Mechanism Design and the Collective Approach to Household Behavior

Carlos E. da Costa, Lucas A. de Lima

URL: https://hdl.handle.net/10438/27929
Os artigos publicados são de inteira responsabilidade de seus autores. As opiniões neles emitidas não exprimem, necessariamente, o ponto de vista da Fundação Getulio Vargas.

EPGE Escola Brasileira de Economia e Finanças

Diretor Geral: Rubens Penha Cysne
Vice-Diretor: Aloisio Araujo
Coordenador de Regulação Institucional: Luis Henrique Bertolino Braido
Coordenadores de Graduação: Luis Henrique Bertolino Braido & André Arruda Villela
Coordenadores de Pós-graduação Acadêmica: Humberto Moreira & Lucas Jóver Maestri
Coordenadores do Mestrado Profissional em Economia e Finanças: Ricardo de Oliveira Cavalcanti & Joisa Campanher Dutra

E. da Costa, Lucas A. de Lima, Carlos
20p. - (Ensaios Econômicos; 808)
Inclui bibliografia.

CDD-330
Mechanism Design and the Collective Approach to Household Behavior*

Carlos E. da Costa               Lucas A. de Lima
EPGE-FGV                       Harvard University

August 6, 2019

Abstract

Do the Revelation and Taxation Principles hold for multi-person households? We provide a positive answer to the former and a negative to the latter if the household decision process is such that no mechanism can lead to inefficient choices. This unconditional version of Chiappori’s (1988) collective approach, offers a household model which can be used in a standard mechanism design approach. Keywords: Collective Approach; Revelation Principle; Taxation Principle. JEL Codes: D13; H21; H31.

1 Introduction

Chiappori’s (1988) highly cited work is representative of a paradigm shift that took place in households economics which replaced the hitherto dominant Unitary model by the so called Collective model of household behavior. By explicitly considering each spouses’ preferences and only restricting choices by the assumption of efficiency, Chiappori (1988) and a large literature that followed – e.g.,

*We thank Leandro Gorno for his invaluable comments, and Pierre-André Chiappori for provoking the discussion that lead to this paper. da Costa thanks CNPq project 301140/2017-0 and INCT for financial support. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. All errors are our responsibility.
Browning and Chiappori (1998); Cherchye et al. (2007); Bourguignon et al. (2009) – has derived testable restrictions for the model, successfully taken it to the data, and shown conditions under which identification is also possible in a Walrasian setting.

The next natural step for the Collective model is to prove its usefulness in addressing the important economic questions for which the Unitary model has fruitfully been applied. An important first step in this direction was taken by Gersbach and Haller (2001) which embed a collective model in a Walrasian economy and ask whether the first and second welfare theorems remain valid. The purpose of this paper is to do the same for the taxation and revelation principles, two essential results for mechanism design.

The Revelation Principle (RP) is, needless to say, the most important tool for characterizing the set of incentive feasible allocations, for any given physical environment. Helping transform theoretical insights into actual policy recommendations, the Taxation Principle (TP), guarantees that any incentive feasible allocation can be implemented with a suitably designed budget set. Do these two important principles which support the whole structure of normative analysis remain valid when we take into account multi-person households?

In this paper we answer this question using a collective approach to household behavior.1 What we are able to show is that, if we restrict household decision processes to those which lead to efficient decisions for any mechanism, then the revelation principle remains valid, but not the taxation principle.

2 What is a collective model?

At a fundamental level, if we respect the methodological individualism, we must start from understanding each person’s objectives. The collective approach recognizes this principle by endowing each household member with a well defined preference over consumption bundles for the household.

More explicitly, assume households are comprised of two spouses only, \( i = f, m \), that \( x_i \) is the spouse \( i \)'s bundle, and that \( x = (x_f, x_m) \) is the household

---

1 Browning et al. (2006) provide a typology for Collective models, based on the type of ‘environmental’ parameters that enter spouses relative weights. Here, we must go beyond this since we must endogenize the environment in which choices are made.
consumption bundle. Then we let \( U_i(x, \theta_i) \) represent spouse \( i \)'s preferences over these household bundles, and \( \theta_i \), a vector of personal characteristics that captures all heterogeneity across individuals. A couple is therefore identified by a vector \( \theta = (\theta_f, \theta_m) \).

Just endowing individuals with interdependent preferences does not yet define a couple. To do so, it is assumed that there is a joint stable decision process that maps the fundamentals of the economy to household outcomes.\(^2\) This is, of course, a minimum requirement for a behavioral model to have any empirical content, and could be justified in our context by a well defined bargaining protocol followed by the spouses.\(^3\)

Finally, the household decision process is restricted to those which lead to efficient outcomes. Recalling that the majority of studies using a Collective approach takes place in a market environment, efficiency is captured that households solve an optimization program of the form

\[
\max_{(u_f, u_m)} \mu u_f + (1 - \mu) u_m
\]

s.t.

\[
(u_f, u_m) \in \mathbb{U}(p, \theta),
\]

where \( p \) is a set of parameters that fully specifies the household utility possibility set, \( \mathbb{U}(p, \theta) \subset U \times U \), and \( \mu \in [0, 1] \) is a Pareto weight.

