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***Inflation and Income Inequality: A Shopping-Time Approach  
(Forthcoming, Journal of Development Economics)***

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# Inflation and Income Inequality: A Shopping-Time Approach (Forthcoming, Journal of Development Economics)\*

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## Abstract

Our work is based on a simplified heterogenous-agent shopping-time economy in which economic agents present distinct productivities in the production of the consumption good, and differentiated access to transacting assets. The purpose of the model is to investigate whether, by focusing the analysis solely on endogenously determined shopping times, one can generate a positive correlation between inflation and income inequality. Our main result is to show that, provided the productivity of the interest-bearing asset in the transacting technology is high enough, it is true that a positive link between inflation and income inequality is generated. Our next step is to show, through analysis of the steady-state equations, that our approach can be interpreted as a mirror image of the usual inflation-tax argument for income concentration. An example is offered to illustrate the mechanism.

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# 1 Introduction

Although there remains some controversy in the empirical literature relating inflation to income distribution (which Galli and Hoeven (2001) call "the inflation-income inequality puzzle"), several works (e.g., Bulir (2001), Romer and Romer (1998), Easterly and Fischer (2001)) present evidence correlating high rates of inflation with income inequality and/or poverty.

This literature, however, still lacks optimizing dynamic models capable of delivering such a result from theoretical standpoints.

The connection between inflation and income inequality is usually made, on descriptive grounds, by claiming that the poor<sup>1</sup>, having more restricted access to interest-bearing moneys, end up paying a higher proportion of their income as inflation tax<sup>2</sup>. Here we do not pursue such a channel directly. We maintain the hypothesis that the poor have more restricted access to financial assets. But instead of focusing on the differentiated (negative) real interest rates paid by transacting balances, we concentrate on the different shares of time allocated as shopping time, by rich and by poor, due to the existence of inflation. Since this shopping time is actually a measure of the welfare costs of inflation (Lucas (2000), Cysne (2003)), our approach can be interpreted as investigating whether the welfare costs of inflation, by affecting the poor more than the rich, can concentrate income.

It turns out that, in equilibrium, the shopping time spent by consumers in order to save on the use of transacting balances, is a mirror image of the amount of real interest payments on these same balances, on account of inflation, in the process of maximizing discounted utility. Intuitively, this fact, noted by Lucas (1993, p. 14, eq. 4.14)<sup>3</sup>, can be explained in terms of an equalization between marginal costs and marginal benefits. Formally, it is presented later, in a generalization of Lucas's argument, by equations (15) and (16). We shall come back to this point in Section 5 of this paper.

The fact that shopping times are, under the formalization described here, mirror images of inflation taxes, implies that our results can be understood from two alternative angles. First, as a direct shopping-time reasoning for income inequality, based on the existence of inflation. Second, as an indirect formalization of the old argument that inflation concentrates income, due to the fact that the poor pay more inflation tax than the rich. This second

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<sup>1</sup>What we mean by rich (or non-poor) and poor consumers is defined later in the text.

<sup>2</sup>Strictly speaking, the usual economic argument refers to the opportunity cost of holding transacting assets, which is defined in terms of the nominal interest rate, rather than in terms of the rate of inflation. We are abusing language here.

<sup>3</sup>See also Lucas (2000), p. 266.

interpretation links our results to the empirical evidence and to the empirical relevance of the inflation-tax argument.

The underlying intuition connecting inflation to income distribution, according to the shopping-time rationale, is that the higher the rate of inflation, the more important the lack of balance between rich and poor consumers, since the rich have access to better transacting technology. When the nominal interest rate is equal to zero, both rich and poor have the same shopping-time, equal to zero. The higher the rate of inflation and the interest rate, though, the higher the opportunity costs of holding monetary assets, and the more monetary assets are substituted by shopping time.

One should therefore expect those with better access to transacting technology to do relatively better when inflation is higher, thereby concentrating income. We shall see that this intuition holds as long as the transacting technology satisfies a certain condition.

An example at the end illustrates this shopping-time mechanism linking inflation to income concentration.

## 2 The model

Our basic model draws upon the homogeneous-agent shopping-time model with interest-bearing deposits presented by Simonsen and Cysne (2001) and Cysne (2003), which in turn draws upon Lucas (2000).

