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FIRM PRODUCTIVITY, INFORMALITY AND LABOR MARKET OUTCOMES

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Dissertação apresentada à Escola de Economia de São Paulo como pré-requisito à obtenção de título de mestre em Economia de Empresas.

Orientador: João Paulo Cordeiro de Noronha Pessoa.

Coorientador: Renata del Tedesco Narita.

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Resumo

Desenvolvemos um modelo de busca por trabalho e postagem de salários com dois setores, no qual as firmas não somente postam salários para atrair trabalhadores como também decidem o quanto investem para elevar sua produtividade. Isso permite que firmas ajustem sua decisões de investimento frente a diferentes cenários do mercado de trabalho, o que abre um novo canal pelo qual intervenções políticas afetam a performance da economia. Apresentamos alguns resultados de estática comparativa que são cruciais para descrever o comportamento ótimo das firmas em equilíbrio, assim como para identificação das funções de produção setoriais. Simulamos o impacto de três diferentes intervenções que procuram diminuir o tamanho do setor informal. As simulações mostram que essas políticas levam a melhoras de bem-estar e produto. A magnitude de cada um desses ganhos depende do quanto cada intervenção permite que novas pequenas firmas formais comecem a operar, assim como o quão relevantes são os custos impostos sobre as maiores firmas informais.

Palavras-chave: Modelos de busca; setor informal; produtividade endógena; postagem de salários, mercado de trabalho.

Abstract

We develop a two-sector job search model in which formal and informal firms not only post contracts to attract workers but also choose how much to invest in order to elevate their productivity. This allows firms to adjust their investment decisions when faced with different labor market scenarios, which opens up a new channel through which policy interventions in the labor market may affect the overall performance of the economy. We provide some theoretical results regarding comparative statics which are crucial for describing the optimal behavior of firms in equilibrium as well as for nonparametric identification of sectoral production functions. We also simulate the equilibrium impact of three different policy interventions aimed at decreasing the size of the informal sector. Our simulations show that such policies are welfare and total output inducing. The magnitude of each these gains depends on how much these policies allow previously non-operating small formal firms to start producing, as well as imposing relevant costs to the largest informal firms.

Keywords: Job search; informal sector; endogenous productivity; wage posting, labor market.

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1 Introduction

An essential feature of labor markets in developing economies is the existence of a large informal sector. In broad terms, the informal sector comprises of every work activity that does not comply with the usual regulation established by labor legislation. This provides more flexibility to resource allocation in a labor market with strict regulations, which results in costs and benefits for the agents. Firms that operate in this sector avoid many of the costs imposed by these regulations such as payroll taxes, taxes on profits and severance pay, while being at risk of being fined if caught by the government. On the other hand, informal workers lose some protections on the job such as the right to unemployment insurance but are able to enter the labor market more easily since informality provides an alternative to unemployment. Understanding these tradeoffs associated with informality, their impacts on the overall performance of the economy and whether informality is desirable or not are important questions for policymaking in developing countries.

We contribute to this debate by investigating how informality may be affecting the overall productivity[1] of an economy, considering that it is such an important determinant of economic growth and performance. We try to achieve this goal by using a job search model with two sectors, in which formal and informal firms not only post contracts, but also choose how much to invest in order to elevate their own productivity. The novelty of this framework is to introduce firms' investment strategies into a wage-posting model and letting this decision take a prominent role in determining labor market outcomes such as wage inequality and equilibrium unemployment.

The framework developed here dialogues with the vast literature that uses job search models[2] to evaluate how frictions, institutions and labor market policies affect the overall performance of an economy characterized by a large informal sector. In our framework, heterogeneity in firm productivity is entirely determined by employers that differ in their inherent capabilities, but unlike previous papers in the literature we characterize how these employers invest to increase productivity of their firms. By taking this into account, optimal investment and productivity of any given firm reacts to variations in institutional

[1] Throughout this entire study, we denote productivity as a synonym to output per worker.
[2] For a detailed survey about search-theoretical models and applications, see Rogerson et al. (2005)

settings such as variations in corporate taxes and regulatory costs as well as to the different degrees of search frictions of each sector, which are known to be crucial sources of wage and productivity dispersion since Burdett and Mortensen (1998). This approach contrasts with papers that follow more closely the tradition of Bontemps et al. (2000), in which the productivity distribution of an economy is endogenously determined entirely by pinning down the bottom line productivity threshold in the support of an exogenously determined productivity distribution, which determines which firms operate or not. Our framework allows for a complete reshape of this endogenous distribution.

Our model builds upon Meghir et al. (2015) (which can be seen as a two-sector version of Bontemps et al. (2000)) by introducing this additional productivity-enhancing investment decision by employers. By doing so, we are capable of providing some additional insights by allowing productivity of each sector to defined not only by compositional means - that is, by the entrance or exit of firms exogenously determined as more or less productive in each sector- but also by variations in investment. This opens a new channel through which informality may affect wages and worker allocation. In their original paper, the authors estimate a model in which firms choose whether to be formal or informal[3] using Brazilian data and are able to replicate the presence of firms with identical productivities in both sectors. They conclude that a more rigid enforcement towards the informal sector leads to better allocation, higher wages and higher output while not increasing unemployment.

Much of the previous literature using equilibrium models with search frictions to study labor market outcomes of economies with informality follow more closely to a search and matching framework as in Mortensen and Pissarides (1994) and output dispersion across firms arises from match-specific productivity outcomes. In these models, wages are usually determined as a result of Nash bargaining. We opt for a wage-posting model because the "take-it-or-leave-it" nature of job opportunities in such a model seems to be a more accurate way of determining wages of low-skilled workers, which are exactly the ones that are allocated in informal jobs. Empirical evidence suggests that bargaining is a more common determinant of wages for more educated workers (Hall and Krueger (2012), Brenzel et al. (2014)).

An example that follows the Mortensen and Pissarides (1994) tradition is Albrecht

[3] This endogenous sector choice by firms is not present in our model. Unlike Meghir et al. (2015), firms in our framework are exogenously determined as formal or informal.

et al. (2009), which provides an important theoretical contribution to how more or less productive workers sort into both sectors. Bosch and Esteban-Pretel (2012) calibrate their model to analyze how labor market policies such as increases in costs of informality can explain cyclical worker flows and unemployment rates found in Brazilian data. A more recent and rich model is present in Haanwinckel and Soares (2016). In their framework, firms differ in the output of high and low skilled employees. The authors estimate the model and are able to reproduce many features of the Brazilian labor market between 2003 and 2012, mainly the reduction in informality observed during this period. Another important contribution is present in Ulyssea (2010) that models an economy with workers searching for opportunities on both sectors simultaneously and find that policies aimed at reducing formal firms' entry costs improves labor market outcomes, including total output.

Accounting for investments on the supply side of the labor market, Bobba et al. (2018) use Mexican data to estimate a search and matching model with Nash bargaining that accounts for schooling investment decisions before entering the market. In their model, workers first choose how much human capital to accumulate and then, after entering the labor force, whether they will search for jobs unemployed or self-employed and in which sector. Their simulations focus on improvements in social security benefits for formal employees and the authors conclude that such policies increase welfare and total output by improving the human capital composition of the workforce. Since the authors focus on investment decisions on the supply-side, we believe that our demand-side centered model is complementary to theirs.

Much of the literature that models how endogenous productivity relates to turnover and labor market frictions using training as the main driver of productivity gains. Differently from paper such as Burdett et al. (2011) and Bagger et al. (2014), in which human capital gains are deterministic and a function of time spent working, our model is more closely related with the branch that models how firms decide to invest and train their employees. Fu (2011) proposes a model with on-the-job search in which firms whether to train it's employees in general training and concludes that this decision accounts for part of the wage dispersion in equilibrium and shows a correlation between pay rates and human capital accumulated. This correlation is also present in Quercioli (2005), which is a theoretical contribution that presents a model in which the amount of firm-specific training provided by the firm is a continuous variable. Flinn et al. (2017) is a recent contribution, that

proposes a framework in which firms invest in general and specific human capital of their workers. This leads to important results regarding how firms and workers sort depending on their types and how increases in the minimum wage lead to different investment choices and reduce the amount of human capital in the economy. Our model presents some differences when compared with these models. The way we approach firm investment is conceptually different from the ones present in these papers - namely, we do not model employee training specifically - but our main contribution to this literature is to incorporate an informal sector into the economy. This allows us to better understand how competition between firms in two different sectors impact resource allocation and productivity.

A different branch of the literature such as D'Erasmo and Boedo (2012) and Ordonez (2014) approaches informality as a source of resource misallocation responsible for negative effects over TFP[4] in developing economies. D'Erasmo and Boedo (2012) develop a firm dynamics model with imperfect credit markets and conclude that the economies with high costs of formal operation are not efficient in terms of resource allocation and informal firms, which are the least productive ones, are responsible large shares of the total output. The latter focuses on imperfect tax enforcement as a source of informality and the entrance of low productive entrepreneurs.

The remainder of this paper proceeds as follows. In Section 2 we present some stylized facts about informality in Brazil that motivate and are incorporated in our study. Section 3 presents the model, establishing workers' and firms' problems along with turnover equations. Section 4 establishes some properties of firms' problem in equilibrium which will be useful for the identification discussion present in Section 5. Section 6 focuses on the empirical application of the model, by presenting the method used to compute the equilibrium, the chosen calibration as well as some counterfactual exercises. Lastly, Section 7 concludes.

[4] Differently from these articles that utilize more aggregate measures of productivity, we refer to output per worker as a synonym for productivity

2 Stylized facts

It is a well-known fact that informal firms are less productive than their formal counterparts, even when controlling for size (La Porta and Shleifer (2008)). In fact, by avoiding the high regulation costs imposed on formal firms, informality can be seen as a subsidy for unproductive firms. This is suggestive that the overall productivity of an economy may be hindered just by employing a large share of the workforce in this unproductive sector.

Studies have argued that the lack of access to credit markets, inability to fully exercise property rights and government enforcement, which impede these firms from becoming large and taking advantage of possible economies of scale, are partially responsible for the lower productivity of the sector (Ulyssea (2010), Fajnzylber et al. (2011)). Furthermore, evidence in La Porta and Shleifer (2008, 2014) also reveals that these firms have less access to public goods, are less capital intensive and are managed by less educated employers. This set of evidence indicates that firm productivity results from a combination between institutional setting (such as enforcement and access to public goods) and the inherent abilities of employers.

Many of these features are present when we look at data from Brazil and we aim to reproduce them in our study. Informal work is historically high among Brazilian workers (Ulyssea (2010), Bosch et al. (2007)) and it has increased in recent years, accounting for more than 40% of the entire workforce (Figure 1)[1]. Data from ECINF[2] also reveals that informal firms are indeed managed by less-educated individuals. About 38% of employers of formal firms with at least 5 workers had a high school degree in 2003, against only 25% for informal ones. Additionally, informal firms are less productive as measured by value-added and profits per worker as proxies (Ulyssea (2018)). While each of these evidences are already well documented (De Paula and Scheinkman (2011)), there is an often overlooked comparison between the two sectors regarding their investment decisions. In Figure 2 we

[1] We compute these shares considering every worker that does not have a "carteira de trabalho" or is self-employed as informal.

[2] Economia Informal Urbana (ECINF) is a survey collected by the Brazilian Bureau of Statistics (IBGE) in 2003, containing micro-level information on small formal and informal urban firms. For a detailed explanation of sampling aspects and information contained in it, see De Paula and Scheinkman (2010)

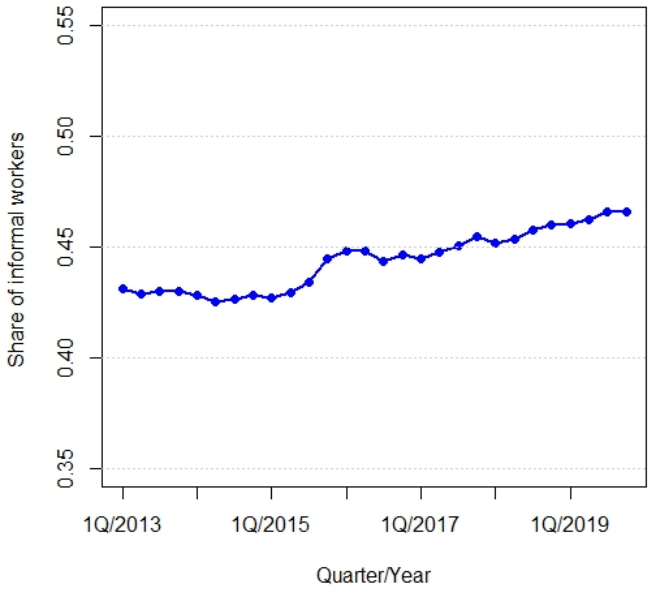

Figure 1 – Evolution of informality in Brazil. Source*: Pnad Contínua*

estimate the distributions of log-investment for both sectors using data from ECINF. We can see that informal firms invest less than their small formal counterparts and that there exists a considerable overlap in the supports of the sectoral distributions, a pattern that is reproduced when looking at the productivity distributions of these same firms (La Porta and Shleifer (2008), La Porta and Shleifer (2014), Meghir et al. (2015), Ulyssea (2018)).

