Essays on concession design
Rodrigo Bomfim de Andrade

Essays on concession design

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This work is dedicated to Nataly, my wonder wife, Bento, my brave boy, and Catarina, my sweet newborn girl.
Abstract

The present thesis is an inquiry into incentive issues related to the theme of concession design. Concessions are a contractual method to procure public goods and services from the private sector, typically used to deliver infrastructure assets and operations. Its main benefit stems from increased economic performance, from accessing the private sector’s flexibility and innovation under an adequate alignment of incentives. Such incentives are mostly provided by the project company’s remuneration mechanism, set out in the concession contract. The design of concessionaire’s compensation is subject to information frictions, in the form of adverse selection and moral hazard, which makes the concession arrangement suitable to be analyzed in the light of principal-agent theory. This thesis’ three essays are explorations of different interaction environments between the procuring authority (principal) and the concessionaire (agent), aimed at deriving applied insights that are relevant to concession policy.

Chapter 1 focuses on the consequences of information dynamics to the optimal design of the concessionaire’s compensation policy. Specifically, it inquires whether the regulator should base the contract on sequential private information about project costs. In the optimal mechanism, the regulator offers a dynamic remuneration menu consisting of an award fee, followed by a contingent menu of linear reimbursement rules, whose incentive power increases in the firm’s forecast optimism. The chapter extends the dynamic framework to allow the possibility of project cancellation, as well as the award of the contract through a competitive process.

Chapter 2 analyzes the effects of exit rights in concession policy. Motivated by the principle of “contractual equilibrium” in concession arrangements, it studies the optimal design of remuneration mechanisms in a dynamic environment when the firm is protected by ex-post limited liability. Such protection result in optimal pooling of first-period private information, i.e. the regulator offers the same incentive scheme regardless of cost forecasts. However, a numerical exercise suggests there is a positive, though non-absolute, optimal level of ex-post protection that maximizes social welfare, which entails relevant policy implications.

Chapter 3 examines the robust design of concession contracts in an environment that requires less knowledge from the principal. Specifically, it assumes the regulator ignores the exact shape of private information, but knows the first two moments of the distribution of project costs. Consequently, the optimal remuneration mechanism depends solely on average costs and cost dispersion, though retaining the incentive properties of conventional Bayesian contracts. Applying the robustness framework to the problem of project selection under moment conditions, the planner obtains a selection criterion that protects her from large losses due to prior misspecification.
Keywords

1. infrastructure economics
2. public-private partnerships
3. concession contracts
4. incentive regulation
5. cost-based procurement
6. dynamic mechanism design
7. ex-post participation constraints
8. limited liability
9. robust contracts
10. project selection
Resumo

Esta tese é uma investigação sobre questões de incentivo relacionadas ao tema do desenho de concessões. Uma concessão é um método de contratação de bens e serviços públicos envolvendo o setor privado, tipicamente usado para contratar a construção e operação de ativos de infraestrutura. Seu principal benefício deriva do desempenho econômico superior, a partir do acesso, sob um adequado alinhamento de incentivos, à flexibilidade e inovação trazidas pelo setor privado. Tais incentivos são proporcionados majoritariamente pelo mecanismo de remuneração da empresa, especificado no contrato de concessão. O desenho desse mecanismo é sujeito a fricções informacionais, na forma de seleção adversa e risco moral, os quais tornam o regime de concessão suscetível ser analisado à luz da teoria do principal-agente. Os três ensaios desta tese exploram diferentes aspectos do ambiente de interação entre o poder concedente (principal) e o concessionário (agente), com o objetivo de obter insights aplicados com relevância para a política de concessões.

O Capítulo 1 concentra-se nas consequências da dinâmica da informação para o desenho ótimo da política de remuneração do concessionário. Especificamente, questiona se o regulador deve basear o contrato em informação privada sequencial sobre os custos do projeto. No mecanismo ótimo, o regulador oferece um menu de remuneração dinâmico que consiste em um valor de outorga, seguido de um menu contingente de esquemas lineares de reembolso, cuja potência de incentivos aumenta conforme o otimismo das projeções da firma. O capítulo estende este arcabouço para permitir a possibilidade de cancelamento do projeto, bem como a adjudicação do contrato por meio de um processo competitivo.

O Capítulo 2 analisa os efeitos de direitos de desistência em uma política de concessão. Motivado pelo princípio do “equilíbrio contratual” nos contratos de concessão, estuda-se o desenho ótimo de mecanismos de remuneração em um ambiente dinâmico, quando a firma é protegida ex-post por responsabilidade limitada. Tal proteção resulta em pooling ótimo de tipos privadas de primeiro período, ou seja, o regulador oferece o mesmo esquema de incentivos independentemente das projeções de custo. No entanto, um exercício numérico sugere a existência de um nível ótimo positivo, porém não-absoluto, de proteção ex-post que maximiza o bem-estar social, o qual tem implicações relevantes de política.

O Capítulo 3 examina o desenho robusto de contratos de concessão em um ambiente que requer menos conhecimento do principal. Especificamente, assume-se que o regulador ignora a forma exata da informação privada, mas conhece os dois primeiros momentos da distribuição de custos do projeto. Consequentemente, o mecanismo ótimo de remuneração depende apenas do custo médio e da dispersão de custos, embora mantenha as propriedades de incentivo de contratos bayesianos convencionais. APLICANDO O ARCAÍSSE DE ROBUSTEZ AO PROBLEMA DE SELEÇÃO DE PROJETOS SOB CONDIÇÕES DE MOMENTO, O PLANEJADOR DISPÕE DE UM CRITÉRIO DE SELEÇÃO QUE O RESGARDE CONTRA PERDAS SEVERAS DECORRENTES DE ERROS DE ESPECIFICAÇÃO.
1. economia da infraestrutura
2. parcerias público-privadas
3. contratos de concessão
4. regulação por incentivos
5. licitações baseadas em custo
6. desenho de mecanismos dinâmicos
7. restrições de participação ex-post
8. responsabilidade limitada
9. contratos robustos
10. seleção de projetos
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Overview

The present thesis is an inquiry into economic issues related to the overarching theme of concession design. The term “concession” refers to a contractual method to procure public assets or delegate the provision of public services to the private sector. It differs from conventional public procurement as it involves the transfer of significant risks and management responsibility to the private partner, or concessionaire, whose remuneration depends on performance. Concessions are an increasingly widespread tool to boost public investment programs in sectors like transport and sanitation. As infrastructure projects tend to be capital intensive and have long durations, the main reason for concessions’ popularity stems from the possibility of tapping private finance, and thus circumvent transitory fiscal constraints to public investment.

More importantly, the concession model has the potential to drive incremental efficiency gains. Under proper incentives, a concession partnership brings the private sector’s flexibility, skills and innovation to project execution and service operation, such that it outperforms conventional public provision (National Audit Office, 2009). However, the mixed experience with concessions in developing countries indicate that the model’s success depends on at least two conditions. First, there must be a legal and institutional framework in place supporting the implementation of a concession program. Second, the concession contract must provide adequate incentives for efficiency through the design of its remuneration mechanism (World Bank, 2017b).

The design of incentives has been the craft of contract theory for at least the past forty years. Since the late 1970s, that subdiscipline of economics has contributed to the understanding of pervasive asymmetries of information in economic interactions. Moreover, contract theory has sought to rationalize, and often prescribe, the corresponding solutions to information frictions devised by economic agents, which typically involved contractual arrangements. Its findings had many applications, including insurance markets, income taxation, public procurement and economic regulation. Concerning the latter, theoretical developments from incentive theory have not only shifted the paradigm of economic regulation theory, but also permeated public policy.

1 Public-private partnership is an alternative nomenclature for essentially the same method, as well as private finance initiative, affermage, and build-operate-transfer contracts. The term concession, however, is more commonly used in Latin America, particularly in Brazil.

2 The perception of concession failure is primarily associated to the incidence of opportunistic contract renegotiation. For an overview of typical concession pitfalls in developing countries, see Guasch (2004).
This thesis draws from the approaches and methods of contract theory, especially the theory of incentive regulation, to address selected issues in concession design. It consists of three essays that analyze economic incentives and tradeoffs pertaining to the contractual relationship between a public authority and the concessionaire. The analysis is founded on a principal-agent modeling of the concession partnership under asymmetric information, focusing on the optimal design of the concessionaire’s remuneration mechanism. The final objective is to derive applied lessons for concession policy that are grounded on formal analysis, though economic models often abstract from details that are relevant in practice. Such has been the tradition of regulation theory, as explored below.

Regulation theory

The theory of economic regulation is a branch of economics dedicated to understand, as well as prescribe, how state intervention in the activities of private agents takes place through rules and contracts, as opposed to direct intervention with command and control instruments like taxes, quotas, and state entrepreneurship. The main motivation for economic regulation is the occurrence of market failures, such as the presence of externalities, informational frictions and the abuse of market power. Thus, the state relies on its coercive power to, in principle, lead society to a more efficient allocation of resources than the one that would prevail in the absence of intervention.

Over the past forty years, regulation theory has been substantially impacted by the development of models concerning strategic interaction under incomplete information, a process also known as the “incentives revolution.” Following the publication of seminal papers like Baron and Myerson (1982) and Laffont and Tirole (1986), the understanding of the interaction between regulator and regulated firms has been revisited to incorporate the effect of information frictions, in the form of adverse selection and moral hazard, which fundamentally altered the paradigm of regulatory practice in industrialized countries.3

Since then, regulation theory has received contributions focused on context-specific issues, in addition to theoretical refinements aimed at giving more realism to economic models.4 Concerning specific issues, for instance, in the 1990s, newly liberalized industries with a history of vertical integration pressed regulation theory to advance issues like the optimal discipline and pricing of access to critical infrastructures. Applied questions included: should an enterprise that controls power plants also be allowed to control transmission lines? Should a railroad operator grant right-of-way to firms that only operate trains? Should a telecommunications company make its network available to its competitors? At what interconnection charge?

3 For example, the period of liberal reforms in the 1980s, led by Margaret Thatcher in the United Kingdom and Ronald Reagan in the United States, was marked by a shift away from regulatory models based on cost-of-service reimbursement, in favor of schemes with higher incentive power like price caps.

4 For a summary of recent developments in regulation theory, refer to Armstrong and Sappington (2007).
As to refinements to the benchmark regulation model, it is worth mentioning the rise of dynamic contract theory. In the late 1980s, a number of contributions were made on the consequences of dynamic environments to contract design, especially concerning imperfect commitment and the possibility of voluntary renegotiation. Moreover, the field of incomplete contracts drew from the property rights literature, among others, and contributed with several insights that explained actual phenomena. For example, one paper argued that the very concession model was devised to remedy the imperfect verifiability of investment quality (Hart, 2003).

Concessions are a form of regulation by contract. Enterprises that are usually structured as concessions, like highways and transmission lines, have natural monopoly characteristics that justify regulatory constraints on pricing and operating decisions. As opposed to the regulation of utilities, which are private enterprises operating public services indefinitely, concessions have a limited term, though long enough to incentivize the firm to operate as if it owned the asset. Hence, the optimal regulation environment of Laffont and Tirole (1993), which entails a one-time interaction, is especially suitable to model the design of concession partnerships.

In that sense, the present thesis also represents an inquiry into the optimal design of regulatory policies. Each chapter proposes a model of optimal regulation under different specifications of the interaction environment between the procuring authority (regulator) and the concessionaire (regulated firm). The investigation adopts principal-agent theory as its main approach, thus relying on mathematical models as representative of strategic interactions. Under hypotheses of rational behavior, the analysis aims to derive optimality results for the design of rules and contracts.

This work does not touch regulatory issues other than the optimal design of cost-based remuneration mechanisms, such as the regulation of access to critical infrastructures, the design of universal service obligations, and risk-based solvency regulation. It is also outside the scope of this thesis to carry out rigorous empirical analysis of structures, conducts and outcomes in matters of regulation. Although there exist pioneering works in this sense, incentive regulation is little studied from an empirical viewpoint. That is so for several reasons, not least due to the lack of systematic databases, as well as the prevalence of latent variables and measurement errors that deem mainstream empirical methods unreliable.5

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5 For instance, it is not possible to consider regulatory change as exogenous for the purpose of its effect. Some recent studies attempt to circumvent this problem; see e.g. Perrigne and Vuong (2011).

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The economics of concession contracts

The standard concession consists of a long-term contractual relationship between the government, usually represented by a procurement agency or ministry, and a private party called the project company, or concessionaire, usually representing a consortium of firms (sponsors). Other stakeholders include service users, debt lenders, and insurance providers, in addition to other mandated government bodies (e.g. audit offices).
The main elements that characterize a concession partnership are as follows:

(i) the bundling, into a single contract, of the construction and operations phases, including maintenance over the project’s lifecycle;

(ii) the allocation of material risks to the private sector, usually through the utilization of private resources (equity and debt) to finance the investment;

(iii) the linkage of the private partner’s remuneration to observed performance; and

(iv) the handback of the infrastructure asset at contract expiry (World Bank, 2017b).

Among all these elements, the most important is the bundling of construction and operations in the concession contract. Infrastructure projects tend to have the characteristic that investments in asset quality are only partially verifiable in the short run. Thus, when construction works are separately sourced, the contractor fails to internalize the impact of asset quality on operation and maintenance costs. The bundling of construction and operation, in turn, aims to achieve the minimization of the project’s lifecycle costs (Engel et al., 2009; Iossa and Martimort, 2014).

In any case, the attributes of a concession partnership make it a strong candidate to be studied under light of the principal-agent model. There is inherent asymmetric information between the procuring agency and the concessionaire, regarding both the efficiency, internal organization and management style of the project company (adverse selection), and the actions, or effort that managers might implement to meet the contract specifications (moral hazard). The risk allocation and the compensation scheme of the concessionaire, defined in the contract, are akin to mechanisms prescribed by incentive theory to optimize well-known tradeoffs, like the one between allocative efficiency and informational rent.

A concession transaction consists of a rite with many phases, reports and interactions between the procuring agency and the potential bidders to the project. It begins with project identification and technical selection, when the agency typically conducts a thorough appraisal to verify whether the project is suitable to be structured as a concession. Afterward, the project is launched to the tendering phase, in which the agency publishes a request for proposals, and awards the contract in a competitive process based on a pre-specified set of criteria. Next comes the construction and operations phase, when the agency monitors the project’s performance according to the terms of the contract, and manages the concessionaire’s remuneration. Finally, the contract expires and the government takes control of the asset, to operate it itself or to renew the concession by means of another procurement process.

Bearing in mind the richness of detail involved in a concession partnership, the present work focuses its attention on the optimal design of the project company’s compensation scheme. The concessionaire’s remuneration is the main incentive mechanism driving the economic efficiency of the concession arrangement, as well as the principal channel for transferring risks. On the other hand, the modeling strategy pursued in this thesis abstracts away from several aspects relevant to real-world concessions, such
as the bundling of building and operations, or even the project’s concession-suitability analysis. Such simplifications are not only key to model tractability, but also permit to isolate the object of analysis and narrow the scope of economic conclusions. Consequently, model results cannot be interpreted literally, but consist of narratives that should prompt reflections and insights onto the shaping of concession policy.

The toolkit employed in this thesis is considerably homogeneous across the chapters. In all cases, the main interest lies in studying the properties of the optimal design of the concessionaire’s remuneration, under the light of incentive theory. Under different setups for the regulator-firm interaction, the three essays depart from the same workhorse model, namely, the principal-agent procurement model with cost observability of Laffont and Tirole (1993, Chapter 1). For reference, the basic Laffont-Tirole model is outlined below.

### The Laffont-Tirole model

The regulator (principal) hires a firm (agent) to execute a project with social value $S > 0$. The project’s total cost is given by $C = \beta - e$, in which $\beta$ denotes an exogenous technological parameter, representing the firm’s efficiency or, alternatively, the project’s intrinsic cost. In turn, the variable $e \geq 0$ denotes the firm’s actions, or effort exerted to reduce total costs. The regulator observes the total cost $C$ realized by the firm, but cannot distinguish between its exogenous and endogenous components. For instance, the principal cannot tell if a high cost observation indicates an intrinsically expensive project or is due to low effort by the firm.

The firm incurs a non-monetary cost (disutility) to exert effort, given by an increasing, convex function $\psi(e)$. Let $T$ represent the total remuneration received by the firm. It is convenient to denote the net revenue in excess of cost reimbursement as $t = T - C$. Thus, the firm’s profit from the partnership can be expressed as $U = t - \psi(e)$.

Both parties are risk neutral, and any contract between them is enforced with full commitment. Under complete information, the regulator would intend to minimize the social cost of executing the project, given by $S - (\beta - e) - \psi(e)$. Assuming $S$ large enough to justify the viability of the project for all possible $\beta$ values, the efficient level of effort would be $\psi'(e^{fb}) = 1$.

Under asymmetric information, the regulator must provide incentives for the firm to exert cost-reducing effort, to a level as close to the first best $e^{fb}$ as possible. However, the design problem has to respect an incentive compatibility constraint: in any feasible mechanism, a firm that execute a type-$\beta$ project cannot prefer to report facing a type-$\beta'$ project. Moreover, the firm’s acceptance of the contract is voluntary; hence, any feasible mechanism must guarantee the firm a payoff at least as large as its opportunity cost (normalized to zero).

The mechanism (contract) offered by the regulator to the firm consists of a remunera-
tion policy that depends on the project’s realized cost, given by the function $T(C)$. By the revelation principle (Myerson, 1979), the problem may be simplified to the analysis of direct mechanisms of the form $(T(\beta), C(\beta))$, in which the regulator asks the firm to report the project’s intrinsic cost (type), and, subsequently, makes a payment of $T(\beta)$ and recommends a cost performance $C(\beta)$ to each reported type $\beta$.

The restriction to direct mechanisms simplifies the expression for the constraints on design problem. For one, incentive compatibility constraints may be written as:

$$U(\beta) \equiv t(\beta) - \psi(\beta - C(\beta)) \geq t(\hat{\beta}) - \psi(\hat{\beta} - C(\hat{\beta})), \quad \forall \beta, \hat{\beta}$$  \quad (IC)

Furthermore, the participation constraints may be expressed as:

$$U(\beta) \geq 0, \quad \forall \beta$$  \quad (IR)

Let $F(\beta)$ denote the probability distribution over possible project types, according to the regulator’s prior belief.\footnote{The assumption on the regulator’s prior includes the distribution’s support $[\underline{\beta}, \overline{\beta}]$, comprising the range of possible project types.} With these elements in hand, one may formulate the contract design problem as the following optimization:

$$\min_{T(\beta), C(\beta)} \int C(\beta) + \psi(\beta - C(\beta))dF(\beta)$$

s.t  (IC), (IR), $C(\beta) \geq 0$

The regulator’s objective is to minimize the expected social cost of executing the project, but subject to the frictions of information asymmetry. The optimal mechanism allocation, i.e. the principal’s effort recommendation to the agent $e^*(\beta)$, is characterized by the following equation:

$$\psi'(e^*(\beta)) = 1 - \frac{F(\beta)}{f(\beta)} \psi''(e^*(\beta))$$

Moreover, the firm’s equilibrium profit (rent) is given by:

$$U^*(\beta) = \int_{\beta}^{\overline{\beta}} \psi'(e^*(z))dz$$

The optimal mechanism resolves the central tradeoff of incentive regulation, namely, between allocative distortions and information rent. The firm commands economic rents due to its superior information, so that truth-telling incentives remain consistent. Consequently, the regulator optimally chooses to set effort recommendations below the efficient level for all project types, i.e. $e^*(\beta) < e^{fb}$. Were it not the case, the regulator would leave excessive rents to more efficient firms.
Finally, the optimal nonlinear remuneration schedule is given by:

\[ T^*(\beta, C) = t^*(\beta) - \psi'(e^*(\beta)) [C - C^*(\beta)] \]

where \( t^*(\beta) = U^*(\beta) + \psi(e^*(\beta)) \)

Laffont and Tirole (1986; 1993) have demonstrated that the optimal schedule stated above has an equivalent implementation, consisting of an infinite menu of linear cost-reimbursement rules. Put another way, the optimal remuneration mechanism entails the regulator offering the firm a range of combinations between fixed payments, denoted \( a(\beta) \), and fractions of the project’s total cost to be borne by the firm, represented by \( b(\beta) \), so the firm’s net payment is expressed as:

\[ t^*(\beta, C) = a(\beta) - b(\beta) \cdot C \]

The parameters \( (a, b) \) are designed to decentralize the effort decision to the firm. Therefore, a firm facing a type-\( \beta \) project self-selects into the option from the menu devised to implement its type’s cost recommendation \( C^*(\beta) \). Moreover, the slope \( b(\beta) \) of the incentive scheme induces the firm to choose the second-best optimal effort choice \( e = e^*(\beta) \).

Table 1 below illustrates an optimal menu of two-part remuneration policies, for a selection of project types from the range \([\beta, \bar{\beta}]\). Firms facing less expensive projects have an incentive to pick higher-powered remuneration plans, featuring higher fixed payments and lower reimbursement shares. Conversely, less fortunate firms (that face expensive projects) feel compelled to choose schemes that progressively approximate a cost-plus contract.

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<th>Remuneration plan</th>
<th>Fixed payment</th>
<th>Cost reimbursement</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Aggressive”</td>
<td>86.11</td>
<td>0.00</td>
</tr>
<tr>
<td>“Moderate”</td>
<td>78.30</td>
<td>0.087</td>
</tr>
<tr>
<td>“Standard”</td>
<td>59.68</td>
<td>0.358</td>
</tr>
<tr>
<td>“Flexible”</td>
<td>33.62</td>
<td>0.661</td>
</tr>
<tr>
<td>“Security”</td>
<td>0.00</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 1: Optimal menu of linear remuneration policies

The Laffont-Tirole model has become a cornerstone of regulation theory. It fits quite expediently into the purpose of analyzing incentive properties of concession remuneration policies, which tend to be performance-based. The following chapters address applied issues in concession design that are of increasing complexity, relying on the strategy of modifying and adapting the present modeling environment.
Summary of thesis contributions

In tune with the overarching theme of concession design under information frictions, the present doctoral thesis aims to address applied questions that are relevant from a policy viewpoint. The main research contributions of the following chapters are summarized below.

Chapter 1. Dynamic contracting based on forecast information

Chapter 1 acknowledges that a concession partnership is a dynamic interaction. At the procurement phase, the operator has imperfect information regarding project costs (i.e. forecasts), whereas at the construction and operation phases costs may reveal to be higher or lower than projected.

The principal may format contract design to exploit the agent’s uncertainty and improve the rent-efficiency tradeoff. This can be accomplished by a dynamic menu, involving the payment of differentiated award fees, and contingent sub-menus of linear remuneration plans, consisting of fixed payments and cost reimbursement fractions.

The dynamic environment can straightforwardly accommodate the competitive bidding process. As more aggressive bidders reveal optimistic forecasts, they should be subject to higher-powered incentive schemes.

Questions of Chapter 1:

- What is the shape of the optimal remuneration policy for the concessionaire when it obtains sequentially more accurate estimates of the project’s true cost?
- How is the dynamic procurement policy different from the benchmark static result, and how is it different from policies that are observed in practice?
- How is the optimal dynamic policy altered when the environment admits the possibility of project cancellation (“shutdown of the firm”), or the award of the contract through a competitive process?

Chapter 2. Effect of exit rights in dynamic contracting

Sequential screening mechanisms, such as in Chapter 1, severely punish the agent in unfavorable states of nature. Such punishment may not be credible or feasible, due to:

- Limited liability protection to the concessionaire, or
- a “financial equilibrium” principle applicable to government contracts.

Chapter 2 models the problem of optimal dynamic contracting under ex-post constraints on the maximum loss the firm is allowed to incur (i.e. ex-post participation).
The results indicate optimal pooling of different forecasts into a single contract, but separation of ex-post types, i.e. the model effectively collapses to the static case.

There is an optimal limit to the firm’s liability from the viewpoint of social welfare maximization, which has implications for the discussion on risk allocation and contractual rebalancing (renegotiation).

**Questions of Chapter 2:**

- How does bestowing exit rights on the firm, like the right to preserve its initial financial equilibrium, affect the optimal design of the regulatory mechanism in a dynamic private-information environment?

- What is the economic effect of exit rights, compared to the case where the firm is fully liable to events that occur after the contract is signed? (For example, finding that the initial project cost forecast was underestimated)

- What is the economic effect of intermediate situations, for example, when the firm is financially liable up to its equity participation in the project?

**Chapter 3. Robust contracts and project selection**

Chapter 3 responds to Wilson’s Critique in the context of the procurement and regulation, i.e. that Bayesian contract design is too sensitive to model primitives. Thus, the model relaxes the assumption that the principal knows the distribution of project costs; in the present setup, she knows only the mean and variance of project types.

The design problem follows a min-max criterion, so the regulator’s optimization considers a worst-case cost distribution. The optimal contract depends solely on the distribution’s moments, and hence, is robust to information the regulator does not have.

The framework is applied to the problem of project selection: how to choose among technologies with different mean-variance profiles? The robust selection criterion insulates the planner from potentially large losses due to prior misspecification.

**Questions of Chapter 3:**

- What is the shape of the robust mechanism when the regulator is ignorant of the full probability distribution over project costs, aside from the first two moments?

- Given the robust regulatory mechanism, how does the planner formulate a criterion for choosing among technological alternatives, or else, how does she robustly rank her preferences over means and variances of project costs?
Chapter 1

Using cost forecasts to regulate concessions

This chapter studies cost-based procurement under dynamic asymmetric information. It presents a principal-agent model addressing how prospective information about project characteristics (forecasts) elicited from the concessionaire can be incorporated into the remuneration mechanism to improve rent extraction and reduce distortions. The optimal mechanism is implemented by a dynamic menu: the agent first chooses a level of concession fee due at contract signature and, after observing the project’s actual cost, chooses a linear remuneration policy from a contingent menu. The latter exhibits increasing incentive power the more aggressive were the previous forecast report, revealed by the fee choice. This chapter also provides extensions of the basic model that (i) consider the option of project cancellation if expected net benefits are negative, and (ii) consider multiple firms bidding for the contract in a competitive process.
1.1 Introduction

The inquiry of the present thesis begins with the study of optimal procurement models in a dynamic setting. The design of concession partnerships is typically concerned with issues of dynamic incentives, such as the presence of evolving private information. Therefore, the analysis of optimal mechanisms in a multi-period principal-agent model, insofar as it departs qualitatively from static benchmark results, should provide valuable insights for concession policy.

Some important parameters for a concession’s economic performance, like construction costs and demand volumes, are subject to substantial uncertainty at the closing of the concession deal, and typically take the form of projections and forecasts. More importantly, such uncertainty is unevenly distributed between the procuring authority and the concessionaire. Not only has the latter better knowledge of its own operational efficiency over time, but also, as the project advances from appraisal to execution, specific technical conditions become clearer to the contractor.

The consequent asymmetry of information imposes restrictions on the economic efficiency attainable by the concession contract, commonly known as the rent-efficiency tradeoff. However, the dynamic mechanism design literature suggests there is a way to alleviate that information friction, and thus improve economic outcomes, by exploiting the agent’s uncertainty with a dynamic incentive scheme (Börgers, 2015, ch. 11). Such strategy, referred to as sequential screening, involves eliciting private information from the concessionaire in multiple periods, and conditioning the firm’s remuneration policy on the entire sequence of reports.