Our economy has an infinite number of homogenous-consumer cohorts classified by how productive their consumers are in producing the consumption good, and distributed in the  $[0, 1]$  interval. Each cohort has the same (large) number of consumers. Work is not mobile among cohorts. The productivity of consumers in cohort  $j \in [0, 1]$  is  $\delta_j > 0$ . We suppose that productivity is non-decreasing in  $j$ . There is a cut-off productivity  $\delta_{\bar{j}}$ ,  $\bar{j} \in (0, 1)$  such that consumers in cohorts with productivity  $\delta_j < \delta_{\bar{j}}$  are called “poor” and consumers in cohorts with productivity  $\delta_j > \delta_{\bar{j}}$  are called “rich”.

The poor only have access to currency ( $M$ ), which they use to make their transactions. The rich can use currency and an interest-bearing asset,  $X$ , to make their transactions<sup>4</sup>.  $X$  pays a nominal interest rate equal to  $i_x$ . The rich also buy bonds ( $B$ ) from the government. Bonds pay an interest rate  $i$  and are not used for transacting operations.

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<sup>4</sup>Mulligan and Sala-i-Martin (1996) have shown that the cost of adopting financial technology is negatively related to the level of education. Such a finding supports our hypothesis of a segmented market for financial assets if we see productivity as being positively correlated with education.

In the remaining presentation of the model, for the sake of notational clarity, we omit the subindex  $j$  that characterizes the (homogeneous) consumers in each cohort.

Both types of consumer, rich and poor, gain utility from the consumption of a single non-storable consumption good, and have a separable utility function

$$\int_0^\infty e^{-gt} U(c_t) dt, \quad c \in C([0, \infty), [0, \infty)). \quad (1)$$

We suppose that  $U$  is continuous, increasing and concave. Consumers in each cohort are endowed with one unit of time that can be used to transact ( $s$ ) or in the production of the consumption good,  $y$ , according to the production function:

$$y = \delta(1 - s) \quad (2)$$

## 2.1 Government and Banks

Our economy is a Fisherian economy with lump-sum taxation, where the government can implement any given interest-rate vector. Besides the productivity parameters  $\delta_j$ , other givens of the model are the rate of monetary expansion and the time-discount parameter  $g$ , which together determine the nominal interest rate through the Fisherian equation. We shall therefore refer to the nominal interest rate as a policy variable.

The government, here consolidated with the Central Bank, is supposed to issue currency and bonds and to collect reserve requirements from the banks. Banks buy bonds from the government and issue  $X$ . The banking system is competitive.  $k$  ( $0 < k < 1$ ) stands for the (non interest-bearing) reserve requirement on  $X$ . The zero-profit condition implies  $i - i_x = ki$ .

$H$  indicates the (exogenous) flow of money transferred to consumers by the government. The way this is done is detailed below.

Make  $P$  = price index,  $\pi = \dot{P}/P$  (inflation rate),  $m = M/P$ ,  $x = X/P$ ,  $b = B/P$  and  $h = H/P$ . In the steady state, consolidation of both government and bank balances leads to:

$$h = \pi m - (i - \pi) b - (i_x - \pi)x \quad (3)$$

We assume that  $h$  is (ex-post) determined by the government in such a way that equation (3) holds separately for each cohort  $j$ .

## 2.2 The rich-consumer maximization problem

With the price of the consumption good indicated by  $P = P(t)$ , the rich consumers in each cohort  $j$  face the budget constraint:

$$\dot{B} + \dot{M} + \dot{X} = iB + i_x X + P (y - c) + H$$

The dot over the variable indicates its time derivative. Each consumer is atomistic in each cohort and takes  $H$  as exogenous in his optimization problem.

Taking into account (2), the budget constraint of the rich consumer (that is, a consumer in a cohort with productivity  $j > \bar{j}$ ) reads:

$$\dot{b} + \dot{m} + \dot{x} = \delta(1 - s) - c + h + (i - \pi)b + (i_x - \pi)x - \pi m \quad (4)$$

Rich consumers have access to a shopping technology  $F(m, x, s)$  where  $m \geq 0$ ,  $x \geq 0$ ,  $s \in [0, 1]$ ,  $F_m > 0$ ,  $F_x > 0$ ,  $F_s > 0$ . Further conditions on the function  $F(\cdot)$  will be introduced later.

