{2} ((\hat{p} - p^e) - (\tilde{p} - p^e))\right] > 0 \]

which holds trivially. Therefore, the lobbies’ surplus loss is greater than the surplus increase of a customs union, thus they are always willing to block the agreement.

\[ \square \]

**Proof of Proposition 5.** We will show that (6.1) does not hold when \( z = \frac{b-d}{b} \) and \( d \) tends to \( b \). To do that, we need to compare the expected welfare increase of a customs union with the expected lobbies’ profit loss.

The difference between the expected welfare of the two regimes is given by

\[ E[W_A(\theta, \tilde{p}^e, p^e) - W_A(\theta, \hat{p}, \tilde{p})] = \frac{b}{2} E \left[ (\hat{p} - p^e)^2 + (\tilde{p} - p^e)^2 + (p^e - p^e)^2 \right] + dE \left[ (p^e - p^e)(\hat{p} - p^e)(\hat{p} - p^e) \right]. \]
Substituting the policies, this becomes

\[
E \left[ W_A (\theta, p^*, p^*) \right] - W_A (\theta, \tilde{p}, \tilde{p}) = E \left[ W_A (\theta, \tilde{p}, \tilde{p}) - W_A (\theta, \hat{p}, \tilde{p}) \right] + \frac{z \delta \left( \frac{b^2 + d^2}{b^2 - d^2} \right)}{2 \lambda \left( 1 + (1 - z) (1 - z) \delta \left( \frac{b + d}{b^2 - d^2} \right) - (b + d) \left( b^2 - d^2 \right) \right)}
\]

where \( \delta = \frac{z b d (\theta_h - \theta_l)}{2 \lambda (1 - z) (b^2 - d^2)} \).

The lobbies’s profit loss is given by

\[
E \left[ \theta (\tilde{p} - p^*) \right].
\]

Substituting (7.7) and (7.8) into (6.1) gives

\[
E \left[ \theta (\hat{p} - p^*) \right] \leq \frac{\lambda E \left[ W_A (\theta, \tilde{p}, \tilde{p}) - W_A (\theta, \hat{p}, \tilde{p}) \right]}{(1 - z) \delta \left( \frac{b + d}{b^2 - d^2} \right) - (b + d) \left( b^2 - d^2 \right)}
\]

which is ambiguous. If the right-hand side is greater than the left-hand side, then the welfare increase of a customs union is big enough and lobbies cannot block the agreement. If the left-hand side is greater than the right-hand side, then lobby’s loss is big enough and they can dissuade the governments to join the customs union.

Assuming that \( z = \frac{b - d}{b} \), the above expression becomes

\[
\left( \frac{b - d}{b} \right)^2 \frac{\theta_h^2 (b - d)}{4 \lambda (b^2 - d^2)} + \frac{d (b - d)}{b^2} \frac{\theta_h^2 b - \theta_h \theta_l d}{4 \lambda (b^2 - d^2)} + \frac{d (b - d)}{b^2} \frac{\theta_l^2 b - \theta_l d \theta_h}{4 \lambda (b^2 - d^2)} + \frac{d^2 (b - d)}{b^2} \frac{\theta_l^2 (b - d)}{4 \lambda (b^2 - d^2)}
\]

\[
\frac{d (b - d)}{b^2} \frac{\theta_h - \theta_l}{2 \lambda (b + d)} \left( \frac{2 b d \theta_h + d \theta_l}{b^2 - d^2} - \frac{(\theta_h - \theta_l) (b - d)}{2 \lambda} \right)
\]

\[
+ \frac{d^2}{b^2} \frac{\theta_h - \theta_l}{2 \lambda (b + d)} \left( \frac{2 \theta_l}{b - d} - \frac{(\theta_h - \theta_l) (b^2 - d^2)}{2 \lambda} \right)
\]
that can be rewritten as
\[(b - d)^3 \frac{\theta_h^2}{4\lambda} + d (b - d) \frac{\theta_h}{4\lambda} (b \theta_h - d \theta_l) + d (b - d) \frac{\theta_l}{4\lambda} (\theta_l - d \theta_h) + d^2 \frac{\theta_l^2}{2} (b - d) \geq \frac{d (b - d) (\theta_h - \theta_l)}{2\lambda} \left( \frac{2b \theta_h + d \theta_l}{b + d} - \frac{(\theta_h - \theta_l) (b - d)}{2\lambda} \right)
+ d^2 \frac{(\theta_h - \theta_l)}{2\lambda} \left( 2\theta_l - \frac{(\theta_h - \theta_l)}{2\lambda} (b + d) (b - d)^2 \right).
\]

When \(d \to b\) the left-hand side of the inequality tends to zero, while the right-hand side tends to \(d^2 \frac{(\theta_h - \theta_l)}{2\lambda} 2\theta_l\). Therefore, there exists \(\hat{d} \in (0, b)\), such that, if \(d > \hat{d}\), then the right-hand side is greater than the left-hand side and the agreement cannot be blocked. □
Bibliography


Part 2

Private costs and the retailing structure
CHAPTER 3

Exclusive dealing with informed manufacturers

1. Introduction

Vertical relationships between manufacturers and retailers are often plagued with information asymmetries. This informational problems can play a key role in determining the structure of the market, for the cost of providing incentives depends on the simultaneous actions of other market participants.

In this paper I compare two different retailing structures. In the first one, manufacturers choose sell their products through an exclusive retailer. This practice has been called exclusive dealing. The second retailing structure consists of two manufacturers selling their products through the same retailer. I refer to this structure as the common agency structure.

This is not the first paper to analyze these two retailing structures, however, the difference of the present analysis is that I consider the case of privately informed manufacturers. To be more precise, each manufacturer is privately informed about her own production cost, but does not know the costs of the other manufacturers. On the other hand, the retailers do not have any private information.

In this framework, the retailing structures differ in two important aspects: they induce different levels of downstream competition and they imply in different costs of providing incentives. In exclusive dealing there is no downstream coordination and the retailers behave competitively, which reduce the overall profits in these markets. However, the vertical relationship between manufacturer and retailer has no informational distortion. Thus, in this model, the contracts are efficient in exclusive dealing.