The starting point of this chapter is to investigate how to apply the sequential screening framework to concession design. The present essay studies a cost-based procurement model under dynamic asymmetric information, in which a risk-neutral contractor has privileged information about project costs over the life of the contract. The basic model is devised to answer (i) how the concessionaire’s remuneration policy should optimally respond to sequential cost information, and (ii) the dependence of the optimal mechanism on the model’s information structure, with special concern to the firm’s forecasting technology.

The analysis is set up in the canonical procurement environment of Laffont and Tirole (1986), where the principal uses cost observations to infer the firm’s private information, namely its innate efficiency and the effort exerted to improve cost performance. The screening of firm types is obtained through a menu of linear remuneration schemes, consisting of fixed payments and cost reimbursement shares. In equilibrium, the firm self-selects into the contract that induces the second-best level of effort desired by the principal. The present essay extends the Laffont-Tirole framework into a dynamic setting of serially correlated private information.

1 Under risk neutrality, the firm’s effort is inferred with certainty from its cost performance under the chosen remuneration scheme, hence the model is said to entail “false moral hazard”.
The basic model has two periods. In the first period, the firm privately observes an informative signal about the project’s intrinsic cost (type), before signing the contract. In the second period, the firm discovers the project’s true cost, and decides whether to exert costly effort to improve cost performance. The procuring agency observes the project’s realized cost, but cannot distinguish between an intrinsically expensive project or low effort by the firm. The contract must incentivize the firm to reveal its private information truthfully in both periods, and induce the firm to voluntarily participate in the partnership.

The firm is able to forecast the project’s type by means of a forecasting technology. Knowledge of such technology permits the principal to consider first-period information when designing the mechanism, by requiring a report of the agent’s project cost estimate and adjusting the shape of the cost-reimbursement schedule accordingly. In the second period, the firm submits an updated cost estimate, and executes the project under its assigned remuneration scheme. Payoffs are realized according to contract terms, so the model abstracts from other dynamic issues like imperfect commitment or enforcement.

The main results are as follows. The optimal mechanism features sequential screening of the firm’s private information, i.e. offering remuneration menus that discriminate firms on both dimensions of cost signals and ex-post project types. The informativeness of the firm’s forecasting technology permits the regulator to attain lower expected procurement costs and a less distorted social surplus under the optimal sequential mechanism, compared to the static Laffont-Tirole contract. The less informative the forecast, the closer the optimal contract gets to the first best, as the environment approaches symmetric information.

The dynamic mechanism allows increased rent extraction because it “toughens the deal” to the firm, in the sense that more aggressive firms are subject to even steeper-sloped incentives than they would in a static contract. Consequently, the mechanism assigns large penalties (rewards), in terms of ex-post losses (profits), to firms that underestimate (overestimate) project costs by a large amount. As the firm cares only about expected profits, it accepts the partnership knowing that it might make heavy losses in some states of nature.

The optimal contract is implemented by a dynamic menu consisting of concession award fees, due at contract signature, and contingent menus of linear reimbursement schemes. The first-period menu is monotonic, i.e. the higher the fee chosen in period one, reflecting more aggressive forecasts, the higher-powered are the plans available to the firm in period two. Further, second-period menus also exhibit a monotonicity property in that firms facing less expensive projects choose remuneration plans with large fixed payments and small fractions of cost reimbursement, and thus have an incentive to exert higher effort.

To improve on the realism of the model, the present chapter provides two extensions. First, it considers that the project’s social benefit might not be large enough to justify its economic viability for all cost scenarios. In such case, the problem also becomes that of
an investment decision under uncertainty. Apart from the rent-efficiency tradeoff, the principal must balance the probabilities of making errors pertaining to false negatives, i.e. failing to implement socially desirable projects, and false positives, i.e. implementing projects with negative net benefits. However, since both parties are risk-neutral, the problem exhibits a convenient separation property between incentives design and defining the threshold cost forecast beyond which the project is canceled.2

Second, this chapter presents an extension of the model to multiple agents. Assuming independent private values, it derives the optimal contract award mechanism, namely an auction based on the highest concession fee. Bidding strategies are characterized in Bayesian Nash Equilibrium, and depend on the firm’s signal, the number of bidders, and the shape of the ex-post incentive scheme. Again, the problem exhibits a useful dichotomy between the design of incentives and the selection of the preferred bidder, which means the ex-post menu is formulated identically to the case with a single firm.3 However, increasing competition both enhances rent extraction and lowers the level of allocative distortion.

Overall, the model captures the insight that it pays to engage potential bidders early in the procurement process, so to extract information that might be useful to set the contractor’s compensation policy. An early engagement strategy has been pursued by Brazilian procurement agencies in the recent past, by means of a process to solicit proposals that included cost and demand forecasts from interested parties.4 Notwithstanding, that strategy had a different motivation: to save on project preparation costs, as Brazil’s rigid procurement legislation hampers the sourcing of feasibility studies. Hence, the alluded policy did not take into account the dynamic incentive issues to which this chapter is devoted. The present analysis suggests that any information unveiling the firm’s private forecasts, like bids at competitive tenders, should be incorporated to toughen the firm’s remuneration mechanism, as it permits to mitigate informational asymmetries, and consequently, improve the project’s net social benefit.

Another related theme in concession policy, and more generally in public procurement, is the prevalence of systematic error in forecasting costs and demand, leading to pervasive cost overruns and contract renegotiation (Flyvbjerg et al., 2002; Guasch, 2004). The extent to which such estimation errors are intrinsic, e.g. a consequence of optimism bias, or deliberate, reflecting contractor’s opportunism, is relevant to determine the appropriate remedy. Though the present chapter does not address the issue directly, as the model forces truth-telling in all periods, the monotonicity property of the optimal mechanism has an important interpretation in that regard. The prescription of providing higher-powered incentives for more optimistic bidders, once violated, fails to guarantee truthful reporting of forecasts in the first period. Hence, the deviation from such monotonicity in practice might explain the occurrence of low-balling and overruns as the consequence of perverse dynamic incentives.

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2 The approach to project cancellation is similar to the shutdown of the firm, presented in Laffont and Tirole (1993), Chapter 1, Section 1.4.5.
3 See Laffont and Tirole (1993), Section 7.4, for a didactic exposition on auctions of incentive contracts. The approach in this essay follows Riordan and Sappington (1987).
4 Known in Portuguese as “Procedimento de Manifestação de Interesse” (PMI).
This chapter is organized as follows. After brief comments on the related literature, Section 1.2 sets up the basic model and characterizes the optimal procurement contract, also discussing the role of forecast informativeness on incentives design. In turn, Section 1.3 presents a numerical exercise to illustrate the main elements of the optimal dynamic mechanism. Section 1.4 extends the basic model to allow project cancellation if cost forecasts are too large, whereas Section 1.5 extends the model to multiple agents and endogenizes the contract’s award criterion. Lastly, Section 1.6 concludes and comments on some limitations. All formal proofs are left to Appendix 1.A.

Related literature

This chapter contributes to the theory of optimal procurement and regulation. Building on the canonical model of Laffont and Tirole (1986), it extends their benchmark results to a dynamic setting and provides additional support to the optimality of incentive schemes that are linear on cost performance. Armstrong and Sappington (2007), in their comprehensive survey on regulation theory, stress the importance of modeling environments that resemble more accurately real-world circumstances and specific regulatory settings. This essay also represents one step in that direction.

The present essay incorporates into the cost-based procurement environment the concept of informativeness of ex-ante information, introduced by Baron and Besanko (1984). The basic model is an application of the sequential screening framework pioneered by Courty and Li (2000) for a single agent, and by Esö and Szentes (2007) for multiple agents, and later generalized by Pavan, Segal, and Toikka (2014). In particular, it is an application of the dynamic adverse selection model to a curvature environment, as in Dai, Lewis, and Lopomo (2006). The innovation here is to consider the specific setting of procurement with adverse selection and (false) moral hazard, which entails its own applied insights and interpretations.

The model is similar to Krähmer and Strausz (2011), who study dynamic procurement under endogenous information acquisition in a quasi-linear environment, in which payoffs are linear in the agent’s type. In contrast, our model considers cost-based procurement, that essentially features curvature in the agent’s payoff function with respect to the private information parameter, leading to results fundamentally different to the binary probability-of-execution mechanism of Krähmer and Strausz. The present results build on an earlier version of the present essay, reported in Andrade (2014).

The contribution of this essay to the study of concession design takes the form of applied theory, due to the difficulty of conducting empirical applications with models of asymmetric information. Such an approach is standard in economics, beseeming to cite Oliver Hart (2003) and Engel, Fischer, and Galetovic (2009) as related works that approach applied questions from a theoretical perspective. Nevertheless, the work of Perrigne and Vuong (2011) propose an empirical framework to principal-agent models and Gagnepain et al. (2013) present an interesting application of a structural empirical model with asymmetric information.
1.2 The model

A procuring agency (regulator) wishes to delegate the execution of a project with social value $S > 0$ to a concessionaire (firm). This section assumes $S$ is sufficiently large to justify the project’s viability in all cost scenarios. The project’s total cost is given by

$$C = \beta - e$$

where $\beta \in B$ represents the project’s intrinsic cost and $e \geq 0$ denotes the effort exerted by the firm to reduce costs to level $C$.

The interaction happens in two periods. In the first period, the contract is signed between the regulator and the firm (“contracting”). In the second period, the project is executed (“production”). At the contracting stage, the firm is uncertain about the project’s intrinsic cost $\beta$, but observes a private signal $s \in S$ which is correlated with $\beta$.

In the production period, the firm learns the project’s actual intrinsic cost $\beta$, and based on the contract’s incentives, chooses the level of cost-reducing effort $e$ to exert. Post completion, the regulator observes the project’s realized cost $C$, but cannot distinguish between an intrinsically expensive project (high $\beta$) and little investment in cost reduction by the firm (low $e$). Hence, the firm’s remuneration must be based on its cost performance $C$ and any information the regulator elicits from the firm along the interaction (e.g. information revealed from the option the firm picks from a menu).

The firm incurs a non-monetary cost to exert effort, denoted $\psi(e)$. Assume the effort disutility function exhibits decreasing returns (curvature), i.e. $\psi', \psi'' > 0$. Assume also that $\psi''' \geq 0$, which is important to rule out random mechanisms.\(^5\)

Let $\bar{C}_s = \mathbb{E}_\beta[C|s]$ denote the firm’s endogenous forecast of the project’s total cost, which depends on the its signal $s$ as well as on the overall incentive structure. The regulator designs a dynamic remuneration mechanism $T(\bar{C}_s, C)$ specifying a menu of nonlinear payment functions conditional on the firm’s reported forecast for the project’s total cost, denoted $\bar{C}_s$, and on effective cost performance $C$.

The regulator’s objective is to minimize the total expected procurement costs:\(^6\)

$$\mathbb{E}_{s,\beta} [T(\bar{C}_s, C)]$$

subject to the firm’s voluntary participation in the mechanism. To induce participation, the regulator must secure the firm an expected payoff $\mathbb{E}_{\beta} [T - C - \psi(e)|s]$ superior to its reservation utility, normalized to zero. It is useful to denote the firm’s payoff in terms of the net transfer $t = T - C$ in excess of cost reimbursement. Both regulator and firm are assumed to be risk neutral, and the contract is enforced with full commitment.

\(^5\) See Laffont and Tirole (1993), Chapter 1, Appendix A1.1.

\(^6\) The present chapter considers primarily “procurement cost minimization”, instead of the “utilitarian welfare maximization” of Laffont and Tirole (1993), in order to focus strictly on incentive issues.
First best. With complete information, the regulator would offer a fixed-price contract to induce the firm’s participation and fully extract its information rent. The first-best effort level follows from the minimization of $E_{\beta}[\beta - e + \psi(e)|s]$, yielding $e_{fb} = \psi^{-1}(1)$ for any combination of $s$ and $\beta$. Denote $k = e_{fb} - \psi(e_{fb})$ the maximum level of cost reduction achievable by an incentive contract. The first-best contract sets the project price equal to $E[\beta|s] - k$.

Information structure

The information structure of the model encompasses (i) the regulator’s prior distribution over signals, denoted $F(s)$, and (ii) the firm’s forecasting technology, denoted $G(\beta|s)$, which corresponds to the distribution of the project’s intrinsic cost $\beta$ conditional on the signal $s$.

Assume the signal $s$ and the project’s intrinsic cost $\beta$ take values, respectively, in supports $S = [s, \infty]$ and $B = [\beta, \overline{\beta}]$, with $\overline{\beta}$ possibly infinite. Assume $[\beta, \overline{\beta}]$ is the same regardless of $s$. We need regularity assumptions on the information structure in order to ensure the optimal mechanism is well behaved.

Assumption 1.1. The distribution of signals satisfies a monotone hazard rate property (MRH), i.e., the function $h(s) = F(s)/f(s)$ is increasing in $s$.

Assumption 1.2. The family of forecasting distributions $\{G(\beta|s)\}_s$ is ranked by order of first-order stochastic dominance (FOSD), i.e. for $s_1 > s_0$, we have $\int \phi(z) dG(z|s_1) \geq \int \phi(z) dG(z|s_0)$ for any increasing function $\phi(.)$. Let $g(\beta|s)$ denote the pdf associated with the distribution $G(.|s)$.

Assume $G(\beta|.)$ is differentiable. A direct consequence of Assumption 1.2 is that $\frac{\partial}{\partial s} G(\beta|s) \leq 0$ for each $\beta$, i.e., a higher $s$ signals higher $\beta$ realizations. In particular, a high signal indicates a high expected intrinsic cost $E[\beta|s]$, so a low $s$ is good news.

Let $\iota(s, \beta)$ denote the informativeness of the firm’s forecasting technology $G(.|s)$, or alternatively, the impulse-response function of the firm’s signal onto the project’s types likelihood:

$$\iota(s, \beta) = -\frac{\partial}{\partial s} G(\beta|s) / g(\beta|s)$$

Assumption 1.3. The informativeness measure satisfies the following properties:

$$\frac{\partial}{\partial s} \iota(s, \beta) \geq 0 \text{ and } \frac{\partial}{\partial \beta} \iota(s, \beta) \geq 0$$

---

7 Therefore, $k$ is the minimum value possible for $\beta$ to ensure positive first-best prices, i.e., $B \subseteq [k, \infty]$.
8 See, for instance, Pavan, Segal, and Toikka (2014), Section 2.1.
Assumption 1.1 is standard in the literature, and is relevant to ensure an ex-ante separating mechanism. Assumption 1.2 imposes an ordering on the set of signals $S$, and is central to guarantee that the optimal mechanism satisfies global incentive compatibility. Assumption 1.3 is technical, and also ensures the monotonicity of the optimal mechanism with respect to second-period types. See the proof of Proposition 1.1.

The design problem

With a large project value $S$, the regulator’s problem is to minimize the expected total payment to the firm $E[t + C]$. The regulator faces asymmetric information, so she must provide incentives for the firm to reveal accurately his private signal $s$, and subsequently, the project’s intrinsic cost $\beta$.

By the revelation principle for multistage games (Myerson, 1986), the problem is equivalent to finding an optimal direct mechanism in a feasible set, characterized by incentive compatibility (IC) and individual rationality constraints (IR). A direct mechanism is expressed as a pair of functions $(t(.), C(.))$ that assigns a net transfer $t(\hat{s}, \hat{\beta})$ and a cost recommendation $C(\hat{s}, \hat{\beta})$ for each sequence of announcements $(\hat{s}, \hat{\beta})$. Alternatively, the mechanism can be expressed in terms of an effort recommendation $e(\hat{s}, \hat{\beta}|\beta) = \beta - C(\hat{s}, \hat{\beta})$.

To be precise, the timing of the direct mechanism is described as follows:

**Period 1: contract signature**

- The firm observes a private signal $s$
- The regulator offers the firm a menu of contingent contracts $(t(s, \beta), C(s, \beta))$
- The firm decides whether to accept the contract and reports signal $\hat{s}$

**Period 2: project execution**

- The firm privately observes the intrinsic cost $\beta$ and submits a second report $\hat{\beta}$
- The regulator recommends a level of cost-reducing effort $e(\hat{s}, \hat{\beta})$, and the firm executes the project obediently to the recommendation
- The regulator reimburses the firm’s cost $C(\hat{s}, \hat{\beta})$ and further compensates it with net transfer $t(\hat{s}, \hat{\beta})$

Given the sequential nature of the model, each period’s truthful revelation is a one-dimensional problem. In the second period, as the signal report is irreversible, the firm’s only relevant decision is which $\beta$ to report. Hence, its ex-post payoff is:

$$u(\hat{\beta}|s, \beta) = t(s, \hat{\beta}) - \psi(e(s, \hat{\beta}|\beta))$$
From the first period viewpoint, note that truth-telling in the second period is the optimal continuation strategy for the firm. Hence, its ex-ante expected payoff is:

\[ U(\hat{s}|s) = \int_{B} u(\beta|\hat{s}, \beta) dG(\beta|s) \]

Let \( U(s) = U(s|s) \) and \( u(s, \beta) = u(\beta|s, \beta) \) denote, respectively, the firm’s ex-ante and ex-post payoffs (rents) from announcing its true types in the first and second periods.

The optimal design problem \((P)\) is defined as an expected cost minimization, subject to ex-ante participation and multi-period incentive compatibility constraints:

\[
\begin{align*}
\min_{\{t(.), C(.)\}} & \int_{S} \int_{B} \left[ t(s, \beta) + C(s, \beta) \right] dG(\beta|s)dF(s) \\
\text{s.t.} & \quad u(s, \beta) \geq u(\hat{\beta}|s, \beta), \quad \forall \hat{\beta}, \beta, s \quad \text{(IC-2)} \\
& \quad U(s) \geq U(\hat{s}|s), \quad \forall \hat{s}, s \quad \text{(IC-1)} \\
& \quad U(s) \geq 0, \quad \forall s \quad \text{(IR)}
\end{align*}
\]

The optimal mechanism

The optimal solution to problem \((P)\) is characterized by the following elements: a monotonic effort recommendation schedule \( e^*(s, \beta) \) that screens the firm’s types sequentially; ex-ante and ex-post incentive compatible rents, \( U^*(s) \) and \( u^*(s, \beta) \), respectively; and a dynamic transfer schedule \( t^*(s, \beta) \). The solution is presented in Proposition 1.1.

**Proposition 1.1.** Under assumptions on payoff and information structures, the optimal direct mechanism is characterized by the following equations:

\[
\begin{align*}
C^*(s, \beta) &= \beta - e^*(s, \beta) \\
\psi'(e^*(s, \beta)) &= 1 - h(s)u(s, \beta)\psi''(e^*(s, \beta)) \\
U^*(s) &= \int_{\beta} \mathbb{E}_\beta [\psi'(e^*(z, \beta))u(z, \beta) | z] dz \\
u^*(s, \beta) &= U^*(s) - \int_{B} \psi'(e^*(s, \beta))G(\beta|s)d\beta + \int_{\beta} \psi'(e^*(s, z))dz \\
t^*(s, \beta) &= \psi(e^*(s, \beta)) + u^*(s, \beta)
\end{align*}
\]

Some comments on the optimal schedules are in order. First, note that the ex-ante information rent \( U^*(s) \) is decreasing, as more optimistic firms must be incentivized not to inflate signal reports by (IC-1).
Second, ex-post information rents \( u^*(s, \beta) \) are monotonic in \( \beta \), as predicated by (IC-2). The optimal mechanism assigns positive ex-post rents if the actual project cost \( \beta \) turns out to be low, and negative for high \( \beta \) realizations. The slope of punishment is steeper the more optimistic the cost forecasts.\(^9\) In particular, at the most expensive project ex-post we have \( u^*(s, \overline{\beta}) < 0 \) for all \( s \), which means all ex-post IR constraints would be violated (see Courty and Li, 2000, fn. 8).

Third, note the optimal effort recommendation entails a second-best distortion proportional to the signal distribution’s hazard rate \( h(s) \) and to the informativeness measure \( \iota(s, \beta) \) of the forecasting technology. The dependence on \( h(s) \) reflects the rent-efficiency tradeoff: the allocation at \( \beta \) is increasingly distorted to limit the rents left to firms that are more efficient in expectation (i.e. more optimistic).

In turn, the dependence on informativeness \( \iota(s, \beta) \) represents the benefit from sequential screening: the more informative is the firm’s signal, the better the regulator can extract its ex-post information rent. The role of informativeness is discussed in more detail below.

Lastly, the power of the incentive scheme increases with forecasting optimism. In particular, the first-best effort recommendation is only optimal for the most optimistic type \( s \), or alternatively, if the firm’s signal is completely uninformative ex-ante, i.e. \( G_s(\beta|s) = 0 \) (see Riordan and Sappington, 1987, Corollary 3).\(^{10}\)

### The role of informativeness

The following examples illustrate the role of the informativeness measure \( \iota(s, \beta) \) in the optimal procurement policy. Recall the optimal mechanism is characterized by equation (1.2):

\[
\psi'(e^*(s, \beta)) = 1 - h(s)e^*(s, \beta)\psi''(e^*(s, \beta))
\]

Note \( \psi'(e^{fb}) = 1 \) corresponds to the first-best level of effort. Therefore, asymmetric information entails a distortion to allocative efficiency that is proportional to (i) the curvature of the firm’s preferences \( \psi'' \), (ii) signals’ hazard rate \( h(s) \), and (iii) the informativeness of the firm’s forecasting technology \( \iota(s, \beta) \). The more informative the signal, the more distorted the optimal mechanism becomes.

**Examples.** Consider the following cases of relationship between the firm’s signal \( s \) and the project’s intrinsic cost \( \beta \). Note that all cases below satisfy a first-order stochastic ordering of the family of conditional distributions \( \{G(.|s)\}_s \) (Assumption 1.2).

---

\(^9\) Formally, \( \partial^2 u(s, \beta)/\partial s \partial \beta \leq 0 \). In the numerical exercise, the cutoff for negative ex-post rents, \( \beta^*(s) \) such that \( u(s, \overline{\beta}, \beta^*(s)) = 0 \), appears to vary with \( s \). See Figure 1.3.

\(^{10}\) In particular, note that \( G(.) \) is insensitive to \( s \) at \( s = \_ \) and \( \overline{\beta} \).
1. IID shock: \( \beta = s + \varepsilon, \varepsilon \sim iid H(.) \)

The simplest case of information acquisition is imposing an iid additive shock to
the signal, so that \( s \) becomes an unbiased estimator for the project’s intrinsic cost \( \beta \). This is the case analyzed by Laffont and Tirole (1986), where they demonstrate
the robustness property of linear contracts to an independent source of uncertainty, irrespective of the shape of \( H(.) \).

With iid shocks, the informativeness measure collapses to 1 regardless of \( H(.) \), and thus the optimal mechanism becomes:

\[
\psi'(e^*(s, \beta)) = 1 - h(s)\psi''(e^*(s, \beta))
\]

which coincides with the static Laffont-Tirole optimal mechanism.

If forecast errors are not independently distributed, they are correlated with the
firm’s average efficiency characteristics such as expertise, experience, or internal
organization. In such case, the signal \( s \) becomes informative of the project type \( \beta \), and the robustness property of the Laffont-Tirole mechanism does not hold.\(^{11}\)

2. Additive structure: \( \beta = \kappa s + (1 - \kappa)\varepsilon, \kappa \in (0, 1), \varepsilon \sim iid H(.) \)

The additive structure was proposed by Courty and Li (2000) and leads to a
global informativeness measure equal to \( \kappa \). The optimal mechanism becomes:

\[
\psi'(e^*(s, \beta)) = 1 - \kappa h(s)\psi''(e^*(s, \beta))
\]

A straightforward analysis of limiting cases ensues:

If \( \kappa \) approaches 0, the optimal mechanism approaches the first-best. Under no
informativeness, the first period incentive constraint becomes innocuous, since
the firm’s expected utility is invariant to \( s \). Hence, the regulator extracts the entire
surplus by reimbursing the firm’s expected cost; risk neutrality ensures it accepts
the contract.

If \( \kappa \) approaches 1, the optimal mechanism approaches the Laffont-Tirole mecha-
nism (although with no uncertainty). The environment’s dynamic structure col-
lapses as the firm knows the project’s cost with certainty from the first period,
thus the error term becomes irrelevant to the analysis.

3. Multiplicative structure: \( \beta = s^\kappa \varepsilon^{(1-\kappa)}, \kappa \in (0, 1), \varepsilon \sim iid H(.) \)

The multiplicative structure was also proposed by Courty and Li (2000), bearing
in mind its amenability to empirical applications. It leads to a variable measure

\(^{11}\)Esö and Szentes (2007) show it is possible to separate new information from the observed cost shock
by means of an orthogonal decomposition, over which the relevant impulse-response schedule may be
computed (see also Pavan et al., 2014).
of informativeness, given by $\iota(s, \beta) = \kappa \frac{\beta}{s}$. The optimal mechanism becomes:

$$\psi'(e^*(s, \beta)) = 1 - \frac{\kappa \beta}{s} h(s) \psi''(e^*(s, \beta))$$

The limiting cases are as follows:

If $\kappa$ approaches 0, the optimal mechanism approaches the first-best. If the regulator can sign the contract when the firm is unaware of the project’s type (or knows a useless signal), then the environment becomes one of symmetric information. Under risk neutrality, the firm accepts any payment that covers its opportunity cost in expectation, and thus, the firm is unable to secure any informational rent.

If $\kappa$ approaches 1, the informativeness measure becomes $\iota(s, \beta) = \frac{\beta}{s}$, which is equivalent to the case when the forecasting technology is an exponential distribution with mean $s$, i.e. $G(\beta|s) = 1 - e^{-\frac{\beta}{s}}$. Note that even when the exponent on the error term $(1 - \kappa)$ approaches zero, it remains beneficial to screen agents’ types sequentially.

4. Normal specification (Esö and Szentes, 2007): $\beta \sim N(s, 1/\tau_\beta), s \sim N(\mu_s, 1/\tau_s)$

The double-normal specification regards the pure marginal distributions of types, instead of the conditional distribution of period-2 types given period-1 type. Such specification is useful as it allows to compute the conditional distribution explicitly, which is given by:

$$G(\beta|s) = N\left(\frac{\tau_\beta \mu_s + \tau_s s}{\tau_\beta + \tau_s}, \frac{1}{\tau_\beta + \tau_s}\right)$$

The global informativeness measure is given by $\iota(s, \beta) = \frac{\tau_s}{\tau_\beta + \tau_s}$, implying the optimal mechanism becomes:

$$\psi'(e^*(s, \beta)) = 1 - \frac{\tau_s}{\tau_\beta + \tau_s} h(s) \psi''(e^*(s, \beta))$$

The case of perfect informativeness happens when $\tau_\beta \to \infty$, i.e., the variance of the $\beta-$marginal distribution becomes zero. Then, the conditional distribution $G(.)$ collapses to the $s$-marginal distribution, and the optimal mechanism becomes identical to Laffont-Tirole.

The case no informativeness happens when $\tau_\beta \to 0$, i.e., the variance of the $\beta$-marginal distribution becomes infinite. Then, the conditional distribution $G(.)$ becomes undefined (i.e. $\beta$ cannot be forecast), and the optimal mechanism approaches the first-best.