### 2.1 Conflict and Cooperation

The Collective approach aims at capturing the conflict/cooperation nature of spouses’ relationship. On the one hand it is assumed that spouses are able to agree on a common objective represented by (1). On the other the Pareto weight \( \mu \) encodes conflict of interests between spouses and, in principle, depends on a set of arguments, which defines how the surplus generated by a couple is to be split.

---

\(^2\)Absent externalities or public goods, outcomes which arise in a ‘competitive model’ where spouses’ choose non-strategically are compatible with the restrictions imposed in a collective approach. Although choice are as if spouses are price takers within the household – see Becker (1991) – this need not mean that decisions are not jointly made. We return to this model in Section 3.

\(^3\)See Browning et al. (2014), p. 110.
The collective approach is, however, agnostic about which variables should be \( \mu \)'s arguments and why.\(^4\) It has, almost invariably, been used to study household choices under pre-defined institutional settings, e.g., competitive goods and marriage markets, etc. In this case, parameters which characterize those institutional settings become natural candidate arguments of \( \mu \). This has allowed a pragmatic approach which dispenses with the necessarily more restrictive modeling of a formal household decision processes determining \( \mu \).\(^5\)

The difficulty raised by this general approach for our purposes is that mechanism design is all about defining the institutional setting. We have no candidate variables to serve as \( \mu \)'s arguments, instead we need to be explicit about the decision process that generates \( \mu \)'s arguments for each mechanism we consider.

### 2.2 Efficient Decision Processes

Although the collective approach is agnostic about the process leading to \( \mu \), we argue that implicit restrictions are imposed on the nature of these processes, since, under the collective approach not everything can be an argument of \( \mu \). According to Browning et al. (2006), an important restriction on Pareto weights to guarantee efficiency is that they should not depend on choice variables.

Allowing labor income, for example, to be a variable in \( \mu \) would lead to inefficient choices as the spouses could engage in wasteful behavior to maximize their Pareto weights. We make this point explicit on the next example.

**Example 2.1.** Let \( \bar{U}(p, \theta, y_f, y_m) \in U \times U \) denote the utility possibility set for spouses restricted to their generating exactly \( y_f \) and \( y_m \). In this case

\[
U(p, \theta) = \bigcup_{(y_f, y_m) \in Y(\theta)} \bar{U}(p, \theta, y_f, y_m),
\]

where \( Y(\theta) \) is the set of labor incomes that household \( \theta \) is capable of producing. Assume also for concreteness that \( \mu = \mu(y_f/y_m) \), and that spouses choose \( y_i \) non-cooperatively to maximize \( u_i \) in (1). Finally let, \( (y_f^*, y_m^*) \) be the Cournot-Nash equilibrium for this non-cooperative game.

\(^4\)In Browning et al.'s (2006) words, “Exactly which variables enter the Pareto weight should depend on an explicit underlying model of the decision process but mostly informal justification is given for the inclusion of one variable or another”.

\(^5\)In all that follows we use institutional setting and mechanism interchangeably.
Then the household solves

$$\max_{(u_f, u_m)} \mu(y_f^* / y_m^*) u_f + (1 - \mu(y_f^* / y_m^*)) u_m$$

(2)

s.t.

$$(u_f, u_m) \in \mathbb{U}(p, \theta, (y_f^*, y_m^*)),$$

Let $\mu^* = \mu(y_f^* / y_m^*)$. Now, consider a counterfactual world in which the household solves

$$\max_{(u_f, u_m)} \mu^* u_f + (1 - \mu^*) u_m$$

(3)

s.t.

$$(u_f, u_m) \in \mathbb{U}(p, \theta).$$

It is clear that the solution to (3) is never dominated by the solution to (2) and the solution to (3) will often dominate the solution to (2). This is the sense in which the presence of choice variables in $\mu(\cdot)$ may lead to inefficient outcomes. Note, however, that it is because $(y_f, y_m)$ affect both $\mu$ and $\mathbb{U}(p, \theta, (y_f^*, y_m^*))$ that inefficiency arises.

Let $\mu = \mu(p, d, d_o)$, where $d$ and $d_o$ are two sets of distribution factors, distinguished by the fact that $d$ is independent of spouses’ choices whereas, $d_o$ is not. As before, $p$ is a set of parameters which define the household utility possibility set.

Browning et al.’s argument suggests that choice variables should not enter $\mu$: the presence of $d_o$ as an argument in $\mu(\cdot)$ should be precluded. Yet, as the example above makes clear, this need not be the case, provided that $d_o$ does not affect $\mathbb{U}(p, \theta)$.

For our purposes it will be important to consider a more general representation for efficiency. Let $\mathbb{U}_c(\theta)$ bet the set of all utility pairs attainable by a couple, in an institutional setting $\mathcal{E}$, a function only of variables that are beyond the control of spouses. Let $\partial \mathbb{U}_c(\theta)$ be the frontier of $\mathbb{U}_c(\theta)$ and $\mathbb{U}_c^d(\theta)$, its interior. Household choices are efficient (resp. inefficient) if they lead to a pair $(u_f, u_m) \in \partial \mathbb{U}_c(\theta)$ (resp. $(u_f, u_m) \in \mathbb{U}_c^d(\theta)$).

