FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

ÂNGELO AVELAR HERMETO MENDES

HETEROGENEITY, FIRM ENTRY, AND DISTORTIONS IN NETWORKED ECONOMIES

São Paulo

2020

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Dissertação apresentada à Escola de Economia de São Paulo como pré-requisito à obtenção de título de mestre em Economia de Empresas.

Orientador: Pierluca Pannella.

Coorientador: Tiago Cavalcanti.

São Paulo

2020

Mendes, Angelo Avelar Hermeto.

Heterogeneity, firm entry, and distortions in networked economies / Angelo Avelar Hermeto Mendes. - 2020.

42 f.

Orientador: Pierluca Pannella.

Co-orientador: Tiago Cavalcanti.

Dissertação (mestrado CMEE) – Fundação Getulio Vargas, Escola de Economia de São Paulo.

1. Economia. 2. Produtividade industrial. 3. Relações intersetoriais. 4. Ciclos econômicos. I. Pannella, Pierluca. II. Cavalcanti, Tiago. III. Dissertação (mestrado CMEE) – Escola de Economia de São Paulo. IV. Fundação Getulio Vargas. V. Título.

CDU 338.312

Ficha Catalográfica elaborada por: Isabele Oliveira dos Santos Garcia CRB SP-010191/O

Biblioteca Karl A. Boedecker da Fundação Getulio Vargas - SP

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Data de aprovação: __/__/____

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Agradecimentos

Agradeço aos meus pais e à minha irmã, por todo apoio, incentivo e amor que me foi dado ao longo de todos esses anos. Agradeço a toda minha família pelo carinho e pelo ótimo ambiente que me proporcionaram.

Aos meus orientadores, Tiago e Pierluca pela paciência, pelas inúmeras conversas incrivelmente proveitosas e por toda a confiança depositada em mim no decorrer de toda a jornada. Ao Prof. Marcelo Santos, por ter aceitado participar da banca e pelos ótimos comentários que me ajudaram a melhorar este trabalho.

Aos meus amigos de BH, por me proporcionarem inúmeros momentos de lazer e descontração nas folgas que conseguia voltar pra casa. A todos os meus colegas da GV, principalmente aos meus amigos da *Famiglia Saligna*, por tornarem o mestrado um ambiente infinitamente mais agradável, por toda a ajuda nos períodos difíceis, por todo o companheirismo pra superar os desafios inerentes ao início da carreira acadêmica. Sem vocês, sem sombra de dúvidas, eu não teria conseguido.

O presente trabalho foi realizado com apoio da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Código de Financiamento 001

Resumo

Nos propomos um modelo teórico de escolha ocupacional em uma economia com relações de insumo-produto, distorções setoriais e agentes heterogêneos. Obtemos soluções analíticas para os objetos de equilíbrio e mostramos que o Teorema de Hulten não é válido em nosso ambiente, exceto em um caso limítrofe. É possível relacionar a validade de tal teorema com a dispersão das produtividades dos agentes e a tecnologia de produção das firmas. Por fim, apresentamos alguns resultados numéricos que ilustram como distorções setoriais afetam a economia.

Palavras-chave: Escolha ocupacional, insumo-produto, Hulten, agentes heterogêneos, distorções.

Abstract

We propose a theoretical model of occupational choice in an economy with input-output links, sectoral wedges, and heterogeneous agents. We provide analytical solutions for equilibrium objects and show that the Hulten's theorem does not hold in our framework. We show the limiting case in which the Hulten's theorem holds. We relate the validity of the theorem with the dispersion of entrepreneurial productivity among agents and diminishing returns to scale in the production technology. We also provide some numerical results of how wedges affect the economy.

Keywords: Occupational choice, Input-Output, Hulten, heterogeneous agents, wedges.

List of Figures

List of Tables

Contents

1 Introduction

Macroeconomists have, in general, abstracted from the complexity of the production process using models with only a few inputs and representative firms. The idea that an economy can produce final goods using only capital and labor is a simplification of a sophisticated input-output (I-O) structure. Some authors have discussed the role played by such structures to analyze the growth and the business cycles of an economy. The one-sector assumption can be particularly critical when considering distorted economies. Since the decision of opening a firm depends intrinsically on individual and sectoral characteristics including, for example, sector-specific wedges (e.g. taxes), the use of representative agent models abstains from relevant features of the economy.

This work addresses the following question: How do sector-specific wedges interact to input-output relations between firms and shape the choice of occupation and sector size in the economy? The idea is that linkages between firms through intermediate goods deliver a multiplier similar to the one associated with capital in a neoclassical growth model. Those links will spread distortions in one sector, affecting profits and optimal size choice in other sectors of the economy.

We develop a theoretical model of occupational choice based on the Lucas Jr (1978) span of control framework with sector-specific wedges and an I-O production structure. This implies that our model is a non-trivial extension of the environment introduced by Bigio and La'o (2017) since we allow for extensive margin misallocation and household heterogeneity.

We can show analytically the amplification generated by the network structure. The production functions are aggregated to a Cobb-Douglas in sectoral level, with parameters depending on the distribution of entrepreneurial productivities. Furthermore, we can show that the result from Hulten (1978) - which states that the effects of shocks on any sector are proportional to its GDP share, independently of networks linkages - does not hold in our heterogeneous agents' framework. We discipline the validity of the theorem associating it with diminishing returns to scale in firm-level, the dispersion of agents' individual productivity, and the absence of sectoral distortions. These results agree with previous findings of the literature that argue that Hulten's theorem may not hold in an economy with firm entry/exit decisions. On the other hand, for the best of our knowledge, our work is the first to provide conditions for the validity of the theorem, relating it to the heterogeneity among agents.

Using data from 2017 US Economic Census, we calibrate a simple version of our model with only two sectors (services and manufacturing) to match the share of entrepreneurs and the average firm size in each of those sector. Then we solve the model and compute the thresholds of entrepreneurial ideas. We find that the minimum productivity to become an entrepreneur is increasing in level of distortions. Moreover, we find that the those thresholds are always higher in networked economies.

The remaining of this project is organized as follows. Section 2 provides a brief review of the literature. Section 3 presents the theoretical model and some of its properties. First, we show that sectoral production functions aggregate to a Cobb-Douglas. Second, we use firms optimality conditions taking labor supply as given to derive sectoral sales/expenditures in equilibrium. Third, we find sectoral prices as function of sectoral sales. Then, we close the economy and solve for general equilibrium using agents' indifference condition between working or managing a firm. Finally, we show that Hulten's theorem does not hold in our model, unless we are in a limiting case of an undistorted economy with constant returns to scale, and no dispersion in managerial abilities. Section 4 presents some of our quantitative exercises. First, we calibrate a simple version of our model using data from 2017 US Economic Census. Second, we find the entrepreneurial thresholds for both sectors, as well as the share of entrepreneurs in economies with different production structures and different levels of distortions. Section 5 does a brief discussion of preliminary results and gives the perspectives to the development of the research.

2 Literature Review

This paper is related to two strands of literature. First, it is related to a growing literature on inter-sectoral production networks. Second, it is associated to the literature on the effects of micro-level distortions on aggregate productivity and firm entry.

2.1 Production Networks

Long Jr and Plosser (1983) and Acemoglu et al. (2012) are considered two canonical multi-sector models to study the aggregate effects of sectoral productivity shocks, and highlighting the importance of central sectors in the economy. Since then, many authors have studied the role played by input-output links in spreading and amplifying misallocation. In this work we abstract from sectoral shocks and consider persistent distortions, similarly to what is done by Jones (2011) who used a simple static model and shows an analytical solution for the effects of sectoral misallocation on aggregate GDP. However, our work has an additional level of aggregation. We start aggregating firms' technology to sectoral production functions. One key characteristic of all those models is the fact that they take the network structure as given. More recent papers (Taschereau-Dumouchel (2017), Oberfield (2018)) have tried to endogenously generate the network structure of the economy. Similarly, Boehm and Oberfield (2018) associates network formation and contract enforcement, in a model with exogenous wedges associated with product differentiation. In this paper, we also consider exogenous distortions and take firms' technology as given. However, in our model the sectoral intermediate inputs use is a result from the first layer of aggregations. Moreover, we allow agents to choose whether to produce managing a firm or to work for a wage.

More related to our work is Baqaee and Farhi (2018) who introduces a model with production networks and heterogeneous agents. However, the authors focus on propagation patterns while we analyze and characterize aggregate outcomes. Furthermore, we allow for endogenous firm opening, which emphasizes the role played by extensive margin misallocation in networked economies. Our paper is also closely related to Bigio and La'o (2017), and Baqaee (2018). The first one builds a model of production networks in distorted economies, and compute the *network multiplier*, calibrating the US economy and using sectoral spreads as the source of financial frictions. However, in their work sectoral size is exogenous since each sector is populated by a representative firm. In our case, agents choose whether to be a worker or an entrepreneur in one of the sectors of the economy, which endogenously generates size discrepancies between sectors. The second paper considers the role played by firm entry/exit in amplifying shocks in a model where sectoral centrality is endogenously determined by the mass of entrants in each industry. Baqaee (2018) also has some important conclusions on the validity of the results from Hulten (1978) in his environment. He shows that in a framework with firm entry/exit the Hulten's theorem does not hold. One key difference to our work is the fact that occupational choice there is also exogenous.