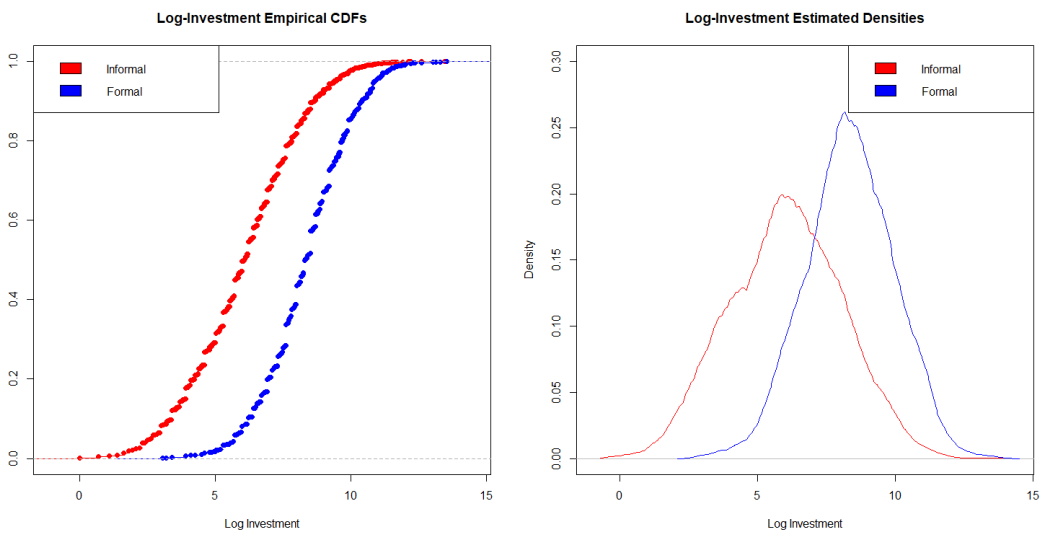

Figure 2 – Log-Investment distributions of Brazilian formal and informal firms. Source: ECINF.

Another feature of the Brazilian labor market incorporated in our framework is the high mobility of workers. This is crucial for our analysis because, as highlighted in the seminal work of Burdett and Mortensen (1998), turnover is key to understanding how firms decide on which contracts to offer: wages must be low enough so that costs of production are small, but high enough that it reduces turnover. By computing monthly probabilities of a given worker moving from a working status to another using data from PME survey[3](Table 1)[4,5], we observe that workers are able to transit on-the-job with relative ease across sectors. In fact, these transitions are documented to be even higher for lower wages (Tannuri-Pianto and Pianto (2002)). Also, it is clear that the informal sector has a higher mobility. Job transitions within the informal sector are almost three times more frequent than within the formal one, the probability of an employee becoming unemployed is much higher in the informal sector and transitions from unemployment into informal work are much more frequent than into formal work. The movement of workers across sectors matters because it indicates that firms from different sectors and that operate under a different set of regulatory costs compete for the same pool of workers, thus affecting their expected profit maximization problem.

Transition	**Probability**
Formal → Unemployment	0.009
Informal → Unemployment	0.023
Formal → Formal	0.005
Informal → Informal	0.020
Formal → Informal	0.046
Informal → Formal	0.071
Unemployment → Formal	0.016
Unemployment → Informal	0.098

Table 1 – Monthly Transition Probabilities (PME 2006 - 2008)

[3] PME is a survey collected by IBGE which consists of a rotating panel of workers sampled from six Brazilian metropolitan regions (Recife, Salvador, Belo Horizonte, Rio de Janeiro, São Paulo e Porto Alegre)

[4] Transition probabilities displayed in Table 1 represent the share of workers in a given state and month that move to another status in the next month. "Formal to formal" transitions represent the share of workers that move from a formal job to a different formal job in the period (the same for informal to informal transitions).

[5] We have only considered the first and second interviews of each worker for these computations.

3 Model

3.1 Environment

We propose a job search model with two sectors - a formal and an informal one. Time is continuous and we focus on steady state. The economy is composed of two kinds of agents: workers and firms. Workers behave identically to those in Meghir et al. (2015). There is a continuum of mass M of homogeneous risk-neutral workers that search for new employment opportunities on and off the job. Workers seek for new jobs in both sectors indiscriminately and can climb the job ladder randomly according to several Poisson processes. Second, there is a mass N_F and a mass N_I of risk-neutral formal and informal firms, respectively. Additionally, firms are also heterogeneous within sectors, differing in their investment capabilities. In our framework, firms decide on job offers and the amount to invest in order to elevate their productivity.

3.2 Workers

Workers are homogeneous, risk-neutral, infinitely lived and discount the future at a rate r. At any point in time, each worker is in one of three possible states: unemployment, formal employment or informal employment. Unemployed workers receive a leisure flow utility b, while formal employees enjoy some benefits associated with their formal contracts such as unemployment insurance (UI) and severance pay (ρ) in case of being fired.

Workers search for jobs in each of the three states and will periodically receive job offers following a Poisson process with arrival rate $\lambda_{ss'}$, where $s \in \{U, F, I\}$ and $s' \in \{F, I\}$ denote, respectively, the state in which the worker is currently in (U for unemployment, F and I for formal and informal employment) and the source of the job offer. Additionally, jobs are destroyed at a rate of λ_{sU}. All values of $\lambda_{ss'}$ are determined exogenously. When a worker receives an offer coming from sector $s \in \{F, I\}$, the contract value offered is drawn from a cumulative distribution F_s with support $[\underline{V}_s, \overline{V}_s]$. Sectoral contract value offer distributions F_F and F_I are taken as given by workers and are endogenously determined in equilibrium by firms. When faced with a job offer, the worker will accept the new contract

if it yields an equal or higher value than the worker's current value. Finally, all offers must be accepted or rejected immediately upon arrival and we assume that there is no recall of past offers.

The value of being in a given state expresses the utility flow associated with wages (for employed workers) and benefits as well as from expected contracts offered in future matches, which allow for workers to climb the job ladder. In this case, the value function that determines how unemployed workers behave optimally in equilibrium is given by:

$$rV_U = b + \lambda_{UF} \int_{\underline{V}_F}^{\overline{V}_F} \max\left\{V - V_U, 0\right\} dF_F(V) + \lambda_{UI} \int_{\underline{V}_I}^{\overline{V}_I} \max\left\{V - V_U, 0\right\} dF_I(V). \qquad [3.1]$$

Notice that the second (third) term in the right-hand side of equation (3.1) captures the flow utility gain from the expected contract offered by formal (informal) firms weighted by the probability of this match. It is easy to see from (3.1) that only contracts that yield a value higher than V_U are accepted. This indicates that in equilibrium, we must have $\underline{V}_F \geqslant V_U$ and $\underline{V}_I \geqslant V_U$ because if a firm opts to offer a contract that yields a value strictly lower than V_U, this contract will never be accepted and the firm will never produce.

The value functions for formal and informal employees are, respectively, given by:

$$\begin{aligned} rV = w_F(V) + \lambda_{FF} \int_{\underline{V}_F}^{\overline{V}_F} \max\left\{V' - V, 0\right\} dF_F(V') + \lambda_{FI} \int_{\underline{V}_I}^{\overline{V}_I} \max\left\{V' - V, 0\right\} dF_I(V') \\ + \lambda_{FU}(V_U + UI + \rho w_F(V) - V) \end{aligned} \qquad [3.2]$$

$$rV = w_I(V) + \lambda_{IF} \int_{\underline{V}_F}^{\overline{V}_F} \max\left\{V' - V, 0\right\} dF_F(V') + \lambda_{II} \int_{\underline{V}_I}^{\overline{V}_I} \max\left\{V' - V, 0\right\} dF_I(V') + \lambda_{IU}(V_U - V) \qquad [3.3]$$

Equation (3.2) displays how the flow utility in the formal sector comprises of the wage rate $w_F(V)$ derived from the current value V, the potential gains associated with a transition from a job to another as well the loss of utility related to the job being destroyed. Job destruction occurs at a rate λ_{FU} and when it happens, the utility loss of a formal worker is attenuated by inflows in the form of the unemployment insurance and severance pay. Equation (3.3) is analog the previous, the only differences are in the contact rates that are sector-specific and the fact that the worker does not have any kind of benefit in case of job destruction. Therefore, since contract values incorporate costs and opportunities of working in each sector, and not only wages, an employee may accept a job that pays a lower wage, but never one that yields a lower contract value.

Comparing equations (3.2) and (3.3), we can see that in the hypothetical case in which the contact rates do not change depending on current status (that is, when $\lambda_{FF} = \lambda_{IF}$, $\lambda_{FI} = \lambda_{II}$ and $\lambda_{FU} = \lambda_{IU}$) we would expect to see higher wages in the informal sector for a given contract value V seeing that the benefits that the formal employees enjoy would compensate a lower wage. To better illustrate this, it is useful to rewrite these equations such that they yield wages as a function of the contract value, which we will denote by $w_s(W)$ for $s \in \{F, I\}$ [1]:

$$w_F(V) = \frac{1}{1+\rho\lambda_{FU}}\Big[rV - \lambda_{FF}\int_V^{\overline{V_F}}\Big[1-F_F(x)\Big]dx - \lambda_{FI}\int_V^{\overline{V_I}}\Big[1-F_I(x)\Big]dx - \lambda_{FU}(V_U + UI - V)\Big] \quad [3.4]$$

$$w_I(V) = rV - \lambda_{IF}\int_V^{\overline{V_F}}\Big[1-F_F(x)\Big]dx - \lambda_{II}\int_V^{\overline{V_I}}\Big[1-F_I(x)\Big]dx - \lambda_{IU}(V_U - V). \quad [3.5]$$

For each V, equations (3.4) and (3.5) indicates the wage rate that is required to achieve a certain value V in each sector. Taking the derivative of these equations relative to V we arrive at

$$w_F'(V) = \frac{1}{1+\rho\lambda_{FU}}\Big[r + \lambda_{FU} + \lambda_{FF}(1-F_F(V)) + \lambda_{FI}(1-F_I(V))\Big] \quad [3.6]$$

$$w_I'(V) = r + \lambda_{IU} + \lambda_{IF}(1-F_F(V)) + \lambda_{II}(1-F_I(V)). \quad [3.7]$$

Notice that (3.6) and (3.7) are strictly positive for any V, this implies that for each sector there is a unique wage rate associated to each contract value. This property is crucial in any empirical application of any wage posting model that follows the tradition of Burdett and Mortensen (1998), since it allows for direct identification of the latent distribution of offered contracts F_s via the distribution of *accepted wages* in an economy with one sector. This step is not so easy when we have workers able to transit freely across sector (this will discussed in more detail in Section 5).

Further investigation of (3.7) reveals that marginal increases in contract values lead to increases in wage rates equal to the discount rate r plus the probability of the match being destroyed [2,3]. This implies that if a contract value is too high, any marginal increase

[1] To write equations wage equations in a more useful form we make use of the fact that we can compute the expected value of any random variable x with cumulative distribution $F(x)$ using its associated survival function, ie

$$\int_{\underline{x}}^{\overline{x}} \max\{x-c, 0\}dF(x) = \int_c^{\overline{x}}(x-c)f(x)dx = \int_c^{\overline{x}} 1 - F(x)dx$$

for any real c.

[2] We will discuss in more detail the properties of destruction rates, denoted by $d_s(V) = \lambda_{sU} + \lambda_{sF}(1 - F_F(x)) + \lambda_{sI}(1 - F_I(x))$ in the next subsection.

[3] Hoffmann and Shi (2016) denote this as the *effective discount rate*.

in V leads to relatively small increase in wages, since the probability of transitioning to better jobs is relatively small.

3.3 Turnover

Our focus is on steady-state equilibrium, therefore we need to describe the conditions so that the masses of workers in each relevant state are constant. Since workers in this model behave identically to those in Meghir et al. (2015), namely, search is undirected and workers transit freely across sectors, turnover will also be described almost identically[4].

First, we must define the job destruction and hiring rates of a firm in sector s offering a contract V. Denote these rates by $d_s(V)$ and $h_s(V)$ respectively, such that:

$$d_s(V) = \lambda_{sU} + \lambda_{sF}[1 - F_F(V)] + \lambda_{sI}[1 - F_I(V)] \tag{3.8}$$

$$h_s(V) = \lambda_{Us}u + \lambda_{Fs}m_F G_F(V) + \lambda_{Is}m_I G_I(V) \tag{3.9}$$

where m_F, m_I, u denote, respectively, the mass of formal, informal and unemployed workers in steady-state, while $G_s(V)$ is the cumulative distribution of *accepted* contracts in sector $s \in \{F, I\}$. As we have already highlighted, the rate at which a job is destroyed is equal to the exogenous probability of transitioning into unemployment plus the endogenous probability of a worker being offered a better contract and moving to this new job. The hiring rate of a firm offering a contract V equals to the probability of a match with any unemployed[5] worker plus the probability of contacting an employed worker whose current job values is lower than V.