The rich consumer maximizes 1 subject to (4) and to

$$0 \leq c \leq F(m, x, s)$$

The first-order conditions for a steady-state solution of the maximization problem are given by:

$$i = \pi + g \quad (6)$$

$$\delta F_m = i F_s \quad (7)$$

$$\delta F_x = (i - i_x) F_s \quad (8)$$

### 2.3 The poor-consumer maximization problem

Poor consumers also have access to the technology  $F(m, x, s)$  but are not allowed to adopt it fully. They are constrained to having  $x = 0$ . The poor consumer maximizes 1 subject to

$$\begin{aligned} 0 &\leq c \leq F(m, 0, s) \\ c + \dot{m} &\leq \delta(1 - s) + h - \pi m \end{aligned}$$

The first-order condition is:

$$\delta F_m(m, 0, s) = i F_s(m, 0, s) \quad (9)$$

### 3 The Steady-State Solutions

From this point onwards, whenever necessary, we shall use the subindexes  $p$  and  $r$ , respectively, for poor and rich. We shall also restrict our analysis to the case of a transacting technology weakly separable in shopping time and monetary assets, by making<sup>5</sup>:

$$F(m, x, s) = G(m, x) s \quad (10)$$

$G(m, x)$  is differentiable, first-degree homogeneous, increasing with respect to each variable, and with  $G_m/G_x$  an increasing function of  $x/m$ .

• **Rich:**

Equations (7) and (8) now read:

$$iG = G_m \delta s \quad (11)$$

$$kiG = G_x \delta s \quad (12)$$

In equilibrium, since the consumption good is non-storable and the government transfers to each cohort match the net amount of real interest payments:

$$\delta(1 - s) = c = G(m, x)s. \quad (13)$$

Given the hypotheses about  $G(m, x)$ , the marginal rate of substitution is an increasing function of the asset ratio  $x/m$ . Taking the inverse function and using (11) and (12):

$$\frac{x_r}{m_r} = J \left( \frac{G_m}{G_x} \right) = J \left( \frac{1}{k} \right) \quad J'(\cdot) > 0 \quad (14)$$

Equations (11), (12) and (13), with  $k$  fixed, determine  $s_r$ ,  $m_r$  and  $x_r$  as a

function of the policy variable  $i$ :

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<sup>5</sup>This transacting technology is a particular case of that used by Simonsen and Cysne (2001). Lucas (1993, 2000) presents the link between first-degree homogenous transacting technologies and the classical inventory-theoretic literature. For the importance of weak separability in another context, see Cysne (2002).

$$\begin{aligned}
s_r(i) &= \frac{-i(1 + kJ(1/k))}{2G(1, J(1/k))} + \sqrt{\frac{i^2(1 + kJ(1/k))^2}{4G(1, J(1/k))^2} + \frac{i(1 + kJ(1/k))}{G(1, J(1/k))}} \\
x_r(i) &= \frac{J(1/k) \delta s_r}{i(1 + kJ(1/k))} \\
m_r(i) &= \frac{\delta s_r}{i(1 + kJ(1/k))}.
\end{aligned}$$

This determination proceeds as follows: Since  $G(m, x) = mG_m + xG_x$  (Euler's theorem), from (11) and (12):

$$s_r = \frac{i(m_r + kx_r)}{\delta}. \quad (15)$$

To obtain the equilibrium variables, use (14) to get  $x_r$  as a function of  $m_r$ . Then use (15) to get  $m_r$  (and  $x_r$ ) as a function of  $s_r$ . Finally, by taking into consideration that  $G(m, x) = mG(1, J(1/k))$ , one can obtain  $s_r$  by using the expressions for  $m_r$  and  $x_r$  in (13).

• **Poor:**

In this case, equations (11) and (13) are still valid, but with  $x = 0$ . The first-degree homogeneity of  $G$  implies  $G(m, 0) = mG(1, 0)$  and  $G_m = G(1, 0)$ . The first-order equation (11) can therefore be rewritten as:

$$\delta s_p = i m_p \quad (16)$$

Following the same procedure as outlined earlier, the solutions for the poor are given by:

$$\begin{aligned}
s_p(i) &= \frac{-i}{2G(1, 0)} + \sqrt{\frac{i^2}{4G(1, 0)^2} + \frac{i}{G(1, 0)}} \\
m_p(i) &= \frac{\delta s_p}{i}.
\end{aligned}$$



## 4 Main Results

Note that  $s_p(0) = s_r(0) = 0$ , since in this case there is no private cost associated with the use of money. Lemma 1 establishes necessary and sufficient conditions for the shopping time of the poor to be greater than that of the rich.

