The Role of Resource Misallocation in Structural Change*

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Abstract

We develop a multi-sector model of economic growth to explain the distinct patterns of structural change in manufacturing sectors in industrialized countries and those in developing and emerging industrial countries. To this end, we include a redistribution mechanism generated by a government because this mechanism affects an economy's relative prices and dynamic of resource allocation. This mechanism can imply changes in the production structure or the perpetuation of structures harmful to economic growth. We also demonstrate that sufficient conditions for the coexistence of structural change and unbalanced aggregate growth in this environment impose fewer restrictions on the elasticity of substitution between intermediate goods, the technological gap across sectors and the household's income effect. Moreover, using micro-founded parameters, our numerical results are consistent with the observed data from Brazilian industry.

Keywords: Economic development, Structural change, Resource misallocation.

JEL Classification: O10, L16, O41.

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1 Introduction

There are meaningful differences in the patterns of structural change across sectors when we compare developed and underdeveloped countries, as documented by McMillan and Rodrik (2011) and UNIDO (2013). The literature on structural change explains the path observed for aggregated sectors (agriculture, industry and services) using alternative explanations from the demand side (e.g., Kongsamut et al. (2001), Foellmi and Zweimuller (2008), and Boppart (2014)) and the supply side (e.g., Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008)). However, these studies do not explain the observed paths of structural changes without considering hierarchical preferences, substantial technological gap and sufficient substitutability across sectors. We propose adding to these models a factor that can fit the different patterns of structural change in industrial sectors: resource misallocation.

The main contribution of this paper is to demonstrate that the interaction between inputs, factor proportions and price distortions across firms leads to unbalanced aggregate growth, while remaining consistent with the observed data. To do so, we propose a multi-sector model of economic growth with structural change. First, in a standard multi-sector model, we introduce a non-rival public goods that increases firm's productivity (e.g., public infrastructure) and a government that can levy identical taxes on these firms to finance the public goods. We demonstrate that this design generates a redistribution mechanism that can change the resource allocation. Our definition of the redistribution mechanism is based on revenue taxation in order to supply a productive public goods but producers' demands for this good can be uneven. Any bias in this redistribution mechanism can then change the distribution of inputs. The pattern of structural change can thus differ between countries because of the resource misallocation implied by biased redistribution mechanism in each country.

Second, we demonstrate that paths of structural change can differ between countries due to the degree of resource misallocation among firms over time. Hsieh and Klenow (2009) suggest that inputs misallocated at the firm-level level affect aggregate productivity and output growth. Following these authors, using a multi-sector model, we introduce an ex ante distortion that affects nominal output and capital costs. These distortions are exogenous and vary across sectors. We assume that the producer's allocative choice depends on the technological level. However, there is an input distortion. The resource allocation would thus result from differences in the input marginal revenue products among firms over time. Although the first model contains an observable effect of misallocation, the extended model captures any

unobservable effect that it could have on relative prices over time (as financial frictions).

Our theoretical results yield an asymptotic equilibrium with an unbalanced growth path, while remaining consistent with the Kaldor facts. Our numerical results generate changes in sectoral input distribution that are comparable to the observed firm-level data. Moreover, the introduction of a measurement of resource misallocation produces relevant issues for economic policy design. Imagine a country with a high tax burden. In this illustration, independent of the technical progress or preferences, the sector that is intensive in public goods would experience the lowest loss over time, since overtaxed firms would lead to the public goods are overproduced. Furthermore, the high tax rate would also change the relative prices. These two effects influence a structural change. The economic transformation could be beneficial or harmful depending on the degree of resource misallocation. In parallel to these issues, it is well-known that inefficient policies and market failures represent big obstacles to efficient allocation in developing countries, which trigger resource misallocation.

Examining changes in the composition of production is an important aspect of the macroeconomic literature, and this paper overlaps with existing studies in several ways. If we compare the framework developed here with those of other studies, such as Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Boppart (2014), we see that our model accurately explains the observed patterns of structural change for three reasons. First, the framework proposed here does not impose any degree of substitutability between goods. With this, we include the general effects of intermediate goods on structural change and growth accounting (Jones, 2011).[1] Second, the asymptotic equilibrium of our model imposes fewer restrictions on the asymmetric technologies across sectors. For instance, unlike to Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007), our model does not require the that low-tech sectors and labor-intensive sectors be the same when the intermediate good sectors are complementary.[2] Third, we do not need to include income effects on structural change based

[1]It is important to understand the effects of the intermediate goods multiplier in the production structure and how these in turn affect the distribution of resources. There is also a strand of the literature that focuses on this effect of intermediate goods and the network effects between such goods (Basu, 1995; Jones, 2011). Moreover, our asymptotic equilibrium results are not dependent to the elasticity of substitution between intermediate goods, while our comparative statics results depend on.

[2]The asymptotic equilibrium conditions in usual models need to assume that the intermediate good sectors are complementary and, simultaneously, low-tech sectors and labor-intensive sectors are the same (Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008; Boppart, 2014). The empirical evidence in the macroeconomic literature suggests that the elasticity of substitution between the sectors is less than one, as computed by Krusell et al. (2000) among others, and consequently the assumption of complementary on the intermediate good sectors is evidence-based. Yet, we do not know final evidences about the other assumption.

on Engel's law (Boppart, 2014).[3] As Kongsamut et al. (2001), we believe that developing and emerging industrial countries are in the early stages of economic development such that the household's income effects are small within manufacturing goods. Furthermore, the distortion effect of government expenditures on structural change would be correlated to the degree of resource misallocation. Moreover, Hsieh and Klenow (2009) and Vasconcelos (2017) suggests that resource misallocation is greatest among some developing countries.

The rest of this paper is organized as follows: Section 2 outlines several empirical facts regarding differences in structural change between countries. Section 3 presents a theoretical model of the relationship between structural change and economic growth path. Section 4 presents an extended theoretical model that includes resource misallocation. The final section concludes this paper.

2 Stylized facts on structural change

In this section, we summarize two empirical facts regarding differences in structural change between countries. We focus on manufacturing sectors because these sectors tend to provide greater opportunities to accumulate capital, exploit economies of scale and acquire new technologies (Jones, 2011). The discussion uses data from two sources. The first source is the sector-level data from the Industrial Statistics Database (INDSTAT 2-digit 2015, ISIC Revision 3) developed by the United Nations Industrial Development (UNIDO). The second source is the firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym). Both databases contain annual time series data.

Following Acemoglu and Guerrieri (2008), we divide the sectors into two groups according to their labor intensity. In each sector-country, we treat the labor share as being equal to the wage and value added rate. We define the cut of the sectoral labor share as the average labor share by country.[4] The sectoral-level data exposed above are the average industry labor share by

[3] Engel's law suggests that as the household's income increases, the fraction of income that it spends on necessity goods declines. The household begins to demand other goods after the threshold of subsistence consumption is met (Ngai and Pissarides, 2007; Boppart, 2014).

[4] Other plausible cut-offs do not significantly affect the observed patterns. For example, we also use the average labor share between sectors and countries as a cutoff. However, the patterns of structural change exposed here remain observable and congruent, as suggested by McMillan and Rodrik (2011) and UNIDO (2013), among others.

country list, where each country share is weighted by the gross value added.[5] Moreover, we use the OECD technological classification according to ISIC Revision 3.1: high-tech, sectors with high R&D intensity; low-tech, sectors with low R&D intensity; and resource based, sectors with almost null R&D intensity. It is important to highlight that, although new countries entered the INDSTAT in the 1980s, the next observed facts remain valid.

See figure 1. For the period between 1963 and 2010, this figure shows the dynamics of the physical capital stock in labor-intensive sectors relative to the aggregate capital stock and the dynamics of the labor force allocated to labor-intensive sectors relative to the aggregate labor factor. In industrialized countries, inputs are moving towards labor-intensive sectors. However, inputs in the manufacturing sectors of developing and emerging industrial countries are moving towards non-labor-intensive sectors. Both patterns are also documented by UNIDO (2013). Furthermore, labor-intensive sectors are not necessarily high-tech sectors, so firms demand fewer resources to produce the same amount as firms from low-tech sectors (see table 2 for a further exposition of the Brazilian industrial case). Based on these facts, the distribution of resources differs between countries and does not necessarily depend on the technological gap across sectors.

The structural change facts only based on the previous sector-country aggregation can be biased and unbelievable. As in other industrial countries, using U.S. industry-level data, Acemoglu and Guerrieri (2008) report that there is structural change in favor of the labor-intensive sectors in U.S. manufacturing sectors. Moreover, using firm-level data from the Brazilian manufacturing sector for the period between 1996 and 2011, figure 2 exhibit the structural change in Brazilian industries. As in other developing and emerging industrial countries, inputs are moving towards the non-labor-intensive sectors in the Brazilian manufacturing sectors. Moreover, figure 2 show a fast structural change in capital distribution. With limitations in mind, facts 1 and 2 summarize these stylized facts on structural change.

Fact 1. *Inputs in the manufacturing sectors of industrialized countries are moving towards the labor-intensive sectors.*

Fact 2. *Inputs in the manufacturing sectors of developing and emerging industrial countries are moving towards the non-labor-intensive sectors.*

Facts 1 and 2 also suggest that in developing and emerging industrial countries, output in the manufacturing sectors with low labor intensity grow more than counterpart in sectors

[5] We use the country group list defined by the UNIDO. See the appendix for further details on the group list.

with high labor intensity. This additional feature does not depend to the technological levels of manufacturing sectors. This remark can be indicative of relative price misalignment and a signal of resource misallocation (Hsieh and Klenow, 2009; Vasconcelos, 2017). Moreover, the cross-industry and time-series evidence evaluated by the macroeconomic literature suggests that the intermediate good sectors are complementary, as evaluated by Krusell et al. (2000), and the observed technological difference within industries is not high (UNIDO, 2013). However, the macroeconomic literature does not explain the uneven patterns of structural change in environments where the intermediate good sectors are complementary, the technical gap is not substantial and without household's income effect, as discussed in Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Boppart (2014).

In the next section, seeking to explain facts 1 and 2, we develop a model with a redistribution mechanism generated by a government. The mechanism changes the dynamic of the resource allocation due to they influence the relative prices. We thus demonstrate that the previous patterns of structural change arise because of redistribution mechanisms that can imply resource misallocation.

3 Model

The model to be presented here relates structural change to the growth rate of aggregate output. We follow the theoretical characterization developed by Acemoglu and Guerrieri (2008). However, we include an agent that acts as a provider of a public goods and a collector of taxes: the government. On one hand, public goods increases firms productivity; on the other hand, to finance the public goods, the government taxes intermediate producers at a common tax rate. Given that the producers' demand for public goods are distinct but taxation is not, this design influence the redistribution of resources between firms. This redistribution can then change relative prices and, in turn, the efficient allocation of resources.

3.1 Environment

Consider an economy with a finite horizon and continuous timing. This economy has a continuum of individuals with standard preferences concerning the consumption of a final good,

and the utility function is

$$U\left(C_t, t\right) = \int_0^{\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} \exp\left(-\rho t\right) dt, \tag{1}$$

where C_t is the aggregate consumption over period t, $\rho \in (0,1)$ is the rate of time preference, and $\theta \in [0,\infty)$ is the coefficient of risk aversion. Again, like Kongsamut et al. (2001), we believe that developing and emerging industrial countries are in the early stages of economic development such that income effects tend to be small enough to be negligible. We thus do not address the assumption of non-homothetic preferences, as in Boppart (2014).

In this economy, a final good Y_t is produced using two intermediate goods, X_{1t} and X_{2t}, which the production functions are given by

$$Y_t = \left[\gamma X_{1t}^{\frac{\epsilon-1}{\epsilon}} + \left(1-\gamma\right) X_{2t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $\epsilon \in (0,\infty)$ is the degree of substitutability between intermediate goods and $\gamma \in (0,1)$ is the proportion of inputs used. Therefore, we need a positive amount of each intermediate good to produce one unit of the final good. Assume that homogeneous firms compose each sector. The production function of the intermediate goods of each sector $s = \{1,2\}$ is

$$X_{st} = M_{st} K_{st}^{1-\alpha_s-\beta_s} L_{st}^{\alpha_s} G_t^{\beta_s}, \tag{3}$$

where $\alpha_s \in (0,1)$ and $\beta_s \in (0,1)$ are the proportions of inputs such that $\alpha_s + \beta_s < 1$. Furthermore, G_t is the public goods; K_{st} is the physical capital; L_{st} is the labor force; and M_{st} is the technological level. The public goods are time-variant and non-rivalrous between firms. Following Slemrod and Gillitzer (2013), when we refer to the public goods, we describe current public expenditures that increase the marginal efficiency of inputs, such as public education, infrastructure and health insurance.

The assumptions on equation 3 imply two features in our model. First, producers take the public goods input as given, the firm's technology thus is of decreasing returns. Second, higher parameter β_s, greater is the decreasing returns in firms from sector s. These features imply that the decentralized market will not be dynamically Pareto optimal and resource misallocation can emerge in an economic environment where there are the producers' demands for public goods. Moreover, if $\alpha_1 = \alpha_2$, then firms in different sectors will have the same labor intensity.