In common agency there is no downstream competition. The unique retailer coordinates the manufacturers’ prices and market outcome is closer to monopoly. However, the retailer is able to extract profits from the manufacturers due to his strategic position in this market structure. He is able to use the information of one manufacturer against the other.

To be more precise, this informational effect is as follows, the manufacturers want to extract all the revenue from the retailer, however, the impact of the price of the manufacturer’s good on the revenue depends on the price of the other good. In turn, the other good’s price depends on the cost of the rival, which

\[ I \text{ refer to the manufacturers with feminine pronouns and to the retailers with masculine pronouns.} \]
the manufacturer does not know. Yet, the retailer learns costs from the contracts, therefore, the manufacturers’ has to screen the rivals’ information from the retailer. Screening allows the retailer to extract some profits. In order to reduce the profits left with the retailer, the manufacturers distort downward their prices.

Each retailing structure has its costs and benefits. Exclusive dealing mitigates incentives problems but induces competition in the downstream market while common agency softens downstream competition but allows the retailer to extract informational rents from the manufacturers.

Then, I analyze the choice of the retailing structure and provide a preliminary characterization for the case of substitute goods. I find similar results to Martimort (1996). When the asymmetric information problem increases relatively to the size of the market, the retailer obtains too much profit under common agency, such that the chosen structure tends to be exclusive dealing. On the other hand when the market is big compared to the informational problem, the chosen retailing structure is common agency, which softens downstream competition and enhances profits.

Related Literature

The literature on exclusive contracts and vertical relations is very extensive and I do not aim in making a detailed description of it. This literature has long been concerned with the possible use exclusivity contracts to undermine competition. Aghion and Bolton (1987) have showed that such contracts can increase the cost entry, therefore, they may be anticompetitive practice of firms. Many others have extended the analysis of Aghion and Bolton (1987), like Segal and Whinston (2000). They showed that when a firm has to cover fixed costs, exclusivity contracts can prevent the entrant from achieving the necessary scale of production.

On the other hand, this work is closer to Bernheim and Whinston (1992) who analyzed the role of incentives problems within the vertical relationship between manufacturers and retailers. They showed that exclusive dealing may be the efficient structure when manufacturers have difficulty in providing incentives for a common retailer. Still closer to this work is Martimort (1996) who compared the two same retailing structure when the retailer have better information about the market demand. Moreover he showed that the retailing structure depended on the costs of providing incentives between the two structures. This paper aims in re-addressing the questions of Martimort (1996) when the origin of asymmetric information is reversed from retailers to manufacturers.

The paper is organized as follows, the next section presents the basic structure of the model. Section 3 analyzes the game under exclusive dealing and Section 4 considers the common agency structure. Section 5 compares the two retailing structures and allows the manufacturers to choose between them. A preliminary characterization of the choice of the structure is presented. Section 6 concludes.
2. The model

Two manufacturers compete in the upstream market. They need to contract a retailer to sell their goods on the final consumer’s markets. I compare two retailing structures, one where each manufacturer chooses a different retailer (exclusive dealing) and another where the two manufacturers choose the same retailer (common agency).

The two manufacturers are, ex-ante, identical. They produce goods $x^1$ and $x^2$, respectively, with constant marginal costs. Their cost function is given by

$$c(x^n, \theta^n) = \theta^nx^n,$$

where the uppercase index $n$ refers to the manufacturer. There are no fixed costs.

The marginal cost $\theta^n$ is the manufacturer’s private information. It can assume two values, $\theta_h$ or $\theta_l$, where $\theta_h > \theta_l$ and the probability of a high type manufacturer ($z$) is common knowledge.

The demands for goods $x^1$ and $x^2$ are given by

$$x^n(p^n, p^{-n}) = a - bp^n + dp^{-n}$$

where $p^n$ is the price of good $n$, $-n$ refers to the other good and $a, b > 0$. If $d$ is positive the goods are substitutes while if $d$ is negative, goods are complements. For simplicity the analysis is restricted to the case of substitutes, although many results hold for the case of complements.

**Assumption 1.** Goods $x^1$ and $x^2$ are substitutes ($d > 0$).

The manufacturers have to choose between different retailers in order to sell their goods in the final consumer’s market. They contract a retailer and require a monetary transfer ($t$) in exchange for the goods. Therefore, the manufacturer’s profit is given by

$$V(\theta^n, p^n, t^n) = t^n - \theta^n x^n(p^n, p^{-n}).$$

The retailer sells the goods and give a monetary transfer in exchange for these goods. For simplicity I assume that the retailers are identical and that they have no costs. His profit is given by

$$U(p^n, t^n) = x^n(p^n, p^{-n}) p^n - t^n$$

when he contracts with manufacturer $n$.

If a retailer is not chosen by a manufacturer, he gets his reserve utility, assumed to be zero.

The following assumptions are sufficient conditions to ensure interior solution of the manufacturer profit maximization.

**Assumption 2.** (i) $a - b\theta_i > 0$,
(ii) $(1 - z) b - d > 0$. 

In this framework, the manufacturers have no incentives to choose more than one retailer for the market in question. Therefore, the retailing contract specifies a monetary transfer \( t^n (\theta^n, p^n) \) from the retailer to the manufacturer conditional on the price charged by the retailer. Therefore, firms compete in price, not in quantities. For simplicity I assume that the manufacturer cannot condition the contract on the price of the rival manufacturer.

With exclusive dealing the retailing structure is a principal-agent problem where the manufacturer is the principal and the retailer is the agent. Each manufacturer/retailer relationship constitutes one hierarchy, thus, the hierarchies compete with each other in the downstream market. On the other hand, if the manufacturers choose the same retailer, then the retailing structure is a common agency game with informed principals.

The timing of the game is:
(0) nature draws the costs of the manufacturers;
(1) manufacturers offer contracts to the retailers;
(2) retailers accept or reject the contracts;
(3) if contracts are accepted, prices are set and transfers are made.

I begin analyzing the exclusive dealing structure.