Market clearing requires that the stock of workers and firms in each part of the contract value distribution is constant. That is, there must be a constant mass of workers employed in jobs that yield a value smaller than V, for every V in the support of F_s, for

[4] The only difference between turnover equations described in this section and the ones in the model in Meghir et al. (2015), in which firms are not segmented, is that we have two different masses, N_F and N_I, of firms in each sector as primitives of the model. In their paper, the authors only need a unique mass of firms that will potentially operate

[5] Matching with any unemployed is sufficient to hire because, in equilibrium, every contract offer will yield a value higher than V_U

$s \in \{F, I\}$. Formally:

$$[\lambda_{FU} + \lambda_{FF}(1 - F_F(V))]m_F G_F(V) + \lambda_{FI} m_I \int_{\underline{V_F}}^{V} 1 - F_I(x) dG_F(x) = \\ = \lambda_{UF} u F_F(V) + \lambda_{IF} m_I \int_{\underline{V_I}}^{V} F_F(V) - F_F(x) dG_I(x) \tag{3.10}$$

$$[\lambda_{IU} + \lambda_{II}(1 - F_I(V))]m_I G_I(V) + \lambda_{IF} m_I \int_{\underline{V_I}}^{V} 1 - F_F(x) dG_I(x) = \\ = \lambda_{UI} u F_I(V) + \lambda_{FI} m_F \int_{\underline{V_F}}^{V} F_I(V) - F_I(x) dG_F(x) \tag{3.11}$$

These equations imply that the outflow of workers employed in a given sector with a contract value smaller than V (left-hand side) must be equal to the flow into this same pool. We skip the explanation of how to solve equations (3.10) and (3.11) since a detailed exposition is present in Meghir et al. (2015). The relation between G_s and F_s implied by these equations is:

$$m_s G_s(V) = \frac{u\left(\lambda_{Us} F_s(V) - \Phi(V)\right)}{d_s(V)}, \text{ for every } V \in supp\ F_s \text{ and } s \in \{I, F\} \tag{3.12}$$

The closed-form of the mapping $\Phi(V)$ is presented in Appendix D and equals zero for any contract value $V \leqslant \max\{\underline{V}_F, \underline{V}_I\}$. Besides providing a connection between the offered and accepted contracts, combining equation (3.12) with the fact that $m_i + m_f + u = 1$ enables us to derive closed-form solutions for equilibrium worker allocation:

$$\frac{m_F}{u} = \frac{\lambda_{UF} - \Phi(\overline{V_F})}{\lambda_{FU} + \lambda_{FI}(1 - F_I(\overline{V_F}))} \tag{3.13}$$

$$\frac{m_I}{u} = \frac{\lambda_{UI} - \Phi(\overline{V_I})}{\lambda_{IU} + \lambda_{IF}(1 - F_F(\overline{V_I}))} \tag{3.14}$$

$$\frac{1}{u} = 1 + \frac{m_F}{u} + \frac{m_I}{u} \tag{3.15}$$

3.4 Firms

In contrast to workers, firms are heterogeneous in two dimensions. First, employers are exogenously determined as formal or informal and cannot switch across sectors. Second, firms differ in firm-specific investment costs, denoted by θ. Thus, firms in sector $s \in \{F, I\}$ are distributed according to a cumulative distribution H_s^0 with support $[\underline{\theta_s}, \overline{\theta_s}] \in \mathbb{R}_{++}$ [6].

[6] Every result still hold if we assume H_s^0 with a semi-infinite support $[\underline{\theta_s}, +\infty)$ together with $\lim_{q \to +\infty} p_s'(q) = 0$ for $s \in \{F, I\}$.

Given a sector s and investment cost θ, firms face two problems in order to maximize steady state profit flow. First, firms must choose an investment level $q \in [0, \overline{q}]$ that will determine their output per worker (denoted by p) according to a sectoral production function $p_s(q) = p$. We assume p_s continuously differentiable with $p_s(q) \geqslant 0, p_s'(q) > 0, p_s'(0) = +\infty$ and $p_s'(\overline{q}) = 0$ for all s and q. In order to invest any amount q, employers must commit and pay a flow maintenance cost θq in order to keep its productivity.

The second problem faced by the firm is to choose which contract value V to post in order to attract workers. This is analog to the fundamental trade-off presented in Burdett and Mortensen (1998) faced by firms, in which employers must choose a wage to post balancing, on one hand, the rate at which workers will be hired and retained, and on the other hand the profit per worker. If wages (or contract value) are relatively small, firms will be smaller but with a high profit margin per worker. Conversely, high wages will imply a higher worker inflow and lower destruction rate (see equations 3.8 and 3.9) but surplus per worker will be smaller.

Since the variations in firm size resulting from variations in the value of the contracts offered play such a crucial role in this fundamental tradeoff, we must define a function that describe the number of employees of each firm, which will be denoted by $l_s(V)$ for each $s \in \{I, F\}$. The most intuitive way to derive this equation is to remember that in steady-state the flows of workers in and out of any given firm must be constant. Therefore, an equilibrium condition of this economy is obtained by equating these flows:

$$\frac{Mh_s(V)}{N_s n_s} = d_s(V) l_s(V). \qquad [3.16]$$

The left-hand side of (3.16) highlights how the total mass of workers M flows into and is distributed equally across the pool of active firms offering a contract V in each sector s, represented by $h_s(V)/N_s n_s$, where $n_s \in [0, 1]$ denotes the share of firms that indeed operate and produce in sector s and will be determined endogenously. The right-hand side illustrates the total flow of employees leaving a firm in sector s offering a contract V and with size $l_s(V)$. Since the only means any given firm has of attracting workers is via contract offers, firms that post the same offer in a given sector must have the same amount of employees.

Trivially, we can derive a closed form for $l_s(V)$ from equation (3.16):

$$l_s(V) = \frac{Mh_s(V)}{N_s n_s d_s(V)}. \qquad [3.17]$$

It is also easy to see that since $h_s(V)$ and $d_s(V)$ are, respectively, increasing and decreasing in V. This implies that $l_s(V)$ must also increase with V. However, we cannot infer anything about the shape nor strict monotonicity of $l_s(V)$ since these properties would rely on additional hypotheses over F_F and F_I, which are endogenously determined by firms.

Now that we have derived how firm size changes in respect to V, we can proceed and describe the firms' maximization problem. Taking as given a cost of θ and a sector of operation s, firms' problem consists of:

$$\max_{V,q} \pi_s(V,q|\theta) \begin{cases} \pi_I(V,q|\theta) = \Big(p_I(q) - w_I(V)\Big)\ell_I(V) - \theta q & \text{if } s = I \quad [3.18] \\ \pi_F(V,q|\theta) = (1-t)\Big[\Big(p_F(q) - (1+\tau+\rho\lambda_{FU})w_F(V)\Big)\ell_F(V) - \theta q\Big] & \text{if } s = F \quad [3.19] \end{cases}$$

Notice by equation (3.18) that informal firms do not incur in some costs and regulations that formal firms do, such as payroll taxes (τ), taxes on profit (t) and severance payments ($\rho w_F(V)$) that must be paid when a worker leaves the job unwillingly (which occurs at rate λ_{FU}). Unlike Meghir et al. (2015), we do not explicitly model a function that incorporates the costs of operation in the informal sector, mainly the risk of paying fines from being caught evading regulations such as not complying with taxes. In their paper, informal profits are subtracted by a function that captures this expected cost. This function is strictly increasing and convex in firm size in order to capture the idea that larger firms are less likely to successfully evade government enforcement. We propose to incorporate these in the production function $p_I(q)$, such that the output of a firm is already discounted by this cost.

Firms will produce only if their profits are positive. Denote by $\Pi_s(\theta)$ the maximum profit achievable by a type-θ firm in sector s. Formally:

$$\Pi_s(\theta) := \sup_{V,q} \pi_s(V,q|\theta) \text{ , for } s \in \{I,F\}$$

Therefore, firms will only operate if $\Pi_s(\theta) \geqslant 0$. Since profit functions are strictly decreasing in investment costs, we conclude that there exists a threshold θ_s^* for each sector s such that $\Pi_s(\theta_s^*) = 0$[7]. Every firm with a cost lower than this threshold can achieve a positive profit and will produce, thus will it hire and invest. From this, we conclude:

$$n_s = H_s^0(\theta_s^*) \text{ , for } s \in \{I,F\} \qquad [3.20]$$

Lastly, it is useful to determine $H_F(\theta)$ and $H_I(\theta)$ as the investment cost distributions

[7] Of course, we are assuming that this threshold θ_s^* lies in the interior of the support of H_s^0

of *active* firms, which is represented by a truncated distribution:

$$H_s(\theta) = \begin{cases} \frac{H_s^0(\theta) - H_s^0(\underline{\theta}_s)}{H_s^0(\theta_s^*) - H_s^0(\underline{\theta}_s)} & \text{, if } \theta < \theta_s^* \\ 1 & \text{, if } \theta \geqslant \theta_s^* \end{cases} \text{, for } s \in \{I, F\} \qquad [3.21]$$

This distinction is important because, as we will discuss later, the truncated distributions $H_F(\theta)$ and $H_I(\theta)$ can be fully identified nonparametrically via offered contracts while H_F^0 and H_I^0 cannot.

4 Properties of firms' problem in equilibrium

In order to better understand how firms behave in this framework, we need to determine the map between firms' inherent investment costs and their optimal contracts and productivity level in each sector. For ease of analysis, we split the firms' profit maximization as a sequential problem in which a decision is taken knowing that subsequent decisions will also be made optimally. In order to keep a stronger resemblance to the rest of the literature, that usually features firms that decide on wages or contract values taking productivity as given, we propose to think the firms' problem as:

1. Optimal investment decision: Firms take sector s and investment cost θ as given in order to choose an investment level knowing that contract offered will be chosen optimally afterward.

2. Optimal contract value offer: Firms take sector s, investment cost θ *and* productivity level $p_s(q)$ as given in order to choose optimal contract values to offer.

A large branch of the literature that explores wage posting models with heterogeneous firms assume an exogenous productivity distribution [Bontemps et al. (2000), Meghir et al. (2015),Bradley et al. (2017)]. By introducing stage 1 in which employers first decide on the productivity level that the firm will operate with, we are able to perceive the economy's equilibrium productivity as a byproduct of firms' investment decision, which is influenced by institutional aspects of the economy as well as the competition between firms for workers.

We proceed the analysis using backward induction in order to solve firms' problem. Our objective now is to provide some theoretical results that characterize the mapping between the primitives of the model and the optimal choices in such a way that allows the model to be identified with minimal additional assumptions. Let's describe the two decisions of the employers as:

Optimal contract value offers: Denote by $K_s(p, \theta)$ the set of optimal contract offers

given $p_s(q)$, s and θ:

$$K_s(p,\theta) := \arg\max_V \pi_s(V|p_s(q),\theta) \ \ , \ \ \text{for } s = \{I,F\}$$

Optimal investment decision: Denote by $\varphi_s(\theta)$ the set of optimal investments given s, θ and knowing that contracts will be offered optimally:

$$\varphi_s(\theta) := \arg\max_q \pi_s(K_s(p,\theta),q|\theta) \ \ , \ \ \text{for } s = \{I,F\}$$

The following are the most important results for describing firms' optimal behavior and state strict monotonicity of the maps $K_s(p,\theta)$ and $\varphi_s(\theta)$:

Lemma 1. *For any $s \in \{I,F\}$, let $q \in \varphi_s(\theta)$ and $q' \in \varphi_s(\theta')$. If $\theta' > \theta$ then $q > q'$.*

Lemma 2. *For any $s \in \{I,F\}$, let $V \in K_s(p,\theta)$ and $V' \in K_s(p',\theta)$. If $p' > p$ then $V' > V$.*

Lemmas 1 and 2 prove the intuitive ideas that a firm that has an inherently higher cost of investment will be less productive and will offer less valuable contracts than a firm with a lower cost. Despite being intuitive, the proof for these results does not rely on the usual arguments of comparative statics. Namely, since profit functions π_F and π_I are not strictly concave, we cannot rely on first-order conditions to provide a unique solution to any of the two decisions of the firm. Thus we cannot apply implicit function theorem to check monotonicity of $K_s(p,\theta)$ and $\varphi_s(\theta)$[1]. However, even without sufficiency of the first-order conditions[2], we are able to prove strict monotonicity of these maps based only on ordinal conditions, as detailed on Edlin and Shannon (1998).

The importance of these results is to guarantee that for any contract value V of this economy in equilibrium, there can only be a unique productivity (and investment) level associated and, analogously, a unique investment cost θ. Notice however that the converse is not true. There may be more than one optimal investment level for a type-θ firm. The following result guarantees that when $H_s^0(\theta)$ is continuously distributed, there may be multiple optimal investment levels only for a set of measure zero of firms.