2.2 Effects of micro-level distortions on aggregate productivity

Hsieh and Klenow (2009) introduced the idea that misallocation can explain 30-50% of TFP losses in the United States and 40-60% in China. Restuccia and Rogerson (2008) considers a broader, abstract set of policies that may affect producers asymmetrically. Their quantitative results are in line with the numbers found in Hsieh and Klenow (2009). Banerjee and Moll (2010) focus on persistence of misallocation. They show that asymptotically, and under pretty mild conditions, there is no misallocation either in the intensive or extensive margin. Midrigan and Xu (2014) build a model with two sectors (traditional and modern), where misallocation of the capital comes from two channels: differences on producers' wealth (*age channel*) and the rigidity that prevents producers from adjusting capital (*adjustment channel*). [1] Differently from our work, all those papers abstract from the existent complexity in the production structure, and how it may interact with misallocation.

Our work is closely related to Buera et al. (2011), which evaluates the effects of financial frictions on two different sectors (manufacturing and services) in a framework with sector-specific non-convexities. They found that manufacturing is more affected by financial frictions due to its higher fixed cost. This paper presents a simpler static model

[1] Besides the traditional misallocation literature, Gabaix (2011) also shows the macroeconomic implications of firm-level shocks. According to this paper, idiosyncratic shocks to large firms can generate aggregate fluctuations when firm size distribution is fat-tailed. This is also closely related to the conclusions obtained by Acemoglu et al. (2012) where the network structure and centrality measures play the same role as large firms.

with exogenous wedges, these features allow us to introduce input-output relations and obtain an analytical characterization for the equilibrium of the model.

3 Model

3.1 Environment

The economy is static and populated by a continuum of individuals of size N who can choose to be either workers or entrepreneurs in a sector $i \in \mathcal{S} = \{1, 2, ..., n\}$ to produce an intermediate good. Each individual is endowed with a vector talent for managing, $\mathbf{z} := (z_i)_{i\in\mathcal{S}}$, drawn from a distribution $\Gamma(\mathbf{z})$ of entrepreneurial ideas and chooses whether to work for a wage, w or to be an entrepreneur in one sector $i \in \mathcal{S}$ and produce an intermediate good (y_i), using labor (l) and intermediary goods (x_{ij}).

3.2 Technologies

3.2.1 Intermediate Good Firms

An entrepreneur of a sector i produces an intermediate good, y_i choosing labor (l) and intermediate goods (x_{ij}), and produces according to a Cobb-Douglas production function:

$$y_i = z_i l^{\theta_i} \prod_{j\in\mathcal{S}} x_{ij}^{\sigma_{ij}}, \qquad \underbrace{\theta_i + \sum_j \sigma_{ij}}_{\eta_i} < 1 \tag{3.1}$$

Let $\sigma_i := \sum_j \sigma_{ij}$ and denote the sectoral scale by $\eta_i := \theta_i + \sigma_i$

The wage is normalized to one. We abuse notation and let $\mathbf{p} \coloneqq (p_i)_{i\in\mathcal{S}}$[1], be the vector of prices of each good j, produced by sector i. The entrepreneur takes all prices ($\mathbf{p}$, w) as given and maximize his profits.

$$\max_{l, x_{ij}} \pi_i(l, x_{ij}, \mathbf{p}, \mathbf{z})$$

where:

$$\pi_i(l, x_{ij}; \mathbf{p}, \mathbf{z}) = \underbrace{(1-\tau_i)}_{\phi_i} p_i \left[z_i l^{\theta_i} \prod_{j\in\mathcal{S}} x_{ij}^{\sigma_{ij}} \right] - \sum_{j\in\mathcal{S}} p_j x_{ij} - l$$

[1] Recall that $\mathcal{S}$ is the set of sectors in our economy. However, since each sector i produces only one intermediate good j, we abuse notation and consider $\mathcal{S}$ the set of goods j produced by each sector $i \in \mathcal{S}$. Another abuse of notation is to write y_i to denote a good produced by sector i. This is without loss of generality since there exists a clear bijection between goods and sectors. More formally, we could consider a set $\mathcal{G}$ of goods, and define $\mathbf{p} \coloneqq (p_j)_{j\in\mathcal{G}}$.

In our model, distortions are rebated to the households. In Bigio and La'o (2017) they can be either rebated to the households or wasted[2]. Since the wage is normalized to one, the indifference condition of being a worker or an entrepreneurs in any sector $i \in \mathcal{S}$ is given by the following equality.

$$\pi_i(\mathbf{z}) = 1$$

3.2.2 Final Good Producer

We have a competitive final good producer that produces according to a Cobb-Douglas production function.

$$Q = \prod_{j \in \mathcal{S}} c_j^{\psi_j}$$

The final good producer chooses c_j to maximize its profits.

$$\max_{c_j} \overline{p}Q - \sum_{j \in \mathcal{S}} p_j c_j$$

3.3 Endowments

Agents draw a vector of entrepreneurial ideas from an independent Pareto distribution $\mathbf{z} \sim \Gamma(\mathbf{z})$. They choose whether to be workers with wage 1, or entrepreneurs with profit $\pi_i(\mathbf{z})$ and consume to maximize their utility.

$$\max U(q) \quad st: \quad pq(\mathbf{z}) = u_0(\mathbf{z}), \quad \text{where} \quad u_0(\mathbf{z}) := \max\{\pi_i(\mathbf{z}) + t, 1 + t\}$$

Agents derive utility from an aggregation of intermediary goods (q) and have a positive income, therefore, the budget constraint binds and agents basically choose their occupation. Distortions are rebated to the agents. Workers and entrepreneurs receive back the same amount of resources, t, such that: $\int t := \sum_{i \in \mathcal{S}} \int_i \tau_i p_i y(\mathbf{z}) \Gamma(d\mathbf{z})$.[3]

3.4 Market Clearing Conditions

The equilibrium of the model is a vector of prices $(p_i)_{i \in \mathcal{S}}$ and policy functions: $o(\mathbf{z})$, $l(\mathbf{z})$, $x_{ij}(\mathbf{z})$, $q(\mathbf{z})$ and c_j, for $i, j \in \mathcal{S}$ such that:

---

2 We can easily extend our model allowing only a fraction δ of the distortions to be wasted.

3 It is easy to allow for a fraction of distortions to be wasted instead of rebated back to households, as in Bigio and La'o (2017). This will not bring any significant difference or intuition to our results, since rebated distortions will not affect decisions of consumption or labor supply as in their case.

(Labor):

$$\frac{L}{N} := \int l(\mathbf{z})\Gamma(d\mathbf{z}) = \int_{\{o(\mathbf{z})=W\}} \Gamma(d\mathbf{z})$$

(Intermediate Goods):

$$c_j + \sum_{i\in\mathcal{S}} \frac{X_{ij}}{N} = \int_{\{o(\mathbf{z})=j\}} \left[z_j l(\mathbf{z})^{\theta_j} \prod_{i\in\mathcal{S}} x_{ji}(\mathbf{z})^{\sigma_{ji}} \right] \Gamma(d\mathbf{z})$$

Where:
$Y_j := N \int_{\{o(\mathbf{z})\in\mathcal{S}\}} y_j \Gamma(d\mathbf{z}) = N \int y_j \Gamma(d\mathbf{z})$ and
$X_{ij} := N \int_{\{o(\mathbf{z})\in\mathcal{S}\}} x_{ij}(\mathbf{z})\Gamma(d\mathbf{z}) = N \int x_{ij}(\mathbf{z})\Gamma(d\mathbf{z})$

(Final Goods:)

$$\int q(\mathbf{z})\Gamma(d\mathbf{z}) = Q$$

3.5 Analytical Results

3.5.1 Sectoral Aggregation

In order to evaluate aggregate impacts of sectoral wedges we aggregate entrepreneurs in a sectoral production function. As in Buera et al. (2011), we can show that each sector produces according to a Cobb-Douglas function with a TFP term. For this aggregation to be possible we assume that active entrepreneurs are a small fraction of the population. This makes the computations much simpler since the measure of agents who are highly productive in more than one sector is zero. Moreover, as in Buera et al. (2011) we impose a technical condition over the parameters: $\zeta > \underline{\zeta} := \max_{i\in\mathcal{S}} 1/(1-\eta_i)$.

Proposition 1: *Assume that entrepreneurial talents for the n sectors follow mutually Pareto distributions with the same tail parameter ζ, $(z_1, ..., z_n) \sim \zeta^n (\prod_{i\in\mathcal{S}} z_i)^{-(\zeta+1)}$ for $z_i \geq 1$, $i, j \in \mathcal{S}$, and the active entrepreneurs are a small fraction of the population. Then the output of a sector equals:*

$$Y_i(L_i, X_{ij}) = A_i N^{\frac{1/\zeta}{1/\zeta+(\theta_i+\sigma_i)}} L_i^{\frac{\theta_i}{1/\zeta+(\theta_i+\sigma_i)}} \prod_{j\in\mathcal{S}} X_{ij}^{\frac{\sigma_{ij}}{1/\zeta+(\theta_i+\sigma_i)}}$$

$$A_i = \left[\frac{\zeta\Big(1-(\theta_i+\sigma_i)\Big)}{\zeta\Big(1-(\theta_i+\sigma_i)\Big)-1} \right]^{\frac{1}{1+\zeta(\theta_i+\sigma_i)}} \left[\frac{\frac{1}{p_i}}{\Big(1-(\theta_i+\sigma_i)\Big)(1-\tau_i)} \right]^{\frac{1-\zeta[1-(\theta_i+\sigma_i)]}{1+\zeta(\theta_i+\sigma_i)}}$$

Where:

$$\bar{\zeta}_i = \frac{1/\zeta}{1/\zeta + (\theta_i + \sigma_i)}$$

$$\bar{\theta}_i = \frac{\theta_i}{1/\zeta + (\theta_i + \sigma_i)} \qquad \varsigma_{ij} = \frac{\sigma_{ij}}{1/\zeta + (\theta_i + \sigma_i)}$$

Proof: See Mathematical Appendix.