3. Exclusive dealing

In exclusive dealing, the market structure consists of two hierarchies that compete in the downstream market. Each hierarchy is viewed as a principal-agent problem where the manufacturer is the principal and the retailer is the agent. I begin analyzing the perfect information benchmark. The analysis is made from the type-i manufacturer since the model is symmetric. The rival’s variables are in bold.

**Perfect information**

In this situation a type-i manufacturer offers a contract \( t (\theta_i, p_{ik}) \) that maximizes her profit

\[
\max_{t(\theta_i, p_{ik}), p_{ik}} t (\theta_i, p_{ik}) - \theta_i x (p_{ik}, \mathbf{p}_{ki})
\]

subject to

\[
x (p_{ik}, \mathbf{p}_{ki}) p_{ik} - t (\theta_i, p_{ik}) \geq 0.
\]  

(3.1)

where the first lowercase index refers to the manufacturer’s own type and the second lowercase index \( k \) refers to the competing manufacturer’s type.

Throughout this paper, I assume that the manufacturer has the bargaining power, hence, the constraint (3.1) is binding and the profit maximization problem becomes

\[
\max_{p_{ik}} x (p_{ik}, \mathbf{p}_{ki}) (p_{ik} - \theta_i).
\]
The first-order conditions of this problem are given by
\[
(a - bp_{ik} + dp_{ki}) - b (p_{ik} - \theta_i) = 0
\]
The prices that solve the profit maximization under exclusive dealing with perfect information are denoted by \( \hat{p}_i \), which is given by
\[
\hat{p}_{ik} = \frac{a + b \theta_i}{2b - d} + \frac{bd (\theta_k - \theta_i)}{4b^2 - d^2}
\]
and a symmetric formula for \( \hat{p}_{ki} \).
Notice that the price is increasing in the manufacturer’s own cost and also on the rival’s cost. Moreover, prices rise if the substitutability between goods increases.

**Private information**

With private information, this game becomes an informed principal problem as analyzed by Maskin and Tirole (1990). They showed that in informed principal problems with quasi-linear preferences and private values the principal does not gain from withdrawing information from the agent. This means that I can restrict the analysis to separating contracts, that is, different types of manufacturer’s offer different contracts and also that separation does not generates distortions. Hence, the retailer learns the type of the manufacturer when he receives the contracts. This implies that the information asymmetry of this game does not generate distortions on the contract.

Nonetheless, there is one information problem in this game. Since firms of one hierarchy do not know the costs of the rival hierarchy, the hierarchies cannot condition their price in the type of the rival and they maximize expected profits considering the rival’s type a random variable. The profit maximization of the type-\( i \) manufacturer is given by
\[
\max_{t(\theta_i,p_i),p_i} E [t (\theta_i, p_i) - x (p_i, p_r) \theta_i] \] (exclusive dealing problem)
subject to
\[
E [x (p_i, p_r) p_i] - t (\theta_i, p_i) \geq 0 \tag{3.2}
\]
where the expectation with respect to taken in the price of the rival.
Notice that there is only one lowercase index because the manufacturer cannot condition her policy on the type of the rival.

The individual rationality constraint of the retailer is binding and I can eliminate the transfers from the problem, which becomes
\[
\max_{p_i} E [x (p_i, p_r) (p_i - \theta_i)] .
\]
The first-order conditions of this problem are given by
\[
E [a - bp_i + dp_r] - b (p_i - \theta_i) = 0. \tag{3.3}
\]
I refer to prices that solve (3.3) as $\hat{p}_i$, which are given by

$$\hat{p}_i = \frac{a + b \theta_i}{2b - d} + \frac{d (\bar{\theta} - \theta_i)}{2(2b - d)}$$

where $\bar{\theta} = E[\theta]$.

Explicitly, these prices are given by

$$\hat{p}_h = \frac{a + b \theta_h}{2b - d} - \frac{(1 - z) d (\theta_h - \theta_i)}{2(2b - d)},$$

$$\hat{p}_l = \frac{a + b \theta_l}{2b - d} + \frac{zd (\theta_h - \theta_i)}{2(2b - d)}.$$

Comparing with the perfect information benchmark I have

$$\hat{p}_{ih} > \hat{p}_i > \hat{p}_{il}.$$  

The price charged in exclusive dealing with privately informed manufacturers is an “average” of the prices under perfect information.

Under exclusive dealing there are no incentive problems between the firms of the hierarchy. The only distortion arises due to the fact that these firms do not know the production’s cost of the competing hierarchy.

4. Common retailer

With a common retailer the retailing structure is a common agency game with informed lobbies. The retailer’s profit is given by

$$U(p_{ik}, t(\theta_i, p_{ik}), p_{ki}, t(\theta_k, p_{ki})) = x(p_{ik}, p_{ki})p_{ik} + x(p_{ki}, p_{ik})p_{ki} - t(\theta_k, p_{ki}) - t(\theta_i, p_{ik}).$$

This structure allows the manufacturers to coordinate their price choices and mitigate competition. The reason for coordination is that the unique retailer receives the revenue of both goods, thus, when a manufacturer increases prices, the retailer’s revenue on both goods are affected. Then, the manufacturer can take into account the effects of her price on the other good’s revenues when asking for the monetary transfer. Thus, coordination increases profits when compared to exclusive dealing.

On the other hand, in this structure, the retailer is able to use the private information of a manufacturer to extract informational profits from the other manufacturer. Therefore, the retailer gains bargaining power in a common agency using the information of one manufacturer against the other. I begin considering the perfect information benchmark.