[1] For a detailed discussion, see Milgrom and Shannon (1994) and Edlin and Shannon (1998).
[2] See Appendix A for the proof

Corollary 1. *If H_s^0 is continuously distributed, then $\varphi_s(\theta)$ is a singleton for almost all $\theta \in supp\, H_s^0$ for each $s \in \{I, F\}$.*

Corollary 1 ensures that firms' investment policy in equilibrium is in pure strategies when H_F^0 and H_I^0 are continuously distributed. This pure strategy equilibrium, which is analogue to the one presented in Bontemps et al. (2000), arises because of firms' ex-ante heterogeneity and contrasts with the classic mixed-strategy equilibrium that characterizes the homogeneous-firm equilibrium in Burdett and Mortensen (1998). Since there is only a unique θ associated to each p and q in equilibrium we will by abuse of notation refer to optimal contracts offered as a function of only p, that is, $K_s(p, \varphi_s^{-1}(p_F^{-1}(p))) = K_s(p)$.

The results previously presented allow us to characterize equilibrium productivity distributions. Once again, it is important to distinguish the distributions of the entire mass of firms from the truncated ones. Therefore, denote $\Gamma_s^0(p)$ and $\Gamma_s(p)$ as the sectoral productivity cumulative distributions that arise from optimal behaviour of every potential firm and from active firms, respectively. Let $p_s^* = p_s(\varphi_s(\theta_s^*))$ and $\overline{p}_s = p_s(\varphi_s(\underline{\theta}_s))$,, then we conclude using Lemma 1 that every producing firm in sector s has a productivity level higher than p_s^*. Therefore, similarly to equation (3.21):

$$\Gamma_s(p) = \begin{cases} 0 & \text{, if } p < p_s^* \\ \frac{\Gamma_s^0(p) - \Gamma_s^0(p_s^*)}{\Gamma_s^0(\overline{p}_s) - \Gamma_s^0(p_s^*)} & \text{, if } p > p_s^* \end{cases} \text{, for } s \in \{I, F\} \qquad [4.1]$$

The following result guarantees the continuity of the productivity cumulative distributions and allows for a direct application of the result present in Bontemps et al. (2000), which is similar to Corollary 1, which guarantees a one-to-one map between contract values and almost all productivity levels.

Corollary 2. *If H_s^0 is continuously distributed, then $\Gamma_s^0(p)$ and $\Gamma_s(p)$ are also continuously distributed for each $s \in \{I, F\}$.*

Corollary 3 (Bontemps et al. (2000)). *If Γ_s^0 is continuously distributed, then $K_s(p)$ is a singleton for almost all $p \in supp\, \Gamma_s^0$ for each $s \in \{I, F\}$.*

The purpose of presenting each of these results is to characterize how wages, contract values, investment, and productivity are determined by the underlying distribution of investment costs. We have shown that in equilibrium, except for a set of measure zero

of firms, there is a one-to-one strictly decreasing map between investment costs and productivity, and an increasing map between productivity and wages offered. The last step to describe the first-order conditions of each of these problems, which are necessarily satisfied in equilibrium.

Derivation of first-order conditions of the optimal contract value problem (equations 4.2 and 4.3) is quite straightforward. On the other hand, derivation for the optimal investment problem(equations 4.6 and 4.7)[3] requires the use of an envelope theorem. Notice however that the correspondence $K_s(p_s(q))$ may not be continuously differentiable, which is a requirement for the usual envelope theorems. To circumvent this obstacle we rely on Milgrom and Segal (2002), which provides a more general envelope theorem for arbitrary choice sets. We provide a better explanation of the process of obtaining each of the following first-order conditions in Appendix A.

1) Maps between p and V

$$p = (1 + \tau + \lambda_{FU}\rho)\left[w'_F(V)\frac{\ell_F(V)}{\ell'_F(V)} + w_F(V)\right] \tag{4.2}$$

$$p = w_I(V) + w'_I(V)\frac{\ell_I(V)}{\ell'_I(V)} \tag{4.3}$$

2) Maps between q and p

$$q = p_F^{-1}(p) \tag{4.4}$$

$$q = p_I^{-1}(p) \tag{4.5}$$

3) Maps between θ and q

$$\theta = p'_F(q)\ell_F(K_F(p_F(q))) \tag{4.6}$$

$$\theta = p'_I(q)\ell_i(K_I(p_I(q))) \tag{4.7}$$

Now that we have fully described firms' equilibrium strategies, we are now able to define the set of equilibria of this economy.

[3] We are in fact abusing notation when writing the first-order conditions as in (4.6) and (4.7) since $K_s(p_s(q))$ may not be a singleton for some elements of the domain. The most accurate way of writing these equations is: $\theta = p'_s(q)\ell_s(V_s^*(q))$ where $V_s^* \in K_s(p_s(q))$ for both $s \in \{I, F\}$. We choose to proceed by abusing this notation for simplicity and since $K_s(p_s(q))$ is not a singleton only for measure-zero set of points of the domain of the correspondence.

Definition 1: Market Equilibrium

Market equilibrium is a list $\{m_F, m_I, u, n_I, n_F, F_F, F_I, G_F, G_I, \Gamma_F, \Gamma_I, H_F, H_I, \theta_F^*, \theta_I^*\}$ *that satisfies:*

Worker optimality:

1. *Unemployed workers' best response to firms' behavior is to accept any offer* V *that satisfies* $V > V_U$*, according to equations (3.1), (3.2) and (3.3).*

2. *Workers' employed with a contract* V *best response to firms' behavior is to accept any offer* V' *that satisfies* $V' > V$*, according to equations (3.1), (3.2) and (3.3).*

Firm optimality:

1. *For both* $s \in \{I, F\}$*,* $\varphi_s(\theta)$ *and* $K_s(p, \theta)$ *maximize firm profit* $\pi_s(V, q|\theta)$ *for* H_s^0*-almost all* $\theta_s \in [\underline{\theta}_s, \overline{\theta}_s]$*, thus satisfy equations (4.2), (4.3), (4.4), (4.5), (4.6) and (4.7)*

2. *For each sector* $s \in \{I, F\}$*, only firms with investment cost lower than* θ_s^* *and productivity higher than* $p_s^* = p_s(\varphi_s(\theta_s^*))$ *operate, and the distribution of their productivities and costs are:*

$$H_s(\theta) = \begin{cases} \frac{H_s^0(\theta) - H_s^0(\underline{\theta}_s)}{H_s^0(\theta_s^*) - H_s^0(\underline{\theta}_s)} & \text{, if } \theta < \theta_s^* \\ 1 & \text{, if } \theta \geqslant \theta_s^* \end{cases}$$

$$\Gamma_s(p) = \begin{cases} 0 & \text{, if } p < p_s^* \\ \frac{\Gamma_s^0(p) - \Gamma_s^0(p_s^*)}{\Gamma_s^0(\overline{p}_s) - \Gamma_s^0(p_s^*)} & \text{, if } p > p_s^* \end{cases}$$

Market Clearing:

1. *For each* $s \in \{I, F\}$*, every* $V \in [\underline{V}_s, \overline{V}_s]$ *must satisfy (3.10) and (3.11) such that* $m_s G_s(V)$ *is constant*

2. *The masses of unemployed, formal and informal workers must satisfy* $m_F + m_I + u = 1$ *and satisfy equations (3.13), (3.14) and (3.15).*

3. *The mass of active firms in each sector* $s \in \{I, F\}$ *satisfy* $n_s = H_s^0(\theta_s^*)$*.*

5 Identification

In this section, we make use of the comparative statics results derived in the previous section to discuss the identification of the model.We limit ourselves to demonstrate how to identify the unobservable parameters given that we know the sectoral contract offer distributions as well as the transitions shocks. Identification of F_F and F_I is rather challenging and it is a common problem in every wage-posting model with workers able to transit directly across sectors. We discuss in more detail these problems and present an alternative to pindown these objects in Appendix C.

The following identification arguments rely on the availability of data containing information about transitions across employment status, duration in each status, wage and firm investment distributions for both formal and informal sectors. Considering Brazilian data, the PME survey, collected by IBGE[1] until 2016, is a longitudinal data source containing almost every information needed. The only missing pieces of information needed are the firm investment distribution in each sector. For that, ECINF[2], also collected by IBGE, is - to our knowledge - the only survey containing micro-level information on small formal and informal urban firms, including information about firm investment[3]. Observation of these distributions (Figure 2) is crucial in order to identify the production functions for each sector, which is one of our main contributions.

Despite the ECINF survey being a representative sample of Brazilian informal firms, this is only true for firms with 5 employees or less. However, it is worth noticing that Brazilian formal firms are indeed very small. In fact, in 2003, 56% of formal firms had 3 or fewer employees and 77% had less than 7 employees.[Ulyssea et al. (2016)]. This indicates that ECINF provides a good representation of a large share of Brazilian formal firms.

[1] Instituto Brasileiro de Geografia e Estatística
[2] Economia Informal Urbana
[3] For a detailed explanation of the sampling aspects, as well as the information contained in it, see De Paula and Scheinkman (2010)

5.1 Productivity distributions and production functions

Given a distribution of offered contract values F_s for each sector $s \in \{F, I\}$ as well as each transition parameter, we can recover the unique associated productivity distributions Γ_s of active firms because of strict monotonicity of $K_s(p)$. An often overlooked fact is that nothing guarantees monotonicity of first-order conditions of the optimal contract value offer problem because the objective function is not strictly concave in V. In fact, this will only be true if, for every V, the marginal increment in output coming from the firm attracting more workers and becoming larger is enough to overcome the costs of paying these additional employees. In other words, first-order conditions may not be sufficient for identification.

However, Lemma 1 guarantees a unique productivity level for each optimal contract, which allows for non-parametric identification of the production distribution without relying on additional hypotheses on profit functions. Thus if we have $F_s(V)$ we can recover $\Gamma_s(p)$ via

$$F_s(K_s(p)) = \Gamma_s(p) \quad , \quad \text{for } s \in \{I, F\}$$

With $\Gamma_s(p)$ on our hands we are able to proceed to identify non-parametrically $p_s(q)$[4] by establishing a relation between sectoral cumulative distributions of productivities and investment levels (which we will denote by $\tilde{\Gamma}_s(q)$ for each s). This relies on two properties: First, $p_s'(q) > 0$ implies a unique investment associated with each productivity level. Secondly, we need $\Gamma_s(p)$ to be continuously distributed, which is guaranteed by Corollary 2 when both H_F^0 and H_I^0 are also continuously distributed. When both are satisfied we can use equations (4.4) and (4.5) to conclude:

$$\Gamma_s(p_s(q)) = \tilde{\Gamma}_s(q) \iff p_s(q) = \Gamma_s^{-1}(\tilde{\Gamma}_s(q)) \quad , \quad \text{for } s \in \{I, F\}. \qquad [5.1]$$

Thus if we are able to observe sectoral investment distributions, or at least estimate it, we are capable of fully identifying $p_s(q)$ without any additional hypothesis regarding concavity nor imposing a closed-form production function. As we have already discussed, ECINF is to our knowledge the only Brazilian data source that contains micro-level information about investments of formal and informal firms which allows for estimation of sectoral cumulative investment distributions.

[4] By this, we mean that we do not need to impose any parametric form to $p_s(q)$ to identify the production function and the complete distribution of investments of active firms

5.2 Investment cost distributions

Having identified F_s, Γ_s and p_s for both sectors, identification argument for H_F and H_I is analogue to the ones already presented. Once again, strict monotonicity plays the central role since it ensures a unique θ associated with each q. Therefore, non-parametric identification is guaranteed by

$$H_s(\theta) = 1 - \tilde{\Gamma}_s(\varphi_s(\theta)) \text{ for } s \in \{I, F\}.$$

Notice that H_s can be identified nonparametrically only because the argument that we presented relies on the fact that we can recover F_s via accepted wages. However, this is not sufficient to identify nonparametrically the untruncated distributions H_s^0. A common obstacle that structural labor market models face is, as Flinn and Heckman (1982) showed, that the knowledge of the truncation point and truncated distribution is not enough to guarantee the uniqueness of the exogenous underlying distribution. In order to satisfy recoverability condition, one should proceed as is usual in the literature and assume that H_s^0 belongs to a recoverable parametric family.

6 Empirical Application

The objective of our empirical application is to conduct some counterfactual exercises and see how the equilibrium economy generated by the model behaves as we vary the costs of operation in each market. In our model, variations in sectoral production functions are equivalent to variations in the costs of operation in the respective sector.

In this section, we explain how to compute numerically the equilibrium, how we calibrate the model's parameters and, finally, conduct some simulations and interpret how the equilibrium economy changes.

6.1 Computing equilibrium

Assume that the underlying investment cost distributions H_F^0 and H_I^0 are known, as well as every transition parameter and all the remaining primitives of the model. The algorithm to compute the model's equilibrium solution is:

1. Guess equilibrium distributions F_F, F_I and masses of active firms n_F and n_I. Assume $V_U = \underline{V}_I$, $n_f < 1$ and $n_I < 1$.

2. Compute $d_F(V)$, $d_I(V)$, $\Phi(V)$, m_F, m_i, u, G_F, G_I, $h_F(V)$ and $h_I(V)$ using equations (3.8), (3.13), (3.14), (3.15), (3.12) and (3.9).

