Serge de Valk

# Essays on Inflation Targeting under Fiscal Fragility

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# Essays on Inflation Targeting under Fiscal Fragility

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# Abstract

Indebted policymakers with limited budgets may face inflationary shocks forcing them to either (i) increase interest rates to have inflation on a pre-announced target or (ii) accept higher inflation to reduce debt costs. We propose a general equilibrium framework to model the inter-temporal trade-off between fiscal and monetary policy when forward-looking, rational, and fully informed agents finance public deficits. We show that doubts about the target's credibility increase expected inflation and pressure an altruistic policymaker to meet fiscal needs with inflation, depressing private consumption and GDP. Continued fiscal indiscipline inevitably causes inflation to overshoot the target as the policymaker fails to coordinate expectations.

**Keywords:** Monetary Policy, Fiscal Policy, Debt Policy

**JEL - Classification:** E52, E62, E63

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# 1 Introduction

The inflation targeting regime has been supporting central banks in their task of coordinating market expectations towards price stability. However, occasionally, inflation expectations may lose their anchor and diverge from the central bank's target. We argue that an altruistic policymaker that maximizes private agents' welfare might find it optimal to deviate from the inflation target in the context of a fragile fiscal situation, marked by high public-debt service and limited tax room.

We model a fiscal and monetary policymaker that may lose credibility and overshoot its target when it becomes fiscally challenging to maintain spending. The framework closely follows (COLE; KEHOE, 1996; COLE; KEHOE, 2000). In our model, we consider a closed economy with two types of agents that form rational expectations with complete information: an altruistic policymaker and private-agents. The policymaker acts jointly as a fiscal authority and as a central bank that targets inflation. Given it is imperfectly committed, it might decide to deviate from the pre-announced inflation target. Such decision is the result of a trade-off between inflating public debt away – to make fiscal room for spending – and keeping inflation on target – to avoid the economic costs caused by deviating. We show that target failures may happen within the fiscal fragility zone – marked by high debt and limited fiscal room for public expenditures. In the said zone, monetary and fiscal authorities are restricted by private-agents rational expectations and exposed to the loss of confidence in their commitment to the target. In the fiscal fragility zone the policymaker is vulnerable to confidence crises resulting in self-fulfilling target failures.

Our work has a similar message with regards to the fiscal limits on possible monetary policy achievements and the interdependence between fiscal discipline and price stability as in (SARGENT; WALLACE, 1981), (LEEPER, 1991), (SIMS, 1994), (WOODFORD, 1995), (LEEPER; LEITH, 2016), and (COCHRANE, 2017). Furthermore, the message with regards to the importance of the target in helping coordinate expectations is related to (ARAUJO; BERRIEL; SANTOS, 2016). The authors argue that a higher target can be beneficial when modelling the central bank as imperfectly committed, as opposed to a perfect commitment framework.

However, to the best of our knowledge, the literature on inflation targeting and confidence crises is relatively thin, although very relevant to countries facing structural fragilities. In canonical NK-DSGE models, such as (CHRISTIANO; EICHENBAUM; EVANS, 2005) and (SMETS; WOUTERS, 2007), central banks have an important role as they help reduce macroeconomic volatility. However, central banks are always committed

to the inflation target through a Taylor type of monetary policy rule and fiscal authorities are always responsible and never default. In the model we propose, however, the altruistic policymaker might find it optimal to abandon such a rule as the costs outweigh the benefits. Recently, (ARELLANO; MIHALACHE; BAI, 2019) have proposed to enrich such models by building on the structure of a NK-DSGE while allowing for sovereign default. Nevertheless, the central bank remains committed to the target within their framework. We hope to contribute to the literature by modeling an inflation targeting central bank that can be exposed to confidence crises and choose to deviate from the inflation target.

Our model is anchored in the literature on multiple equilibria. We closely follow models used in the literature on confidence crises in external sovereign debt markets as in (COLE; KEHOE, 1996; COLE; KEHOE, 2000) and (CALVO, 1988). Although we have chosen perfect information, there exists a large literature on the possibility of multiple equilibria in imperfect information settings such as (MORRIS; SHIN, 2002) and (ANGELETOS; WERNING, 2006).

As an example of such confidence crises and their impact on monetary policy we use the case of the presidential election race in Brazil in 2002. In the same year, Brazilian policymakers faced inflationary pressures when it became clear that the presidential candidate Lula da Silva would win. The perception was that Lula's victory would change the prior policy-framework that had sustained the transition from hyperinflation towards controlled and well behaved levels. Consequently, inflation expectations overshot the target's upper-bounds at all horizons relevant to the central bank's monetary policy. We interpret this episode as a confidence crisis triggered by the reduced credibility of the policymaker.

Based on this episode we calibrate our model in a numerical exercise. We show how a perceived change in preferences from private agents' consumption towards public spending can tilt a policymaker into the fiscal fragility zone where debt levels no longer give the necessary fiscal support to monetary policy. In said fiscal fragility zone, the policymaker may loose credibility and decide to deviate from the inflation target.

In response to rising inflation expectations, the outgoing and new administrations took several steps: i) in order to coordinate inflation expectations in the short run, they increased the target for 2003 in an extra meeting held on 6/27/2002 and unofficially again on 1/21/2003. ii) In 2003, responsible macroeconomic policies were sustained by public debt reduction, and iii) changes in the debt mix away from indexed bonds. Ultimately inflation expectations converged back to the target. The policy decisions made in Brazil in 2002-2003 to face the confidence crisis closely mirror the policy prescriptions of our model.

Finally, we perform several econometric exercises to verify some of the conclusions of our theoretical model. In the case of inflation expectations from professional forecasters in Brazil we do not find any evidence of debt being a relevant variable in the formation

of expectations. However, the primary fiscal balance was found to be relevant. A result similar to (CERISOLA; GELOS, 2005). Moving to a panel of countries that adopted inflation targeting regimes, we find some evidence of higher debt, in countries with lower revenues, causing higher deviations from the inflation target. Something that might not necessarily be restricted to inflation targeting regimes given that (KWON; MCFARLANE; ROBINSON, 2008) have found empirical evidence of inflation and debt growth being more positively correlated in countries with already high debt levels. We also find evidence that higher targets do, on average, reduce deviations and the probability of overshooting the target's upper-bounds.

In section 2 we set out the model and derive the recursive equilibrium. In our main theorem we show that an altruistic policymaker may choose an optimal inflation level above the target when in the fiscal fragility zone that is marked by high debt servicing costs and limited tax room. Moving to section 3, we specify functional forms and calibrate our model to the 2002 confidence crisis in Brazil. In this section we show how a perceived shock to preferences can cause the policymaker to fall into the fiscal fragility zone, leading private agents to adapt their inflation expectations to take into account the possibility of deviations from the inflation target. We also show that higher inflation targets can help the policymaker regain credibility. Notwithstanding the focus of the paper on coordination, we show that the numerical results are robust to including welfare costs of inflation. The econometric tests of the model are performed in section 4. We show evidence of the impact of fiscal indicators and the level of the inflation target on both deviations and the probability of overshooting the upper-bound of the target. Finally, section 5 presents concluding remarks.

# 2 Model

## 2.1 Basic Setup

We consider a closed economy with two types of agents living infinite periods and forming rational expectations with complete information: a policymaker and private-agents. The policymaker acts as a fiscal and monetary authority, choosing current inflation and one-period debt hold by private agents to finance itself. The policymaker is altruistic and maximizes the private agent's welfare and chooses each period whether to deviate from the target or not. Private agents receive a stream of fixed endowments. Each period they choose how much debt to hold and form expectations about next-period inflation taking into account an exogenous announced inflation target and the current debt level. When multiple equilibria are possible a sunspot variable determines the equilibrium.

### 2.1.1 Private-agents

We assume a continuum of infinitely lived private-agents that choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{2.1}$$

where $c_t$ is the private-agents' consumption in period $t$, $g_t$ is government spending on public goods, and $\beta$ is the inter-temporal discount rate. The usual non-negative consumption restriction applies. We assume that $0 < \beta < 1$ and make the following assumptions about the utility function:

**Assumption 1** *The utility function $u(c, g)$ is continuously differentiable and monotonically increasing, and strictly concave, in $g$ and linear in $c$. Furthermore, it satisfies $\lim_{g \to 0^+} u(g) = -\infty$.*

Each period, private agents receive a deterministic endowment $e$, which is taxed at a constant rate $\tau$ by the government, and receive payments on their bond holdings. The private-agents' budget constraint is given by:

$$c_t + d_{t+1} \leq (1 + r_t)d_t + \alpha_t(1 - \tau)e \tag{2.2}$$

where $d_{t+1}$ are one-period bonds bought in $t$ and $d_t$ are the previous period bond holdings paying the gross interest rate $(1 + r_t)$. The productivity factor $\alpha_t$ should be understood as a reduced form capturing the cost of deviating from the inflation target on economic

activity. It will be properly defined further on. For now we will limit ourselfs to defining the penalty $\alpha(.)$ as a function of the inflation target $\pi^a$, current inflation $\pi_t$, and dependent on its past value $\alpha_{-1}$. We furthermore state that the penalty function will have the following properties:

**Assumption 2** *The penalty $\alpha(\pi - \pi^a)$ is continually differentiable on $\pi \neq \pi^a$, strictly decreasing in the absolute deviation $|\pi - \pi^a|$ such that $0 < \alpha_t < 1$ when $\pi \neq \pi^a$ and $\alpha_t = 1$ when $\pi = \pi^a$.*

As usual, a no-Ponzi condition is imposed on debt holdings. Given a linear utility in $c$, in equilibrium the ex-post real interest rate will be given by:

$$r_t = \frac{1 + \pi_t^e}{1 + \pi_t} \frac{1}{\beta} - 1 \tag{2.3}$$

where $\pi_t^e = \mathbb{E}(\pi_t | t - 1)$ is the expected inflation for period $t$ that private-agents formed in period $t - 1$ and where $\pi_t$ is the current period inflation. In the case of multiple equilibria, private agents will form expectations over the probability of each outcome happening. In the case of multiple equilibria the policymaker either deviates or keeps inflation on target. Inflation expectations will therefore be of the form $\pi_t^e = f\pi^D + (1 - f)\pi^a$ where $f$ is the probability of the policymaker deciding to deviate from the inflation target due to an adverse situation.

When the current inflation rate is equal to expectation, $\pi_t = \pi_t^e$, the gross ex-post real interest rate will be equal to the inverse of the inter-temporal discount rate, $1/\beta$. An inflationary surprise, defined by $\pi_t > \pi_t^e$, reduces the ex-post real interest rate and consequently the payments the policymaker has to make on its debt. Such a partial default on debt gives additional fiscal room for government spending.

### 2.1.2 Policymaker

We assume an altruistic policymaker that chooses both fiscal and monetary policies to maximize private agents' utility. As a monetary authority, the policymaker chooses the inflation rate $\pi_t$ and, as a fiscal authority, next period's debt $D_{t+1}$:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{2.4}$$

subject to the budget constraint:

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \alpha_t \tau e \tag{2.5}$$

where $D_t$ is last period's debt. Each period, non-negative spending $g_t$ and the repayments on the previous period obligations are financed through taxes $\tau e$ and the issuance of new

debt $D_{t+1}$. If the monetary authority deviates from inflation target $\pi_a$, the economy suffers a permanent negative shock to the endowments through $\alpha_t$ such that a deviation from the target, $\pi_t > \pi_a$, implies $0 < \alpha_t < 1$. Furthermore, assumption 2 gives us an increasing cost in deviations as opposed to a fixed cost as in (ARAUJO; BERRIEL; SANTOS, 2016).

In each period, the policymaker can satisfy the budget constraint by: i) adjusting expenditures, ii) issuing new debt $D_{t+1}$; iii) partially defaulting on debt through an inflationary surprise ($\pi_t > \pi_t^e$) and rolling-over the remaining debt. Rational expectations govern the strategic interactions between the policymaker and private agents. As in (COLE; KEHOE, 2000) self fulfilling multiple equilibria may occur. Conditional on the debt level the best response from the policymaker's perspective may depend on the expectations of private agents. More specifically, if they expect a deviation from the target, the best response will be to deviate, and if they expect no deviations the best response will be to keep inflation on target. In this case we consider an exogenous sunspot variable $\zeta_t$ to determine the selection of the equilibrium.

In the absence of uncertainty it can be shown that the policymaker's optimal fiscal policy is to keep debt constant as stated in proposition 1.

**Proposition 1** *Let $u(c, g)$ satisfy assumption 1. Then the optimal fiscal policy in the absence of uncertainty is to maintain debt constant.*

Proof: See appendix A.1.

We assume that private-agents behave competitively in making their choice of $d_{t+1}$ and that they do not think that their individual actions affect next-period's aggregate state. We therefore distinguish between the individual decision $d_{t+1}$ and the aggregate value $D_{t+1}$. In equilibrium, because all consumers are identical and start with $d_0 = D_0$, $d_{t+1}$ matches $D_{t+1}$ for any $t$, making debt markets clear. We, however, have to impose restrictions, from a technical perspective, on the initial debt in order for the policymaker to not have an empty choice set $\{\pi^t, D_{t+1}\}$:

**Assumption 3** *The initial debt level $D_0 \in [0, \overline{D}]$ should be such that $\overline{D}$ leaves the policymaker with a non-empty set of feasible choices $\{\pi^t, D_{t+1}\}$.*

The need to impose this restriction comes from the positive spending and consumption restrictions together with a non-Ponzi condition on debt. For very high initial debt levels the policymaker could be left with no option. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, leaving no space for spending. Even if the policymaker were to partially default on its debt payments, it would still not be able to meet future positive spending restrictions due to the high future debt servicing costs and the inability to make use of inflationary surprises again.

### 2.1.3 Timing

At the beginning of each period, the policymaker implements the actual inflation rate $\pi_t$ and chooses how much debt $D_{t+1}$ to sell. Thereafter, private-agents form their expectations $\pi^e_{t+1}$ about next-period's inflation and decide how much debt to hold $d_{t+1}$. Finally, next-period's uncertainty is resolved through the realization of the sunspot variable $\zeta_{t+1}$ selecting next period's equilibrium as detailed below:

$1^{st}$: Policymaker chooses actual inflation, $\pi_t$;

$2^{nd}$: Policymaker chooses next debt-level, $D_{t+1}$;

$3^{rd}$: Private-agents form next-period inflation expectation, $\pi^e_{t+1}$, and choose the amount of next-period debt $d_{t+1}$ to hold;

$4^{th}$: Next-period sunspot variable, $\zeta_{t+1}$, is realized.

Given this timing, private-agents may face uncertainty about which equilibrium will be selected next-period when forming their inflation expectations.

## 2.2 Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker that might choose an optimal inflation level to transfer resources for increasing public spending in the face of high debt servicing costs. In the literature several different channels through which debt impacts inflation can be identified. (CALVO; GUIDOTTI, 1990) model a policymaker that can finance spending either through lump-sum taxes or inflation. Having concave desutility in both, the policymaker may opt for more inflation as the burden of debt increases. In (AGUIAR et al., 2013) a policymaker can inflate away local currency debt hold by non-residents in order to increase domestic consumption. (COCHRANE, 2001) models a policymaker choosing nominal debt and private agents forming expectations about real surpluses. Inflation arises as the price of nominal debt needs to equate the net present value of future surpluses.

In each period the policymaker may choose to deviate from the inflation target $\pi^a$. Private agents understand this when forming their expectations $\pi^e_t$. The discretionary inflation chosen by the policymaker is the result of a trade-off between increasing spending against the costs of reducing consumption and the loses due to the costs of deviating from the inflation target. Let $\pi^D$ be the endogenous and optimal level of discretionary inflation chosen in $T$.

When deviating from the inflation target we make the simplifying assumption that the policymaker will roll over debt as stated in assumption 4. This assumption limits the use of inflation to increasing current spending through inflation. In the appendix A.2 we

show that the policymaker might prefer to use inflation to also slightly decrease future debt, and debt servicing costs, in order to have higher future spending. Hence, by taking away an additional incentive to use inflation this assumption actually works against the policymaker choosing inflation above the inflation target as we want to show later on.

**Assumption 4** *When the policymaker decides to deviate at time $T$ from the inflation target $\pi^a$, then debt will be rolled over such that $D_t = D_T, \forall t > T$.*

Once the policymaker deviates from the inflation target we assume that the private-agents loose confidence in the commitment of the policymaker to the inflation target. Private-agents update the probability given to the policymaker deviating next period, as stated in in assumption 5, by setting it equal to 1.