**Perfect information**

The type-$i$ manufacturer maximizes her profits

$$\max_{t(\theta_i, p_{ik}), p_{ik}} t(\theta_i, p_{ik}) - \theta_i x(p_{ik}, p_{ki})$$
subject to
\[ x (p_{ik}, p_{ki}) p_{ik} + x (p_{ki}, p_{ik}) p_{ki} - t (\theta_k, p_{ki}) - t (\theta_i, p_{ik}) \geq 0. \] (IR_{ik})

Since the manufacturer has the bargaining power, the constraint (IR_{ik}) is binding and this problem becomes
\[ \max_{p_{ik}} x (p_{ik}, p_{ki}) (p_{ik} - \theta_i) + x (p_{ki}, p_{ik}) p_{ki} - t (\theta_k, p_{ki}) \]
and the first-order conditions are given by
\[ (a - bp_{ik} + dp_{ki}) - b (p_{ik} - \theta_i) + dp_{ki} = 0. \] (4.1)

I denote the prices that solve (4.1) for the two manufactures by \( \bar{p} \), which is given by
\[ \bar{p}_{ik} = \frac{a + b \theta_i}{2 (b - d)} + \frac{bd (\theta_k - \theta_i)}{2 (b^2 - d^2)}. \]

Comparing with the prices in exclusive dealing I have
\[ \bar{p}_{ik} \geq \hat{p}_{ik}. \]

Thus, under perfect information, a common retailer softens the competition between the manufacturers and prices are higher than under exclusive dealing.

**Private information**

When manufacturers choose the same retailer, the retailing structure is a common agency game with informed principals. In this structure the asymmetric information introduces a new element on the contracting between manufacturers and the retailer. The manufacturers try to extract profit from the retailer, yet the retailer’s revenue depends on both prices. In a perfect information framework, the manufacturers anticipate the impact of her good’s price and requires all the revenue. However, with informed manufacturers, one manufacturer cannot anticipate the price of the other, which means they do not know the impact of their price on the retailers’ revenue.

Therefore, the retailer can lie and say that the revenue increase is small because there is strong competition (the rival is low type) when the competition is actually weak. This means that, in the view of one manufacturer, the private information of the rival becomes the private information of the retailer. Thus each manufacturer has to screen her rival’s information from the retailer.

This game structure is a little more complex than the previous ones. Thus, some preliminary discussion of the technical issues of this approach is needed.

To tackle the manufacturer’s profit maximization problem, I take as given the offer \( t (\theta_i, p_{ki}) \) of the rival retailer. Yet I assume that this offer is separating in the rival’s type, i.e., different types offer different contracts. More strongly, the analysis is restricted to equilibria that satisfy

**Condition 1.** The rival’s prices are increasing in the rival’s cost.
4. COMMON RETAILER

**CONDITION 2.** The retailer’s profit is non-decreasing in the rival’s type, i.e.,

\[
U(p, t, p_h, t_h) \geq U(p, t, p_l, t_l).
\]

Given these conditions on the rival’s offer, I tackle the manufacturer’s profit maximization as an informed principal problem. Since this is a private value model, Maskin and Tirole (1992) showed that I can restrict the analysis to contract offers that reveal the type of the manufacturer. That is, there is no signaling problem.

So, in this game, the manufacturer has no advantage in holding private information. However, she looses from not knowing her rival’s type since she has to screen this information from the retailer. Screening requires incentive compatibility constraints on the contract offer, that assure that the retailer chooses the price according to the true marginal revenue of her good. For a type-\(i\) manufacturer, they are given by

\[
x(p_{ik}, p_{ki}) p_{ik} + x(p_{ki}, p_{ik}) p_{ki} - t_{ik} - t_{ki} \geq x(p_{il(-k)}, p_{ki}) p_{il(-k)} + x(p_{ki}, p_{il(-k)}) p_{hi} - t_{il(-k)} - t_{hi}\]

where \(-k \neq k\) is the rival’s false type. That is, in the right-hand side of the constraint \((IC_{ik})\) the retailer is lying to the manufacturer about the type of her rival.

I refer to the manufacturer’s profit maximization as the informed manufacturer problem, which is given by

\[
\max_{t(i; p_i)} E [t (\theta_i, p_i) - \theta_i x (p_i, p_i)] \quad \text{(informed manufacturer’s problem)}
\]

subject to \((IR_{ik})\) and \((IC_{ik})\) for all \(k\).

It is necessary to identify the set of constraints that are binding in the informed manufacturer’s problem. Since goods are substitutes, when the manufacturer increases her price, the revenue increase is big if the price of the substitute good is high, while the revenue increase is small if the price of the substitute good is low. Yet, once the manufacturer does not know the price of the substitute good, the retailer is prompt to lie and say that the revenue increase is small when it truly is big. Therefore, the constraints \((IC_{ih})\) and \((IR_{il})\) should be binding in equilibrium. Therefore, it follows:

**LEMMA 1.** The first-order conditions of the informed manufacturer’s problem are given by

\[
a - 2bp_{ih} + 2dp_{hi} + b\theta_i = 0 \quad (4.2)
\]

\[
a - 2bp_{il} + 2dp_{li} + b\theta_i + \frac{z}{1 - z} 2d(p_{hi} - p_{hi}) = 0. \quad (4.3)
\]

Notice that (4.2) is identical to (4.1) while (4.3) has an additional term that accounts for the informational rents the manufacturer must leave to the retailer.
In order to save on this rent, the manufacturer reduces the price charged when she faces an efficient rival.

Then, it follows:

**Proposition 1.** The equilibrium prices in a common agency with informed manufacturers are given by

\[
\begin{align*}
    p_{hh}^* &= \hat{p}_{hh} \\
    p_{hl}^* &= \hat{p}_{hl} - \frac{zb^3d (\theta_h - \theta_l)}{2 ((1 - z)b^2 - d^2)(b^2 - d^2)} \\
    p_{lh}^* &= \hat{p}_{lh} - \frac{zb^2d^2 (\theta_h - \theta_l)}{2 ((1 - z)b^2 - d^2)(b^2 - d^2)} \\
    p_{ll}^* &= \hat{p}_{ll} - \frac{zb^2d (b + d) (\theta_h - \theta_l)}{2 ((1 - z)b^2 - d^2)(b^2 - d^2)}
\end{align*}
\]

where the uppercase asterisk refers to the equilibrium policies.

Notice that the presence of private information decreases the price when compared to the common agency structure under perfect information. Moreover, the distortion increases with \(z\) and also with the difference between the high and low marginal costs.

Comparing with the exclusive dealing prices gives

\[p_{hh}^* > \hat{p}_{hh},\]

and the result is ambiguous in the other states.