3. Compute $w_F(V)$, $w_I(V)$, $l_F(V)$ and $l_I(V)$ using equations (3.4), (3.5) and (3.17).

4. Compute $K_F^{-1}(V)$ and $K_I^{-1}(V)$ in order to obtain the predicted $\Gamma_s(p)$ for both $s \in \{I, F\}$.

5. Compute $p_F^{-1}(p)$, $p_I^{-1}(p)$, $\varphi_F^{-1}(q)$ and $\varphi_I^{-1}(q)$ in order obtain the predicted θ_s^* and $H_s(\theta)$ for both $s \in \{I, F\}$

6. Check if:

$$n_F = H_F^0(\theta_F^*)$$

$$n_I = H_I^0(\theta_I^*)$$

$$H_F(\theta) = \begin{cases} \frac{H_F^0(\theta) - H_F^0(\underline{\theta}_F)}{H_F^0(\theta_F^*) - H_F^0(\underline{\theta}_F)} & \text{, if } \theta < \theta_F^* \\ 1 & \text{, if } \theta \geqslant \theta_F^* \end{cases}$$

$$H_I(\theta) = \begin{cases} \frac{H_I^0(\theta) - H_I^0(\underline{\theta}_I)}{H_I^0(\theta_I^*) - H_I^0(\underline{\theta}_I)} & \text{, if } \theta < \theta_I^* \\ 1 & \text{, if } \theta \geqslant \theta_I^* \end{cases}$$

If these conditions are not satisfied, return to step 1 with a new guess for F_F, F_I, n_F and n_I. If conditions are satisfied, this guess is the equilibrium for that set of primitives.

The objective of such an algorithm is to compute each endogenous equilibrium object given a set of primitives. The rationale behind the steps of the algorithm is to hypothesize an equilibrium and check which underlying distributions of investment costs are associated with this guess. If these underlying distributions related to this guessed equilibrium are equal (or at least, very similar) to the ones that were set as a model primitive, then we conclude that the guess is indeed an equilibrium.

The "trial and error" nature of this process might seem a bit counterintuitive but it is convenient to calculate firm size. Notice that computation of $l_s(V)$ requires n_s and F_s for both sectors, which are objects defined in equilibrium by firms' optimal behavior. But, in its turn, firms' objective function requires $l_s(V)$. We overcome this adversity by guessing the equilibrium - which allows us to compute firm sizes - and then checking if this equilibrium makes sense with the primitives - namely the underlying distributions H_F^0 and H_I^0.

We implement in practice this algorithm by assuming parametric forms for F_s and $p_s(q)$ for $s \in \{I, F\}$. We also assume parameters on the production function that allows us to utilize first-order conditions to solve steps 5 and 6. These assumptions will be discussed in detail in Section 6.2. Also, we discretize the support of each distribution F_F and F_I by interpolating them with 100 Chebyshev nodes on the interval $[-1, 1]$ and then solving every other endogenous function for these points. Finally, we consider as a solution

the guess $\{F_F, F_I, n_F, n_i\}$ that minimizes the mean squared error between the primitive underlying distributions and the ones generated by the guess.

6.2 Calibration

The first step of the algorithm described previously is to guess equilibrium distributions F_F and F_I. Despite not requiring a parametric form, for simplicity purposes, we choose to follow Meghir et al. (2015) and suppose that they follow a non-standard Beta distribution:

$$F_s \sim Beta\left(\frac{v - \underline{V}_s}{\overline{V}_s - \underline{V}_s}, \alpha_s, \beta_s\right) \text{ , with } \underline{V}_s \leqslant v \leqslant \overline{V}_s \quad \text{for } s \in \{I, F\}$$

Assuming a Beta distribution is advantageous because, besides being very flexible, this parametric form is continuously differentiable. This is important in order to compute m_F, m_I and u[1]. Therefore, our initial guess for F_F and F_I comprises of an eight parameter guess $(\alpha_F, \beta_F, \overline{V}_F, \underline{V}_F, \alpha_I, \beta_I, \overline{V}_I, \underline{V}_I)$.

We also assume a simple closed form for the sectoral production functions:

$$p_s(q) = \nu_s q^{\eta_s} \quad \text{, for } s \in \{I, F\}.$$

Despite having proven that strict concavity of the production is not necessary in order to establish a one-to-one map between q and θ, for simplicity purposes, we simulate the model for small values of η_F and η_I. Remember that first-order conditions are not sufficient for solving firms' problem, because the maps described by (4.6 and 4.7) are monotonically decreasing only if the marginal increase in firms' output due to the better contracts offered (as a consequence of the higher productivity) is not enough to surmount the costs associated to this productivity increment. By assuming small enough values for the output elasticities η_F and η_I, the marginal benefit of investing becomes monotonically decreasing, which allows us to use equations (4.6) and (4.7) as sufficient conditions. It's important to notice that even values close to zero are not enough to guarantee the sufficiency of first-order conditions for every guess of F_F and F_I. However, this was true almost always for the chosen baseline calibration, as well for the counterfactual calibrations.

[1] Computation of Φ requires the probability density functions F_F' and F_I'. See Appendix and Meghir et al. (2015).

As we have discussed in Section 5, for identification purposes, we must impose a parametric form for the underlying distributions H_F^0 and H_I^0. For the present exercise, we assume that investment costs are lognormally distributed:

$$H_s^0 \sim \textit{Log-normal}\ (\theta, \mu_s, \sigma_s) \quad \text{, for } s \in \{I, F\}.$$

The remaining parameters are calibrated according to the estimates present in Meghir et al. (2015)[2]. We present a table containing all the calibrated parameters in Appendix E.

6.3 Counterfactual Exercises

We conduct three exercises and analyse how equilibrium economy behave[3]. The objective is to compare the baseline economy's equilibrium (common to all three exercises) with the ones generated as we vary firms' costs of operation. The first two simulations reflect policy interventions that increase costs of informal operation, while the last one reflects a policy that decreases formal costs of operation. The three exercises are:

Policy 1 : Reducing returns to investment of informal firms: The first exercise consists of reducing informal firms' marginal return to investment. We denote the calibrated parameter of the baseline economy as η_I^B. We simulate the model with $\eta_I = 0.75\eta_I^B$ and $\eta_I = 0.5\eta_I^B$ - that is, considering that the investment marginal return of informal firms is three quarters and half of the baseline economy.

Policy 2: Imposing a fixed cost of hiring informally: For the second exercise, we impose a cost c of hiring an informal worker. The idea is to capture the notion that informal firms may be caught by regulations and must pay fines if so. We model this by imposing a cost c in the production function of informal firms, that is

$$p_I(q) = \nu_I q^{\eta_I} - c$$

The baseline economy has $c = 0$. We then simulate the model for $c = 50$ and $c = 100$. These two values represent, approximately, 20% and 40% of the output per worker of the baseline economy's least productive firm.

[2] The authors estimate their model for a range of different samples. The parameters we borrow are from their estimates for low educated males in São Paulo

[3] In Appendix F we present an additional simulation in which we vary the scale parameter ν_i

This additional cost affects directly the first order condition of the optimal contract problem (equation 4.3). For this problem, the necessary condition for for profit maximization is given by:

$$p = w_I(V) + c + w_I'(V)\frac{\ell_I(V)}{\ell_I'(V)}$$

Policy 3: Reducing payroll taxes: The last simulation consists of reducing payroll taxes payed by formal firms. Denote the baseline economy's payroll tax as τ^B. We simulate the model proposing reductions of 5 percentage points at a time, that is, with $\tau = 0.235$ and $\tau = 0.185$.

6.4 Results

Equilibrium worker allocation and sector size for each exercise are displayed in Table 2. First, the unemployment level remains almost constant across the two exercises. This indicates that, despite some informal firms ceasing operations, the employees of these firms are able to reallocate in the formal sector. Indeed, the mass of formal workers increases. This result contrasts with evidence present in Almeida and Carneiro (2012) showing that increasing enforcement raises unemployment rates, but is in line with recent papers that argue that there is no tradeoff between lower informality and high unemployment rate [Ulyssea (2010), Meghir et al. (2015)]. The mechanism here appears to be similar to the one described in Meghir et al. (2015), in which informal firms seem to be creating an over-representation of low productive jobs instead of actually creating lots of employment opportunities that would not exist otherwise. Our model seems to predict that, as the smaller and less productive firms are removed from the market, these employees seem to relocate with ease into the firms that keep operating and the new small formal firms that were able to enter the market, produce and hire.

The effects over productivity distributions are shown in Table 3 and contains our main results. The first thing worth noticing is that in every scenario we observe an overlap in productivity between the two sectors. This is a well established stylized fact about economies with a large informal sector since La Porta and Shleifer (2008, 2014).

By imposing higher costs of operation on informal firms, we are affecting the productivity distribution of informal firms in two ways. The first is a compositional effect.

Table 2 – Equilibrium worker and firms allocation by share

	Policy 1			**Policy 2**			**Policy 3**		
	Baseline	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$\tau = 0.235$	$\tau = 0.185$
Unemployment	0.066	0.066	0.062	0.066	0.066	0.066	0.066	0.066	0.065
Formal workers	0.367	0.369	0.434	0.367	0.369	0.370	0.367	0.372	0.385
Informal workers	0.567	0.565	0.503	0.567	0.565	0.564	0.567	0.562	0.550
Formal firms	0.132	0.147	0.155	0.132	0.149	0.158	0.132	0.148	0.159
Informal firms	0.868	0.853	0.845	0.868	0.851	0.842	0.868	0.852	0.841

Note: Each row is computed, respectively, as: u, m_F, m_I, $N_F n_F/(N_F n_F + N_I n_I)$ *and* $N_I n_I/(N_F n_F + N_I n_I)$

Firms with the highest investment costs, that were barely making positive profits cannot be as productive as before and must cease operation. As we know, these firms with high investment costs are also at the bottom of the productivity distribution. Therefore, by removing these firms with high θ from the market, only firms with lower investment costs and relatively higher productivity keep operating. This compositional effect, by itself, should positively affect the informal sector's productivity distribution. But since our model presents an endogenous investment decision, we have a second effect taking place. By increasing the costs of operation, we are affecting the incentives of investing in the entire mass of informal employers that are operating. Firms now have fewer resources to allocate to wages and investments, and as a result, the productivity of these informal firms is negatively affected.

However, it is important to note that our model shuts down a possible important channel through which overall productivity may be affected. By assuming that firms are exogenously determined as formal or informal, we cannot address how firms sort between sectors. Indeed the largest and more productive informal firms are exactly the ones that more likely would transit to the formal sector if faced with higher operation costs (or a less burdensome formal regulation), and this transitions may have important outcomes (for more details on how firms sort between the two sectors, see Meghir et al. (2015) and Ulyssea (2018)) Let's start by focusing on the policies that focus on increasing the costs of informal operation (Panels A and B). Notice that the effects of each policy intervention affect the mass of informal firms differently. A policy that decreases the returns to investment (Policy 1) greatly affects the firms on the top of the productivity distribution, while a policy that introduces a fixed cost per informal worker (Policy 2) is especially harmful to the smaller firms and less productive firms, which are the ones barely making enough profit keep operating. These two contrasting effects are reflected when we observe Table 3.

Table 3 – Mean and percentiles of log-productivity distributions

Panel A		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$
1st	6.13	6.08	6.21	5.63	5.57	5.53	5.63	5.57	5.53
10th	6.34	6.30	6.44	5.75	5.70	5.64	5.75	5.70	5.64
25th	6.55	6.52	6.68	6.05	6.01	5.92	6.18	6.13	5.92
50th	6.90	6.89	7.00	6.34	6.31	6.20	6.44	6.41	6.33
75th	7.22	7.21	7.26	6.59	6.57	6.45	6.79	6.77	6.68
90th	7.55	7.54	7.46	6.96	6.95	6.81	7.10	7.11	7.09
95th	7.69	7.83	7.72	7.10	7.09	6.96	7.32	7.42	7.31
Mean									
$\mathbb{E}(\ln(p))$	6.92	6.92	6.99	6.36	6.33	6.22	6.43	6.42	6.34
Panel B		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$c = 50$	$c = 100$
1st	6.13	6.08	6.07	5.63	5.74	5.91	5.63	5.74	5.91
10th	6.34	6.30	6.30	5.75	5.85	6.00	5.75	5.85	6.07
25th	6.55	6.52	6.53	6.05	6.13	6.24	6.18	6.21	6.30
50th	6.90	6.89	6.89	6.34	6.40	6.49	6.44	6.41	6.53
75th	7.22	7.21	7.21	6.59	6.64	6.71	6.79	6.77	6.89
90th	7.55	7.54	7.54	6.96	6.99	7.05	7.10	7.13	7.18
95th	7.69	7.83	7.83	7.10	7.13	7.18	7.32	7.42	7.42
Mean									
$\mathbb{E}(\ln(p))$	6.92	6.92	6.92	6.36	6.42	6.51	6.43	6.50	6.58
Panel C		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$\tau = .235$	$\tau = .185$	**Baseline**	$\tau = .235$	$\tau = .185$	**Baseline**	$\tau = .235$	$\tau = .185$
1st	6.13	6.03	5.85	5.63	5.62	5.53	5.63	5.62	5.61
10th	6.34	6.26	6.14	5.75	5.74	5.73	5.75	5.74	5.85
25th	6.55	6.49	6.40	6.05	6.04	6.03	6.18	6.16	6.14
50th	6.90	6.85	6.81	6.34	6.33	6.32	6.44	6.37	6.40
75th	7.22	7.18	7.14	6.59	6.59	6.58	6.79	6.73	6.70
90th	7.55	7.50	7.44	6.96	6.96	6.96	7.10	7.11	7.12
95th	7.69	7.79	7.71	7.10	7.11	7.12	7.32	7.38	7.34
Mean									
$\mathbb{E}(\ln(p))$	6.92	6.89	6.85	6.36	6.35	6.32	6.43	6.43	6.41

The first striking difference between Panels A and B of Table 3 are the different effects over the productivity distribution of informal firms. Reductions in the investment returns indeed remove the least productive from the market, but the effect on the firms that keep operating overcomes this compositional effect, thus reducing mean productivity in the sector. The opposite is observed in Panel B, where the massive removal of the least productive firms increases average firm productivity in the sector. These two divergent outcomes are only possible because of the endogenous firm productivity of our framework, which allows for a complete reshape of the productivity distribution. Otherwise, we would observe the average informal productivity in both policy interventions due to the compositional effect[4].