**Lemma 1**  $s_p > s_r$  if and only if the transaction technology and the parameter  $k$  are such that

$$G(1, J(1/k)) > G(1, 0)(1 + kJ(1/k)) \quad (17)$$

**Proof.**  $s_r$  and  $s_p$  are determined, respectively, as roots of the quadratic equations:

$$f_r(s) = s^2 + \frac{(1 + kJ(1/k))i}{G(1, J(1/k))}s - \frac{(1 + kJ(1/k))i}{G(1, J(1/k))} \quad (18)$$

and

$$f_p(s) = s^2 + \frac{i}{G(1, 0)}s - \frac{i}{G(1, 0)} \quad (19)$$

The family of quadratic equations  $g(x; b) = x^2 + bx - b$ ,  $b > 0$  always presents a real root  $x_1$  such that  $0 < x_1 < 1$ <sup>6</sup>. Besides, since this root satisfies  $x_1^2 + bx_1 - b = 0$ , it follows from the implicit function theorem that:

$$\frac{dx_1}{db} = \frac{1 - x_1}{\sqrt{b^2 + 4b}} > 0$$

The demonstration is complete since one realizes that (17) is equivalent to having

$$\frac{(1 + kJ(1/k))i}{G(1, J(1/k))} < \frac{i}{G(1, 0)}$$

in (18) and (19). ■

Condition (17) is satisfied if the productivity of the transacting technology with respect to  $x$  is high enough for all values of  $x$  and  $m$ . Indeed, this underscores the disadvantage of the poor not having access to this asset, which leads them to spend more time shopping. Example 1 below shows that this condition is satisfied, for instance, when  $G(m, x)$  is a *CES* function.

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<sup>6</sup>Indeed,  $g(0; b) = -b < 0$  and  $g(1; b) = 1 > 0$ . The other root is negative and can be disregarded, because  $s > 0$ .

- The Gini Coefficient and the Rate of Inflation

To measure the inequality in the income distribution, we use the Gini coefficient of income distribution. The Gini coefficient  $I$  is given by:

$$I = 1 - 2 \int_0^1 L(j) dj \quad (20)$$

where

$$L(j) = \frac{\int_0^j c_u du}{\int_0^1 c_u du}$$

stands for the Lorenz curve. The Lorenz curve measures the proportion of the total income of the economy that is received by the lowest 100j% of the consumers. The Gini coefficient expresses the area between the Lorenz curve and the Lorenz curve for an economy where everyone receives the same income.

We proceed to calculate the Gini coefficient for our economy. Note that  $s_r$  and  $s_p$  do not depend on the productivity coefficient  $\delta_j$ . It will be notationally convenient to define

$$\Delta_j = \int_0^j \delta_u du$$

and

$$\Gamma_j = \int_0^j \Delta_u du$$

The first step is to calculate  $\int_0^j c_u du$ . If  $j < \bar{j}$ ,

$$\int_0^j c_u du = \int_0^j c_u^p du = \int_0^j \delta_u (1 - s_p) du = (1 - s_p) \Delta_j$$

If  $j > \bar{j}$ ,

$$\int_0^j c_u du = \int_0^{\bar{j}} c_u^p du + \int_{\bar{j}}^j c_u^r du = (1 - s_p) \Delta_{\bar{j}} + (1 - s_r) (\Delta_j - \Delta_{\bar{j}}).$$

Thus, from (20):

$$I = 1 - 2 \frac{\int_0^{\bar{j}} (1 - s_p) \Delta_j dj + \int_{\bar{j}}^1 ((1 - s_p) \Delta_{\bar{j}} + (1 - s_r) (\Delta_j - \Delta_{\bar{j}})) dj}{(1 - s_p) \Delta_{\bar{j}} + (1 - s_r) (\Delta_1 - \Delta_{\bar{j}})}$$

$$I = 1 - 2 \frac{(1 - s_p) \Gamma_{\bar{j}} + (1 - s_p) \Delta_{\bar{j}} (1 - \bar{j}) + (1 - s_r) (\Gamma_1 - \Gamma_{\bar{j}}) - (1 - s_r) (1 - \bar{j}) \Delta_{\bar{j}}}{(1 - s_p) \Delta_{\bar{j}} + (1 - s_r) (\Delta_1 - \Delta_{\bar{j}})}$$

$$I = 1 - 2 \frac{(1 - s_p) (\Gamma_{\bar{j}} + \Delta_{\bar{j}} (1 - \bar{j})) + (1 - s_r) (\Gamma_1 - \Gamma_{\bar{j}} - \Delta_{\bar{j}} (1 - \bar{j}))}{(1 - s_p) \Delta_{\bar{j}} + (1 - s_r) (\Delta_1 - \Delta_{\bar{j}})} \quad (21)$$