However, if $\beta_1 \neq \beta_2$, firms in different sectors will differ in their capital intensity and demand for the public goods. If $\alpha_1 \neq \alpha_2$ and $\beta_1 = \beta_2$, then this environment would be similar to that constructed by Acemoglu and Guerrieri (2008), and the introduction of public expenditure represents a decomposition of a technological measure. We thus propose assumption 1.

Assumption 1. $\alpha_1 > \alpha_2$ *and* $\beta_1 \neq \beta_2$.

Assumption 1 implies that sector 1 is labor intensive, sector 2 is non-labor intensive and the producers' demands for the public goods differ. We then have six possible states of nature. Assume that intermediate goods are complementary, $\epsilon \leq 1$. First, if $\beta_1 > \beta_2$, then sector 1 will be intensive in the public goods and sector 2 will be capital intensive. Second, if $\alpha_1 - \alpha_2 < \beta_2 - \beta_1$, then sector 2 will be capital intensive and intensive in the public goods. If $\alpha_1 - \alpha_2 > \beta_2 - \beta_1 > 0$, then sector 1 will be capital intensive, and sector 2 will be intensive in the public goods. This final case would refer to firms from sectors with a high demand for public infrastructure, for example. Hence, these sectors have a high β. Assuming that the intermediate goods were substitutes, $\epsilon > 1$, we would also have the three possibilities above.

The technological progress of each sector $s = \{1, 2\}$ is

$$\frac{\dot{M}_s}{M_{st}} = m_{st}, \tag{4}$$

where m_{st} is exogenous and has a stochastic process. The sectoral resource constraints are

$$K_{1t} + K_{2t} \leq K_t, \tag{5a}$$

$$L_{1t} + L_{2t} \leq L_t, \tag{5b}$$

where K_t is the aggregate capital and L_t is the aggregate labor in each period. The growth rate of the labor force is exogenous and is $n \in (0, \infty)$. The resource constraint is standard and given by

$$\dot{K} + \delta K_t + C_t \leq Y_t, \tag{6}$$

where $\delta \in [0, 1)$ is the depreciation rate of the physical capital stock. Here, the public goods increases firms' current productivity and is fully availed in each period. Hence, the public goods influences the resource constraint only through changes in the final good.

We have a government that can levy identical taxes on the intermediate producers to finance the public goods. This tax rate is $\tau \in (0, 1)$ and assessed on the nominal product of the intermediate goods and is sector- and time-invariant. The government budget constraint is thus

$$G_t \leq \tau P_{1t} X_{1t} + \tau P_{2t} X_{2t}, \tag{7}$$

where P_{st} is the price of the intermediate good of each sector in each period. It is important to highlight that the focus is to understand forces that can drive structural change. Although a micro-founded evaluation of the effects of economic policies on structural change is an important avenue for future research, we do not study it here. To do so, we take τ as given and derive G_t in equilibrium, provided that $\beta_1 \neq \beta_2$.

3.2 Decentralized equilibrium

The determination of equilibrium will be divided into two parts. First, we solve the static problem. In each period, the producers maximize the value of their profits subject to input constraints. These producers then determine their input allocation, given the technological level. Assuming that the government budget constraint is binding, we endogenously obtain the quantity of the public goods. Second, we solve the dynamic problem. We maximize an objective function such that the dynamics of future capital and of current consumption are chosen according to the optimal allocation of inputs in each period.

3.2.1 Static equilibrium

The producer's problem of the final good determines the amounts of intermediate goods X_{1t} and X_{2t} demanded. We define the final good price as the numeraire. Solving the problem above, we obtain the (inverse) demands for the intermediate goods:

$$P_{1t} = \gamma \left(\frac{X_{1t}}{Y_t} \right)^{-\frac{1}{\epsilon}}, \tag{8a}$$

$$P_{2t} = (1 - \gamma) \left(\frac{X_{2t}}{Y_t} \right)^{-\frac{1}{\epsilon}}. \tag{8b}$$

Given assumption 1, equations 8a and 8b imply lemma 1.

Lemma 1. *The relative price is such that*

$$\frac{P_{1t}}{P_{2t}} = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{X_{1t}}{X_{2t}}\right)^{-\frac{1}{\epsilon}}.$$

Lemma 1 proposes that the relative price is time-variant and is an implicit function of the technological gap across sectors, the resource distribution, and the realized use of the public goods. This lemma suggests that the tax rate influences the relative prices because the revenue from this tax finances the public goods, given that the producers' demands for the public goods differ.

In each sector $s = \{1, 2\}$, intermediate goods firms choose inputs K_{st} and L_{st} to maximize their profits

$$(1-\tau)P_{st}X_{st} - R_tK_{st} - W_tL_{st},$$

given P_{st} for each sector and τ for both. Suppose that there are no financial and labor frictions across firms. The competitive environment thus ensures that the cost of labor (W_t) and the cost of capital (R_t) do not differ between firms in equilibrium. We relax this assumption in the next section. Given this setup and equations 8a and 8b, the marginal returns equalize within sectors $s = \{1, 2\}$ in each period, and this condition implies that

$$\frac{K_{st}}{L_{st}} = \frac{1-\alpha_s-\beta_s}{\alpha_s}. \tag{9}$$

In turn, the value of the marginal return of each input equalizes across sectors in each period. We define $\kappa_t \equiv K_{1t}/K_t$ as the share of aggregate capital allocated to the labor-intensive sector, $\kappa_t \in (0, 1)$, and $\lambda_t \equiv L_{1t}/L_t$ as the share of the aggregate labor force allocated to the labor-intensive sector, $\lambda \in (0, 1)$, such that the optimal allocation of labor across sectors imply

$$\lambda_t = \left[1 + \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{1-\alpha_1-\beta_1}{1-\alpha_2-\beta_2}\right)\left(\frac{1}{\kappa_t} - 1\right)\right]^{-1}. \tag{10}$$

The parameter λ is strictly increasing in κ, which implies that capital and labor are reallocated towards the same sector over time. We define $k_t \equiv K_t/(L_tM_{1t}^{1/\alpha_1})$ as the aggregate capital-to-labor ratio normalized by the augmented technology of the labor-intensive sector, $k \in (0, \infty)$. Although the sectoral capital-to-labor ratio is not time-variant, we can obtain that the aggregate capital-to-labor ratio can be time-variant. We define $\phi_t \equiv G_t/K_t$ as the ratio of the public goods to the aggregate capital, $\phi \in (0, \infty)$. Given assumption 1, lemma 1 and the

above-defined variables, the optimal allocation of capital across sectors imply lemma 2.

Lemma 2. *The output gap between sectors is*

$$\frac{X_{1t}}{X_{2t}} = \left[\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1-\alpha_1-\beta_1}{1-\alpha_2-\beta_2}\right)\left(\frac{1}{\kappa_t}-1\right)\right]^{\frac{\epsilon}{1-\epsilon}}.$$

Lemma 2 suggests that the output gap between sectors results primarily from differences in the resource allocation and implicitly from differences in the technological level and the demand for the public goods. Given the technical gap across sectors and the uneven demands for the public goods, the ratio X_1/X_2 is strictly decreasing in κ when $\epsilon < 1$, which implies that production move away from the labor-intensive sector to non-labor sector. The opposite is true when $\epsilon \geq 1$.

For equations 2, 7, 8a and 8b, we obtain that

$$G_t = \tau Y_t. \tag{11}$$

The ratio of the public goods to the final good is thus not time-variant and exogenous in equilibrium. Finally, we define the static equilibrium below.

Definition 1. *In each period, the static partial equilibrium defines a system of prices* $\{R_t, W_t, P_{1t}, P_{2t}\}$ *and a feasible allocation* $\{L_{1t}, L_{2t}, K_{1t}, K_{2t}\}$ *such that firms maximize their earnings, the public goods are produced, and markets clear.*

3.2.2 Comparative statics analysis

According to equation 11, a time-invariant tax rate on the nominal output of the intermediate goods or the final good would generate the same provision of the public goods. Moreover, G/Y cannot be time-variant when G/K and K/Y are time-variant on the dynamic transition path.[6] This condition implies that structural change will have ambiguous effects on the provision of the public goods in the dynamic transition path. We rewrite the ratio between the public goods

[6] If K/Y is time-variant in long-run equilibrium, the economy would tend towards a trajectory of explosive growth or slow growth such that it is not consistent with the Kaldor facts. In this case, G/K would be constant in long-run equilibrium.

and aggregate capital as

$$\begin{aligned}\phi_t &= \tau\Bigg[\gamma\left(\kappa_t^{1-\alpha_1-\beta_1}\lambda_t^{\alpha_1}k_t^{-\alpha_1}\phi_t^{\beta_1}\right)^{1-\frac{1}{\epsilon}} \\ &+ (1-\gamma)\left(M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}(1-\kappa_t)^{1-\alpha_2-\beta_2}(1-\lambda_t)^{\alpha_2}k_t^{-\alpha_2}\phi_t^{\beta_2}\right)^{1-\frac{1}{\epsilon}}\Bigg]^{\frac{\epsilon}{\epsilon-1}}. \qquad (12)\end{aligned}$$

In proposition 1, we describe the relationship between the share of aggregate capital that is allocated to the labor-intensive sector and the ratio between the public goods and aggregate capital. See the appendix for further details on this proposition.

Proposition 1. *If κ increases over time, we obtain*

$$\frac{\partial\phi_t}{\partial\kappa_t}\begin{cases}>0, & \text{if } \phi_t > \left[\tau k^{*-\alpha_1}\left(\beta_1\gamma\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{1-\beta_1}}, \\ \leq 0, & \text{otherwise}.\end{cases}$$

If κ decreases over time, we obtain

$$\frac{\partial\phi_t}{\partial\kappa_t}\begin{cases}>0, & \text{if } \phi_t < \left[\tau M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}k^{*-\alpha_2}\left(\beta_2(1-\gamma)\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{1-\beta_2}}, \\ \leq 0, & \text{otherwise}.\end{cases}$$

Proposition 1 implies that a structural change might accompany a change in the role of the public goods. The process of economic growth can imply that the growth in the provision of the public goods occurs at the lower levels of this input and the reduction in the provision of the public goods occurs at the higher levels of this input. Figure 3 illustrates this proposition and possible structural change paths based on certain possible states of nature. For example, imagine an economy with poor public infrastructure. The structural change in production that accompanies economic development can influence the growth in the provision of both goods. In the development process, this economy will tend to demand less growth in the provision of the public goods. This transition can imply changes in the production structure or the perpetuation of structures harmful to economic growth. A key point is that the role of public expenditure depends on the technological level, the demand for the public goods, and economic development.

It is important to highlight that this model assumes exogenous restrictions in the policy space, such as linear taxes, and ignores a distortion observed in real-world political processes:

policymakers have an incentive to tax minorities to benefit the majority. In developing and emerging industrial countries, high-tech firms are minorities. The effect of a technological gap across sectors on resource allocation can thus be altered, depending on the degree of substitutability between goods and the power of the political process. With this issue in the mind, the static equilibrium suggests propositions 2 and 3. See the appendix for further details on these propositions.

Proposition 2. *If* $\epsilon < 1$,

$$\frac{\partial \kappa_t}{\partial M_{2t}/M_{1t}} > 0.$$

The opposite is true when $\epsilon \geq 1$.

Proposition 3. *If* $\epsilon < 1$,

$$\frac{\partial \kappa_t}{\partial \tau} \begin{cases} < 0, & \textit{if} \quad \beta_1 > \beta_2 \\ \geq 0, & \textit{otherwise}. \end{cases}$$

The opposite is true when $\epsilon \geq 1$.

Proposition 2 is similar to that obtained by Acemoglu and Guerrieri (2008). If sectors have uneven technological levels and the intermediate goods are complementary, then the fraction of total inputs allocated to the high-tech sector decreases. As desired, efficient firms use fewer resources to produce the same amount. The inputs can thus be reallocated to firms in the low-tech sector. Note that the illustration presented in figure 3 remains valid should the technological gap across sectors increase or decrease. The change in the role of the public goods depends on the technological gap across firms.

Although our goal is not evaluate economic policies, proposition 3 demonstrates the effect of the tax rate change in resource allocation. If intermediate goods sectors were complementary, tax growth would accelerate the reallocation of inputs (capital and labor) in the sector with lower demand for the public goods. This phenomenon would occur because the tax rate finances the public goods, which implies the same effect as a sector-specific technological shock. The sector with greater demand for the public goods becomes more efficient and requires fewer inputs. However, despite the presence of the effects described above, we will have a substantial redistributive effect on the prices of intermediate goods if the goods are substitutes because the relative increase in sectoral output also affects the relative prices. In this case, the increased

tax rate can cause the inputs to move towards the sector with greater demand for the public goods. The tax rate can thus influence the distribution of inputs, depending on the degree of substitutability between goods and the sector's demand for the public goods.

3.2.3 Dynamic equilibrium

We define the dynamic equilibrium below.