As a result of these different effects, outcomes on the formal side of the economy

[4] In a model in which employers shift across sectors, these effects could be possible even without endogenous firm productivity since firms on the top of the informal distribution could transit to the formal sector.

are also different. In both policies, the imposition of additional costs to informal firms hinders the value of offered contracts in the sector. This means that formal contracts become relatively more appealing and firms are able to attract informal employees more easily, thus increasing the flow of workers into the formal sector. This explains why m_F increases as η_I decreases (or c increases) in Table 2. Since Policy 1 is able to greatly affect the largest firms in the informal, these firms cannot employ as much neither can pay as much. Therefore, a fraction of those workers employed on higher percentiles of the wage distribution of the sector must relocate. This greatly increases the flow of workers into the formal sector, which reflects on average firm size (see the increase in average firm size in Table 4), expected profits and for productivity enhancements of formal firms. In the end, this effect over the mean productivity of formal firms is so strong that it overcomes the negative effect due to the small and unproductive firms that starting to operate. On the other hand, the introduction of a fixed cost (Policy 2) has minimal effects over the firms that are not so close to the threshold of operation. In this case, we barely observe any effect on formal productivity distribution. In the end, outcomes on the overall distribution are dictated by the effect observed in the informal sector, since the vast majority of firms are informal in every scenario.

When we turn to Panel C, we see that reductions in formal labor costs decrease mean productivity in both sectors. In the formal sector, this is due to the compositional effect caused by the huge entrance of new formal firms being so big that it overcomes the positive effect of increased investments of the firms that were already operating. Because these firms are less now taxed and can invest more and offer higher wages (as we will later discuss in Table 5), informal firms are relatively less attractive to workers. This affects the expected profits of informal employees which, in turn, hinders investment and productivity.

Despite clarifying the effects over the sectoral and overall productivity distributions, we cannot use Table 3 to infer about the effects of such policies over the total output of an economy. There are two important aspects that must be taken into consideration when analyzing how the total output of an economy changes after each policy intervention. First, since firm productivity is defined as output per employee in our context, we need to take into account how firm size varies. Secondly, despite reducing the mean productivity of active firms, the entrance of new and less productive firms means that more employers are hiring and producing, which should improve total output. Table 4 present these results

as well as total output per worker of the economy. Remember that the mass of workers remains constant across each simulation.

Table 4 – Mean firm size and productivity, share of active firms and total output per worker

	Policy 1			**Policy 2**			**Policy 3**		
	Baseline	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$\tau = .235$	$\tau = .185$
Share of active firms (formal)	0.83	0.87	0.90	0.83	0.87	0.87	0.83	0.91	0.99
Share of active firms (informal)	0.91	0.84	0.82	0.91	0.83	0.77	0.91	0.87	0.87
Avg. firm size (formal)	25.50	24.43	27.83	25.50	24.43	24.54	25.50	23.58	22.41
Avg. firm size (informal)	5.95	6.44	5.87	5.95	6.51	7.00	5.95	6.18	6.05
Mean log-productivity (formal)	6.92	6.92	6.99	6.92	6.92	6.92	6.92	6.89	6.85
Mean log-productivity (informal)	6.36	6.33	6.22	6.36	6.42	6.51	6.36	6.35	6.32
Log-output per worker	**3.12**	**3.13**	**3.56**	**3.12**	**3.13**	**3.14**	**3.12**	**3.14**	**3.20**

N*ote: Each row is computed, respectively, as:* n_F, n_I, $\mathbb{E}_{F_F}(\ell_F)$, $\mathbb{E}_{F_I}(\ell_I)$, $\mathbb{E}_{\Gamma_F}(\ln(p))$, $\mathbb{E}_{\Gamma_I}(\ln(p))$, $\sum_{s=I,F} N_s n_s \mathbb{E}_{\Gamma_s}(\ln(p))\mathbb{E}_{F_s}(\ell_s)/M$

It becomes clearer that in terms of total output, the three policies that decreased the share of informal firms in the market had positive effects. It is worth noticing that the two policies that decreased overall productivity (Panels A and C in Table 3) were the ones with the highest positive effects on total output. Reductions in returns to investment (Policy 1) harm informal firms of all sizes, especially the large ones with the highest investment. Those firms cannot be as large anymore, and the informal employees relocate to the formal size, which greatly increases the average size of formal firms. This ultimately has a huge positive on total output. On the other hand, reductions in labor taxes (Policy 3) operate through a different channel. These reductions in payroll taxes promote a mass entrance of small formal firms, decreasing their average size. However, there are so many more producing firms that total output increases.

Essentially, Table 4 shows that policies aimed at decreasing the share of informal firms in the market induce gains of output. However, there are two main drivers of output gains. First, the intensity of the entrance of new formal firms. Second, how much formal employers are benefited in terms of firm size. This is determined by how much the flow of workers into the sector increases after a policy intervention, which by its turn, is partially explained by how much the informal firms that are not so close to the threshold of operation are affected. Notice that Policy 2 does not achieve either of these aspects with much intensity. This explains why this is the scenario in which we observe the smallest gain in total output.

Increasing costs of operation also decrease mean wages in the economy (Table 5).

Table 5 – Mean, percentiles and inequality measures of log-wages

Panel A		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$0.75\eta_I^B$	$0.5\eta_I^B$
10st	5.96	5.90	5.89	4.97	4.82	4.98	5.51	5.43	5.27
25th	6.50	6.47	6.43	5.86	5.80	5.75	6.11	6.07	6.11
50th	6.72	6.71	6.76	6.46	6.43	6.31	6.59	6.57	6.43
75th	6.98	6.98	6.94	6.70	6.68	6.55	6.80	6.88	6.74
90th	7.13	7.20	7.16	6.90	6.88	6.74	7.06	7.05	6.99
95th	7.27	7.27	7.22	7.07	7.06	6.91	7.13	7.20	7.14
Mean									
$\mathbb{E}(\ln(w))$	6.75	6.76	6.75	6.42	6.40	6.25	6.55	6.54	6.48
Panel B		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$c = 50$	$c = 100$
10st	5.96	5.90	5.91	4.97	4.82	4.91	5.51	5.43	5.48
25th	6.50	6.47	6.47	5.86	5.80	5.83	6.11	6.07	6.09
50th	6.72	6.71	6.71	6.46	6.43	6.44	6.59	6.57	6.58
75th	6.98	6.98	6.98	6.70	6.68	6.69	6.80	6.88	6.79
90th	7.13	7.20	7.20	6.90	6.88	6.89	7.06	7.05	7.06
95th	7.27	7.27	7.27	7.07	7.06	7.07	7.13	7.20	7.20
Mean									
$\mathbb{E}(\ln(w))$	6.75	6.76	6.76	6.42	6.40	6.40	6.55	6.54	6.54
Panel C		**Formal**			**Informal**			**Overall**	
Percentiles	**Baseline**	$\tau = .235$	$\tau = .185$	**Baseline**	$\tau = .235$	$\tau = .185$	**Baseline**	$\tau = .235$	$\tau = .185$
10st	5.96	5.92	6.10	4.97	5.00	5.06	5.51	5.52	5.50
25th	6.50	6.48	6.45	5.86	5.86	5.88	6.11	6.11	6.10
50th	6.72	6.71	6.80	6.46	6.46	6.45	6.59	6.59	6.58
75th	6.98	6.98	7.05	6.70	6.70	6.69	6.80	6.80	6.89
90th	7.13	7.20	7.20	6.90	6.90	6.89	7.06	7.06	7.05
95th	7.27	7.27	7.33	7.07	7.07	6.98	7.13	7.20	7.20
Mean									
$\mathbb{E}(\ln(w))$	6.75	6.76	6.79	6.42	6.41	6.38	6.55	6.55	6.55

As we have seen, such policies deteriorate informal productivity, narrowing the interval between output per worker and reservation wages. As a result, informal firms cannot offer wage rates as high as before. And since the majority of workers are still employed in this sector, the effect on overall wage distribution is negative for both policies, although this effect is minimal for Policy 2.

When we turn to Panel C, we notice that reductions in payroll taxes have opposing effects on the sectoral wage distributions. Informal wages follow decrease because, as we have already argued, informal jobs become relatively less attractive since formal firms are now less taxed, which enables them to offer better contracts. In the end, the gains and losses in average wages in each sector balance each other, and overall mean wages remain practically unaltered.

Lastly, we evaluate each policy intervention in terms of welfare implications using Table 6. Worker welfare reflects not only current wages but also the possibility of climbing the job ladder expected contracts. Firm welfare reflects the expected profit per employee

in the sector. Notice that in every scenario simulated, total welfare is higher than in the baseline economy. This shows that any policy that aims at improving welfare should relocate workers with low-value contracts to jobs that yield a higher value and allow for the entrance of new formal firms. Relocation of workers from the informal sector towards the formal, where they can climb to higher contracts on the job ladder more easily, is a key driving force of the welfare gain. The ability to match in the future with employees that offer better contracts is an important component of the overall value of a current given contract. Considering that transitions across sectors are much less common than within sector ones, being employed in the formal sector, where contracts are more valuable and the probability of losing the job is lower, greatly enhances the odds of climbing higher in the job ladder.

However, welfare improves even if relocation from low-valued to higher-valued contracts occurs within sectors, as it is more prominent after Policy 2. Notice that this intervention, which provided important welfare gains, presented an increase average size of informal firms, no increase in unemployment rates and an intense removal from the market of the least productive firms in the sector (and in the economy). This indicates that workers employed in these less productive firms relocated not only across sectors but also to higher valued jobs within the informal sector.

Table 6 – Welfare measures and total welfare per worker

	Policy 1			**Policy 2**			**Policy 3**		
	Baseline	$0.75\eta_I^B$	$0.5\eta_I^B$	**Baseline**	$c = 50$	$c = 100$	**Baseline**	$\tau = .235$	$\tau = .185$
Unemployed	417.38	423.60	384.90	417.38	423.60	421.66	417.38	419.78	421.47
Formal workers	677.07	686.70	661.54	677.07	686.70	686.48	677.07	686.25	700.28
Informal workers	502.00	504.35	452.69	502.00	504.35	503.25	502.00	502.17	498.92
Formal firms	679.73	754.43	745.47	679.73	762.17	810.47	679.73	728.19	736.90
Informal firms	274.64	310.87	275.03	274.64	269.07	255.26	274.64	266.78	255.83
Welfare per worker	**965.87**	**1020.04**	**1001.42**	**965.87**	**999.27**	**1009.78**	**965.87**	**986.13**	**995.58**

N*ote: Last row indicates total welfare per worker, excluding the government. Remember that the mass of workers is constant across all exercises. Welfare measures for workers are given by the expected contract value of each sector, while welfare of firms are given by total profits per employee for each sector. Thus, each row is computed, respectively, as:* rV_u, $r\mathbb{E}_{F_F}(V)$, $r\mathbb{E}_{F_I}(V)$, $\mathbb{E}_{F_F}(\pi)N_F n_F/Mm_F$, $\mathbb{E}_{F_I}(\pi)N_I n_I/Mm_I$ *and* $rV_u + r\mathbb{E}_{F_F}(V) + r\mathbb{E}_{F_I}(V) + \mathbb{E}_{F_F}(\pi)N_F n_F/M + \mathbb{E}_{F_I}(\pi)N_I n_I/Mm_I$.

This by itself is an argument in favor of policies that aim to increase formality. However, it is worth noticing that the means to achieve higher formality rates dictate how much total welfare per worker actually increases. Notice that when reductions in the returns to investment are too severe (i.e, when we reduce from $0.75\eta_I^B$ to $0.5\eta_I^B$), total

welfare starts to decline. This happens because, even in this scenario, informality still represents the majority of the labor market and the loss of welfare of this sector overcomes the positive effects of formalization. Despite Policy 2 presenting the highest gains of welfare, it is unlikely that these gains would still be present for costs much higher than $c = 100$ since it would start to overly harm the larger informal firms. Reductions in payroll taxes also feature increasing welfare outcomes.