**Proposition 2** *If condition (17) is satisfied, then the Gini coefficient is a non-decreasing function of the nominal interest rate (or, equivalently, of the rate of inflation), at the point  $i = 0$ .*

**Proof.** We proceed by calculating the derivative of the Gini coefficient with respect to the interest rate at  $i = 0$ .

$$I(0) = 1 - 2 \frac{\Gamma_{\bar{j}} + \Delta_{\bar{j}} (1 - \bar{j}) + \Gamma_1 - \Gamma_{\bar{j}} - \Delta_{\bar{j}} (1 - \bar{j})}{\Delta_1} = 1 - 2 \frac{\Gamma_1}{\Delta_1} \quad (22)$$

$$\begin{aligned} \frac{I(i) - I(0)}{2} &= \frac{-\Delta_1 (1 - s_p) (\Gamma_{\bar{j}} + \Delta_{\bar{j}} (1 - \bar{j})) - \Delta_1 (1 - s_r) (\Gamma_1 - \Gamma_{\bar{j}} - \Delta_{\bar{j}} (1 - \bar{j}))}{\Delta_1 [\Delta_{\bar{j}} (1 - s_p) + (\Delta_1 - \Delta_{\bar{j}}) (1 - s_r)]} \\ &\quad + \frac{\Gamma_1 \Delta_{\bar{j}} (1 - s_p) + \Gamma_1 (\Delta_1 - \Delta_{\bar{j}}) (1 - s_r)}{\Delta_1 [\Delta_{\bar{j}} (1 - s_p) + (\Delta_1 - \Delta_{\bar{j}}) (1 - s_r)]} \\ &= \frac{[\Gamma_1 \Delta_{\bar{j}} - \Delta_1 (\Gamma_{\bar{j}} + \Delta_{\bar{j}} (1 - \bar{j}))] (s_r - s_p)}{\Delta_1 [\Delta_{\bar{j}} (1 - s_p) + (\Delta_1 - \Delta_{\bar{j}}) (1 - s_r)]} \end{aligned}$$

Since

$$\Gamma_1 = \Gamma_{\bar{j}} + \int_{\bar{j}}^1 \Delta_u du < \Gamma_{\bar{j}} + \Delta_1 (1 - \bar{j})$$

we have

$$[\Gamma_1 \Delta_{\bar{j}} - \Delta_1 (\Gamma_{\bar{j}} + \Delta_{\bar{j}} (1 - \bar{j}))] < 0$$

Therefore,

$$\frac{I(i) - I(0)}{2} > 0 \Leftrightarrow s_p(i) > s_r(i)$$

Taking the limit as  $i \rightarrow 0$ ,

$$s_p(i) > s_r(i) \Rightarrow I'(i) |_{i=0} \geq 0$$

The Proposition follows from Lemma 1. ■

**Example 1** We consider the transacting technology  $F(m, x, s) = G(m, x)s = A(m^a + x^a)^{1/a}s$ ,  $A > 0$ ,  $0 < a < 1$  and productivities  $\delta_j = \delta$  if  $j \leq \bar{j}$ ,  $\delta_j = \lambda\delta$ , if  $j > \bar{j}$  ( $\lambda \geq 1$ ). Using (14):

$$kJ(1/k) = k^{a/(a-1)}$$

$$G(1, J(1/k)) = A(1 + k^{a/(a-1)})^{1/a}$$

It can be easily checked that condition (17) and the previous assumptions about  $G$  are satisfied. In this case, ( $\delta$  can be taken as equal to one in the calculations of  $I$  because it cancels out):