The restriction which this imposes to study the questions that motivate this paper is that depending on the decision processes used by households a mechanism may be designed which leads to choices affecting both Pareto weights and the utility possibility set.
The next example makes this point clear.

**Example 2.2.** Assume that the share of non-labor income is a distribution factor. Consider first a mechanism in which wives are asked to make a report. Depending on the report either the wife or the husband will receive 100 monetary units. If power is affected by one’s share of total household non-labor income, the wife will choose to receive the money herself, but no inefficiency is created by her choice.

Consider now an alternative mechanism such that depending on the report made by the wife the household will be entitled to 100 monetary units to be received equally by the two spouses or 90 monetary units to be received by the wife only. Imagine now a mechanism in which only wives are asked to make a report. Depending on the report the household will be entitled to 100 monetary units to be received equally by the two spouses or 90 monetary units to be received by the wife only. A decision process at the confront stage which maximize equilibrium utility leads to non-efficient choices in this case.

To overcome the difficulty highlighted in Example 2.2, we specify a decision process imposing as little structure as possible, while at the same time capturing both the efficiency of household decision and the conflict in their preferences among all the feasible efficient choices.

In example 2.1 we did not make explicit the institutional environment, but simply assumed that, whatever it is, it leads the ratio of labor earnings to determine the specific pair \((u_f, u_m) \in \mathbb{U}(p, \theta, (y_f^*, y_m^*))\) which is chosen. The implicit assumption is that the underlying process is such that under the current institutional setting, \(y_f^*/y_m^*\) affects the relative power of spouses, thus leading to inefficiency.

Mechanism design is essentially choosing the institutional setting. In this sense, to guarantee efficiency is to make sure that a situation like the one in example 2.1 does not arise in any institutional setting: for all \(\mathcal{E}, (u_f, u_m) \in \partial \mathbb{U}_\mathcal{E}(\theta)\).

The question is whether we can restrict decision processes to be such that, under no mechanism inefficiencies should arise.

**Admissible Decision Processes** To keep the discussion well grounded, we frame our discussion within an example of a decision process that meets the requirements above: Nash’s (1953) two person bargain with endogenous threats.\(^6\)

Nash (1953) started with a normal form game \(G = (S_1, S_2; U_1, U_2)\) and asked what would happen if any player could either insist on an action \(s_i\) or or sign a

\(^6\)Our presentation of this model follows Abreu and Pearce (2015).
binding contract proposed by either player that, when signed by both, determines their utility. He considered both a strategic and an axiomatic analysis of the game and showed that both yielded the same unique solution. Here, we only present the axiomatic one.

Two important sets for Nash’s game are the set of payoffs which are feasible without cooperation \( \Pi = \text{co}\{U(s_1, s_2); (s_1, s_2) \in S_1 \times S_2\} \) and the set \( B \) of payoffs that are attainable through cooperation.\(^7\) For any nonempty, compact, convex \( B \subset \mathbb{R}^2 \) and threat points \( (u_1, u_2) \) the Nash bargaining solution is the pair of utilities solving \( \max_{(u_1, u_2) \in B} (u_1 - u_1)(u_2 - u_2) \), if there is any \( (u_1, u_2) \gg (u_1, u_2) \), and \( (u_1, u_2) \) if there does not exist such pair, \( (u_1, u_2) \).

Note how the model captures conflict in the choice of \( s_i \) and cooperation in the maximization of the Nash product.

If, spouse \( i \) can increase \( u_i \) by anticipating his or her labor income choice, \( y_i \). Then, when the joint decision arises, spouses will choose \( (u_f, u_m) \) from the restricted set \( U_c(\theta, (y_f^*, y_m^*)) \) possibly leading to \( (u_f, u_m) \in U_c(\theta) \).

The preceding discussion and examples 2.1 and 2.2 convey an important message. If an institutional setting is such that the choice situation which determines \( \mu \) can also affect the utility possibility set of agents, then one cannot guarantee efficiency of household choices. Hence, for there to be no mechanism under which choices are inefficient, we must rule out this possibility.

Toward this end we offer the following definition.

**Definition 2.1.** An economy is manipulable in equilibrium if the planner can induce in equilibrium the choice of variables under the control of spouses that affect the Pareto weight, \( \mu \).

There are two important qualifications here. First, we say that the planner can ‘induce in equilibrium’ to say that the planner has the instruments to make it optimal for spouses to actually make choices that determine this variable. The ‘actually’ part is crucial here. We need not preclude the manipulation of off-equilibrium choices which may \( \mu \).

In the Nash bargain example, it was crucial that spouses could make a decision regarding \( y_i \), before they reached an agreement. This is in contrast with the more commonly held view that spouses commit to a certain behavior *in case no agreement is reached*; something that never happens in equilibrium.

