On the positive correlation between income inequality and unemployment

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Abstract

Several empirical studies in the literature have documented the existence of a positive correlation between income inequality and unemployment. I provide a theoretical framework under which this correlation can be better understood. The analysis is based on a dynamic job search under uncertainty. I start by proving the uniqueness of a stationary distribution of wages in the economy. Drawing upon this distribution, I provide a general expression for the Gini coefficient of income inequality. The expression has the advantage of not requiring a particular specification of the distribution of wage offers. Next, I show how the Gini coefficient varies as a function of the parameters of the model, and how it can be expected to be positively correlated with the rate of unemployment. Two examples are offered. The first, of a technical nature, to show that the convergence of the measures implied by the underlying Markov process can fail in some cases. The second, to provide a quantitative assessment of the model and of the mechanism linking unemployment and inequality.

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1 Introduction

Several empirical studies in the literature have documented the existence of a positive correlation between income inequality and unemployment. Nolan (1986) measured the impact of changes in the level of unemployment on the UK size distribution of annual income using cross-section data of the Family Expenditure Survey. He documented that unemployment lead to a shift in the shape of the income distribution, with a rise in the top decile, the effect of unemployment on the deterioration of the income distribution being very significative. Cardoso (1993) and Cardoso et alli (1995) found the same positive correlation when studying data of Brazil in the 80s. Using monthly data for the six largest metropolitan areas, these authors concluded that inequality responded very clearly to the sharp oscillations in employment. Quoting one of the main conclusions of the paper: "unemployment increases inequality". Mirer (1973) arrives at similar conclusions through simulations of income experiences of the US population under alternative macroeconomic conditions. Blinder and Esaki (1978) use a time-series approach and conclude, as well, that changes in the level of unemployment have a discernible impact on the size distribution of income. Other references which have arrived at similar conclusions are Beach (1977) and Budd and Whiteman (1978).

The literature on income distribution, though, still lacks theoretical formalizations able to deliver such a result in a setting in which consumers maximize utility intertemporally, subject to uncertainty. The purpose of this paper is filling in this gap. The basic framework used here is a variation of McCall’s (1970) job-search model. The presentation follows the approach to this model offered in Stokey and Lucas (1989). The givens of the model are the distribution of wage offers, the probability of layoffs ($\theta$) and the probability that a worker does not find a job offer next period ($\alpha$). Elsewhere [Cysne (2004)] I use the same model to argue in defense of inequality measures freed from the variations in the rate of unemployment of the type I investigate here.

I draw on the stationary distribution of wage offers determined in Cysne (2004) to analyze the correlation between unemployment and inequality.

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1Stokey and Lucas (1989, c. 10) find the stationary distribution for the case in which $\alpha = 0$. These authors, though, provide no proofs (before taking limits) that the sequence of measures converge. Cysne (2004) provides a (pathological) example in which this limit does not exist.
This is done by: i) using the stationary distribution to provide a general expression for the Gini coefficient of income inequality\(^2\) which does not require the previous specification of the distribution of job offers; ii) showing how the Gini coefficient varies as a function of the parameters of the model; iii) showing the channel by means of which the Gini coefficient, as one concludes from the empirical evidence, can be expected to be positively correlated with the rate of unemployment and, finally; iv) by providing two examples to illustrate the method.

The paper proceeds as follows. Section 2 presents the basic model. Section 3 is used to obtain the expression for the Gini coefficient of income inequality as a function of the distribution of wage offers, the probability of layoffs, and the probability of having job offers. Section 4 provides a theoretical support for the empirical evidence showing the positive correlation between unemployment and inequality. Section 5 concludes.

2 The Model

Consider an economy populated by a group of homogenous workers represented by the measurable space \(\left([0,1], B_{[0,1]}, \mathcal{L}\right), B_{[0,1]}\) denoting the borelians in \([0,1]\) and \(\mathcal{L}\) the Lebesgue measure. This economy can be imagined as a small economy in which all workers are contracted by foreign firms. For \(0 < D < \infty\), consider also the second measurable space \((\Omega, \mathcal{F}, M)\) and, in this space, the measure \(m_w\) induced by the wage-offer function \(w: \Omega \to [0,D]\). In the induced space \((\left([0,D], B_{[0,D]}, m_w\right), \mathcal{F}\), denote by \(F_w(t)\) the distribution function that \((m_w A e. -uniquely)\) determines the measure \(m_w\):

\[ F_w(t) = M \{ w \leq t \} . \]