**Assumption 5** *Private agents will expect $f = 1, \forall t > T$ after the policymaker deviates from the inflation target $\pi^a$ in period $T$.*

Consequently, after the policymaker deviates, the economy enters a steady state as there is no longer any uncertainty to be resolved making maintaining debt constant the optimal fiscal policy as stated in proposition 1. Finally, the penalty function takes the value $\alpha(\pi)$ when deviating and remains so thereafter. The problem the policymaker resolves, when defining the level of discretionary inflation, can therefore be written as:

$$\pi^D = \underset{\pi}{\operatorname{argmax}} \quad \underbrace{u(c_T, g_T)}_{\text{possibly increasing in } \pi} + \underbrace{\frac{\beta}{1-\beta} u(c, g)}_{\text{decreasing in } \pi}$$

subject to

$$\begin{aligned} g_T &= D\left(1 - \frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta}\right) + \alpha(\pi)\tau e \\ g &= D\left(1 - \frac{1}{\beta}\right) + \alpha(\pi)\tau e \\ c_T &= \left(\frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e \\ c &= \left(\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e \end{aligned} \tag{2.6}$$

Given rational expectations, in equilibrium, $\pi^D$ is optimal given $\pi^e$ and vice versa. We later numerically solve this problem by writing it as as a fixed point. Proposition 2 states that the problem has a solution.

**Proposition 2** *Let the utility function $u(c, g)$ and the penalty function $\alpha(\pi - \pi^a)$ satisfy respectively assumptions 1 and 2. If the universe of possible inflation choices is defined on*

*the compact set* $[0, \overline{\pi}]$ *where* $\overline{\pi} > 0$ *is some upper limit. Then there exists a discretionary inflation level* $\pi^D$ *such that* $\pi^D$ *is optimal given private agents' inflation expectations* $\pi^e$ *and vice versa.*

Proof: see appendix A.3.

The first order condition of the policymaker's problem for choosing discretionary inflation will be given by:

$$u_{c_T}\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'(1-\tau)e\right)+u_{g_T}\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'\tau e\right) \\ +\frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e+u_g\alpha'\tau e\right)=0 \tag{2.7}$$

where $\alpha'$ is the first order derivative of the penalty function with respect to the deviation $\pi - \pi^a$. The first term represents the short-term marginal loses stemming from reduced consumption caused by i) the inflationary surprise and ii) the productivity shock to endowments. The second term may be positive after discounting marginal losses due to lower tax revenues from the marginal benefits of higher spending through the inflationary surprise. Finally, the last term illustrates the long lasting effects of the productivity shock to the economy causing both a loss of consumption and government spending.

The policymaker will choose discretionary inflation levels above the inflation target when the benefits outweigh the costs. If there were no costs to transferring resources through inflation, a benevolent policymaker would strive to equate both the marginal utility of consumption to the marginal utility for government spending. However, the policymaker will have to take into account the costs of deviating from the target when choosing the optimal allocation between spending and consumption. Theorem 1 gives the conditions for the policymaker to choose $\pi^D > \pi^A$. Intuitively, whether a higher marginal utility for public spending $g$ is sufficient will depend on the marginal costs of deviating. The lower the marginal costs, the more likely the policymaker will choose to use discretionary inflation to reduce debt servicing costs. A higher debt level will furthermore pressure the available space for spending and increase the marginal utility for $g$ given private agents' strictly concave utility in $g$ as specified in assumption 1.

**Main Theorem 1** *Suppose the utility function, penalty, and the initial debt level respectively satisfy assumptions 1, 2, and 3. For sufficiently high marginal utility of spending, debt levels, and a sufficiently low marginal cost of deviating the policymaker will choose a discretionary inflation higher than the inflation target.*

Proof: see appendix A.4.

An altruistic policymaker, maximizing private agent's welfare, may hence choose to deviate from the inflation target when it finds itself with limited fiscal room to finance public spending.

## 2.3 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private-agents choose their actions sequentially. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \alpha_{-1})$ is public since the aggregate debt $D$, the expected inflation $\pi^e$ for the current period, the realization of the sunspot variable $\zeta$, and the past penalty $\alpha_{-1}$ have all been determined in the previous period. The policy choices $(\pi, D')$, the expected inflation $(\pi^{e\prime})$ for next period, the individual debt holdings $d'$ for next period, jointly with $s$, determine the equilibrium. We denote by $\pi(.)$ and $D(.)$ the inflation and debt policy functions, and by $\pi^e(.)$ the inflation expectation-function, all yet to be defined.

To define a recursive equilibrium, we work the timing of actions in each period backward. We start the definition of a recursive equilibrium with private-agents as they move last. When forming expectation $\pi^{e\prime}$ at the end of any period, private agents know all the parameters of the economy, his individual public debt holding $d$, the aggregate state $s$, the policymaker's offering of new debt $D'$, the current period inflation $\pi$, and the policymaker's optimal policy functions. The private agent's value function is defined by the following functional equation:

$$
\begin{aligned}
V^{pa}(s, d, \pi, D') = & \max_{c,d'} u(c, g) + \beta \mathbb{E} V^{pa}(s', d', \pi', D'') \\
& \text{subject to} \\
& \qquad c + d' \leq \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} d + \alpha(\pi, \pi^a, \alpha_{-1})(1-\tau)e \\
& \qquad s' = \Big(D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta'\Big) \\
& \qquad \pi' = \pi(s') \\
& \qquad D'' = D(s') \\
& \qquad c \geq 0 \\
& \qquad d' \geq -A \\
& \qquad \zeta' \sim U(0, 1)
\end{aligned}
\tag{2.8}
$$

in which $A > 0$, rules out Ponzi schemes in favor of private-agents but does not bind in equilibrium for any positive initial debt condition. The penalty function $\alpha(.)$ is a function of its previous value $\alpha_{-1}$, the inflation target $\pi^a$, and current inflation $\pi$. Finally, the sunspot variable selecting next period's equilibrium is uniformly distributed on the interval $(0, 1)$ such that $\zeta' < f$, with $0 < f < 1$, selects one equilibrium and $\zeta' \geq f$ the other. The

parameter $f$ can therefore be seen as the exogenous probability of the first equilibrium occurring.

Each period, after the policymaker decided on how much debt it offered $D'$ and defined the period's inflation$\pi$, private agents decide on how much debt to hold. Let $d'(s, d, \pi, D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, $\pi$ is perfectly anticipated and the real return is always $1/\beta$. However, when multiple equilibria are possible, private-agents do not know the realization of the sunspot variable when forming expectations and what the policymaker will opt to do.

When forming inflation expectations private agents look at what the policymaker could do next period. Their expectations can be written as $\pi^e(s, d, \pi, D') = \mathbb{E}\pi(s')$. When forming expectations, the set $(D', \pi^{e\prime}, \alpha) \in s'$ is known to private agents. Hence, the only unknown variable on which private-agents form their expectations is the realization of the sunspot variable $\zeta'$. Integrating out the sunspot variable, whose distribution is commonly known, we obtain:

$$\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \alpha) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \alpha) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{2.9}$$

where $f$ is the exogenous probability of the adverse equilibrium occurring.

The policymaker chooses, at the beginning of the period, inflation $\pi$ and debt issuance $D'$ given the state $s$. The policymaker knows that next period's debt level affects the private-agents' inflation expectations and will resolve the following problem:

$$\begin{aligned} V^p(s) = & \max_{\pi, D'} u\Big(c(s, d, \pi, D'), g\Big) + \beta \mathbb{E} V^p(s') \\ \text{subject to} & \\ & g + \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} D \leq D' + \alpha(\pi, \pi^a, \alpha_{-1})\tau e \\ & s' = (D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta') \\ & g \geq 0 \\ & \zeta' \sim U(0, 1) \end{aligned} \tag{2.10}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions $V$ for the representative private-agent, $V^{pa}$, and for the representative policymaker, $V^p$; functions $c()$ and $d'()$ for the private-agents' consumption and saving decisions; functions $\pi()$ and $D'()$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e()$; and an equation of motion for the aggregate debt level $D'$ such that the following holds:

- Given $D'$ and $\pi$, $V^{pa}$ is the value function for the solution to the representative private-agent's problem, and $c$, $d'$ and $\pi^{e\prime}$ are the maximizing choices when $d' = D'$;
- Given $\pi^e$, $V^p$ is the value function for the solution to the policymaker's problem, and both $D'$ and $\pi$ are the maximizing choices;
- $D'(s)$ equals $d'(s, d, \pi, D')$.

Our definition of an equilibrium is similar to (COLE; KEHOE, 1996) and (COLE; KEHOE, 2000) and is restricted to a Markov equilibrium. Hence, the agents' future conditional plans can be derived from their policy functions.

## 2.4 The Fiscal Fragility Zone

In this section, we will show that the ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming inflation has always been on target, then 3 different scenarios can be drawn[1]. When conditioning the policymaker's value function on private-agents' expectations and on its own choice to deliver or not the target, we define the following 3 scenarios:

a) $V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D) \rightarrow \pi^e = \pi = \pi^a$

b) $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot

c) $V^p(D|\pi = \pi^a, \pi^e = \pi^a) \leq V^p(D|\pi = \pi^D, \pi^e = \pi^a) \rightarrow \pi^e = \pi = \pi^D$

In the first case, the policymaker finds it better to always keep inflation on target, even when private agents think it will not. Consequently, there is only one equilibrium possible where private-agents have faith in the policymaker delivering on-target inflation. Conditional on $(\pi^e, \alpha_{-1})$ and given that the sunspot $\zeta$ is simply disregarded, as only one equilibrium is possible, the only important variable defining the policymaker's value function $V^p$ is the debt level $D$. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

The more interesting scenario is when multiple equilibria, akin to self-fulfilling target failures, are possible. If private-agents believe the target will be delivered, then the policymaker will indeed prefer to do so. On the contrary, in the face of adverse expectations the policymaker chooses to deviate. The interval of debt $\left(\underline{\mathrm{D}}\ \overline{\mathrm{D}}\right)$, for which there exist multiple equilibria, is what we call the fiscal fragility zone. In this zone, private agents have doubts about the commitment of the monetary authority to the target. For debt levels under $\underline{\mathrm{D}}$ the target is perfectly supported.

[1] Given the simplifying assumptions we made, a past deviation implies a loss of credibility in all subsequent periods.

Interestingly, higher inflation targets change the tradeoffs the policymaker has to make. To see this let us recall the real interest rate on bonds from equation 2.3: $r_t = \frac{1+\pi_t^e}{1+\pi_t}\frac{1}{\beta} - 1$. In the fiscal fragility zone the inflation expectations will be given by $\pi_t^e = f\pi_t^D + (1-f)\pi^a$. Hence, the real interest rate in the fiscal fragility zone, when the policymaker deviates, will be given by:

$$r_t = \frac{1 + f\pi_t^D + (1-f)\pi^a}{1+\pi_t^D}\frac{1}{\beta} - 1 \tag{2.11}$$

Suppose that the probability of a crisis occurring is not certain, that is $f < 1$. Then the partial equilibrium effects of increasing the target on the marginal real interest rate will be given by:

$$\frac{\partial}{\partial \pi^a}\frac{\partial r_t}{\partial \pi_t^D} = -\frac{1-f}{(1+\pi_t^D)^2}\frac{1}{\beta} < 0 \tag{2.12}$$

The marginal ability of the policymaker to transfer resources through inflation therefore decreases in the target, changing the trade-off determining discretionary inflation. The marginal benefits of using discretionary inflation will be reduced given the lower marginal capacity to transfer resources. Given certainty properties of the utility and penalty functions, increasing the target can therefore help reduce deviations and consequently the real interest rate as stated in proposition 3.

**Proposition 3** *Let $u(c,g)$ satisfy assumption 1 and also be separable and linear in $c$. Let the penalty function $\alpha(\pi - \pi^a)$ satisfy assumption 2 and do not present decreasing marginal costs such that $\alpha'' \leq 0$. Then for some reasonable, but sufficiently high probability of deviation $f$ and for a marginal utility in spending that does not decrease too fast, increasing the inflation target $\pi^A$ decreases the real interest rates in the fiscal fragility zone when the policymaker keeps inflation on target.*

Proof: See appendix A.5.

A policymaker with a higher inflation target will choose a smaller deviation from the target when private-agents start doubting the target. Consequently, as deviations decrease, it will face lower real interest rates on its bonds in the fiscal fragility zone.

## 2.5 Inflation-Indexed Debt

It is not unusual for governments to issue inflation indexed bonds. We will here look at the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the actions' timing in order to give private agents all the needed information to perfectly anticipate the policymaker's decisions. By

allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will simply pay real interest rate $\frac{1}{\beta}$ in all states of nature.

$1^{st}$: Policymaker chooses actual inflation, $\pi_t$;

$2^{nd}$: Policymaker chooses next debt-level, $D_{t+1}$;

$3^{rd}$: Next-period sunspot variable, $\zeta_{t+1}$, is realized;

$4^{th}$: Private-agents form next-period inflation expectation, $\pi^e_{t+1}$, and choose the amount of next-period debt $d_{t+1}$ to hold.

With this new timing, private agents' information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations $\pi^e$, given information set $s'$, will be such that $\pi^e(s') = \pi(s')$ which is the policymaker's next period inflation choice. As the policymaker's choices are anticipated, it is no longer possible to transfer resources from the private-agents to itself in the event of a bad sunspot.

An equilibrium for this model and given this timing is obtained through a penalty on endowments that is increasing in the deviation size as in assumption 2. Larger deviations result in larger endowment losses and, if sufficiently high, the discretionary inflation level will be well defined. However, it is clear that in the absence of the penalty $\alpha$ there would not be any solution for the discretionary inflation problem from 2.6.

We will exploit the differences between indexed and nominal bonds in next section's numerical exercises.

## 2.6 Conclusion

Indebted policymakers have limited budgets and private-agents taking into account fiscal fundamentals may come to doubt their commitment to the inflation target. When policymakers find themselves in such a fiscally fragile situation they can either (i) accept a higher real interest rate to have inflation on the pre-announced target (ii) accept higher inflation. Coordination with fiscal authorities to ensure credible monetary policy is already a well-established result. In our model, high debt levels may lead the policymaker to become vulnerable to adverse shocks. Although the policymaker is altruistic, the optimal policy might be to deviate from the inflation target. Finally, when monetary policy is being doubted by private agent's, a higher target may help coordinate expectations and decrease real interest rates.

# 3 Quantitative Analysis

In this section we calibrate the model, previously defined, based on the 2002 confidence crisis in Brazil. The presidential election race of 2002 is an interesting study case in that the candidate most likely to win was seen as running on a platform that could deteriorate the fiscal situation. Professional forecasters, surveyed by the central bank, saw inflation overshooting the target for all horizons. The loss of credibility of the inflation target, in the face of a perceived fiscally fragile situation, is the type of event that our model aims to capture.

Based on the calibrated model we show that an altruistic, but indebted, policymaker may choose discretionary inflation above the target. When monetary policy is susceptible to self-fulfilling target failures, the policymaker may either prefer to i) run down debt, ii) keep debt constant, or iii) run up debt depending on its fiscal situation. We show that when debt levels are sufficiently low the policymaker's optimal policy is to exit the fiscal fragility zone, marked by high debt and little tax room, by reducing debt levels. By so doing it will eventually be able to avoid the higher real interest rate on sovereign bonds and the non-null probability to suffer an adverse expectations shock. However, if debt gets too high, the optimal fiscal policy is to run-up debt. Eventually the policymaker will be subjected to an adverse shock where private agents loose confidence in the inflation target. Such debt levels do, therefore, no longer support monetary policy credibility.

Higher inflation targets are shown to remain perfectly credible up to higher levels of debts. Furthermore, we show that deviations are decreasing in the inflation target. Consequently, the real interest rate in the fiscal fragility zone will be lower for higher targets. An indebted policymaker with pressured fiscal space to finance public spending could therefore benefit from a higher and more credible inflation target.

Importantly, these results based on deviation costs are robust to introducing a welfare cost of inflation. The economy will not only be penalized when the policymaker deviates, but also from the overall inflation level even when on target. The welfare gains from a higher, but more credible, inflation targets will however be offset at some point by the cost of higher inflation on the economy.

Finally, we show that an MIT shock to preferences can result in inflation expectations overshooting the target as observed during the 2002 confidence crisis in Brazil. We, furthermore, show that the successfully implemented measures to regain a credible monetary policy are also those suggested based on our calibrated model.