\(^7\) \( \text{co} \) stands for the convex hull of a set.
Second, the planner may still control variables that affect Pareto weights. In fact, policy can still affect both \( \mu \) and \( \mathbb{U}_E(\theta) \), but not through variables that are under spouses’ control.

**Proposition 2.1.** Under regularity conditions, the equilibrium outcome is efficient for any possible mechanism if and only if the economy is not manipulable in equilibrium.

*Proof.* Roughly, the planner can induce the same variable to show up on both utility and \( \mu \), by linking a variable that affects utility with the variable that affects \( \mu \), which exists because the economy is manipulable in equilibrium. \( \Box \)

In a sense, a variable that is manipulable in equilibrium produces information that the can be used to affect the utility possibility set of agents. Naturally, to preclude inefficient outcomes, the planner must not be able to use information produced under the choice situation determining \( \mu \) to control (or influence) choices made by spouses in equilibrium.

If choices at a conflict stage, in which \( \mu \) is defined, are off-equilibrium, then a mechanism can only use reports made at a point where spouses are maximizing a common objective. This is the main possibility for the assumption that the economy is not manipulable in equilibrium to be satisfied.

Note that it is not necessary for the planner to assess this information. Fundamentally, what matters is whether choices made in disagreement commits spouses to choice in agreement which affect their utility possibility set.

Based on the previous discussion, we restrict our analysis to decision processes in which: \( i \) there is an adversarial situation which determines \( \mu \); \( ii \) the adversarial situation is either previous to marriage or an off-equilibrium state, and; \( iii \) actual choices made by the household are efficient. Such decision processes guarantee that under no mechanism choices made in conflict can affect \( \mathbb{U}_E(\theta) \). The information produced off-equilibrium by a couple cannot be used by the planner to change the allocations for the specific couple in agreement. Of course the mechanism can still affect these off-equilibrium choices, but the latter cannot feedback into \( \mathbb{U}_E(\theta) \). Our assumption about the decision process has the immediate consequence of precluding the type of deviation from a collective model found in Example 2.2, and is similar to that used in *da Costa and de Lima* (2019).
Before we move to the main section of this paper, an important comment is due. The notion of efficiency is conditional on decisions made prior to marriage. This may not be the correct one, as changes in the mechanism may have an effect on previous decisions. For instance, changes in the mechanism may affect the education decision before marriage, and these choices should be taken into account. But, by the previous result, it seems essential to think conditional on past decisions to keep within the Collective Model framework.

3 Revelation and Taxation Principles?

Now, with decision processes restricted by the conditions in Section 2 does the revelation principle (RP) apply? How about the taxation principle (TP)?

To answer these questions we first specify the informational structure of a household model. We follow da Costa and de Lima (2019) in assuming that spouses from a \((\theta_f, \theta_m)\) couple know \((\theta_f, \theta_m)\) and whether the couple is in a situation of conflict or not. Neither is known by the planner.

Absence of asymmetric information does not guarantee efficiency neither is required for it, but it does make the case for efficiency more plausible. We start our discussion with a very general model, then, at the end of this section we discuss two relevant special cases often considered in the literature.

Given the decision process defined above, our next task is to characterize allocations chosen under a general mechanism. An important assumption we make is that, whatever the mechanism, it can only influence agents’ payoff through the bundles assigned.\(^8\) Choices can be made in agreement or (potentially) in conflict and we allow payoffs associated with each bundle to vary according to whether the couple is in agreement or in conflict. That is, although we maintain the assumption that the payoff that spouse \(i\) with characteristics \(\theta_i\) attains with household consumption \(x\) in agreement is \(u_i = U_i(x, \theta_i)\), we allow for a different mapping from bundles to final payoffs if the household is in conflict.

Allowing for different payoffs captures the misallocation of transferable goods across spouses, the under provision of public goods, the mis-allocation of time in household production, and other potential consequences of non-cooperative be-

\(^8\)That is, only the consequences of choices matter, not the process through which these choices are made.
behavior. Most importantly it allows us to include the important situation explored by Nash (1953), in which each spouse chooses with the specific purpose of influencing the Pareto weights, instead of increasing his or her disagreement utility, as in Example 2.2.

To represent these possibilities we use $w_i$ to denote the utility attained by spouses if choices are made in conflict and $u_i$ if they are made in agreement. A general mechanism ask for reports, $r_i \in \Sigma_i$, $i = f, m$, from both spouses and map these reports into some payoffs $(u_f, u_m)$, if spouses are in agreement and $(w_f, w_m)$ if they are in disagreement.\footnote{Of course the payoff associated with the report $(r_f^*(\theta), r_m^*(\theta))$ which solves (4), below, must be mapped into $(u_f(\theta), u_m(\theta))$.}

A mechanism, $\mathcal{M}$, is comprised of message spaces $\Sigma_f$ and $\Sigma_m$ with typical elements $r_f$ and $r_m$ and outcome function, $\gamma : \Sigma_f \times \Sigma_f \mapsto X \times X$, mapping reports into consumption bundles.