The term ς_{ij} is an entry of the Input-Output matrix, denoting the share of good j in the total intermediate input use of sector i. Note that sectoral wedges, τ_i explicitly reduce the TFP of sector i. For a given $\mathbf{z}$, these distortions reduce the optimal choice of inputs (*intensive margin* misallocation), reducing output and profits. Lower profits for any level of entrepreneurial productivity increase the share of workers the population, generating *extensive margin* misallocation in our model.

3.5.2 Equilibrium Characterization

We now provide an equilibrium characterization of the model. We start taking the threshold of entrepreneurial ideas, $\hat{\mathbf{z}}$, as given to find sectoral sales and expenditures in equilibrium. Note that taking $\hat{\mathbf{z}}$ as given is exactly the same as considering an exogenous supply of labor. The proceeding for the characterization mentioned is the following: (i) solve the optimization problem of intermediate good firms; (ii) solve the optimization problem of the final good producer; (iii) stack market clearing conditions for the intermediate goods.

Lemma 1*: Let $\circ$ denote the Hadamard (entrywise) product. For any labor supply, determined by a vector of thresholds $\hat{\mathbf{z}}$, sectoral sales (G) and sectoral expenditures (U) are linear in labor supply.*

$$G = \left[\mathbb{I}_n - \Sigma' \circ \left(\mathbb{1}\phi'\right) - \psi\left(1 - \left(\phi \circ (\theta + \Sigma 1)\right)\right)'\right]^{-1} \psi F(\hat{\mathbf{z}})$$

$$U = \phi \circ (\theta + \Sigma 1) \circ G$$

Proof: See Mathematical Appendix.

As in Bigio and La'o (2017) sectoral sales/expenditures are linear functions of the aggregate labor supply. However, in our model the supply of labor will be defined by the share of agents whose potential profits are lower than the wage (normalized to one). Our next step is to solve for equilibrium prices using firms production function and sectoral aggregations from ***Proposition 1***.

Proposition 2 (Equilibrium Prices): *The equilibrium prices in the economy can be written as a function of sectoral sales:*

$$\log \mathbf{p} = \Big[I_n \circ \Xi 1' - \Omega\Big]^{-1}\Big(\log G \circ (1 - \overline{\theta} - \Omega 1) - (\overline{\theta} + \Omega 1) \circ \log \phi - \tilde{\kappa}\Big)$$

$$\log \overline{p} = \psi'(\log \mathbf{p} - \log \psi)$$

Where:

$$\Xi_i = \frac{\zeta}{1 + \zeta(\theta_i + \sigma_i)}$$

$$\tilde{\kappa}_i = (\overline{\theta}_i + \sum_j \varsigma_{ij}) \log \phi_i + \overline{\theta}_i \log \theta_i + \sum_j \varsigma_{ij} \log \sigma_{ij} + \frac{1}{1 + \zeta(\theta_i + \sigma_i)} \log\left[\frac{\zeta(1 - \theta_i - \sigma_i)}{\zeta(1 - \theta_i - \sigma_i) - 1}\right]$$
$$- \frac{1 - \zeta(1 - \theta_i - \sigma_i)}{1 + \zeta(\theta_i + \sigma_i)} \log(1 - \theta_i - \sigma_i)$$

Proof: See Mathematical Appendix.

Notice that prices are also a function of the level of sectoral distortions and aggregate input output relations. The first term in the right hand side in the above equation captures the entire hierarchy of infinite-order network effects of distortions in prices.[4]

Next, we close the economy and solve for general equilibrium using prices, sectoral sales, and agents' indifference condition. Our strategy allow us to evaluate the size of each sector in the economy, since we can solve for general equilibrium and find the entrepreneurial thresholds for each sector using the indifference condition $\pi_i(\hat{\mathbf{z}}) = 1$.

Proposition 3 (Occupational Choice)*: The equilibrium thresholds of entrepreneurial ideas, and sectoral sales (**Lemma 1**) are determined by the following non-linear system.*

$$\log \hat{\mathbf{z}} = -\Big(I_n - \Sigma\Big)\Big[I_n \circ \Xi 1' - \Omega\Big]^{-1}\Big(\log G \circ (1 - \overline{\theta} - \Omega 1) + \tilde{\kappa}\Big) - \mathbf{T}$$

$$G = \Big[\mathbb{I}_n - \Sigma' \circ \Big(\mathbb{1}\phi'\Big) - \psi\Big(1 - \Big(\phi \circ (\theta + \Sigma 1)\Big)\Big)'\Big]^{-1} \psi F(\hat{\mathbf{z}})$$

Where:

$$\mathbf{T_i} = \log \phi_i + \theta_i \log \theta_i + (1 - \theta_i - \sigma_i) \log(1 - \theta_i - \sigma_i) + \sum_j \sigma_{ij} \log \sigma_{ij}$$

Proof: See Mathematical Appendix.

4 This term is closely related to the Leontief inverse matrix, which will be detailed in the next subsection.

This result is the key difference between our model and the current work in the literature. The above non-linear system fully determines the thresholds of occupational choice as a function of the distortions in one sector, its labor shares, and its input-output relationships. We numerically show that entrepreneurial thresholds increase as the economy becomes more distorted. The first equation describing the occupational choice of the agents follows from the stacked indifference conditions. The second equation was already deduced in ***Lemma 1*** and expresses sectoral sales as a function of a given labor supply.

While in Bigio and La'o (2017) there is only *intensive margin* misallocation since the choice of labor and intermediate inputs is distorted from the optimal, our model allow for *extensive margin* misallocation since productive agents may choose to become workers due to the high level of sectoral distortions. ***Proposition 3*** also allow us to calculate average firm size in each sector of the economy.

3.5.3 Hulten's Theorem

The Hulten's Theorem (Hulten (1978)) states that the aggregate effect of any sector's TFP shock on aggregate GDP is proportional to its equilibrium share of sales (Domar weight). It is a well know fact in the literature that Hulten's Theorem does not hold in distorted economies. However, we can also discipline the validity of the theorem in an heterogeneous agents framework.

Starting from sectoral production functions, we can find an analytical solution for total output. This closed-form solution allows us to analyze the effects of sectoral distortions on aggregate output.

Proposition 3: *In the competitive equilibrium, the solution for total production of aggregate final good is:*

$$Q = A^{\tilde{\mu}} N^{\tilde{\zeta}} L^{\tilde{\theta}} \epsilon$$

Where:

$\mu' = \psi'(I - \Omega)^{-1}$

$\tilde{\mu} = 1'\mu$

$\tilde{\zeta} = \psi'(I - \Omega)^{-1}\overline{\zeta}$

$\tilde{\theta} = \psi'(I - \Omega)^{-1}\overline{\theta}$

$\epsilon = \psi'(I - \Omega)^{-1}\chi + \psi'\xi_c$

Ω *is the input-output matrix, with entries* ς_{ij}*, and* $\overline{\zeta}$*, and* $\overline{\theta}$ *were defined in* ***Proposition***

1.

Proof: See Mathematical Appendix.

The term $(I - \Omega)$ is the Leontief inverse matrix. Each entry of this matrix gives the effect of a change in the productivity of sector j on the sector's i output. It also considers all indirect effects. Multiplying the Leontief inverse matrix by the vector ψ we account for the weight of this sector in the production of the final good, the resulting vector is denoted by μ. A similar vector is also found by Acemoglu et al. (2012) and Bigio and La'o (2017), which they call the **influence vector**, summing the entries of this vector we get the input-output multiplier, $\tilde{\mu}$. We should note that μ reveals the effects of sectoral TFP shocks on the aggregate product of this economy.

$$\frac{d \log Q}{d \log A_i} = \mu_i, \quad \forall i \in \mathcal{S}$$

This result shows that Hulten's Theorem **does not hold** for our economy[5], otherwise the effect of sectoral TFP shocks on GDP would be given by the Domar weight of sector i. While network literature usually solves the problem of a representative firm in each sector i, our model allows heterogeneity of firms within the sector, throughout entrepreneurs' vector of ideas $\mathbf{z}$. Heterogeneity plays an important role here, since firms' technology differs from sectoral technologies. While the vector of Domar weights is such that: $N\lambda = \psi'(I - \Sigma')^{-1}$, the vector of sectoral multipliers is $\mu' = \psi'(I - \Omega)^{-1}$. Σ represents the share of good j used by a firm in sector i, with entries σ_{ij} and Ω is the I-O matrix of the whole economy, with entries ς_{ij}. By definition, $\mu = N\lambda$ holds when $\sigma_{ij} = \varsigma_{ij}$, or, $1/\zeta + \theta_i + \sum_j \sigma_{ij} = 1$. For more details see the proof of Proposition 4 in the Appendix.

Corollary 1: *The Hulten's theorem holds if (i) $\zeta \to \infty$ (representative economy); (ii) firms have constant returns to scale, ie, $\theta_i + \sum_j \sigma_{ij} = 1$; and (iii) $\phi = 1$ (undistorted economy)*

Proof: See Mathematical Appendix.