## 3.1 Functional Forms

We first start by defining the functional forms of both the private agent's utility and the penalty functions. We define the private agent's utility as a weighted average of a linear consumption and logarithmic government spending utility, quite similar to what can be found in (COLE; KEHOE, 2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as some relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) log(g_t) \tag{3.1}$$

The penalty function for deviating mirrors observations made in the literature on the non linear impacts of inflation on GDP. More specifically, (BARRO, 1995) and (SAREL, 1996) found negative impact of high inflation levels on economic growth and (BOYD; LEVINE; SMITH, 2001) on the financial sector. We furthermore define a lower bound for the penalty, captured by the parameter $\kappa$, to keep the size of the economy above $(1 - \kappa)e$ as $\lim_{\pi \to \infty} \alpha(\pi, .) = (1 - \kappa)$. The penalty $\alpha_t$ is defined as:

$$\alpha_t = \begin{cases} 1 & \text{if } \alpha_{t-1} = 1, \pi_t = \pi_a \\ (1 - \kappa) + \kappa e^{-(c_0 + c_1(\pi_t - \pi_a)^2)} & \text{if } \alpha_{t-1} = 1, \pi_t \neq \pi_a \\ \alpha_{t-1} & \text{otherwise} \end{cases} \tag{3.2}$$

As is usual in the literature on defaults we assume a perpetual penalty after deviating from the target. Hence, the penalty is conditional on its previous value: if $\alpha_{t-1} \neq 1$ then $\alpha_t = \alpha_{t-1}$. Including a fixed cost to deviating changes the policymaker's optimal fiscal policy by increasing the incentives to exit the fiscal fragility zone and regain perfectly credibility for its inflation target.

## 3.2 Calibration

Our model is calibrated on yearly data in order to match the usual time-frame targeted by central banks. For the discount factor, we use the historical average of the ex-post real interest rate for the period 1996-2019 [1]. The 2002 general government revenue over GDP is used as a proxy for the imposition rate on the economy's endowments. The exogenous crisis probability is calibrated on the country risk, captured by the EMBI + Brazil, around election time. The endowments $e$ were chosen to represent a relatively poor government looking to increase spending. For the baseline exercises, we choose the neutral value of parameter $\rho$, that is $\rho = \frac{1}{2}$. The remaining parameters referring to the penalty function were chosen so as to obtain a crisis zone around 70% of debt, matching gross debt levels in 2002, and reasonable levels of discretionary inflation $\pi^D$.

[1] Using inflation indexed bonds, such as NTN-C or NTN-B, around 2002 would give similar results.

| Parameter | Value | Meaning | Calibration |
|---|---|---|---|
| $\beta$ | .915 | Discount factor | Ex-post 1996-2019 real interest rate |
| $\tau$ | 35% | Tax rate | General gov. revenue in % of GDP |
| $\pi_a$ | 3.5% | Inflation target | 2002 BCB target |
| f | 20% | Crisis prob. | EMBI+ Brazil on 10/2002 |
| e | 1.5 | Endowment | Expansive government |
| $\rho$ | .5 | Pref. for consumpt. | Neutral value |
| $\kappa$ | 20% | Limit to TFP cost | Crisis zone at a 70% debt level |
| $c_0$ | .05 | Fixed cost | Crisis zone at a 70% debt level |
| $c_1$ | .5 | Variable cost | Crisis zone at a 70% debt level |

Table 1 – Parameters of the Baseline Model

## 3.3 Results

### 3.3.1 Fiscal Fragility Zone and Nominal Interest Rates

An indebted and altruistic policymaker that chooses inflation optimally may decide to deviate from the target in the event of an inflationary shock. With our calibrated model the policymaker becomes subject to such expectation shocks after it reaches a debt level of 70%. We furthermore obtain discretionary inflation spanning the interval $\pi^D \in [4, 20]$. Up to 70% of debt/GDP, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower-bound, multiple equilibria are possible as the policymaker may decide to deviate. Taking this probability into account, private-agents will demand higher nominal interest rates on government bonds once the policymaker exceeds a certain debt level as depicted in figure 1.

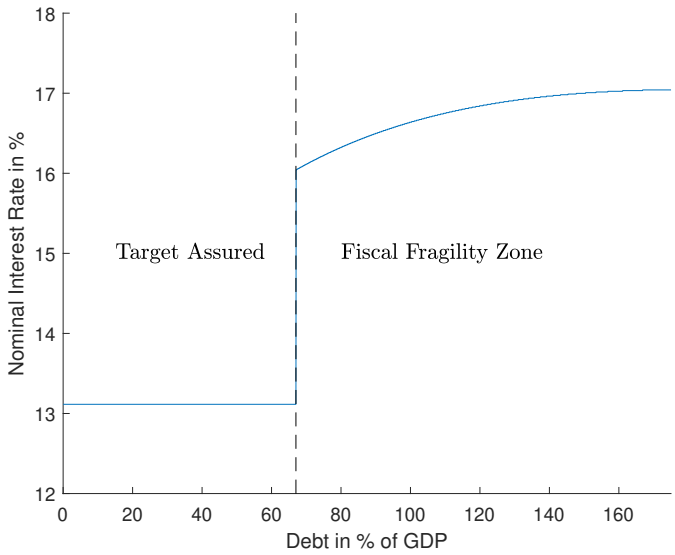


Figure 1 – Nominal Interest Rates and the Fiscal Fragility Zone

### 3.3.2 Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the fiscal fragility zone, it prefers to maintain debt levels constant as shown in proposition 1. Within the fiscal fragility zone, it might either: i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain debt levels constant; or iii) increase its debt in order to maintain a given spending level (see figure 2). Those results are similar to (COLE; KEHOE, 2000).

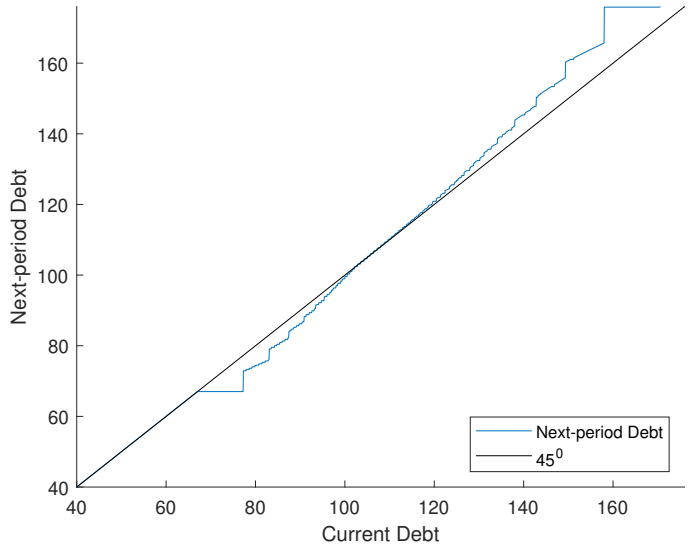


Figure 2 – Debt Policy Function

The policymaker chooses a fiscally responsible debt path in order to avoid the expected endowment loss from deviating from the inflation target in the eventuality of adverse inflation expectation. Nevertheless, the fiscal room available to the policymaker may shrink to an extend where it becomes more interesting to run-up debt in order to maintain spending. As it runs up debt, it eventually suffers an adverse shock and looses credibility. By opting to run up debt the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

### 3.3.3 Coordinating Expectations Through the Target

Higher inflation targets may help improve credibility of monetary policy as they increase the costs of deviating to attain a given inflationary transfer of resources. The level of the inflation target can therefore help coordinate private agents' expectations. First, as the cost of deviating depends on the target level. Second, as private agents use the inflation target to form expectations in the fiscal fragility zone. The target, therefore, functions as a nominal anchor for expectations. Proposition A.5 also stated that under some conditions deviations $\pi^D - \pi^A$ would decrease in the inflation target, thereby reducing the real interest

rate in the fiscal fragility zone. In figure 3 we show a sensitivity analysis of the deviations to changes in the inflation target.

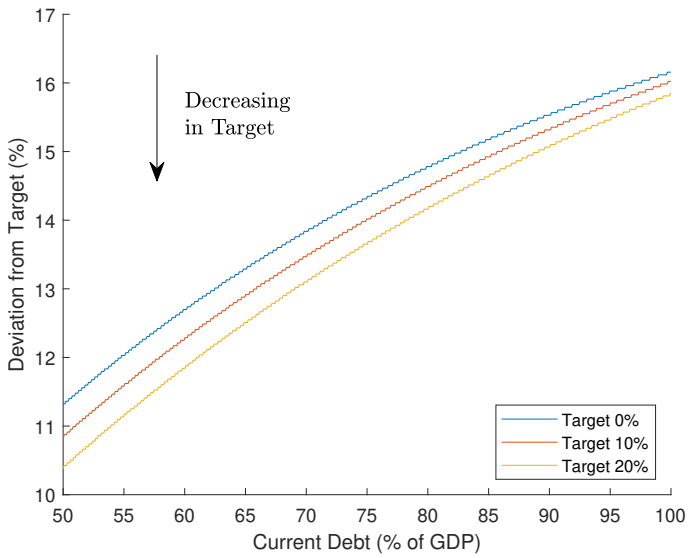


Figure 3 – Sensitivity: Deviations to Inflation Target

Let us now look at what happens to the fiscal fragility zone's lower bound $\underline{D}$ defined by $V^p(\underline{D}|\pi = \pi^a, \pi^e = \pi^D) = V^p(\underline{D}|\pi = \pi^D, \pi^e = \pi^D)$. For initial debt levels below the lower bound $\underline{D}$ the policymaker will have a perfectly credible target as it will prefer to keep inflation on target regardless of the private-agent's expectations. Above the lower bound private-agents may doubt its commitment. As deviations decrease in the target, it becomes less costly to keep inflation controlled for a given level of debt. This effect in turn increases the credibility of the inflation target as it remains fully assured up to higher levels of debt as shown in figure 4.

### 3.3.4 Inflation-Indexed Debt

Indexed debt was defined by taking away the uncertainty about which equilibrium would be selected next period by revealing the realisation of the sunspot variable to the private-agents when forming expectations. Using such a timing, private-agents are always able to correctly anticipate inflation and hence obtain a constant real interest rate on their bond holdings. We show that indexed debt, so defined, comes with higher inflation. The policymaker, unable to use inflation to partially default when subjected to a negative expectations shock nonetheless attempts to transfer resources. Private agents' correctly anticipating this adapt their expectations, leading to higher levels of discretionary inflation. The difference in the optimal inflation chosen by the policymaker when its debt stock is either nominal or inflation indexed is depicted in figure 5.

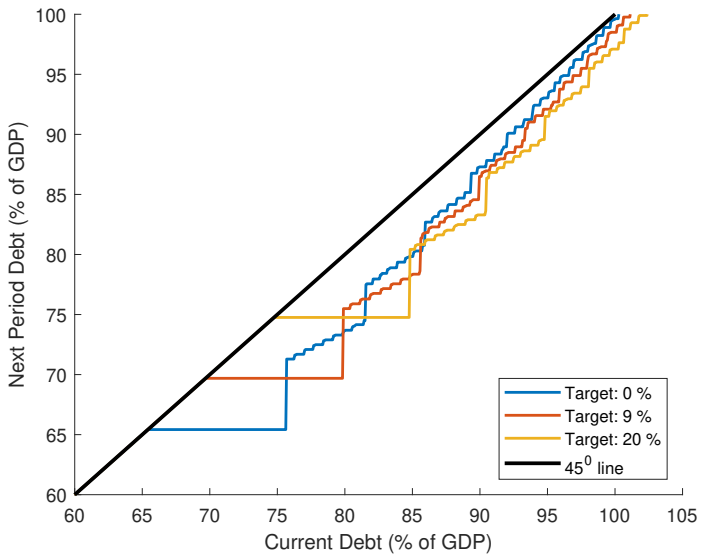


Figure 4 – Sensitivity: Inflation Target and the Fiscal Fragility Zone

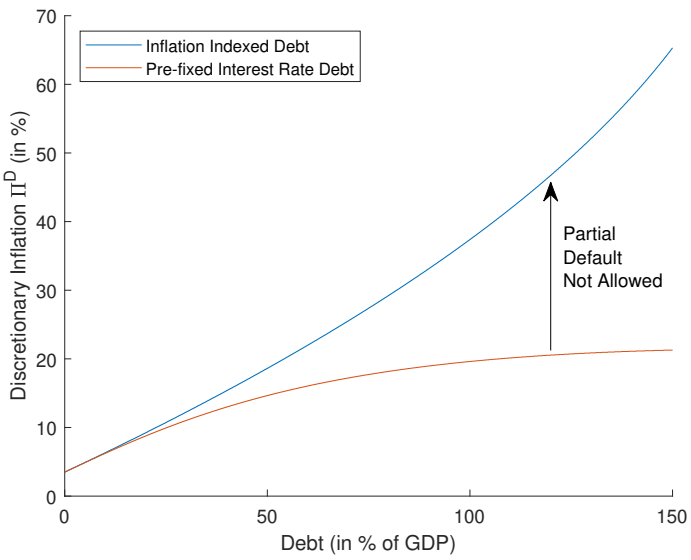


Figure 5 – Discretionary Inflation

Intuitively, the higher discretionary inflation resulting from this timing may change the credibility of the policymaker's inflation target, for a an initial debt stock, as the cost of maintaining the target increases in discretionary inflation under adverse expectations. The policymaker facing higher inflation expectations would have to pay a higher real interest rate to keep inflation on target.

Debt levels $D \in \{D : V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D)\}$ support the inflation target with certainty. It is easy to see that the costs of keeping inflation on target when private agents expect a deviation is much higher. The reduced fiscal room, as the policymaker faces higher debt servicing costs, results in lower spending. Given

theorem A.4 we have higher marginal utility for spending than for consumption when the policymaker deviates. Hence, a decrease in spending would mean a decrease in welfare: $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$ therefore decreases. This effect is, given our calibration, much more pronounced than the loss of welfare related to the higher productivity shock from the higher deviation. Hence, for any $D$, $V^p(D|\pi = \pi^D, \pi^e = \pi^D)$ lowers, but less than the reduction in $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$. Consequently the lower bound of the fiscal fragility zone $\underline{D}$ decreases when debt is indexed, indicating a reduced credibility of the target as it is fully supported by a reduced set of initial debt levels.

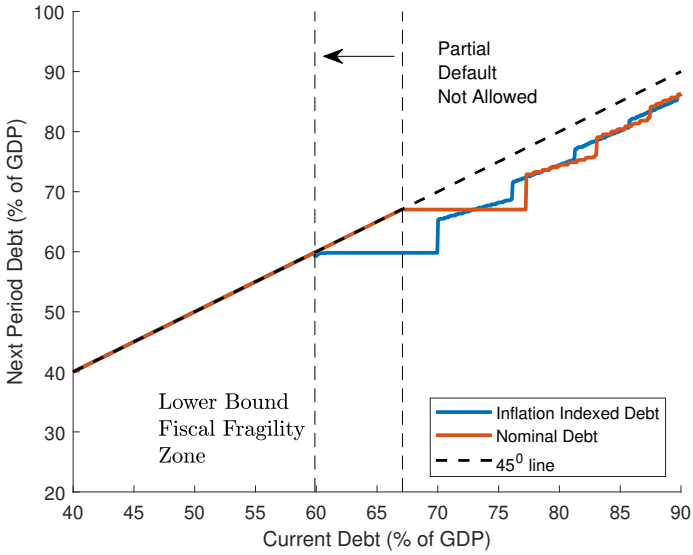


Figure 6 – Sensitivity: Lower Bounds

### 3.3.5 Preference for Spending

In this sensitivity analysis we will analyse the impact of a shock to preferences. This will later be used to connect our model with the situation observed in Brazil during the 2002 confidence crisis.

Suppose that the policymaker's preferences shift towards giving more weight to public spending. Based on the utility function defined in 3.1, decreasing $\rho$ would be tantamount to increasing the weight given to public spending. As this changes marginal utilities, so does it change the optimal allocation of resources with an increasing share going to public spending. It is easy to see that this will lead the altruistic policymaker to choose higher discretionary inflation levels given the changes in the optimal allocation of resources. For an initial stock of nominal debt accounting for 70% to GDP, figure 7 depicts the change of discretionary inflation in the preference parameter $\rho$.