Assume that under the mechanism we evaluate, the Nash equilibrium for choices in conflict lead to a value $\mu^*$ for the Pareto weights, and let

$$x(r_f^*(\theta), r_m^*(\theta), \theta) = (x_f(r_f^*(\theta), r_m^*(\theta), \theta), x_m(r_f^*(\theta), r_m^*(\theta)))$$

be the allocation associated with the reports, $(r_f^*(\theta), r_m^*(\theta))$, that solve\footnote{We are assuming dominant strategy implementation for simplicity.}

$$\max_{r_f, r_m} \mu^* \mathcal{U}_f(x(r_f, r_m, \theta)) + (1 - \mu^*) \mathcal{U}_m(x(r_f, r_m, \theta)).$$

Then,

$$(u_f(\theta), u_m(\theta)) = \left(\mathcal{U}_f(x(r_f^*(\theta), r_m^*(\theta), \theta), \theta_f), \mathcal{U}_m(x(r_f^*(\theta), r_m^*(\theta), \theta), \theta_m)\right)$$

Can this allocation be implemented as a truthful equilibrium of a direct mechanism?

The type of a gender $i = f, m$ spouse in a $\theta$ couple is $\tau_i = (i, \theta, \{a, d\})$, where $a$ is the cooperative (agreement) situation and $d$ is the conflict (disagreement) situation. We assume that $i$ is public information, hence, in a direct mechanism, a report by such a spouse should be of the form $\hat{r}_i = (\hat{\theta}_f, \hat{\theta}_m, \iota)$ for $\iota \in \{a, d\}$. 

\begin{equation}
\max_{r_f, r_m} \mu^* \mathcal{U}_f(x(r_f, r_m, \theta)) + (1 - \mu^*) \mathcal{U}_m(x(r_f, r_m, \theta)).
\end{equation}
Assume that $\mu^* = \mu(\tilde r^f, \tilde r^m)$, where $\tilde r^f$ and $\tilde r^m$ are the reports that spouses would have given were they in conflict. That is, the reports that would arise as the Nash equilibrium of the disagreement game. Then, a direct mechanism must, at the same time, induce reports $(r^*_f(\theta), r^*_m(\theta))$ in agreement and $(\tilde r^f, \tilde r^m)$ in disagreement. Can this always be done?

Starting with choices in agreement, let us say that the mechanism has induced $\mu^*$ in the sense explained in the last paragraph. If each report $(\theta_f, \theta_m, \alpha)$ is associated with an allocation $\alpha(x^*_f(\theta), r^*_m(\theta), \theta)$ then the standard proof for the revelation principle remains valid.

The question is how can the same mechanism induce $\mu^*$? Under our assumption of a non-cooperative disagreement game, $(\tilde r^f(\theta), \tilde r^m(\theta))$ are the Nash equilibrium reports under the mechanism if household $\theta$ were to find itself in such state. Each report solves $\max_i w_i(r_i, \tilde r_{-i}, \theta)$. Can such payoffs be induced as a truthful equilibrium of a direct mechanism which requires reports $\tilde r_i = (\tilde r_f, \tilde r_m, \nu)$?

A complete characterization for the mechanism requires a specification for the outcome function. First, no mechanism directly defines $(w_f, w_m)$. Instead, some bundle $x(r_f, r_m) = (x_f(r_f, r_m), x_m(r_f, r_m))$, as a function of reports is assigned leading, and the latter is mapped into a pair $(w_f, w_m)$. In other words, for each $\theta$, there is a function $\omega(\cdot, \theta) = (\omega_f(\cdot, \theta), \omega_m(\cdot, \theta))$ such that for all $(r_f, r_m)$ and all $\theta$,

$$(w_f(r_f, r_m, \theta), w_m(r_f, r_m, \theta)) = (\omega_f(x(r_f, r_m, \theta), \omega_m(x(r_f, r_m, \theta))).$$

Define the outcome function for the direct mechanism as

$$\gamma(\theta, \nu, \theta', \nu') = (x_f(r_f(\theta, \nu), r_m(\theta', \nu')), x_m(r_f(\theta, \nu), r_m(\theta', \nu'))),$$

where $r^f(\theta, \nu) := \tilde r(\theta)$ if the couple is playing a non-cooperative game, while $r^f(\theta, \alpha) := r^*(\theta)$ if the choice is made by spouses in agreement.

Assume that $m$ from a $\theta$ couple is telling the truth in the non-cooperative game. Then, if there is an announcement $(\theta', \nu')$ which increases $f'$‘s utility it is because the report $r^f(\theta', \nu')$ which was available in the original mechanism, since chosen in equilibrium by spouse $f$ in couple $\theta'$ in state $\nu'$ would be chosen when $m$ chose $\tilde r_m(\theta) = r^m(\theta, d)$. A contradiction with $(\tilde r_m(\theta), \tilde r_m(\theta))$ being an equilibrium. That is, truth telling is an equilibrium if spouses are in the disagreement situation too.
As for the taxation principle, it is clear that it cannot be satisfied. The fact that different payoffs must be offered by couples in agreement and disagreement shows that a single budget set cannot handle implementation.