---

\[12\] It simplifies notation to express the variance in terms of its reciprocal, the precision parameter $\tau = 1/\sigma^2$. 

21
Remark. If the regulator’s prior is assumed to be normal, the hazard rate $h(s)$ explodes for high values of $s$, as the density $f(s)$ converges to zero at the upper tail of the normal distribution. Consequently, the optimal mechanism likely exhibits extreme distortions for the most inefficient types, such as negative effort recommendations. Such an unrealistic outcome entails that the normal-prior assumption does not work well without an analysis of the shutdown option, which is introduced in Section 1.4.

Discussion. The collapse of the dynamic mechanism into a static, Laffont-Tirole mechanism in the case of perfect informativeness requires qualification. By static we mean that it is useless to condition the mechanism on ex-post information, since the regulator cannot exploit the firm’s uncertainty to extract additional rent. The best she can do is to design the contract as if there were only one relevant period.

Then again, Krähmer and Strausz (2015) argue that, under ex-post participation constraints, the best the regulator can do is to ignore the first-period report (i.e. pooling over period-1 types) and condition the mechanism on second-period types only, even if the informativeness is not perfect. Theirs is an altogether different environment, and the static contract in that context is understood as the dependence on ex-post-only private information; the regulator’s prior becomes the unconditional (convoluted) distribution

$$\varphi(\beta) = \int f(s)g(\beta|s)ds.$$  

In this chapter, we compare the following two situations: (i) a static Laffont-Tirole mechanism conditioned only on first-period types, which is robust to independent additive shocks, and (ii) a dynamic mechanism conditioned on the reported first-period type, and also on the realization of the shock, when uncertainty is correlated to the first-period type. In the latter case, the regulator behaves sub-optimally if she fails to exploit such informativeness.

Dynamic implementation

The previous characterization refers to the optimal direct mechanism. The next step is to translate the optimal schedules in Proposition 1.1 in terms of their equivalent indirect implementation, based on information the regulator actually observes. The indirect mechanism is a dynamic menu of contracts, where the firm chooses a fee $\phi$ to pay the regulator in period 1, which, in turn, restricts the menu of linear contracts the firm faces in period 2 to $\{a(\phi), b(\phi)\}$, consisting of a fixed payment $a$ and a fraction of cost reimbursement $(1-b)$.

Denote $\bar{s} = \bar{C}_s^{-1}(C)$ as the function that assigns each signal $s$ to a level of the endogenous reported cost forecast $\bar{C}$. Moreover, fixing $s$, denote $\hat{\beta}_s = C^*-1(s, \beta)$ as the inverse mechanism allocation that retrieves the project’s intrinsic cost $\beta$ for each level of observed cost performance $C$. Proposition 1.2 presents the main implementation result.

---

13 Assumption 1.2 and the ex-ante monotonicity of the optimal mechanism imply that the endogenous forecast $\bar{C}_s = \int C^*(s, \beta)dG(\beta|s)$ is increasing in $s$. See the proof of Proposition 1.1.
**Proposition 1.2.** The optimal direct mechanism is implemented by a system of transfers based on the firm’s reported cost forecast $C$ and the project’s observed cost performance $C$:

$$T(\bar{C}, C) = \phi(\bar{C}) + \hat{T}(\bar{C}, C)$$

where $\phi(\bar{C})$ is an ex-ante franchise fee, given by

$$\phi(\bar{C}) = U^*(\tilde{s}) - \int_{B} \psi'(e^*(\tilde{s}, \beta))G(\beta|\tilde{s})d\beta < 0$$

and $\hat{T}(\bar{C}, C)$ is a menu of linear reimbursement rules, given by

$$\hat{T}(\bar{C}, C) = a(s, \beta) - b(s, \beta) \cdot C$$

with $b(s, \beta) = \psi'(e^*(s, \beta)) \in [0, 1]$

and $a(s, \beta) = \psi(e^*(s, \beta)) + \int_{\beta}^{\infty} \psi'(e^*(s, z))dz + b(s, \beta)C^*(s, \beta)$

The first-period part of the transfer, $\phi$, is a fixed payment conditioned solely on ex-ante information, namely the firm’s project cost forecast $\bar{C}$. As $\phi < 0$ in equilibrium, it may be interpreted as a franchise fee or a surety bond issued by the contractor to guarantee the project’s completion. The lesson here is that $\phi$ is a potential screening instrument, so it should be optimally designed as part of a menu.

The second-period part of the transfer scheme, $\hat{T}$, is a contingent menu of linear contracts that decentralizes the effort decision to the firm, similar to Laffont and Tirole (1986). The firm chooses its preferred remuneration scheme $(a, b)$ after it learns the project’s actual intrinsic cost $\beta$, and, subsequently, chooses its optimal level of effort under the respective scheme. Nevertheless, the menu available to the firm in period 2 becomes increasingly high-powered as the firm reports a more optimistic forecast in period 1. See Figure 1.6 for an illustration.

### 1.3 Numerical exercise

This section presents a numerical illustration of the optimal sequential procurement policy. Consider a project whose cost forecasts $s$ are uniformly distributed over interval $[s, \bar{s}]$. The firm’s cost-reduction technology is assumed to be a quadratic function:

$$\psi(e) = \alpha e^2 / 2, \quad e \geq 0$$

$$\alpha \in \left[0, \frac{1}{s-\bar{s}}\right]$$

Hence, the first-best optimal effort is $e^{fb} = 1/\alpha$ and the maximum cost-savings from incentives is $k = 1/2\alpha$. The firm is assumed to forecast the project’s cost with a shifted exponential distribution:

$$G(\beta|s) = 1 - \exp\left(-\frac{\beta-\beta}{s}\right), \quad \beta \geq \bar{\beta}$$
Such assumption yields a conditional mean $\mathbb{E}[\beta|s] = \beta + s \equiv \mu_s$ and an informativeness measure $\iota(s, \beta) = (\beta - \bar{\beta}) / s$, so it facilitates the computation of explicit forms for the optimal schedules. Figure 1.1 illustrates the forecasting technology.

![Figure 1.1: Forecasting technologies with exponential assumption](image)

The optimal effort recommendation is given by the first order condition in (1.2). Under the proposed parametrization, the optimal schedule becomes:

$$ e^*(s, \beta) = \frac{1}{\alpha} - \frac{\beta - \bar{\beta}}{s}(s - \bar{s}) $$

The following exercise illustrates the optimal mechanism for parameters $\alpha = 0.025$, $\beta = 80$, $s \sim U[10, 30]$. Such parametrization results in an expected intrinsic cost $\mu_s$ uniformly distributed over interval [90, 110].

**Ex-post optimal schedules.** Figure 1.2 presents the computation of the optimal direct mechanism schedules for the proposed parametrization. The diagrams depict the mechanism’s assignment of effort recommendations $e^*(s, \beta)$ and ex-post utilities $u^*(s, \beta)$ to each project type $\beta$, for various levels of the firm’s signal.

Note the optimal dynamic mechanism always yields negative payoffs in some states of nature, especially when the firm underestimates the project’s cost by a large amount.
Negative net transfers for high $\beta$-reports illustrate how the regulator actually punishes the firm for under-performance. In contrast, better-than-expected performance is highly rewarded, with steeper utility increases for more optimistic firms.

One interesting question concerns the behavior of the mechanism’s assignment of ex-post rewards and punishments, i.e. the cutoff type $\beta^*(s)$ such that $u^*(s, \beta^*(s)) = 0$. Indeed, the optimal mechanism violates ex-post individual rationality constraints for all types more expensive than $\beta^*(s)$. Under the proposed parametrization, the cutoff level is presented in Figure 1.3.

Figure 1.3 also plots a line corresponding to the conditional expectation for $\beta$ given $s$. Intuitively, the dynamic mechanism could be guessed to reward overestimation and punish underestimation errors, the case where the cutoff level $\beta^*(s)$ would coincide with the $\mathbb{E}[\beta|s]$ curve. This is clearly not the case in the numerical exercise.
Ex-ante optimal schedules. Figure 1.4 presents the optimal mechanism from an ex-ante perspective, depicting the optimal assignment of expected rent levels $U^*(s)$ and franchise fees $\phi(s)$ to the reported signal $s$.

In line with mainstream screening mechanisms’ properties, the firm’s expected rent is positive for all signals, which is required for the decision to participate in the contract. The firm’s utility from the mechanism attains maximal value at the most optimistic signal $s$, and decreases monotonically until reaching value zero for the upper limit of the signal range, meaning zero rent for the most pessimistic firm.

The franchise fee is depicted as a negative transfer charged to all firm types. For the present parametrization, the optimal mechanism assigns a rather large fee for the most optimistic types. The lesson here is that very optimistic forecasts should be subject to a disproportional level of skin in the game, so to discourage under-reporting of expected costs.
The equilibrium cost forecast is illustrated in Figure 1.5. The difference between the conditional mean for the project’s intrinsic cost $E[\beta|s]$ and the endogenous cost forecast $\bar{C}(s)$ corresponds to the firm’s anticipated effort given the dynamic incentive structure.

![Figure 1.5: Equilibrium project cost forecasts](image)

**Optimal second-period menu.** Figure 1.6 presents the period-2 menu implementation of the optimal mechanism, which consists of a combination of fixed payments and cost reimbursement fractions. The menu enables the decentralization of the effort decision to the firm, which coincides, in equilibrium, to the optimal effort recommendation of the direct mechanism.

![Figure 1.6: Period 2 menu of linear contracts](image)
Full-powered incentives are only available to firm $x_1$ that provided the most optimistic cost estimates in period 1. For all other types, the regulator reimburses some fraction of the realized cost, at an increasing proportion the more expensive the project’s actual intrinsic cost $\beta$.

Table 1.1 illustrates what a dynamic remuneration policy looks like. The first column presents the first-period menu. It displays franchise fees one of which the firm must choose at contract signature. The present menu considers a discrete selection of signal types, i.e. such that $\mu_s \in \{90, 95, 100, 105, 110\}$.

<table>
<thead>
<tr>
<th>Franchise fee</th>
<th>Remuneration plan</th>
<th>Fixed payment</th>
<th>Reimbursement share</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 298.64$</td>
<td>“Premium”</td>
<td>380.25</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>“Standard”</td>
<td>380.25</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>“Flexible”</td>
<td>380.25</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>“Security”</td>
<td>380.25</td>
<td>0.000</td>
</tr>
<tr>
<td>Expected cost</td>
<td>$\bar{C} = 49.99$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 39.85$</td>
<td>“Premium”</td>
<td>120.08</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>“Standard”</td>
<td>118.03</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>“Flexible”</td>
<td>113.71</td>
<td>0.129</td>
</tr>
<tr>
<td></td>
<td>“Security”</td>
<td>94.32</td>
<td>0.370</td>
</tr>
<tr>
<td>Expected cost</td>
<td>$\bar{C} = 59.99$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 21.07$</td>
<td>“Premium”</td>
<td>99.73</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>“Standard”</td>
<td>95.29</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>“Flexible”</td>
<td>85.99</td>
<td>0.256</td>
</tr>
<tr>
<td></td>
<td>“Security”</td>
<td>37.12</td>
<td>0.743</td>
</tr>
<tr>
<td>Expected cost</td>
<td>$\bar{C} = 69.81$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 15.45$</td>
<td>“Premium”</td>
<td>92.96</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>“Standard”</td>
<td>85.92</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>“Flexible”</td>
<td>69.77</td>
<td>0.390</td>
</tr>
<tr>
<td></td>
<td>“Security”</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>Expected cost</td>
<td>$\bar{C} = 78.95$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 12.95$</td>
<td>“Premium”</td>
<td>89.21</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>“Standard”</td>
<td>79.69</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td>“Flexible”</td>
<td>56.10</td>
<td>0.516</td>
</tr>
<tr>
<td></td>
<td>“Security”</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>Expected cost</td>
<td>$\bar{C} = 87.28$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Dynamic menu of linear contracts with exponential forecasting technology
Each fee option leads to a distinct second-period menu of linear contracts (remuneration policies), made available at the beginning of the production stage. The firm then chooses a combination of fixed payment (third column) and cost reimbursement fraction (fourth column). Such remuneration plans consider a selection of project types, i.e. such that the conditional percentiles for each covered signal are 

\[ G(\beta|s) = \{0.05, 0.35, 0.65, 0.95\} \]

An optimistic firm might feel comfortable to pick the high-fee menu, but a less optimistic one should not like to risk a high fee and the obligation to execute an ex-post expensive project under high-powered incentives.

### 1.4 Project cancellation

An important extension to the optimal procurement model is the consideration of the firm shutdown option. Such extension represents the cost-benefit analysis undertaken by the regulator to assess the project’s viability, that is, so the project’s expected cost under asymmetric information does not exceed its social benefit.

This section relaxes the assumption that the project’s social value \( S \) is sufficiently large. Suppose the regulator may choose a threshold in the signal space \( s^* \in [\underline{s}, \bar{s}] \) that restricts project implementation to a subset of firm types \( s \leq s^* \) (firms such that \( s > s^* \) are excluded from the partnership).

Let \( \tilde{f}(s) \) denote the regulator’s prior distribution over cost signals truncated at \( s^* \), i.e. \( \tilde{f}(s) = f(s)/F(s^*) \) for \( \underline{s} \leq s \leq s^* \). The level of welfare attained under this restriction is given by

\[
W(S, s^*) = \int_{\underline{s}}^{s^*} \left[ S - \int_{\beta}^{\bar{\beta}} C(s, \beta) + \psi(e(s, \beta))dG(\beta|s) - U(s, s^*) \right] \tilde{f}(s)ds
\]

(1.6)

where \( U(s, s^*) \) denotes firm \( s \) expected rent when the cutoff signal is \( s^* \). The regulator’s problem is similar to (P), but with an objective function given by (1.6), an additional optimization variable \( s^* \), the same incentive constraints, and a modified participation constraint:

\[
U(s, s^*) \geq 0, \quad \forall s \leq s^*
\]

The present result is analogous to Laffont and Tirole (1993, Section 1.4.5). Even with shutdown, optimal effort recommendation remains the same as characterized by first-order condition (1.2), since the hazard rate associated to the regulator’s prior is invariant to upward truncations.\(^\text{14}\)

\[^{14}\text{That is, for } s < s^*, \tilde{h}(s) = \tilde{F}(s)/\tilde{f}(s) = [F(s)/F(s^*)]/[f(s)/F(s^*)] = h(s).\]
The firm’s expected informational rent does depend on the truncation point $s^*$. As incentive constraints are the same as in (P), it follows the rent function $U(s, s^*)$ decreases with $s$. Hence, the truncated participation constraint implies that, optimally, $U(s^*, s^*) = 0$, and from the envelope condition it obtains:

$$U(s, s^*) = \int_s^{s^*} \mathbb{E}_\beta [\psi'(e^*(z, \beta))\nu(z, \beta)|z]dz$$

The separability between the incentives design problem and the shutdown decision entails that the optimal cutoff is characterized by a simple first-order condition, presented in Proposition 1.3.

**Proposition 1.3.** Given the project’s social value $S > 0$, the regulator optimally shuts down firms with signals $s > s^*$, where the cutoff level $s^*$ is given by:

$$f(s^*) \left[ S - \int_\beta C^*(s^*, \beta) + \psi(e^*(s^*, \beta))dG(\beta|s^*) \right] = F(s^*)\mathbb{E}_\beta [\psi'(e^*(s^*, \beta))\nu(s^*, \beta)|s^*]$$

(1.7)

Moreover, the optimal cutoff $s^*$ increases with $S$, and attains $s^* = \bar{s}$ for $S$ large enough.

Figure 1.7 illustrates the response of the forecast threshold $s^*$ to the project’s social value. Note that, for the present parametrization (same as in Section 1.3), no firm is shut down if $S \geq 102.0$.

![Figure 1.7: Optimal shutdown value $s^*$ as a function of $S$](image-url)
The optimal cutoff \( s^* \) may be implemented by a ceiling on expected costs at entry, given by \( \bar{C}(s^*) \), or, alternatively, by a minimum franchise fee, given by:

\[
\phi(s) = \min \left\{ \phi_{\text{min}}, U^*(s, s^*) - \int \psi'(e^*(s, \beta)) G(\beta|s) d\beta \right\}
\]

\[
\phi_{\text{min}} = - \int \psi'(e^*(s^*, \beta)) G(\beta|s^*) d\beta
\]

The comparison between optimal franchise fees with and without the shutdown option is illustrated in Figure 1.8. In the present parametrization, accounting for shutdown entails significantly greater rent extraction, rejection of nonviable projects, and little alteration to the optimal fee menu.

**Figure 1.8: Expected rents and franchise fees with \( S = 90.0 \) \( (s^* = 23.0) \)**

**Remark.** Note the example in Figure 1.8 is parametrized with \( S = 90 \), which is less than the mean intrinsic cost \( \mathbb{E}[\beta] = 100 \). However, the mean social cost of executing the project takes into account the cost savings from the firm’s effort, so the net social benefit is \( S - \beta + e - \psi(e) \). Such discrepancy could be interpreted as the benefit from delegating the project to the private sector, compared to the government executing the project on its own. Under full information, the project is viable provided the firm deploys its cost reduction technology, since the expected net benefit is \( S - \mathbb{E}[\beta] + k = 10.0 \).

**Project implementation errors.** Unlike the static model, the dynamic procurement policy might lead to errors in the project implementation decision. Errors can be “Type I”, when the regulator implements a project that turns out to have negative net benefit (cost overrun), or “Type II”, when the regulator shuts down a firm that had positive probability of executing a project with positive social value (incorrect cancellation).\(^{15}\)

An interesting extension to the dynamic model with shutdown would be to introduce loss functions (which amount to a form of risk aversion) in the regulator’s objective

\(^{15}\) The terminology of Type I and Type II errors in dynamic procurement was introduced by Krähmer and Strausz (2011).
to analyze the tradeoff between both error types. However, under risk neutrality, the optimal tradeoff is given by equation (1.7).

**Multistage cancellation rules.** A further relevant extension to the dynamic procurement model would be to analyze the shutdown option in period 2. Suppose the regulator, in addition to defining a cutoff signal $s^*$, can also define a project cancellation threshold $\beta^*$ after eliciting information on the project’s type. The analysis is not so simple because, unlike the hazard rate associated to the regulator’s prior $F(s)/f(s)$, the informativeness measure $G(s|\beta)/g(\beta|s)$ is not invariant to an upward truncation at $\beta^*$. Consequently, the regulator’s problem is not separable into incentives design and project shutdown, and we leave it for further research.

The closest work to this proposal is Arvan and Leite (1990), who develop a multi-period model of procurement contracting with dynamic cancellation rules. However, their environment features serially uncorrelated types (zero informativeness) and inability of the regulator to commit not to expropriate the firm. This chapter, in turn, assumed full commitment and serial type correlation. In the absence of the former, the model leads to the ratchet effect studied in Laffont and Tirole (1988). In the absence of the latter, the optimal mechanism is static, as discussed in Section 1.2 (false dynamics).

### 1.5 Competitive bidding

Procurement transactions typically involve a competitive process to select the partner that will execute the project or supply the procured good or service. In concessions, particularly, it is common to award the contract by means of an auction, where bidders submit proposals on service tariffs or award fees. The procuring agency then selects the winner based on a pre-specified criterion, such as the lowest tariff, the highest fee, or a scoring rule.

This section extends the dynamic procurement model to an environment of multiple agents. Suppose there are $n \geq 1$ symmetric bidders interested in the tender process, each of whom observes an independent private signal $s_i$, $i \in \{1, \ldots, n\}$, drawn from the same distribution $F(s)$ defined on $[s, \overline{s}]$. Bidders are uncertain about the project’s intrinsic cost $\beta$, but the observed signal is useful to forecast costs with technology $G(s|\beta)$, defined on a common support $[\beta, \overline{\beta}]$ for all $s_i$. Assume low signals are indicative of low costs, i.e., the forecasting technology satisfies FOSD (Assumption 1.2).

The project’s total cost is $C = \beta - e$, where $e$ denotes the effort exerted by the winning bidder at the production stage. Cost-reduction activities cost the firm $\psi(e)$, which is also homogeneous among bidders. The regulator and the bidders are risk neutral, and the latter choose their bidding strategies in a symmetric Bayesian Nash Equilibrium. The mechanism is enforced with full commitment.

---

16 The idea of capturing rents by means of introducing competition for the market is attributed to Demsetz (1968).
Suppose the project’s benefit $S$ is large. The regulator must simultaneously design a selection mechanism and a remuneration policy in order to minimize expected procurement costs (the total payment to the winning firm). Motivated by results in Proposition 1.2, assume the optimal auction awards the contract to the bidder who submits the highest franchise fee $\max_i \phi(s_i)$. By the revelation principle, this mechanism is equivalent to selecting the firm who reports the most optimistic signal $\min_{i \leq n} s_i$.\(^{17}\)

The winning firm’s ex-post profit, as in Section 1.2, is given by the net transfer from the regulator (in excess of cost reimbursement) minus the disutility of effort:

$$u(\hat{\beta}|s_i, \beta) = t(s_i, \hat{\beta}) - \psi(e(s_i, \hat{\beta}|\beta))$$ \hspace{1cm} (1.8)

where $\hat{\beta}$ is the firm’s cost announcement in the production stage.

In the first period, a type-$i$ firm announces signal $\hat{s}_i$ and obtains an expected profit given by:

$$U_n(\hat{s}_i|s_i) = \left[1 - F(\hat{s}_i)\right]^{n-1} \int_{\beta} \bar{\beta} u(\beta|\hat{s}_i, \beta) dG(\beta|s_i)$$ \hspace{1cm} (1.9)

The term in brackets represents the probability of winning the tender, $\Pr(\hat{s}_i < \min_{j \neq i} s_j)$. Let $T(s, \beta) = C(s, \beta) + \psi(e(s, \beta)) + u(s, \beta)$ denote the firm’s ex-post IC remuneration policy. The regulator’s total expected payment to the firm takes into account that the contract is awarded to the lowest signal bidder: \(^{18}\)

$$\int_{s_i}^{\bar{s}} \int_{\beta} T(s, \beta) dG(\beta) ds \left[1 - F(s)\right]^{n-1} ds$$ \hspace{1cm} (1.10)

Hence, the design problem consists of minimizing (1.10) subject to ex-ante individual rationality and multi-period incentive constraints:

$$\beta \in \arg\max_\beta u(\hat{\beta}|s_i, \beta) \quad \forall \hat{\beta}, \beta, s_i$$ \hspace{1cm} (IC-2)

$$s_i \in \arg\max_{s_i} U_n(\hat{s}_i|s_i) \quad \forall \hat{s}_i, s_i$$ \hspace{1cm} (IC-1)

$$U_n(s_i) \geq 0, \quad \forall s_i$$ \hspace{1cm} (IR)

The optimal mechanism exhibits an useful dichotomy property, established independently by McAfee and McMillan (1987), Riordan and Sappington (1987), and Laffont and Tirole (1987). Namely, the incentives design problem is separable from bidder selection, so the optimal allocation is the same as if there was a single firm. The effect of competition is simply to reduce the total payment (and utility) left to the winning bidder, by means of increasing the expected value of the maximum franchise fee. Proposition 1.4 summarizes the optimal mechanism characterization.

\(^{17}\) It is a standard result in the literature that an efficient auction selects a bidder with the valuation most beneficial to the principal; see, for instance, Laffont and Tirole (1987).

\(^{18}\) Recall the density of the first order statistic in an iid sample $X_i \sim f_X(\cdot)$, $i \in \{1, ..., n\}$, is given by $f_{X(i)}(x) = nf_X(x)[1 - F_X(x)]^{n-1}$.
Proposition 1.4. The optimal concession mechanism implements the same effort recommendation \( e^*(s, \beta) \) given by equation (1.2) and the same ex-post remuneration policy \( \hat{T}(\bar{C}, C) \) described in Proposition 1.2.

Moreover, the firm’s optimal bidding strategy is:

\[
\phi^*(s_i) = \int_{s_i}^{s} \left[ \frac{1 - F(z)}{1 - F(s_i)} \right]^{n-1} \int_{\beta}^{\beta} \psi'(e^*(z, \beta)) \iota(z, \beta) dG(\beta|z) dz - \int_{\beta}^{\beta} \psi'(e^*(s_i, \beta)) G(\beta|s_i) d\beta
\]

The optimal bid (1.11) may be interpreted as the value of the contract to the second bidder, conditional on \( i \) being the winner (first term), minus the gain from screening arising from the fact that the optimal allocation depends on the winning bid (second term).\(^{19}\) Figure 1.9 illustrates the firm’s expected profit and equilibrium bidding behavior in response to its endogenous cost forecast \( C \).

![Figure 1.9: Optimal profit and bid strategy as a function of expected project costs](image)

In the present example, competition makes little difference for the firm’s optimal strategy, impacting the average bid by less than 5% for each additional bidder. On the other hand, the effect of competition on rent extraction is expressive, though the benefits of competition are largely realized with a small number of bidders: 67% of the average rent is extracted with \( n = 4 \). The following corollary formalizes the limiting analysis.

Corollary. As the number of bidders diverges, the winning bidder’s profit converges to zero. Moreover, the regulator’s expected procurement cost converges to the most optimistic first best:

\[
\lim_{n \to \infty} \mathbb{E}[T_n(\bar{C}, C)] = \mathbb{E}[\beta|s] - k
\]

\(^{19}\) This result is analogous to Riordan and Sappington (1987, Corollary 6).
The response of the winning bidder’s expected rent and the regulator’s expected payment is illustrated in Figure 1.10.

![Figure 1.10: Limiting behavior of average profits and expected procurement costs](image)

**1.6 Conclusion**

This chapter developed a model of cost-based procurement under sequential asymmetric information, to learn about the incentive properties of optimal dynamic mechanisms and derive lessons applicable to concession design. The main insights are twofold: (i) the optimal mechanism screens private information sequentially to exploit the informativeness of the firm’s forecast, and (ii) the mechanism is implemented by a dynamic menu of contracts. At contract signature, the firm pays the regulator a franchise fee, which depends monotonically on the firm’s endogenous cost forecast. In the production period, the firm chooses a remuneration plan from a menu of linear contracts, i.e., a combination of fixed payment and cost reimbursement fraction, designed to induce second-best effort levels. The incentive power of the second-period scheme responds to first-period type revelation: the more optimistic the firm reveals itself to be through its fee choice, the higher-powered the second-period menu becomes.

Furthermore, this chapter explored two important extensions. First, it explored the design problem with a shutdown option: when the benefit of the project is not large, the regulator may not wish to implement the project for all cost forecasts. Consequently, the extended setting gives rise to the possibility of implementation errors, which are optimally traded-off at the cutoff signal. The optimal cancellation rule may be implemented by either a ceiling on expected costs or a minimum franchise fee.