In the Mathematical Appendix, we prove that the three conditions are sufficient to equalize the effects of sectoral TFP shocks to the Domar weights. Note that distortions have a direct effect on the sales vector (G) that is not present in the influence vector. If we

5 One can define the Hulten's Theorem in our framework as the effect of shocks in sectoral distortions on aggregate output: $\frac{d \log Q}{d \log \phi_i}$

set $\phi = 1$, those two vectors become almost identical, except for the differences between sectoral input shares and firms' input shares.

The above corollary disciplines the validity of Hulten's theorem. Although previous works have already shown that the theorem may be valid for efficient economies, which in our case are expressed by $\phi = 1$, our result imposes additional requirements for it to be true. One of the additional assumptions of **Corollary 1** is that $\zeta \to \infty$, and the Pareto distribution collapses to a Dirac, and the entrepreneurial ideas are homogeneously distributed. Moreover, our model assumes diminishing returns to scale for the firms. This is recurrent in the firm dynamics literature since it allows firms to have positive profits, otherwise, if we had constant returns to scale firms would make zero profits and any positive wage, w, would rule out the decision of opening new firms. When sectoral scale approaches one, we are closer to a constant returns-to-scale economy when only the most productive agent becomes an entrepreneur. Taking those assumptions together, we are approximating our model to a representative agent economy.

4 Numerical Results

We start calibrating and solving our model for a simple version with two sectors (services and manufactures). A more detailed calibration with more sectors is exhibited in the Appendix.

4.1 Calibration

We follow Buera et al. (2011) in targeting moments of the US economy. Specifically, we use data from the 2017 US Economic Census. However, differently from the model in Buera et al. (2011), our closed form solutions apply to both undistorted and distorted economies. Therefore, alternative calibrations may also be ran with data from developing countries. We start by a the calibration of the input-output economy, in which firms use labor and intermediate goods to produce its own good.

Table 1: Calibration - I-O Economy

Targeted Moments	US Data	Model	Parameters
Share of entrepreneurs	5%	4.39%	$\zeta = 3.18$
Average Firm Size - Services	16	14	$\eta_S = 0.625$
Average Firm Size - Manufactures	40	40	$\eta_M = 0.681$
Agg. Labor share - Services ($\overline{\theta}_S$)	0.31	0.35	$\theta_S = 0.325$
Agg. Labor share - Manufactures ($\overline{\theta}_M$)	0.18	0.22	$\theta_M = 0.221$
Agg. Services Share - Services (ς_{SS})	0.31	0.29	$\sigma_{SS} = 0.27$
Agg. Manufacturing Share - Services (ς_{SM})	0.03	0.03	$\sigma_{SM} = 0.03$
Agg. Services Share - Manufacturing (ς_{MS})	0.19	0.19	$\sigma_{MS} = 0.19$
Agg. Manufacturing Share - Manufacturing (ς_{MM})	0.35	0.27	$\sigma_{MM} = 0.27$

The parameter ζ controls dispersion of entrepreneurial talent. As $\zeta \to \infty$, the mass of low talented agents approaches one.[1] On the other hand, whether ζ goes to its lower bound, the distribution of entrepreneurial talent becomes more disperse, and the heterogeneity among agents grows.[2] Sectoral scale determines the average size of firms in each sector. Fixing the sum of technological parameters we calibrate θ_i to match aggregate

[1] More specifically, the mass of agents with the **lowest** possible productivity level approaches one.
[2] The lower bound of ζ is a technical condition that allows the aggregation process.

labor share in services and manufacturing sector. We then used results from Valentinyi and Herrendorf (2008) to calibrate σ_{ij}. We also consider the scenario of an Horizontal Economy. In this case, we keep the scale parameters constant, that is: $\eta_i^{I-O} = \theta_i^{Horizontal}$.

Table 2: Naive Calibration - Horizontal Economy

Parameters	Value
Pareto Parameter (ζ)	3.18
Scale Services ($\eta_S^{I-O} = \theta_S^{Horizontal}$)	0.625
Scale Manufacturing ($\eta_M^{I-O} = \theta_M^{Horizontal}$)	0.681

4.2 Horizontal vs. I-O economy

In this section we show the differences between the horizontal economy and an economy with all possible input-output linkages. In the former, the optimal choice of inputs (labor) is distorted as long as $\phi_i \in (0, 1)$, $\forall i \in \mathcal{S}$. However, the absence of sectoral linkages will not spread distortions from one sector to another, and primary inputs have a unique path from primary suppliers to households.

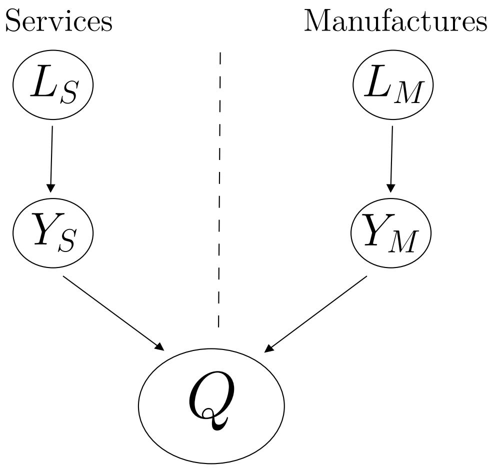

Figure 1: No I-O/Horizontal Economies

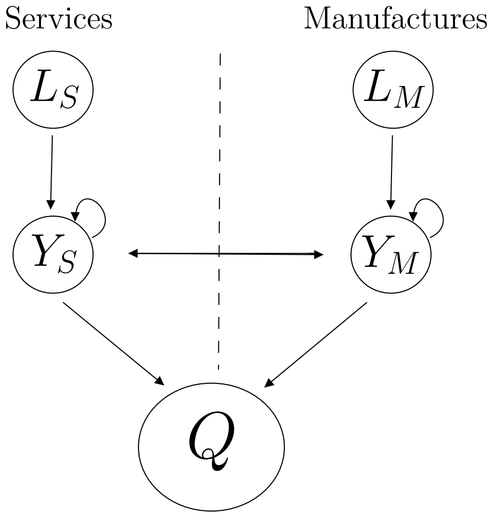

Figure 2: I-O Economies

On the other hand, in input-output economies as one sector uses intermediate goods as input in the production process, distortions that affect one sector will spread throughout network linkages. In this case, the allocative failures are amplified.

Horizontal Economy:

$$y_S = l^{\theta_S + \sum_j \sigma_{Sj}} \quad \text{and} \quad y_M = l^{\theta_M + \sum_j \sigma_{Mj}} \tag{4.1}$$

Input-Output Economy:

$$y_S = l^{\theta_S} \prod_j x_{Sj}^{\sigma_{Sj}} \quad \text{and} \quad y_M = l^{\theta_M} \prod_j x_{Mj}^{\sigma_{Mj}} \tag{4.2}$$

The indifference condition between managing a firm and working for a wage will define two thresholds for each value of distortion. Both panels express a similar pattern. The curves are increasing in the value of the distortion (decreasing in ϕ_i), indicating less entrepreneurs in more distorted economies.

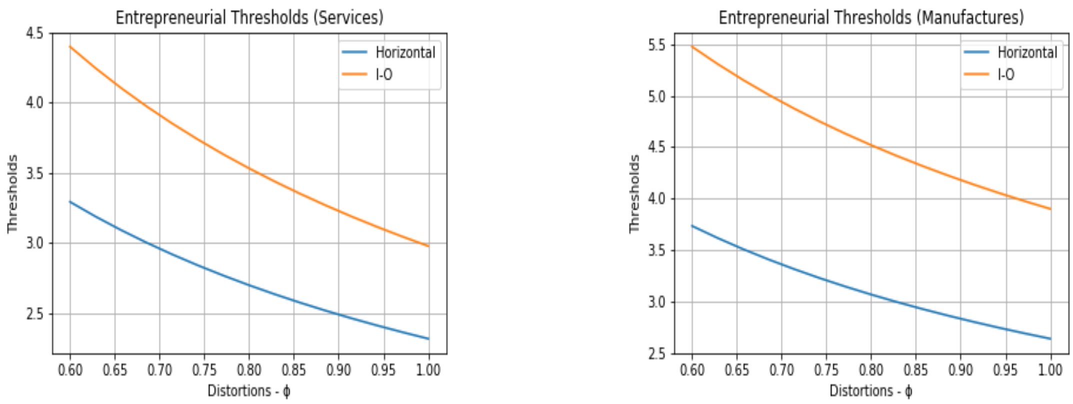

Figure 3: Entrepreneurial Thresholds

The above results indicate that the share of agents above the lines in the horizontal economy, that is, those who choose to manage a firm, is greater than in the networked economies. Next we show the share of entrepreneurs in both economies.

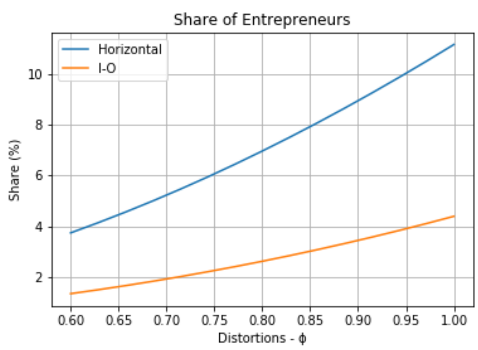

Figure 4: Share of Entrepreneurs - I-O vs. Horizontal

Figure (4) is another evidence of extensive margin misallocation in networked economies. As the thresholds are greater in I-O economies for any level of distortion, we observe that the absence of sectoral links may be an important factor to firm entry/exit. Note that for any level of sectoral wedges, the I-O economy has less entrepreneurs than the horizontal economy. This reflects our primary intuition on the role played by input-output links in amplifying distortions. Also, the share of entrepreneurs falls more in the horizontal case than in the I-O economy.