When analyzing these welfare gains, it is important to address once again a limitation of the present work. If our model allowed for firm relocation across sectors, we suspect that the welfare gains of each of these policies would be higher. Informal firms with the lowest investment costs are the ones that most likely would become formal when faced with lower costs of formalization (such as lower taxes), and these are exactly the firms with the highest profits and that would employ more formal workers.

Table 3 and Table 6 together show that policies aimed at reducing the informal sector are output and welfare inducing. However, the different ways of achieving this dictate the intensities in which each of these two outcomes are affected. As we have already highlighted, the two main drivers of output gains are the intensity of entrance of new formal firms and how much formal employers are benefited in terms of firm size. Welfare gains, on the other hand, are dictated by how intense is the relocation of workers from low-valued jobs to high-valued ones, either within or across sectors.

7 Conclusion

This paper contributes to the vast literature that uses equilibrium models with search frictions to evaluate how labor market policies affect the overall performance of an economy with a large informal sector. With that goal in mind, we developed a framework in which firms maximize expected profits not only by choosing which contracts to offer to attract workers, but also by deciding on how much to invest optimally in order to elevate productivity. This contrasts with papers that follow the tradition of Bontemps et al. (2000), which assumes that firms' heterogeneity in output per worker is exogenously determined. By endogenizing this variable, our framework allows for a complete reshape of sectoral productivity distributions as a result of a variety of policy interventions.

We have shown that if employers' investment costs are continuously distributed, equilibrium is characterized by pure strategies for almost every firm. Even without assuming concavity or any other parametric form of production functions, we are able to demonstrate that the map between investment costs and optimal investment is strictly decreasing, while the map between productivity and contract offers is strictly increasing. These features are crucial for establishing identification of the model, including non-parametric identification of sectoral production functions. Each of these arguments do not rely on first-order conditions, which are not sufficient to characterize optimal behavior by firms in this context. Instead, these comparative statics results depend solely on ordinal conditions.

Our simulations show that policies that aim at reducing informality, either via increasing costs of informal operation or via reductions in labor taxes, are desirable approaches to increase total welfare and output per worker in the economy. The main driver of welfare improvements in the economy is the transition of workers from jobs that yield low-value to higher valued ones. This relocation can occur within or across sectors. However, if such policies become overly harmful to those informal firms that keep operating, welfare gains can be lowered because even after such interventions, informality still represents the largest share of the economy

We also find in our exercises that there two main determinants of gains in output. First, policy interventions should aim at allowing for small formal firms to enter the market,

despite decreasing the average productivity in the sector. Secondly, these interventions should allow for formal firms to grow and become larger. This can be achieved by imposing relevant costs of operation to the larger informal firms forcing them to shrink in size, which improves the flow of workers into the formal sector. Additionally, when we simulate a policy that decreases the returns to informal investment, we observe an increase in mean formal productivity, despite the entrance of previously inactive firms. The fact that this could not happen in an environment such as Bontemps et al. (2000) where output per worker of any firm is taken as given, suggests that we may lose some important factors for policymaking in developing countries should we not consider endogenous investment decisions.

We believe there are many ways to further enrich the present work. An obvious one is to estimate the model presented here using Brazilian data, namely ECINF and PME surveys. To do so, one would require a maximization algorithm to solve the firms' problem, rather than relying solely on first order conditions - as we have done here. Second, it would be interesting to model this economy using only one underlying distribution of investment cost instead of imposing one for each sector. This would lead to insights on how firms sort into each sector and how investments respond when employers transit across sectors. Lastly, a model that fully contemplates the interactions between informality and the productivity level of the economy should not only account for investment decisions of employers but also of workers. The presence of a large informal sector, characterized by unproductive firms, low-wage workers and that offers slim possibilities of climbing high in the job ladder are likely to be important determinants of workers' schooling decisions and, ultimately, the overall productivity of the economy.

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APPENDIX A - Derivation of the first order conditions

Optimal contract value offer

The process of obtaining equations (4.2) and (4.3) is as follows. At this point, firms take productivity, investment and investment costs as given. At this point, first order conditions are easily derived.

Remember the problem of informal firms. Productivity $p_I(q) = p$ is given and first-order conditions are necessary for profit maximization.

$$\max_V \quad (p - w_I(v))\,\ell_I(V) - \theta q$$

Imposing the derivative to equal zero we arrive at equation (4.3):

$$(p - w_I(V))\ell_I'(V) - w_I'(V)\ell_i(V) = 0$$
$$p = w_I(V) + w_I'(V)\frac{\ell_I(V)}{\ell_I'(V)}$$

We follow analogously for the formal case. The maximization problem is:

$$\max_V \quad (1-t)\Big[\Big(p - (1+\tau+\rho\lambda_{FU})w_F(V)\Big)\ell_F(V) - \theta q\Big]$$

Taking the derivative, we obtain the first-order condition in equation (4.2):

$$(1-t)\left[(p - (1+\tau+\rho\lambda_{FU})w_F(V))\ell_F'(V) - (1+\tau+\rho\lambda_{FU})w_F'(V)\ell_F(V)\right] = 0$$
$$p = (1+\tau+\lambda_{FU}\rho)\left[w_F'(V)\frac{\ell_F(V)}{\ell_F'(V)} + w_F(V)\right]$$

Optimal investment decision

At this point, firms only take investment costs as taken. Offered contracts are defined optimally via the map $K_s(p_s(q))$.

Notice however that the correspondence $K_s(p_s(q))$ may not be continously differentiable at every point in the domain, which is a necessary condition for the most common envelope theorems. Milgrom and Segal (2002) provides a proof for a more general

envelope theorem that does not require any structure of the choice set $K_s(p_s(q))$ apart from non-emptiness.

Denote:

$$\pi_s^*(q|\theta) = \sup_V \pi_s(V, q|\theta) \qquad \text{for } s \in \{I, F\}$$
$$V_s^*(q) \in K_s(p_s(q)) \qquad \text{for } s \in \{I, F\}$$

Since $\pi_s(V, q|\theta)$ is continuously differentiable in q for both $s \in \{I, F\}$, Theorem 1 in Milgrom and Segal (2002) guarantees that

$$\frac{\partial \pi_s^*(q|\theta)}{\partial q} = \frac{\partial \pi_s(V_s^*, q|\theta)}{\partial q}(V^*, q) \qquad \text{for } s \in \{I, F\}$$

When these derivatives are equal to zero, we obtain the first-order conditions of the problem (equations (4.4) and (4.5)).

$$\frac{\partial \pi_F^*(q|\theta)}{\partial q} = p_F'(q)\ell_F(V_F^*(q)) - \theta = 0$$
$$\frac{\partial \pi_I^*(q|\theta)}{\partial q} = p_I'(q)\ell_i((V_I^*(q)) - \theta = 0$$

APPENDIX B - Omitted proofs

Lemma 1

For any $s \in \{I, F\}$***, let*** $q \in \varphi_s(\theta)$ ***and*** $q' \in \varphi_s(\theta')$***. If*** $\theta' > \theta$ ***then*** $q > q'$***.***

Proof: For any $s \in \{I, F\}$, suppose $q \in \varphi_s(\theta)$, $q' \in \varphi_s(\theta')$ and $\theta' > \theta$. Also, denote by π_s^* the supremum profit that a type-θ firm with investment level q, that is:

$$\pi_s^*(q, \theta) = \sup_V \pi_s(V, q|\theta).$$

Also, denote by $V_s^*(q, \theta)$ the optimal contract of firm with q and θ as its investment level and costs. That is:

$$V_s^*(q, \theta) \in K_s(p_s(q), \theta)$$

Since the profit function is differentiable in q, application of Theorem 1 of Milgrom and Segal (2002), which is an envelope theorem that does not require $K_s(p)$ to be continuously differentiable, yields

$$\frac{\partial \pi_s^*(q, \theta)}{\partial q} = \frac{\partial \pi_s(V, q|\theta)}{\partial q}(V_s^*(q, \theta), q)$$

Furthermore, if we proceed by differentiating in θ we conclude $\partial^2 \pi_s^* / \partial q \partial \theta = -1$, that is, the marginal variation in the maximum output of a firm in relation to variations in the investment is strictly decreasing in relation to investment costs θ.

The objective is to show that $q > q'$. Let's start by assuming by way of contradiction $q' > q$. Since q' is, by hypothesis, a profit maximizing investment for a type-θ' firm, we must have

$$\pi_s^*(q', \theta') - \pi_s^*(q, \theta') \geqslant 0 \qquad \text{[A.1]}$$

But notice that when $q' > q$, $\partial^2 \pi_s^* / \partial q \partial \theta < 0$ implies

$$\pi_s^*(q', \theta) - \pi_s^*(q, \theta) > \pi_s^*(q', \theta') - \pi_s^*(q, \theta') \qquad \text{[A.2]}$$

Combining equations (A.1) and (A.2) we conclude that

$$\pi_s^*(q', \theta) - \pi_s^*(q, \theta) > 0$$

which contradicts the fact that $q \in \varphi_s(\theta)$. Thus we cannot have $q' > q$.

Now, let's assume by way of contradiction that $q' = q$. Remember that $p_s'(0) = \infty$ and $p_s'(\overline{q}) = 0$ imply that optimal investment allocation is interior, thus

$$\frac{\partial \pi_s^*}{\partial q}(q', \theta') = 0$$

Additionally, $\partial^2 \pi_s^* / \partial q \partial \theta < 0$ means that the partial derivative $\partial \pi_s^* / \partial q$ is strictly monotonic, which implies that

$$\frac{\partial \pi_s^*}{\partial q}(q', \theta) \neq 0$$

Therefore we must have $q' \notin \varphi_s(\theta)$, which contradicts $q = q'$. Thus we conclude that $q > q'$. □

Lemma 2

For any $s \in \{I, F\}$, let $V \in K_s(p, \theta)$ and $V' \in K_s(p', \theta)$. If $p' > p$ then $V' > V$.

Proof: Proof of Lemma 2 is very similar to the proof of Lemma 1. The are only two differences: First, there is no need of an general envelope theorem as in Lemma 1. Second, since the mixed second order partial derivatives of the profit function in relation to V and q are positive (instead of negative, as in Lemma 1), we only need to change the sign of the arguments.

For any $s \in \{I, F\}$, suppose $V \in K_s(p, \theta)$, $V' \in K_s(p', \theta)$. Also, let $p' = p_s(q')$, $p = p_s(q)$ and $p' > p$. Remember that strict monotonicity of the production functions implies that $q' > q$. Following similarly to Lemma 1, we can mixed second order partial derivatives of the profit function and obtain $\partial \pi_s(V, q) / \partial V \partial q > 0$.

The objective is to show that $V' > V$. Start by assuming by way of contradiction $V > V'$. Since V is, by hypothesis, a profit maximizing investment for a firm with investment q, we must have

$$\pi_s(V, q|\theta) - \pi_s(V', q|\theta) \geqslant 0 \qquad \text{[A.3]}$$

But notice that when $V > V'$, $\partial^2 \pi / \partial V \partial q < 0$ implies that:

$$\pi_s(V, q'|\theta) - \pi_s(V', q'|\theta) > \pi_s(V, q|\theta) - \pi(V', q|\theta) \qquad \text{[A.4]}$$

However, combining equations (A.3) and (A.4) implies that

$$\pi_s(V, q'|\theta) - \pi_s(V', q'|\theta) > 0$$

which contradicts the fact that $V' \in K_s(p', \theta)$. Thus we cannot have $V > V'$.

Now assume by way of contradiction that $V = V'$. $V' \in K_s(p', \theta)$ means that first order conditions must be valid:

$$\frac{\partial \pi_s}{\partial V}(V', q') = 0$$

Additionally, $\partial \pi_s(V, q)/\partial V \partial q > 0$ means that:

$$\frac{\partial \pi_s}{\partial V}(V', q) \neq 0$$

Therefore we must have $V' \notin K_s(p, \theta)$. Thus we conclude $V' > V$.

Corollary 1

If H_s^0 is continuously distributed, then $\varphi_s(\theta)$ is a singleton for almost all $\theta \in supp\, H_s^0$ for each $s \in \{I, F\}$.

Proof: The argument is identical for both sectors. Define $\Theta_s := \{\theta \in supp\, H_s^0 : \varphi_s(\theta)\ is\ not\ a\ singleton\}$ and assume by way of contradiction that Θ_s is uncountable. Next, define the function $f_s(\theta)$ that assigns to each element θ of Θ_s an arbitrary rational number in the interval $[\inf \varphi_s(\theta), \sup \varphi_s(\theta)]$.

Now denote by θ_1 and θ_2 two distinct arbitrary elements of Θ_s. By Lemma 1, strict monotonocity of $\varphi_s(\theta)$ guarantees $\varphi_s(\theta_1) \cap \varphi_s(\theta_2) = \emptyset$. This means that $f_s(\theta_1) \neq f_s(\theta_2)$. Therefore we conclude that f_s must be a bijective function between Θ_s and a subset of rational numbers. This implies a bijection between a countable and an uncountable set, which is a contradiction. Thus we must have Θ_s countable. □

Corollary 2

If H_s^0 is continuously distributed, then $\Gamma_s^0(p)$ and $\Gamma_s(p)$ are also continuously distributed for each $s \in \{I, F\}$.