Second, this chapter characterized the optimal mechanism with multiple agents. The contract award mechanism was found to be an efficient auction based on the highest franchise fee, where the winner executes the project under a remuneration policy chosen from a menu at the production stage. Bidders know from the outset the link between the winning bid and the shape of the second-period scheme.
From the previous analysis, it is worth stressing an important lesson for concession design. The model suggests that incorporating forecast information into the concessionaire’s compensation rule leads to better social outcomes, as the regulator exploits the imperfect informativeness of the agent’s signal to attain higher rent extraction. However, when considering forecasts provided by the firm, the monotonicity property should be respected, that is, contracts should provide higher-powered incentives to more optimistic cost forecasts. This means, for example, that firms bidding over-optimistic estimates of the project’s viability (e.g. low-balling) should operate under rigorous fixed-price contracts. Failing to commit to such prescription leads to the violation of ex-ante incentive compatibility, so firms might find it profitable to deviate from truthful reporting of their cost signal.20

Two limitations of the analysis in this chapter should be mentioned. First, the model considers only ex-ante participation constraints. Each risk-neutral firm compares its expected profit from the mechanism to its reservation utility in order to accept the contract, knowing that in some states of nature it might make heavy losses. In the real world, the concession sponsor might walk away under bankruptcy rules (limited liability) or invoke the legislation’s protection of his “financial equilibrium principle”, which amounts to an ex-post participation constraint. While beyond the scope of the present chapter, such assumption might lead to starkly different results. The matter of ex-post individual rationality is addressed in Chapter 2.

Secondly, the model assumes common knowledge of agents’ preferences and information structure, which includes the shape of forecasting technologies. Knowledge of type distributions, for instance, could turn out to be a strong assumption for projects being procured for the first time, or in sectors with rapidly changing technology, in which the regulator is unable to form a prior over project costs. The sensitivity of optimal mechanisms to details of the environment, referred to in the literature as Wilson’s Critique, raises questions as to how the principal might design mechanisms that are robust to uncertainty about model primitives. Such questions motivate the analysis of Chapter 3.

As for further research, there is one additional suggested venue. The issue of determining reservation prices in concession auctions could benefit from an analysis grounded on incentive theory, particularly with respect to the tradeoff between rent extraction and fostering entry. We suggest a framework that integrates the shutdown analysis of Section 1.4 and the competitive bidding of Section 1.5 to address normative questions in that regard.

---

20 This is a concern in jurisdictions where there is pervasive renegotiation of concession contracts; see, for instance, Guasch (2004).
1.A Proofs of Chapter 1

Proof of Proposition 1.1. In order to characterize the optimal direct mechanism, there are some steps to make problem \((P)\) more tractable.

By standard arguments,\(^{21}\) the second incentive constraint \((\text{IC-2})\) is equivalent to an envelope equation for the ex-post rent function and a monotonicity condition for the cost recommendation:

\[
w_2(s, \beta) = -\psi'(\beta - C(s, \beta)), \quad \forall s \quad \text{(IC-2')}\]

\(C(s, \beta)\) is increasing in \(\beta\), \(\forall s \quad \text{(MON)}\)

We follow the first-order approach for dynamic mechanism design problems (Pavan et al., 2014), according to which the first incentive constraint \((\text{IC-1})\) is replaced by its envelope condition:

\[
U'(s) = \int_\beta^\beta u(s, \beta) \frac{d}{ds} g(\beta|s) d\beta = u(s, \beta) \frac{d}{ds} G(\beta|s) \bigg|_{\beta} - \int_\beta^\beta \frac{d}{ds} u(s, \beta) \frac{d}{ds} G(\beta|s) d\beta
\]

(1.12)

where the last equality follows from integration by parts. Notice \(G(\beta|s)\) becomes constant for \(\beta \in \{\hat{\beta}, \bar{\beta}\}\), hence the derivative \(G_s(\beta) = G_s(\bar{\beta}) = 0\). Integrating (1.12), it obtains the expression for the incentive-compatible expected rents:

\[
U(s) = U(\bar{s}) + \int_s^\bar{s} \int_\beta^\beta \psi'(e(z, \beta)) G_s(\beta|z) d\beta dz
\]

(1.13)

Notice from (1.12) that, under \((\text{IC-2')}\) and Assumption 1.2, the firm’s expected rent decreases with its signal, i.e. \(U'(s) \leq 0\). Thus, constraint \((\text{IR})\) is satisfied if the contract secures a positive rent to the least optimistic agent:

\[U(\bar{s}) \geq 0\]

We eliminate transfers from the regulator’s objective function using the definition of the firm’s ex-post rent:

\[t(s, \beta) = \psi(e(s, \beta)) + u(s, \beta)\]

so the interim expected value of the regulator’s procurement cost becomes:

\[E_{\beta} [t(s, \beta) + C(s, \beta)|s] = \int_\beta^\beta C(s, \beta) + \psi(e(s, \beta)) dG(\beta|s) + U(s)\]

\(^{21}\)See Courty and Li (2000), Lemma 3.1.
Thus, the regulator’s global objective function is expressed as:

\[
\begin{align*}
E_{s, \beta} [t(s, \beta) + C(s, \beta)] &= \int_0^\pi \left\{ \int_\beta^\pi C(s, \beta) + \psi(e(s, \beta))dG(\beta|s) + U(s) \right\} f(s)ds \\
&= \int_0^\pi \left\{ \int_\beta^\pi C(s, \beta) + \psi(e(s, \beta))dG(\beta|s) + \int_\beta^\pi \psi'(e(s, \beta))G_s(\beta|s) \frac{F(s)}{f(s)}d\beta \right\} f(s)ds \\
&\quad + U(\bar{s}) 
\end{align*}
\]

(1.14)

where the last equality follows from integration by parts. The expression in brackets in (1.14) corresponds to the virtual social cost of the project.

We analyze a relaxed version of the design problem, denoted \((R)\), which considers only necessary conditions for incentive compatibility in both periods (incorporated into the regulator’s objective):

\[
\begin{align*}
\min_{\{e()\}} & \int_0^\pi \int_\beta^\pi \left\{ \beta - e(s, \beta) + \psi(e(s, \beta)) \\
&\quad + \psi'(e(s, \beta))G_s(\beta|s) \frac{F(s)}{f(s)}d\beta \right\} dG(\beta|s)f(s)ds + U(\bar{s}) \\
\text{s.t. } & U(\bar{s}) \geq 0
\end{align*}
\]

(\(R\))

If the optimal solution to \((R)\) also satisfies the neglected constraints (MON) and global (IC-1), then it is also an optimum of problem \((P)\). Note any optimal mechanism assigns \(U(\bar{s}) = 0\). Computing problem \((R)\)’s pointwise first-order conditions yields:

\[
\psi'(e^*(s, \beta)) = 1 - \frac{F(s)}{f(s)} \left[ -\frac{G_s(\beta|s)}{g(\beta|s)} \psi''(e^*(s, \beta)) \right], \quad \text{for all } s, \beta
\]

(1.15)

Using the definitions of \(h(s)\) and \(\iota(s, \beta)\), (1.2) obtains. Equation (1.1) follows immediately from the identity \(C^*(s, \beta) = \beta - e^*(s, \beta)\), and the firm’s equilibrium expected profit (1.3) is obtained from plugging \(e^*(s, \beta)\) into (1.13).

To verify that the allocation \(e^*(s, \beta)\) solves problem \((P)\), we first need to show \(C^*(s, \beta)\) is monotonic with respect to both arguments, i.e.

\[
\nabla C^*(s, \beta) = \begin{bmatrix} -\frac{\partial}{\partial s}e^*(s, \beta) \\ 1 - \frac{\partial}{\partial \beta} e^*(s, \beta) \end{bmatrix} \geq 0
\]

Differentiating (1.15) with respect to \(s\) yields

\[
\begin{align*}
\psi''(e^*) \frac{\partial e^*}{\partial s} &= -\left[ \frac{\partial h}{\partial s} \cdot \iota \cdot \psi''(e^*) + h \cdot \frac{\partial \iota}{\partial s} \cdot \psi''(e^*) + h \cdot \iota \cdot \psi'''(e^*) \right] \\
\Rightarrow \frac{\partial e^*}{\partial s} &= -\frac{\psi''(e^*) \left[ \frac{\partial h}{\partial s} \cdot \iota + h \cdot \frac{\partial \iota}{\partial s} \right]}{\psi''(e^*) + h \cdot \iota \cdot \psi'''(e^*)} \leq 0
\end{align*}
\]

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The sign of the derivative \( \partial e^*/\partial s \) follows from the assumptions of convexity of \( \psi(.) \), \( \partial r/\partial s \geq 0 \) (Assumption 1.3) and \( F/f \) increasing (Assumption 1.1).\(^{22}\)

Differentiating the (1.15) with respect to \( \beta \) yields

\[
\psi''(e) \frac{\partial e^*}{\partial \beta} = -h \left[ \frac{\partial \psi''(e^*)}{\partial \beta} + i \cdot \psi'''(e^*) \right] \frac{\partial e^*}{\partial \beta}
\]

\[
\Rightarrow \frac{\partial e^*}{\partial \beta} = -\frac{h \cdot \psi''(e^*) \frac{\partial e^*}{\partial \beta}}{\psi''(e^*) + h \cdot i \cdot \psi'''(e^*)} \leq 0
\]

The sign of the derivative \( \partial e^*/\partial \beta \) follows from the assumptions of convexity of \( \psi(.) \) and \( \partial r/\partial \beta \geq 0 \) (Assumption 1.3).

In dynamic mechanism design problems, verifying monotonicity is sufficient to guarantee that (IC-2) is satisfied, but the same is not true for (IC-1). Therefore, to ensure the optimality of \( e^*(s, \beta) \), we need to show that:

\[
\Delta \equiv U(s) - U(\hat{s}|s) \geq 0 \quad \forall s, \hat{s}
\]

First, notice that:

\[
\Delta = U(s) - U(\hat{s}) + U(\hat{s}|\hat{s}) - U(\hat{s}|s) = \int_{\hat{s}}^{s} U'(z) - \frac{\partial}{\partial s} U(\hat{s}|z) dz
\]

By the envelope condition (1.12), it follows that:

\[
U'(s) = -\int \psi'(e(s, \beta)) \frac{\partial}{\partial s} G(\beta|s) d\beta
\]

Moreover, it is true that:

\[
\frac{\partial}{\partial s} U(\hat{s}|s) = \frac{\partial}{\partial s} \int [t(\hat{s}, \beta) - \psi(e(\hat{s}, \beta))] dG(\beta|s)
\]

\[
= \int u(\hat{s}, \beta) \frac{\partial g(\beta|s)}{\partial s} d\beta
\]

\[
= -\int \psi'(e(\hat{s}, \beta)) \frac{\partial G(\beta|s)}{\partial s} d\beta
\]

Therefore, it obtains that:

\[
\Delta = \int_{\hat{s}}^{s} \int_{\beta}^{\eta} \left[ \psi'(e(\hat{s}, \beta)) - \psi'(e(z, \beta)) \right] \frac{\partial}{\partial s} G(\beta|s) d\beta dz
\]

\(^{22}\) Laffont and Tirole (1993) make the additional assumption that \( \psi'''(e) \geq 0 \) to rule out optimal stochastic mechanisms. Such assumption is sufficient for the monotonicity of \( e^*(.,) \), but not necessary.
Suppose \( s > \hat{s} \). Note that \( z \geq \hat{s} \) for all \( z \in [\hat{s}, s] \). Recall that Assumption 1.2 implied \( \partial G/\partial s \leq 0 \), thus we need the term in brackets to be non-positive to obtain \( \Delta \geq 0 \). But, from the monotonicity of \( e^*(s, \beta) \) with respect to \( s \), in addition to \( \psi' > 0 \), it follows that \( \psi'(e^*(\hat{s}, \beta)) \leq \psi'(e^*(z, \beta)) \). If \( s < \hat{s} \), a similar argument applies.

To compute optimal transfers \( t^*(s, \beta) \) in (1.5), we need to make explicit the expression for ex-post profits. Integrating the envelope condition (IC-2'), it obtains

\[
\psi' \left( e^* \left( s, \beta \right) \right) G(\beta | s) d\beta \\
+ \psi(e^*(s, \beta)) + \int_{\beta}^{\bar{\beta}} \psi'(e^*(s, z)) dz \\
\]

The natural condition pinning down \( u^*(s, \beta) \) is that the expectation over ex-post profits equal the firm’s ex-ante rent, i.e.

\[
U^*(s) = \int u^*(s, \beta) dG(\beta | s) \\
= \int \left[ \int_{\beta}^{\bar{\beta}} \psi'(e^*(s, z)) dz \right] g(\beta | s) d\beta + u^*(s, \bar{\beta}) \\
= \int \psi'(e^*(s, \beta)) G(\beta | s) d\beta + u^*(s, \bar{\beta})
\]

where the last equality follows from standard integration by parts. Therefore, the ex-post profit left to the most expensive project is

\[
u^*(s, \beta) = U^*(s) - \int \psi'(e^*(s, \beta)) G(\beta | s) d\beta
\]

and the firm’s ex-post profit in equilibrium is given by

\[
u^*(s, \beta) = U^*(s) - \int \psi'(e^*(s, \beta)) G(\beta | s) d\beta + \int_{\beta}^{\bar{\beta}} \psi'(e^*(s, z)) dz
\]

and equation (1.4) obtains.

Proof of Proposition 1.2. Notice the optimal net transfer in (1.5) can be separated into two components, each depending on each period’s history of reports:

\[
t^*(s, \beta) = \int_{\beta}^{\bar{\beta}} \left[ \int_{\beta}^{\bar{\beta}} \psi'(e^*(z, \beta)) \nu(z, \beta) dG(\beta | z) \right] dz \\
+ \psi(e^*(s, \beta)) + \int_{\beta}^{\bar{\beta}} \psi'(e^*(s, z)) dz
\]
First, consider the first-period transfer $t_1^*(s)$. Recall that, in period 1, the agent anticipates that obedience to the recommended effort is an optimal continuation strategy. Hence, the project’s expected cost to a signal-$s$ firm is given by:

$$\bar{C}_s \equiv \mathbb{E}[C|s] = \int \beta - e^*(s, \beta) dG(\beta|s)$$

From Assumption 1.2 and the monotonicity of the optimal effort $e^*(s, \beta)$ with respect to $s$, it follows that the equilibrium expected cost is monotonic in the agent’s signal.

Denote $\tilde{s} = \bar{C}_s^{-1}(\bar{C})$ as the function that assigns each signal $s$ to a level of the endogenous reported cost forecast $\bar{C}$. A simple rewriting of the expression for the first-period transfer yields:

$$t_1^*(\tilde{s}(\bar{C})) = U(\tilde{s}(\bar{C})) - \int \psi'(e^*(\tilde{s}(\bar{C}), \beta)) G(\beta|\tilde{s}(\bar{C})) d\beta = \phi(\bar{C})$$

To see that $\phi(\bar{C}_s)$ is always negative, i.e. a payment from the firm to the regulator, notice first that:

$$\phi(\bar{C}_s) = -\int \psi'(e^*(\bar{s}, \beta)) G(\beta|\bar{s}) d\beta < 0$$

Differentiating $t_1^*(s)$ with respect to $s$:

$$\frac{d}{ds} t_1^*(s) = U'(s) - \frac{d}{ds} \int \psi'(e^*(s, \beta)) G(\beta|s) d\beta$$

$$= -\int \psi'(e^*(s, \beta)) \frac{\partial G(\beta|s)}{g(\beta|s)} dG(\beta|s)$$

$$- \int [\psi''(e^*(s, \beta)) \frac{\partial}{\partial s} e^*(s, \beta) G(\beta|s) + \psi'(e^*(s, \beta)) \frac{\partial}{\partial s} G(\beta|s)] d\beta$$

$$\geq 0$$

since $\psi'' > 0$ and $e^*(s, \beta)$ decreases in $s$. Hence, $\phi(\bar{C}_s) < 0$ for all $s$.

Now, consider the optimal second-period transfer $t_2^*(s, \beta)$. Fix $s$ and let $\beta = \beta^*(s, C)$ denote the optimal mechanism’s type-assignment function. The existence of the inverse allocation follows from the monotonicity of $C^*(s, \beta)$ with respect to $\beta$, for any $s$. Substituting the inverse allocation in the optimal net transfer yields:

$$T_2(s, C) = \psi(e^*(s, \beta^*(s, C))) + \int_{\beta^*(s, C)}^{\beta} \psi'(e^*(s, z)) dz$$
It turns out that, for a given $s$, $T_2(s,.)$ is a convex function, since:

$$\frac{\partial T_2}{\partial C} = \frac{\partial T_2^*}{\partial \beta} = -\psi'(\beta - C^*(s, \beta))$$

$$\frac{\partial^2 T_2}{\partial C^2} = -\psi''(\beta^*(s, c) - C) \left( \frac{1}{\partial C^*/\partial \beta} - 1 \right) = -\psi''(e^*(s, \beta)) \frac{\partial}{\partial \beta} e^*(s, \beta)(C^*(s, \beta)) \geq 0$$

where the last inequality follows from the monotonicity of the optimal allocation, i.e., $\frac{\partial}{\partial \beta} C^*(s, \beta) \geq 0$ and $\frac{\partial}{\partial \beta} e^*(s, \beta) \leq 0$.

Because the indirect transfer is convex, it can be replaced by its family of tangents, which represent a menu of contracts that are linear in realized costs:

$$t_2(s, \hat{\beta}, C) = t_2^*(s, \hat{\beta}) - \psi'(e^*(s, \hat{\beta})) \left[ C - C^*(s, \hat{\beta}) \right]$$

Given the optimal menu $t_2(s, \hat{\beta}, C)$, the firm’s problem in the second period becomes:

$$\max_{\{\hat{\beta}, e\}} t_2^*(s, \hat{\beta}) - \psi'(e^*(s, \hat{\beta})) \left[ \beta - e - \hat{\beta} + e^*(s, \hat{\beta}) \right] - \psi(e)$$

The respective first-order conditions are:

(i) $$-\psi'(e^*(s, \hat{\beta})) \left[ 1 - \frac{\partial}{\partial \beta} e^*(s, \hat{\beta}) \right] - \psi'(e^*(s, \hat{\beta})) \left[ -1 + \frac{\partial}{\partial \beta} e^*(s, \hat{\beta}) \right]$$

(ii) $$\psi'(e) = \psi'(e^*(s, \hat{\beta}))$$

Hence, the optimal choice for the firm is precisely $e = e^*(s, \hat{\beta})$ and $\hat{\beta} = \beta$, as second-order conditions are satisfied from the concavity of the problem.

The menu of linear contracts can be expressed as a choice of parameters $a(s, \beta)$ and $b(s, \beta)$, where $a(.)$ is a fixed payment and $b(.)$ is a fraction of reimbursed cost, corresponding, respectively, to the intercept and slope of the incentive scheme.

To summarize, the second-period transfer selected by a firm facing project type $\beta$, given first-period signal $s$, is given by:

$$T(s, \beta, C) = a(s, \beta) - b(s, \beta) \cdot C$$

where

$$a(s, \beta) = t_2^*(s, \beta) + \psi'(e^*(s, \beta))C^*(s, \beta)$$

$$b(s, \beta) = \psi'(e^*(s, \beta))$$

23 The convexity of indirect transfers obtains from the assumption that the disutility of effort $\psi(.)$ has the same curvature regardless of the firm’s innate efficiency. Chu and Sappington (2009) show that other types of incentive contracts are optimal (e.g. concave in realized costs) if that assumption fails.
Proof of Proposition 1.3. Denote $\varphi(s) = E_\beta[\psi(e^*(s, \beta))\iota(s, \beta)|s]$ as the signal-$s$ firm’s expected marginal rent. Thus, if the threshold signal is $s^*$, the expected profit of a signal-$s$ firm becomes:

$$U(s, s^*) = \int_s^{s^*} \varphi(z)dz$$

Taking expectations with respect to $s$ using the truncated distribution $\tilde{f}(s)$ and integrating by parts yields the following:

$$\int_s^{s^*} U(s, s^*)\tilde{f}(s)ds = U(s, s^*)\tilde{F}(s)\bigg|_s^{s^*} + \int_s^{s^*} \frac{\partial}{\partial s} U(s, s^*)\tilde{F}(s)ds$$

$$= \int_s^{s^*} \varphi(s)h(s)\tilde{f}(s)ds$$

where the last equality uses the equivalence of hazard rates between the original and truncated distributions (see footnote 14). Substituting in the expression for social welfare (1.6) yields:

$$W(S, s^*) = \int_s^{s^*} \left[ S - \int_\beta C(s, \beta) + \psi(e(s, \beta))dG(\beta|s) - \varphi(s)h(s) \right] \tilde{f}(s)ds$$

Differentiating with respect to $e(s, \beta)$ yields the same pointwise first-order condition as in (1.2). In turn, differentiating with respect to $s^*$ yields:

$$\left[ S - \int_\beta C(s^*, \beta) + \psi(e(s^*, \beta))dG(\beta|s^*) - \varphi(s^*)h(s^*) \right] \tilde{f}(s^*)$$

$$= \left[ S - \int_\beta C(s^*, \beta) + \psi(e(s^*, \beta))dG(\beta|s^*) \right] \frac{f(s^*)}{\tilde{F}(s^*)} - \varphi(s^*)$$

and the first-order condition (1.7) obtains.

To see that $s^*$ increases with the project’s social value $S$, let $S_0 < S_1$. By revealed preference, it holds that:

$$W(S_0, s_0^*) \geq W(S_0, s_1^*) \quad \text{and} \quad W(S_1, s_1^*) \geq W(S_1, s_0^*)$$

Adding up both inequalities show that:

$$\int_{s_0^*}^{s_1^*} \int_{S_0}^{S_1} \frac{\partial^2 W}{\partial S \partial s^*} \geq 0$$

But $\frac{\partial^2 W}{\partial S \partial s^*} = f(s^*) > 0$. Therefore, $s_0^* \leq s_1^*$. \hfill \qed
Proof of Proposition 1.4. Throughout the proof we omit the subscript \( i \) to save on notation. Denote \( u(s, \beta) = u(\beta|s, \beta) \) and \( U(s) = U(s|s) \) denote, respectively, the ex-post and ex-ante truth-telling utilities. Applying the envelope theorem to (1.9), it holds that:

\[
U'(s) = \int_{\beta}^{\overline{\beta}} u(s, \beta) g_s(\beta|z) d\beta \cdot (1 - F(s))^{n-1} \tag{1.16}
\]

Integration by parts show that:

\[
\int_{\beta}^{\overline{\beta}} u(z, \beta) g_s(\beta|z) d\beta = u(s, \beta) G_s(\beta|z) |_{\beta}^{\overline{\beta}} - \int_{\beta}^{\overline{\beta}} \frac{\partial}{\partial \beta} u(s, \beta) G_s(\beta|s) d\beta
\]

\[
= \int_{\beta}^{\overline{\beta}} \psi'(e(s, \beta)) G_s(\beta|s) d\beta \equiv -\varphi(s) \tag{1.17}
\]

where the last equality follows from the envelope theorem applied to (1.8). The term \( \varphi(s) \) is the type-\( s \) agent’s marginal rent, as defined in the proof of Proposition 1.3. Plugging (1.17) into (1.16) yields:

\[
U'(s) = \varphi(s) (1 - F(s))^{n-1}
\]

Expanding the expected procurement cost from the regulator’s objective in (1.10), using the definition of expected profit in (1.9):

\[
\int_{s}^{\overline{s}} \int_{\beta}^{\overline{\beta}} [C(s, \beta) + \psi(e(s, \beta)) + u(s, \beta)] dG(\beta|s) f_s(1)(s) ds
\]

\[
= \int_{s}^{\overline{s}} \left\{ \int_{\beta}^{\overline{\beta}} [C(s, \beta) + \psi(e(s, \beta))] dG(\beta|s) + \frac{U(s)}{(1 - F(s))^{n-1}} \right\} f_s(1)(s) ds
\]

Integrating by parts once more shows that:

\[
\int_{s}^{\overline{s}} \frac{U(s)}{(1 - F(s))^{n-1}} f_s(1)(s) ds = n \int_{s}^{\overline{s}} U(s) f(s) ds
\]

\[
= n \left[ U(s) F(s) |_{s}^{\overline{s}} + \int_{s}^{\overline{s}} \varphi(s) (1 - F(s))^{n-1} F(s) ds \right]
\]

\[
= \int_{s}^{\overline{s}} \varphi(s) \frac{F(s)}{f(s)} \frac{n(1 - F(s))^{n-1} f(s)}{f_s(1)(s)} ds \tag{1.18}
\]

Plugging (1.18) into the objective function obtains the following:

\[
\int_{s}^{\overline{s}} \left\{ \int_{\beta}^{\overline{\beta}} [C(s, \beta) + \psi(e(s, \beta))] dG(\beta|s) + \varphi(s) \frac{F(s)}{f(s)} \right\} f_s(1)(s) ds
\]
Therefore, pointwise optimization yields the same first-order condition as in (1.2) and the main result obtains.

**Optimal transfers.** To compute the equilibrium bidding strategy, first note that, by definition, the following equations hold:

\[ U^*(s) = (1 - F(s))^{n-1} \int_{\beta}^\overline{\beta} u^*(s, \beta)dG(\beta|s) \]

\[ u^*(s, \beta) = u^*(s, \overline{\beta}) + \int_\beta^\overline{\beta} \psi'(e^*(s, z))dz \]

Therefore, it follows that:

\[ \frac{U^*(s)}{(1 - F(s))^{n-1}} = u^*(s, \overline{\beta}) + \int_\beta^\overline{\beta} \left[ \int_\beta^\overline{\beta} \psi'(e^*(s, z))dz \right] g(\beta|s)d\beta \]

Rearranging the previous expression and integrating by parts yields:

\[ u^*(s, \overline{\beta}) = \frac{U^*(s)}{(1 - F(s))^{n-1}} - \int_\beta^\overline{\beta} \psi'(e^*(s, \beta))G(\beta|s)dz \quad (1.19) \]

Integrating (1.16) from \( s \) to \( \overline{s} \), it obtains:

\[ U^*(s) = \int_s^\overline{s} (1 - F(z))^{n-1} \varphi(z)dz \quad (1.20) \]

Plug (1.20) into (1.19) and denote \( u^*(s, \overline{\beta}) = \phi^*(s) \), and the optimal bidding strategy (1.11) obtains. 

\[ \square \]
Chapter 2

The effect of exit rights on cost-based procurement contracts

This chapter studies the problem of concession design under ex-post exit rights. It builds on the dynamic environment of Chapter 1, modeling the optimal design of a procurement partnership with evolving information about project cost. However, the firm is entitled to withdraw from the project after discovering its true cost. Thus, contract design must respect ex-post individual rationality, which is interpreted as limited liability in procurement. The optimal mechanism pools first-period signals into a single contract, and conditions the incentive scheme solely on second-period reports. The effects of exit rights on distortions, on the regulator’s value function, and on social welfare is computed in a numerical exercise. The analysis suggests there is an optimal level of limited liability protection, associated to the threshold of optimality for the pooling mechanism.
2.1 Introduction

This chapter studies the problem of concession design under dynamic asymmetric information and ex-post exit rights. It builds on the environment set up in Chapter 1, which models the optimal design of a procurement partnership with evolving private information about project costs. The main difference, though, is the presence of ex-post participation constraints to contract design: the firm is entitled to withdraw from the project after discovering its true cost, should it calculate that profits under the current incentive scheme fall below an ex-post reservation utility.