Definition 2. *In the competitive equilibrium, households' choices are* $\{C_t, K_{t+1}\}_{t=0}^{\infty}$*, and they maximize their utility over time subject to a system of prices* $\{R_t, W_t, P_{2t}, P_{2t}\}_{t=0}^{\infty}$*, a feasible allocation* $\{L_{1t}, L_{2t}, K_{1t}, K_{2t}\}_{t=0}^{\infty}$*, and the provision of the public goods* $\{G_t\}_{t=0}^{\infty}$*. We thus maximize equation 1 and restrict equations 2, 3, 4, 5, 6, and 7 for a given list of initial conditions* $\{\kappa_0, \lambda_0, K_0, G_0, L_0, M_{10}, M_{20}\}_{t=0}$*.*

Because the utility function is continuous and strictly concave, and the restrictions are convex, the equilibrium has a unique solution that corresponds to a unique competitive equilibrium. The households' first-order conditions imply that the intertemporal condition is given by

$$\frac{\dot{C}}{C_t} = \frac{1}{\theta}\left[\gamma(1-\alpha_1-\beta_1)\left(\frac{X_{1t}}{Y_t}\right)^{-\frac{1}{\epsilon}}\frac{X_{1t}}{K_{1t}} - \delta - \rho\right].$$

As in Acemoglu and Guerrieri (2008), we define the consumption per worker normalized by the augmented technology of the labor-intensive sector as $c_t \equiv C_t/(L_t M_{1t}^{1/\alpha_1})$ and obtain

$$\frac{\dot{c}}{c_t} = \frac{1}{\theta}\left[\gamma\left(1-\alpha_1-\beta_1\right)\left(\frac{\lambda_t^{\alpha_1}\phi_t^{\beta_1}}{\eta_t^{\frac{1}{\epsilon}}\kappa_t^{\alpha_1+\beta_1}k_t^{\alpha_1}}\right) - \delta - \rho\right] - n - \frac{m_{1t}}{\alpha_1}, \tag{13}$$

where $\eta_t \equiv X_{1t}/Y_t$. Note that ϕ is implicitly given by equation 12 as a function of the list of parameters $\{\tau, \beta, \epsilon\}$ and the list of variables $\{\kappa, c, k\}$. Furthermore, market clearing implies that $C = Y$ such that the dynamic of ϕ is

$$\frac{\dot{\phi}}{\phi_t} = \frac{\dot{c}}{c_t} - \frac{\kappa_t^{1-\alpha_1-\beta_1}\lambda^{\alpha_1}\phi_t^{\beta_1}}{\eta_t k_t^{\alpha_1}} + \frac{c_t}{k_t} + \delta. \tag{14}$$

The ratio of the public goods to aggregate capital thus depends on the resource allocation, the output per worker and the aggregate capital-to-labor ratio.

Using the dynamics of aggregate capital and the definition of k_t, we obtain

$$\frac{\dot{k}}{k_t} = \frac{\kappa_t^{1-\alpha_1-\beta_1}\lambda^{\alpha_1}\phi_t^{\beta_1}}{\eta_t k_t^{\alpha_1}} - \frac{c_t}{k_t} - \delta - n - \frac{m_{1t}}{\alpha_1}, \tag{15}$$

and using the definition of κ,

$$\frac{\dot{\kappa}}{\kappa_t} = \Omega\left(\kappa_t\right)\left[m_{2t} - \frac{\alpha_2}{\alpha_1}m_{1t} + \left(\alpha_1 - \alpha_2\right)\frac{\dot{k}}{k_t} - \left(\beta_1 - \beta_2\right)\frac{\dot{\phi}}{\phi_t}\right], \tag{16}$$

where

$$\Omega(\kappa_t) = \frac{-(1-\kappa_t)}{1/(\epsilon-1) + \beta_1 + \left(\alpha_1 - \alpha_2\right)\left(\lambda_t - \kappa_t\right) - \left(\beta_1 - \beta_2\right)\kappa_t}.$$

The transversality condition is

$$\lim_{t\to\infty} k_t \exp\left[-\left(\rho - n - (1-\theta)\frac{m_{1t}}{\alpha_1}\right)t\right] = 0. \tag{17}$$

Equations 10, 12, 13, 15, 16, and 17 determine the dynamic equilibrium path. The first represents the evolution of the share of labor allocated to the various sectors over time, and the second is the ratio of the public goods to aggregate capital. The third is the standard Euler equation. The fourth is the law of motion of the aggregate capital-to-labor ratio normalized by the augmented technology of the labor-intensive sector. The fifth is the evolution of the share of capital allocated to the various sectors over time. The last equation represents the standard transversality condition. These equations are based on assumptions 1, 2 and 3.

Assumption 2.

$$\frac{m_{1t}}{\alpha_1} \leq \frac{\rho - n}{1-\theta}$$

when t approaches infinity.

Assumption 3.

$$\frac{m_{1t}}{\alpha_1} < \frac{m_{2t}}{\alpha_2}$$

when t approaches infinity.

Assumption 2 implies that the No-Ponzi condition is satisfied. Assumption 3 is a result of the asymptotic stability of differential equations. See the appendix for further details on assumption 3. According to this assumption, one intermediate sector asymptotically dominates, given the technical gap across sectors and the uneven demands for the public goods. The

asymptotic dominance does not necessarily favor high-tech firms. This suggestion is congruent with propositions 2 and 3. Unlike those in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008), here the asymptotic stability of equilibrium equations does not depend on the elasticity of substitution between the sectors.

Under assumptions 1, 2, and 3 and given the initial conditions $\{k_0, \kappa_0, \phi_0, c_0\}$, the solutions to the dynamic problem satisfy the system of differential equations 13, 14, 15 and 16, and λ is given by equation 10.

3.2.4 Constant Growth Path

Definition 3. *The Constant Growth Path (CGP) is a dynamic competitive equilibrium characterized by constant growth in the aggregate key variables.*

With definition 3 and given assumptions 1, 2, and 3, the CGP implies proposition 4.

Proposition 4. *There is a unique stable CGP and a unique CGP implied by equations 10, 12, 13, 15, 16, and 17 such that*

$$\frac{\dot{c}}{c_t} = \frac{\dot{k}}{k_t} = 0.$$

From proposition 4, we obtain

$$\frac{\dot{C}}{C_t} = n + \frac{m_1^*}{\alpha_1} = \nu, \tag{18}$$

where we define m_1^* as the technological progress in the labor-intensive sector in the asymptotic equilibrium and $x^* = \lim_{t\to\infty} x_t$ such that x is a key variable. If $\kappa^* = 1$, aggregate capital is allocated towards the labor-intensive sector. Because capital and labor move in the same direction, $\kappa^* = 1$ implies that $\lambda^* = 1$, and the labor force moves towards the labor-intensive sector. The opposite scenario is true when $\kappa^* = 0$. These cases occur only at the asymptotic limit because the production of the final good requires a positive quantity of each intermediate good. There can be $\kappa^*, \lambda^* \in (0, 1)$ such that there would be a structural change in which one sector sufficiently dominates, but this sector does not fully dominate the economy. Without loss of generality, we present here the theoretical case in which $\kappa^* = 1$.

The definition of the aggregate capital-to-labor ratio implies that

$$k^* = \frac{1 - \alpha_1 - \beta_1}{\alpha_1}, \tag{19}$$

and using equations 13, 15, and 19, we obtain

$$c^* = \left(\frac{\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho}{\alpha_1}\right) + (\delta + n)\left(\frac{1 - \alpha_1 - \beta_1}{\alpha_1}\right), \tag{20}$$

$$\phi^* = \left(\frac{\gamma^{\frac{\epsilon}{1-\epsilon}}\left(\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho\right)}{1 - \alpha_1 - \beta_1}\right)\left(\frac{1 - \alpha_1 - \beta_1}{\alpha_1}\right)^{\alpha_1}. \tag{21}$$

Equations 19 and 20 determine the aggregate capital-to-labor ratio and the consumption per worker in the asymptotic equilibrium. Equation 21 determines the ratio between the public goods and aggregate capital in the asymptotic equilibrium. Proposition 5 summarizes the asymptotic CGP. See the appendix for further details on this proposition.

Proposition 5. *Under assumptions 1, 2 and 3, there exists a non-trivial asymptotic CGP such that*

$$\begin{aligned}
\kappa^* &= \lambda^* = 1, \\
k^* &= \frac{1 - \alpha_1 - \beta_1}{\alpha_1}, \\
c^* &= \left(\frac{\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho}{\alpha_1}\right) + (\delta + n)\left(\frac{1 - \alpha_1 - \beta_1}{\alpha_1}\right), \\
\phi^* &= \left(\frac{\gamma^{\frac{\epsilon}{1-\epsilon}}\left(\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho\right)}{1 - \alpha_1 - \beta_1}\right)\left(\frac{1 - \alpha_1 - \beta_1}{\alpha_1}\right)^{\alpha_1}, \\
\frac{\dot{Y}}{Y_t} &= \frac{\dot{K}}{K_t} = \frac{\dot{X}_1}{X_{1t}} = \frac{\dot{K}_1}{K_{1t}} = n + \frac{m_1^*}{\alpha_1} = \nu, \\
\frac{\dot{X}_2}{X_{2t}} &= \nu + m_2^* + \frac{m_2^* - \alpha_2 m_1^*/\alpha_1}{1/(\epsilon - 1) + \beta_1} \lesseqgtr \nu, \\
\frac{\dot{K}_2}{K_{2t}} &= \nu + \frac{m_2^* - \alpha_2 m_1^*/\alpha_1}{1/(\epsilon - 1) + \beta_1} < \nu, \\
\frac{\dot{L}_1}{L_{1t}} &= \frac{\dot{L}}{L_t} = n < \nu, \\
\frac{\dot{L}_2}{L_{2t}} &= n + \frac{m_2^* - \alpha_2 m_1^*/\alpha_1}{1/(\epsilon - 1) + \beta_1} < n < \nu.
\end{aligned}$$

Based on definition 2 and proposition 5, we find that the aggregate capital-to-labor ratio and the consumption per worker are constant in the asymptotic equilibrium. This proposition indicates that the main theoretical results are asymptotically consistent with the aggregate Kaldor facts. In contrast to other studies, our model has a structural change in an environment with a redistribution mechanism and is asymptotically consistent with the Kaldor facts without considering large differences in preferences, substantial technological

gap or sufficient substitutability across sectors. On one hand, this structural change reduces consumption and capital per worker because it reduces the available amount of goods. On the other hand, this change can increase the consumption and capital per worker when these factors increase firm efficiency.[7]

3.3 Additional remarks on the model

Similar to the results obtained by Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008), in the model developed here, the economic growth path depends on the elasticities of inputs and the technological progress of the dominant sector. However, it also now depends on the redistribution mechanism generated by producers' demand for the public goods. Before proceeding, we need to highlight several points.

First, although technological progress is exogenous in the model, it is a major driver of structural change in the real world (Kongsamut et al., 2001). The world experiences a global wave of technical progress with the potential to create a widespread transformation of how economies work. Without endogenous technology, the effect of technological progress on employment is negative because it decreases the use of the labor factor. However, innovation usually favors structural change through shifts in production processes and the creation of new goods, but its effect on employment varies. The aim of most process innovation is to increase efficiency or save on inputs and thereby reduce labor, but product innovation normally leads to more jobs. What matters for employment is the net effect of these two dimensions. The high-tech sector and the labor-intensive sector are then not necessarily the same. Proposition 3 takes account of this fact.

Second, an implicit issue in this paper is that we may want to distinguish between outcome inequality, which has thus far been the focus of the discussion of optimal policy, and political inequality, in which the inequality of outcomes creates greater political power for a specific sector, enabling its actors to reinforce a biased redistribution mechanism. This phenomenon could be highly inefficient and might have negative effects on structural change (Slemrod and Gillitzer, 2013). An example of income redistribution that represents an inefficient mechanism

[7] In additional, when the growth rate of aggregate capital is greater than the growth rate of aggregate labor, as in proposition 5, we observe a stylized fact of the Rybczynski theorem; at constant relative goods prices, a rise in the endowment of one factor will lead to a more-than-proportional expansion of output in the sector that intensively uses that factor and an absolute decline in the output of the other good. Although the growth rate of aggregate capital is greater than that of aggregate labor, according to figures 1 and 2, relative goods prices are time-variant in the observed data.

is low-tech firms' greater demand for the public goods. In this case, the redistribution mechanism changes relative prices and might discourage the reallocation of resources. It is difficult to argue that this form of redistribution corrects the market failures. In the above example, price supports are Pareto inefficient in the sense that the earnings of low-tech firms could be sustained, and all other firms made better off, by a redistribution mechanism that does not involve resource misallocation.

Third, it is useful to distinguish between two categories of inefficient policies observed in developing and emerging industrial countries. The first category is inefficient targeting, which discourages the productive agents. An example of this targeting is the provision of temporary subsidies to low-tech firms instead of to current high-tech firms. This policy can be inefficient when it encourages firms to enter a sector in which their productivity is likely to be low. The second category is inefficient conditioning, which distorts producers' marginal production decisions. An example is the price subsidies rather than a constant transfer to low-tech firms; price subsidies encourage production more than the efficient amount. Public policies could influence structural change, and this topic is an important avenue for future research.

The macroeconomic literature is concerned primarily with inefficient targeting, but our framework above provides a rationale for inefficient conditioning. Although low-tech firms do not want their numbers to decline, they simultaneously do not want many more agents to enter the sector and reduce firms' profits. When a redistribution mechanism provides a constant transfer to firms, some firms might claim to be low-tech firms. Policies that condition production redistribution, such as an increased tax rate on a product, might be a way to prevent excess entry. Therefore, these two categories tend to affect the asymptotic CGP through the same channel: the redistribution mechanism affects the relative price and, in turn, the path of structural change.