Without changing the initial debt level an MIT shock could push the policymaker into the fiscal fragility zone. A sufficiently high shock to the parameter $\rho$ could result in the

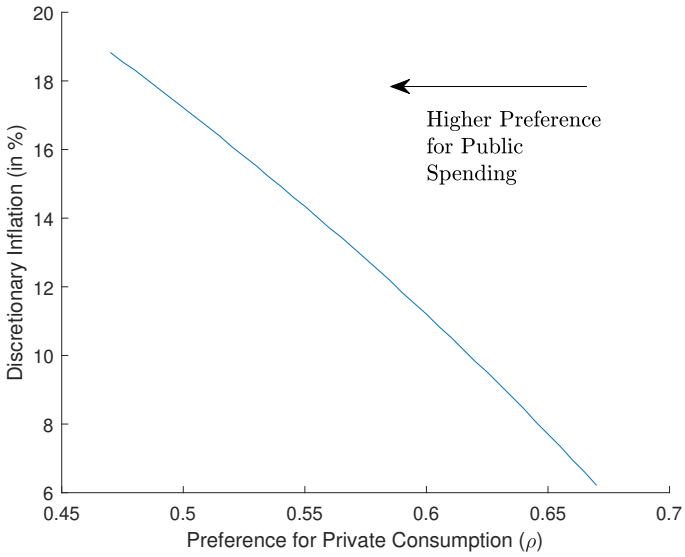


Figure 7 – Sensitivity: Preference for Public Spending

loss of credibility of the target under adverse expectations. Private-agents understanding this adapt their inflation expectations. A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured as shown in figure 8. Although it will be discussed later on, we use such an MIT shock to explain how expectations can suddenly overshoot the target as happened in Brazil in 2002.

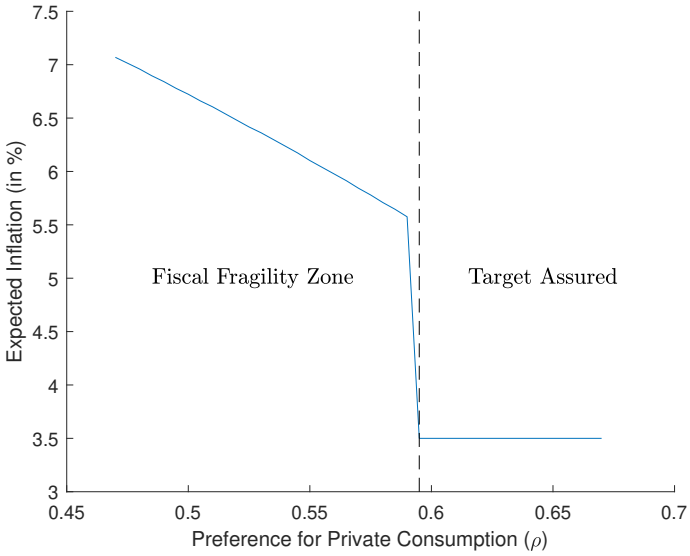


Figure 8 – Inflation Expectations and Private Consumption Preferences

As a last sensitivity exercise we will show that changing preferences has an impact on optimal fiscal policy and consequently on the debt levels that support monetary policy. The policymaker's optimal fiscal policy is to run down debt for a range of initial debt. The upper limit of that range shifts downwards as the policymaker's preference increases

towards more public spending. In figure 9 we show that this upper limit, that can be thought of as an inflection point, decreases in the preference for public spending.

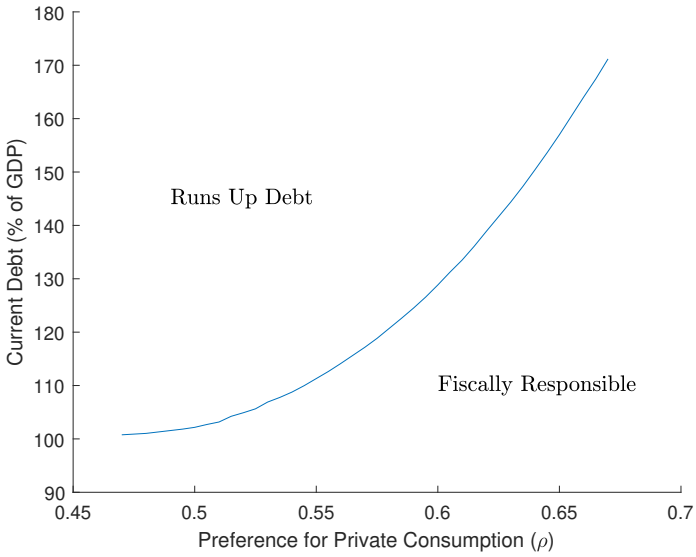


Figure 9 – Debt Policy Function Inflection Point

## 3.4 Welfare Cost of Inflation

Up to this point we have defined the penalty $\alpha$ as a function of deviations from the inflation target. This choice was made in order to focus on coordination dynamics around the inflation target. In the following numerical exercise we will include a cost of inflation to take possible negative impacts of inflation on the economy into account.

Expanding on the work of (BAILEY, 1956) and (LUCAS, 2000), (CYSNE, 2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model. Estimating the model for Brazil, (CAMPOS; CYSNE, 2018) find a 0.24% cost of GDP for an average annual inflation of 6.63%. In the same ballpark as (AIYAGARI; BRAUN; ECKSTEIN, 1998), in the case of the US, with a cost of 0.5% for a 10% nominal interest rate. Using a linear approximation to the cost of inflation from (CAMPOS; CYSNE, 2018) one obtains:

$$cost(\pi_t) = 0.0272\pi_t + 0.0007 \tag{3.3}$$

The linear function $cost(\pi_t)$ so defined is not bounded. Nevertheless, for limited positive inflation rates this might still be an adequate approximation. Combining both the cost of inflation together with the cost of deviating we define the new penalty function:

$$\tilde{\alpha}_t(\pi_t, \pi^a, \alpha_{t-1}) = \alpha_t(\pi_t - \pi^a, \alpha_{t-1}) + cost(\pi_t) \tag{3.4}$$

where $\alpha_t$ is as previously defined in equation 3.2. Using the calibrated model it can be verified that the results and the various sensitivity analyses continue to hold. In figure 10 we show that the dynamics of the optimal fiscal policy remain similar. The lower-bound of the fiscal fragility zone shifted upwards due to the higher costs of choosing discretionary, making the target fully credible up to higher levels of debt. Within the fiscal fragility zone the policymaker's optimal fiscal policy continues to depend on the initial debt stock. For sufficiently low levels within the fiscal fragility zone, the policymaker will prefer to run down debt in order to exit the crisis zone. However, for high initial stocks the optimal policy is to slowly increase debt until the occurrence of a confidence crisis. Such debt levels, therefore, do not support the inflation target.

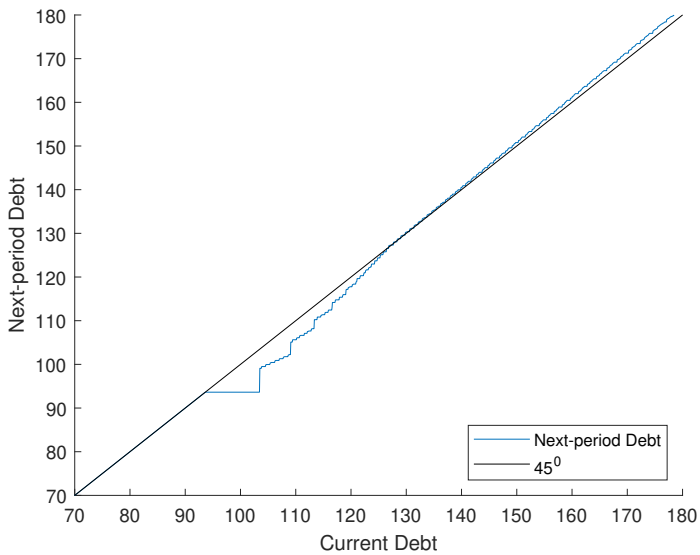


Figure 10 – Optimal Fiscal policy

Increasing the inflation target increases the lower-bound of the fiscal fragility zone as depicted in figure 11. A result similar to what was found without the welfare costs of inflation. Higher targets are therefore fully credible up to higher debt levels.

By including welfare costs for inflation it is possible to make a welfare analysis about the optimal inflation target for an initial debt range. By including welfare costs, there now exists a trade-off between costs associated with higher levels of inflation against the benefits of a more credible higher target. Let us define the debt interval $D_0 \in [80, 110]$ that includes the lower-bounds of the fiscal fragility zone for inflation targets $\pi^a \in [0, 20\%]$ as can be seen in figure 11.

Let W be the private agents' welfare on the discrete debt interval:

$$W = \sum_{D=80\%GDP}^{110\%GDP} V^p(s) \tag{3.5}$$

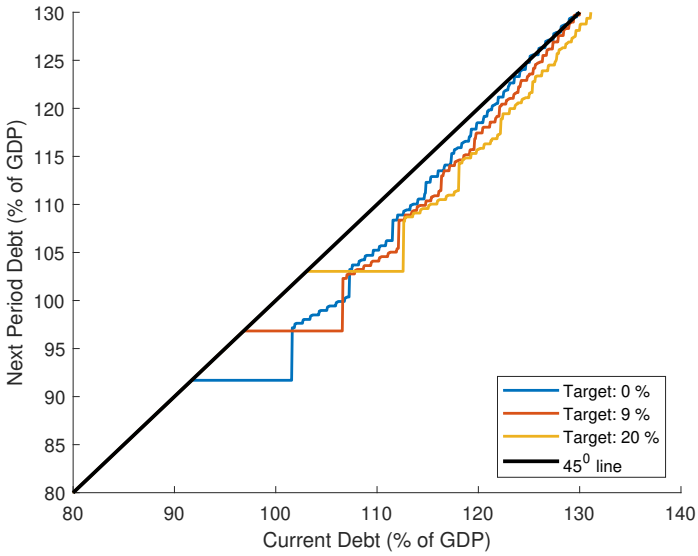


Figure 11 – Sensitivity: Optimal Fiscal policy for Different Targets

where $s = (D, \pi^e, \zeta, \alpha_{-1})$. The value function $V^p(s)$ has been previously defined in equation 2.10. Private agents' welfare defined on the initial debt interval is depicted in figure 12. Given the linear costs in inflation, welfare is upward bounded. Maximizing welfare on the interval with respect to a pre-announced inflation target would therefore yield an optimal choice.

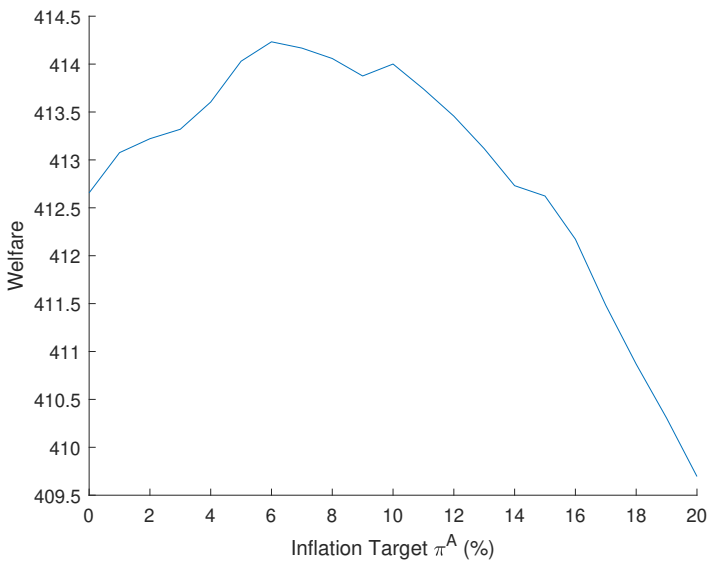


Figure 12 – Sensitivity: Welfare Around Fiscal Fragility Zone

## 3.5 2002 Confidence Crisis in Brazil

In 2002 and 2003, Brazilian policymakers faced inflationary pressures when it became clear that the presidential candidate Lula da Silva would win. The perception was that Lula's victory could come with the implementation of a new policy-framework that could undermine the previous one that helped reduce inflation. Consequently, inflation expectations overshoot the target's upper-bounds at all horizons relevant to the central bank as shown in figure 13.

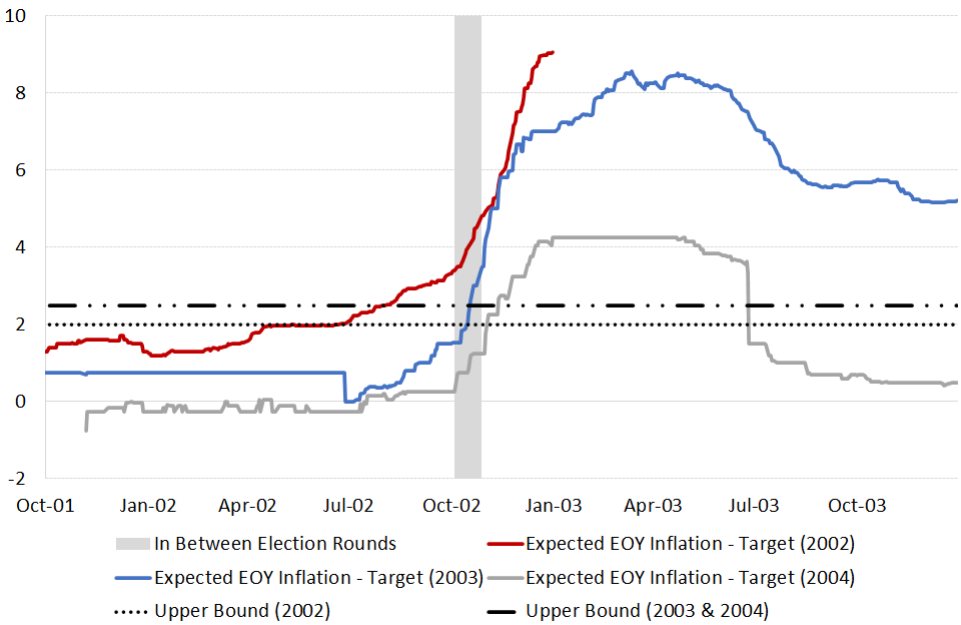


Figure 13 – Expectation Shock

Through an MIT shock to the policymaker's preferences we showed that for a given initial debt stock the target could loose credibility. By favoring more public spending the policymaker could become vulnerable to adverse shocks that would make it deviate from the inflation target. Private-agents taking this probability into account when forming expectations would increase their forecasts of future inflation, which is what was observed in 2002.

In response to rising inflation expectations the outgoing and new administrations took several steps: i) in order to coordinate inflation expectations in the short run, they increased the target for 2003 in an extra meeting held on 6/27/2002 and unofficially again on 1/21/2003, ii) in 2003 responsible macroeconomic policies were sustained by public debt reduction, and iii) changes in the debt mix away from indexed bonds (especially foreign exchange) were implemented. Ultimately inflation expectations converged back to the target. Those policy responses closely mirror the prescriptions suggested by the various sensitivity analyses we performed. We will now look at those policies in further detail.