3.1 Specifying $w_i$

To complete our analysis we discuss the meaning of $w_i$, i.e. we explain by means of examples how $w_i$ is used in different models to capture spouses’ objectives in a disagreement situation. Beyond explaining the objective we must further specify how, for each mechanism, announcements are translated into choices and how these choices lead to $w_i$.

In Nash (1953) spouse’s objective in the conflict state is to maximize the utility they attain in the Nash product maximization. Nash (1953) assumes that the set of utilities which are attainable without cooperation, $\Pi$, is a subset of all which can be attained without cooperation, $B$. It is not clear whether this is due to restrictions in the set of feasible bundles or inefficient allocation of feasible bundles. Indeed, the mapping from bundles to payoffs plays no role in Nash (1953).

In contrast, Nash (1950) does not specify how threat points are determined. A great deal of da Costa and de Lima’s (2019) work is aimed exactly at specify how, for each mechanism, announcements are translated into choices and how these choices lead to $w_i$ using Nash’s (1950) framework. da Costa and de Lima (2019), assume that spouses’ lack the type of commitment required to sustain the threats devised by Nash (1953). Instead, they assume that, in disagreement, spouses maximize their disagreement utility. Mechanisms map reports into transactions and these are mapped into different levels of utility depending on whether spouses are in agreement or disagreement. That is, mechanisms affect payoffs only insofar as they determine the bundles to which spouses are entitled. The focus on transactions allows one to consider the presence of non-assignable goods, through a mapping from transaction to utility possibility sets. Utilities (the threat points in the Nash product) are given by a reduced form function mapping transactions into pairs of utility. The properties of such mapping are, arguably, representative of various sensible forms of non-cooperative distribution of household resources. In particular, they assume that, for any transaction, utilities generated by this function are lower than those generated in agreement.

Note that da Costa and de Lima’s (2019) mapping from transactions to threat
points is not in conflict with Nash’s (1953) approach. Spouse’s objectives in disagreement is. In Appendix A we borrow the threat point structure from da Costa and de Lima (2019) and apply it to Nash’s (1953) model.

4 Alternative Household Decision Models

Although Nash (1950) is the paradigm for cooperative choices involving conflicting objectives, other models are also compatible with efficient decisions by spouses who are in opposition to each other regarding the specific efficient outcome to choose.

4.1 Market approaches and ‘exogenous’ $\mu$

In Gayle and Shephard (2019) power is determined exclusively by the marriage market equilibrium, which is, in turn influenced by the tax system.

There, as in many other marriage market models, no individual can change the Pareto weights that he or she will be entitled to for any given type to whom he or she marries. This is true in Gayle and Shephard (2019) before or after the marriage market, e.g. schooling is taken as a demographic not a choice potentially impacted by the tax system. For our purposes, what matters for us to include it as a particular collective model is whether these variables that may enter the household program (1) both through $\mu$ and the utility possibility set are chosen prior or after marriage.

Assuming that all variables of this type are chosen prior to marriage markets open, then all arguments in $\mu$ are associated with some equilibrium aggregate choices which affect power, but which do not depend on the decisions of a specific individual. That is Pareto weights only depend on aggregate variables, of which we assume the planner to have statistic knowledge, are therefore ‘exogenous’ to spouses.

The Competitive Household Model

In the competitive model, due to Becker (1991); Grossbard-Shechtman (1993), a household is essentially a small open competitive economy, where each spouse
chooses his or her consumption bundles taking prices of both market and household goods as given. Absent public goods and externalities this leads to efficiency, as required by the collective approach.\footnote{An interesting aspect of this market model in the absence of public goods and externalities is that efficient outcomes do not require an appeal to symmetric information. Price taking behavior suffices. The problem is that public goods, like children, are in many cases the main reason for cohabitation. In this case, price-taking behavior alone will not guarantee efficiency, and some form of joint optimization will be needed. The choice of a common objective cannot be escaped in this case, and the assumption of a decision process leading to endogenous \( \mu \) may need to be used.}

To fully specify household choices, each spouse’s budget set must be specified, e.g., by assuming that they can use all their full income. Becker (1991) takes the view that it is the condition of marriage markets that define the share of household full income that goes to each spouse. In his words, “…bargain within marriages takes place in the shadow of competition in marriage markets…” In this sense, the competitive approach is closer in spirit to Gayle and Shephard’s (2019) marriage market model which we discuss in the next section.

More importantly, however appealing the competitive approach is in a market setting, absent public goods or externalities, it does not offer the right language to think about mechanism design. Indeed, although there are empirical benefits to think that spouses choose \textit{as if} in a competitive setting, it is hard to take seriously it as describing an actual decision process.

Note that both in Gayle and Shephard (2019) and in Becker (1991); Grossbard-Shechtman (1993) Browning et al.’s (2006) restriction that no choice variable enter \( \mu \), the ‘exogenous \( \mu \)’ case, is satisfied.

Because agents cannot ‘manipulate’ \( \mu \) through non-cooperative choices, the revelation principle is easily shown to hold using the preferences defined by (1). As for the taxation principle, if all variables in \( \mu \) are the result of aggregate equilibrium choices, then the taxation principle is also valid.