The introduction of exit rights in a dynamic procurement model primarily aims to capture the effect of limited liability protection on contract design, and its interplay with asymmetric information. The possibility to file for bankruptcy before completing the project represents an ex-post outside option to the firm, as the law usually restricts the procuring agency’s ability to extract payments or seize assets from an insolvent supplier. The requirement of performance bonds, which are paid upfront and returned to the firm upon project completion, is a common solution to this problem, but has limited scope due to cash constraints or imperfect capital markets.

The maximum loss level the firm stands to bear, in case of unfavorable ex-post private information, typically corresponds to its equity stake in the project. However, in certain regulated sectors, the concept of limited liability is furthered to a “financial equilibrium principle”, under which the firm is entitled to a change in contract terms when it is unable to earn expected returns (Guasch, 2004). For instance, such principle was enacted into Brazilian federal legislation,\(^1\) under a rationale of risk allocation, i.e. the government should not remunerate private firms’ risk premium for adverse contingencies that constitute extraordinary events.\(^2\) Nonetheless, the legislation’s vague wording and myriad judicial interpretations have often led to abuses of the principle, in which contractors demanded renegotiation grounded on debatable events, like demand shortfalls and cost overruns.

The present essay begins the investigation focusing on the consequences of such absolute view of limited liability for concession design. In particular, we depart from the dynamic cost-based procurement model of Chapter 1 (based on Laffont and Tirole, 1986), and introduce ex-post participation constraints, such that ex-ante and ex-post reservation utilities coincide. The analysis of the model entails a theoretical contribution concerning the optimality of static vs. dynamic contracts, connecting the findings to an ongoing discussion in the literature. To simplify the exposition, the analysis restricts attention to the case of two ex-post types.

The essay then proceeds to compare model outcomes between the cases with and without exit rights, focusing on the extent of efficiency distortions, rent extraction, expected

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\(^{1}\) Federal Law no. 8.666, of 1993 (Administrative Contracts Law), art. 65, subsection I, item “a”; and Federal Law no. 8987, of 1995 (Concession Law), art. 9, paragraph 2, and art. 10.

\(^{2}\) For example, extraordinary events can be those classified as material adverse government action, change in law, and force majeure (World Bank, 2017a).
procurement costs and social welfare. Lastly, it runs a numerical exercise to compute the path of the optimal mechanism in response to relaxing ex-post participation constraints, in order to assess the effect of not-so-severe forms of limited liability.

The main result of this chapter is the finding of optimal pooling of firms’ signals under exit rights. If the concessionaire is entitled to earn at least its initial rate of return throughout the contract, as claimed by the absolute version of the financial equilibrium principle, the regulator finds it no longer beneficial to consider forecasts in the design of the remuneration policy. Moreover, the effect of exit rights on distortions, procurement costs and welfare are negative in the present simple model. The analysis suggests that the social costs of such extreme form of contractors’ protection should be seriously evaluated in public procurement policy.

A second finding of this chapter comes from the numerical exercise. Results show that, in line with intuition, the regulator’s objective function (i.e. expected procurement costs) improves from relaxing ex-post participation constraints. However, the effect thereof on firms’ average profit is detrimental only up to the point where ex-ante participation starts to bind. From that point on, as worse signals must be incentivized to accept the contract, the pooling mechanism must leave increased rents to better signals in order to respect incentive compatibility. If exit rights are relaxed further, such effect accumulates until the regulator finds it optimal to begin screening signals to increase rent extraction. At the limit, when ex-post participation is no longer binding, the firm’s expected profit attains the value in Chapter 1’s model, i.e. full sequential screening with heavy losses in some states.

Consequently, the effect on social welfare is non-monotonic: it attains a maximum value at the threshold of optimality for the pooling mechanism. This suggests there is an optimal level of limited liability protection, which holds important implications for concession design. However, the optimal mechanism at such maximum point still pools firms’ signals into one contract, i.e. it maintains the prescription that the regulator should avoid conditioning the remuneration policy on ex-ante private information.

This chapter is organized as follows. After some brief comments on the relevant literature, Section 2.2 presents the model environment and formally states the procurement design problem under ex-post individual rationality. Section 2.3 provides formal propositions on the optimal solution for the “absolute exit rights” case, in which the firm bears no penalty for leaving the project. Section 2.4 analyzes the effect of exit rights on optimal mechanisms and welfare, by comparing model outcomes with and without ex-post participation. Moreover, it provides the results of a numerical exercise for different levels of the ex-post opportunity cost. Lastly, Section 2.5 concludes and proposes directions for future research. Appendix 2.A presents a further discussion on the optimality of pooling mechanisms under exit rights. Omitted proofs are left to Appendix 2.B and the numerical code is listed in Appendix 2.C.

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3 Social welfare in this chapter is understood as the utilitarian sum of the regulator and the firm’s value functions, similarly to Laffont and Tirole (1993). Meantime, the regulator’s objective is narrowly defined as the project’s net benefit maximization.
Related literature

The sequential screening literature was formally introduced by Courty and Li (2000), who modeled optimal sales of an indivisible unit good under risk neutrality and sequential private information. In that model, the principal had to respect only ex-ante individual rationality, and as a result, optimally offered a menu of option contracts.\textsuperscript{4} Previously, Baron and Besanko (1984) studied a multi-period regulation problem with serially correlated types, focusing on the role of the informativeness of early private information. For a recent survey of the dynamic mechanism design literature, refer to Börgers (2015, chapter 11).

Krähmer and Strausz (2015) pioneered the investigation on ex-post individual rationality, motivated by the occurrence of withdrawal rights in e-commerce markets. The authors stated sufficient conditions for an optimal sales policy to be a static contract, i.e. a mechanism that ignores any first-period information the agent might report and conditions the allocation solely on ex-post private valuations. Bergemann, Castro, and Weintraub (2017) furthered their result, finding necessary and sufficient conditions for optimal pooling vs. sequential separation with two ex-ante signals. Both papers are set up an environment of linear payoffs, in which players’ utilities depend linearly on payoff-relevant private information. Linear environments give rise to threshold mechanisms, i.e. mechanisms that allocate the good with probability one to types above a certain threshold on the type space.\textsuperscript{5}

In contrast, Krähmer and Strausz (2016) examine ex-post participation an environment with curvature, similar to the nonlinear pricing model of Mussa and Rosen (1978). The authors find the optimality of static contracts breaks down if the principal can sell multiple units. They argue that variable quantities afford the principal an additional screening instrument, so discriminating ex-ante information becomes profitable again.

This chapter contributes to the theory of dynamic mechanisms by arguing that the failure of static contracts in Krähmer and Strausz (2016) is a consequence of their particular assumption of discrete signals and continuous ex-post valuations. We find that static contracts are again optimal under ex-post participation constraints. The result holds for two ex-post types regardless of the number of ex-ante signals, and without imposing any assumption on the model’s information structure.

We delineate the following explanation: in a nonlinear environment, the principal finds it too expensive, in terms of information rents, to screen the agents’ signals if there are few ex-post types, so she prefers to offer a static contract. However, if there are many types to few signals, as in the continuous-types by discrete-signals environment of Krähmer and Strausz (2016), signals convey precious information to attenuate the ex-post rent-efficiency tradeoff, so the principal prefers to screen sequentially.

\textsuperscript{4} Option contracts refer to an arrangement where the buyer pays an upfront fee, and, when he later discovers his true valuation, he may decide to actually buy the good, or else, withdraw from the contract with a pre-specified refund.

\textsuperscript{5} Hence the relevance of the discussion on the optimality of “deterministic” vs. “stochastic” contracts, where the latter means a mechanism that allocates the good randomly to certain types.
This chapter also contributes to understanding the effect of limited liability on optimal procurement. In a related work, Calveras et al. (2004) motivates the occurrence of abnormally low bids in procurement auctions if bidders can file for bankruptcy ex post. Moreover, Gottlieb and Moreira (2017) find a simplicity result from the interplay of limited liability with moral hazard in procurement. In a way, this chapter’s optimal pooling of cost signals may also be interpreted as a simplicity result arising from the presence of limited liability.

2.2 Environment

The procuring agency (regulator) is required to delegate the execution of a project to a concessionaire (firm). The project has a social benefit level assumed to be high enough to ensure the project’s viability in all cost scenarios. The project’s total cost is given by

$$C = \beta - e$$

where $e \geq 0$ denotes the effort exerted by the firm to reduce cost to level $C$ and $\beta \in \{\beta_L, \beta_H\}$ are the two possible levels for the project’s intrinsic cost (low and high). Let $\Delta \beta = \beta_H - \beta_L > 0$.

As in Chapter 1, the interaction happens in two periods. In the first period, the contract is signed between the regulator and the firm; in the second period, the project is executed. At the contracting stage, the regulator announces the incentive scheme to the firm, who accepts it if expected profits from the partnership exceed the initial reservation utility, normalized to zero. The firm is uncertain about the project’s intrinsic cost $\beta$, but observes a private signal $s \in [s, \bar{s}]$ which conveys information about the likelihood of $\beta$. Specifically, a firm who observes signal $s$ has probability $\Pr(\beta_L|s) = q_s$ of facing a low-cost project and $\Pr(\beta_H|s) = 1 - q_s$ of a high-cost project.

In the production period, the firm privately learns the project’s actual cost $\beta$. However, if the contract specifications result in a profit lower than the firm’s ex-post outside option $\bar{u} \leq 0$, it has the right to exit the partnership. Such right imposes an ex-post participation constraint on contract design.

If the firms decides to complete the project, it chooses its optimal level of cost-reducing effort. The firm has an effort disutility $\psi(e)$, assumed to be increasing ($\psi' > 0$) and convex ($\psi'' > 0$). After the project is delivered, the regulator observes the project’s realized cost $C$, but cannot distinguish between the intrinsic cost $\beta$ and cost-reduction effort $e$.

The regulator must design a dynamic mechanism $T(s, C)$, specifying a nonlinear remuneration schedule that depends on the firm’s reported signal and on effective cost performance $C$, to minimize expected procurement costs. It is useful to denote payoffs in terms of the net transfer $t = T - C$ in excess of cost reimbursement. Both regulator and firm are risk neutral, and the contract is enforced with full commitment.
By the dynamic revelation principle (Myerson, 1986), the problem is equivalent to finding an optimal direct mechanism in an incentive compatible and individually rational set. A direct mechanism is expressed as \(\{(t_L(s), C_L(s)), (t_H(s), C_H(s))\}\) that assigns a net transfer \(t_j(\hat{s})\) and a cost recommendation \(C_j(\hat{s})\) for each sequence of announcements \((\hat{s}, \beta_j)\).\(^6\)

**Incentive compatibility.** In the second period, when the signal report is irreversible, the firm's only relevant decision is which \(\beta_j, j \in \{L, H\}\), to report. Hence, the firm's ex-post profit from the direct mechanism is:

\[
u_j(s|i) = t_j(s) - \psi(\beta_i - C_j(s))
\]

Let \(u_i(s) = u_i(s|i)\) denote the firm's profit from reporting the project's true type in second period. For every signal \(s\), ex-post incentive compatibility is given by:

\[
\begin{align*}
u_L(s) &\geq t_H(s) - \psi(\beta_L - C_H(s)) \\
u_H(s) &\geq t_L(s) - \psi(\beta_H - C_L(s))
\end{align*}
\]

(\(IC-2\))

**Lemma 2.1.** For all \(s\), incentive constraints (\(IC-2\)) are equivalent to

\[
\begin{align*}
u_L(s) &\geq \phi(e_H(s)) + u_H(s) \quad (IC-2') \\
C_H(s) &\geq C_L(s) \quad (MON)
\end{align*}
\]

where \(\phi(e) \equiv \psi(e) - \psi(e - \Delta \beta)\) denotes the information rent of a firm with project \(\beta_L\).

From the first period viewpoint, the second-period optimal continuation strategy for the firm is to report \(\beta_j = \beta_i, i = L, H\). Hence, the firm's ex-ante expected payoff is:

\[
U(\hat{s}|s) = q_s u_L(\hat{s}) + (1 - q_s) u_H(\hat{s})
\]

Let \(U(s) = U(s|s)\) denote the firm's expected profit from reporting its signal accurately in the first period. Ex-ante incentive compatibility is thus given by:

\[
U(s) \geq U(\hat{s}|s), \quad \forall \hat{s}, s \quad (IC-1_{\hat{s}, s})
\]

**Individual rationality.** The direct mechanism satisfies multi-period voluntary participation if it secures ex-ante and ex-post profits larger than the firm’s opportunity cost in each period, i.e.\(^7\)

\[
\begin{align*}
U(s) &\geq 0, \quad \forall s \quad (IR-1) \\
u_H(s) &\geq \bar{u}, \quad \forall s \quad (IR-2)
\end{align*}
\]

\(^6\) The mechanism can be alternatively expressed in terms of the regulator's effort recommendation to the firm: \(e_j(\hat{s}|i) = \beta_i - C_j(\hat{s})\).

\(^7\) Under (\(IC-2\)), ex-post profits are decreasing with \(\beta\), i.e. \(u_L(s) \geq u_H(s), \forall s\), so the only relevant ex-post IR constraint concerns the expensive project type \(\beta_H\).
The ex-post constraint (IR-2) aims to capture the concept of limited liability in procurement contracting, as the firm is entitled to stop its loss at $\bar{u} \leq 0$ in unfavorable states. Moreover, the case $\bar{u} = 0$ is interpreted as an absolute form of the “financial equilibrium principle”, under which the firm is entitled to its ex-ante opportunity cost in all states.

**Design problem.** Suppose the regulator has prior belief that firm signals are distributed according to $F(s)$, with density $f(s) > 0$, $\forall s \in [s, \bar{s}]$. We make no assumption on the information structure other than a relabeling of signals, such that a low $s$ signifies a higher probability of an inexpensive project $\beta_L$ (“good news”), i.e., $q_s$ decreases with $s$. Such assumption is without loss of generality.

Denote $T_i(s) = t_i(s) + C_i(s)$ the ex-post payment to the firm. As the regulator wishes to minimize the expected procurement cost, the design problem is:

$$\mathcal{P} : \min_{\{t_i(s), C_i(s)\} \in (L,H)} \int_{\bar{s}}^{\bar{s}} \{q_s T_L(s) + (1 - q_s) T_H(s)\} dF(s)$$

s.t. (IC-1, $s$), (IC-2), (IR-1), (IR-2)

Unless stated otherwise, we assume $\bar{u} = 0$, so the firm’s ex-ante and ex-post outside options are equal. Consequently, constraint (IR-1) becomes redundant under (IR-2).

There is a number of transformations that make the problem more amenable to analysis. First, note the ex-post payment is equivalently expressed as

$$T_i(s) = \beta_i - e_i(s) + \psi(e_i(s)) + u_i(s)$$

$$= \beta_i - e_i(s) + \psi(e_i(s)) + u_H(s) + \phi(e_H(s)) \mathbb{1}_{i = L}$$

where the indicator function equals one when the project type is $\beta_L$. The last equality follows from the fact that (IC-2’) binds at the optimum, since $u_i(s)$ enters linearly in the regulator’s minimization. Also, under (IC-2’), the constraint (IC-1, $s$) can be written as

$$q_s [\phi(e_H(s)) + u_H(s)] + (1 - q_s) u_H(s) \geq q_s [\phi(e_H(\tilde{s})) + u_H(\tilde{s})] + (1 - q_s) u_H(\tilde{s})$$

$$\Leftrightarrow q_s [\phi(e_H(s)) - \phi(e_H(\tilde{s}))] + u_H(s) - u_H(\tilde{s}) \geq 0$$

Thus, the regulator’s design problem can be restated as follows:

$$\mathcal{P} : \min_{\{e_L(s), e_H(s), u_H(s)\}} \int_{\bar{s}}^{\bar{s}} \{q_s [\beta_L - e_L(s) + \psi(e_L(s)) + \phi(e_H(s))]$$

$$+ (1 - q_s) [\beta_H - e_H(s) + \psi(e_H(s))] + u_H(s)\} f(s) ds$$

s.t. $q_s [\phi(e_H(s)) - \phi(e_H(\tilde{s}))] + u_H(s) - u_H(\tilde{s}) \geq 0, \ \forall \tilde{s}, s$ (IC-$s$)

$u_H(s) \geq 0, \ \forall s$ (IR-$s$)

$\Delta \beta \geq e_H(s) - e_L(s), \ \forall s$ (MON-$s$)
2.3 Model analysis

This section addresses the design problem with a guess-and-verify algorithm: we mean to show that the pooling mechanism defined below is an optimal solution to problem (P). Such guess is not coincidental; Krähmer and Strausz (2015) stated sufficient conditions for optimal pooling of ex-ante information in sequential screening models with ex-post participation. Further, Bergemann et al. (2017) have found necessary and sufficient conditions for optimal pooling vs. separation in a model with two ex-ante signals.\footnote{The environment analyzed by these authors features a continuum of ex-post types and, crucially, payoffs that are linear in payoff-relevant private information. In contrast, the present model features two ex-post types and payoffs that exhibit curvature, despite the likewise finding that pooling is optimal. See the discussion at the end of this section.}

**Definition** (pooling mechanism). The mechanism that assigns the same ex-post effort recommendation to all firm signals \( s \in [\underline{s}, \bar{s}] \) is denoted \( \{\bar{e}_L, \bar{e}_H\} \), given by:

\[
\begin{align*}
\psi'(\bar{e}_L) &= 1 \\
\psi'(\bar{e}_H) &= 1 - \frac{\bar{q}}{1 - \bar{q}} \phi(e_H)
\end{align*}
\]  

(2.1)

where \( \bar{q} = \int_{\underline{s}}^{\bar{s}} q_s f(s) ds \) is the unconditional probability of an inexpensive project \( \beta_L \).

The effort recommendation in (2.1) coincides with the solution to the optimal procurement problem with two project types (Laffont and Tirole, 1993, Section 1.3), in which the regulator believes the low-cost project \( \beta_L \) happens with probability \( \bar{q} \). Hence, the pooling allocation is referred to as static, since it ignores first-period information and conditions the mechanism solely on second-period reports.

Notice the pooling mechanism described in (2.1) satisfies all (IC-1,\( s, \bar{s} \)) constraints with equality. Also, it immediately follows that \( \{\bar{e}_L, \bar{e}_H\} \) strictly satisfies monotonicity constraints (MON). Hence, to our purpose it suffices to focus on ex-post participation constraints (IR,\( s \)) and actual optimality conditions.

The present solution algorithm proceeds according to the following steps:

1. State a relaxed version of the regulator’s problem, denoted (\( R \)), which exogenously set ex-post profits \( u_H(s) \) to zero;
2. Guess-and-verify the set of active (IC-1) constraints at the pooling solution. Define a further relaxed problem that ignores the remaining inactive constraints, denoted (\( R^0 \));
3. Construct problem (\( R^0 \))’s Lagrangian and present a Kuhn-Tucker argument to the solution taking the form of pooling mechanism (2.1); and
4. Extend the Lagrangian to include ex-post IR constraints and find multipliers that sustain the pooling mechanism (2.1) and \( u^*_H(s) = 0 \) as an optimal solution.
Step 1. Suppose, at first, that the optimal mechanism indeed satisfies $u_H(s) = 0$. Then, the (relaxed) regulator’s design problem becomes:

$$
\mathcal{R} : \min_{\{e_L(\cdot), e_H(\cdot)\}} \int_2^\pi \left\{ q_s [\beta_L - e_L(s) + \psi(e_L(s)) + \phi(e_H(s))] \\
+ (1 - q_s) [\beta_H - e_H(s) + \psi(e_H(s))] \right\} f(s) ds
$$

s.t. $q_s [\phi(e_H(s)) - \phi(e_H(\tilde{s}))] \geq 0, \ \forall \tilde{s}, s$

Step 2. To improve the tractability of problem ($\mathcal{R}$), we restrict the set of first-period incentive constraints to pairs of signals $s$ and $\tilde{s}$ for which the constraints (IC-1$_{s, \tilde{s}}$) are active at the optimum. If the pooling mechanism (2.1) is optimal under this restricted set, then it solves problem ($\mathcal{R}$), as it trivially satisfies all neglected IC-1 constraints.\(^9\)

Lemma 2.2. Let $s^\ast$ denote the cutoff signal such that $q_{s^\ast} = \bar{q}$. Denote IC$^\ast$ as the set of incentive constraints in which a firm with below-average signal (“good type”) does not envy the allocation assigned to an above-average signal (“bad type”), i.e.

$$
IC^\ast = \{ IC_{s, \tilde{s}} : q_s > \bar{q} > q_{\tilde{s}} \}
$$

If the pooling mechanism $\{\bar{e}_L, \bar{e}_H\}$, as defined in (2.1), solves problem ($\mathcal{R}$) restricted to IC$^\ast$, then it solves the global problem ($\mathcal{R}$).

Hence, it suffices to solve the following version of the relaxed problem:

$$
\mathcal{R}^0 : \min_{\{e_L(\cdot), e_H(\cdot)\}} \int_2^\pi \left\{ q_a [\beta_L - e_L(s) + \psi(e_L(s)) + \phi(e_H(s))] \\
+ (1 - q_s) [\beta_H - e_H(s) + \psi(e_H(s))] \right\} f(s) ds
$$

s.t. $q_a [\phi(e_H(a)) - \phi(e_H(b))] \geq 0, \ \forall a < s^\ast < b$

Step 3. Note that the Kuhn-Tucker theorem for function spaces (Luenberger, 1997, p.220) applies to concave optimization problems (i.e. convex objectives and feasibility sets for minimization problems). However, the constraints (IC-1) in problem ($\mathcal{R}^0$), being a difference of convex functions $\phi(.)$, are not necessarily convex. The following Lemma 2.3 establishes sufficient conditions for the applicability of Kuhn-Tucker conditions to the optimization problem ($\mathcal{R}^0$).

Lemma 2.3. Problem ($\mathcal{R}^0$)’s objective function is concave, and its (IC) constraints are quasi-concave functions.

\(^9\) This technique was first proposed by Krähmer and Strausz (2015).
Invoking the quasi-concave version of the Kuhn-Tucker theorem (Arrow and Enthoven, 1961, Theorem 1), a mechanism \( \{e_L(s), e_H(s)\} \) is a solution to problem (\( R \)) if there exist positive multipliers \( \lambda: [\underline{s}, s^*] \times [s^*, \bar{s}] \rightarrow \mathbb{R}_+ \) so that \( \{e_L(s), e_H(s)\} \) minimizes the Lagrangian:

\[
\mathcal{L} = \int_\underline{s}^\bar{s} \{ q_s [\beta_L - e_L(s) + \psi(e_L(s)) + \phi(e_H(s))] + (1 - q_s) [\beta_H - e_H(s) + \psi(e_H(s))] \} f(s)ds \\
- \int_\underline{s}^s \int_\underline{s}^\bar{s} \lambda(a, b) q_a [\phi(e_H(a)) - \phi(e_H(b))] dbda
\]

The minimizer of \( \mathcal{L} \) must satisfy the following pointwise first-order conditions (FOC):

\[
[e_L(s)] : \quad f(s)q_s (-1 + \psi'(e_L(s))) = 0 \\
[e_H(s)] : \begin{cases} 
  f(s)(1 - q_s) (-1 + \psi'(e_H(s))) + f(s)q_s \phi'(e_H(s)) - \int_\underline{s}^\bar{s} \lambda(s, b) q_s \phi'(e_H(s)) db = 0 & \text{if } s \in [\underline{s}, s^*] \\
  f(s)(1 - q_s) (-1 + \psi'(e_H(s))) + f(s)q_s \phi'(e_H(s)) + \int_\underline{s}^s \lambda(a, s) q_a \phi'(e_H(s)) da = 0 & \text{if } s \in [s^*, \bar{s}] 
\end{cases}
\]

The existence of a multiplier function \( \lambda(a, b) \geq 0 \) that satisfies the preceding FOC system is established in the following Proposition 2.1. Therefore, by Lemma 2.2, the pooling mechanism also solves problem (\( R \)).

**Proposition 2.1.** There exists a function \( \lambda^*: [\underline{s}, s^*] \times [s^*, \bar{s}] \rightarrow \mathbb{R}_+ \) such that the pooling allocation \( \{\bar{e}_L, \bar{e}_H\} \) is an optimal solution to problem (\( R^0 \)), with multipliers given by \( \lambda^*(s, \hat{s}) \).

**Step 4.** Now, consider the Lagrangian that explicitly incorporates ex-post participation constraints (IR-2):

\[
\mathcal{L}^* = \int_\underline{s}^\bar{s} \{ q_s [\beta_L - e_L(s) + \psi(e_L(s)) + \phi(e_H(s))] \\
+ (1 - q_s) [\beta_H - e_H(s) + \psi(e_H(s)) + u_H(s)] \} f(s)ds \\
- \int_\underline{s}^\bar{s} \int_\underline{s}^\bar{s} \lambda(a, b) \{ q_a [\phi(e_H(a)) - \phi(e_H(b))] + u_H(a) - u_H(b) \} dbda \\
- \int_\underline{s}^\bar{s} \mu(s) u_H(s) ds
\]

By the same argument in Arrow and Enthoven (1961), if there exist positive multipliers \( \lambda(a, b) \geq 0 \) and \( \mu(s) \geq 0 \) such that Lagrangian \( \mathcal{L}^* \) is minimized at \( \{\bar{e}_L, \bar{e}_H\} \) and \( u_H'(s) = 0, \forall s \), then the pooling mechanism is a solution to problem (\( P \)). The existence of such multipliers is established in Proposition 2.2.
Proposition 2.2. There exist functions $\lambda^* : [\bar{s}, s^*] \times [s^*, \bar{s}] \to \mathbb{R}_+$ and $\mu^* : [\bar{s}, \bar{s}] \to \mathbb{R}_+$ such that the pooling allocation $\{\hat{e}_L, \hat{e}_H\}$ and the ex-post profit level $u^*_H = 0$ are an optimal solution to problem $(P)$, with multipliers given by $\lambda^*(s, \hat{s})$ and $\mu^*(s)$.

Therefore, the optimal procurement policy under exit rights pools firms’ signals into a single contract. In other words, the regulator does not charge different franchise fees to different signals, thereby offering the same ex-post menu of incentive contracts irrespective of the firm’s forecast. This result stands in contrast to the optimal policy in Chapter 1, where the regulator was able to achieve an improved rent-efficiency tradeoff from screening ex-ante signals.

Discussion

The finding of optimal pooling is novel for a sequential mechanism design model with ex-post participation constraints and payoff curvature. It stands in sharp contrast to the result in Krähmer and Strausz (2016), namely, that sequential screening is always optimal with multiple units for sale (i.e. nonlinear payoffs). Those authors argue that multiple units represent an additional screening instrument available to the principal, differently from the indivisible-unit case, where linear payoffs lead to an optimal threshold mechanism. As it turns out, the cardinality of type-spaces seems to be an important factor determining the optimality of pooling. The Krähmer and Strausz (2016) result is replicated to the procurement model in Appendix 2.A, confirming their finding that pooling generically fails to be optimal. That result may be interpreted as follows: with a finite number of ex-ante signals and a continuum of ex-post types, signals are too precious to ignore, even though a signal-separating contract leaves increased rents to the firm due to exit rights. Meanwhile, this chapter inverts the setting to a continuum of signals by two ex-post project types; the findings are accordingly opposite. With few ex-post project types, signals loose their value, as it becomes prohibitively expensive to pay the ex-ante rents required by a sequential screening contract.