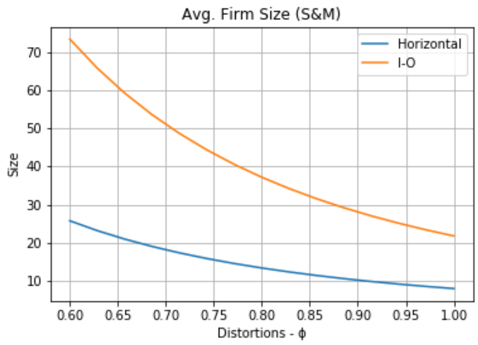

Figure 5: Average Firm Size

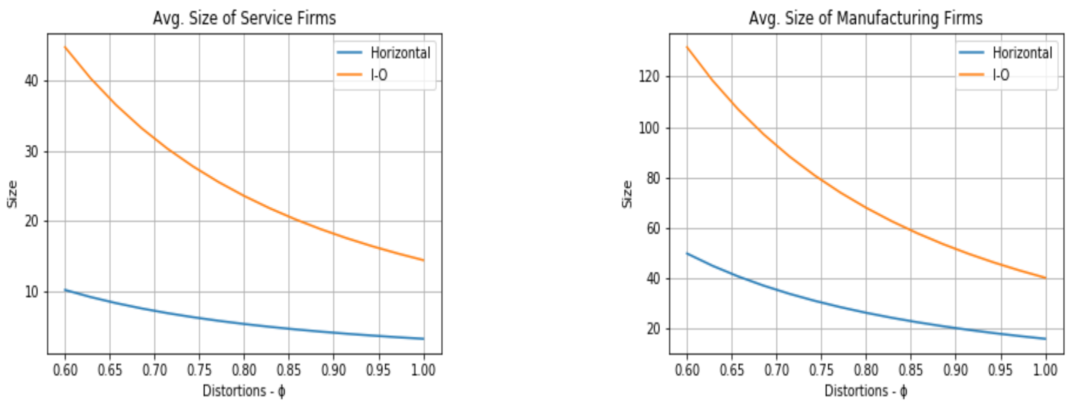

Figure 6: Average Firm Size - Services vs. Manufactures

Distortions have two opposite effects in firm size. First, it reduces the optimal choice of labor for existent firms (intensive margin). Second, in a less distorted economy profits are higher, which leads to a higher share of entrepreneurs, that is, more firms in both sectors (extensive margin) which may reduce average firm size. According to Figure (6), the second effect is stronger than the first one, that is, the number of new firms more than compensate the increase in hired labor by incumbent firms in both sectors, reducing average firm size. The higher share of entrepreneurs in the horizontal economy may also explain the higher average firm size in Input-Output economies. Moreover, we should note that the slope of average firm size as a function of distortions is higher in economies with sectoral interlinks. This is an evidence that networked economies are more sensible to shocks, since those shocks are amplified by the input-output structure spreading through the sectoral links.

Next, we evaluate the effects of distortions in aggregate TFP. These distortions can generate effects on two different directions. First, in distorted economies firms are not choosing inputs optimally. From **Proposition 1** we can observe this negative partial equilibrium effect.

$$\frac{d \log A_i}{d\phi_i} = \frac{1 - \zeta(1 - \eta_i)}{1 + \zeta\eta_i} \frac{1}{\phi_i} > 0$$

Second, distorted economies increase the entrepreneurial thresholds. In this case, only the most productive agents (the largest firms) choose to manage a firm, which increases the productivity of the pool of active firms. According to Figure (7) the effect on selection of entrepreneurs is dominated by the direct effects of distortions, since the TFP losses are attenuated in general equilibrium. [3]

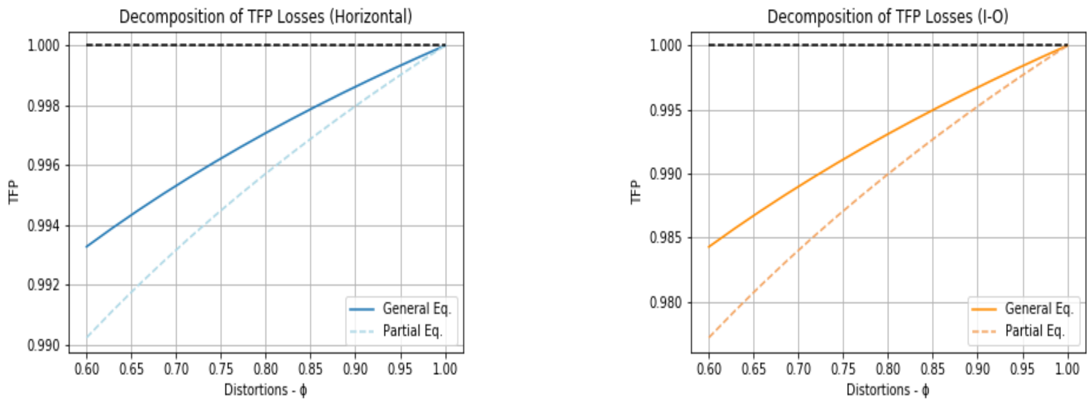

Figure 7: Decomposition - Partial Eq. vs. General Eq.

[3] In Figure (7) we normalized the TFP in the undistorted economy to one.

5 Conclusion

We have developed a model linking sectoral distortions, occupational choice, and sectoral interlinks. Our model allows for two layers of aggregation. First, we can find analytical results for sectoral production functions depending on sectoral TFP terms. Second, we find analytical solutions for equilibrium objects such as sectoral sales, sectoral expenditures, prices, and entrepreneurial thresholds. We numerically compare entrepreneurial thresholds in a *horizontal economy* and in a *input-output economy*. Furthermore, we can compare the share of entrepreneurs and firm size in each sector for those two different production structures.

Our second layer of aggregation shows that Hulten's theorem does not hold in our framework even when we consider an undistorted economy, characterized by $\phi = 1$. Moreover, we can fully discipline its validity relating it to the Pareto distribution of agents' entrepreneurial ideas and diminishing returns to scale at the firm level. When the tail parameter of the Pareto distribution goes to infinity, and sectoral scale goes to one we are approximating our model to a representative agent economy, such as most of the previous papers of the I-O literature.

Our quantitative exercises indicate that the share of entrepreneurs in horizontal economies is higher than in networked economies, holding the scale parameters constant. This result highlights the importance of *extensive-margin* misallocation in networked economies. Moreover, we discuss two opposite effects of sectoral distortions on aggregate TFP. First, it reduces the optimal choice of inputs and firms' profits. Second, it increases the productivity on the pool of active entrepreneurs. Numerical exercises indicate that the second effect is stronger in the model, and the effects are different according to the production structure.

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Appendix

Appendix

Mathematical Appendix

Proof of Proposition 1

Solving for optimal choice of inputs we have:

$$l = (\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{1-\sum_j \sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

$$x_{ij} = (\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\sigma_{im}}{p_m}\right)^{\frac{1-\theta_i-\sum_{j\neq m} \sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

$$\prod_{j\neq m} \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

Claim 1: $l(z_i) = \left(\frac{z_i}{Z_i}\right)^{\frac{1}{1--\theta_i-\sum_j \sigma_{ij}}} \left(\frac{L_i}{N}\right)$ (analogous to x_{ij}).

Proof of Claim 1:

$$\frac{l(z_i)}{z_i^{\frac{1}{1--\theta_i-\sum_j \sigma_{ij}}}} = (\phi_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

$$\int_{\{o(\mathbf{z})\in\mathcal{S}\}} l(z_i) = \int_{\{o(\mathbf{z})\in\mathcal{S}\}} (\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

$$\frac{L_i}{N} = (\phi_i Z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}}$$

$$\frac{L_i}{N} = Z_i^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \frac{l(z_i)}{z_i^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}}} \quad \square$$

Solving for the optimal profit we get that:

$$\pi_i = (\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \left(1-\theta_i-\sum_j \sigma_{ij}\right) \quad (1)$$

As in Buera et al. (2011) there exist two threshold for entrepreneurial ideas, $\hat{z}_i(z_{-i})$, dividing the occupational choice space into workers and entrepreneurs (services and manufactures) and defined by two indifference conditions:

$$(\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \left(1-\theta_i-\sum_j \sigma_{ij}\right) = w \tag{2}$$

$$\begin{gathered}(\phi_i z_i p_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{\theta_i}{w}\right)^{\frac{\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j \left(\frac{\sigma_{ij}}{p_j}\right)^{\frac{\sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \left(1-\theta_i-\sum_j \sigma_{ij}\right) \\ = \\ (\phi_{-i} z_{-i} p_{-i})^{\frac{1}{1-\theta_{-i}-\sum_j \sigma_{-ij}}} \left(\frac{\theta_{-i}}{w}\right)^{\frac{\theta_{-i}}{1-\theta_{-i}-\sum_j \sigma_{-ij}}} \prod_j \left(\frac{\sigma_{-ij}}{p_j}\right)^{\frac{\sigma_{-ij}}{1-\theta_{-i}-\sum_j \sigma_{-ij}}} \\ \left(1-\theta_{-i}-\sum_j \sigma_{-ij}\right)\end{gathered} \tag{3}$$