### 4.2 Kalai-Smorodinsky and Efficiency

Consider the case in which households choose through a bargaining process which solution satisfies Kalai and Smorodinsky’s (1975) axioms. The appeal of such a solution concept for the collective approach is, as Nash’s (1953) solution, assumption of Pareto efficiency.
In contrast with Nash (1953), the final outcome depends not only on threat points but also on the shape of the utility possibility set, which allows us to highlight how the specific way we define efficiency is important for establishing whether we can still call the household ‘collective’.

In example 2.1, we have defined two utility possibility sets, $\mathcal{U}_c(\theta, y_f, y_m)$ and $\mathcal{U}_c(\theta)$. If we consider that the relevant utility possibility set is the one after some decisions, e.g. labor supply decisions, $\mathcal{U}_c(\theta, y_f, y_m)$, then we fall under the conditions for Proposition 2.1 and the allocation can be inefficient, and we need to define how the couple arrive to the decision prior to the bargaining. On the other hand, if we consider that the relevant utility set is the one before all decisions, $\mathcal{U}_c(\theta)$ then we get an efficient outcome.

The Kalai-Smorodinsky solution can be modelled by adding two extra stages, one in which the husband has all the bargain power ($\mu = 0$), and another in which the wife has all the power ($\mu = 1$). And, with this assumption, no equilibrium variable affects their outcome.

What is important to keep from this discussion is that the notion of efficiency is tightly related to which variables affect the Pareto weights, and it can be hard empirically to disentangle situation where you have equilibrium choices affecting it, which can generate inefficient outcomes, from situations where out-of-equilibrium choices are the relevant ones, and efficiency is preserved. For instance, one of the appeals from the Kalai-Smorodinsky solution is to justify the use of full income in as a distribution factor in empirical works. This situation would allow for inefficient outcomes. But if we slightly alter this relation to say that the out-of-equilibrium full income is the one that affects the distribution factor, then we restore efficiency. And these two variables, equilibrium and out-of-equilibrium full income, should be closely correlated.

Another issue of having equilibrium variables affecting the Pareto weights is the need to specify how these variables are decided, on top of the out-of-equilibrium variables.

4.3 Further Considerations

Collective or Unitary? According to Browning et al.’s (2006) definition, what distinguishes empirically a collective model from an unitary model is choices made by a collective household cannot be rationalized as if they were made by
an individual. In a competitive market setting, the matrix of household compensated demands is not symmetric and negative semi-definite. Instead it is the sum of a symmetric, semi-definite matrix and a rank one matrix – the SR1 property in Browning and Chiappori (1998). Generically, this will take place whenever \( \mu \) depends on prices.

Following Browning et al. (2006), if \( \mu(p, d, d_0) = \tilde{\mu}(d) \) despite the existence of distribution factors, household choices may still be rationalized as if it were made by a single agent. It is therefore still a unitary model.

5 Conclusion

Recent developments in household economics point to the inadequacy of adopting a unitary view which simply endows households with well defined preferences and relies on revealed preference arguments to make welfare assessments.

Departing from rationality raises a host of novel difficulties for the use of mechanism design for characterizing the set of implementable allocations.

In this paper, we assume that spouses are rational and follow a well defined decision protocol to add structure to household choices. We also restrict our analysis to household decision processes which always, i.e., under any conceivable mechanism, respect the efficiency restrictions imposed by the collective approach. An alternative view is that the collective model refers not to a decision process itself but to a combination of household choices and institutional environments that lead to efficient outcomes. How much more can be said about implementation beyond de Clippel’s (2014) work which imposes only the existence of well define choice correspondence in this case? This is an interesting question we leave for future work.

References


A NB with endogenous threat points

In this appendix we take advantage of da Costa and de Lima’s (2019) setup to offer a fully specified version of the model explained in Section 3.

Da Costa and de Lima (2019), assume that whether in agreement or disagreement, the flow utility attained by spouses is a function of transactions $z = (z_f, z_m)$
conducted by the household. Transactions in agreement define utility possibility sets which, to guarantee that the conditions in Nash (1953) are always valid contains the utilities that can be attained with the same transactions in disagreement. To fully specify the disagreement game played by spouses, they define \( \Phi : Z \times Z \times \Theta \times \Theta \mapsto U \times U \), a function mapping spouses’ transactions \( z \) into a pair of disagreement utilities. Hence, for a \( \theta \) couple, transactions \( z \) leads to \((\bar{u}_f, \bar{u}_m)\) according to

\[
\begin{pmatrix}
\bar{u}_f \\
\bar{u}_m
\end{pmatrix} = \begin{pmatrix}
\phi_f(z_f, z_m, \theta) \\
\phi_f(z_f, z_m, \theta)
\end{pmatrix} = \Phi(z, \theta).
\]

The next important definition is what they call an institutional environment \( E \) or mechanism under which choices are made. Any institutional setting, \( E \), affects disagreement utilities by defining different choice spaces for spouses.