What happens in intermediate cases is an open question. Appendix 2.A presents an attempt to shed light on conditions for optimal pooling vs. separation in sequential environments with ex-post participation and curvature, by analyzing a simple 2-signal by 3-types model. The solution depends on the information structure, the richness of which is somewhat lost in the present version of the model with two ex-post types. Nonetheless, the result in Bergemann et al. (2017) appears to point in the right direction. Those authors argue that the optimality of ex-ante screening depends on how “different” forecasting technologies are from each other, as measured by a transformation of the conditional distributions’ “cross-hazard rates” (see Appendix 2.A).

10 Krähmer and Strausz (2015) have shown that, under ex-post participation constraints, the optimal selling mechanism is a threshold mechanism (posted price), which pools first-period information.
2.4 The effect of exit rights

This section presents an application of the dynamic procurement model under exit rights. We have shown that the pooling mechanism is an optimal solution to problem \( P \), which set \( \bar{u} = 0 \). For comparison purposes, now we consider the problem of dynamic procurement design without exit rights (i.e. \( \bar{u} \to -\infty \)), adapted from the model developed in Chapter 1.

Suppose there are only two signals, \( s_L \) and \( s_H \) (low and high), with probabilities \( \nu_L = \nu \) and \( \nu_H = 1 - \nu \), respectively. A firm observing signal \( s_L \) expects the project to be low-cost with probability \( q_L > q_H \).

Let \( S > 0 \) denote the project’s gross benefit, assumed sufficiently large to grant the project’s viability in all cost scenarios. The regulator’s design problem under ex-ante participation is:

\[
S : \min_{\{e_L(\cdot), e_H(\cdot)\}} \sum_{s \in \{L, H\}} \nu_s \left\{ q_s [\beta_L - e_L(s)] + \psi(e_L(s)) \right\} + (1 - q_s) [\beta_H - e_H(s)] + U_s \]  

\[ \begin{align*}
\text{s.t.} & \quad U_s \geq q_s \phi(e_H(r)) + u_H(r), \quad r \neq s \quad \text{(IC}_{s,r}\text{)} \\
& \quad \Delta \beta \geq e_H(s) - e_L(s), \quad s = L, H \quad \text{(MON}_s\text{)} \\
& \quad U_s \geq 0, \quad s = L, H \quad \text{(IR}_s\text{)}
\end{align*}
\]

Assume the effort disutility takes a quadratic form, i.e. \( \psi(e) = \alpha e^2/2 \), with curvature parameter \( \alpha > 0 \) (such that \( e^{fb} = 1/\alpha \)). Based on a discrete version of equations (1.2)-(1.4) in Chapter 1, we compute effort recommendations, expected profits, and realized profits that characterize the optimal solution to problem \( S \), as follows:

\[
\begin{align*}
e_L(s_L) &= e^{fb} \\
e_L(s_H) &= e^{fb} \\
e_H(s_L) &= e^{fb} \\
e_H(s_H) &= e^{fb} - \frac{\nu}{1 - \nu} \frac{q_L - q_H}{1 - q_H} \Delta \beta \\
U(s_L) &= \phi(e_H(s_H))(q_L - q_H) \\
U(s_H) &= 0 \\
u_L(s_L) &= U(s_L) + \phi(e_H(s_L))q_H \\
u_L(s_H) &= \phi(e_H(s_H))q_L \\
u_H(s_L) &= U(s_L) - \phi(e_H(s_L))(1 - q_H) \\
u_H(s_H) &= -\phi(e_H(s_H))(1 - q_L)
\end{align*}
\]

\[11\] In this simple \( 2 \times 2 \) case, \( q_L > q_H \) represents to the first-order stochastic dominance (FOSD) assumption on forecasting technologies.

\[12\] The project’s benefit is relevant to compute the regulator’s value function (net benefit) and the level of social welfare attained by the optimal mechanism. More on that below.
Recall from Proposition 2.2 that, under (IR-2) with $\bar{u} = 0$, the optimal mechanism is given by the pooling allocation (2.1) and binding participation constraints:

$$\bar{e}_L = e_{fb}$$
$$\bar{e}_H = e_{fb} - \frac{\bar{q}}{1 - \bar{q}} \Delta \beta$$
$$\bar{u}_L = \phi(\bar{e}_H)$$
$$\bar{u}_H = 0$$

Finally, we need a definition of social welfare. Following Laffont and Tirole (1993), denote $V = \mathbb{E}_s [S - (t + C)]$ as the project’s expected net benefit, i.e. the regulator’s narrow objective. Adding it up with firms’ average profit $\mathbb{E}_s [U]$, define the utilitarian social welfare as:

$$W = \mathbb{E}_s [V + U]$$

To assess the effect of exit rights, we compute the optimal mechanism numerically for both versions of the model. Numerical outcomes for models $(P)$ and $(S)$ are compared in the following Table 2.1. The exercise parameters are as follows:

- Social value: $S = 100$
- Intrinsic costs: $\beta \in \{80, 100\}$
- Curvature parameter: $\alpha = 0.025$ (i.e. $e_{fb} = 40$)
- Information structure: $\nu = 0.5$, $q_L = 0.75$ and $q_H = 0.25$

<table>
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<th>IR ex-post</th>
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<td>40.00</td>
</tr>
<tr>
<td>effort($\beta_H$)</td>
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<td></td>
<td>avg = 2.08</td>
<td>avg = 2.50</td>
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<tr>
<td>$\mathbb{E}[\text{payment}</td>
<td>s]$</td>
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<td></td>
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<td>avg = 75.00</td>
</tr>
<tr>
<td>social welfare</td>
<td>29.17</td>
<td>27.50</td>
</tr>
</tbody>
</table>

Table 2.1: Optimal mechanism outcomes under ex-ante and ex-post participation

Intuitively, the model where (IR-2) is absent attains a better value function for the regulator (lower expected payment). By symmetry, the unconstrained model yields worse average payoffs for the firm, albeit gains from exit rights are relatively small for the present parametrization.
As for social welfare, the unconstrained model also dominates the exit-rights case. The normative implication in this simple model is clear: absolute claims to financial equilibrium are detrimental to welfare. It benefits the concessionaire at the expense of users, while also generating a deadweight loss. Thus, the firm should bear some liability for bad states of nature in a concession partnership.

Relaxing ex-post participation

The following exercise computes numerically the optimal solution to problem \((P)\) for a range of ex-post opportunity cost levels \(\bar{u} \in [-10, 0]\).\(^\text{13}\) A remarkable result from the exercise is that relaxing (IR-2) gives rise to three relevant thresholds:

- \(\bar{u}_1\): the ex-ante participation constraint (IR-1) becomes binding;
- \(\bar{u}_2\): the pooling solution \(\{\bar{e}_L, \bar{e}_H\}\) breaks down, so the optimal allocation entails some sequential screening; and
- \(\bar{u}_3\): the ex-post participation constraint (IR-2) becomes redundant.

Figure 2.1 displays the three aforementioned thresholds. At the level of ex-post outside option \(\bar{u}_1\), the expected rent of the high-signal firm attains zero, so (IR-1) becomes active. The informational rent left to the low-signal firm increases sharply so that the pooling mechanism remains consistent with (IC-1s, s).

Figure 2.1: Optimal ex-ante profits in response to \(\bar{u}\)

\(^{13}\) The Matlab codes that implement the global optimization \((P)\) are presented in Appendix 2.C.
The pooling mechanism is no longer optimal from $\bar{u}_2$ onwards, as the regulator finds it more profitable to separate each signal’s allocation and begin to extract some rent from type $s_L$. At $\bar{u}_3$, the scope for rent extraction ceases as (IR-2) becomes inactive, and $U(s)$ attains the optimal level for problem $(S)$, corresponding to (2.4).

The optimal ex-post profit levels $u_i(s)$ are shown in Figure 2.2.

![Figure 2.2: Optimal ex-post profit in response to $\bar{u}$](image)

The lines move in parallel while the ex-ante constraint (IR-1) is inactive, as the difference $u_L - u_H$ is constant and given by (IC-2'). When (IR-1) becomes active, the two ex-post profit levels begin to distance one from another, so to respect non-negativity of the expected profit $U(s)$. When pooling breaks down, the regulator begins to discriminate profit levels among all type sequences $(s, \beta_i)$. Finally, when (IR-2) becomes inactive, ex-post profits stabilize at the level characterized by (2.5).

The behavior of the problem’s optimal Lagrange multipliers in response to shifting the ex-post outside option is depicted in Figure 2.3. The plots clearly indicate the points where constraints (IR-1) and (IR-2), respectively, become active (at point $\bar{u}_1$) and inactive (at point $\bar{u}_3$). Note that for $\bar{u} \geq \bar{u}_1$, the multipliers for (IC-1) and (IR-2) are indeterminate, as there are multiple solutions that satisfy the Lagrangian’s system of first-order conditions (see Appendix 2.A). Finally, note that the multiplier for type-$H$’s ex-post participation becomes zero at $\bar{u}_2$, meaning the optimality of pooling breaks down precisely when (IR-2$_H$) constraint becomes slack.
Figure 2.3: Behavior of Lagrange multipliers in response to $\bar{u}$
Figure 2.4 plots the optimal mechanism’s effort recommendation schedule. Similarly to ex-post utilities, optimal efforts move in parallel for small values of \( \bar{u} \) such that (IR-1) is inactive.\(^{14}\) Beyond \( \bar{u}_1 \), although signal-pooling is still optimal, the wedge \( \bar{e}_L - \bar{e}_H \) diminishes to support a widening gap \( \bar{u}_L - \bar{u}_H \), needed to ensure \( U(s) \geq 0 \). Notice this amounts to a decrease in aggregate distortions. Beyond \( \bar{u}_2 \), pooling is no longer optimal, so the regulator starts to recommend different efforts for different signals, reducing distortions for \( s_L \) and increasing for \( s_H \). Lastly, beyond \( \bar{u}_3 \) effort levels are the optimal solution to problem \((S)\), given by (2.3).

Figure 2.5 presents the regulator’s net expected value from the partnership, given by \( V \), as well as expected social welfare defined as \( W = \mathbb{E}_s[V + U] \). Presumably, the regulator’s value function \( V(\bar{u}) \) increases monotonically (though at decreasing rates) as we relax the ex-post participation constraint (IR-2), attaining the optimal value of the unconstrained problem \((S)\) at \( \bar{u}_3 \).

A more striking finding from the numerical exercise, seen from Figure 2.5, is that the expected social welfare \( W(\bar{u}) \) peaks at \( \bar{u}_2 \), as it corresponds to the point of maximum average profit ex ante (see Figure 2.1). The level of ex-post loss \( \bar{u}_2 \) corresponds to the limit on the firm’s liability that would be optimally set by a benevolent, utilitarian planner. Such point represents the threshold of optimality for the pooling mechanism \( \{\bar{e}_L, \bar{e}_H\} \), where both ex-ante and ex-post participation constraints are active. Besides, it represents the point where the high-signal’s ex-post participation constraint \((IR - 2_H)\) becomes inactive, as seen from Figure 2.3.

\(^{14}\) The optimal effort for the low-cost project \( \beta_L \) is always equal to the first-best.
For the present parametrization, the benefit for social welfare from setting the ex-post outside option at $\bar{u}_2$ is not overwhelmingly significant: the optimal welfare $W(\bar{u}_2)$ only exceeds the welfare from full sequential screening $W(\bar{u}_3)$ by about 0.03%. However, an interesting question for future research concerns the generalization of the present numerical result.

**Policy implications.** The preceding analysis offers relevant lessons for concession design. The ex-post opportunity cost $\bar{u}$ is intended to capture the extent of limited liability protecting the concession operator. At the limit $\bar{u} = 0$, the firm is entitled to the same profit level expected from the outset of the contract. However, such a policy entails a significant cost as it attains the lowest social value in the model, be it measured in terms of welfare or the project’s net benefit.

Further, from the procuring agency’s narrow viewpoint, the concessionaire’s liability would preferably be unlimited, so she could screen all private information in the name of efficiency and rent extraction. Notwithstanding, the numerical exercise suggests the aggregate surplus is maximal for some positive, but finite level of ex-post protection (Figure 2.5). Applied to infrastructure industries, such recommendation amounts to requiring a minimum equity stake from the project’s sponsors, beyond which the consortium is allowed to file for bankruptcy without bearing further losses.

---

15 Also known as the “principle of financial equilibrium”, under which the concessionaire expects a change in contract terms when the project’s rate of return frustrates expectations (Guasch, 2004).
2.5 Conclusion

This chapter developed a model of optimal procurement with dynamic cost information and ex-post exit rights. The motivation for ex-post individual rationality comes from the observation that procurement relationships often entail some extent of limited liability to the contractor. In its extreme form, it becomes an absolute view of the “financial equilibrium principle”, under which the firm is entitled to zero losses no matter what. In our stylized model with two ex-post types, the associated optimal policy exhibits pooling of ex-ante signals. Hence, the regulator prefers to ignore informative cost forecasts in contract design, as exit rights increase the burden of ex-ante information rents.

The main result, however, does not hold for general type spaces. We presented a discussion on the role of type-space cardinality, as well as details of the information structure, in determining the optimality of pooling vs. sequential screening in curvature environments. The extension of the present results to more general contracting environments would be a welcome development from a theoretical standpoint, and remains an active agenda for future research.

The essay also explored the effect of exit rights on optimal procurement contracts in a numerical exercise, by assessing model outcomes in response to shifting the firm’s ex-post outside option. The absolute financial equilibrium case is clearly dominated by pure sequential screening, be it in terms of social welfare or the project’s net benefit. Full protection against losses entails a deadweight loss, and hence, benefits the concessionaire at the expense of society. Results also suggest there is an optimal level of limited liability protection from a welfare standpoint. The optimal level corresponds to the threshold point such that, for higher losses allowable to the firm, the pooling contract ceases to be optimal. Such point happens coincidentally with the slackness of the worst type’s ex-post participation constraint.

For further research, we suggest a formal analysis that endogenizes the ex-post outside option in contract design. The idea is to check if the finding of an optimal limit on the firm’s liability coupled with optimal pooling of ex-ante signals holds in general. Such a model could be motivated by a recurrent concern in infrastructure policy, about the extent to which the regulator should require the concessionaire to assume an equity stake in the project. The characterization of optimal liability motivated by dynamic incentive issues would bring additional light to the ongoing debate about the early termination of concession contracts.

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16 For linear environments, Bergemann et al. (2017) show that signal-pooling under ex-post participation fails to be optimal if forecasting technologies are sufficiently “different” from each other.
2.A Alternative information structures

This Appendix extends the discussion presented in Section 2.3. First, we replicate the result in Krähmer and Strausz (2016) for the procurement setting. Then we present a simple version of the model, featuring 2 signals by 3 ex-post types, to illustrate the information structure’s role in determining the optimality of pooling contracts.

Case: 2 signals and continuous ex-post types

Consider the environment set up in Section 2.2, but with two signals \( s \in \{s_L, s_H\} \) and a continuum of project costs \( \beta \in [\beta, \bar{\beta}] \). Suppose the signal probabilities are \( \Pr(s_i) = \nu_i \) and forecasting technologies are given by \( G_i(\beta) \), with \( G_L(\beta) \geq G_H(\beta) \).

First, note ex-post incentive constraints with continuous project types are stated similarly to Chapter 1:

\[
\begin{align*}
&\text{Ex-ante incentive constraints are given by:} \\
&\int_\beta^{\bar{\beta}} \left[ u_{s_i}(\beta) - u_{s_j}(\beta) \right] dG_i(\beta) \quad \forall i, j \\
&\quad \Leftrightarrow \int_\beta^{\bar{\beta}} \left[ u'_{s_i}(\beta) = -\psi(e_{s_i}(\beta)) \\& C_{s_i}(\beta) \text{ monotonic in } \beta \right] , \forall s_i
\end{align*}
\]

Plug the expression for ex-post IC information rents in the regulator’s objective function; the design problem under ex-post participation constraints is stated as follows:

\[
\begin{align*}
\min_{\{e_{s_i}(\cdot), u_{s_i}(\cdot)\}_{s_i \in \{L, H\}}} \sum_{i \in \{L, H\}} \nu_i \left\{ \int_\beta^{\bar{\beta}} \left[ \beta - e_{s_i}(\beta) + \psi(e_{s_i}(\beta)) + \psi'(e_{s_i}(\beta)) \frac{G_i(\beta)}{g_i(\beta)} \right] dG_i(\beta) + u_{s_i}(\bar{\beta}) \right\} \\
\text{s.t. } (IC_{ij}), \quad u_{s_i}(\bar{\beta}) \geq 0, \quad e'_{s_i}(\beta) \leq 1 \\
                   \forall i, j \in \{L, H\}
\end{align*}
\]
The problem’s Lagrangian (ignoring monotonicity constraints) becomes:

\[
\mathcal{L}^* = \sum_{i \in \{L,H\}} \nu_i \left\{ \int_\beta^\beta \left[ \beta - e_i(\beta) + \psi(e_i(\beta)) + \psi'(e_i(\beta)) \frac{G_i(\beta)}{g_i(\beta)} \right] dG_i(\beta) + u_i(\beta) \right\} \\
- \lambda_{iH} \int_\beta^\beta [\psi'(e_i(\beta)) - \psi'(e_H(\beta))] G_L(\beta) d\beta \\
- \lambda_{iL} \int_\beta^\beta [\psi'(e_i(\beta)) - \psi'(e_H(\beta))] G_H(\beta) d\beta \\
+ u_i(\beta) \left[ \nu_i - \lambda_{iH} + \lambda_{iL} - \mu_i \right]
\]

For a mechanism to be optimal there must exist multipliers \((\lambda_{iH}, \lambda_{iL}, \mu_i, \mu_H) \geq 0\), such that \{e_i(\cdot), u_i(\beta)\}_{i \in \{L,H\}} satisfies the following first-order system for \(j \neq i \in \{L, H\}\):

\[
\nu_i \left( -1 + \psi'(e_i(\beta)) + \psi''(e_i(\beta)) \frac{G_i(\beta)}{g_i(\beta)} \right) \\
- \lambda_{ij} \psi''(e_i(\beta)) \frac{G_j(\beta)}{g_i(\beta)} + \lambda_{ji} \psi''(e_i(\beta)) \frac{G_j(\beta)}{g_i(\beta)} = 0 \tag{2.6}
\]

\[
\mu_i \leq \nu_i - \lambda_{ij} + \lambda_{ji} \tag{2.7}
\]

Let \(\tilde{G}(\beta) = \sum_i \nu_i G_i(\beta)\) denote the unconditional \(\beta\)-distribution and denote the pooling mechanism by \(\tilde{u}(\beta) = 0\) and \(\tilde{e}(\beta)\) such that

\[
\psi'(\tilde{e}(\beta)) = 1 - \psi''(\tilde{e}(\beta)) \frac{\tilde{G}(\beta)}{\tilde{g}(\beta)} \tag{2.8}
\]

Substituting the pooling mechanism (2.8) in first-order condition (2.6) it obtains:

\[
\nu_i \left( \frac{\tilde{G}(\beta)}{\tilde{g}(\beta)} - \frac{G_i(\beta)}{g_i(\beta)} \right) = -\lambda_{ij} \frac{G_j(\beta)}{g_i(\beta)} + \lambda_{ji} \frac{G_j(\beta)}{g_i(\beta)} \tag{2.9}
\]

Generically, there exists no \((\lambda_{iH}, \lambda_{iL}) \in \mathbb{R}_+^2\) such that the nonlinear equation (2.9) is satisfied for all \(\beta\). Therefore, the case of two signals and continuous project types does not support the pooling mechanism as an optimal solution.\(^{17}\)

**Case: 2 signals and 3 ex-post types**

The case of \(2 \times 2\)-dimensional type-space is a special case of the model presented in Section 2.2, so it results in optimal signal-pooling regardless of the information structure.

\(^{17}\) The result readily extends to any finite number of signals (Kröhmer and Strausz, 2016).
Now, consider the case of 2 signals $s_i, i \in \{L, H\}$ and a 3 project types $\beta \in [\beta_0, \beta_1, \beta_2]$. Assume $\Pr(s_i) = \nu_i, \Pr(\beta_k|i) = q_{i,k}$, and $\beta_2 - \beta_1 = \beta_1 - \beta_0 = \Delta \beta$.

Let $e_{i,k}$ denote the level of cost-reducing effort recommended to a firm who reports signal $s_i$ and project type $\beta_k$, and $\phi(e_i) \equiv \psi(e) - \psi(e - \Delta \beta)$ as in Section 2.2. In the discrete case, the (binding) ex-post individual rationality constraints are given by:

\[ u_i(\beta_0) = \phi(e_{i,1}) + \phi(e_{i,0}) \]
\[ u_i(\beta_1) = \phi(e_{i,0}) \]
\[ u_i(\beta_2) = 0, \quad i = L, H \]

Plugging into the ex-ante individual rationality constraint for signal $i$, it obtains:

\[ \sum_k q_{i,k} [u_i(\beta_k) - u_j(\beta_k)] \geq 0, \quad j \neq i \]
\[ \Leftrightarrow q_{i,0} [\phi(e_{i,1}) - \phi(e_{i,0})] + (q_{i,0} + q_{i,1}) [\phi(e_{i,2}) - \phi(e_{i,1})] \geq 0 \quad (\text{IC}_{i,j}) \]

Denote the unconditional $\beta$-distribution as $\bar{q}_k = \nu_Lq_{L,k} + \nu_Hq_{H,k}$, and the pooling mechanism as $\bar{u}(\beta_2) = 0$ and $\bar{e}_k, k \in \{0, 1, 2\}$, such that:

\[ \psi'(\bar{e}_0) = 1 \]
\[ \psi'(\bar{e}_1) = 1 - \frac{\bar{q}_0}{\bar{q}_1} \phi'(e_{1,k}) \]
\[ \psi'(\bar{e}_2) = 1 - \frac{\bar{q}_0 + \bar{q}_1}{\bar{q}_2} \phi'(e_{2,k}) \quad (2.10) \]

Note the pooling mechanism in (2.10) satisfies monotonicity constraints $\bar{e}_k - \bar{e}_{k-1} \leq \Delta \beta$, for $k = 1, 2$, under the conventional monotone hazard rate assumption: \(^{18}\)

\[ \frac{\bar{q}_0}{\bar{q}_1} \leq \frac{\bar{q}_0 + \bar{q}_1}{\bar{q}_2} \]

The problem’s Lagrangian under ex-post participation constraints becomes as follows:

\[ \mathcal{L}^* = \sum_{i \in \{L, H\}} \nu_i \left\{ \sum_{k \in \{0,1,2\}} q_{i,k} \left[ \beta_k - e_{i,k} + \psi(e_{i,k}) \right] + q_{i,0} \phi(e_{i,1}) + (q_{i,0} + q_{i,1}) \phi(e_{i,2}) \right\} \]
\[ - \nu_L \lambda_{LH} \left\{ q_{L,0} \left[ \phi(e_{L,1}) - \phi(e_{H,1}) \right] + (q_{L,0} + q_{L,1}) \left[ \phi(e_{L,2}) - \phi(e_{H,2}) \right] \right\} \]
\[ - \nu_H \lambda_{HL} \left\{ q_{H,0} \left[ \phi(e_{H,1}) - \phi(e_{L,1}) \right] + (q_{H,0} + q_{H,1}) \left[ \phi(e_{H,2}) - \phi(e_{L,2}) \right] \right\} \]
\[ + \nu_L u_L(\beta_2) [1 - \lambda_{LH} + \lambda_{HL} - \mu_L] + \nu_H u_H(\beta_2) [1 - \lambda_{HL} + \lambda_{LH} - \mu_H] \]

\(^{18}\) This is the discrete version of $d|G(\beta)/g(\beta)|/d\beta \geq 0$, required for the monotonicity of optimal mechanisms with more than two types (see Laffont and Tirole, 1993, section 1.4).
with associated first-order conditions, for \( i \in \{L, H\} \):

\[
\begin{align*}
q_{i,0} \left[ 1 - \psi'(e_{i,0}) \right] &= 0 \\
q_{i,1} \left[ 1 - \psi'(e_{i,1}) \right] + q_{i,0} \phi'(e_{i,1}) - \lambda_{ij} q_{i,1} \phi'(e_{i,1}) + \lambda_{ij} q_{j,1} \phi'(e_{i,1}) &= 0 \\
q_{i,2} \left[ 1 - \psi'(e_{i,2}) \right] + (q_{i,0} + q_{i,1}) \phi'(e_{i,2}) - \lambda_{ij} (q_{i,0} + q_{i,1}) \phi'(e_{i,2}) + \lambda_{ij} (q_{j,0} + q_{j,1}) \phi'(e_{i,2}) &= 0 \\
u_i(\beta_2) \left[ 1 - \lambda_{ij} + \lambda_{ji} - \mu_i \right] &\geq 0
\end{align*}
\] (2.11) (2.12) (2.13) (2.14)

Notice from summing (2.14) for \( i \in \{L, H\} \) that there exist \( \mu_L, \mu_H \geq 0 \) that support active ex-post participation constraints if, and only if

\[
\lambda_{LH}, \lambda_{HL} \in (0, 1)
\]

Moreover, the pooling mechanism (2.10) always satisfies the FOC (2.11), as it recommends first-best effort to the least expensive project type. Evaluating (2.12) and (2.13) at the pooling allocations \( \bar{e}_1 \) and \( \bar{e}_2 \), and canceling out the \( \phi'(\cdot) \) terms, it obtains, for \( i \in \{L, H\} \):

\[
\frac{q_{i,0}}{q_{i,1}} \lambda_{ij} - \frac{q_{j,0}}{q_{j,1}} \lambda_{ji} = \frac{q_{i,0} - \bar{q}_0}{q_{i,1} - \bar{q}_1} \\
\frac{q_{i,0} + q_{i,1}}{q_{i,2}} \lambda_{ij} - \frac{q_{j,0} + q_{j,1}}{q_{j,2}} \lambda_{ji} = \frac{q_{i,0} + q_{i,1} - \bar{q}_0 + \bar{q}_1}{q_{i,2} - \bar{q}_2}
\]

We may restrict attention to the previous system for \( i = L \), as there are only two linearly independent equations. Denote the hazard rates \( h(\beta_k) = \sum_{l=0}^{k-1} \bar{q}_l / \bar{q}_k \) and \( h_{i,j}(\beta_k) = \sum_{l=0}^{k-1} q_{j,l} / q_{i,k} \), with \( h_i(\beta) \equiv h_{i,i}(\beta) \).

Stating the FOC system in matrix form, we get:

\[
\begin{bmatrix}
h_{L}(\beta_1) & -h_{L,H}(\beta_1) \\
h_{L}(\beta_2) & -h_{L,H}(\beta_2)
\end{bmatrix}
\begin{bmatrix}
\lambda_{LH} \\
\lambda_{HL}
\end{bmatrix}
=
\begin{bmatrix}
h_{L}(\beta_1) - \bar{h}(\beta_1) \\
h_{L}(\beta_2) - \bar{h}(\beta_2)
\end{bmatrix}
\]

Operating the linear system above using Cramer’s rule, one may check that

\[
\frac{h_{L}(\beta_2)}{h_{L}(\beta_1)} \geq \frac{\bar{h}(\beta_2)}{\bar{h}(\beta_1)} \geq \frac{h_{L,H}(\beta_2)}{h_{L,H}(\beta_1)}
\] (2.15)

is a sufficient condition for the solution \( \lambda^*_{LH}, \lambda^*_{HL} \in (0, 1) \). Therefore, if the information structure satisfies (2.15), the pooling mechanism (2.10) is an optimal solution to the regulator’s design problem under exit rights.\(^{20}\)

\(^{19}\) The term \( h_{i,j}(\cdot) \) corresponds to Krähmer and Strausz (2015)’s definition of “cross-hazard rate”.