Fourth, while the model here considers a closed economy, several developing and emerging industrial countries, such as Brazil, made important trade reforms in the 1980s and 1990s. From this perspective, Dix-Carneiro (2014) suggests that trade reforms in Brazil will lead to labor reallocation, but the period of transition might take several years. For example, assuming imperfect capital mobility, this process might take more than 30 years. Furthermore, unlike the analysis performed here, Dix-Carneiro (2014) assume that the marginal productivity of capital is identical across sectors and equal to the economy-wide rental price of capital. With this assumption, we cannot discuss to the role of resource misallocation in structural change

because the uneven marginal productivity of inputs across sectors is the source of the structural change process. The structural change in an open economic environment according to labor and financial frictions is thus an important direction for future research.

3.4 Numerical exercises

We now examine whether the model above is consistent with facts 1 and 2, mainly the last. The equilibrium paths of the key variables of the model will be computed from equations 10, 12, 13, 15, 16, and 17. In table 1, we describe sixteen parameters and ten initial conditions that characterize the model. Given the specificity of certain parameters, we calibrate the model to the firm-level data from the Brazilian manufacturing sector. To do so, we utilize the fourth-order Runge-Kutta method. This method consists of discretizing the differential equation system and the use of a fourth-order Taylor approximation that minimizes the error arising from the discretization time relative to the exact solution, as detailed in Judd (1998).

3.4.1 Characterization of parameters

Table 1 presents the parameters and initial conditions used in the calibration. In this process, we seek to ascertain whether the proposal explains the theoretical characterization proposed in facts 1 and 2. For this, we use firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. This survey obtains information on employees, wages and salaries, revenues, costs and expenses, investments, depreciation, output, and intermediate consumption, among other variables.

We use the perpetual inventory method to estimate firms' gross fixed capital stock. To do so, we compute initial capital stocks from reported depreciation and investment in the first observation. In addition, the firm's investment is deflated by the Brazilian sectoral producer price index (IPA, Portuguese acronym). Labor remuneration is the annual wages paid to blue- and white-collar workers and adjusted by the operation days of each firm. The firm's labor remuneration is deflated by the Brazilian consumer price index (INPC, Portuguese acronym). The value added is defined as the industrial gross values of output minus the industrial operating cost. Therefore, the value added is deflated by the Brazilian producer price index (IGP-OG, Portuguese acronym). Moreover, we reclassify firms using the International Standard Industries Classification (ISIC) Code Rev 3.1 at the 4-digit level. We exclude the top 1% of firms by

production in each sector-time and the bottom 1% of firms by production.

We reclassify the sectors by the labor share. Again, we consider the labor share to be equal to the wage and value added rate, similar to Acemoglu and Guerrieri (2008). The sectors can be separated into two groups: labor intensive and non-labor intensive. We employ the key variables in current values for this reclassification and account for each sector by the weighted average of products. We then define the cutoff of the sectoral labor share as the average labor share. It is important to highlight that other plausible cutoffs do not significantly affect the following results (because this, we omit these results here). Table 2 presents the sectors with this reclassification. Based in this table, we compute the labor share of the labor-intensive sector as .321 and the labor share of the non-labor-intensive sector as .2. We use the IPA to measure the price P_{st} for each sector. We thus obtain X_{st} as the real value added for each sector. Moreover, we adopt .02 as the discount rate, and the depreciation rate is .05.

We define the public goods as the total expenditure of the government on infrastructure, health and public education because these expenses increase the productivity of inputs. For the Brazilian economy, this expenditure data was obtained from the Government Financial Statistics from the International Monetary Fund. We do not fully observe the effects of the public goods on the firms' production, and our process may still be subject to some measurement error. For example, imagine that the value added generated by a skilled worker is higher than the expenditure per worker for the provision of the public education. Therefore, the effect of this public goods would be underestimated. Moreover, the public goods can affect the stock more than the flow (as public education, health or infrastructure). We then measure the observable part of the current provision of the public goods. The initial stock of the public goods will also be represented by data for 1996 for calibration. Moreover, in table 2, assigning values from the mean ratio of the tax among sectors, we compute the tax rate τ.

The parameters β_1 and β_2 are another problem because producer's demand for the public goods is an unobservable variable. In the theoretical framework, we find that the firm's production function implies that

$$\beta_s = \left(\frac{\partial X_{st}}{\partial \tau}\right)\left(\frac{\tau}{X_{st}}\right).$$

Table 2 presents the mean ratio of tax/value added, which differs across sectors. The problem consists of accounting how $\partial X_{st}/\partial \tau$ differs across firms. From the production function of each sector, the firm-level data of the Brazilian manufacturing sector, and the restriction of values

αs is the ratio of wage/value added, we thus estimate

$$\log X_{st} = \omega_0 + \omega_1 \log M_{st} + \omega_2 \log K_{st} + \omega_3 \log L_{st} + \omega_4 \log G_t + \Gamma_{st} + u_{st},$$

where, for each firm in sector s, $\omega_2 \equiv (1 - \alpha - \beta)$, $\omega_4 \equiv \beta$, and $\omega_3 = \alpha$. Moreover, Γ_{st} is a vector of time-fixed effect and of firm-fixed effect in each group sector s and u_{st} is the error component that has usual properties. We use Total Factor Productivity at firm-level constructed by Vasconcelos (2017) to generate a sectoral technology measurement M_{st}. With this, we obtain $\beta_1 = .114$ and $\beta_2 = .198$, which means that the labor-intensive sector demands the public goods less intensively than the non-labor intensive sector. In a country where the labor force is large and unskilled whilst public infrastructure is scarce, the previous inference seems reasonable. However, this imputation is not a perfect measurement of the demand for the public goods. Public expenditure may involve multiplier effects and include unobservable factors (Slemrod and Gillitzer, 2013). With this limitation in mind, the robustness of the subsequent numerical results will be checked.

Other important parameters are the elasticity of substitution between sectors and the proportion of intermediate goods used in production of the final good. For equation 8, we obtain that

$$\log \left(\frac{P_{1t} X_{1t}}{P_{2t} X_{2t}} \right) = \zeta_0 + \zeta_1 \log \left(\frac{X_{1t}}{X_{2t}} \right) + \Gamma_{st} + u_{st},$$

where $\zeta_0 \equiv \log(\frac{\gamma}{1-\gamma})$ and $\zeta_1 \equiv (1 - 1/\epsilon)$. Using firm-level data, we estimate $\epsilon = .424$ and $\gamma = .466$. In our economy, the elasticity of substitution between products is less than one. This imputation is consistent with other cross-industry evidences evaluated in the macroeconomic literature, as in Krusell et al. (2000) for US firms. Finally, we use 1996 as the initial time and, with this, the definition of κ_t then implies that $\kappa_0 = K_{10}/K_0$, and by the same mode, we obtain $\lambda_0 = L_{10}/L_0$. We impute the initial conditions K_{10}, K_0, L_{10} and L_0 from the firm-level data for 1996. For a given K_{10}, K_{20}, X_{10} and X_{20} and employing equation 3, we can determine M_{10} and M_{20}.

3.4.2 Calibration

Figure 4 presents the results of our calibration. The four panels depict the share of capital in the labor-intensive sector, the share of labor in the labor-intensive sector, the capital-to-labor ratio, and the ratio of the public goods to aggregate capital for the first 20 years. The period

corresponds to 1996-2016. The model calibration is consistent, as presented in figure 1, while the benchmark model calibration is not (where $\beta_1 = \beta_2 = 0$). The economy is far from the asymptotic equilibrium in which $\kappa^* = \lambda^* = 0$. In fact, as in Acemoglu and Guerrieri (2008), the economy takes a long time – over one hundred years – to reach the asymptotic equilibrium. Regarding the imperfect capital mobility, Dix-Carneiro (2014) also suggests that the labor reallocation is slow in Brazil in the 1990s. However, unlike those in Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007), the model developed here can explain the direction of resource reallocation in an environment in which the elasticity of substitution between sectors is less than one ($\epsilon = .424$) and the technological gap across sectors is not substantial ($m_1^* - m_2^* = 1\%$).

In table 3, we compare these numerical results (figure 4) to the observed data (figure 2). This table reports the Brazilian firm-level data and the values implied by our calibrations for 1996 and 2011 on the relative quantity of inputs and relative output of the labor-intensive versus non-labor-intensive sectors. Columns 1 and 2 of this table display the patterns shown in figure 2; production, capital and employment grow more quickly in non-labor-intensive industries. Column 3 shows that the benchmark model calibration is not consistent with this pattern (such that κ and λ increases), while column 4 shows that the model calibration is consistent. As in Dix-Carneiro (2014), the labor reallocation is slower. Here, the interaction between inputs amplifies the duration of the structural change.

Table 4 reports alternative calibrations of our model economy when we consider different values for the elasticity of substitution. Given the parameters used here, if we use the lower elasticity of substitution, the variables κ and X_1/X_2 are the best-fitting values for the observed data. However, if we use a higher elasticity of substitution such that it is less than one, the variable λ is the best-fitting value for the observed data. Moreover, the checked exercises for the elasticity of substitution in table 4 generate similar responses to the main results.

The dynamic behavior of the demand for the public goods seems to have an ambiguous relationship with capital intensity. According to proposition 1, for the parameters calibrated here, the relative provision of the public goods increases at low levels of aggregate capital and decreases at high levels of aggregate capital. These numerical results corroborate proposition 1. Furthermore, the magnitude of these effects is smaller than that for the growth rate of capital. There is an inverted-U-shaped relationship between the public goods and resource allocation, and the ratio of the public goods to aggregate capital asymptotically approaches zero.

We also check the robustness of the imputation in τ, β_1 and β_2. Again, we focus on understanding the forces that can drive structural change in developing and emerging economies. On one hand, our numerical check assesses that the change in the tax rate influences the speed of structural change. This check corroborates proposition 3. However, it cannot change the direction of structural change without considerable changes in the tax rate, and the acceleration effect on structural changes is not substantial when we impute some plausible changes in the tax rate ($\tau < .5$). Moreover, from proposition 5, the tax rate does not influence the key variables in asymptotic CGP. Our numerical check also corroborates this prediction (for these reasons, we omit this robustness check from the main text).

On the other hand, producers' demand for the public goods can influence the asymptotic CGP, and we perform a robustness check on β_1 and β_2 in table 5. In general, with greater β_1 and lower β_2, the variables κ and X_1/X_2 are the best-fitting values for the data, while λ is not. Furthermore, given that the values on α_1 and α_2 are constant and changing only β_1 and β_2, without micro-founded values, it is possible to obtain the best-fitting values for λ. For example, if $\beta_1 = .114$ and $\beta_2 = .144$ or $\beta_1 = .16$ and $\beta_2 = .198$, the variable λ is the best-fitting value for the observed data.

Using micro-founded parameters, our numerical results show that inputs and production move away from the labor-intensive sector to non-labor sector, consistent with the Brazilian manufacturing data. The model presented here can thus explain fact 2 and suggests that developing and emerging industrial countries (at least Brazil) have an uneven patterns of structural change because of the resource misallocation implied by biased redistribution mechanisms.

4 Extended model

As found in the previous section, the redistribution mechanism can influence the dynamics of resource allocation and, hence, economic growth path. In the real world, harmful redistribution mechanisms tend to be intensified by the market failures and inefficient policies. For example, if the inequality of outcomes creates greater political power for a given interest group – and thereby allows it to adopt sector-specific policies – inefficiency might result. The redistribution mechanism can thus generate resource misallocation that dynamically affects the distribution of production. Hsieh and Klenow (2009) argue that inputs misallocated at the

firm-level level disincentive the efficient use of inputs, which affects aggregate productivity and therefore aggregate output. Based on this argument, we show here that the productive sectors in developing and emerging industrial countries exhibit a pattern of structural change that is distinct from the pattern observed in industrialized countries because the degree of resource misallocation is higher in developing and emerging industrial countries. In this context, the resource misallocation effect on structural change represents an alternative explanation to the previous effect of redistribution mechanism generated by the demand for the public goods.

4.1 Environment

As in the previous environment, consider an economy with a finite horizon and continuous timing. This economy has a continuum of individuals with standard preferences concerning the consumption of a final good C_t, and their the utility function is given by equation 1. A final good Y_t is produced using two intermediate goods, X_{1t} and X_{2t}, the production functions of which are given by equation 2. The production function of the intermediate goods of each sector $s = \{1, 2\}$ is now

$$X_{st} = M_{st} K_{st}^{1-\alpha_s} L_{st}^{\alpha_s}, \tag{22}$$

where $\alpha_s \in (0, 1)$. Unlike equation 3, equation 22 imply that firms' technology is of constant returns. With this, only exogenous factors would imply that the decentralized market will not be dynamically Pareto optimal. Moreover, replacing assumption 1, assumption 4 implies that sector 1 is labor intensive and sector 2 is non-labor intensive.

Assumption 4. $\alpha_1 > \alpha_2$.