The analysis of the job search can be made as a function of just two states regarding the consumer’s optimization problem: call it state “\(w\)” and state “\(0\)”. State \(w\) corresponds to a job offer of \(w\) at hand, and state \(0\) to no job offer. In state \(w\) the worker can accept or turn down the offer. If he accepts it, by assumption he stays employed with that wage till he is laid off, which can happen, in each period, with probability \(\theta\). If he does not accept the offer or if he gets no offer, he remains in state \(0\). Being in state zero the only

\(^2\)I assume throughout the whole paper that the only source of income of each worker/consumer is the wage income. Transfers and capital income usually represent only a small fraction of most households’ total income. For the United States, for instance, following the 1992 SCF (Survey of Consumer Finances), transfers and capital income account in average for only around 28% of the total income of the households surveyed. This percentage tends to be even lower in developing countries.
thing he can do is wait again for a job offer next period, which happens with probability $1 - \alpha$. By assumption\(^3\):

\begin{align*}
0 < \theta < 1 \\
0 < \alpha < 1
\end{align*}

(1)

Note that it makes economic sense excluding zero and one of the set in which theta and alpha takes values.

The individual is not allowed to search while in the job. Going to the job market again requires first quitting the job and then waiting for a new offer next period, which can be easily proved to make this option valueless. The job offers are independent and drawn according to the measure $m_w$, which is supposed to be known by all workers. The worker is not allowed to borrow or to lend. His consumption, $c_t$, is equal to his income, $w_t$, in each period.

Consumers maximize the expected present value of their consumption:

$$E \left( \sum_{t=0}^{\infty} \beta^t c_t \right), \quad 0 < \beta < 1$$

As pointed out by Cysne (2004), the solution to this problem is given by the existence of a reservation wage, above which one accepts the offer, and below which one turns it down. The reservation wage, $\bar{w}$, which by definition makes the consumer indifferent between accepting and rejecting the offer, is determined by:

$$\bar{w} = \frac{\beta(1 - \alpha)}{1 - \beta(1 - \theta)} \int_{[w,D]} (w' - \bar{w})dF_w(w')$$

(2)

Cysne (2004) proves the uniqueness and finds the stationary distribution of wages of the economy. Make $f_p(s)ds$ represent the number of people earning income in the range $(s, s + ds)$. Taking into consideration that all

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\(^3\)It follows from the developments to be made below that having $\alpha = 1$ would imply the only ergodic set associated with the transition function defined by the job-search mechanism to be the set $\{0\}$, and the (only) invariant distribution of the associated Markov process to the degenerated distribution with mass one at this point. I rule out this case by having $\alpha$ strictly less than one. Regarding theta, since I am allowing the measure of wage offers to be of mass one at $w = D$, having $\theta = 1$ in this case would imply a cyclical behavior of the state, alternating between the points $\{0\}$ and $\{D\}$, which I want to rule out (see example 1).
wage offers in between zero and the reservation wage implies a wage equal to zero, the invariant measure of wages reads:

\[
F_p(s) = \begin{cases} 
\frac{\theta}{\theta + (1-\alpha)m_w(A)} & \text{if } s = 0 \\
\frac{(1-\alpha)dF_w(s)}{\theta + (1-\alpha)m_w(A)} & \text{if } \bar{w} \leq s < D
\end{cases}
\] (3)

By the law of large numbers, since we are considering a large numbers of workers drawing from the same distribution, this measure represents the cross-sectional distribution of wages in the economy.

Note that the cross-sectional average wage in this economy is:

\[
s_A = \int_{[\bar{w}, D]} \frac{s(1-\alpha)dF_w(s)}{\theta + (1-\alpha)m_w(A)}
\] (4)

where \(\bar{w}\) follows from (2) and \(A\) is the set the lower bound of which is the reservation wage, and the upper bound of which is \(D\).