\(^{20}\) We conjecture (2.15) is also necessary for the optimality of pooling.
2.B Proofs of Chapter 2

Proof of Lemma 2.1. Suppose (IC-2) holds. Using the identity \( C_i(s) = \beta_i - e_i(s) \) to manipulate the first equation, we get:

\[
\begin{align*}
u_L(s) & \geq t_H(s) - \psi(\beta_L - C_H(s)) \pm u_H(s) \\
& = u_H(s) + \psi(\beta_H - C_H(s)) - \psi(\beta_L - C_H(s) \pm \beta_H)
\end{align*}
\]

which results in (IC-2'). Moreover, adding up the two equations in (IC-2) yields

\[
\psi(\beta_L - C_H) + \psi(\beta_H - C_L) - \psi(\beta_H - C_H) - \psi(\beta_L - C_L) \geq 0
\]

\[
\iff \int_{C_L}^{C_H} \int_{\beta_L}^{\beta_H} \psi''(\beta - C) d\beta dC \geq 0
\]

which, together with \( \psi'' > 0 \) and \( \beta_H > \beta_L \), yields \( C_H \geq C_L \).

Conversely, suppose conditions (IC-2') and (MON) hold. The first equation in (IC-2) holds immediately. As for the second equation, suppose, to the contrary, that

\[
\begin{align*}
u_H(s) & < t_L(s) - \psi(\beta_H - C_L(s)) \pm u_L(s) \\
& = u_L(s) + \psi(\beta_L - C_L(s)) - \psi(\beta_H - C_L(s) \pm \beta_H)
\end{align*}
\]

\[
\iff \nu_H(s) - \nu_L(s) = \phi(\beta_H - C_H(s)) < \phi(\beta_L - C_L(s))
\]

Since \( \phi' > 0 \) and \( \beta_H > \beta_L \), it follows that \( C_L > C_H \), a contradiction. \( \square \)

Proof of Lemma 2.3. Let \( x > x' \), and thus \( \phi(x) > \phi(x') \). Take \( \theta \in [0, 1] \) and suppose that \( \phi(\theta x + (1 - \theta)x') < \phi(x') \). The contradiction follows immediately from the assumptions \( \psi', \psi'' > 0 \) and from the definition of \( \phi(.) \). The result obtains from the fact that any affine transformation of a quasi-concave function is quasi-concave. \( \square \)

Proof of Proposition 2.1. Substituting the pooling mechanism \( \{\bar{e}_L, \bar{e}_H\} \) defined in (2.1) in the FOC equations, the equation for \( e_L(s) \) becomes immediately satisfied (first-best). In turn, all terms \( \phi'(\bar{e}_H) \) cancel out in the equations for \( e_H(s) \), so they become:

\[
\text{for } s \in [\bar{s}, s^*] : 
\]

\[
f(s)(1 - q_s) \left[ \frac{q_s}{1 - q_s} - \frac{\bar{q}}{1 - \bar{q}} \right] = \int_{s^*}^{\bar{s}} \lambda(a, s) q_a da \quad (2.16a)
\]

\[
\text{for } s \in [s^*, \bar{s}] : 
\]

\[
f(s)(1 - q_s) \left[ \frac{\bar{q}}{1 - \bar{q}} - \frac{q_s}{1 - q_s} \right] = \int_{s^*}^{\bar{s}} \lambda(s, b) q_s db \quad (2.16b)
\]
Let the left-hand side of equations (2.16a) and (2.16b) be defined as $r(s)$ and $\tau(s)$, respectively. Reorganizing the expressions, it obtains

\[
r(s) = \frac{f(s)}{1 - \bar{q}} (q_s - \bar{q}) > 0, \quad \forall s < s^*
\]

\[
\tau(s) = \frac{f(s)}{1 - \bar{q}} (\bar{q} - q_s) > 0, \quad \forall s > s^*
\]

Notice the terms $\int_s^{s^*} \tau(a) da$ and $\int_{s^*}^{s} r(b) db$ are equal, and denote them by $r^*$. Defining the Lagrange multipliers as

\[
\lambda(a, b) = \frac{\tau(b)r(a)}{q_a r^*} \geq 0
\]

we have first-order conditions (2.16) satisfied. \qed

**Proof of Proposition 2.2.** Reorganize the Lagrangian of problem $(P^*)$ to obtain:

\[
\mathcal{L}^* = \mathcal{L} + \int_{s^*}^{s} u_H(s) \left[ f(s) - \int_{s^*}^{s} \lambda(s, b) db - \mu(s) \right] ds
\]

\[
+ \int_{s}^{s^*} u_H(s) \left[ f(s) - \int_{s}^{s^*} \lambda(a, s) da - \mu(s) \right] ds
\]

Note that $\mathcal{L}^*$ coincides with $\mathcal{L}$ if we define:

\[
\mu^*(s) = \begin{cases} 
  f(s) - \int_{s^*}^{s} \lambda^*(s, b) db & \text{if } s \in [\underline{s}, s^*] \\
  f(s) + \int_{s}^{s^*} \lambda^*(a, s) da & \text{if } s \in [s^*, \overline{s}]
\end{cases}
\]

To see that the proposed multiplier $\mu^*$ is positive, note that, for $s < s^*$:

\[
\mu^*(s) = f(s) - \int_{s^*}^{s} \frac{\tau(b)r(a)}{q_a r^*} db = \frac{f(s)}{1 - \bar{q}} \left[ 1 - \bar{q} + 1 - \frac{q_s}{\bar{q}} \right] \geq 0
\]

Moreover, for $s > s^*$:

\[
\mu^*(s) = f(s) + \int_{s}^{s^*} \lambda^*(a, s) da \geq 0
\]

\qed
2.C Matlab codes

Main code implementing the optimization problem stated in $(P)$. The algorithm is based on Matlab’s \texttt{fmincon}; it searches for a global minimum of \texttt{obj_function} subject to \texttt{ineq_constr}, given an initial guess.

```matlab
global alpha beta_L beta_H

%% Parameters
n = 2; % number of signals
S = 100;
alpha = 0.025;
beta_H = 100;
beta_L = 80;

efb = 1/alpha; % first−best effort
delta = beta_H − beta_L;
psi = @(x) alpha*x.^2/2;
phi = @(x) psi(x) − psi(x−delta);

ubar = −10:0.1:0; % ex−post outside options
firstbest = [ones(n,2)*efb zeros(n,1)];

%% Information structure
q = [0.75 0.25];
f = [0.50 0.50];

%% Iteration begins
for iter = 1:length(ubar)

% Objective function
obj = @(optvar) obj_function(optvar,f,q);

% Set of inequality constraints
restr = @(optvar) ineq_constr(optvar,q);

% Initial guess
init = cat(1,firstbest(:,1),firstbest(:,2),firstbest(:,3));
```

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% Non-negativity constraints

expostIR = ones(n,1)*ubar(iter);
lowerbound = cat(1,zeros(2*n,1),expostIR);

% Optimization options

options = ...
optimoptions('fmincon','Algorithm','sqp','Display','off');

% Call solver

nlcon = @(optvar) deal(restr(optvar),[]);
[optvar,fval] = ...
fmincon(obj,init,[],[],[],lowerbound,[],nlcon,options);

% Optimal mechanism

e_L(:,iter) = optvar(1:n);
e_H(:,iter) = optvar(n+1:2*n);
u_H(:,iter) = optvar(2*n+1:end);
u_L(:,iter) = u_H(:,iter) + phi(e_H(:,iter));

% Expected profits

expect(iter) = fval;
for s = 1:n
    rent(s,iter) = q(s)*u_L(s,iter)+(1-q(s))*u_H(s,iter);
end
Erent(iter) = f*rent(:,iter);

fprintf('Iteration_%d_of_%d
',iter,length(ubar))
end

Auxiliary function evaluating the objective function for input levels of optimization variables \{e_L(\cdot),e_H(\cdot),u_H(\cdot)\} and information structure \{f,q(\cdot)\}.

function value = obj_function(optvar,f,q)

global alpha beta_L beta_H

n = length(q);
delta = beta_H - beta_L;
psi = @(x) alpha*x.^2/2;
phi = @(x) psi(x) - psi(x-delta);

%% Retrieve optimization variables

e_L = optvar(1:n);
e_H = optvar(n+1:2*n);
u = optvar(2*n+1:end);

%% Value of objective

T = zeros(n,1);
for s = 1:n
    T(s) = q(s)*(beta_L-e_L(s)+psi(e_L(s))+phi(e_H(s))...
    + (1-q(s))*(beta_H-e_H(s)+psi(e_H(s))) +u(s);
end

value = f*T; % minimization problem objective

end

Auxiliary function evaluating inequality constraints (IC-1,s), (MON) and (IR-1) for input levels of optimization variables \{e_L(.),e_H(.),u_H(.)\} and information structure \{f,q(.)\}.

function value = ineq_constr(optvar, q)

global alpha beta_L beta_H

n = length(q);
delta = beta_H - beta_L;
psi = @(x) alpha*x.^2/2;
phi = @(x) psi(x) - psi(x-delta);

%% Retrieve optimization variables

e_L = optvar(1:n);
e_H = optvar(n+1:2*n);
u = optvar(2*n+1:end);

%% Value of LHS of each IC_ij constraint (including i=j)

IC = zeros(n);
for a = 1:n

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for \( b = 1 : n \)
\[
IC(a,b) = q(a) \ast (\phi(e_H(a)) - \phi(e_H(b))) + u(a) - u(b);
\]
end

IC = -IC; \% nlcon format: \( g(x) \leq 0 \)

\% Stack all constraint values into a single array

value = IC(:,1);
for \( i = 2 : n \)
\[
value = \text{cat}(1, value, IC(:, i));
\]
end \% so far there have been \( n^2 \) IC-1 constraints

\% Monotonicity constraints

mon = zeros(n,1);
for \( s = 1 : n \)
\[
mon(s) = e_H(s) - e_L(s) - \delta; \% format: mon(s) \leq 0
\]
end
value = cat(1, value, mon(:));

\% Ex-ante IR constraints

IR = zeros(n,1);
for \( s = 1 : n \)
\[
IR(s) = q(s) \ast \phi(e_H(s)) + u(s);
\]
end
IR = -IR; \% nlcon format: \( g(x) \leq 0 \)
value = cat(1, value, IR(:));

end
Chapter 3

Robust project selection under mean and variance conditions

This chapter studies robust procurement mechanisms under cost observability. It presents a model set up in Laffont and Tirole’s (1986) canonical environment, in which the procuring authority optimizes the rent-efficiency tradeoff with a menu of linear reimbursement schemes. However, the principal is assumed to be non-Bayesian, in that she cannot form a prior belief over the shape of asymmetric information, but only knows the first two moments of the type-distribution. We call the solution to the associated min-max problem a “robust procurement contract”, and characterize it analytically for a special case formulation. The principal’s min-max value function is proposed as a robust project selection criterion, i.e. a template for choosing among production technologies with different mean-variance profiles, so to avoid potentially large losses from prior misspecification.
3.1 Introduction

Expanding infrastructure services often involves technological tradeoffs. For example, in the near future, electricity sector planners are expected to face a choice between alternative forms of implementing the smart grid technologies. In transportation, the alternatives might include dynamic toll pricing technologies, pavements equipped with electrical induction sections, and intelligent transportation systems. These examples have in common that there is little prior information on the economic performance of infrastructure services that adopt such technologies. Therefore, the associated planning decisions must be grounded on the rough estimates available to the authorities.

The design of concessions for infrastructure services in the presence of novel or untested technologies, like the examples mentioned above, cannot benefit from a long-running experience and registry of the regulated sector. However, the theory of optimal procurement under asymmetric information, from which this thesis have drawn so far, assumes a great degree of knowledge about the environment, and, more importantly, features a striking sensitivity to model primitives. In other words, the optimal tradeoff between rents and efficiency sought by the procuring authority relies heavily on accurate, Bayesian knowledge of firms’ preferences and the distribution of the project’s intrinsic cost. Small changes to the principal’s beliefs, for example, in terms of the highly nonlinear type-distribution’s hazard rates, could dramatically alter the structure of optimal remuneration menus. Such limitation has been referred to in the literature as Wilson’s Critique (Carroll, 2018).

Consequently, Bayesian contract design is prone to significant misspecification losses if prior information is unreliable. Consider the problem of project selection. If the criterion for ranking among alternatives concerns total procurement costs, and these, in turn, depend on the extent of information frictions, then the misspecification of the shape of asymmetric information could lead not only to a suboptimal incentive mechanism, but also to an incorrect preference ordering. The social value forgone from a potentially incorrect project selection motivates an approach to contract design that is robust to such ignorance about model primitives.

In an attempt to address these issues, the present chapter proposes a model of cost-based procurement with an extended scope for the principal’s ignorance about the environment, namely, concerning the distribution of the project’s intrinsic cost. It assumes the procuring authority cannot form a complete prior over the cost distribution in advance of awarding the contract, but she has access to estimates of the mean and variance of different technologies’ costs. The essay proposes a “robust” framework for selecting among technological alternatives, i.e. that takes into account the principal’s

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1 “Smart grid” is a generic term used to denote a set of upgrades to transmission and distribution networks, in terms of equipment, software, and operational model, aimed at enabling demand response mechanisms and distributed generation (Castro and Dutra, 2013).

2 The definition of intelligent transportation systems, according to the Brazilian land transportation regulator, includes traffic monitoring, real-time position tracking, weather sensors and other applications aimed at enhanced data collection and improved management of traffic operations (ANTT, 2010).
lack of information on cost distributions aside from the first two moments. Hence, the principal is able to minimize the misspecification losses from assuming an incorrect prior. The analysis also characterizes the optimal robust incentive contract governing the subsequent relationship between firm and regulator.

The model is set up as a static principal-agent relationship between the procuring authority and the concessionaire. It departs from the dynamic environment of previous chapters, to focus on the consequences of the principal’s ignorance to contract design. The regulator offers an incentive scheme to the firm, who must be incentivized to participate and to reveal the project’s type truthfully, through its choice of remuneration plan. The contract design criterion takes the form of a saddle-point problem, in which the principal optimizes the rent-efficiency tradeoff under a worst-case scenario for the distribution of project costs. We approach the associated min-max problem by defining and auxiliary zero-sum game between the regulator and an adversarial nature, who chooses the most detrimental distribution that respects moment conditions.\footnote{The analysis takes inspiration from Carrasco, Luz, Monteiro, and Moreira (2018), who first proposed the alluded Nash equilibrium approach to the robust design problem.}

The first main result is the characterization, for a special case formulation, of a robust remuneration mechanism that depends solely on information the procuring agency has access to, namely, the mean and variance of project costs and the firm’s cost reduction technology (disutility of effort). The robust mechanism guarantees that expected procurement costs are minimized when considering a worst-case distribution of project costs. The firm chooses a remuneration plan from a menu devised to screen its private information, and that provides increasingly powerful incentives, much like the Bayesian optimal contract of Laffont and Tirole (1986). The difference, in this chapter, is that the cost-reimbursement menu is based on a much more limited set of information about the environment, so that the expected surplus from the project is made immune to distributional assumptions.

Furthermore, the essay applies the robustness framework to construct the principal’s preference ordering over mean and variance of project costs, based on the min-max value function. The latter can be interpreted as a robust project selection criterion, i.e. a template that allows to rank different technologies using data on average costs and cost variability. Alternatively, the value function implies an indifference map that permits to assess the principal’s willingness to accept, under asymmetric information, a more expensive technology, on average, in exchange for a lower cost dispersion.

This chapter is organized as follows. After a brief overview of the relevant literature, Section 3.2 presents the general setup for the principal-agent relationship and formalizes the regulator’s min-max problem. Section 3.3 derives the main results for a special case formulation (quadratic agent’s preferences), whereas Section 3.4 applies that particular framework to obtain the principal’s robust project selection criterion. Section 3.5 concludes and suggests some venues for further research. Appendix 3.A presents a discussion on theoretical analogies of the present setting to a model of robust nonlinear pricing. Proofs to propositions in the main text are left to Appendix 3.B.
Related literature

This chapter builds on Laffont and Tirole’s (1986; 1993) canonical model of optimal procurement under cost observability. Their main result is an optimal menu of remuneration schemes that are linear in cost performance, designed to decentralize operational decisions to the firm and induce second-best efficiency. Such result has become a hallmark of procurement and regulation theory. However, the optimal contractual format derived in the Laffont-Tirole model is seldom seen in practice. Empirical studies on procurement and regulatory contracts found predominance of simple schemes, such as dichotomous menus of pure fixed-price and cost-reimbursement regimes (Wolak, 1994; Gagnepain and Ivaldi, 2002; Gagnepain, Ivaldi, and Martimort, 2013).

More generally, a strand of contract theory has tried to address the dissonance between the optimal mechanisms proposed in the literature and actual contracts observed in practice. In an attempt to address the so-called Wilson’s Critique, several works have sought to investigate optimal contracting when parties have a more limited set of information available, compared to Bayesian mechanism design. Hence, such literature has been dubbed “robust mechanism design” (Carroll, 2018). Some recent papers in this strand have emphasized the simplicity of robust mechanisms (Chung and Ely, 2007; Carroll, 2015; Garrett, 2014), whereas others, while also examining the properties of robust mechanisms, have not (Lopez-Cunat, 2000; Bergemann and Schlag, 2011; Idrogo, 2015). All these papers have in common an environment of Knightian uncertainty, in which the principal selects contracts according to a max-min criterion.

Concerning robust procurement, Rogerson (2003) proposed a version of the Laffont-Tirole procurement model restricted to a simple menu of contracts, namely a fixed-price/cost-reimbursement menu (FPCR). That optimal result, in addition being simpler, benefited from being less demanding on the principal’s available information. Later on, Garrett (2014) formalized the argument that the FPCR contract is actually robust optimal when the principal has uncertainty over the shape of firms’ preferences. One partially related work is Gottlieb and Moreira (2017), who derive a simple procurement mechanism in a Bayesian environment featuring interplay between adverse selection, moral hazard, and limited liability.

This chapter is closely related to the robust mechanism design model in Carrasco, Luz, Kos, Messner, Monteiro, and Moreira (2018), who study a monopoly pricing problem when the seller knows the first \( n \) moments of the distribution of consumer types, but ignores the full distribution. In turn, the present essay innovates in that it analyzes a design environment with curvature, which is a non-trivial extension to robust mechanisms with moment conditions. In that respect, this chapter draws from the methodology proposed in Carrasco, Luz, Monteiro, and Moreira (2018), who study a likewise robust design problem with one moment condition and curved payoffs. That paper first suggested to approach the max-min solution by constructing a Nash equilibrium of a zero-sum game between the principal and an adversarial nature.

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4 McAffee and McMillan (1987) have independently arrived at equivalent results.
3.2 The model

The procuring agency (regulator) is required to delegate the execution of a project to a concessionaire (firm). The project has a social benefit level assumed to be high enough to ensure the project’s viability in all cost scenarios. Post completion, the regulator observes the project’s realized cost:

\[ C = \beta - e \]

However, the regulator cannot distinguish between the intrinsic cost level of the project \( \beta \in B \) and the effort exerted by the firm to reduce cost to level \( C \), denoted \( e \geq 0 \).

The firm observes the project’s intrinsic cost \( \beta \in B \) and has access to a cost reduction technology, represented by a non-monetary cost to exert effort \( \psi(e) \). Assume this effort disutility function exhibits decreasing returns (curvature), i.e., \( \psi', \psi'' > 0 \).

The regulator designs a remuneration mechanism \( T(C) \), specifying a nonlinear payment function conditional on the firm’s realized cost performance. To optimally design this mechanism, the regulator would need information on the firm’s technology \( \psi(.) \) and on the distribution of the project’s intrinsic cost, denoted \( F(\beta) \). However, the regulator is unable to form a full prior over \( F(.) \), and has to make do with information restricted to the mean \( \mu \) and variance \( \sigma^2 \) of the intrinsic cost distribution.

The regulator’s objective is to minimize the expected procurement costs \( \mathbb{E}[T(C)] \), subject to the firm’s alignment of incentives and voluntary participation in the mechanism. The firm’s participation obtains from securing it profit level:

\[ U = T - C - \psi(\beta - C) \]

superior to its outside option, normalized to zero\(^6\). Both regulator and firm are assumed to be risk neutral, and the contract is enforced with full commitment.

First best. Under complete information, the regulator would offer a fixed-price contract to the firm to induce participation and fully extract its information rent. The first-best effort level follows from the minimization of \( \beta - e + \psi(e) \), yielding \( e^{fb} = \psi^{-1}(1) \). Denote \( k = e^{fb} - \psi(e^{fb}) \) the maximum level of cost reduction achievable by an incentive contract. The first-best contract sets the project price equal to \( \beta - k \).

\(^5\) The model also requires the technical assumption \( \psi''' \geq 0 \), which is important to rule out random mechanisms (Laffont and Tirole, 1993).

\(^6\) It is useful to denote \( t = T - C \) as the regulator’s net transfer to the firm in addition to reimbursing the project’s realized cost.
Optimal Bayesian procurement under mean and variance

To motivate the robustness framework, we illustrate, with examples, how optimal contract design responds to different Bayesian assumptions over the distribution of asymmetric information in the procurement setting.

Suppose the regulator knows that the project’s intrinsic cost $\beta$ is distributed with mean $\mu = 80$ and standard deviation $\sigma = 10$. She could make a parametric assumption on $F(\beta)$ and design the optimal contract according to the Bayesian criterion described in Laffont and Tirole (1993, chapter 1). Moreover, Bayesian procurement design also needs an assumption on the firm’s cost-reduction technology $\psi(.)$.

Figure 3.1a depicts four examples of distributional assumptions that could underpin the regulator’s mechanism; all of them are calibrated to produce the same mean and variance. Figure 3.1b illustrates the sensitivity of the optimal cost-reducing effort implemented by the mechanism to different assumptions about $F(\beta)$.

![Figure 3.1a](image1.png)  
(a) Density assumptions  
![Figure 3.1b](image2.png)  
(b) Effort recommendation ($e^{fb} = 20.0$)

Figure 3.1: Sensitivity to distributional assumptions with $\mu = 80$ and $\sigma = 10.0$

The Bayesian optimal menus of linear remuneration policies computed for each distributional assumption are presented in Table 3.1. The calibration of remuneration plans varies significantly with the assumed prior, implementing incentive contracts with starkly different slopes in each case. For instance, if the principal assumes $F \sim$ Log-normal, but the true distribution is Pareto, then she offers contracts that pay too much, both in terms of fixed portions and cost-reimbursements, compared to the actually optimal mechanism (fifth column).

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7 In the present example, assume $\psi(e) = 0.025e^2$.
8 Optimal menus are approximated for a selection of project types, corresponding to the quartiles of the type distribution, in addition to the 1st and 99th percentiles.
Note also that, in the present example, there is some variation in expected procurement costs arising from each distributional assumption in Table 3.1. Such variation might result in misspecification losses, which jeopardize a project selection decisions, as discussed in more detail in Section 3.4. For instance, if the true distribution is Uniform but the regulator works with a Pareto assumption, she incurs an expected misspecification loss of 0.98%.

### The robust design problem

In the presence of asymmetric information, the robust procurement mechanism must respect incentive compatibility in addition to voluntary participation. By the Revelation Principle, one can restrict attention to an equivalent direct mechanism of the form \((T(\beta), C(\beta))\). In the direct mechanism, the firm is asked to announce its observed cost parameter \(\beta\), and, subsequently, the mechanism assigns the firm a total payment \(T(\beta)\) and recommends that it realizes a level of cost \(C(\beta)\).

**Incentive compatibility.** Denote \(U(\hat{\beta} | \beta) = T(\hat{\beta}) - C(\hat{\beta}) - \phi(\beta - C(\hat{\beta}))\) as the profit the firm obtains from announcing a project of type \(\hat{\beta}\) in the direct mechanism. The mechanism is incentive compatible if, for each project type \(\beta\):

\[
U(\beta) \equiv U(\beta | \beta) \geq U(\hat{\beta} | \beta), \quad \forall \hat{\beta}, \beta \in \mathcal{B}
\]  

(IC)

It is convenient to express incentive compatibility in an equivalent form, namely, an envelope condition and a monotonicity condition (Laffont and Tirole, 1993):

\[
\text{(IC)} \iff \begin{cases} 
    U(\beta) = U(\bar{\beta}) + \int_{\beta}^{\bar{\beta}} \phi'(z - C(z)) dz \\
    C(.) \text{ is non-decreasing in } \beta
\end{cases}
\]

where \(\bar{\beta}\) is the most expensive project type considered in the mechanism.
**Voluntary participation.** The direct mechanism induces the firm to accept the contract if, for each project type $\beta$, the profit obtained by the firm is at least its outside option:

$$U(\beta) \geq 0, \quad \forall \beta \in B$$ (IR)

Noting that (IC) implies that the firm’s profit $U(\beta)$ is decreasing on the project’s intrinsic cost $\beta$, the constraint (IR) can be equivalently stated as $U(\beta) \geq 0$.

It is also convenient to express the regulator’s objective in terms of the firm’s truth-telling profit $U(\beta)$, or informational rent:

$$T^C(\beta) = C(\beta) + \psi(\beta - C(\beta)) + U(\beta)$$

$$= C(\beta) + \psi(\beta - C(\beta)) + \int_\beta^\infty \psi'(z - C(z))dz$$

where the last form incorporates the envelope condition from the constraint (IC) and the optimal setting of $U(\beta) = 0$ in the minimization problem.

The optimal mechanism $C(.)$ is set to minimize the expected payment $\mathbb{E}[T^C(\beta)]$, but the regulator does not have a full prior over $\beta$ realizations on which to base the expectation. She has, though, exact knowledge of the mean project’s intrinsic cost $\mathbb{E}[\beta] = \mu$, and the upper bound\(^9\) for the variance $\mathbb{E}[(\beta - \mu)^2] \leq \sigma^2$.

The robustness of the optimal mechanism to the regulator’s partial ignorance about the distribution of private information is achieved when the regulator considers the worst possible distribution in the family:\(^{10}\)

$$\mathcal{F} \equiv \left\{ F \in \Delta([k, \infty]) : \int \beta dF = \mu, \int (\beta - \mu)^2 dF \leq \sigma^2 \right\}$$

Hence, the regulator’s problem is to design a robust direct mechanism $C(\beta)$ that minimizes the expected procurement cost, considering the worst-case scenario for the distribution of the private information parameter:\(^{11}\)

$$\inf_{C(\cdot) \in C} \sup_{F(\cdot) \in \mathcal{F}} \int_B \left[ C(\beta) + \psi(\beta - C(\beta)) + \int_\beta^\infty \psi'(z - C(z))dz \right] dF(\beta)$$ (P)

where $C = \{ C : B \to \mathbb{R}_+ : C \text{ is non-decreasing} \}$

---

\(^9\) The inequality condition for the 2nd moment is to ensure the compactness of nature’s choice set in the auxiliary game defined below; see Carrasco, Luz, Kos, Messner, Monteiro, and Moreira (2018).