### 3.5.1 Fiscal Policy

After the 2002 election gross public debt was gradually reduced. As can be seen in table 2, gross debt went down from almost 80% of GDP in 2002 to close to 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations, a signal of fiscal responsibility. From the perspective of our model, such fiscal policy is compatible with the policymaker trying to exit the fiscal fragility zone and give the needed fiscal support to its inflation target.

| | Gross Debt (% of GDP) | Primary Result (% of GDP) |
|---|---|---|
| 2000 | 66.7 | 1.73 |
| 2001 | 70.2 | 1.69 |
| 2002 | 79.8 | 2.16 |
| 2003 | 74.7 | 2.28 |
| 2004 | 70.6 | 2.70 |
| 2005 | 69.2 | 2.60 |

Table 2 – Brazil - Fiscal Policy

### 3.5.2 Inflation Target

Before the October elections, the 2003 target was exceptionally revised upwards from a previously announced 3.25 to 4%. Similarly, the upper and lower bounds widened from -/+ 2 to 2.5%. In January of 2003, the ministry of Finance send a letter stating that the adjusted target for the same year would be of 8.5% and 5.5% in 2004. The latter was confirmed by the national monetary committee as the inflation target for 2004 on the 25 of June 2003 as can be seen in table 3. From the perspective of our model an indebted policymaker with a higher inflation target might be more credible. Serving as a nominal anchor the higher and more credible inflation target makes private agents readjust their inflation expectations.

| Year | Date When Set | Target | Bounds |
|---|---|---|---|
| 2002 | 28/6/2000 | 3.5 | 2 |
| 2003 | 28/6/2001 | 3.25 | 2 |
| | 27/6/2002 | 4 | 2.5 |
| | 21/1/2003 | 8.5 | |
| 2004 | 27/6/2002 | 3.75 | 2.5 |
| | 21/1/2003 | 5.5 | |
| | 25/6/2003 | 5.5 | 2.5 |
| 2005 | 25/6/2003 | 4.5 | 2.5 |

Table 3 – Brazil - Official Inflation Targets

### 3.5.3 Debt Management

The lion share of Brazilian public securities were indexed to some benchmark prior to 2002. Such benchmarks include consumer price inflation, exchange rates, and the targeted policy interest rate. On the one hand, debt indexed to inflation and exchange rates made up 56% of outstanding debt in 2002 and were gradually reduced to 32.5% in 2005. On the other hand, pre-fixed securities and those indexed to the Selic made up close to 44% in 2002 and increased to over 67% of outstanding debt in 2005 as described in table 4. The latter type of debt can be thought of as debt on which partial defaults are possible. Using this classification as a proxy for the nominal and indexed debt denominations used in the model, we suggest that reducing indexed debt could help towards providing support to the inflation target.

| | Pre -fixed | Selic | Debt Allowing Partial Default | Exchange rate | Price Index | Other | Indexed Debt |
|---|---|---|---|---|---|---|---|
| | A | B | A+B | C | D | E | C+D+E |
| 2002 | 1.5 | 42.4 | 43.9 | 45.8 | 8.8 | 1.4 | 56 |
| 2003 | 9.5 | 46.5 | 56 | 32.4 | 10.3 | 1.4 | 44.1 |
| 2004 | 16.1 | 45.7 | 61.8 | 24.2 | 11.9 | 2.1 | 38.2 |
| 2005 | 23.6 | 43.9 | 67.5 | 17.6 | 13.1 | 1.8 | 32.5 |

Table 4 – Brazil - Debt Type (in % of Debt)

## 3.6 Conclusion

An altruistic policymaker's optimal policy might be to deviate from the inflation target when debt levels are too high. We show that in a model calibrated to the 2002 confidence crisis in Brazil the target can be perfectly assured up to around 70% of GDP. Through an MIT shock to preferences, in favor of increased spending, the policymaker could be liable to fall into the fiscal fragility zone where, in the event of an adverse inflationary shock, the inflation target will no longer be credible. We also show that the optimal fiscal policy depends on the initial debt stock. Up to a given debt level the policymaker prefers fiscal responsibility and will run down debt in order to exist the fiscal fragility zone. However, when the policymaker is too indebted, it chooses to run-up debt and thereby fails to give the needed fiscal support to its monetary policy. We also show that the indebted policymaker may benefit from a higher inflation target to regain credibility and reduce real interest rates in the fiscal fragility zone. We also observe that those conclusions continue to hold when including welfare costs of inflation to the penalty function.

Finally, we show through a sensitivity analysis that the policy measures implemented by Brazilian policymakers to make inflation expectations converge back to the center of

the target closely mirror prescriptions based on the calibrated model. More specifically the Brazilian policymakers i) run primary fiscal surpluses to help reduce debt levels, ii) increased the previously low inflation target to more credible levels, and iii) reduced the share of indexed debt.

# 4 The Model: an Empirical Look

In this chapter we test some of the results from our theoretical model. With regards to the impact of debt, we will focus on two different channels: i) we will look at whether debt has impacted inflation expectations in Brazil and ii) if the size of deviations and the probability of overshooting targets are impacted by debt levels in a panel of countries with inflation targeting regimes. Furthermore, with regards to the latter, we will look at whether higher inflation targets have the expected impacts on both deviations and the probability of overshooting targets.

In our theoretical model an altruistic policymaker might want to transfer resources through inflation from private agents to itself in order to finance an optimal increase in public spending, even though deviating from the target would result in a negative productivity shock. Private agents anticipating the possibility of this occurring adapt their inflation expectations. This channel we aim to empirically test for Brazil. The latter is an interesting study case for two reasons. First, the central bank has continuously and consistently surveyed professional forecasters since the early 2000's. Second, the sustainability of public finances has been a recurring concern for analysts and policymakers. On the one hand, we do not find any relevant impact of debt on expectations. On the other hand, the primary fiscal balance has a statistically significant impact on inflation expectations and could be interpreted in a way similar to debt levels. This result was already observed for Brazil by (CERISOLA; GELOS, 2005) and for a larger set of countries by (GELOS; PRATI; CELASUN, 2004). This highlights the fact that while debt could be a necessary condition in explaining forecaster's beliefs about the commitment and ability of policymakers to keeping inflation on target, it certainly is not sufficient.

Moving away from the expectations channel to observed inflation, we further test the model's predictions in a data panel of inflation targeting regime countries with at least 15 years of experience over the 2000-2019 period. More specifically, we further investigate whether there is empirical evidence for the effects of debt on i) observed deviations and ii) the probability of overshooting the inflation target's upper-bound. We find that deviations are positively related to debt/GDP stocks for countries with low revenues, as is the case for all middle-income countries in our sample. The results are robust to including variables and shocks known to impact inflation. We do, however, not find any significant impact of debt when looking at the probability of overshooting the target.

Finally, using a calibrated model, we showed that an indebted policymaker could benefit from a higher inflation target when the policymaker is susceptible to confidence crises. We find that both deviations from the target and the probability to overshoot them

are, on average, negatively related to the target's level. This suggests that a policymaker consistently overshooting its target might benefit from a higher more credible target. The results are robust to the inclusion of different variables and shocks impacting inflation.

In the first section we evaluate the impact of debt on the formation of inflation expectations of professional forecasters in Brazil. In a second section we evaluate the model's predictions in a data panel of 20 countries with IT regimes. Finally, the last section offers some concluding remarks.

## 4.1 Debt Levels and Inflation Expectations: the Case of Brazil

### 4.1.1 Inflation Expectations in Brazil

#### 4.1.1.1 CB Survey of Professional Forecasters

When looking at expectations we will restrict ourself to the inflation expectations of surveyed professional forecasters by the Central Bank of Brazil. Other alternatives, such as a survey of consumers from FGV or implicit inflation expectations extracted from government securities, are available. However, consumers might likely give more weight to certain items from the reference CPI basket such as food stuff and may not possess the same reporting incentives as in the other two cases. With regards to implicit inflation, one might have to correct for different types of risk and liquidity premia necessitating the use of an economic model, as in (VICENTE; GRAMINHO, 2014), in order to obtain inflation expectations. Based on such caveats we have chosen to limit this analysis to expectations from the central bank's survey.

Figure 14 shows the expected deviation from the inflation target for different forecast horizons. In the absence of monthly forecasts for horizons exceeding 12 months we have used the end-of-year forecasts for next year, two subsequent years, and 3 subsequent years. Since short-term forecasts are more sensitive to short-term shocks they are more likely to deviate more from the target as can be seen in the figure. It is, however, worth noting that the expected deviations for the different horizons seem to be highly correlated.

#### 4.1.1.2 Literature Review

In this section we will identify some important dynamics of inflation expectations and relevant variables used in inflation forecasts based on the literature. The aim being to avoid biases caused by omitted variables when estimating the impact of debt on inflation expectations.

Most empirical works focus on the short-term 12 months ahead inflation forecast. We will, however, also evaluate longer term forecasts from the survey such as the end-of-year inflation for 1,2,and 3 years ahead. In testing the rationality of professional forecasters

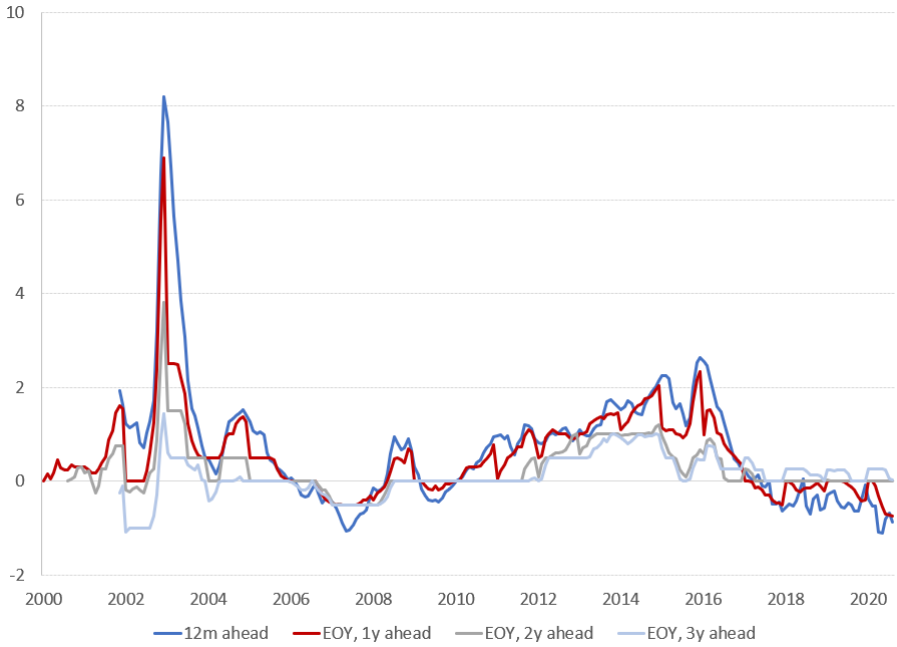


Figure 14 – Brazil - Expected Deviations from the Inflation Target (%)

(CARVALHO; MINELLA, 2012) and (KOHLSCHEEN, 2012) have found that forecast errors are strongly correlated. Similarly, (ARAUJO; GAGLIANONE, 2010) points to the high persistence of inflation expectations over time. As could be expected in an inflation targeting regime, (BEVILAQUA; MESQUITA; MINELLA, 2008) find that the inflation target anchors expectations. However, it is worth noting that they use the so called adjusted targets [1].

Using the small-scale inflation forecast models of the central bank as a benchmark one may also include several other variables. Those may include GDP gap estimates, exchange rates, especially USD/BRL since virtually all of the invoicing is done in USD (GERSTEN, 2015), commodity prices, real interest rates to capture monetary policy effects, and climatic shocks that impact agricultural production.

Finally, looking at fiscal indicators, (CERISOLA; GELOS, 2005) find that the primary balance matters as an increase in deficits causes an increase in inflation expectations. The fact that an important share of the debt was indexed to the USD/BRL exchange rate may have strengthened the 2002 confidence crisis according to (BLANCHARD, 2004). We depicted the evolution of both series in figure 15. Given the short span of the series we cannot readily evaluate the historical relationship. Nevertheless, it is clear that after the 2002 confidence crisis the policymakers reduced virtually to zero the stock of USD indexed debt.

[1] After the initial inflationary shock, a resolution from June 2002 changed the 2003 and 2004 targets to respectively 4% and 3.75%. An open letter from January 2003 from the ministry of finance defined the adjusted targets as 8.5 and 5.5% respectively. The latter was officially set, by the CMN, as the inflation target for 2004 through a June 2003 resolution.

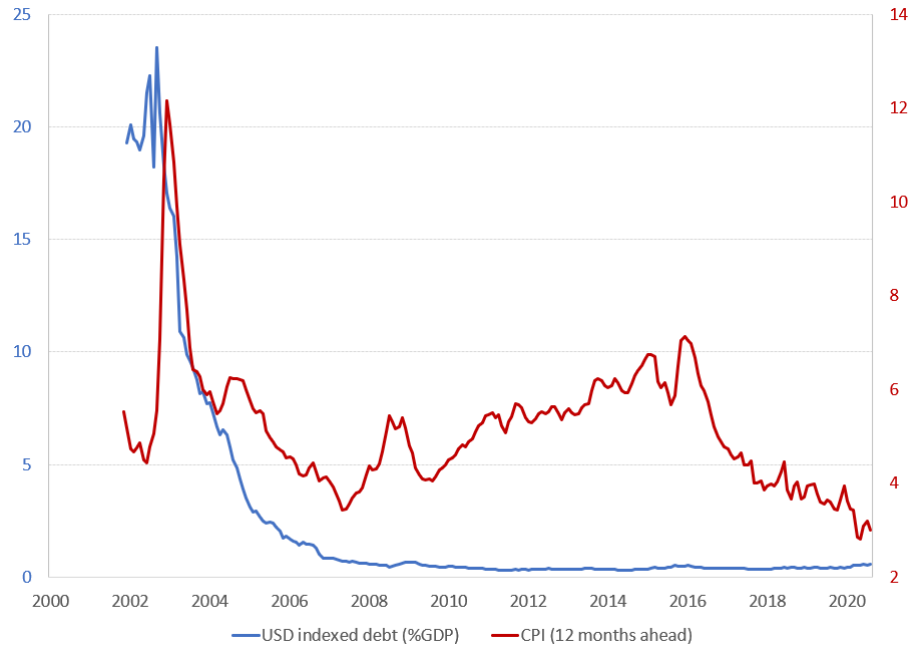


Figure 15 – Brazil - USD indexed debt and CPI expectations

### 4.1.2 Empirical Tests

#### 4.1.2.1 The Equation

Let us define the equation that will be estimated in order to evaluate the impact of debt on inflation expectations as follows:

$$\begin{aligned}\pi^e_{t,t+12} &= \alpha_1\pi^e_{t-1,t+11} + \alpha_2\pi_{t-13,t-1} + \alpha_3\pi^a_{t,t+12} + \alpha_4\Delta(i_t - \pi_{t-12,t}) \\ &+ \sum_{i=0}\alpha_{5,i}\Delta_{12}debt_{t-i} + \sum_{i=0}\alpha_{6,i}usd_{t-i} + \sum_{i=0}\alpha_{7,i}h_{t-i} + \sum_{i=0}\alpha_{8,i}debt^{usd}_{t-i} \\ &+ \sum_{i=0}\alpha_{9,i}pb_{t-12-i,t-i} + \sum_{i=0}\alpha_{10,i}X_{t-i} + u_t\end{aligned} \tag{4.1}$$

where $\pi^e_{t,t+12}$ is the 12 month ahead CPI inflation expectations and $\pi_{t-13,t-1}$ is the past inflation accumulated over the last 12 months. $\pi^a_{t,t+12}$ is the average inflation target over the next 12 months and $i_t - \pi_{t-13,t-1}$ is the real interest rate expressed as the annualized overnight interest rate on government bonds minus past inflation accumulated over the last 12 months. $\Delta_{12}debt_t$ is the yearly difference in monthly gross debt to GDP levels, while $debt^{usd}_t$ is the gross debt to GDP indexed to the USD. $pb_{t-12,t}$ is the accumulated primary fiscal balance over the last 12 months. Positive values indicate deficits. $h_t$ is the GDP gap. $usd_t$ is the accumulated percentage change in the USD/BRL exchange rate over the last 3 months. Finally, $X_t$ gathers different types of shocks such as commodities and climate. For the former we use the commodity index IC-Br and for the latter the ONI index from NOAA.

#### 4.1.2.2 Data

The data used in estimating equation 4.1 is further described in table 5. The debt statistics used are those for the central government using the pre-2007 methodology due to better availability of data. The quarterly GDP gap estimates are from the *Instituição Fiscal Independente* (IFI) and are based on a production function capturing slack in both capital and labor markets [2]. In order to obtain monthly estimates of the GDP gap we linearly interpolate the original quarterly series. With regards to the climate anomalies we obted to use squared anomalies in order to better capture the lack/excess of rain on crops through the La Nina/El Nino effects.

| Variable | Series | Transformation | Source |
|---|---|---|---|
| $\pi^e_{t,t+12}$ | Inflation expectations | 12 months ahead | Focus |
| $\pi_{t-12,t}$ | IPCA | YoY | IBGE |
| $\pi^a_{t,t+12}$ | Inflation target | 12 months ahead | BCB |
| $i_t$ | SELIC | Annualized rate | BCB |
| $debt_t$ | Gross central government debt (%GDP) | - | BCB |
| $usd_t$ | USD/BRL MoM | Accumulated 3 months | BCB |
| $h_t$ | GDP gap | Linearly interpolated | IFI |
| $debt^{usd}_t$ | Debt indexed to USD (%GDP) | - | BCB |
| $pb_{t-12,t}$ | Monthly primary balance (%GDP) | Accumulated 12 months | BCB |
| $ICBr_t$ | Commodity Index IC-Br MoM | Accumulated 3 months | BCB |
| $ONI^2_t$ | Climate anomalies ONI | Squared | NOAA |

Table 5 – Series Used in Estimation

#### 4.1.2.3 Results for 12-month Ahead Inflation Expectations

We find most of the results from the literature, but do not find any statistically significant impact of debt on inflation expectations even though the coefficient is always of the expected sign. The sample used is limited by the available data series and runs from November 2002 to April 2020. Equation 4.1 is estimated with small variations in order to test the robustness of the results. Furthermore, due to potential problems related to the auto-correlation of error term $u_t$ we report Newey-West standard errors. The results are summarized in table 6.