3 A General Expression for the Gini Coefficient

The Gini coefficient \((G)\) is a ratio between two areas. The first area is the one between the the curves \(k(j) = j\) and the Lorenz curve \(L(j)\), to be defined below. The second area is the one between the curves \(k(j) = j\) and \(k_1(j) = 0\). In all cases, \(j\) runs from 0 to 1. By integrating:

\[
G = 1 - 2 \int_{[0,1]} L(j) dj
\] (5)

The Lorenz curve expresses the fraction of income earned by a fraction of the population, when this population is ordered from the poorer to the richer. Given the income density function (3), the fraction of the population earning income less or equal to \(s^*\) is given by the distribution function:

\[
F_p(s^*) = \int_{0}^{s^*} f_p(u) du = \begin{cases} 
\frac{\theta}{\theta + (1-\alpha)m_w(A)} & \text{if } 0 \leq s < \bar{w} \\
\frac{\theta}{\theta + (1-\alpha)m_w(A)} + \int_{[\bar{w}, s^*]} \frac{(1-\alpha)dF_w(u)}{\theta + (1-\alpha)m_w(A)} & \text{if } \bar{w} \leq s \leq D
\end{cases}
\] (6)
and the fraction of income earned by workers with income less or equal to \( s^* \) by:

\[
F_s(s^*) = \frac{1}{s A} \int_0^{s^*} sf_p(u) du = \begin{cases} 
0 & \text{if } 0 \leq s < \bar{w} \\
\int_{[\bar{w}, s^*]} \frac{(1-\alpha)dF_w(u)}{\theta + (1-\alpha)m_w(A)} & \bar{w} \leq s \leq D 
\end{cases}
\]  

(7)

The Lorenz curve given by is given by the function \( F_s(F_p) \) when \( s^* \) runs from 0 to \( D \).

Note in (6) that if the reservation wage is less than the lower bound of the distribution \( F_w \), then \( m_w(A) = 1 \). From (5), it is easy to see that an increase of the income inequality can be characterized by a decrease of the area under the Lorenz curve.

The next lines pursue an expression for the Lorenz curve as a function of \( f_p \), which is known. This will allow us to characterize what happens with the Gini coefficient of income inequality when the parameters \( \theta \) and \( \alpha \) change, given the measure \( m_w \). The procedure detailed by Levine and Singer (1970) can be useful here.

Taking the derivative in (6) and (7) above, one concludes that the slope of the Lorenz curve is given by \( s/sA^4 \), which by integration with respect to the Lebesgue-Stieltjes measure \( F_p(s^*) \) yields \( F_s(s^*) = \frac{1}{sA} \int_0^{s^*} udF_p(u) \). Using integration by parts and (6):

\[
F_s(s^*) = \frac{1}{sA} \left[ s^* \int_{[0, s^*]} f_p(u) du - \int_{[0, s^*]} (\int_{[0, u]} f_p(v) dv) du \right] 
\]  

(8)

We are interested in the area under the Lorenz curve between 0 and a certain wage \( W \in [0, D] \). Call it \( A_L(0, W) \).

\[
A_L(0, W) = \int_{[0, W]} F_s(F_p(s^*)) f_p(s^*) ds^* 
\]

Using \( F_s(F_p(s^*)) \) given by (8):

\[
A_L(0, W) = \frac{1}{sA} \int_{[0, W]} f_p(s^*) ds^* \left[ s^* \int_{[0, s^*]} f_p(u) du - \int_{[0, s^*]} (\int_{[0, u]} f_p(v) dv) du \right] 
\]

Finally, by integrating the last double integral by parts:

\[
\]
\[ A_L(0, W) = \frac{1}{S_A} \int_{[0,W]} f_p(s^*) ds^* \int_{[0,s^*]} u f_p(u) du \quad (9) \]

**Proposition 1** If an economy follows the rules described in Section 2, the area under the Lorenz curve associated with the long-run wage distribution is given by:

\[ A_L(0, 1) = \frac{(1 - \alpha)}{(\theta + (1 - \alpha)(1 - F(\bar{w})) \int_{[\bar{w},D]} u F_w(u)} \sum_{[\bar{w},s^*]} udF_w(u) \quad (10) \]

and the Gini coefficient of income distribution by:

\[ G = 1 - 2 \frac{(1 - \alpha)}{(\theta + (1 - \alpha)(1 - F(\bar{w})) \int_{[\bar{w},D]} u F_w(u)} \sum_{[\bar{w},s^*]} udF_w(u) \quad (11) \]

**Proof.** The first part follows from (3), (4) and (9). The second part follows from (5). \[ \blacksquare \]

Proposition 1 allows for a direct calculation of the (short run) income distribution within a certain group, once the distribution of wage offers is known.