## 5. The choice of retailing structure

In this section I analyze the choice of the retailing structure. I introduce a previous stage in the game where the manufacturers commit to a retailer. The timing of this new game is

1. manufacturers choose the retailing structure;
2. nature draws the manufacturers costs;
3. manufacturers offer contracts to the chosen retailer;
4. the retailer(s) accepts or rejects contracts;
5. if contracts were accepted, prices are set and transfers are made.

Notice that nature only draws the manufacturers’ types after the choice of the retailing structure. If otherwise, the manufacturers could reveal her type through the choice of the regime. That would make the game quite complicated and moreover it would not be possible to compare the solution of this game with the results of the previous sections.

There are two forces that influence the choice of the retailing structure, the first one is the coordination in the downstream market. Under exclusive dealing there is no coordination while in common agency the retailer coordinate actions and firms
can obtain a profit closer to the monopoly profit. Therefore, downstream coordination favors a common agency structure. Yet, with informed manufacturers, the common retailer is able to extract informational rents from the manufacturers by using the information of a manufacturer against the other. Under exclusive dealing, on the other hand, such problem does not arise because the exclusive retailer does not learn the information of the rival manufacturer. Therefore, under exclusive dealing there are no incentive problems in the bilateral relationship between manufacturer and retailer, which favors this structure.

The following propositions are a first step towards the characterization of this game.

**Proposition 2.** Under perfect information, common agency is the chosen retailing structure.

Proposition 2 shows that common agency dominates exclusive dealing in absence of incentive problems. However, with informed manufacturers there are incentive problems on the common agency structure and I have a preliminary result that points to the direction of the effects in force:

**Proposition 3.** Let $z$ be equal to $\frac{b-d}{b}$. Then, there exists $d$ such that when $d > d$ either

i. exclusive dealing is the retailing structure chosen if

$$a^2 - b^2\theta_1^2 - 2bd(\theta_h - \theta_l)^2 < 0.$$  \hspace{1cm} (5.1)

ii. common agency in the retailing structure chosen if (5.1) is reversed.

The intuition Proposition 3 is the following, I set $z$ to the highest value that ensures interior solution on the manufacturer’s problem and increases the substitutability of the goods. When the substitutability increases, the gains from downstream coordination increase. However, so does the incentives problem in the common agency structure, thus, the result depends on the sign of (5.1).

Notice that (5.1) tends to be negative as the difference between the high and low marginal cost increase relatively to the size of the market. As $(\theta_h - \theta_l)$ increases, the incentive problem in the common agency increases and this structure becomes less attractive. Therefore, when the cost of uncertainty is big the market structure tends to exclusive dealing.

These results are similar to those in Martimort (1996), which shows that changing the origin of the informational problem generates a similar effects. However the motives are quite different. In Martimort (1996) the retailing structure differ first on the level of downstream competition but also on the cost of extracting rents from the retailer. But the retailers have a bargaining power since they observe a parameter of the demand, i.e., they hold the private information.

On the other hand, in our model the retailers do not have any bargaining power. However, a coordination problem between manufacturers arises in common
agency: since the manufacturers do not reveal their information to one another, they allow the retailer to extract rents from them. If the manufacturer revealed their information to each other, the retailer would not be able to extract any rents and the common agency with informed manufacturers would be as the perfect information case. This is the intuition for the following

PROPOSITION 4. The equilibrium of the common agency with informed manufacturers is not interim efficient.

Proposition 4 states that a benevolent and uninformed planner could implement contracts that generate higher profits in the common agency retailing structure. Thus, the inefficiency of this game could be avoided if the manufacturers coordinated their actions and revealed their information to each other.

Moreover, in this framework, the contract inefficiencies under common agency arise for the case of complementary goods as well. This shall generate a similar result to Proposition 3. However, in Martimort (1996), when goods are complements, common agency is always the chosen retailing structure. Hence, this model may generate results different than Martimort (1996) for the case of complementary goods.

6. Conclusion

I assumed that the manufacturer’s costs were private information and we compared two retailing structures, one where manufacturers practice exclusive dealing and the other where firms choose the same retailer. Then, I provided a preliminary result that identifies the key elements for the choice of the retailing structure.

Under exclusive dealing there is no downstream coordination since the retailers of each hierarchy compete with each other. Yet, the manufacturers reveal their information to the retailer without any cost. Hence, there are no informational distortion in this vertical relationship, contracts are efficient and maximize the hierarchy’s profits.

On the other hand, in common agency, the unique retailer coordinates the actions and softens the downstream competition, increasing the profits. However, the retailer is able to use the private information of one manufacturer against the other to obtain profits. The intuition for this informational effect is the following: the manufacturers try to extract all the profit from retailer. However, the revenue of their product depends on the price of the substitute good, which they do not know. Hence they have to screen the retailer. Screening gives informational profits to the retailer and also makes the manufacturers distort their prices downward.

Then, I analyzed the choice of the retailing structure. This choice weights the profits from downstream coordination versus the costs of the informational problems in the vertical relationship. We provide a partial characterization which shows that when the information asymmetry problem \((\theta_n - \theta_l)\) increases, the market tends to be organized through exclusive dealing while if the size of the market
(a) is relatively big compared to the costs, the market tends to be organized in common agency.

This shows that the results of Martimort (1996) also hold when the source of the informational problem is reversed. However, the intuition suggests that our results shall be replicated for the case of complementary goods, which might generate different results than Martimort (1996).

Obviously, much is yet to be done. Although Proposition 3 already points to the direction of the results, we have to provide a full characterization of the choice of the retailing structure. Two other important improvements are to investigate the case of complementary goods and also to analyze equilibria where Condition 2 does not hold. Breaking down this condition allows countervailing incentives in the informed manufacturer problem, which may be plausible in this particular context. Countervailing incentives change the directions of the price distortions and induce manufacturers to ask for prices higher than under perfect information.