The difference between I and II is the use of the adjusted inflation target as in (BEVILAQUA; MESQUITA; MINELLA, 2008) for 2003 and 2004 versus the official inflation target. The argument for the adjusted target is that first round effects such as the inertia of CPI and backward-looking shocks to regulated items should not be neutralized by the central bank and should hence be added to the official target (FRAGA; GOLDFAJN; MINELLA, 2003). Including debt indexed to the USD (III) and an auto-regressive term as well as a climate shock (IV) does not fundamentally change the results. We verify findings

[2] Using estimates from IPEA does not change the results.

| | I | II | III | IV |
|---|---|---|---|---|
| $\pi^{a}_{t,t+12}$ | 0.573*** | | 0.699*** | 0.141** |
| | (0.063) | | (0.066) | (0.054) |
| $\pi^{a,adjusted}_{t,t+12}$ | | 0.762*** | | |
| | | (0.127) | | |
| $\pi_{t-13,t-1}$ | 0.439*** | 0.327*** | 0.357*** | 0.003 |
| | (0.044) | (0.089) | (0.033) | (0.021) |
| $pb_{t-12,t}$ | 0.177 | 0.400** | 0.436** | 0.140** |
| | (0.147) | (0.176) | (0.182) | (0.061) |
| $\Delta_{12}debt_t$ | 0.047 | 0.015 | 0.048 | 0.002 |
| | (0.040) | (0.037) | (0.041) | (0.012) |
| $\Delta(i_t - \pi_{t-12,t})$ | −0.673*** | −0.409*** | −0.442*** | −0.140*** |
| | (0.093) | (0.122) | (0.112) | (0.039) |
| $h_t$ | 0.155* | 0.237** | 0.270*** | 0.079*** |
| | (0.084) | (0.108) | (0.095) | (0.024) |
| $usd_t$ | 0.020** | 0.024*** | 0.022*** | 0.011*** |
| | (0.009) | (0.009) | (0.007) | (0.003) |
| $ICBr_t$ | 0.036*** | 0.027 | 0.025** | 0.005* |
| | (0.012) | (0.017) | (0.012) | (0.003) |
| $debt^{usd}_t$ | | | 0.153*** | 0.074*** |
| | | | (0.054) | (0.014) |
| $\pi^{e}_{t-1,t+11}$ | | | | 0.866*** |
| | | | | (0.056) |
| $ONI^2_t$ | | | | 0.070*** |
| | | | | (0.025) |
| *Note:* | | | *p<0.1; **p<0.05; ***p<0.01 | |
| | | | Newey-West standard errors | |

Table 6 – Results - Inflation Expectations 12 Months Ahead

already documented in the literature as inflation expectations are positively related to the inflation target (official or adjusted), devaluations of the BRL versus the USD, the GDP gap, reductions in the real interest rate, climate shocks, the fiscal balance, past inflation, and lagged values of itself. With regards to debt and inflation expectations, we find a positive, albeit not statistically significant, relationship in all settings. Changing the lags of the variables in order to respect the availability of data to the professional forecasters does not change the results.

#### 4.1.2.4 Results for Long Term Inflation Forecasts

In the literature the inter-temporal choice of inflation from the policymaker might be impacted by the maturity of debt as in (COCHRANE, 2001). Intuitively the policymaker could spread the inflationary surprise over several periods in order to reduce the real value of nominal debt. This highlights a specific channel through which confidence about fiscal

solvency may also impact longer-term inflation forecasts. We follow a similar strategy and estimate equation 4.1 for the end-of-year inflation in 2 years time. Given the change in horizons we tested different transformations of the variables and settled for a 12-month accumulated change for the USD/BRL exchange rate. The inflation target will be the official target announced 2 years prior to the forecast period. The results are summarized in table 7. Newey-West standard errors are reported.

While the coefficient on changes in debt levels was previously found to be positive, albeit not significant, we do now observe a sign inversion in case I and III. There is therefore no evidence that professional forecasters change longer term inflation forecasts depending on changes in debt levels. Replacing the variable with debt levels, as well as accounting for non linear effects through the interaction of debt and a threshold indicator, does not change the results. The primary fiscal balance does however remain statistically significant and of the expected sign. Although it can signal the state of public finances, and the future evolution thereof, it can also capture the effect of increasing public spending on inflation. However, given the time frame the latter seems to be less likely than the former. There therefore seems to be some evidence of forecasters looking at fiscal indicators even for longer term forecasts. The more short term shocks, as those captured by the commodity index and climate anomalies, are no longer significant as could be expected. Perhaps more surprising is that foreign exchange shocks remain relevant in all settings.

We did not find any statistically significant impact of changes in debt to GDP levels on the inflation expectations of professional forecasters in Brazil. This result is robust to changes in the forecast horizon. However, we have found the primary fiscal balance to be relevant at all tested horizons with deficits increasing inflation expectations. Professional inflation forecasters may prefer to look at the future stream of fiscal balances as an indication of the fiscal situation than at changes in debt.

## 4.2 Missing the Inflation Target

### 4.2.1 Deviations from the Target

#### 4.2.1.1 The Equation

The first order condition of the discretionary inflation problem, from equation 2.6, relates the deviation of inflation $\pi_{i,t}$ from the inflation target$\pi^A_{i,t}$ to observable and latent variables for each country $i$. In appendix B.1 we show how the equation 4.2 is related to the first order condition of the discretionary inflation problem from equation 2.7.

$$\begin{aligned}\pi_{i,t} - \pi^A_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^A_{i,t} \\ + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \quad t = 1, ..., T\end{aligned} \tag{4.2}$$

| | I | II | III |
|---|---|---|---|
| $\pi^{a}_{t+24}$ | 1.121*** | 0.232*** | 0.231*** |
| | (0.066) | (0.051) | (0.052) |
| $pb_{t-12,t}$ | 0.247* | 0.079*** | 0.081*** |
| | (0.132) | (0.029) | (0.030) |
| $\Delta_{12}debt_t$ | -0.017 | 0.0005 | -0.0002 |
| | (0.022) | (0.006) | (0.007) |
| $debt^{usd}_t$ | 0.145*** | 0.040*** | 0.042*** |
| | (0.028) | (0.009) | (0.010) |
| $\Delta(i_t - \pi_{t-12,t})$ | -0.040 | -0.003 | -0.003 |
| | (0.030) | (0.005) | (0.005) |
| $usd_t$ | 0.010*** | 0.003*** | 0.003*** |
| | (0.003) | (0.001) | (0.001) |
| $h_t$ | 0.114 | 0.045*** | 0.045*** |
| | (0.074) | (0.013) | (0.014) |
| $\pi^{e,EOY}_{t+23}$ | | 0.787*** | 0.788*** |
| | | (0.047) | (0.047) |
| $ONI^2_t$ | | | 0.005 |
| | | | (0.012) |
| $ICBr_t$ | | | -0.0004 |
| | | | (0.001) |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 | | |
| | Newey-West standard errors | | |

Table 7 – Results - EOY Inflation Expectations 2 Years Ahead

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0, t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time $t$ and for country $i$. The unobserved variable $c_i$ captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator as it seems reasonable to assume that their choices of debt, revenue and inflation target - $X_{i,t}$ are related to the unobserved characteristics of each country $c_i$. In other words we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0, \forall t$ as required by a random effect estimator [3].

In terms of interpretation we expect the net impact of debt to be positive. Given higher levels of debt the policymaker will have higher incentives for using discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, we expect on average the deviation to increase in debt levels as the policymaker will be i) more likely to deviate and ii) will choose higher discretionary inflation when doing so. Given the presence of an interaction term in equation 4.2 one would have to look at the joint impact captured by $\beta_2$ and $\beta_4$ for a given level of revenues to GDP.

We also expect the coefficient on the inflation target to be negative as the policyma-

[3] A Haussman test between a fixed and random effect estimator similarly suggests the use of the former.

ker could help coordinate private agents' expectations by adopting a more credible (higher) inflation target in given situations. Were inflation perfectly anchored, then changing the target would not result in changes in the expected deviation. In other words, the coefficient $\beta_3$ would be equal to zero.

Finally, higher revenues mean that the policymaker has more fiscal room available for spending. This decreases the incentives to transfer resources through discretionary inflation and we would therefore expect the net impact of revenues to be negative. Given the interaction term between debt and revenue we would expect the joint impact captured by $\beta_1$ and $\beta_4$ to be negative for a given level of debt.

#### 4.2.1.2 Data

The variables and parameters of the model are mapped into both observed series and latent variables as can be seen in table 8. The dataset includes 20 countries with at least 15 years of inflation targeting [4] covering the period between 2000 and 2019.

| Model | Serie |
|---|---|
| D | Gross debt (%GDP) |
| $\tau e$ | Revenue (%GDP) |
| $\pi^A$ | Inflation Target |
| $e, f, c_1, c_2$ | Country Dummy |

Table 8 – Map of Model Variables into Timeseries

The variables present both inter and intra-country variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the change however came from middle income countries [5] as can be seen in figure 16 for the period 2004-2019. Targets are those reported by the respective central banks. Inflation, gross debt- and revenue to GDP statistics are from the IMF. With regards to inflation, we used end-of-year consumer price inflation as most central banks target said benchmark. Some general statistics are reported in table 9.

| | Debt/GDP | Revenue/GDP | CPI EOY | CPI target |
|---|---|---|---|---|
| Average | 45.2 | 32.9 | 3.9 | 3.2 |
| Min | 13.4 | 16.4 | 1.5 | 1.5 |
| Max | 80.8 | 56.1 | 15.4 | 8.2 |

Table 9 – Data - Description Timeseries

[4] The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[5] We used the World Bank classification

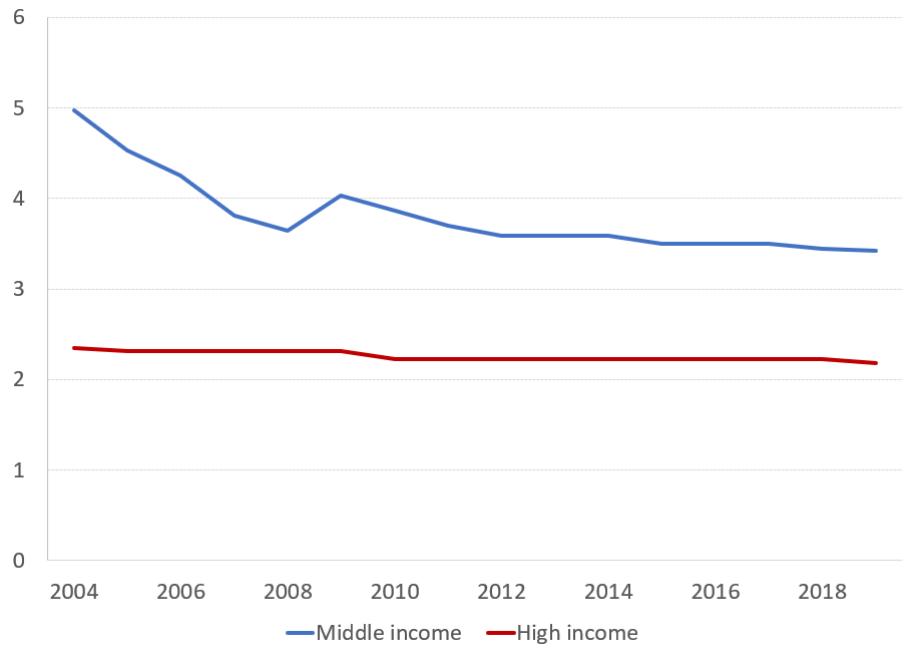


Figure 16 – Average Inflation Targets

We also use Real Effective Exchange Rate (Reer) statistics and GDP gap estimates to perform robustness checks. When Reer statistics were not available from the IMF other sources were used [6]. To construct the GDP gap estimates we used quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, we used the nonadjusted equivalent and seasonally adjusted the series using the Arima X-11 procedure[7]. The quarterly GDP gap statistics are obtained by applying an HP filter with a smoothing parameter of 1600. In order to mitigate the endpoint bias of the filter, at the beginning of each series, we estimated the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

#### 4.2.1.3 Results

Estimation I from table 10 is the basic model from equation 4.2. The remaining estimations, II-V, are robustness checks.

Looking at estimation I we have a positive and significant coefficient on debt while a negative and significant coefficient on the interaction term between debt and revenues. This can be interpreted as the fact that, on average, higher debt means higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP the net impact of debt is positive. This applies to the middle-income countries in our sample. The result goes in the direction of what the theoretical model predicted as both the probability of deviating and deviations from the target were positively related to debt levels. Hence

[6] BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.
[7] This was the case for Peru and Turkey

one would expect, on average, countries with higher debt levels to have higher deviations from their inflation target.

Looking at estimation I we see that deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation one would expect the coefficient to not be statistically different from zero. However, targets might not always be credible given the fiscal situation. A middle income country with high debt levels and low revenues could on average consistently overshoot the target given the results from table 10. In this case a higher target might be more credible and reduce the deviations as suggested by the negative coefficient on the target.

The coefficient on revenues is however positive in all settings, although not always statistically significant. Given the interaction term with debt, the net impact of revenues is positive up to debt levels of 88%, which is above the maximum in our sample. Hence, on average, the impact of higher revenues is to increase the deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenues could be correlated to preferences for public spending that in turn could lead to inflationary pressures.

The results remain when accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II we include a time fixed effect in order to account for global shocks such as prices in commodities. As noted by (HA; KOSE; OHNSORGE, 2019) domestic inflation has been significantly influenced by global shocks after 2001. In our sample 2008 stands out as many countries overshot their inflation targets after the great financial crisis. The time dummies are meant to help take into account such global co-movements in inflation [8]. In estimation III we also include shocks to the real effective exchange rate and in estimation V we add the impact of deviations from potential GDP on inflation.

### 4.2.2 Probability of Overshooting the Target

#### 4.2.2.1 The Equation

The policymaker overshoots the inflation target when end-of-year inflation exceeds the upper-bound of the target such that $\pi_t > \pi_t^a + tolerance := \overline{\pi}_t^A$ where $\overline{\pi}_t^A$ is the upper-bound. In the theoretical model the policymaker had more incentives to overshoot the target when it had limited fiscal space due to high debt servicing costs. We will estimate a similar equation to the one in 4.2, but this time with regards to the probability

[8] Using the first principal component extracted from the yearly inflation panel, more in the spirit of (HA; KOSE; OHNSORGE, 2019), and letting the impact be country specific does not change the results.

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| Revenue | 0.171** | 0.098 | 0.063 | 0.125* | 0.087 |
| | (0.076) | (0.076) | (0.078) | (0.070) | (0.072) |
| Debt | 0.069* | 0.074** | 0.073** | 0.062** | 0.058* |
| | (0.035) | (0.034) | (0.034) | (0.031) | (0.032) |
| Debt * Revenue/100 | −0.194* | −0.168* | -0.149 | −0.163* | -0.136 |
| | (0.099) | (0.096) | (0.096) | (0.088) | (0.089) |
| Target | −0.403*** | −0.458*** | −0.441*** | −0.360*** | −0.342*** |
| | (0.063) | (0.062) | (0.062) | (0.059) | (0.058) |
| GDP Gap | | | 0.363*** | | 0.342*** |
| | | | (0.102) | | (0.095) |
| Reer YoY | | | | −13.956*** | −13.645*** |
| | | | | (1.648) | (1.653) |
| FE | Country | Country & Time | Country & Time | Country & Time | Country & Time |
| $R^2$ | 0.290 | 0.408 | 0.433 | 0.515 | 0.537 |
| Num. obs. | 382 | 382 | 374 | 372 | 364 |
| *Note:* | | | | | *p<0.1; **p<0.05; ***p<0.01 |

Table 10 – Results - Deviations from the Inflation Target

of overshooting the target. Equation 4.3 specifies the relationship that we will test.

$$\begin{aligned} I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} &= \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} \\ &+ \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t}, \quad t = 1, ..., T \end{aligned} \tag{4.3}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country $i$ at time $t$. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ overshoots the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0, t = 1, ..., T$. The probability of overshooting the target will then be given by:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \tag{4.4}$$

The expected results and dynamics are quite similar to those described in the previous section with an expected net positive impact of debt, negative impact of the inflation target and of revenues on the probability of overshooting the target.

#### 4.2.2.2 Data

Some countries adopt point wise targets instead of tolerance bounds. This is for instance the case of the UK and Norway. In such cases we used the average upper tolerance limit from the rest of the sample (1.2%). Each year at least 2 countries overshot their respective inflation target in our sample. The years 2007 and 2008 stand out as over 50% of the countries in our sample overshoot their inflation target as can be seen in figure 17. A time dummy is likely to capture this fact.