As in the previous section, assume that each sector has homogeneous firms or that there is one representative firm in each sector. Suppose that there are some market failures in the economy and that these failures generate distortions with uneven effects on different sectors. We define distortions that increase the marginal products of capital and labor by the same proportion as output distortions τ_{X_s}. In turn, we define distortions that increase the marginal product of capital relative to that of labor as capital distortions τ_{K_s}. These parameters vary across sector group and may be change over time (we omit t-subscript only for clarity of exposure). Now these distortions imply that the decentralized market will not be dynamically Pareto optimal and resource misallocation can thus influence the structural change path. An increase in the parameters represents actual Pareto worsening.

In this context, producers' profit from intermediate goods is

$$(1-\tau_{X_s})\,P_{st}X_{st} - W_tL_{st} - (1+\tau_{K_s})\,R_tK_{st}.$$

We implicitly assume that there is only misallocation of resources within-sectors, disregarding misallocation of resources between-sectors. We do it because, according to Hsieh and Klenow (2009) and Vasconcelos (2017), the main source of resource misallocation is the within-sector effect. Moreover, this assumption renders the model tractable without a lack of substantial generalization. Assume that the allocative choice depends only on the technological level. However, the values of inputs can be distorted (Hsieh and Klenow, 2009). Resource allocation would thus result from differences in the marginal revenue product of inputs among firms. Specifically, for each sector $s = \{1, 2\}$, the marginal revenue product of labor is

$$W_t = \alpha_s\,(1-\tau_{X_s})\,\frac{X_{st}}{L_{st}},$$

and the marginal revenue product of capital is

$$R_t = (1-\alpha_s)\left(\frac{1-\tau_{X_s}}{1+\tau_{K_s}}\right)\frac{X_{st}}{K_{st}}.$$

The marginal revenues of inputs equalize ex post. However, the marginal revenues of inputs are small ex ante for a share of firms and large ex ante for other firms. Here, the parameters τ_{X_s} and τ_{K_s} capture more than the previous effects of redistribution mechanism. These parameters capture several factors, such as uneven credit restrictions, a biased tax system, poor market regulations, and barriers to trade. Affecting the relative prices over time, these factors dynamically reduce the incentives to efficiently allocate resources.

4.2 Decentralized equilibrium

The determination of equilibrium is similar to the previous section. First, we solve the static problem. In each period, the producers maximize the value of their profits subject to input constraints. These producers then determine their input allocation, given the technological level and distortions. Second, we solve the dynamic problem. We maximize an objective function such that the dynamics of future capital and of current consumption are chosen according to the optimal allocation of inputs in each period.

4.2.1 Static equilibrium

As in the previous section, we must solve the static maximization problem of the production of the final good, which implies that (inverse) demands are set by the system of equations 8. The problem of the intermediate goods sectors now implies that the marginal returns are equalized within sectors in each period. Using equations 8a and 8b, for each sector $s = \{1, 2\}$, we obtain

$$\frac{K_{st}}{L_{st}} = \left(\frac{1-\alpha_s}{\alpha_s}\right)(1+\tau_{K_s}). \tag{23}$$

Equation 23 implies that the sectoral capital-to-labor ratio might differ across sectors not only because of the uneven shares of inputs across sectors but also because of the uneven extents of resource misallocation. We obtain that the aggregate capital-to-labor ratio can be time-variant in the dynamic transition when the degree of resource misallocation is time-variant. Given equations 22 and 23, the demand established by system of relative prices (equation 8) implies that lemma 1 remains valid; however, it's interpretation will change. Lemma 1 now indicates that the relative price is an implicit function of the technological gap across sectors, the capital distribution, and the resource misallocation.

As in the previous section, the value of the marginal return of each input equalizes across sectors in each period. The definition of variables κ, λ, k and c remain the same. The optimal allocation of labor across sectors now imply

$$\lambda_t = \left[1 + \left(\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}\right)(1-\tau_K)\left(\frac{1}{\kappa_t}-1\right)\right]^{-1}, \tag{24}$$

where we define the degrees of resource misallocation between sectors generated by capital distortions as

$$1-\tau_K = \frac{1+\tau_{K_2}}{1+\tau_{K_1}},$$

and by output distortions as

$$1+\tau_X = \frac{1-\tau_{X_2}}{1-\tau_{X_1}}.$$

Given assumption 4, lemma 1 and the above-defined variables, the optimal allocation of capital across sectors imply lemma 3.

Lemma 3. *The output gap between sectors is*

$$\frac{X_{1t}}{X_{2t}} = \left[\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1-\alpha_1}{1-\alpha_2}\right)\left(\frac{1-\tau_K}{1+\tau_X}\right)\left(\frac{1}{\kappa_t}-1\right)\right]^{\frac{\epsilon}{1-\epsilon}}.$$

Lemma 3 replaces lemma 2 and suggests that the output gap between sectors results from differences in the resource misallocation and the technological level. Given the technical gap and the degree of resource misallocation, the ratio X_1/X_2 is strictly decreasing in κ when $\epsilon < 1$. Similar to lemma 2, lemma 3 implies that production move away from the labor-intensive sector to non-labor sector. The opposite is true when $\epsilon \geq 1$. However, the distribution of resources now is a function of differences in the extent of misallocation across sectors. As the difference in the extent of misallocation across sectors increases, the effect of misallocation on the dynamic reallocation of inputs increases. This effect arises because the values of the inputs of production are ex ante distorted.

We define the static equilibrium below.

Definition 4. *In each period, the static equilibrium is defined as a system of prices* $\{R_t, W_t, P_{1t}, P_{2t}\}_{t=0}^{\infty}$ *and a feasible allocation* $\{L_{1t}, L_{2t}, K_{1t}, K_{2t}\}_{t=0}^{\infty}$ *such that firms maximize their earnings and markets clear for a given degree of resource misallocation* $\{\tau_X, \tau_K\}$.

4.2.2 Comparative statics analysis

In this extended model, proposition 2 remains valid and static equilibrium now implies proposition 6.

Proposition 6. *In the allocative equilibrium,*

$$\frac{\partial \kappa_t}{\partial \tau_X} \leq 0 \quad \textit{and} \quad \frac{\partial \kappa_t}{\partial \tau_K} \leq 0.$$

Proposition 6 suggests that more resource misallocation among firms in labor intensive sector relative to firms in non-labor intensive sector results in a less favorable reallocation of inputs for labor intensive sector. See the appendix for calculation details. The elevated extension of resource misallocation in one group sector relative to other group sector implies that the price is distorted and the marginal value of inputs is affected. With this, the current resource misallocation can influence structural change and may even imply a new equilibrium path of economic growth. It is important to highlight that this result does not depend on the

elasticity of substitution between sectors.

4.2.3 Dynamic equilibrium

We define the dynamic equilibrium below.

Definition 5. *The competitive equilibrium is defined such that households' choices are* $\{C_t, K_{t+1}\}_{t=0}^{\infty}$*, and households maximize their utility over time and subject to a system of prices* $\{R_t, W_t, P_{1t}, P_{2t}\}_{t=0}^{\infty}$*, and feasible allocation* $\{L_{1t}, L_{2t}, K_{1t}, K_{2t}\}_{t=0}^{\infty}$*, for a given degree of resource misallocation* $\{\tau_X, \tau_K\}$*. We thus maximize equation 1 and restrict equations 2, 4, 5, 6, and 22 to a given list of initial conditions* $\{\kappa_0, K_0, L_0, M_{10}, M_{20}, \tau_{X_0}, \tau_{K_0}\}_{t=0}$*.*

Because the utility function is continuous and strictly concave, and the restrictions are convex, the competitive equilibrium has a unique solution that corresponds to a unique competitive equilibrium. The household's first-order conditions imply that the intratemporal condition is given by equation 23 and that the intertemporal condition of consumption is

$$\frac{\dot{c}}{c_t} = \frac{1}{\theta}\left[\frac{\gamma(1-\alpha_1)}{\eta_t^{1/\epsilon}}\left(\frac{\lambda_t}{\kappa_t k_t}\right)^{\alpha_1} - \delta - \rho\right] - n - \frac{m_{1t}}{\alpha_1}. \tag{25}$$

From the dynamics of the aggregate capital and the definition of k_t, we obtain

$$\frac{\dot{k}}{k_t} = (1+\tau_{K_1})\frac{\kappa_t^{1-\alpha_1}\lambda^{\alpha_1}}{\eta_t k_t^{\alpha_1}} - \frac{c_t}{k_t} - \delta - n - \frac{m_{1t}}{\alpha_1}, \tag{26}$$

and using the definition of κ

$$\frac{\dot{\kappa}}{\kappa_t} = \bar{\Omega}(\kappa_t)\left[\epsilon\left(\frac{\dot{\tau}_K}{1-\tau_K} + \frac{\dot{\tau}_X}{1+\tau_X}\right) + (\epsilon-1)\left(m_{2t} - \frac{\alpha_2}{\alpha_1}m_{1t} + (\alpha_1-\alpha_2)\frac{\dot{k}}{k_t}\right)\right], \tag{27}$$

where

$$\bar{\Omega}(\kappa_t) = \frac{-(1-\kappa_t)}{(\epsilon-1)+(\alpha_1-\alpha_2)(\lambda_t-\kappa_t)}.$$

The $\dot{\tau}_K$ and $\dot{\tau}_X$ are respectively exogenous change in τ_K and τ_K, representing a change in the degree of resource misallocation (for example, a sector-specific inefficient policy). We do not study evaluation of the effects of economic policies on structural change, but the time-variant distortions allow this check. The standard transversality condition is identical to equation 17.

Equations 25, 26, and 27 are based on assumptions 2, 3, and 4. As in the previous section, assumption 2 implies that the No-Ponzi condition is satisfied. Although the details of the

calculation of assumption 3 are trivially different, this assumption implies a similar condition for the asymptotic stability of differential equations: a sector asymptotically dominates, given the technical gap and the degree of resource misallocation. In this extension, demonstration of assumption 3 follows the same steps presented in the appendix, and thus we will omit this demonstration. Under assumptions 2, 3, and 4 and given the initial conditions $\{k_0, \kappa_0, c_0\}$, the solutions to the dynamic problem satisfy the system of differential equations 25, 26, and 27, λ is given by equation 24, and τ_K and τ_X are given.

From equations 25, 26, and 27, if $\tau_{K_{10}} = \tau_{K_{20}} = \tau_{X_{10}} = \tau_{X_{20}} = 0$ and $\dot{\tau}_K = \dot{\tau}_X = 0$ for all period, the degree of resource misallocation among sectors does not change the input allocation dynamics. In this case, the dynamics of capital allocation are identical to those developed by Acemoglu and Guerrieri (2008). If at least one distortion is not zero, the degree of resource misallocation between sectors can change the dynamics of input allocation. The change in the degree of misallocation represents the speed of structural change and, depending on the magnitude of this degree, can change the economic growth path. Moreover, this effect can be potentate if $\dot{\tau}_K$ or $\dot{\tau}_K$ exogenously increase. Remember that the extent of misallocation in each sector is ex ante unobservable to firms, and the degree of misallocation is thus unobservable.

When there is a substantial degree of resource misallocation among producers, output growth can be lower than the output growth implied by the optimal resource allocation. Therefore, technological progress and resource misallocation influence the input allocation at the disaggregated level and in turn the structural change and the economic growth path. The growth path generated cannot be socially optimal when the degree of resource misallocation leads to adverse structural change and a suboptimal standard of living in the economy.

4.2.4 Constant Growth Path

We can set the CGP to exhibit the same pattern as in definition 3. With definition 3 and given assumptions 2, 3, and 4, the CGP implies proposition 7.

Proposition 7. *There is a unique, stable CGP and a unique CGP implied by equations 17, 24, 25, 26, and 27 such that*

$$\frac{\dot{c}}{c_t} = \frac{\dot{k}}{k_t} = 0.$$

From proposition 7, we obtain

$$\frac{\dot{C}}{C_t} = n + \frac{m_1^*}{\alpha_1} = \nu, \tag{28}$$

where we again define m_1^* as the technological progress in the labor-intensive sector in the asymptotic equilibrium and $x^* = \lim_{t\to\infty} x_t$ such that x is a key variable. The definition of the aggregate capital-to-labor ratio implies that

$$k^* = \frac{1-\alpha_1}{\alpha_1(1+\tau_{K_1})}, \tag{29}$$

and using equations 25, 26 and 29, we obtain

$$c^* = \left(\frac{\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho}{\alpha_1}\right) + (\delta + n)\left(\frac{1-\alpha_1}{\alpha_1\left(1+\tau_{K_1}^*\right)}\right). \tag{30}$$

Equations 29 and 30 determine the aggregate capital-to-labor ratio and the consumption per worker in the asymptotic equilibrium. Proposition 8 summarizes the asymptotic CGP remarks in the extended model. The details of this proposition are similar to those of proposition 5.