Let us start with a general mechanism, \( \mathcal{M} \), comprised of message spaces \( \mu^f \) and \( \mu^m \) and an outcome function mapping reports \((r^f, r^m) \in \mu^f \times \mu^m \) into transactions, \( z \).

Hence, spouses choose transactions through the report they send. The mapping from reports to threat points is the composite of the outcome function mapping reports \( r^f \in \mu^f, r^m \in \mu^m \) to transactions

\[
\begin{pmatrix}
z_f \\
z_m
\end{pmatrix} = \begin{pmatrix}
z_f(r_f, r_m) \\
z_m(r_f, r_m)
\end{pmatrix}
\]

and \( \Phi \), mapping transactions to threat points.

da Costa and de Lima (2019) focus on the case in which, in disagreement, spouses objective is to maximize disagreement utility, i.e., the threat points themselves. Here we consider Nash’s (1953) assumption, that spouses can commit to harm themselves if needed be to choose a threat point which maximizes their agreement utility.

To represent this objective, define

\[
\left( u^*_f(\bar{u}_f, \bar{u}_m), u^*_m(\bar{u}_f, \bar{u}_m) \right)
\]

as the pair of utilities that maximize the Nash product for threat points \( \bar{u}_f \) and \( \bar{u}_m \). Of course, this pair depends on the agreement utility possibility set as well,
but, under the assumptions we have made regarding the decision process, they are not affected by the choices made in disagreement.

Now we can define the relevant reaction functions,

\[
\begin{pmatrix}
  r_f(r_m) \\
  r_m(r_f)
\end{pmatrix}
= \begin{pmatrix}
  \arg\max_{r_f \in \mu_f} u_f^*(\phi_f(z(r_f, r_m), \theta), \phi_m(z(r_f, r_m), \theta)) \\
  \arg\max_{r_m \in \mu_m} u_m^*(\phi_f(z(r_f, r_m), \theta), \phi_m(z(r_f, r_m), \theta))
\end{pmatrix}
\]

An equilibrium for this disagreement game is a pair \((r_f^*, r_m^*)\) such that

\[
\begin{pmatrix}
  r_f^* \\
  r_m^*
\end{pmatrix}
= \begin{pmatrix}
  r_f(r_m^*) \\
  r_m(r_f^*)
\end{pmatrix}.
\]

Does a pure strategy equilibrium always exist? Existence depends not only on the properties of \(\Phi\) but on those of the mechanism itself. We shall simply assume it does, noting that, beyond an increase in notational burden, we could have instead considered a mixture over reports and used a mixed strategy equilibrium.

If \((r_f^*(\theta), r_m^*(\theta))\) is an equilibrium for the disagreement game for household then there is no deviation in the space of messages that increases the utility of one spouse if the other spouse does not change his or her report. Now, for all agents replace this general report by the direct report \(\{\theta, \iota\}, \iota \in \{a, d\}\). da Costa and de Lima (2019) show how the outcome function for this direct mechanism can be chosen in such a way that deviations from truth telling is optimal only if they were in the original mechanism reports that increased one spouse’s payoff holding the other spouse’s report fixed. This means that the original reports could not be a Nash equilibrium, thus proving the revelation principle.

As for the taxation principle, let \(\zeta_i^d(z^{-i})\) denote the transaction set for spouse \(i\) in the institutional setting \(E\), defined as a market allocation with a possibly non-linear unique budget set. If one’s spouse, \(-i\), has chosen \(z^{-i}\) and letting \(\bar{U}_E\) be the associated utility possibility set

\[
\bar{U}_E := \left\{ \left( \frac{\bar{u}_f}{\bar{u}_m} \right) \in U \times U \mid \exists z \in \zeta_i^d \text{ s.t. } \left( \frac{\bar{u}_f}{\bar{u}_m} \right) = \Phi(z, \theta) \right\}
\]
one can define the two reaction functions

\[
\begin{pmatrix}
  z_{E}^f(z_m) \\
  z_{E}^p(z_f)
\end{pmatrix}
= \begin{pmatrix}
  \arg \max_{z_f \in \Xi^d_{E}(z_m)} u_f^*(\phi_f(z_f, z_m, \theta), \phi_m(z_f, z_m, \theta)) \\
  \arg \max_{z_m \in \Xi^d_{E}(z_f)} u_m^*(\phi_f(z_f, z_m, \theta), \phi_m(z_f, z_m, \theta))
\end{pmatrix}
\]

Note that spouses now choose transactions, i.e., we restricted the space of messages to be the desired transactions. As forcefully put by da Costa and de Lima (2019), the main restriction this creates is that, the mechanism does not distinguish between agreement and disagreement choices. And despite not being the only restriction it suffices to invalidate the taxation principle.

Two important differences remain from da Costa and de Lima (2019) and this version of Nash (1953). First, even if choices in disagreement are made in a different set, because the objective depends on the set in which choices in agreement will be made, the existence of a fall back tax schedule does not retrieve the Unitary model.

Second. Because agents are only concerned with the agreement utility, the extreme punishment threat that was used for characterization need not work here.