Virtually all countries, except two, overshot their target at least once during the period, with some countries like Turkey close to being serial overshooters. Overall, middle income countries overshot the target more often than high-income countries as can be seen in table 11. Nevertheless, high income countries still overshot the target some 39 times in our sample.

| | Overshooting | Not-overshooting | Total |
|---|---|---|---|
| Middle-Income | 56 | 109 | 165 |
| High-Income | 39 | 173 | 212 |
| Total | 95 | 282 | 377 |

Table 11 – Data - Description Overshooting Episodes

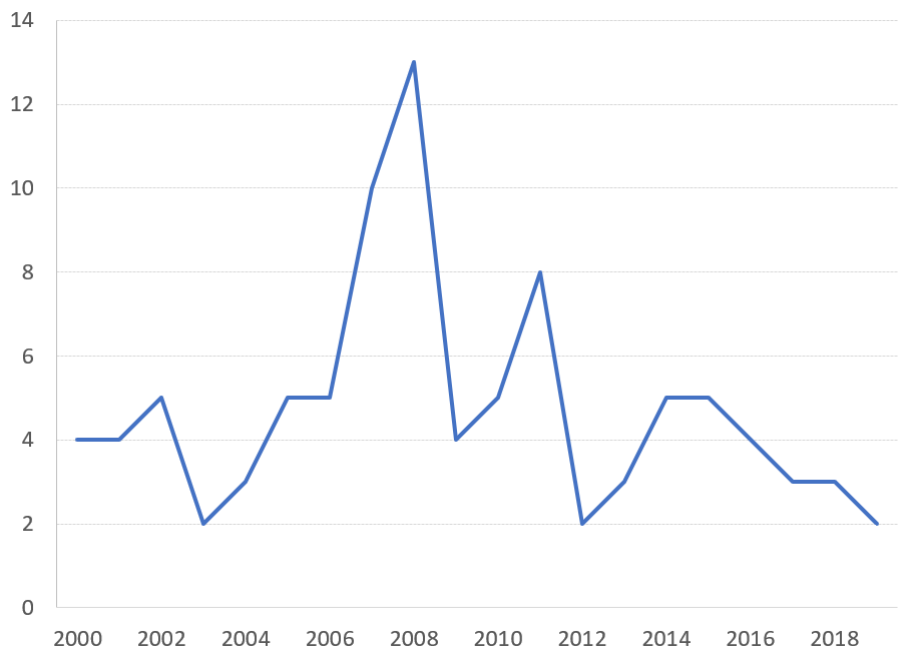


Figure 17 – Number of Countries Overshooting Inflation Targets (out of 20)

#### 4.2.2.3 Results

The first column of table 12 is the baseline model while the remaining columns represent robustness checks similar in spirit to the the previous section. When looking at the net impact of debt on the probability to overshoot the target the coefficients have similar signs as to the previous estimations with regards to deviations from the target. Estimation I has the most restrictive condition for the net effect of debt to be positive. For revenues over 30% of GDP the net effect of debt stops being positive. Not all middle-income countries in our sample do however have revenues below such a level. However, the effects are not statistically significant in any of the settings. There therefore doesn't seem to be clear evidence of debt increasing, on average, the probability of overshooting the target.

The net impact of revenues remains positive for the debt levels within our sample and does not go in the same direction as what was predicted by our theoretical model. We predicted a negative impact based on the increased fiscal room, higher revenues provide, decreasing incentives to use discretionary inflation to transfer resources away from private agents' holding debt. However, it might be correlated with some other factors such as a higher preference for government spending that increases incentives to use inflation for making transfers of resources. Similarly, higher revenues might be correlated with a larger presence of the state in the economy that might have consequence on the formation of prices.

The probability of overshooting the target is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, leaving the door open to overshoot the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. However, given the previous results, this tendency to overshoot the target does not seem to come from either revenue or debt but from some other source of fragility be it fiscal or other.

The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to overshoot their inflation target while the output gap no longer is statistically significant.

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| Revenue | 0.145 | 0.108 | 0.084 | 0.115 | 0.082 |
| | (0.091) | (0.105) | (0.109) | (0.110) | (0.113) |
| Debt | 0.034 | 0.055 | 0.053 | 0.050 | 0.042 |
| | (0.044) | (0.052) | (0.053) | (0.053) | (0.054) |
| Debt*Revenue/100 | -0.114 | -0.107 | -0.088 | -0.121 | -0.085 |
| | (0.125) | (0.149) | (0.151) | (0.151) | (0.154) |
| Target | $-0.624^{**}$ | $-1.242^{***}$ | $-1.207^{***}$ | $-0.990^{**}$ | $-0.936^{**}$ |
| | (0.263) | (0.376) | (0.376) | (0.390) | (0.386) |
| GDP Gap | | | 0.206 | | 0.218 |
| | | | (0.158) | | (0.167) |
| Reer YoY | | | | $-10.493^{***}$ | $-10.262^{***}$ |
| | | | | (3.062) | (3.101) |
| Num. obs. | 377 | 377 | 369 | 368 | 360 |
| Log Likelihood | −178.526 | −151.367 | −149.281 | −139.619 | −137.954 |
| *Note:* | | | | | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 |

Table 12 – Results - Probability of Overshooting the Target

## 4.3 Conclusion

In this chapter we aimed to test some of the conclusions from the theoretical model previously introduced. We show that debt levels do on average increase deviations from

the target for countries with revenue levels compatible with middle-income countries. Furthermore, both deviations and the probability of overshooting the upper-bound of the target are negatively related to the target level. Hinting that countries consistently overshooting the target and with on average positive deviations could reduce those by increasing targets. Those results are shown to be robust to including various shocks commonly impacting inflation. Finally, we found primary fiscal balances to impact the formation of inflation expectations in Brazil at both the 12 month ahead and longer term horizons. With deficits being associated with higher inflation expectations.

# 5 Conclusion

Oftentimes, models describing monetary policy rules assume a perfectly credible inflation target. This is for instance the case of the much used Taylor rule. We argue that the assumption of a perfectly credible inflation target might not always be realistic. In this paper, we show that an indebted and altruistic policymaker's optimal choice might be to deviate from a pre-announced inflation target when debt servicing costs become too high. High debt levels open the door to inflationary shocks in what we call the fiscal fragility zone.

In a model calibrated to the 2002 Brazilian confidence crisis, we show that the policymaker's optimal fiscal policy is to reduce debt in order to work towards assuring full credibility of monetary policy. This, however, is only true up to a given debt level. If the latter were too high, the policymaker prefers to run up debt, thereby taking away the target's fiscal support. Furthermore, higher targets are also shown to be fully credible up to higher levels of debt. An indebted policymaker might therefore be more credible with a higher target.

Finally, we have found evidence of inflation expectations being impacted by primary fiscal balances for different forecast horizons in Brazil. With deficits being associated with higher inflation expectations, even in the case of long-term forecasts. Moving to a panel of inflation targeting countries, we showed that higher debt, together with revenues compatible with middle income countries, came with higher deviations from the inflation target. Similarly, higher targets were associated with lower deviations and lower probabilities of overshooting the target's upper bounds.

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# Appendix

# APPENDIX A – Theoretical Model

## A.1 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the fiscal fragility zone there is only a unique inflation equilibrium making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\max_{D_{t+1}} \sum_t \beta^t u(c_t, g_t)$$
$$\text{s.t. } c_t = \frac{1}{\beta} D_t + \alpha(1-\tau)e - D_{t+1}$$
$$g_t = D_{t+1} + \alpha\tau e - \frac{1}{\beta} D_t$$

The first order condition for $D_{t+1}$ gives:

$$u_{g_t} - u_{c_t} = u_{g_{t+1}} - u_{c_{t+1}}$$

Supposing a utility function linear in $c$ we must have:

$$u_{g_{t+1}} = u_{g_t}$$

Given $u(c, g)$ strictly concave in $g$:

$$g_{t+1} = g_t$$

Replacing $g_t$ and $g_{t+1}$:

$$D_{t+2} - D_{t+1} = \frac{1}{\beta}\left(D_{t+1} - D_t\right)$$

Substituting forward and taking limits we obtain:

$$\lim_{t\to\infty} D_t = \sum_{i=0}^{\infty} \left(\frac{1}{\beta}\right)^i \left(D_{t+1} - D_t\right) + D_{t+1}$$

Suppose that $D_{t+1} \neq D_t$, then the policymaker will either run up infinite debt or credit and violate the no Ponzi condition. Hence, outside of the fiscal fragility zone we must have that $D_{t+1} = D_t$.

## A.2 Debt Roll-over Assumption

We will show that the debt roll-over assumption 4 reduces the incentives to use inflation and therefore works against the argument that the policymaker would like to

deviate for high enough debt levels. Suppose that the policymaker chooses a given inflation level $\pi > \pi^a$, when deviating, and either keeps debt constant at $D$ or decreases debt to $D'$. Given assumption 1, let the utility function $u(c, g)$ be given by $u(c, g) = c + h(g)$ with $h$ continuously differentiable and strictly concave.

If debt is constant, then the discounted utility for private agents would be given by:

$$\begin{aligned}&\left(\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}-1\right)D+\alpha(1-\tau)e+h\left(\left(1-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}\right)D+\alpha\tau e\right)\\&+\frac{\beta}{1-\beta}\left(\left(\frac{1}{\beta}-1\right)D+\alpha(1-\tau)e+h\left(\left(1-\frac{1}{\beta}\right)D+\alpha\tau e\right)\right)\end{aligned} \tag{A.1}$$

If the policymaker chooses debt $D' < D$, then the discounted utility would be given by:

$$\begin{aligned}&\left(\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}\right)D-D'+\alpha(1-\tau)e+h\left(D'-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D+\alpha\tau e\right)\\&+\frac{\beta}{1-\beta}\left(\left(\frac{1}{\beta}-1\right)D'+\alpha(1-\tau)e+h\left(\left(1-\frac{1}{\beta}\right)D'+\alpha\tau e\right)\right)\end{aligned} \tag{A.2}$$

Subtracting A.1 from A.2 we obtain:

$$\begin{aligned}&h\left(D'-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D+\alpha\tau e\right)-h\left(\left(1-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}\right)D+\alpha\tau e\right)\\&+\frac{\beta}{1-\beta}\left(h\left(\left(1-\frac{1}{\beta}\right)D'+\alpha\tau e\right)-h\left(\left(1-\frac{1}{\beta}\right)D+\alpha\tau e\right)\right)\end{aligned} \tag{A.3}$$

where the first line is negative given the higher current spending when maintaining debt constant. The second line is positive given the higher utility of increased future spending through lower debt servicing costs. Observe that if $D' = D$ then A.3 will be zero. Let us see what happens to A.3 when we slightly decrease $D'$.

The partial derivative with respect to $D'$ of A.3 will be given by:

$$h_g\left(D'-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D+\alpha\tau e\right)-h_g\left(\frac{\beta-1}{\beta}D'+\alpha\tau e\right) \tag{A.4}$$

The marginal impact of decreasing $D'$ would hence be captured by:

$$-h_g\left(D'-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D+\alpha\tau e\right)+h_g\left(\frac{\beta-1}{\beta}D'+\alpha\tau e\right) \tag{A.5}$$

In order to evaluate A.5 take $D' < D$, but sufficiently close to $D$. Suppose that $\pi^e < \pi$, then we have that:

$$D'-\frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D+\alpha\tau e > \frac{\beta-1}{\beta}D'+\alpha\tau e$$

Given $h$ strictly increasing:

$$h\left(D' - \frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D + \alpha\tau e\right) > h\left(\frac{\beta-1}{\beta}D' + \alpha\tau e\right)$$

Given $h$ continuously differentialbe and strictly concave:

$$h_g\left(D' - \frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D + \alpha\tau e\right) < h_g\left(\frac{\beta-1}{\beta}D' + \alpha\tau e\right)$$

Which gives us:

$$-h_g\left(D' - \frac{1+\pi^e}{1+\pi}\frac{1}{\beta}D + \alpha\tau e\right) + h_g\left(\frac{\beta-1}{\beta}D' + \alpha\tau e\right) > 0$$

Consequently, the policymaker would have preferred to slightly decrease future debt over maintaining it constant. It is easy to see that the policymaker would benefit from slightly higher inflation would it also be able to decrease future debt levels instead of just being able to roll-over debt.

## A.3 Existence Solution Discretionary Inflation

In order to prove that there exists a discretionary inflation level $\pi^D$ such that $\pi^D$ is optimal given $\pi^e$, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in equation 2.6.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi\Big(f\pi^D + (1-f)\pi^a\Big) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by hypothesis we know that the penalty function $\alpha : [0, \overline{\pi}] \to (0, 1)$ mapping discretionary inflation into total factor productivity is continuous. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by hypothesis.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is easy to see that the map of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous. Which is what we wanted to show.

## A.4 Above-Target Discretionary Inflation

We want to show that there exists a discretionary inflation $\pi^D$ such that $\pi^D > \pi^a$ under certain initial conditions. From the first order conditions of the policymaker's problem given by equation 2.7 we have:

$$u_{c_T}\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'(1-\tau)e\right)+u_{g_T}\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'\tau e\right)$$
$$+\frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e+u_g\alpha'\tau e\right)=0$$

Rearranging:

$$\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D(u_{g_T}-u_{c_T}) = -u_{c_T}\alpha'(1-\tau)e-u_{g_T}\alpha'\tau e-\frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e+u_g\alpha'\tau e\right)$$
$$=\kappa_1>0$$

since assumption 2 gives us $\alpha'<0$ and assumption 1 gives us $u_c, u_g > 0$. Which gives us:

$$(1+\pi)^2=\frac{1+\pi_T^e}{\kappa_1}\frac{1}{\beta}D(u_{g_T}-u_{c_T})$$

We know that $\pi_T^e$ satisfies $\pi_T^e = f\pi+(1-f)\pi^a$ where $f\in[0,1]$ and suppose that $u_{g_T}-u_{c_T}>0$ then:

$$(1+\pi)^2=\frac{1+f\pi+(1-f)\pi^a}{\kappa_1}\frac{1}{\beta}D(u_{g_T}-u_{c_T})$$

From which we obtain the following quadratic problem:

$$\pi^2+\pi\left(2-\frac{f}{\kappa_1}\frac{1}{\beta}D(u_{g_T}-u_{c_T})\right)+1-\frac{1+(1-f)\pi^a}{\kappa_1}\frac{1}{\beta}D(u_{g_T}-u_{c_T})=0$$

Simplifying notation once again:

$$\pi^2+\pi(2-f\kappa_2)+1-(1+\pi^a-f\pi^a)\kappa_2=0$$

Which gives the solution:

$$\pi=\frac{-(2-f\kappa_2)\pm\sqrt{(2-f\kappa_2)^2-4(1-(1+\pi^a-f\pi^a)\kappa_2)}}{2}$$
$$=\frac{-(2-f\kappa_2)\pm\sqrt{4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2}}{2}$$

Observe that $4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2>0$ since $\kappa_2>0$. For $\pi$ to be higher than the inflation target $\pi^a$ we must have:

$$\pi=\frac{-(2-f\kappa_2)\pm\sqrt{4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2}}{2}>\pi^a$$

Rewriting:

$$\pm\sqrt{4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2} > 2\pi^a + (2-f\kappa_2)$$

If $2\pi^a + (2-f\kappa_2) < 0$ than there is always at least one solution such that $\pi > \pi^a$ since $\sqrt{4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2} > 0 > 2\pi^a + (2-f\kappa_2)$. If, however, $2\pi^a + (2-f\kappa_2) \geq 0$, for there to be at most one possible solution we need to prove that $\sqrt{4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2} > 2\pi^a + (2-f\kappa_2)$. Since both terms are defined on $\mathbb{R}_+$ we can use the square operator and keep the inequality preserved:

$$4\kappa_2(1-f)(1+\pi^a)+(f\kappa_2)^2 > (2\pi^a + (2-f\kappa_2))^2$$

Expanding:

$$4\kappa_2 - 4f\kappa_2 + (f\kappa_2)^2 + 4\pi^a\kappa_2 - 4\pi^a f\kappa_2 > 4(\pi^a)^2 + 8\pi^a - 4\pi^a f\kappa_2 + 4 - 4f\kappa_2 + (f\kappa_2)^2$$

Simplifying:

$$\kappa_2 > 1 + \pi^a$$

Replacing $\kappa_2$:

$$\frac{D(u_{g_T} - u_{c_T})}{-u_{c_T}\alpha'(1-\tau)e - u_{g_T}\alpha'\tau e - \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right)}\frac{1}{\beta} > 1 + \pi^a$$

Rewriting:

$$\begin{aligned}
u_{g_T} &> u_{c_T} + \frac{\beta(1+\pi^a)\left(-u_{c_T}\alpha' e - \frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right)\right)}{\beta(1+\pi^A)(\alpha'\tau e) + D} \\
&= u_{c_T}\left(1 + \frac{-\alpha' e\beta(1+\pi^a)}{\beta(1+\pi^A)(\alpha'\tau e) + D}\right) + \frac{\beta(1+\pi^a)\left(-\frac{\beta}{1-\beta}\left(u_c\alpha'(1-\tau)e + u_g\alpha'\tau e\right)\right)}{\beta(1+\pi^A)(\alpha'\tau e) + D} \\
&= u_{c_T}\left(1 - \frac{\beta(1+\pi^a)(\alpha' e)}{\beta(1+\pi^A)(\alpha'\tau e) + D}\right) - \frac{\beta(1+\pi^A)\left(\alpha'\tau e\right)\left(u_g\frac{\beta}{1-\beta}\right)}{\beta(1+\pi^A)(\alpha'\tau e) + D} - \frac{\beta(1+\pi^a)\left(\alpha'(1-\tau)e\right)\left(u_c\frac{\beta}{1-\beta}\right)}{\beta(1+\pi^A)(\alpha'\tau e) + D}
\end{aligned}$$

Suppose debt $D$ is sufficiently high to have $\beta(1+\pi^A)(\alpha'\tau e) + D > 0$ then taking limits in the marginal deviation we have:

$$\lim_{\alpha'\to 0^-} u_{g_T} = u_{g_T} > u_{c_T}\left(1 + \frac{1}{D}\right) + \frac{1}{D} + \frac{1}{D}$$

Hence for a sufficiently low marginal penalty combined with a sufficiently high level of debt $D$ and marginal utility of spending the policymaker will chose a discretionary inflation level above the inflation target.