Proposition 8. *Under assumptions 2, 3 and 4, there exists is a non-trivial asymptotic CGP such that*

$$\begin{gathered}
\kappa^* = \lambda^* = 1, \\
k^* = \frac{1-\alpha_1}{\alpha_1(1+\tau_{K_1})}, \\
c^* = \left(\frac{\theta\left(n + m_1^*/\alpha_1\right) + \delta + \rho}{\alpha_1}\right) + (\delta + n)\left(\frac{1-\alpha_1}{\alpha_1\left(1+\tau_{K_1}\right)}\right), \\
\dot{\tau}_K = \dot{\tau}_X = 0, \\
\frac{\dot{Y}}{Y_t} = \frac{\dot{K}}{K_t} = \frac{\dot{X}_1}{X_{1t}} = \frac{\dot{K}_1}{K_{1t}} = n + \frac{m_1^*}{\alpha_1} = \nu, \\
\frac{\dot{X}_2}{X_{2t}} = \nu + m_2^* + (\epsilon - 1)\left(m_2^* - m_1^*\frac{\alpha_2}{\alpha_1}\right) \lesseqgtr \nu, \\
\frac{\dot{K}_2}{K_{2t}} = \nu + (\epsilon - 1)\left(m_2^* - m_1^*\frac{\alpha_2}{\alpha_1}\right) < \nu, \frac{\dot{L}_1}{L_{1t}} = \frac{\dot{L}}{L_t} = n < \nu, \\
\frac{\dot{L}_2}{L_{2t}} = n + (\epsilon - 1)\left(m_2^* - m_1^*\frac{\alpha_2}{\alpha_1}\right) < n < \nu.
\end{gathered}$$

Resource misallocation can thus influence the structural change in the worst form. The consumption per worker and the aggregate capital-to-labor ratio are smaller than those in the case without misallocation. When the degree of resource misallocation in the dominant sector increases, the consumption per worker and the aggregate capital-to-labor ratio decreases. Moreover, the aggregate capital-to-labor ratio and the consumption per worker are constant

in the asymptotic equilibrium. Proposition 8 shows that the extended theoretical results are asymptotically consistent with the aggregate Kaldor facts. The introduction of resource misallocation reduces consumption and capital per worker because it reduces the allocative efficiency of inputs. Moreover, this effect can be potentate if there is exogenous factors (as sector-specific inefficient policy) that increase resource misallocation in the extensive margin and the intensive margin.

4.3 Numerical exercises

We investigate whether the extended model is consistent with the observable data. The equilibrium paths of the key variables of the model will be computed from equations 17, 24, 25, 26, and 27. As in previous section, we calibrate the model to the firm-level data from the Brazilian manufacturing sector and describe parameters and initial conditions in table 1. Here, the exercise is technically similar to the previous numerical exercises. We utilize the fourth-order Runge-Kutta method such that we solve a system of differential equations.

4.3.1 Additional characterization of parameters

The parameter characterization is similar to that shown in table 1, excluding the degrees of resource misallocation, the government and producers' demand for the public goods. More specifically, the parameter list $\{\theta, \delta, \rho, \alpha_1, \alpha_2, \gamma, \epsilon, \nu, n, m_1^*, m_2^*\}$ and the intial conditions $\{\kappa_0, \lambda_0, \phi_0, K_0, L_0, X_{10}, X_{20}\}$ are the same described in the previous section, while these parameters not depend to the public goods variables or the tax measurement. We use the micro-founded results from Vasconcelos (2017) to obtain τ_K and τ_X. According to this study, the degree of resource misallocation within industries was high, persistent and time-variant in the Brazilian manufacturing sector between 1996 and 2011.[8] However, according to the model, τ_K and τ_X are not time-variant in asymptotic equilibrium even though they can be time-variant in the dynamic transition path. We thus use the average distortions and will check changes in these imputations. Calculating the parameter equivalents, we impute that the average output distortion is $.447$ and the average capital distortion is $.310$.

[8]Using the similar metrics of Hsieh and Klenow (2014), Vasconcelos (2017) find that the resource misallocation falls between 1996 and 2005 and then tends to grow in the Brazilian manufacturing sector. Moreover, we find that the Brazilian manufacturing sector operates at about 50% of its efficient product.

4.3.2 Calibration

Observing figure 4, we see that the extended model can also explain the direction of structural change in the Brazilian manufacturing sector. This result is also further calibrated in table 3, which reports the Brazilian firm-level data and the numbers implied by our benchmark calibration for 2011 with respect to the relative quantity of inputs and the relative output of the labor-intensive versus non-labor-intensive sectors. Again, columns 1 and 2 of table 3 confirm the patterns depicted in figure 2; quantity and employment grow more quickly in non-labor-intensive industries. Column 5 of this table shows that the extended model calibration is broadly consistent with this pattern. The observed ratio of real output between labor-intensive and non-labor-intensive sectors is comparable to the model with resource misallocation.

As Vasconcelos (2017) shows that the degree of resource misallocation within industries was time-variant in the Brazilian manufacturing sector, we check the robustness of our numerical results in table 6. In resume, when τ_K and τ_X differ, there are greater changes in the key variables. Given our extended model, as τ_K and τ_X increase, the resource misallocation for the firms in labor intensive sector relative to the firms in non-labor intensive sector increases. In general, given the parameters used here, if we use the lower τ_K and the higher τ_X, the variable λ is the best-fitting value for the observed data. If we use the higher τ_K and the lower τ_X, the variables κ and X_1/X_2 are the best-fitting values for the observed data. Furthermore, changing only τ_K and τ_X without micro-founded values, it is possible to obtain only the best-fitting values of each key variable.

Our numerical exercises indicate that the mechanism proposed in the extended model generates changes in the sectoral composition of output that are comparable to the changes that we observe in the Brazilian manufacturing sector. The dynamics of resource reallocation are faster than those presented in the numerical exercises in the previous section. The extended model can thus explain fact 2 and suggests that developing and emerging industrial countries (at least Brazil) have an uneven patterns of structural changes primarily because the degree of resource misallocation can be high and persistent.

5 Concluding remarks

There is a need to examine the pattern of structural change occurring in different countries. Figure 1 report that the pattern of input distribution in developing and emerging industrial countries differs from that observed in industrialized countries, as figure 2 exhibits to Brazilian case. Our main contribution is demonstrating that the interaction between inputs, factor proportions and price distortions across firms will lead to unbalanced growth while remaining consistent with the observed data. To do so, we propose adding to a multi-sector model of economic growth an ingredient that can fit the different patterns of structural change in industrial sectors: resource misallocation. Using micro-founded parameters for Brazil, our numerical exercises corroborate the theoretical predictions presented here.

In the case of the developing and emerging industrial countries, the restricted credit market, scarce supplies of skilled labor and poor public infrastructure could result in a high degree of resource misallocation (Hsieh and Klenow, 2009; Vasconcelos, 2017). This fact undermines the process of economic development and can imply worse structural change. In this study, we not evaluate the dynamic welfare gains. Thus, another important avenues for future research is the evaluation of welfare gains of structural change process and the relationship between economic policies, resource misallocation and structural change in the developing and emerging industrial countries.

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Figure 1: Resource allocation to manufacturing sectors (1963-2010)

——- The share of aggregate capital allocated to the labor-intensive sectors

– – – The share of aggregate labor force allocated to the labor-intensive sectors

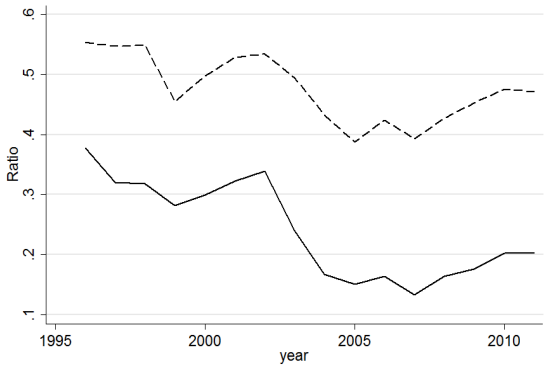
(a) Industrialized countries (b) Developing and emerging industrial countries

Note: We use the sector-level data from the Industrial Statistics Database (INDSTAT 2-digit 2015, ISIC Revision 3) developed by the United Nations Industrial Development (UNIDO) between 1963 and 2010. Like Acemoglu and Guerrieri (2008), we divide the sectors into two groups according to their labor share. In each sector-country, we treat the labor share as being equal to the wage and value added rate. The cutoff of the labor share is defined as the average industry labor share by country. The data exposed above are the average industry labor share by country list, where each country share is weighted by the gross value added.

Figure 2: Resource allocation to the Brazilian manufacturing sectors (1996-2011)

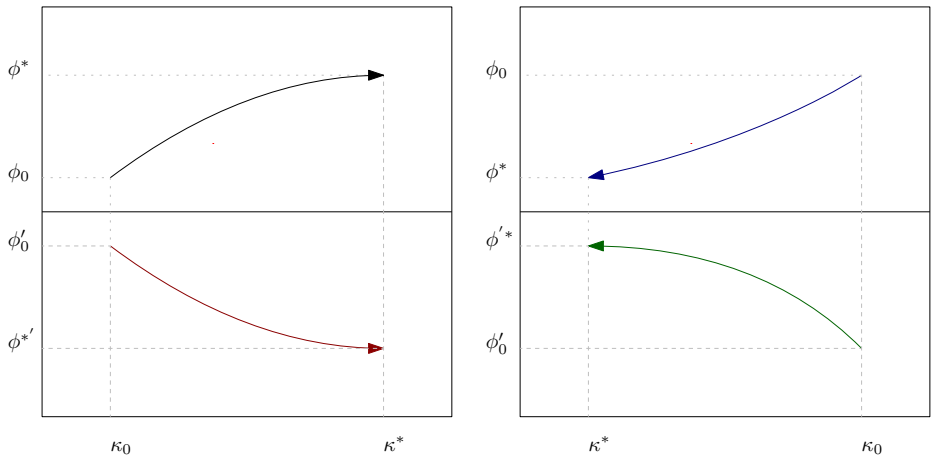
——- The share of aggregate capital allocated to the labor-intensive sectors

– – – The share of aggregate labor force allocated to the labor-intensive sectors



Note: We use the firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. As Acemoglu and Guerrieri (2008), we divide the sectors into two groups according to their labor share. We treat the labor share as being equal to the wage and value added rate. The cutoff is equal to 0.26 in the Brazilian manufacturing sector (see table 2 for further details).

Figure 3: Illustration of proposition 1



Note: This figure illustration proposition 1. We define κ as the share of aggregate capital allocated to the labor-intensive sector and ϕ as the ratio of the public good to the aggregate capital.

Table 1: Parameters

θ	4.000	δ	.050	ρ	.020	α_1	.321
α_2	.200	β_1	.114	β_2	.198	τ	.097
γ	.466	ϵ	.424	ν	.059	n	.032
m_1^*	.037	m_2^*	.027	κ_0	.375	λ_0	.555
ϕ_0	.044	G_0	78.03	K_0	1790	L_0	5.144
X_{10}	26.87	X_{20}	29.14	τ_K	.447	τ_X	.310

Note: The parameters θ, δ, and ρ are defaults in the macroeconomic literature. The unit measure of variables K_0, G_0, X_{10}, and X_{20} is one billion Brazilian reales and L_0 is one million workers. We adopt the public good as the annual total expenditure of government on infrastructure, health and public education. We obtain the expenditures from the data of the Government Financial Statistics from the International Monetary Fund. We use the firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011 to calculate other parameters. The imputation of parameters is described in the main text.

Table 2: Facts on the Brazilian manufacturing sector (1996-2011)

Manufacturing sector	Labor group	Wage / Value added	Tax / Value added	Technology group
Coke, refined petroleum products and nuclear fuel	non-labor intensive	.095	.042	low-tech
Tobacco products	non-labor intensive	.146	.061	resource based
Basic metals	non-labor intensive	.185	.073	low-tech
Food products and beverages	non-labor intensive	.202	.076	resource based
Paper and paper products	non-labor intensive	.202	.076	resource based
Chemicals and chemical products	non-labor intensive	.212	.083	high-tech
Radio, television and communication equipment and apparatus	non-labor intensive	.218	.092	high-tech
Other non-metallic mineral products	non-labor intensive	.232	.086	low-tech
Publishing, printing and reproduction of recorded media	non-labor intensive	.251	.089	resource based
Other transport equipment	non-labor intensive	.257	.096	high-tech
Office, accounting and computing machinery	labor intensive	.267	.104	high-tech
Motor vehicles, trailers and semi-trailers	labor intensive	.280	.103	high-tech
Wood and products of wood and cork, except furniture	labor intensive	.283	.095	resource based
Rubber and plastics products	labor intensive	.300	.110	low-tech
Textiles	labor intensive	.314	.117	resource based
Medical, precision and optical instruments, watches and clocks	labor intensive	.316	.113	high-tech
Machinery and equipment n.e.c.	labor intensive	.319	.117	high-tech
Fabricated metal products, except machinery and equipment	labor intensive	.319	.114	low-tech
Recycling	labor intensive	.325	.113	resource based
Electrical machinery and apparatus n.e.c.	labor intensive	.325	.118	high-tech
Furniture; manufacturing n.e.c.	labor intensive	.326	.114	resource based
Tanning and dressing of leather	labor intensive	.352	.116	resource based
Wearing apparel; dressing and dyeing of fur	labor intensive	.392	.115	resource based