Appendix

**Proof of Lemma 1.** It is necessary to identify which constraints are binding in the informed manufacturer’s problem. Condition 2 gives

\[ x(p_i, p_{hi}) p_i + x(p_{hi}, p_i) p_{hi} - t(\theta_i, p_{hi}) - t(\theta_i, p_i) \geq x(p_i, p_{hi}) p_i + x(p_{hi}, p_i) p_{hi} - t(\theta_i, p_{hi}) - t(\theta_i, p_i). \]

Replacing \( p \) and \( t(\theta, p) \) by \( p_{id} \) and \( t(\theta_i, p_{id}) \) gives

\[ x(p_{id}, p_{hi}) p_{id} + x(p_{hi}, p_{id}) p_{hi} - t(\theta_i, p_{hi}) - t(\theta_i, p_{id}) \geq x(p_{id}, p_i) p_{id} + x(p_i, p_{id}) p_i - t(\theta_i, p_{id}) - t(\theta_i, p_i). \]

From the constraint \((IR_{ih})\), we have

\[ x(p_{id}, p_i) p_{id} + x(p_i, p_{id}) p_i - t(\theta_i, p_{id}) - t(\theta_i, p_i) \geq 0. \]

Combining the two inequalities above gives

\[ x(p_{id}, p_{hi}) p_{id} + x(p_{hi}, p_{id}) p_{hi} - t(\theta_i, p_{hi}) - t(\theta_i, p_{id}) \geq x(p_{id}, p_i) p_{id} + x(p_i, p_{id}) p_i - t(\theta_i, p_{id}) - t(\theta_i, p_i) \geq 0. \]

But this implies that

\[ x(p_{ihi}, p_{hi}) p_{ihi} + x(p_{hi}, p_{ihi}) p_{hi} - t(\theta_i, p_{hi}) - t(\theta_i, p_{ihi}) \geq x(p_{id}, p_{ihi}) p_{id} + x(p_{ihi}, p_{id}) p_{ihi} - t(\theta_i, p_{ihi}) - t(\theta_i, p_{id}) \geq 0. \]

Therefore, the constraint \((IC_{ih})\) assures that the constraint \((IR_{ih})\) is not violated, thus we can ignore this last constraint. The two incentive compatibility constraints can be written as

\[
\begin{align*}
  t_{ih} - t_{id} & \leq x(p_{ih}, p_{hi}) p_{ih} + x(p_{hi}, p_{ih}) p_{hi} - x(p_{id}, p_{hi}) p_{id} - x(p_{hi}, p_{id}) p_{hi} \\
  t_{ih} - t_{id} & \geq x(p_{ihi}, p_{hi}) p_{ihi} + x(p_{hi}, p_{ihi}) p_{ihi} - x(p_{id}, p_{hi}) p_{id} - x(p_{ihi}, p_{id}) p_{ihi}
\end{align*}
\]

where \( t(\theta_i, p_{ik}) = t_{ik} \).
The constraint \((IR_{il})\) is binding. If not, the we could propose another contract, with higher transfers in both states, such that both (6.1) and (6.2) would not be violated and the manufacturer would obtain higher profits.

Given that the constraint \((IR_{il})\) is binding, the constraint \((IC_{ih})\) is also binding. If not, we could propose a contract with a higher transfer \(t_{ih}\), which respects (6.2) and gives higher profits to the manufacturer.

Provided that \((IC_{ih})\) and \((IR_{il})\) are binding, we can eliminate transfers from informed manufacturer’s problem and maximize it in respect to \(p_{ik}\). That is,

\[
\max_{p_{il}, p_{ih}} \left[ x(p_{ih}, p_{hi})(p_{ih} - \theta_i) + x(p_{hi}, p_{ih})p_{hi} - x(p_{il}, p_{hi})p_{il} - x(p_{hi}, p_{il})p_{hi} \right] \\
+ (1 - z) \left[ x(p_{il}, p_{li})(p_{il} - \theta_i) + x(p_{li}, p_{il})p_{li} \right]
\]

The first-order conditions of this problem are given by

\[
z[a - 2bp_{ih} + 2dp_{hi} + b\theta_i] = 0,
\]

\[
(1 - z) [a - 2bp_{il} + 2dp_{hi} + b\theta_i] + z[a - 2bp_{il} - 2dp_{hi} - a + 2dp_{il}] = 0.
\]

Dividing both first order condition by \(z\) and \((1 - z)\) gives respectively (4.2) and (4.3).

**Proof of Proposition 1.** The first-order conditions (4.2) and (4.3), for the two manufacturers, constitute a system of linear equations that, for state \(\{\theta_{h}, \theta_{h}\}\), can be written in matrix form

\[
\begin{bmatrix}
-2b & 2d \\
2d & -2b
\end{bmatrix}
\begin{bmatrix}
p_{1h}^h \\
p_{2h}^h
\end{bmatrix}
= \begin{bmatrix}
b\theta_i^h - a \\
b\theta_i^h - a
\end{bmatrix}.
\]

The determinant of this system is given by \((2b)^2 - (2d)^2\), which is positive, from Assumption 2. Hence, this system has a solution.

Provided the solution of this system, \(p_{hh}^h\), we can compute the system for state \(\{\theta_{h}, \theta_{l}\}\), which is given by

\[
\begin{bmatrix}
-2b(1 - z) & 2d \\
2d & -2b
\end{bmatrix}
\begin{bmatrix}
p_{1h}^l \\
p_{2h}^l
\end{bmatrix}
= \begin{bmatrix}
-b\theta_i^l + a (1 - z) + z2dp_{hl}^l \\
-b\theta_i^l - a
\end{bmatrix}.
\]

and we have a symmetric system for state \(\{\theta_{l}, \theta_{h}\}\).

From Assumption 2, the determinant of these systems are also positive.