$$s_r(i) = -\frac{i}{2A}(1 + k^{\frac{a}{a-1}})^{1-1/a} + \sqrt{\frac{i^2}{4A^2}(1 + k^{\frac{a}{a-1}})^{2(1-1/a)} + \frac{i}{A}(1 + k^{\frac{a}{a-1}})^{1-1/a}}$$

$$s_p(i) = -\frac{i}{2A} + \sqrt{\frac{i^2}{4A^2} + \frac{i}{A}}$$

$$\Delta_j = \begin{cases} j & \text{if } j \leq \bar{j} \\ \lambda j - (\lambda - 1)\bar{j} & \text{if } j > \bar{j} \end{cases}$$

$$\Gamma_j = \begin{cases} j^2/2 & \text{if } j \leq \bar{j} \\ (j - \bar{j})^2\lambda/2 + (j - \bar{j})\bar{j} + \bar{j}^2/2 & \text{if } j > \bar{j} \end{cases}$$

The value of the Gini coefficient for different values of the interest rate can be obtained using (21) and the above expressions. Table 1 presents the values of  $s_r$ ,  $s_p$  and of the Gini coefficient for the parameter values  $a = 0.3$ ,  $A = 1$ ,  $k = 0.25$ ,  $\bar{j} = 0.75$ <sup>7</sup>,  $\lambda = 1$  and interest rates equal to 0, 100%, 500% and 1000%.

<b>Table 1, <math>\lambda = 1</math></b>				
Interest Rate (%)	0	100	500	1000
$s_r$	0	0.2579	0.4819	0.5993
$s_p$	0	0.6180	0.8541	0.9161
Gini Coefficient ( $I$ )	0	0.1431	0.2921	0.3641

Figure 1 presents the evolution of the shopping time of rich and poor, as well as of the Gini coefficient, when interest rates assume values between zero and a thousand percent. The two upper graphs present the case in which  $\lambda = 1$  (as in Table 1), and the remaining graphs the case with  $\lambda = 10$ .

<sup>7</sup>Note that, as one should expect,  $G = 0$  for  $\bar{j} = 0$  or  $\bar{j} = 1$ .

Some points are worth noting with regard to the data generated in this example. First, from equation (22), the value of the Gini coefficient when the interest rate is equal to zero reads:

$$I(0) = \frac{(\lambda - 1)\bar{j}(1 - \bar{j})}{\bar{j} + \lambda(1 - \bar{j})} \quad (23)$$

Table 1 has been constructed under the assumption that  $\lambda = 1$ , implying, by the above formula, that  $G(0) = 0$ , as one can read at the bottom of the second column of the table. In this case ( $\lambda = 1$ ), all inequality, which ranges from zero to 0.3641, is generated by inflation. Higher or lower values of the variables in Table 1 can be generated by allowing the parameter  $A$  to assume lower or higher values, respectively.

Figure 1 also includes the data when  $\lambda = 10$ . Note that the Gini coefficient is different from zero (its value is given by (23)) when the interest rate is equal to zero. Indeed, in this case we are assuming that a part of the population is more productive than the other, something which did not happen in the case  $\lambda = 1$ .

Note also that the shopping times of the rich and of the poor do not depend on  $\lambda$ .

## 5 Inflation Tax and Shopping Time

So far, our theoretical analysis has focused solely on a shopping-time reasoning in studying the link between inflation and inequality.

The purpose of this section is to show that in our formalization of the problem, shopping times of both the rich and the poor read as a constant times the inflation tax they pay. To express it differently, our examination of inequality based on the shopping-time rationale can be understood as a mirror image of the usual inflation-tax argument that links inflation to inequality.

Indeed, for the sake of generality, make:

$$F(m, x, s) = G(m, x) s^\mu, \quad 0 < \mu \leq 1$$

in (10). Note that  $\mu$  can assume the value 1, used throughout the text, or another value less than one (see footnote 3 and also the discussion at the bottom of page 265, in Lucas (2000)). Under this specification, equations (15) and (16) read, respectively:

$$s_r = \frac{\mu}{\delta} i (m_r + kx_r) = \frac{\mu}{\delta} (i m_r + (i - i_x)x_r) \quad (24)$$

and

$$s_p = \frac{\mu}{\delta} i m_p \quad (25)$$

Equation (24) shows that, in equilibrium, the fraction of time spent by the rich as shopping time is actually a constant ( $\frac{\mu}{\delta}$ ) times the inflation tax that they pay<sup>8</sup>. Equation (25) leads to the same conclusion, this time regarding the poor.