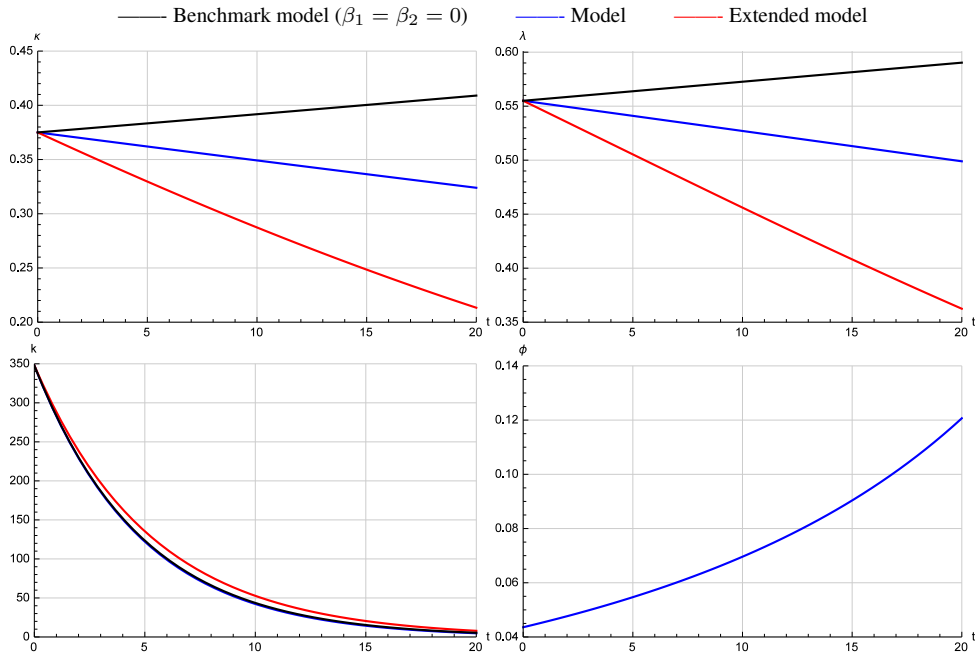
Note: We employ firm-level data from the Brazilian Survey of Industries constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011 to calculate the information above. We consider the labor share to equal the wage and value added rate such that the sectors can be separated into two groups: labor intensive and non-labor intensive. We define the cutoff of the sectoral labor share as the average labor share. We use the OECD technological classification according to ISIC Revision 3.1: high-tech, sectors with high R&D intensity; low-tech, sectors with low R&D intensity; and resource based, sectors with almost null R&D intensity.

Table 3: Data and model calibration

Variables	Data 1996	Data 2011	Benchmark model 2011	Model 2011	Extended model 2011
κ	.375	.200	.400	.336	.248
λ	.555	.480	.558	.464	.385
X_1/X_2	.922	.122	.286	.274	.138

Note: We define κ as the share of aggregate capital that is allocated to the labor-intensive sector, λ as the share of the aggregate labor force that is allocated to the labor-intensive sector, and X_1/X_2 as the output ratio of high- to low-labor-intensive sectors. We employ firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. The main text describes this exercise.

Figure 4: Numerical exercise (1996-2011)



Note: We define κ as the share of aggregate capital that is allocated to the labor-intensive sector, λ as the share of the aggregate labor force that is allocated to the labor-intensive sector, k as the aggregate capital-to-labor ratio normalized by the augmented technology of the labor-intensive sector (in thousands of reales per worker), and ϕ as the ratio of the public good to the aggregate capital. We employ firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. The main text describes this exercise.

Table 4: Robustness I

Variables	Data	Model		Extended model	
		$\epsilon = .323$	$\epsilon = .523$	$\epsilon = .323$	$\epsilon = .523$
κ	.200	.329	.343	.268	.241
λ	.480	.456	.472	.409	.375
X_1/X_2	.122	.266	.281	.152	.134

Note: We define κ as the share of aggregate capital that is allocated to the labor-intensive sector, λ as the share of the aggregate labor force that is allocated to the labor-intensive sector, and X_1/X_2 as the output ratio of high- to low-labor-intensive sectors. Moreover, we define ϵ as the degree of substitutability between intermediate goods. We employ firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. Here, we focus on 2011. The main text describes this numerical exercise.

Table 5: Robustness II

Variables	Data	Model					
		$\beta_1 = .0$	$\beta_1 = .014$	$\beta_1 = .214$	$\beta_1 = .114$	$\beta_1 = .114$	$\beta_1 = .114$
		$\beta_2 = .198$	$\beta_2 = .198$	$\beta_2 = .198$	$\beta_2 = .0$	$\beta_2 = .098$	$\beta_2 = .298$
κ	.200	.349	.348	.324	.319	.327	.346
λ	.480	.433	.437	.499	.515	.492	.430
X_1/X_2	.122	.319	.313	.242	.184	.228	.329

Note: We define κ as the share of aggregate capital that is allocated to the labor-intensive sector, λ as the share of the aggregate labor force that is allocated to the labor-intensive sector, and X_1/X_2 as the output ratio of high- to low-labor-intensive sectors. Moreover, we define β_1 and β_2 as the producer's demand for the public good in sectors 1 and 2, respectively. We employ firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. Here, we focus on 2011. The main text describes this numerical exercise.

Table 6: Robustness III

Variables	Data	Extended model			
		$\tau_K = .347$	$\tau_K = .547$	$\tau_X = .210$	$\tau_X = 0.410$
κ	.200	.328	.162	.227	.275
λ	.480	.480	.268	.357	.418
X_1/X_2	.122	.198	.083	.124	.158

Note: We define κ as the share of aggregate capital that is allocated to the labor-intensive sector, λ as the share of the aggregate labor force that is allocated to the labor-intensive sector, and X_1/X_2 as the output ratio of high- to low-labor-intensive sectors. Moreover, $\bar{\tau}_X$ assess the distortions that increase the marginal products of capital and labor by the same proportion, and $\bar{\tau}_K$ assess the distortions that increase the marginal product of capital relative to that of labor. We employ firm-level data from the Brazilian Survey of Industries (PIA, Portuguese acronym) constructed by the Brazilian Institute of Geography and Statistics (IBGE, Portuguese acronym) between 1996 and 2011. Here, we focus on 2011. The main text describes this exercise.

Supplementary appendix (not for publication)

A Additional remarks

In the main text, we use the country group lists defined by the UNIDO. Industrialized countries are: Australia, Austria, Belgium, Canada, Taiwan Province, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Ireland, Israel, Italy, Japan, Kuwait, Lithuania, Luxembourg, Malaysia, Malta, Netherlands, Norway, Portugal, Qatar, Republic of Korea, Russian Federation, Singapore, Slovakia, Slovenia, Spain, Sweden, United Kingdom, and United States of America. Developing and emerging industrial countries are: Argentina, Azerbaijan, Bahamas, Bangladesh, Bolivia (Plurinational State of), Brazil, Bulgaria, Chile, China, Colombia, Cyprus, Ecuador, Egypt, Eritrea, Ethiopia, Fiji, Georgia, Ghana, Greece, India, Indonesia, Iran (Islamic Republic of), Jordan, Kenya, Latvia, Madagascar, Malawi, Mexico, Mongolia, Morocco, Nigeria, Oman, Pakistan, Panama, Peru, Philippines, Poland, Republic of Moldova, Romania, Senegal, South Africa, Sri Lanka, State of Palestine, Thailand, The f. Yugosl. Rep of Macedonia, Trinidad and Tobago, Turkey, and United Republic of Tanzania.

B Mathematical issues

B.1 Proof of proposition 1

First, note that

$$\frac{\partial \lambda_t}{\partial \kappa_t} = \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{1-\alpha_1-\beta_1}{1-\alpha_2-\beta_2}\right)\left(\frac{\lambda_t}{\kappa_t}\right)^2 > 0, \tag{B.1}$$

$$\frac{\partial k_t}{\partial \kappa_t} = \left(\frac{1-\beta_1}{\alpha_1} - \frac{1-\beta_2}{\alpha_2}\right)\frac{\partial \lambda_t}{\partial \kappa_t}, \tag{B.2}$$

as $\alpha_1 > \alpha_2$, then

$$\frac{\partial k_t}{\partial \kappa_t} = \begin{cases} > 0, & \text{if } \beta_2 > \beta_1; \\ \leq 0, & \text{otherwise.} \end{cases}$$

We define

$$A_t = \kappa_t^{1-\alpha_1-\beta_1}\lambda_t^{\alpha_1}k_t^{-\alpha_1}\phi_t^{\beta_1},$$

and

$$B_t = M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}(1-\kappa_t)^{1-\alpha_2-\beta_2}(1-\lambda_t)^{\alpha_2}k_t^{-\alpha_2}\phi_t^{\beta_2}.$$

Thus, from equation 12 we obtain

$$\begin{aligned}\frac{\partial\phi_t}{\partial\kappa_t} = &\left[\frac{1}{\tau^{\frac{1-\epsilon}{\epsilon}}\phi_t^{-\frac{1}{\epsilon}} - \phi_t^{-1}\left(\beta_1\gamma A_t^{\frac{\epsilon-1}{\epsilon}} + \beta_1(1-\gamma)B_t^{\frac{\epsilon-1}{\epsilon}}\right)}\right] \\ &\left[\gamma A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{1-\alpha_1-\beta_1}{\kappa_t} + \frac{\alpha_1}{\lambda_t}\frac{\partial\lambda_t}{\kappa_t} - \frac{\alpha_1}{k_t}\frac{\partial k_t}{\partial\kappa_t}\right)\right. \\ &\left.-(1-\gamma)B_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{1-\alpha_2-\beta_2}{1-\kappa_t} + \frac{\alpha_2}{1-\lambda_t}\frac{\partial\lambda_t}{\kappa_t} + \frac{\alpha_2}{k_t}\frac{\partial k_t}{\partial\kappa_t}\right)\right].\end{aligned}$$

The first term above is positive if

$$\phi_t > \tau\left(\beta_1\gamma A_t^{\frac{\epsilon-1}{\epsilon}} + \beta_1(1-\gamma)B_t^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

and the second term is positive if

$$\begin{aligned}&\gamma A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{1-\alpha_1-\beta_1}{\kappa_t} + \frac{\partial\lambda_t}{\partial\kappa_t}\left(\frac{(1-\alpha_2-\beta_2)\alpha_1^2}{(1-\alpha_1-\beta_1)\alpha_2\lambda_t + (1-\alpha_2-\beta_2)\alpha_1(1-\lambda_t)\lambda_t}\right)\right) \\ &-(1-\gamma)B_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{1-\alpha_2-\beta_2}{1-\kappa_t} + \right. \\ &\left.\frac{\partial\lambda_t}{\partial\kappa_t}\left(\frac{(1-\alpha_1-\beta_1)\alpha_2^2}{(1-\alpha_1-\beta_1)\alpha_2\lambda_t(1-\lambda_t) + (1-\alpha_2-\beta_2)(1-\lambda_t)^2\lambda_t}\alpha_1\right)\right) > 0.\end{aligned}$$

We see the extreme points for κ. Assume that $\kappa^* = 1$, and thus these inequalities can be rewritten as

$$\begin{aligned}\phi_t &> \tau\left(\beta_1\gamma\left(k^{*-\alpha_1}\phi_t^{\beta_1}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \\ \phi_t &> \underline{\phi} = \left[\tau k^{*-\alpha_1}\left(\beta_1\gamma\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{1-\beta_1}},\end{aligned} \tag{B.3}$$

and

$$\gamma\left(k^{*-\alpha_1}\phi_t^{\beta_1}\right)^{\frac{\epsilon-1}{\epsilon}}\left(1-\alpha_1-\beta_1+\frac{\partial\lambda_t}{\partial\kappa_t}\left(\frac{(1-\alpha_2-\beta_2)\alpha_1^2}{(1-\alpha_1-\beta_1)\alpha_2}\right)\right)>0,$$
$$\gamma\left(k^{*-\alpha_1}\phi_t^{\beta_1}\right)^{\frac{\epsilon-1}{\epsilon}}(1-\beta_1)>0. \tag{B.4}$$

Thus, assuming that $\lim_{t\to}\kappa_t=1$, if ϕ is sufficiently greater, and thus, the resources allocated to the labor-intensive sector is higher, the higher the ratio of the public good to aggregate capital is. Note that the opposite, $(\partial\phi_t/\partial\kappa_t\leq 0)$, is true when $\phi_t\leq\underline{\phi}$.

Suppose now that $\kappa^*=0$, and thus, previous inequalities can be rewritten as

$$\phi_t<\tau\left(\beta_1(1-\gamma)\left(M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}k^{*-\alpha_2}\phi_t^{\beta_2}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$
$$\phi_t<\bar{\phi}=\left[\tau M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}k^{*-\alpha_2}\left(\beta_2(1-\gamma)\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{1-\beta_1}}, \tag{B.5}$$

and

$$-(1-\gamma)\left(M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}k^{*-\alpha_2}\phi_t^{\beta_2}\right)^{\frac{\epsilon-1}{\epsilon}}\left(1-\alpha_2-\beta_2+\frac{\partial\lambda_t}{\partial\kappa_t}\left(\frac{(1-\alpha_2-\beta_2)\alpha_1^2}{(1-\alpha_1-\beta_1)\alpha_2}\right)\right)<0,$$
$$-(1-\gamma)\left(M_{2t}M_{1t}^{-\frac{\alpha_2}{\alpha_1}}k^{*-\alpha_2}\phi_t^{\beta_2}\right)^{\frac{\epsilon-1}{\epsilon}}(1-\beta_2)<0. \tag{B.6}$$

Thus, assuming that $\lim_{t\to}\kappa_t=0$, if ϕ is sufficiently smaller, and thus greater resources are allocated to the labor-intensive sector, the higher the ratio of the public good to aggregate capital is. Note that the opposite, $(\partial\phi_t/\partial\kappa_t\leq 0)$, is true when $\phi_t\geq\bar{\phi}$. From equation B.3 (B.5), if κ_t grows (decreases) over time, the ratio of the public good to aggregate capital increases for ϕ sufficiently large (small), and it decreases (increases) when ϕ is considerably smaller (greater). The above equations determine the possible effects described in proposition 1.