Given the solution of these systems, we can compute the system of first-order conditions for state \(\{\theta_{l}, \theta_{l}\}\), which is given by

\[
\begin{bmatrix}
-2b(1 - z) & 2d \\
2d & -2b(1 - z)
\end{bmatrix}
\begin{bmatrix}
p_{1l}^l \\
p_{2l}^l
\end{bmatrix}
= \begin{bmatrix}
-b\theta_i^l + a (1 - z) + z2dp_{hl}^l \\
-b\theta_i^l + a (1 - z) + z2dp_{hl}^l
\end{bmatrix}.
\]

Again from Assumption 2, this system has a positive determinant, thus it has a unique solution. The solution of all these systems is the equilibrium of the retailing game in common agency.
APPENDIX

To compute the equilibrium prices, it is much simpler to work with the best response functions, which for state \( \theta_h, \theta_h \) are given by

\[
p_{hh}^1 = f_{hh} \left( p_{hh}^2 \right) = \frac{a + 2dp_{hh}^2 + b\theta_h}{2b},
\]

\[
p_{hh}^2 = f_{hh} \left( p_{hh}^1 \right) = \frac{a + 2dp_{hh}^1 + b\theta_h}{2b}.
\]

Substituting one into the other gives

\[
p_{hh}^* = f_{hh} \left( f_{hh} \left( p_{hh}^* \right) \right) = \frac{a + 2d \left( \frac{a+2dp_{hh}^* + b\theta_h}{2b} \right) + b\theta_h}{2b}
\]

which becomes

\[
p_{hh}^* = f_{hh} \left( f_{hh} \left( p_{hh}^* \right) \right) = \frac{a \left( b + d \right) + 2d^2 p_{hh}^* + bd\theta_h + b^2\theta_h}{2b^2}
\]

which, finally, gives

\[
p_{hh}^* = \frac{a + b\theta_h}{2 \left( b - d \right)}.
\]

The prices in the other states are found with a similar procedure. \( \square \)

**Proof of Proposition 2.** This is simply a restatement of Bernheim and Whinston (1986b) result of efficiency of the truthful equilibrium in common agency games.

If the manufacturers undertook a centralized decision to maximize joint profits, they would solve

\[
\max_{p_{ik}, p_{ki}} x \left( p_{ik}, p_{ki} \right) \left( p_{ik} - \theta_i \right) + x \left( p_{ki}, p_{ik} \right) \left( p_{ki} - \theta_k \right)
\]

independently the retailing structure (the contributions would compensate the reserve utility of the retailer(s)).

The first-order conditions of this problem are given by

\[
a - 2bp_{ik} + 2dp_{ki} + b\theta_i = 0
\]

\[
a - 2bp_{ki} - 2dp_{ik} + b\theta_k = 0
\]

which are identical to (4.1).

Therefore, the prices charged by this coalition would be the same as in common agency under perfect information. If prices are the same, the quantities and the profits are also the same. Therefore, the prices in common agency maximize the joint profits of the manufacturers. Thus, these profits must be greater than the profits in exclusive dealing. \( \square \)

**Proof of Proposition 3.** We need to show which retailing structure generates higher profits for the manufacturers when \( z = \frac{b - d}{b} \) and \( d \to b \). The expected manufacturer’s profit in exclusive dealing is given by

\[
\Pi^{ED} = E \left[ x \left( \hat{p}, \hat{p} \right) \left( \hat{p} - \theta \right) \right].
\]
Substituting the equilibrium prices, this profit is given by

\[
\Pi^{ED} = z \left[ \left( b \left( \frac{a - b \theta_h}{2b - d} \right) \right)^2 - \frac{b}{2} \left( \frac{(1 - z) \theta_h - \theta_l}{(2b - d)} \right)^2 \right] + z \left[ \frac{b d^2 \theta h}{(2b - d)^2} + \frac{a + b \theta_h}{2b - d} \left( \frac{bd (\theta + \theta_h)}{2b - d} \right) \right] + z \left[ \frac{b d d^2 \theta h}{2b - d} \left( \frac{a + b \theta_h}{2b - d} \right) \left( \frac{bd (\theta + \theta_h)}{2b - d} \right) \right] + (1 - z) \left[ \frac{b d^2 \theta h}{(2b - d)^2} + (a + b \theta_h) \left( \frac{bd (\theta + \theta_h)}{2b - d} \right) \right] + (1 - z)^2 \left[ \frac{b d^2 \theta h}{2b - d} \right] + \frac{b d^2 \theta h}{2b - d} \right] \right] .
\]

Given the equilibrium contributions and prices, the profit under common agency is given by

\[
\Pi^{CA} = E \left[ x(p^*, p^*) (p^* - \theta) - IP \right]
\]

where \( IP \) refers to the informational rent of the retailer.

Explicitly, these terms are given by

\[
E \left[ x(p^*, p^*) (p^* - \theta) \right] = z \left[ \frac{(a + b \theta_h) (a - b \theta_h)}{2} - \theta_h \frac{(a - b \theta_h)}{2} \right] + z \left( 1 - z \right) \left[ \frac{b^2 d (\theta_h - \theta_l)^2}{2} \right] + \left( 1 - z \right) \left[ \frac{(a + b \theta_h) (a - b \theta_h)}{2} - \theta_h \frac{(a - b \theta_h)}{2} \right] + \left( 1 - z \right) \left[ \frac{b^2 d (\theta_h - \theta_l)^2}{2} \right]
\]

where \( \delta = \frac{z b^2 d (\theta_h - \theta_l)}{(1 - z) b^2 - d^2} \).

The informational rent is given by

\[
E \left[ -IP \right] = \left[ x(p_hh, p_hh) p_hh - x(p_hl, p_hh) p_hl + x(p_hl, p_hl) p_hl - x(p_hh, p_hl) p_hh \right] + \left[ x(p_hl, p_hl) p_hl - x(p_hl, p_hl) p_hl \right]
\]
which, explicitly, is given by

\[ E[-IP] = z^2 \left[ -\frac{b + d}{z} \frac{a + b\theta_h}{2} \delta + \delta \frac{bd(\theta_h - \theta_i)}{2z} - \frac{\delta^2 b(b^2 - d^2)}{z} - \frac{(b + d) b(a + b\theta_h)}{2} \right] \\
+ z^2 \left[ \frac{(a + b\theta_h) bd^2 (\theta_h - \theta_i)}{2(b - d) 2 (b^2 - d^2)} + \frac{b^2 d^3 (\theta_h - \theta_i)}{2 (b^2 - d^2)^2} - \frac{bd(\theta_h - \theta_i)}{2 (b^2 - d^2)} \delta (b^2 - bd - d^2) \right] \\
+ z^2 \left[ -\delta^2 (b^2 - bd - d^2) + \frac{b^2 d (\theta_h - \theta_i)}{2 (b^2 - d^2) (b - d)} + \frac{(a + b\theta_h) b^2 \delta - \delta(b + d)(a - b\theta_h)}{2} \right] \\
+ z^2 \left[ \frac{(a - b\theta_i) b^2 (\theta_h - \theta_i)}{2 (b - d) 2 (b^2 - d^2)} - \delta^2 b^2 (b + d) - \frac{0}{2 (b - d)} (b^2 - d^2) \delta + (b^2 - d^2) \delta^2 \right] \\
+ z^2 \left[ \frac{b^2 d^2 (\theta_h - \theta_i)}{2 (b^2 - d^2)} \delta + \frac{(a + b\theta_h) b^2 (b^2 - bd - d^2)}{2 (b - d) \delta (b^2 - bd - d^2)} \right]. \\