Proof: Start by defining the set $\mathcal{S}_s(p) := \{\theta \in int\ (supp\, H_s^0) : p_s^{-1}(p) \in \varphi_s(\theta)\}$, that is, let $\mathcal{S}_s(p)$ be the set of every investment cost that belongs to the interior of the distribution H_s^0 whose optimal investment yields a producitivity level p. Assume by way of contradiction that $\Gamma_s^0(p)$ is not continuously distributed. This implies that there exists

a p^* such that the set $\mathcal{S}_s(p^*)$ has strictly positive probability measure. But by strict monotonicity of $\varphi_s(\theta)$ (Lemma 1), we know that $\varphi_s(\theta) \cap \varphi_s(\theta') = \emptyset$ for all $\theta \neq \theta'$, which implies $\mathcal{S}_s(p^*)$ must be a singleton. Furthermore, the fact that $H_s^0(\theta)$ is continuously distributed means that the probability measure of $\mathcal{S}_s(p^*)$ must be zero. Thus we arrive at a contradiction.

Continuity of $\Gamma_s(p)$ is trivial since it is defined as the quotient between two continuous functions (see equation 4.1). $\square$

APPENDIX C - Identification of contracts offered and transition shocks

The identification problem

The identification argument for the entire model is rather challenging. We cannot use the same arguments as in Burdett and Mortensen (1998) or Bontemps et al. (2000) because workers are free to transit across two different sectors in our framework. The fact that a worker must balance the possibility of receiving offers from both sectors and climbing the job ladder in order to accept any given contract causes the four distributions of offered and accepted contracts to be deeply intertwined, complicating identification of F_F and F_I. This problem is common to any wage-posting model in which workers are able to directly transit across sectors.

To illustrate this difficulty, take the transition parameters as given. Identification of contracts offered is conditional on knowledge of these parameters. Remember that what is observable via data are the sectoral distributions of accepted wages (or at least, we can directly estimate them from data), which we denote as $G_s^*(w)$. Next, notice that equations (3.4) and (3.5) provide one-to-one maps between contract values and wages as such:

$$G_s^*(w_s^{-1}(w)) = G_s(V) \ , \ \text{ for } s = \{I, F\}$$

This step is not trivial, because in order to fully recover which wage is associated with which contract, we need F_F and F_I (see equations (3.4) and (3.5)). Combining this with the equations that provide the link between accepted and offered contract distributions (equation (3.12)), distributions F_F and F_I are identified if and only if there is a unique pair $\{F_F, F_I\}$ that satisfies the following equation for both sectors:

$$m_s G_F^*(w_s^{-1}(w)) = \frac{u\left(\lambda_{Us} F_s(w_s^{-1}(w)) - \Phi(w_s^{-1}(w))\right)}{d_s(w_s^{-1}(w))} \quad \text{for every } w \in supp\, G_s^* \text{ and } s \in \{I, F\}$$

Since both sides of this equation depend on the pair $\{F_F, F_I\}$, this statement can be rewritten as a fixed point problem on $\mathbb{R}^\infty$. Identification is guaranteed if there is a unique fixed point solution to this rather complicated problem that we were unable to solve.

Pinnning down F_F, F_I and transition shocks

Since we cannot pin down analytically a pair $\{F_F, F_I\}$ that is associated with the observed pair $\{G_F^*, G_I^*\}$, we present an algorithm that can possibly pin down numerically these distributions together with the transition parameters. This algorithm is used on the estimation strategy of Meghir et al. (2015). Briefly, the following algorithm consists of searching for a combination of contract offer distributions and transition parameters that most accurately reproduces the wage and transition patterns observed in data.

First, notice that from panel data such as the PME survey, one can estimate the cumulative probability of a worker moving from one employment status to another during a period T of time. Denote these estimates by $\hat{D}_{ss'}$ with $s, s' \in \{U, F, I\}$. Now notice that if we fix a pair $\{F_F, F_I\}$ we can compute the same cumulative probabilities of moving to another jobs or into unemployment predicted by the model using the following exponential cumulative distribution expressions to compare with the estimates $\hat{D}_{ss'}$:

$$D_{Us} = \frac{\lambda_{Us}}{\lambda_{UF} + \lambda_{UI}}(1 - e^{-(\lambda_{UF}+\lambda_{UI})T}) \ \text{, for } s \in \{I, F\} \qquad \text{[A.1]}$$

$$D_{sU} = \int_{\underline{V}_s}^{\overline{V}_s} \frac{\lambda_{sU}}{d_s(x)}(1 - e^{-d_s(x)T})dG_s(x) \ \text{, for } s \in \{I, F\} \qquad \text{[A.2]}$$

$$D_{ss'} = \int_{\underline{V}_s}^{\overline{V}_s} \frac{\lambda_{ss'}(1 - F_{s'}(V))}{d_s(x)}(1 - e^{-d_s(x)T})dG_s(x) \ \text{, for } s \in \{I, F\} \text{ and } s' \in \{I, F, U\} \qquad \text{[A.3]}$$

Secondly, take the set of transition parameters as given. Notice that despite not being able to guarantee that there exists a unique $\{F_F, F_I\}$ for each $\{G_F^*, G_I^*\}$ as we have pointed, the converse is indeed possible. Therefore for any guess $\{F_F, F_I\}$ we are able to compute the (unique) set of associated accepted wage distributions and compare them with the ones observed or estimated via data, which we denote by $\{\hat{G}_F^*, \hat{G}_I^*\}$.

With that in mind, the following algorithm consists of iterating these two steps until convergence[1]:

1. Fix a initial set of transition parameters $\Lambda^0 = \{\lambda_{UF}^0, \lambda_{UI}^0, \lambda_{FF}^0, \lambda_{FI}^0, \lambda_{IF}^0, \lambda_{II}^0, \lambda_{FU}^0, \lambda_{IU}^0\}$ and a pair of contract offer distributions $F^0 = \{F_F^0, F_I^0\}$.

2. Given Λ^0, compute the pair of sectoral contract offer distributions that minimizes the distance between the associated accepted wages generated by the model (using equations (3.4), (3.5) and (3.12)) and $\{\hat{G}_F^*, \hat{G}_I^*\}$. Denote this pair as F^1.

[1] Of course, we are assuming that this algorithm indeed converges.

3. Taking F^1 as given, compute the set of transition parameters that minimizes the distance between the transition probabilities generated by the model (equations (A.1), (A.2) and (A.3)) and their empirical counterparts $\hat{D}_{ss'}$. Denote this set of parameters as Λ^1.

4. If $F^0 = F^1$ and $\Lambda^0 = \Lambda^1$ stop. If not, return to step 1 updating F^1 and Λ^1 as the new set of initial guesses instead of F^0 and Λ^0.

In practice, one would need to discretize the set of initial guesses to search for the solution as well as impose a precision criterion for convergence. Relying on these pinned down objects we can proceed with the identification arguments for the remaining objects.

APPENDIX D - Φ Operator

$$\Phi(V) = \frac{\int_{V_U}^{V} A(x) e^{\int B(x)dx'} dx + c}{e^{\int_{V_U}^{V} B(x)dx'}}$$

where $A(V)$ and $B(V)$ are:

$$A = \lambda_{UI} F_I(V) \left(\frac{-\lambda_{IF} F_F'(V)}{d_I(V)} \right) + \lambda_{UF} F_F(V) \left(\frac{\lambda_{FI} F_I'(V)}{d_F(V)} \right)$$

$$B = \frac{\lambda_{IF} F_F'(V)}{d_I(V)} + \frac{\lambda_{FI} F_I'(V)}{d_F(V)}$$

APPENDIX E - Calibrated parameters

Table 7 – Calibrated parameters for the baseline economy

Parameters	Value	Comment
Transition Parameters		
λ_{UF}	0.0261	Estimated by Meghir et al. (2015)
λ_{UI}	0.1271	Estimated by Meghir et al. (2015)
λ_{FF}	0.0175	Estimated by Meghir et al. (2015)
λ_{FI}	0.0533	Estimated by Meghir et al. (2015)
λ_{FU}	0.0051	Estimated by Meghir et al. (2015)
λ_{IF}	0.0018	Estimated by Meghir et al. (2015)
λ_{II}	0.0725	Estimated by Meghir et al. (2015)
λ_{IU}	0.0146	Estimated by Meghir et al. (2015)
Production Functions		
η_f	0.20	Calibrated to allow for the use of the first-order conditions
η_i	0.15	
ν_f	240	
ν_i	300	
H distributions		
μ_f	1.0000	Calibrated to approximate unemployment rate in Meghir et al. (2015)
μ_i	-0.2714	
σ_f	0.9	
σ_i	3.31	
Additional Parameters		
r	0.005	Estimated by Meghir et al. (2015)
t	0.23	Estimated by Meghir et al. (2015)
τ	0.285	Estimated by Meghir et al. (2015)
ρ	0.1275	Estimated by Meghir et al. (2015)
b	-1308.03	Estimated by Meghir et al. (2015)
UI	1085.1655	Estimated by Meghir et al. (2015)
M/N_f	57.4	Calibrated to approximate firm size in Meghir et al. (2015)
M/N_i	9.56	Calibrated to approximate firm size in Meghir et al. (2015)

APPENDIX F - Additional simulation (reductions in ν_I)

The following tables show the results of the additional simulation we have made. This policy intervention consists of reducing the scale parameter of the informal production function from the baseline value $\nu_i^B = 300$ to 255 and then 210, which represent respectively $0.85\nu_i^B$ and $0.7\nu_i^B$.

Table 8 – Equilibrium worker and firms allocation by share (Appendix E)

	Baseline	$0.85\nu_i^B$	$0.7\nu_i^B$
Unemployment	0.066	0.066	0.065
Formal workers	0.367	0.370	0.385
Informal workers	0.567	0.564	0.550
Formal firms	0.132	0.153	0.177
Informal firms	0.868	0.847	0.823

Table 9 – Mean and percentiles of log-productivity distributions (Appendix E)

	Formal			**Informal**			**Overall**		
Percentiles	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$
1st	6.13	6.07	5.98	5.63	5.60	5.40	5.63	5.60	5.40
10th	6.34	6.30	6.33	5.75	5.72	5.56	5.75	5.72	5.73
25th	6.55	6.53	6.60	6.05	6.03	5.91	6.18	6.12	5.98
50th	6.90	6.89	6.98	6.34	6.32	6.24	6.44	6.41	6.38
75th	7.22	7.21	7.29	6.59	6.58	6.52	6.79	6.77	6.78
90th	7.55	7.54	7.62	6.96	6.95	6.91	7.10	7.11	7.19
95th	7.69	7.83	7.76	7.10	7.10	7.06	7.32	7.42	7.42
Mean									
$\mathbb{E}(\ln(p))$	6.92	6.92	6.94	6.36	6.34	6.25	6.43	6.43	6.37

Table 10 – Mean firm size and productivity. share of active firms and total output per worker (Appendix E)

	Baseline	$0.85\nu_i^B$	$0.7\nu_i^B$
Share of active (formal)	0,83	0,87	0,97
Share of active firms (informal)	0,91	0,80	0,75
Avg. firm size (formal)	25,50	24,54	22,86
Avg. firm size (informal)	5,95	6,74	7,02
Mean log-productivity (informal)	6,92	6,92	6,94
Mean log-productivity (informal)	6,36	6,34	6,25
Total output per worker	**3,12**	**3,14**	**3,23**

Table 11 – Mean. percentiles and inequality measures of log-wages (Appendix E)

	Formal			**Informal**			**Overall**		
Percentiles	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$	**Baseline**	$0.85\nu_i^B$	$0.7\nu_i^B$
1st	5.96	5.91	6.03	4.97	4.91	4.43	5.51	5.48	5.23
10th	6.50	6.47	6.41	5.86	5.83	5.67	6.11	6.09	5.97
25th	6.72	6.71	6.78	6.46	6.44	6.36	6.59	6.58	6.51
50th	6.98	6.98	7.04	6.70	6.69	6.63	6.80	6.79	6.84
75th	7.13	7.20	7.19	6.90	6.89	6.84	7.06	7.06	7.03
90th	7.27	7.27	7.32	7.07	7.07	7.03	7.13	7.20	7.19
95th	7.46	7.46	7.51	7.23	7.23	7.27	7.38	7.38	7.42
Mean									
$\mathbb{E}(\ln(w))$	6.75	6.76	6.78	6.42	6.40	6.33	6.55	6.54	6.52

Table 12 – Welfare measures (Appendix E)

	Baseline	$0.85\nu_i^B$	$0.7\nu_i^B$
Unemployed	417.38	421.66	435.58
Formal Workers	677.07	686.48	706.92
Informal Workers	502.00	503.25	504.10
Formal Firms	679.73	784.74	887.61
Informal Firms	274.64	269.51	271.53
Total welfare per worker	**965.87**	**1008.28**	**1068.50**