B.2 Proof of proposition 2

From lemma 2, we obtain

$$\frac{1}{\kappa_t}-1=\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2-\beta_2}{1-\alpha_1-\beta_1}\right)\left(\frac{X_{2t}}{X_{1t}}\right)^{\frac{\epsilon-1}{\epsilon}}. \tag{B.7}$$

Using equation B.7, we obtain

$$
\begin{aligned}
-\frac{1}{\kappa_t^2}\frac{\partial \kappa_t}{\partial M_{2t}/M_{1t}} &= \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2-\beta_2}{1-\alpha_1-\beta_1}\right)\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{X_{2t}}{X_{1t}}\right)^{-\frac{1}{\epsilon}}\left(\frac{\partial X_{2t}/X_{1t}}{\partial M_{2t}/M_{1t}}\right) \\
\frac{\partial \kappa_t}{\partial M_{2t}/M_{1t}} &= \kappa_t\left(1-\kappa_t\right)\left(\frac{1-\epsilon}{\epsilon}\right)\left(\frac{M_{1t}^{\frac{\alpha_2}{\alpha_1}}}{M_{2t}}\right) \\
\frac{\partial \kappa_t}{\partial M_{2t}/M_{1t}} &\propto 1-\epsilon. \qquad \text{(B.8)}
\end{aligned}
$$

This equation determines the effects described in proposition 2.

B.3 Proof of proposition 3

Using equation B.7, we obtain

$$
\begin{aligned}
-\frac{1}{\kappa_t^2}\frac{\partial \kappa_t}{\partial \tau} &= \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2-\beta_2}{1-\alpha_1-\beta_1}\right)\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{X_{2t}}{X_{1t}}\right)^{-\frac{1}{\epsilon}}\left(\frac{\partial X_{2t}}{\partial G_t}\frac{\partial G_t}{\partial \tau}\frac{1}{X_{1t}}-\frac{\partial X_{1t}}{\partial G_t}\frac{\partial G_t}{\partial \tau}\frac{X_{2t}}{X_{1t}^2}\right) \\
\frac{\partial \kappa_t}{\partial \tau} &= -\kappa_t^2\left(\frac{1-\kappa_t}{\kappa_t}\right)\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{\beta_2}{G_t}\left(P_{1t}X_{1t}+P_{2t}X_{2t}\right)-\frac{\beta_1}{G_t}\left(P_{1t}X_{1t}+P_{2t}X_{2t}\right)\right) \\
\frac{\partial \kappa_t}{\partial \tau} &= \kappa_t\left(1-\kappa_t\right)\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{\beta_1-\beta_2}{\tau}\right) \\
\frac{\partial \kappa_t}{\partial \tau} &\propto \left(\epsilon-1\right)\left(\beta_1-\beta_2\right), \qquad \text{(B.9)}
\end{aligned}
$$

where $G_t = \tau P_{1t}X_{1t} + \tau P_{2t}X_{2t}$ for definition. This equation determines the effects described in proposition 3.

B.4 Proof of proposition 5

Assume that the curve $\dot{k}/k_t$ is constant. Differentiating equation κ, we obtain a positive relationship between k and κ along the constant schedule $\dot{k}/k_t$:

$$
\frac{\partial \kappa_t}{\partial k_t} = \frac{\alpha_1(1-\beta_1)}{k_t}\left(\frac{1-\alpha_1-\beta_1}{\kappa_t}+\frac{\alpha_1}{\lambda_t}\frac{\partial \lambda_t}{\partial \kappa_t}-\frac{1}{\eta_t}\frac{\partial \eta_t}{\partial \kappa_t}\right)^{-1}. \qquad \text{(B.10)}
$$

From equations 30 and 31, and using the fact that

$$\frac{\partial \eta_t}{\partial \kappa_t} = \left(\frac{\epsilon}{\epsilon - 1}\right) \eta_t^{2-\frac{1}{\epsilon}} \gamma \left(\frac{1-\alpha_1-\beta_1}{1-\alpha_2-\beta_2}\right) \left(\frac{1}{\kappa_t} - 1\right)$$
$$\frac{\partial \eta_t}{\partial \kappa_t} \propto \epsilon - 1,$$

we know the previous equation is positive if $\epsilon \geq 1$. Thus, this schedule is upward-sloping in the space (k, κ). The curve $\dot{k}/k = 0$ meets $\kappa^* = 1$. Below this curve, we have $\dot{k}/k < 0$. Setting 16 equal to zero requires that $\Omega(k_t) = 0$ or

$$m_2^* - \frac{\alpha_2 m_1^*}{\alpha_1} + (\alpha_1 - \alpha_2)\frac{\dot{k}}{k_t} - (\beta_1 - \beta_2)\frac{\dot{\phi}}{\phi_t} = 0,$$

which implies that either $\kappa = 1$ or

$$m_2^* - \frac{\alpha_2 m_1^*}{\alpha_1} = -(\alpha_1 - \alpha_2)\frac{\dot{k}}{k_t} + (\beta_1 - \beta_2)\frac{\dot{\phi}}{\phi_t}.$$

The latter case cannot be a steady state. Therefore, the only non-trivial steady-state allocation of capital is $\kappa^* = 1$. In other words, the relevant schedule $\dot{k}/k_t = 0$ that determines the steady state is the horizontal line $\kappa = 1$ in the space (k, κ). Since the constant schedule $\dot{k}/k_t$ is strictly increasing in k, there exists a unique non-trivial steady state given by equation 15.

The growth rates of the asymptotically dominant sector are derived by combining the steady-state solutions and the growth rates of the exogenous variables with κ, k, and η. The growth rates for sector 2 are given by the solution of the system of three equations using the growth rates given by equations 3, 5 and 6.

$$\begin{bmatrix} \dot{k} \\ \dot{\kappa} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{k}}{\partial k_t}(k = k^*, \kappa = 1) & \frac{\partial \dot{k}}{\partial \kappa_t}(k = k^*, \kappa = 1) \\ \frac{\partial \dot{\kappa}}{\partial k_t}(k = k^*, \kappa = 1) & \frac{\partial \dot{\kappa}}{\partial \kappa_t}(k = k^*, \kappa = 1) \end{bmatrix} \begin{bmatrix} k - k^* \\ \kappa - 1 \end{bmatrix}, \tag{B.11}$$

where

$$\frac{\partial \dot{k}}{\partial k_t} = \frac{c^*}{k^*} > 0, \tag{B.12a}$$

$$\frac{\partial \dot{k}}{\partial \kappa_t} = \alpha_1 (\delta + n) \left(\frac{\partial \kappa_t}{\partial k_t} - 1 \right) + \frac{c^*}{k^*} \left(\alpha_1 \frac{\partial \kappa_t}{\partial k_t} + 1 - \alpha_1 \right), \tag{B.12b}$$

$$\frac{\partial \dot{\kappa}}{\partial k_t} = 0, \tag{B.12c}$$

$$\frac{\partial \dot{\kappa}}{\partial \kappa_t} < 0. \tag{B.12d}$$

It follows that the determinant of the Jacobian is positive and its trace negative. As a result, the linear system has two negative eigenvalues, and the steady state associated with this CGP is locally stable. Assumption 1 implies that the determinant of the Jacobian would be negative, and therefore, the steady state exhibits the saddle point property. Nonetheless, the steady state under evaluation, $\kappa^* = \lambda^* = 1$, was characterized under the assumption that sector 1 is asymptotically dominant. When one reverses the labor intensities, one changes the asymptotically dominant sector and therefore the stability properties of the steady state characterized under the initial ranking of factor intensities. A global analysis using arrows shows that paths converging to the steady state (k^*, κ^*) can display non-monotone behaviour. With this, we proof proposition 5.

B.5 Dynamic stability - Assumption 3

We determine here the dynamic stability of the basic framework. Equations 13, 14, 15, and 16 can be rewritten as

$$\dot{x} = f(x),$$

where $x = [C \quad k \quad \kappa \quad \phi]'$. For a given τ, ϕ is a monotonic function of k and c such that we can simply to $x = [C \quad k \quad \kappa]'$ without change the dynamic stability. To investigate the dynamic stability in the neighborhood of the steady state, we consider a linear system

$$\dot{z} = J^*(x)z,$$

where $z \equiv x - x^*$, x^* such that $f(x^*) = 0$ and $J(x^*)$ is the Jacobian matrix of $f(x)$ when they assume x^* values. However, we obtain

$$J(x) = \begin{bmatrix} a_{cc} & a_{ck} & a_{c\kappa} \\ a_{kc} & a_{kk} & a_{k\kappa} \\ a_{\kappa c} & a_{\kappa k} & a_{\kappa\kappa} \end{bmatrix},$$

where $a_{cc} = a_{\kappa c} = 0$. This fact implies that $det\,(J(x^*)) = a_{kc}(a_{\kappa k}a_{c\kappa} - a_{\kappa\kappa}a_{ck})$. Moreover, given a

$$a_{kc} = -\frac{1}{k^*} < 0,$$

dynamic stability would depend fundamentally on κ^*. If $\kappa^* = 1$, then $a_{\kappa k} = 0$ because $\Omega(1) = 0$ and

$$a_{ck} = -\frac{1}{\theta}\alpha_1(1-\alpha_1-\beta_1)\gamma^{\frac{\epsilon}{\epsilon-1}}(\tau m_1^*)^{\frac{\beta_1}{1-\beta_1}}(1-\tau)^{1+\frac{1}{\epsilon}}\frac{1}{k^{*1+\alpha_1}} < 0,$$
$$a_{\kappa\kappa} = \left[\frac{1}{1/(\epsilon-1)+\beta_2}\right]\left[m_2^* - \frac{\alpha_2 m_1^*}{\alpha_1}\right].$$

According to assumption 3, there is a stable equilibrium if the determinant of $J(x^*)$ is positive. This is true if $a_{\kappa\kappa} < 0$, and thus, $m_1^*/\alpha_1 > m_2^*/\alpha_2$ and $\epsilon < 1 - 1/\beta_2$ or $m_1^*/\alpha_1 < m_2^*/\alpha_2$ and $\epsilon > 1 - 1/\beta_2$. However, $1 - 1/\beta_2 < 0$ and $\epsilon > 0$, thus $m_1^*/\alpha_1 < m_2^*/\alpha_2$. If $\beta_1 = \beta_2 = 0$, we have that the conditions for dynamic stability are identical to those obtained by Acemoglu and Guerrieri (2008). In this work, $\beta_1 \neq \beta_2$ and we can have a stable equilibrium without needing to impose some degree of complementarity on path of dynamic stability.

B.6 Proof of proposition 6

From lemma 3, we obtain

$$\frac{1}{\kappa_t} - 1 = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1+\tau_X}{1-\tau_K}\right)\left(\frac{X_{1t}}{X_{2t}}\right)^{\frac{1-\epsilon}{\epsilon}}. \tag{B.13}$$

Using equation B.13, we obtain

$$
\begin{aligned}
-\frac{1}{\kappa_t^2}\frac{\partial \kappa_t}{\partial \tau_X} &= \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{1-\tau_K}\right)\left(\frac{X_{1t}}{X_{2t}}\right)^{\frac{1-\epsilon}{\epsilon}} \\
\frac{\partial \kappa_t}{\partial \tau_X} &= -\kappa_t^2\left(\frac{1}{\kappa_t}-1\right)\left(\frac{1}{1+\tau_X}\right) \\
\frac{\partial \kappa_t}{\partial \tau_X} &= -\kappa_t(1-\kappa_t)\left(\frac{1}{1+\tau_X}\right) \leq 0.
\end{aligned} \tag{B.14}
$$

Analogously, we obtain

$$
\begin{aligned}
-\frac{1}{\kappa_t^2}\frac{\partial \kappa_t}{\partial \tau_K} &= \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\left(\frac{1+\tau_X}{(1-\tau_K)^2}\right)\left(\frac{X_{1t}}{X_{2t}}\right)^{\frac{1-\epsilon}{\epsilon}} \\
\frac{\partial \kappa_t}{\partial \tau_K} &= -\kappa_t^2\left(\frac{1}{\kappa_t}-1\right)\left(\frac{1}{1-\tau_K}\right) \\
\frac{\partial \kappa_t}{\partial \tau_K} &= -\kappa_t(1-\kappa_t)\left(\frac{1}{1-\tau_K}\right) \leq 0.
\end{aligned} \tag{B.15}
$$

These equations determine the effects described in proposition 6. Remember that the extent of misallocation in each sector is ex ante unobservable to firms. Thus, the variables X_1 and X_2 are not differentiable at τ_K.