## 4 Unemployment and Inequality

The purpose of this section is analyzing how the Gini coefficient varies as a function of the givens of the model, and how it relates to the rate of unemployment.

Below, denote by \( a_L \) the lower bound of the support of the distribution of wage offers, and by \( E_{mw} w \) the expected value of the distribution characterized by the measure \( m_w \). The reservation wage \( \bar{w} \) is usually a function of theta and alpha. In the case in which:

\[ \frac{\beta}{1 + \beta(1 - \alpha)} E_{mw} w < a_L \quad (12) \]

though, that does not happen. where.

Proposition 3, below, establishes the mechanism by means of which one can better understand the positive correlation between unemployment and inequality, as documented by the empirical evidences mentioned in the introduction. Under (12), and for small variations of \( \theta \) and \( \alpha \), one has:
Proposition 2 Suppose that an economy is characterized as in Section 2, and obeys condition (12). Then, regardless of the initial distribution of wage offers, the Gini coefficient of income distribution is an increasing function of the probability of layoff ($\theta$) and an increasing function of the probability that the worker does not get a job offer ($\alpha$). Moreover, since the unemployment rate $\frac{\theta}{\theta + (1 - \alpha)(1 - F(\bar{w}))}$ is an increasing function of theta and alpha, increases in anyone of these parameters generate a positive correlation between unemployment and inequality.

Proof. This is a consequence of (11).

Remark 1 When condition does not apply, a qualitative analysis of the problem shows the following. The y-coordinate of the Lorenz remains at zero till the population reaches mass $\frac{\theta}{\theta + (1 - \alpha)(1 - F(\bar{w}))}$. Since this is an increasing function of theta, for the Gini coefficient not to be an increasing function of theta it is necessary that the Lorenz curve with a lower theta crosses the Lorenz curve with the higher theta from above (note that the slope of the Lorenz curve is given by $w/w_A$ and $w_A$ is a decreasing function of theta). Example 1 below shows that condition (12) is not a necessary condition for the result of Proposition 1 to be true.

Next, I illustrate the result of Propositions 1 and 2 with an example of the calculation of the within-group income inequality. First, I calculate the area under the Lorenz curve by the usual method. Then I show that one gets the same answer for the area under the Lorenz curve when (10) is used.

Example 1 Suppose the measure $m_w$ is given by the Lebesgue measure in $[0, 1]$. Note that this measure does not obey condition (12). Using (3):

$$f_p(s) = \begin{cases} \frac{\theta}{\theta + (1 - \alpha)(1 - F(\bar{w}))}, & w = 0 \\ 0, & 0 < s < \bar{w} \\ \frac{1 - \alpha}{\theta + (1 - \alpha)(1 - F(\bar{w}))}, & \bar{w} \leq s \end{cases}$$

which leads to the expression for the fraction of the population with income less or equal than $s$:

$$F_p(s) = \begin{cases} \frac{\theta}{\theta + (1 - \alpha)(1 - F(\bar{w}))}, & 0 \leq s < \bar{w} \\ \frac{\theta + (s - \bar{w})(1 - \alpha)}{\theta + (1 - \alpha)(1 - F(\bar{w}))}, & \bar{w} \leq s \leq 1 \end{cases}$$

The usual method, when feasible, uses a parameter $s$ to write the fraction of the population that earns income less or equal than $s$, does the same regarding the fraction of total income earned by workers with income less or equal than $s$, and then proceeds to the elimination of the parameter. See, e.g., Kendall and Stuart (1963).
From (7):

\[ F_s(s) = \frac{1}{sA} \int_0^s u f_p(u) \, du = \begin{cases} 
0 & \text{if } 0 \leq s < \bar{w} \\
\frac{(s^2 - \bar{w}^2)}{1 - \bar{w}^2} & \bar{w} \leq s \leq 1
\end{cases} \]  \quad (15)

Solve for \( s \) in the second term in (14) and substitute into (15) to get the expression for the Lorenz curve:

\[ L(j) = \begin{cases} 
0, & 0 \leq j < \frac{\theta}{\theta + (1 - \alpha)(1 - \bar{w})} \\
\frac{\left[ j\left(\frac{\theta + (1 - \alpha)(1 - \bar{w})}{1 - \bar{w}}\right) + \bar{w}(1 - \alpha) - \theta \right]^2 - \bar{w}^2}{(1 - \bar{w}^2)}, & \frac{\theta}{\theta + (1 - \alpha)(1 - \bar{w})} \leq j \leq 1
\end{cases} \]  \quad (16)