Where $-i \in \mathcal{S}/\{i\}$. We want to aggregate firms output to get an expression for the output of sector i. Using **Claim 1** we have:

$$Y_i = N \int_{\hat{z}_i}^{\infty} \int_1^{\hat{z}_{-i}(z_i)} z_i l(z_i)^{\theta_i} \prod_j x_{ij}^{\sigma_{ij}} \Gamma(d\mathbf{z})$$

Where

$$Y_i = N \int_{\hat{z}_i}^{\infty} \int_1^{\hat{z}_{-i}(z_i)} z_i \left(\frac{z_i}{Z_i}\right)^{\frac{\theta_i+\sum_j \sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \left(\frac{L_i}{N}\right)^{\theta_i} \prod_j \left(\frac{X_{ij}}{N}\right)^{\sigma_{ij}}$$

$$Y_i = Z_i N^{1-\theta_i-\sum_j \sigma_{ij}} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}$$

Using the first order conditions, market clearing conditions, the optimal input choices, assuming $\Gamma(d\mathbf{z}) = \zeta^n (\prod_{i\in\mathcal{S}} z_i)^{-(\zeta+1)}$, for $z_i \geq 1$, and that entrepreneurs are a small fraction of the population, i.e, $\hat{z}_i$ is large for $i \in \mathcal{S}$, we get a closed solution for the threshold $\hat{z}_i$.

$$Z_i = \left[\int_{\hat{z}_i}^{\infty} \int_1^{\hat{z}_{-i}(z_i)} z^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} \Gamma(d\mathbf{z})\right]^{1-\theta_i-\sum_j \sigma_{ij}} \approx \left[\int_{\hat{z}_i}^{\infty} \zeta z_i^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}-(\zeta+1)} dz_j\right]^{1-\theta_i-\sum_j \sigma_{ij}}$$

Using properties of the Pareto distribution:

$$Z_i = \left[\frac{\zeta(1-\theta_i - \sum_j \sigma_{ij})}{\zeta(1-\theta_i - \sum_j \sigma_{ij}) - 1} z_i^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}} - \zeta} \right]^{1-\theta_i-\sum_j \sigma_{ij}} \tag{4}$$

Claim 2: $\theta_i/w = (L_i^{1-\theta_i}/\prod_j X_{ij}^{\sigma_{ij}} N^{1-\theta_i-\sum_j \sigma_{ij}})(1/\phi_i p_i Z_i)$ (analogous to L_i and X_{ij})

Proof of Claim 2:

$$\frac{\theta_i}{w} = \frac{l^{1-\theta_i}}{\phi_i p_i z_i \prod_j x_{ij}^{\sigma_{ij}}} = \frac{(z_i/Z_i)^{\frac{1-\theta_i}{1-\theta_i-\sum_j \sigma_{ij}}} (L_i/N)^{1-\theta_i}}{\phi_i p_i z_i (z_i/Z_i)^{\frac{\sum_j \sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} \prod_j X_{ij}^{\sigma_{ij}}} = \frac{L_i^{1-\theta_i}}{\phi_i p_i Z_i \prod_j X_{ij}^{\sigma_{ij}} N^{1-\theta_i-\sum_j \sigma_{ij}}} \qquad \square$$

Using **Claim 2** and the indifference between being worker or an entrepreneur in sector i:

$$(1-\theta_i - \sum_j \sigma_{ij})(\phi_i p_i z_i)^{\frac{1}{1-\theta_i-\sum_j \sigma_{ij}}} Z_i^{\frac{-\theta_i-\sum_j \sigma_{ij}}{1-\theta_i-\sum_j \sigma_{ij}}} N^{-\theta_i-\sum_j \sigma_{ij}} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} = w$$

Using 4:

$$\hat{z}_i = \left[\frac{\left(\frac{w}{p_i}\right)\left(\frac{\zeta(1-\theta_i-\sum_j \sigma_{ij})}{\zeta(1-\theta_i-\sum_j \sigma_{ij})-1}\right)^{\theta_i+\sum_j \sigma_{ij}}}{\phi_i(1-\theta_i-\sum_j \sigma_{ij}) N^{-\theta_i-\sum_j \sigma_{ij}} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}} \right]^{\frac{1}{1+\zeta(\theta_i+\sum_j \sigma_{ij})}} \tag{5}$$

Using 4, 5 and $Y_i = Z_i N^{1-\theta_i-\sum_j \sigma_{ij}} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}$ we get the desired result. ■

Equilibrium Characterization

Solution to Firm's Problem

$$\max \phi_i p_i z_i l^{\theta_i} \prod_{j\in\mathcal{S}} x_{ij}^{\sigma_{ij}} - l - \sum_{j\in\mathcal{S}} p_j x_{ij}$$

FOC:

$$(l): \quad \phi_i \theta_i \frac{p_i y_i(z)}{l_i(z)} = 1 \Longrightarrow l(z) = \phi_i \theta_i g_i(z)$$

$$(x_{ij}): \quad \phi \sigma_{ij} p_i y_i(z) = p_j x_{ij} \Longrightarrow p_j x_{ij} = \phi_i \sigma_{ij} g_i(z)$$

Define the expenditure of sector i, G_i

$$U_i = \int_i u(z) = \int_i [l(z) + \sum_j p_j x_{ij}] = \int_i \phi_i g(z)(\theta_i + \sigma_i) = \phi_i G_i(\theta_i + \sigma_i)$$

Proof of Lemma 1

We start by the market clearing condition:

$$\int y_j(\mathbf{z}) = c_j + \sum_{i=1}^{n} \int_{\{o(a,\mathbf{z})=i\}} x_{ij}(\mathbf{z})$$

$$\int p_j y_j(\mathbf{z}) = p_j c_j + \sum_{i=1}^{n} \int_{\{o(a,\mathbf{z})=i\}} p_j x_{ij}(\mathbf{z})$$

$$\underbrace{\int g_j(z)}_{\text{Revenue from product } j} = \underbrace{\psi_j \int u_0(z)}_{\text{Consumption of product j}} + \underbrace{\sum_{i=1}^{n} \int_{\{o(\mathbf{z})=i\}} p_j x_{ij}}_{\text{Expenditure on good } j \text{, used as input}}$$

Recall that $\int u_0(z)$ is the total “household" expenditure of this economy. [1]

$$\int u_0(z) = \underbrace{w}_{1} \int_{\{o(\mathbf{z})=W\}} \Gamma(d\mathbf{z}) + \int_{\{o(\mathbf{z})=S\}} \pi_S(\mathbf{z})\Gamma(d\mathbf{z}) + \int_{\{o(\mathbf{z})=M\}} \pi_M(\mathbf{z})\Gamma(d\mathbf{z}) + \sum_{i=1}^{n} \int_i \tau_i g_i(\mathbf{z})$$

$$U_0 = F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i}) + \sum_{i=1}^{n} \int_{\{o(\mathbf{z})=i\}} \pi(\mathbf{z})\Gamma(d\mathbf{z}) + \sum_{i=1}^{n} \int_{\{o(\mathbf{z})=i\}} \tau_i g_i(\mathbf{z})\Gamma(d\mathbf{z})$$

$$U_0 = F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i}) + \sum_{i=1}^{n} \int_{\{o(\mathbf{z})=i\}} [g_i(\mathbf{z}) - u_i(\mathbf{z})]$$

$$U_0 = F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i}) + \sum_{i=1}^{n} (G_i - U_i)$$

Therefore:

$$U_0 = F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i}) + \mathbb{1}'(G - U)$$

$$\begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix} U_0 + \underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \dots \sigma_{3n} \\ \vdots & \vdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots \sigma_{nn} \end{bmatrix}'}_{\Sigma'} \begin{bmatrix} \phi_1 G_1 \\ \phi_2 G_2 \\ \vdots \\ \phi_n G_n \end{bmatrix}$$

$$G = \psi F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i}) + \psi 1'(G - U) + \Sigma'(\phi \circ G)$$

From firm optimality, we have that firm expenditures satisfy $u_i(z) = \phi_i g_i(z)$, integrating over z we get the expenditures from one specific sector i, stacking all those sectors we have:

$$U = \phi \circ (\theta + \Sigma 1) \circ G$$

[1] u_0 is the consumption expenditure of our economy.

$$G = \Sigma'(\phi \circ G) + \psi 1'(G - (\phi \circ (\theta + \Sigma 1) \circ G)) + \psi F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i})$$

$$G = \left[\Sigma' \circ \left(1\phi'\right) + \psi\left(1 - \left(\phi \circ (\theta + \Sigma 1)\right)\right)'\right] G + \psi F(z_i < \hat{z}_i, z_{-i} < \hat{z}_{-i})$$

$$G = \left[\mathbb{I}_n - \Sigma' \circ \left(\mathbb{1}\phi'\right) - \psi\left(1 - \left(\phi \circ (\theta + \Sigma 1)\right)\right)'\right]^{-1} \psi F(\hat{\mathbf{z}}) \quad \blacksquare \tag{6}$$

Proceeding as Bigio and La'o (2017) we can get U_0 as a linear function of labor supply. We are still holding labor supply as fixed, taking occupational choice as exogenous.

Proposition 2 - Equilibrium Prices

This proposition characterizes the prices of the economy as a function of sectoral distortions and the network. After having a closed form solution for prices we can use those prices to calculate the profits as a function of parameters only and then get the working/entrepreneurship occupational threshold, $\hat{z}$.