This point, namely that the welfare costs of inflation under the shopping-time rationale can be seen as a mirror image of the inflation tax, is not new in the literature. As we mentioned in the introduction, it has been noted by Lucas (1993, p. 14, eq. 4.14) and also appears in Lucas (2000, p. 266)<sup>9</sup>. Equation (24) generalizes Lucas's finding for the case in which there is a second transacting balance in the economy.

The connection made by equations (24) and (25) adds a new dimension to our results here, by allowing a link between our approach to inflation and inequality, focusing solely on the shopping-time argument, and the more conventional argument, based on the differentiated inflation tax paid by both rich and poor. It also connects our results to the empirical evidence concerning the usual inflation-tax argument.

## 6 Conclusions

We have developed a simplified model, based on a shopping-time rationale, to investigate the effect of inflation on the Gini coefficient of income distribution. A basic assumption of the model is that some (cohorts of) consumers have access to a better transacting technology than others.

Our main conclusion is that under such assumptions a formal link between inflation and the Gini coefficient of income distribution can be theoretically proved. For transacting technologies in which the productivity of the interest-bearing asset is high enough, an increase of the inflation rate unequivocally leads to a deterioration of the income distribution. A link between our approach to the problem and the usual inflation-tax reasoning, which connects inflation to income inequality, has also been presented, as well as an example to illustrate our point.

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<sup>8</sup>The argument actually refers to the opportunity cost of holding the transacting assets  $m$  (given by  $i m_{r,p}$ ) and  $x$  (given by  $(i - i_x)x_r$ ), compared to holding bonds. But these are exactly the quantities to which the usual argument linking inflation and inequality refers. See also footnote 2.

<sup>9</sup>A non-numbered equation in page 266 of Lucas (2000) is equal to equation (25), for the case in which  $\mu/\delta = 1$ .

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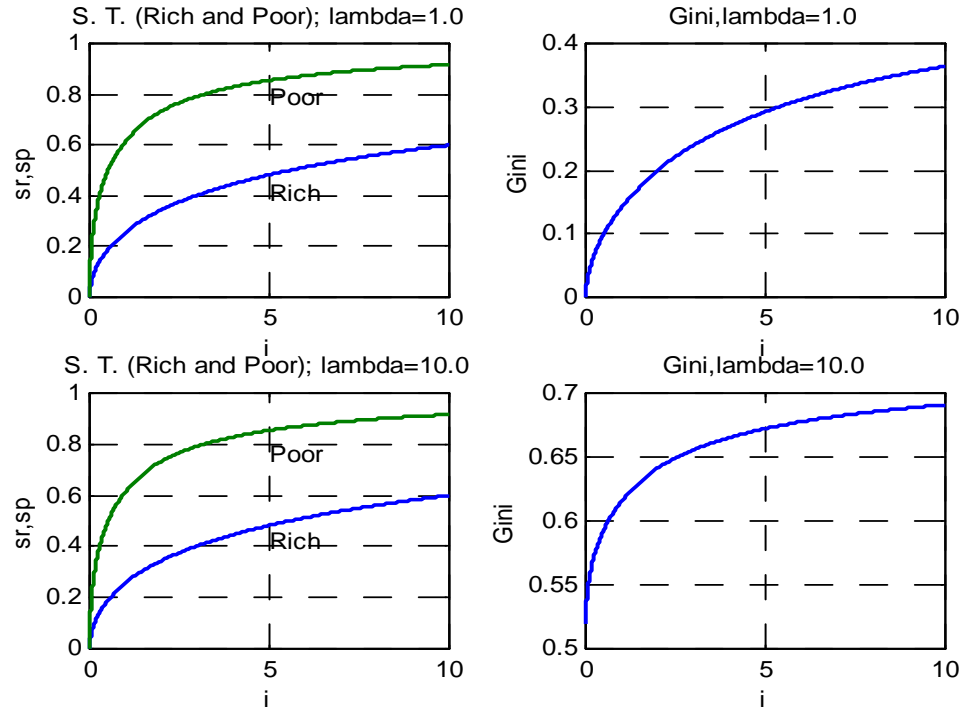


Figure 1: Shopping Times of Rich and Poor and Gini Coefficient as a Function of Nominal Interest Rates,  $\Lambda = 1$  and  $\Lambda = 10$ .



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