To calculate the area under the Lorenz curve, make:

\[ U = \int_0^1 L(j) \, dj = \int_0^1 \frac{\left[ j\left(\frac{\theta + (1 - \alpha)(1 - \bar{w})}{1 - \bar{w}}\right) + \bar{w}(1 - \alpha) - \theta \right]^2 - \bar{w}^2}{(1 - \bar{w}^2)} \, dj
\]

Making \( u = \frac{j\left(\frac{\theta + (1 - \alpha)(1 - \bar{w})}{1 - \bar{w}}\right) + \bar{w}(1 - \alpha) - \theta}{1 - \bar{w}} \), the above integral reads:

\[ U = \int_0^1 \frac{(1 - \alpha)(u^2 - \bar{w}^2)}{(1 - \bar{w}^2)(\theta + (1 - \alpha)(1 - \bar{w}))} \, du
\]

By integration:

\[ U = \frac{(1 - \alpha)(1 - 3\bar{w}^2 + 2\bar{w}^3)}{3(1 - \bar{w}^2)(\theta + (1 - \alpha)(1 - \bar{w}))} \quad (17)
\]

By using (2) and (5), (17) leads to the closed-form solution to the Gini coefficient. To compare this expression with the one given by (10), and show that both expressions deliver the same result, note that, in this case, in (10), \( 1 - F(\bar{w}) = 1 - \bar{w} \) and:

\[ \int_{[\bar{w},1]} u dF_u(u) = \frac{1 - \bar{w}^2}{2}
\]

\[ \int_{[\bar{w},1]} d\bar{s} \int_{[\bar{w},s^*]} u dF_u(u) = \frac{1 - 3\bar{w}^2 + 2\bar{w}^3}{6}
\]

from which (17) follows trivially.

Let’s proceed to find the Gini coefficient in this case, as a function of both theta and alpha. Using (17), the only thing we have to do is calculating the reservation wage as a function of both alpha and theta. From (2) we get:

\[ \bar{w}(\theta, \alpha) = \frac{1 - \beta \alpha + \beta \theta}{\beta - \beta \alpha} - \sqrt{\left(\frac{1 - \beta \alpha + \beta \theta}{\beta - \beta \alpha}\right)^2 - 1} \quad (18)
\]
The final expression for the Gini coefficient as a function of the parameters theta and alpha can be obtained by substituting (18) in (17) and using (5). Figure 1 and 2 present the Gini coefficient, for $\beta = 0.98$, when theta and alpha are allowed to vary in $(0, 1)$. Note that, in this case, the Gini coefficient is an increasing function of both theta and alpha, a concave function of theta, and a convex function of alpha. Note also that the rate of unemployment of this group of workers is given by $\frac{1}{3} + (1 - \alpha)(1 - \omega)$, which is an increasing function of both theta and alpha.

When alpha is very close to one (but not one), the Gini is very close to one as well, because a very small percentage of the population happens to get job offers. All the remaining workers have no offers and a wage equal to zero. Having theta close to one, though, does not imply a Gini coefficient tending towards one. The reason is that, even when theta is equal to one, those workers who were not employed last period are allowed (with probability 1 - $\alpha$) to get job offers and, possibly, to accept them.

5 Conclusions

The empirical evidence shows that the income inequality increases when unemployment increases. This link, though, still lacks theoretical formalizations of a dynamic and stochastic nature, by means of which it can be better understood. In this paper have tried to add to the understanding of this problem.

The analysis was based on a job-search model characterized by a unique invariant distribution of wages in the economy. Under this setting, I have derived an expression which allows for the (invariant-distribution) calculation of the Gini coefficient of income inequality under any initial distribution of wage offers. Next, I have drawn on this measure to study how inequality varied with the parameters of the model, and with the rate of unemployment.

Two examples have been offered in the paper. The first, actually a counterexample, to show that the convergence of the measures defined by the Markov process implicit in the problem is not trivial. The second, to illustrate, the connection between inequality, the probability of layoff, and the probability of finding a job offer. Since an increase in anyone of these parameters leads to an increase in the rate of unemployment, the example also allows for a quantitative assessment of the positive correlation between inequality and unemployment.
References


The Gini Coefficient as a Function of Theta and Alpha; Beta=0.98

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