## A.5 Lower Real Interest Rates through Higher Target

The proof will be constructed as follows. We will first show the conditions under which deviations decrease in the inflation target. We will then show that this implies a lower real interest rate in the fiscal fragility zone.

The first order condition of the discretionary inflation problem is given by:

$$u_c^T\left[\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}\right)D+\alpha'(1-\tau)e\right]+u_g^T\left[\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}\right)D+\alpha'\tau e\right]$$
$$+\frac{\beta}{1-\beta}\left[u_c\alpha'(1-\tau)e+u_g\alpha'\tau e\right]=0$$

where the subscript $T$ in $u_c^T$ and $u_g^T$ indicates that the marginal utilities are evaluated at time $T$ when the policymaker chooses to deviate from the target. After the policymaker chooses discretionary inflation the economy goes to a steady-state and we therefore drop all time subscripts.

Further assuming that utility is linear in $c$ the first order condition for $\pi$ can be rewritten as:

$$\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D(u_g^T-1)+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

Using the rational expectations assumptions we have expected inflation $\pi_T^e = f\pi+(1-f)\pi^A$. Replacing $\pi_T^e$ in the above equation we obtain:

$$f\frac{1}{1+\pi}\frac{1}{\beta}D\left(u_g^T-1\right)+(1-f)\frac{1+\pi^A}{(1+\pi)^2}\frac{1}{\beta}D\left(u_g^T-1\right)$$
$$+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

We can find $\frac{\partial\pi}{\partial\pi^A}$ by taking the implicit derivative of the above equation with respect to $\pi^A$. Let us first evaluate the derivative of $u_g$ with respect to $\pi^A$ at time $T$. Using the fact that the cross terms are 0 for separable utilities, we get:

$$\begin{aligned}
\frac{\partial u_g^T}{\partial\pi^A} &= u_{gg}^T\frac{\partial g^T}{\partial\pi^A}\\
&= u_{gg}^T\left[-\frac{f\frac{\partial\pi}{\partial\pi^A}+(1-f)}{1+\pi}\frac{1}{\beta}D+\frac{\partial\pi}{\partial\pi^A}\frac{1+f\pi+(1-f)\pi^A}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'\left(\frac{\partial\pi}{\partial\pi^A}-1\right)\tau e\right]\\
&= \frac{\partial\pi}{\partial\pi^A}u_{gg}^T\left[\frac{-f}{1+\pi}\frac{1}{\beta}D+\frac{1+f\pi+(1-f)\pi^A}{(1+\pi)^2}\frac{1}{\beta}D+\alpha'\tau e\right]-u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\\
&= \frac{\partial\pi}{\partial\pi^A}u_{gg}^T\left[\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right]-u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)
\end{aligned}$$

and at times $t > T$ we simply get:

$$\frac{\partial u_g}{\partial \pi^A} = u_{gg}\alpha'\tau e\left(\frac{\partial \pi}{\partial \pi^A} - 1\right)$$

Taking the derivative of the first order conditions of the discretionary inflation problem with respect to $\pi^A$:

$$\begin{aligned}
&f\frac{D}{\beta}\left[\frac{\frac{\partial u_g^T}{\partial \pi^A}}{1+\pi} - \frac{(u_g^T-1)\frac{\partial \pi}{\partial \pi^A}}{(1+\pi)^2}\right] \\
&+(1-f)\frac{D}{\beta}\left[\frac{(u_g^T-1)+(1+\pi^A)\frac{\partial u_g^T}{\partial \pi^A}}{(1+\pi)^2} - \frac{2(1+\pi^A)(u_g^T-1)\frac{\partial \pi}{\partial \pi^A}}{(1+\pi)^3}\right] \\
&+\tau e\left(\frac{\partial u_g^T}{\partial \pi^A}\alpha' + u_g^T\alpha''\left(\frac{\partial \pi}{\partial \pi^A}-1\right)\right) + \frac{1}{1-\beta}(1-\tau)e\alpha''\left(\frac{\partial \pi}{\partial \pi^A}-1\right) \\
&+\frac{\beta}{1-\beta}\tau e\left(\frac{\partial u_g}{\partial \pi^A}\alpha' + u_g\alpha''\left(\frac{\partial \pi}{\partial \pi^A}-1\right)\right) = 0
\end{aligned}$$

Reordering the terms we obtain:

$$\begin{aligned}
&\frac{\partial \pi}{\partial \pi^A}\left[-f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - (1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3} + \alpha''\left(u_g^T\tau e + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}u_g\tau e\right)\right] \\
&+\frac{\partial u_g^T}{\partial \pi^A}\left[f\frac{D}{\beta}\frac{1}{1+\pi} + (1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2} + \alpha'\tau e\right] \\
&+\frac{\partial u_g}{\partial \pi^A}\left[\frac{\beta}{1-\beta}\alpha'\tau e\right] \\
&+(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - \alpha''\left(\tau e u_g^T + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}\tau e u_g\right) = 0
\end{aligned}$$

Replacing $\frac{\partial u_g^T}{\partial \pi^A}$ and $\frac{\partial u_g}{\partial \pi^A}$ in the above equation:

$$\begin{aligned}
&\frac{\partial \pi}{\partial \pi^A}\left[-f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - (1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3} + \alpha''\left(u_g^T\tau e + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}u_g\tau e\right)\right] \\
&+\left(\frac{\partial \pi}{\partial \pi^A}u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta} + \alpha'\tau e\right) - u_{gg}^T\left(\alpha'\tau e + \frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\right) \\
&\times\left[f\frac{D}{\beta}\frac{1}{1+\pi} + (1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2} + \alpha'\tau e\right] \\
&+u_{gg}\alpha'\tau e\left(\frac{\partial \pi}{\partial \pi^A}-1\right)\left[\frac{\beta}{1-\beta}\alpha'\tau e\right] \\
&+(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - \alpha''\left(\tau e u_g^T + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}\tau e u_g\right) = 0
\end{aligned}$$

Reordering again:

$$\frac{\partial\pi}{\partial\pi^A}\Bigg[-f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}-(1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3}+\alpha''\left(u_g^T\tau e+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}u_g\tau e\right)$$
$$+u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}\Bigg]$$
$$-u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)$$
$$-\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}+(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}-\alpha''\left(\tau eu_g^T+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}\tau eu_g\right)=0$$

For the sake of notation let us write:

$$\frac{\partial\pi}{\partial\pi^A}\,[\text{term 1}]=\text{term 2}$$

In order to find the conditions under which $\frac{\partial\pi}{\partial\pi^A}<1$ we have to prove that $\frac{\text{term 2}}{\text{term 1}}<1$. Suppose for now that term $1<0$, then we have:

$$\text{term 1}<\text{term 2}$$

Replacing terms:

$$-f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}-(1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3}+\alpha''\left(u_g^T\tau e+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}u_g\tau e\right)$$
$$+u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta}+\alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}$$
$$<u_{gg}^T\left(\alpha'\tau e+\frac{1-f}{1+\pi}\frac{1}{\beta}D\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi}+(1-f)\frac{D}{\beta}\frac{1+\pi^A}{(1+\pi)^2}+\alpha'\tau e\right)$$
$$+\frac{\beta}{1-\beta}(\alpha'\tau e)^2u_{gg}-(1-f)\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2}+\alpha''\left(\tau eu_g^T+\frac{1}{1-\beta}(1-\tau)e+\frac{\beta}{1-\beta}\tau eu_g\right)$$

Eliminating terms on both sides and rearranging:

$$-(1-f)\frac{2(1+\pi^A)}{1+\pi}+(1-2f)<0$$

Isolating $f$:

$$f\left(\frac{2(1+\pi^A)}{1+\pi}-2\right)<\frac{2(1+\pi^A)}{1+\pi}-1$$

If $\pi>\pi^A$ then:

$$f>\frac{\frac{2(1+\pi^A)}{1+\pi}-1}{\left(\frac{2(1+\pi^A)}{1+\pi}-2\right)}$$
$$=\frac{1+2\pi^A-\pi}{2(\pi^A-\pi)}$$

To see that the restriction on the probability of default is reasonable, suppose that discretionary inflation is below 100% and that the inflation target is low. Then the restriction $f > \frac{1+2\pi^A-\pi}{2(\pi^A-\pi)}$ would not be binding. Only for higher levels of discretionary inflation would this restriction become binding. At the limit, when $\pi \to \infty$, the restriction requires $f > \frac{1}{2}$.

We supposed that term $1 < 0$. Let us look at the conditions necessary for this to be true.

$$\begin{aligned} term1 = &- f\frac{D}{\beta}\frac{(u_g^T-1)}{(1+\pi)^2} - (1-f)\frac{D}{\beta}\frac{2(1+\pi^A)(u_g^T-1)}{(1+\pi)^3} + \frac{\beta}{1-\beta}(\alpha'\tau e)^2 u_{gg} \\ &+ \alpha''\left(u_g^T\tau e + \frac{1}{1-\beta}(1-\tau)e + \frac{\beta}{1-\beta}u_g\tau e\right) \\ &+ u_{gg}^T\left(\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta} + \alpha'\tau e\right)\left(f\frac{D}{\beta}\frac{1}{1+\pi} + \frac{D}{\beta}\frac{1-f)(1+\pi^A)}{(1+\pi)^2} + \alpha'\tau e\right) \end{aligned}$$

The first line is negative. The only uncertainty about the sign of term 1 therefore comes from the last two lines. Suppose that $\alpha'' \leq 0$, we then either want $\frac{(1-f)(1+\pi^A)}{(1+\pi)^2}\frac{D}{\beta} + \alpha'\tau e$ and $f\frac{D}{\beta}\frac{1}{1+\pi} + \frac{D}{\beta}\frac{1-f)(1+\pi^A)}{(1+\pi)^2} + \alpha'\tau e$ to be of the same sign or $u_{gg}^T$ to be limited in such a way for term 1 to remain negative. The former is true when $\alpha' \in \mathbb{R}_- \setminus \left(-\frac{D}{\beta\tau e}\left(f\frac{1}{1+\pi} + (1-f)\frac{1+\pi^A}{(1+\pi)^4}\right), -\frac{D}{\beta\tau e}(1-f)\frac{(1+\pi^A)}{(1+\pi)^2}\right)$. This restriction does not have any easy economic interpretation. We will therefore focus on the latter. There exists $\epsilon < 0$ such that $\alpha'' \leq 0$ and $\epsilon < u_{gg}^T$ imply that term 1 is negative.

Let us now look at the implication on the real interest rate in the fiscal fragility zone when multiple equilibria are possible. The real interest rate on bonds is given by:

$$i = \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} - 1$$

where $\pi = \pi^A$ and $\pi^e = f\pi^D + (1-f)\pi^A$ in the fiscal fragility zone. Assuming $\frac{\partial \pi^D}{\partial \pi^A} < 1$ we will show that $\frac{\partial i}{\partial \pi^A} < 0$.

$$\begin{aligned} \frac{\partial i}{\partial \pi^A} &= \frac{1}{\beta}\left(\frac{f\frac{\partial \pi^D}{\partial \pi^A} + (1-f)}{1+\pi^A} - \frac{1+f\pi^D+(1-f)\pi^A}{(1+\pi^A)^2}\right) \\ &< \frac{1}{\beta}\left(\frac{1}{1+\pi^A} - \frac{1+f\pi^D+(1-f)\pi^A}{(1+\pi^A)^2}\right) \\ &= \frac{1}{\beta}\left(\frac{f(\pi^A-\pi^D)}{(1+\pi^A)^2}\right) \\ &< 0 \end{aligned}$$

since $\pi^A - \pi^D < 0$.

Summarizing, if:

1. $\alpha'' \leq 0$
2. $f > \frac{1+2\pi^A-\pi}{2(\pi^A-\pi)}$

then there exists some $\epsilon < 0$ such that $\epsilon < u_{gg}^T$ implies that the deviation is decreasing in the target, $\frac{\partial \pi}{\partial \pi^A} < 1$, which in turn implies that the real interest rate in the fiscal fragility zone will be decreasing in the target.

# APPENDIX B – Empirics

## B.1 Testing the FOC

The first order condition of the discretionary inflation problem from equation 2.7 when assuming linear utility in consumption is given by:

$$\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\frac{1}{\beta}D(u_g^T-1)+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

where $u_g^T$ is the marginal utility of spending when deviating at time $T$ and $u_g$ is the ensuing steady state marginal utility. $\pi_T^e$ are the private agents' expectations at time $T$ and $\pi_T^D$ is the optimal discretionary inflation chosen by the policymaker when deviating from the target at time $T$. $D$ is the level of debt and $\tau e$ the policymaker's revenues. Finally, $\alpha'$ is the marginal productivity shock when deviating.

The equation can be rewritten as:

$$D\left(u_g^T-1\right)\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\right)=\tau e\alpha'\left[\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right]$$

Taking logs we obtain:

$$\begin{aligned}&d+\log(u_g^T-1)+\log(1+\pi_T^e)-2\log(1+\pi^D)\\&=\log(\tau e)+\log(\alpha')+\log\left(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right)\end{aligned}$$

where $d=\log(D)$. Using the approximation for $\log(1+x)\simeq x$ for small $x$ we have $\log(1+\pi_T^e)-2\log(1+\pi^D)=(2-f)(\pi^A-\pi^D)-\pi^A$ when replacing expectations $\pi_T^e=f\pi^D+(1-f)\pi^A$. Hence:

$$\pi^D-\pi^A=-\frac{\log(\tau e)}{2-f}+\frac{d}{2-f}-\frac{\pi^A}{2-f}-\frac{c}{2-f}$$

Where $c=\log(\alpha')-\log(u_g^T-1)+\log(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T)$ will also capture effects of debt levels $d$ and revenue $\tau e$ through the marginal utility of government spending. This unfortunately makes the coefficients less straightforward to interpret without any prior calibration and initial conditions. We propose to model the relationship for country $i$ as the following approximation:

$$\begin{aligned}\pi_{i,t}-\pi_{i,t}^A=\beta_{0,i}&+\beta_1\text{revenue}_{i,t}+\beta_2\text{debt}_{i,t}+\beta_3\text{target}_{i,t}\\&+\beta_4\text{revenue}_{i,t}*\text{debt}_{i,t}+u_{i,t}\end{aligned}$$

where the interaction term between revenue and public debt is meant to capture the dynamics of the marginal utility of government spending at time $t$. The idiosyncratic error term $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0, t = 1, ..., T$. Coefficient $\beta_{0,i}$ captures a country fixed effect while all other coefficients are common to all countries.