Starting from the production function:

$$y_i = z_i l^{\theta_i} \prod_j x_{ij}^{\sigma_{ij}}, \qquad \theta_i + \sum_j \sigma_{ij} < 1$$

$$\int y_i = \int z_i l^{\theta_i} \prod_j x_{ij}^{\sigma_{ij}}$$

$$p_i Y_i = p_i A_i L^{\overline{\theta}_i} \prod_j X_{ij}^{\varsigma_{ij}}$$

By the FOC:

$$\int l(z) = \int \theta_i \phi_i g(z) \implies \frac{L_i}{1} = \theta_i \phi_i G_i$$

$$\int x_{ij}(z) = \int \sigma_{ij} \phi_i g(z) \implies \frac{X_{ij}}{1} = \sigma_{ij} \phi_i \frac{G_i}{p_j}$$

Hence we have:

$$p_i Y_i = p_i A_i \left(\frac{L_i}{1}\right)^{\overline{\theta}_i} \prod_j \left(\frac{X_{ij}}{1}\right)^{\varsigma_{ij}}$$

$$G_i = p_i A_i \left(\phi_i \theta_i G_i\right)^{\overline{\theta}_i} \prod_j \left(\phi_i \sigma_{ij} \frac{G_i}{p_j}\right)^{\varsigma_{ij}}$$

$$\log G_i = \log p_i + \log A_i + \overline{\theta}_i \left[\log \phi_i + \log \theta_i + \log G_i\right] + \sum_j \varsigma_{ij} \log \sigma_{ij} + \sum_j \varsigma_{ij} \log \phi_i$$
$$+ \sum_j \varsigma_{ij} \log G_i - \sum_j \varsigma_{ij} \log p_j$$

$$\log G_i = \log p_i + \log A_i + \overline{\theta}_i \log G_i + \sum_j \varsigma_{ij} \log G_i - \sum_j \varsigma_{ij} \log p_j + (\overline{\theta}_i + \sum_j \varsigma_{ij}) \log \phi_i + \log \kappa_i$$

Where: $\log \kappa_i = \overline{\theta}_i \log \theta_i + \sum_j \varsigma_{ij} \log \sigma_{ij}$

But note that A_i also depend on prices (following the derivation of Buera et al. (2011)):

$$A_i = \left[\frac{\zeta(1-\theta_i-\sigma_i)}{\zeta(1-\theta_i-\sigma_i)-1}\right]^{\frac{1}{1+\zeta(\theta_i+\sigma_i)}} \left[\frac{\frac{1}{p_i}}{1-\theta_i-\sigma_i}\right]^{\frac{1-\zeta(1-\theta_i-\sigma_i)}{1+\zeta(\theta_i+\sigma_i)}}$$

$$\log A_i = -\frac{1-\zeta(1-\theta_i-\sigma_i)}{1+\zeta(\theta_i+\sigma_i)} \log p_i + cte$$

Where $cte = \frac{1}{1+\zeta(\theta_i+\sigma_i)} \log\left[\frac{\zeta(1-\theta_i-\sigma_i)}{\zeta(1-\theta_i-\sigma_i)-1}\right] - \frac{1-\zeta(1-\theta_i-\sigma_i)}{1+\zeta(\theta_i+\sigma_i)} \log(1-\theta_i-\sigma_i)$ Therefore:

$$\log G_i = \log p_i - \frac{1-\zeta(1-\theta_i-\sigma_i)}{1+\zeta(\theta_i+\sigma_i)} \log p_i + \log G_i \left(\overline{\theta}_i + \sum_j \varsigma_{ij}\right) - \sum_j \varsigma_{ij} \log p_j + \tilde{\kappa}_i$$

$$\log G_i = \underbrace{\frac{\zeta}{1+\zeta(\theta_i+\sigma_i)}}_{\Xi_i} \log p_i + \log G_i \left(\overline{\theta}_i + \sum_j \varsigma_{ij}\right) - \sum_j \varsigma_{ij} \log p_j + (\overline{\theta}_i + \sum_j \varsigma_{ij}) \log \phi_i + \tilde{\kappa}_i$$

Where:

$$\tilde{\kappa}_i = \overline{\theta}_i \log \theta_i + \sum_j \varsigma_{ij} \log \sigma_{ij} + \frac{1}{1+\zeta(\theta_i+\sigma_i)} \log\left[\frac{\zeta(1-\theta_i-\sigma_i)}{\zeta(1-\theta_i-\sigma_i)-1}\right]$$
$$-\frac{1-\zeta(1-\theta_i-\sigma_i)}{1+\zeta(\theta_i+\sigma_i)} \log(1-\theta_i-\sigma_i)$$

$$\log G = \Xi \log p + \log G \circ \left(\overline{\theta} + \Omega 1\right) - \Omega \log p + (\overline{\theta} + \Omega 1) \circ \log \phi + \tilde{\kappa}$$

$$\left[I_n \circ \Xi 1' - \Omega\right] \log p = \log G - \log G \circ (\overline{\theta} + \Omega 1) - (\overline{\theta} + \Omega 1) \circ \log \phi - \tilde{\kappa}$$

$$\log p = \left[I_n \circ \Xi 1' - \Omega\right]^{-1} \left(\log G \circ (1 - \overline{\theta} - \Omega 1) - (\overline{\theta} + \Omega 1) \circ \log \phi - \tilde{\kappa}\right) \tag{7}$$

Now we solve for the price of the final good.

$$\overline{p} Q = \min_{c_j} \sum_{j \in \mathcal{S}} p_j c_j \quad \text{st:} \quad Q = \prod_{j \in \mathcal{S}} c_j^{\psi_j}$$

By FOC:

$$p_j - \lambda_0 \psi_j \frac{Q}{c_j} = 0 \Longrightarrow p_j c_j = \lambda_0 \psi_j Q$$

Summing all j goods we have:

$$\overline{p}Q = \sum_{j \in \mathcal{S}} p_j c_j = \lambda_0 Q \Longrightarrow \overline{p} = \lambda_0$$

Thus:

$$Q = \prod_{j \in \mathcal{S}} c_j^{\psi_j} = \prod_{j \in \mathcal{S}} \left(\frac{\psi_j \overline{p} Q}{p_j} \right)^{\psi_j} = \overline{p} Q \prod_{j \in \mathcal{S}} \left(\frac{\psi_j}{p_j} \right)^{\psi_j}$$

$$\overline{p} = \prod_{j \in \mathcal{S}} \left(\frac{p_j}{\psi_j} \right)^{\psi_j} \quad \blacksquare$$

Proof of Proposition 3 - Occupational Choice

Using the indifference condition we have that:

$$\pi_i(\hat{z}_i) = 1$$

$$(\phi_i \hat{z}_i p_i)^{\frac{1}{1-\theta_i-\sigma_i}} \theta_i^{\frac{\theta_i}{1-\theta_i-\sigma_i}} \prod_j \left(\frac{\sigma_{ij}}{p_j} \right)^{\frac{\sigma_{ij}}{1-\theta_i-\sigma_i}} \Big(1 - \theta_i - \sigma_i\Big) = 1$$

Taking the logs and multiplying both sides by $(1 - \theta_i - \sigma_i)$:

$$\log \hat{z}_i + \log p_i - \sum_j \sigma_{ij} \log p_j + \log \phi_i + \underbrace{\theta_i \log \theta_i + (1 - \theta_i - \sigma_i) \log(1 - \theta_i - \sigma_i) + \sum_j \sigma_{ij} \log \sigma_{ij}}_{\mathbf{T}_i} = 0$$

$$\log \hat{z}_i + \log p_i - \sum_j \sigma_{ij} \log p_j + \log \phi_i + \mathbf{T}_i = 0$$

$$\log \hat{z} + \Big(I_n - \Sigma\Big) \log p + \log \phi + \mathbf{T} = 0$$

$$\log \hat{z} + \Big(I_n - \Sigma\Big) \Big[I_n \circ \Xi 1' - \Omega\Big]^{-1} \Big(\log G \circ (1 - \overline{\theta} - \Omega 1) - (\overline{\theta} + \Omega 1) \circ \log \phi - \tilde{\kappa} \Big) + \log \phi_i + \mathbf{T} = 0$$

$$\log \hat{z} = -\Big(I_n - \Sigma\Big) \Big[I_n \circ \Xi 1' - \Omega\Big]^{-1} \Big(\log G \circ (1 - \overline{\theta} - \Omega 1) - (\overline{\theta} + \Omega 1) \circ \log \phi - \tilde{\kappa} \Big) - \log \phi - \mathbf{T} \quad (8)$$

Proof of Proposition 4 - Hulten's Theorem Validity

We can rewrite the market clearing conditions for intermediate good j such as:

$$c_j + \sum_{i \in \mathcal{S}} \int_{\{o(\mathbf{z}) \in \mathcal{S}\}} x_{ij}(\mathbf{z}) = \underbrace{\int_{\{o(\mathbf{z}) \in \mathcal{S}\}} y_j(\mathbf{z})}_{Y_j/N}$$

$$p_j c_j + \sum_{i \in \mathcal{S}} \int_{\{o(\mathbf{z}) \in \mathcal{S}\}} p_j x_{ij}(\mathbf{z}) = \int_{\{o(\mathbf{z}) \in \mathcal{S}\}} p_j y_j(\mathbf{z})$$