Thus, we must show what happens to the following inequality

\[ \Pi_{EE} \leq \Pi_{CA} \] 

(6.3)

when \( z = \frac{b - d}{b} \) and \( d \to b \).

When \( z = \frac{b - d}{b} \), the value of \( \delta \) is \( \frac{b(\theta_h - \theta_i)}{b^2 - d^2} \). Thus, when \( d \to b \), the profits on exclusive dealing (\( \Pi_{ED} \)) and the informational rent (\( IP \)) do not explode. However, the following term from the expected profit in common agency explodes

\[ \frac{d}{b} \frac{1}{2 (b - d)} \left[ (a + b\theta_i) - 2db(\theta_h - \theta_i)^2 \right] \] 

(6.4)

because \( \frac{1}{b - d} \to \infty \).

Since the informational rent (\( IP \)) and the exclusive dealing profits (\( \Pi_{ED} \)) do not explode and \( E[x(p^*, p^*) (p^* - \theta)] \) explodes, then (6.4) determines the sign of (6.3).

The sign of (6.4) depends on the sign of the term inside the brackets, which is exactly (5.1). If it is positive the profits on common agency dominate are higher than the profits of exclusive dealing. On the other hand, if it is negative, the profits of common agency are smaller than the profits in exclusive dealing. \( \Box \)

**Proof of Proposition 4.** To show that the equilibrium of the retailing game under with private information is not interim efficient, I will show that an uninformed planner could propose different contracts \( (\hat{t}(), \hat{p}_{ik}) \) that give the same payoff for the retailer and manufacturer 2, but that give a strictly higher payoff for the manufacturer 1.

If this new contract gives manufacturer 2 the same payoff it must be that

\[ t^* (\theta_k, p^*_{ki}) - x (p^*_{ki}, p^*_{ik}) \theta_i = \hat{t} (\theta_k, \hat{p}_{ki}) - x (\hat{p}_{ki}, \hat{p}_{ik}) \theta_k \] 

(6.5)

for all \( i \) and \( k \).
Since the new contract gives the same payoff for the retailer, it must also be that
\[ x(p_{ki}^*, p_{ik}^*) p_{ki}^* + x(p_{ik}^*, p_{ki}^*) p_{ik}^* - t^*(\theta_k, p_{ki}) - t^*(\theta_i, p_{ik}) = x(\hat{p}_{ki}, \hat{p}_{ik}) \hat{p}_{ki} + x(\hat{p}_{ik}, \hat{p}_{ki}) \hat{p}_{ik} - \hat{t}(\theta_k, \hat{p}_{ki}) - \hat{t}(\theta_i, \hat{p}_{ik}) \]
or all \(i\) and \(k\).

Substituting (6.5) into (??) gives
\[ x(p_{ki}^*, p_{ik}^*) (p_{ki}^* - \theta_k) + x(p_{ik}^*, p_{ki}^*) p_{ik}^* - t^*(\theta_i, p_{ik}^*) = x(p_{ki}, \hat{p}_{ik}) (\hat{p}_{ki} - \theta_k) + x(\hat{p}_{ik}, \hat{p}_{ki}) \hat{p}_{ik} - \hat{t}(\theta_i, \hat{p}_{ik}) \]
(6.6)

The objective of the uninformed planner is to maximize the profit of manufacturer 1, therefore, the proposed new contract is
\[ \{\hat{t}(\theta_i, \hat{p}_{ik})\} \in \arg\max_{t(\theta_i, p_{ik})} t(\theta_i, p_{ik}) = x(p_{ik}, p_{ki}) \theta_i \]
(6.7)
subject to (6.6) and
\[ t(\theta_{i}, p_{ik}) - x(p_{i-ik}, p_{k-i}) \theta_{i} \geq t(\theta_i, p_{ik}) - x(p_{ik}, p_{ki}) \theta_i \quad (MIC_{-i}) \]

Notice that \((MIC_{-i})\) prevents the manufacturer 1 from lying to the uninformed planner.

However, Lemma 3 of Chapter 1 can be extended to this framework, thus, we can ignore the \((MIC_{-i})\) constraint. Thus, from (6.6) I can eliminate \(\hat{t}()\) from (6.7), which becomes
\[ \{\hat{t}(\theta_i, \hat{p}_{ik})\} \in \arg\max_{t(\theta_i, p_{ik})} x(p_{ik}, p_{ki}) (p_{ik} - \theta_i) + x(p_{ki}, p_{ik}) (p_{ki} - \theta_i) - \Omega^* \]
(6.8)
where \(\Omega^* = x(p_{ki}^*, p_{ik}^*) (p_{ki}^* - \theta_k) + x(p_{ik}^*, p_{ki}^*) p_{ik}^* - t^*(\theta_i, p_{ik}^*)\).

The first order conditions of this problem are given by
\[ a - 2bp_{ik} + 2dp_{ki} - \theta_i = 0 \]
\[ a - 2bp_{ki} + dp_{ki} - \theta_k = 0 \]
which are identical to the solution of the common agency under perfect information.

Therefore, the uninformed planner can implement the truthful equilibrium, which clearly gives higher profits than the equilibrium of the common agency with informed manufacturers. Thus the manufacturer 1 is strictly better-off while the retailer and manufacturer 2 are receive the same profits. Thus the equilibrium of the common agency retailing game with informed manufacturers is not interim efficient. \(\square\)
Bibliography