By firms' first order condition:

$$p_j c_j + \sum_{i \in \mathcal{S}} \int_{\{o(\mathbf{z}) \in \mathcal{S}\}} p_i \sigma_{ij} y_i(\mathbf{z}) = \int_{\{o(\mathbf{z}) \in \mathcal{S}\}} p_j y_j(\mathbf{z})$$

$$p_j c_j + \sum_{i \in \mathcal{S}} \sigma_{ij} p_i \frac{Y_i}{N} = p_j \frac{Y_j}{N}$$

Using the fact that $p_i = \psi_i \overline{p} Q / c_i$, which follows from final producers' first order condition, we have:

$$\psi_j \overline{p} Q + \sum_{i \in \mathcal{S}} \sigma_{ij} p_i \frac{Y_i}{N} = p_j \frac{Y_j}{N} \quad (\div \overline{p} Q)$$

$$\psi_j + \sum_{i \in \mathcal{S}} \sigma_{ij} \frac{\psi_i Y_i / N}{c_i} = \frac{\psi_j Y_j / N}{c_j}$$

Now define $v_i = \frac{\psi_i Y_i / N}{c_i}$:

$$\psi + \Sigma' v = v \Longrightarrow v^* = \psi' (I - \Sigma)^{-1} =: \lambda$$

Where $\lambda_i = \frac{1}{N} p_i Y_i / \overline{p} Q$ is the **Domar weight** of sector i and Σ is the **firm** I-O matrix. Recall that the first order condition of x_{ij}: $x_{ij} = \sigma_{ij} \frac{p_i}{p_j} y_i$. Since $p_i = \frac{\psi_i \overline{p} Q}{c_i}$:

$$x_{ij}(\mathbf{z}) = \sigma_{ij} \frac{\psi_i}{\psi_j} \frac{c_j}{c_i} y_i(\mathbf{z})$$

We should note that this is the **firm's optimization problem**, integrating over all firms:

$$\int_{\{o(\mathbf{z}) = i\}} x_{ij}(\mathbf{z}) \Gamma(d\mathbf{z}) = \sigma_{ij} \frac{\psi_i}{\psi_j} \frac{c_j}{c_i} \int_{\{o(\mathbf{z}) = i\}} y_i(\mathbf{z}) \Gamma(d\mathbf{z})$$

$$X_{ij} = \sigma_{ij}\frac{\psi_i Y_i}{c_i}\frac{c_j}{\psi_j Y_j}Y_j \implies X_{ij} = \sigma_{ij}\frac{\lambda_i}{\lambda_j}\frac{Y_j}{N}$$

Now we should use ***Proposition 1*** to get $Y_i(Y_j)$

$$Y_i = A_i N^{\bar{\zeta}} L_i^{\bar{\theta}_i} \prod_{j\in\mathcal{S}} X_{ij}^{\varsigma_{ij}}$$

Where:

$$\bar{\zeta} = \frac{1/\zeta}{1/\zeta + \theta_i + \sum_j \sigma_{ij}}$$

$$\bar{\theta}_i = \frac{\theta_i}{1/\zeta + \theta_i + \sum_j \sigma_{ij}} \qquad \varsigma_{ij} = \frac{\sigma_{ij}}{1/\zeta + \theta_i + \sum_j \sigma_{ij}}$$

$$Y_i = A_i N^{\bar{\zeta}} L_i^{\bar{\theta}_i} \prod_{j\in\mathcal{S}} \left(\sigma_{ij}\frac{\lambda_i}{\lambda_j}\frac{Y_j}{N}\right)^{\varsigma_{ij}}$$

The above equation relates the output of all sectors of this economy. Taking logs of this expression:

$$\mathbf{y}_i = \underbrace{\log A_i}_{\mathbf{a}_i} + (\bar{\zeta} - \varsigma_{ij})\underbrace{\log N}_{\mathbf{n}} + \bar{\theta}_i \log L_i + \underbrace{\sum_{j\in\{M,S\}} \varsigma_{ij}\log\left(\sigma_{ij}\frac{\lambda_i}{\lambda_j}\right)}_{\chi_i} + \sum_{j\in\{M,S\}} \varsigma_{ij}\mathbf{y}_j$$

We can rewrite $\mathbf{y}_i$ in a matrix notation to get:

$$\mathbf{y} = \mathbf{a} + ((\bar{\zeta} - \varsigma_{ij})\mathbf{n} + \bar{\theta}\log L + \chi + w\mathbf{y}$$

$$\mathbf{y} = (I - \Omega)^{-1}(\mathbf{a} + (\bar{\zeta} - \varsigma_{ij})\mathbf{n} + \bar{\theta}\log L + \chi)$$

Remember the FOC of the final good producer: $c_i = \psi_i Y_i/\lambda_i$. Taking logs and rewriting in matricial notation:

$$\mathbf{c} = \xi_c + \mathbf{y}$$

By final good producer's production function:

$$\log Q = \psi'\mathbf{c} = \psi'\xi_c + \psi'\mathbf{y}$$

$$\log Q = \psi'\xi_c + \psi'(I - \Omega)^{-1}(\mathbf{a} + (\bar{\zeta} - \varsigma_{ij})\mathbf{n} + \bar{\theta}\log L + \chi)$$

$$\log Q = \underbrace{\underbrace{\psi'(I - \Omega)^{-1}}_{\mu'}\mathbf{a}}_{\log TFP} + \underbrace{\psi'(I - \Omega)^{-1}(I\bar{\zeta} - \Omega)}_{\tilde{\zeta}} n + \underbrace{\psi'(I - \Omega)^{-1}\bar{\theta}}_{\tilde{\theta}}\log L$$

$$+\underbrace{\psi'(I-\Omega)^{-1}\chi+\psi'\xi_c}_{\epsilon}$$

$$\log Q = \mu'\mathbf{a} + \tilde{\zeta}n + \tilde{\theta}\log L + \epsilon$$

$$Q = A^{\tilde{\mu}} N^{\tilde{\zeta}} L^{\tilde{\theta}} \epsilon$$

Where $\tilde{\mu} = 1'\mu$ is the network multiplier.

Comparing the influence vector and the equilibrium sales vector from ***Lemma 1***:

$$\left[\mathbb{I}_n - \Sigma' \circ \left(\mathbb{1}\phi'\right) - \psi\left(1 - \left(\phi \circ (\theta + \Sigma 1)\right)\right)'\right]^{-1}\psi$$

We can observe that when $\phi = 1$ (undistorted economy) and if $\theta + \Sigma 1 = 1$ (constant returns-to-scale)we have that:

$$(I_n - \Sigma)^{-1} = (I_n - \Omega)^{-1}$$

Analogously. Using the production function for gross output product we have the effect on sectoral TFP distortions on GDP:

$$\frac{\mathrm{d}\log Q}{\mathrm{d}\log A_i} = \mu_i$$

Using the definition of μ_i and the definition of the Domar weights, $N\lambda_i$ we get that a condition for the validity of the Hulten's theorem in our case is:

$$\sigma_{ij} = \varsigma_{ij} \Longrightarrow \frac{\sigma_{ij}}{1/\zeta + \theta_i + \sum_j \sigma_{ij}} = \sigma_{ij} \Longrightarrow 1/\zeta + \theta_i + \sum_j \sigma_{ij} = 1$$

The heterogeneity, expressed by the tail distribution parameter, ζ, and the diminishing returns to scale (θ_i and σ_{ij}) determine whether the Hulten's theorem holds. Note that if we have constant returns to scale, $\theta_i + \sum_j \sigma_{ij} = 1$, and firms make zero profit. In this case the theorem holds when $\zeta \to \infty$, and the Pareto distribution colapses to a Dirac, approximating our model to a representative agent economy. ■

Data Appendix

Here we present another possible calibration in a more general case considering an economy with more sectors. [2]

[2] Data used

Table 3: Alternative Calibration

Target Moments	US Data	Model	Parameters
Share of entrepreneurs - Total	5%		ζ
Average Firm Size - Services	16		η_S
Average Firm Size - Manufactures	40		η_M
Average Firm Size - Agriculture	7		η_A
Average Firm Size - Mining	22		η_{Mi}
Average Firm Size - Utilities	34		η_U
Average Firm Size - Construction	9		η_C

Table 4: Allocation of Entrepreneurs

Target Moments	US Data
Share of entrepreneurs - Services	86.32%
Share of entrepreneurs - Manufacturing	3.70%
Share of entrepreneurs - Agriculture	0.30%
Share of entrepreneurs - Mining	0.33%
Share of entrepreneurs - Utilities	0.24%
Share of entrepreneurs - Construction	9.11%

Table 5: Labor Shares - Alternative calibration

Sector	Labor share ($\bar{\theta}_i$)
Manufacturing	0.18
Services	0.31
Agriculture	0.13
Mining	0.14
Utilities	0.18
Construction	0.34

Table 6: Input-Output Table (ς_{ij}) - Alternative calibration

	Manufacturing	Services	Agriculture	Mining	Utilities	Construction
Manufacturing	0.5357	0.2842	0.0257	0.0196	0.0069	0.1279
Services	0.0718	0.8836	0.0057	0.0132	0.0072	0.0185
Agriculture	0.7044	0.0520	0.2355	0.0004	0.0000	0.0077
Mining	0.7388	0.0103	0.0048	0.1143	0.0881	0.0437
Utilities	0.1683	0.7125	0.0136	0.0225	0.0695	0.0135
Construction	0.0737	0.8617	0.0087	0.0197	0.0352	0.0010