\(^{10}\) The restriction $B \subseteq [k, \infty]$ is to ensure positive first-best prices for the project.

\(^{11}\) Gilboa and Schmeidler (1989) provide a motivation for a max-min expected payoff structure for problems without an exactly defined prior.
Model analysis

The approach to solving the min-max problem \((P)\) is based on the work of Carrasco, Luz, Monteiro, and Moreira (2018). The authors study an analogous problem: finding a robust nonlinear pricing mechanism with just one moment condition and bounded support, in an environment with payoff curvature. They define the solution to the associated max-min problem as the saddle point of a zero-sum game between the principal and an adversarial nature.

Likewise, we define an auxiliary game in strategic form with the following structure:

- The regulator \((R)\) chooses a feasible mechanism \(C \in \mathcal{C}\);
- Nature \((N)\) chooses a feasible distribution \(F \in \mathcal{F}\);
- Players obtain payoffs respectively equal to
  \[ V_R(C, F) = \int_B T^C(\beta) dF(\beta) \]
  and
  \[ V_N(C, F) = -V_R(C, F). \]

If \((C^*, F^*)\) is a Nash equilibrium of the auxiliary game, then \(C^*\) solves problem \((P)\) and \(V_R(C^*, F^*)\) is the regulator’s robust expected procurement cost. Moreover, it is possible to compute the worst-case distribution \(F^*\) on which the regulator bases his payoff guarantee, given \(\mu\) and \(\sigma^2\).

Next, we outline a strategy to construct such Nash equilibrium:

1. Set up a hypothesis (guess) as to the robust mechanism, namely, that it induces the principal’s objective \(T^C(\beta)\) to coincide with a quadratic polynomial over an endogenous support \([\beta, \bar{\beta}]\);
2. Find a mechanism \(\tilde{C} \in \mathcal{C}\) that induces an ex-post payment function \(T^\tilde{C}(\beta)\) consistent with the hypothesis in step 1, by means of an ordinary differential equation (ODE);
3. Show that \(T^\tilde{C}(\beta)\) induces nature to choose any \(F \in \mathcal{F}\) such that \(\text{supp}(F) \subseteq [\beta, \bar{\beta}]\), since the quadratic hypothesis makes the game’s expected value depend only on \(\mu\) and \(\sigma^2\);
4. Construct an appropriate worst-case distribution \(\tilde{F}^\tilde{C}\) that supports \(\tilde{C} \in \mathcal{C}\) as the regulator’s best reply;
5. Show that the pair of best replies \((C^*, F^*) = (\tilde{C}, \tilde{F}^\tilde{C})\) constitute a Nash equilibrium of the auxiliary game, by establishing feasibility of the candidate worst-case distribution, i.e. \(\tilde{F}^\tilde{C} \in \mathcal{F}\) if the nonlinear system of moment conditions is solved by the model’s free parameters.
**Hypothesis.** Let $T^C(\beta)$ denote the regulator’s payment to a firm that reports an intrinsic cost of $\beta$. We conjecture that the regulator chooses a robust cost recommendation $C(\beta)$ that induces the function $T^C(\beta)$ to coincide, over some interval $[\beta, \overline{\beta}]$, with the monotonic hull of a second-degree convex polynomial, denoted

$$\pi(\beta) \equiv \xi_0 + \xi_1\beta + \xi_2\beta^2, \quad \text{with } \xi_2 > 0$$

$$= T^C(\beta), \quad \text{for } \underline{\beta} \leq \beta \leq \overline{\beta} \quad (3.1)$$

The range $[\beta, \overline{\beta}] \ni \mu$ is endogenous to the model, and is assumed to lie in the intersection of the following sets:

$$\{\beta : \pi'(\beta) \geq 0\} \cap \{\beta : \pi(\beta) \leq \beta\} \quad (3.2)$$

For $\beta \leq \underline{\beta}$, suppose the robust mechanism assigns a fixed payment $T^C(\beta) = \min_{\beta} \pi(\beta)$. Conversely, for $\beta \geq \overline{\beta}$, the robust mechanism assigns a cost-plus contract $T^C(\beta) = \beta$. The rationale for both assignments is explained below. Figure 3.2 illustrates the hypothesis for the candidate mechanism.

![Figure 3.2: Hypothesis for the regulator’s ex-post payment to the firm](image)

Combining parts (3.1) and (3.2) of the hypothesis, notice that

1. $\underline{\beta} = \arg\min \pi(\beta)$, so it follows $\pi'(\underline{\beta}) = 0$. As a consequence,

$$\xi_1 = -2\xi_2\underline{\beta}$$
ii. \[ \overline{\beta} \in \{ \beta : \pi(\beta) - \beta = 0 \} \]; combined with \[ \underline{\beta} \leq \overline{\beta} \] by definition, it follows that
\[ \xi_0 = \overline{\beta} - \xi_2 \left( \overline{\beta}^2 - 2\underline{\beta}\overline{\beta} \right) \]

Let \( \xi \equiv \xi_2 \). Therefore, the hypothesis for the ex-post payment function may equivalently be expressed in a simpler form:

\[
T^C(\beta) = \begin{cases} 
\pi(\beta), & \beta < \underline{\beta} \\
\xi \left( \beta^2 - \overline{\beta}^2 - 2\overline{\beta}(\beta - \overline{\beta}) \right) + \overline{\beta}, & \beta \leq \beta \leq \overline{\beta} \\
\beta, & \overline{\beta} < \beta 
\end{cases}
\tag{3.3}
\]

such that \( 0 < \xi < \frac{\overline{\beta}}{(\overline{\beta} - \underline{\beta})^2} \)

The quadratic form of ex-post payoffs makes nature indifferent to any feasible distribution with support contained in \([\beta, \overline{\beta}]\), and thus, neutralizes the sensitivity of the regulator’s objective to uncertainty about \( F \). Consequently, the game’s expected value depends solely on the moments \( \mu \) and \( \sigma^2 \), and on the regulator’s beliefs about the firm’s technology \( \psi(.) \) (e.g. the degree of curvature).

**Robust mechanism.** In order to characterize the candidate robust mechanism, recall that
\[ T^C(\beta) = C(\beta) + \psi(\beta - C(\beta)) + \int_{\beta}^{\overline{\beta}} \psi(z - C(z))dz \]

Taking the derivative with respect to \( \beta \), hypothesis (3.3) implies:

\[ C'(\beta) \left[ 1 - \psi'(\beta - C(\beta)) \right] = 2\xi \left( \beta - \underline{\beta} \right) \]

Therefore, the robust cost recommendation is defined as a solution to the following first-order ordinary differential equation:

\[
C'(\beta) = \frac{2\xi \left( \beta - \underline{\beta} \right)}{1 - \psi'(\beta - C(\beta))} \\
C(\underline{\beta}) = C^{fb}(\underline{\beta}) \equiv \underline{\beta} - e^{fb}
\tag{3.4}
\]

**Lemma 3.1.** For each pair \( (\xi, \beta) \), there exists a solution to ODE (3.4) denoted \( \tilde{C} = C^{\xi,\beta} : [\underline{\beta}, \overline{\beta}] \to \mathbb{R}^+ \), for some \( \tilde{\beta}^\xi,\beta \geq \beta \), satisfying the following properties:

1. Cost-plus for the most expensive project: \( \tilde{C}(\overline{\beta}) = \overline{\beta} \)
2. Strict monotonicity: \( \tilde{C}'(\beta) > 0 \) for all \( \beta \in (\underline{\beta}, \overline{\beta}) \)

\(^{12}\)The technical condition \( \xi < \overline{\beta}/(\overline{\beta} - \underline{\beta})^2 \) is required to ensure the polynomial has no real roots, so ex-post payments are always positive.
3. **Parameter dependence:** $C_{\xi,\beta}(\cdot)$ is continuous and decreasing in $\xi$, for all $\beta$

4. **Limiting behavior (i):** for each $\beta$, $\lim_{\xi \to 0} C_{\xi,\beta}(\beta) = C_{fb}(\beta)$

5. **Limiting behavior (ii):** for each $\beta$, $\lim_{\xi \to \infty} \beta \cdot \xi = \beta$

The boundary condition pinning down ODE (3.4) is based on the part of hypothesis (3.3) that assigns a fixed-price contract to project types $\beta \leq \bar{\beta}$. Such contract induces the first-best effort level $e_{fb}$ to a firm facing type $\beta$, the equilibrium range’s least expensive project. The rationale is that, being the most profitable project type from the regulator’s viewpoint, there are no types “more efficient” than $\bar{\beta}$ to leave rents for, so there is no reason to distort the effort recommendation at $\bar{\beta}$.

Moreover, the parameter $\bar{\beta}$ is endogenous to the ODE solution as the fixed-point $\bar{C}(\beta) = \bar{\beta}$, since hypothesis (3.3) assigns a cost-plus contract to project types $\beta \geq \bar{\beta}$. The rationale is that, as $\pi(\beta) > \beta$ for $\beta > \bar{\beta}$, the regulator would not recommend the firm to exceed the project’s intrinsic cost, i.e. $C(\beta) > \beta$ can never be optimal.

Notably, the candidate robust mechanism pools project types $\beta < \bar{\beta}$ and $\beta > \bar{\beta}$ into a fixed-price and a cost-plus contract, respectively. In any case, these types lie outside the equilibrium support, and hence, are not assigned probability mass in the worst-case distribution.

The following Lemma rules out singularity at the boundary condition of ODE (3.4).

**Lemma 3.2.** The first derivative of the robust cost recommendation is bounded at the lower limit of the equilibrium support, i.e.

$$ R \equiv \lim_{\beta \to \bar{\beta}} \bar{C}'(\beta) < \infty $$

Moreover, the limiting value for the first derivative at $\bar{\beta}$ is

$$ R = \frac{1}{2} + \left( \frac{1}{4} + \frac{2\xi}{\psi''(e_{fb})} \right)^{\frac{1}{2}} > 1 $$

The robust mechanism $\bar{C}$, which induces an ex-post quadratic payment function $T_{\bar{C}}$, makes nature’s problem relatively simple, as shown in the following proposition.

**Proposition 3.1 (nature’s best reply).** Suppose the regulator chooses a robust mechanism $\bar{C} \in \mathcal{C}$ as defined in Lemma 3.1, such that $\bar{\beta} < \mu < \bar{\beta}$. Then nature optimally chooses any $\bar{F} \in \mathcal{F}$ such that $\text{supp} (\bar{F}) \subseteq [\beta, \bar{\beta}]$.

**Corollary.** The robust expected procurement cost is

$$ \mathbb{E} \left[ T_{\bar{C}}(\beta) \right] = \xi \left( \mu_2 - \bar{\beta}^2 + 2\bar{\beta}(\mu - \bar{\beta}) \right) + \bar{\beta} > 0 \quad (3.5) $$

where $\mu_2 = \sigma^2 + \mu^2$.
**Worst-case distribution.** The distribution that supports $\hat{C}$, defined in Lemma 3.1, as the regulator’s best reply is constructed to induce a hazard rate coinciding with the following function:

$$\gamma(\beta) = \frac{\psi''(\beta - \hat{C}(\beta))}{1 - \psi'(\beta - \hat{C}(\beta))}$$

Solving the simple differential equation given by $F'/F = \gamma$ with boundary condition $F(\beta) = 1$, it obtains:

$$\hat{F}(\beta) = \begin{cases} 
0, & \text{if } \beta < \beta \\
\exp\left(-\int_\beta^\beta \gamma(z)dz\right), & \text{if } \beta \in [\beta, \beta] \\
1, & \text{if } \beta > \beta
\end{cases}$$

(3.6)

Note there might be positive mass assigned to $\beta$, for example, if $\hat{F}(\beta^+) - \hat{F}(\beta) > 0$. The following proposition establishes the optimality of $\hat{C}$ given the present worst-case distribution.

**Proposition 3.2 (regulator’s best reply).** Suppose nature chooses a distribution $\hat{F}$ as defined in (3.6), not necessarily in $F$. Then the regulator optimally chooses the mechanism $\hat{C}$ as defined in Lemma 3.1.

**Nash equilibrium.** For the profile $(\hat{C}, \hat{F})$ to constitute a Nash equilibrium of the auxiliary game, there remains to show that $\hat{F} \in F$, that is, there exists a pair of parameters $\xi > 0$ and $\beta \in [\kappa, \mu]$ such that the moment conditions are satisfied:

$$\int_B \beta dF^\xi_{\beta}(\beta) = \mu \quad (3.7a)$$
$$\int_B (\beta - \mu)^2 dF^\xi_{\beta}(\beta) \leq \sigma^2 \quad (3.7b)$$

The following conjecture points to the primary theoretical milestone of this chapter. However, it currently lacks a complete proof.

**Conjecture.** For all values of $(\mu, \sigma^2)$, there exists a pair $(\xi^*, \beta^*)$ such that moment conditions (3.7a) and (3.7b) are satisfied.

Appendix 3.A presents an analogous model of robust nonlinear pricing with moment conditions. The discussion therein concerns the emergence of a “maximum admissible variance” in the model with two moments and curvature, i.e. there exists an upper bound on the second moment $\sigma^*$, beyond which the system given by (3.7) fails to be satisfied with equality. However, that need not mean there exists no robust mechanism; it could simply mean nature cannot play distributions whose dispersion exceeds $\sigma^*$.

---

13 The exception is the case in which $\int \gamma(z)dz$ takes a logarithmic form.
Moreover, the procurement model is further complicated by a specific feature: more profitable types, from the viewpoint of the principal (decreasing $\beta$ values), have a natural lower bound at zero. Consequently, the principal cannot expand the equilibrium support indefinitely (downwards) in order to accommodate a higher $\sigma^2$, as done in the nonlinear pricing model. See, for instance, the proof of Proposition 3.9.

### 3.3 Quadratic cost reduction technology

This section characterizes a solution to the robust procurement model by assuming a functional specification for the firm’s cost reduction technology $\psi(.)$.

**Assumption 3.1.** The firm’s cost reduction technology has a quadratic form:

$$\psi(e) = \alpha e^2 / 2, \quad \alpha > 0$$

Note that $\psi''(e) = \alpha$ is a constant measure of curvature in the firm’s payoff function. Moreover, the level of first-best effort and the maximum cost savings from incentives are, respectively, $e_{fb} = 1 / \alpha$ and $k = 1 / 2\alpha$.

**Robust mechanism.** The characterization of the robust cost recommendation becomes much simpler with Assumption 3.1. Under constant curvature, the ODE (3.4) admits a linear solution:

$$\bar{C}(\beta) = R \cdot (\beta - \beta) + \frac{\beta - e_{fb}}{R - 1}, \quad \beta \in [\bar{\beta}, \bar{\beta}]$$

$$R = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2e_{fb}}{\alpha}} > 1$$

(3.8)

The upper limit of the equilibrium range $\bar{\beta}$ is obtained from the intersection $\bar{C}(\bar{\beta}) = \bar{\beta}$:

$$\bar{\beta} = \beta + \frac{e_{fb}}{R - 1}$$

(3.9)

Additionally, in order to match hypothesis (3.3) the robust mechanism assigns:

- full-powered incentives $\tilde{C}(\beta) = \beta - e_{fb}$ for project types $\beta < \bar{\beta}$; and
- a cost-plus scheme $\tilde{C}(\beta) = \beta$ for project types $\beta > \bar{\beta}$.

The robust mechanism can be alternatively expressed in terms of the regulator’s effort recommendation to the firm, given by:

$$\tilde{e}(\beta) = \begin{cases} e_{fb}, & \beta < \bar{\beta} \\ e_{fb} - (R - 1) \left( \beta - \bar{\beta} \right), & \bar{\beta} \in [\beta, \bar{\beta}] \\ 0, & \bar{\beta} < \beta \end{cases}$$

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The term \((R - 1)\) corresponds to the slope of incentives in the robust procurement contract under quadratic effort disutility. In equilibrium, it must hold that \(R^* > 1\); such condition is related to the emergence of a maximum admissible variance \(\sigma^*\), as will be seen from the exposition below. Figure 3.3 illustrates the shapes of the robust mechanism’s allocations.

\[\gamma(\beta) = \frac{1}{(R - 1)(\beta - \bar{\beta})}\]

Integrating from \(\beta\) to \(\bar{\beta}\), it obtains

\[\int_{\beta}^{\bar{\beta}} \gamma(z)dz = \frac{1}{(R - 1)} \ln \left( \frac{\beta - \bar{\beta}}{\bar{\beta} - \beta} \right)\]

Hence, the worst-case distribution has an explicit form

\[\tilde{F}(\beta) = \left( \frac{\beta - \beta}{\bar{\beta} - \beta} \right)^{\frac{1}{R - 1}}, \quad \beta \in [\beta, \bar{\beta}]\]

with zero probability mass assigned to the lower boundary of the support, i.e \(\tilde{F}(\beta^+) = 0\). Further, the worst-case density may also be computed explicitly from \(f = \gamma \cdot \tilde{F}\):

\[\tilde{f}(\beta) = \frac{1}{(R - 1)(\beta - \bar{\beta})} \left( \frac{\beta - \beta}{\bar{\beta} - \beta} \right)^{\frac{1}{R - 1}}, \quad \beta \in [\beta, \bar{\beta}]\]
Remark. The slope of the worst-case pdf $\tilde{f}(\beta)$ or, alternatively, the concavity or convexity of the cdf $\tilde{F}(\beta)$ is determined by the equilibrium value of $R$:

$$\frac{d}{d\beta} \tilde{f}(\beta) = A \cdot \left( \frac{1}{R - 1} - 1 \right) (\beta - \beta) \frac{1}{R - 1 - 2}$$

where $A > 0$ is a constant

Therefore, for $\beta \in [\beta, \beta]$, $\tilde{f}(\beta)$ is increasing if $R < 2$, and decreasing otherwise. Note there is a threshold case ($R = 2$), for which the worst-case distribution is uniform over $[\beta, \beta]$, as illustrated in Figure 3.4.

Figure 3.4: Worst-case distributions for different $R$ values
At first, it makes little intuitive sense that an antagonist nature would assign higher probability to more favorable $\beta$ realizations. Nonetheless, as $R > 2$ implies a higher slope for the effort recommendation, it signifies higher marginal rents. Hence, nature punishes the regulator by making high-rent, less expensive projects more likely.

**Moment conditions.** The quadratic technology case enables an explicit computation of the worst-case distribution’s moments:

\[
M_1(R, \beta, \tilde{\beta}) \equiv \int_{\beta}^{\tilde{\beta}} \beta \hat{f}(\beta)d\beta = \beta + \frac{\beta - \beta}{R} \tag{3.10a}
\]

\[
M_2(R, \beta, \tilde{\beta}) \equiv \int_{\beta}^{\tilde{\beta}} \beta^2 \hat{f}(\beta)d\beta = \beta^2 + 2\beta \left( \frac{\beta - \beta}{R} \right) + \frac{(\beta - \beta)^2}{2R - 1} \tag{3.10b}
\]

There remains to find parameters $(R, \beta, \tilde{\beta})$ such that (3.10a) and (3.10b) are equal to $\mu$ and $\sigma^2 + \mu^2$, respectively. Developing the moment conditions further, and using the expression for endogenous $\tilde{\beta}$ in (3.9), it obtains the following nonlinear system:

\[
R(\mu - \beta) = \tilde{\beta} - \beta \tag{3.11a}
\]

\[
(2R - 1)\sigma^2 = (\tilde{\beta} - \mu)^2 \tag{3.11b}
\]

\[
(R - 1)(\beta - \beta) = 1/\alpha \tag{3.11c}
\]

The solution to system (3.11) defines $(\tilde{C}, \tilde{F})$ as a Nash equilibrium of the auxiliary game, and therefore, an optimal solution to the min-max problem. Proposition 3.3 establishes the existence of such solution for a restricted subset of the moment space.

**Proposition 3.3.** Suppose $\sigma < 1/\alpha$. There exist a unique set of scalars $\beta^* < \mu < \beta^*$ and $R^* > 1$ that solve the nonlinear system (3.11).

Proposition 3.3 implies that, if $\sigma \geq 1/\alpha$, there is no robust solution that satisfies the second moment condition with equality. Such result is a consequence of the curvature environment, though it does not mean a robust contract does not exist (see the discussion in Appendix 3.A). In contrast, linear environments produce robust mechanisms that always meet moment conditions with equality (Carrasco, Luz, Kos, Messner, Monteiro, and Moreira, 2018).

**Proposition 3.4 (comparative statics).** The robust mechanism’s linear coefficient $R^*$ exhibits the following behavior with respect to model primitives:

\[
\frac{d}{d\sigma} R^* < 0 \quad \text{and} \quad \frac{d}{d\alpha} R^* < 0
\]

Proposition 3.4 suggests an intuitive economic interpretation. The regulator implements less powerful incentives when he faces a high dispersion $\sigma$, because, for each project type $\beta$, the recommended effort $e^*(\beta)$ is proportional to $R^*$. Put another way,
the severity of asymmetric information leads to higher levels of distortion. Moreover, the more punitive the technology’s curvature $\alpha$, the more expensive it becomes to exert effort, and the robust mechanism responds implementing less powerful incentives.

**Optimal schedules.** Suppose then $\sigma < 1/\alpha$. From the robust cost recommendation $C^*(\cdot)$ defined in (3.8), it is possible compute the optimal rent and net transfer functions:

$$U^*(\beta) = \frac{\xi^*}{R^*}(\overline{\beta} - \beta)^2$$

$$t^*(\beta) = \xi^*(\overline{\beta} - \beta)^2$$

where $\xi^* = \alpha R^* (R^* - 1)/2$

From the monotonicity of $C^*(\cdot)$, it is possible to construct an inverse mechanism (type-assignment function) given by $\beta^*(C) = C^{*-1}(C)$:

$$\beta^*(C) = \beta + \frac{C - C^{fb}(\beta)}{R^*}$$

Let $C = C^{fb}(\beta)$. Substituting in the expression for the robust net transfer, it obtains:

$$t^*(\beta^*(C)) = \frac{R^*-1}{R^*}(C - C)^2 - (C - C) + \frac{R^*}{R^*-1} (e^{fb})^2$$

for $C \leq C \leq \overline{C}$

The robust remuneration policy as a function of the firm’s realized cost is given by $T^*(C) = t^*(\beta^*(C)) + C$, illustrated in Figure 3.5.

![Figure 3.5: Robust remuneration policy with $\alpha = 0.05, \mu = 100$ and $\sigma = 10$](image)
Implementation. The robust contract \( T^*(C) \) coincides with the Bayesian optimal contract with a prior precisely equal to the worst-case distribution \( F^*(\beta) \). Therefore, it is straightforward to apply Laffont and Tirole’s (1993) implementation result, namely, the decentralization of the effort decision to the firm by means of a menu of linear reimbursement contracts:

\[
T = a + (1 - b)C
\]

A contract from the menu consists of a fixed payment \( a \) and a fraction \( b \) of the realized cost borne by the firm. Thus, a type-\( \beta \) firm chooses \((a(\beta), b(\beta))\) such that:

\[
a(\beta) = \alpha(R^* - 1)(\bar{\beta} - \beta) - \beta(\bar{\beta} - \beta)^2
\]

\[
b(\beta) = \alpha(R^* - 1)(\bar{\beta} - \beta)
\]

Proposition 3.5. The optimal robust contract \( T^*(C) \) is implemented with a menu of linear remuneration functions

\[
T(\beta, C) = \begin{cases} 
  a(\beta), & C < \underline{C} \\
  a(\beta) + (1 - b(\beta))C, & \underline{C} \leq C \leq \bar{\beta} \\
  C, & C > \bar{\beta}
\end{cases}
\]

with parameters \((a, b)\) given by (3.12a) and (3.12b).

The optimal robust menu offers fixed-price and cost-plus contracts (the two extremes in terms of incentive power) to project types below and above the equilibrium support, respectively. In the middle ground, it offers sliding scale contracts, as illustrated in Figure 3.6.

![Figure 3.6: Robust menu of linear contracts with \( \alpha = 0.05, \mu = 100 \) and \( \sigma = 10 \)]
3.4 Robust project selection

This section deploys the robustness framework (with quadratic technology) to the problem of project selection with moment conditions and uncertainty about the distribution of asymmetric information. The regulator remains ignorant when it comes to procure the project, so she anticipates that contract design will follow the solution to the min-max problem ($\mathcal{P}$).

Suppose the regulator faces a planning decision: she must choose one technology among $n$ alternatives to implement the project, under asymmetric information concerning the project’s cost and the firm’s cost-reducing effort. The alternatives have different mean-variance profiles for the distribution of asymmetric information, i.e. $\{(\mu_1, \sigma_1), ..., (\mu_n, \sigma_n)\}$.

For example, suppose the regulator has access to the following estimates about technological alternatives’ cost distributions:

<table>
<thead>
<tr>
<th>Technology</th>
<th>Average cost</th>
<th>Cost dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>82.0</td>
<td>5.0</td>
</tr>
<tr>
<td>B</td>
<td>80.0</td>
<td>10.0</td>
</tr>
<tr>
<td>C</td>
<td>78.0</td>
<td>22.5</td>
</tr>
<tr>
<td>D</td>
<td>76.0</td>
<td>37.5</td>
</tr>
</tbody>
</table>

Table 3.2: Mean-variance profiles of technological alternatives

The Bayesian selection problem requires that the principal assumes a specific distribution $F(\beta)$ in order to design the optimal remuneration mechanism. The expected procurement cost under the optimal contract is given by:

$$E[T^*(C)] = \int_B [C^*(\beta) + \psi(e^*(\beta)) + U^*(\beta)] dF(\beta)$$

Table 3.3 below presents the regulator’s optimal value (expected procurement cost) from implementing optimal Bayesian mechanisms, computed for technologies $A$–$D$ under different specifications for $F(\beta)$:

<table>
<thead>
<tr>
<th>Project</th>
<th>Uniform</th>
<th>Triangular</th>
<th>Log-normal</th>
<th>Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>65.20</td>
<td>69.89</td>
<td>69.75</td>
<td>69.82</td>
</tr>
<tr>
<td>B</td>
<td>61.70</td>
<td>71.96</td>
<td>71.56</td>
<td>69.66</td>
</tr>
<tr>
<td>C</td>
<td>61.12</td>
<td>74.72</td>
<td>73.46</td>
<td>68.87</td>
</tr>
<tr>
<td>D</td>
<td>61.38</td>
<td>73.96</td>
<td>70.14</td>
<td>65.16</td>
</tr>
</tbody>
</table>

Table 3.3: Expected procurement costs with $\alpha = 0.025$
Therefore, the assumptions on cost distribution \( F(\beta) \) that respect moment conditions \( \mu \) and \( \sigma \) yield the following preference orderings among technological alternatives:

- **Uniform**: \( C \succ D \succ B \succ A \)
- **Log-normal**: \( A \succ D \succ B \succ C \)
- **Triangular**: \( A \succ B \succ D \succ C \)
- **Pareto**: \( D \succ C \succ B \succ A \)

How should the regulator proceed? In this example, the misspecification penalty may reach 6.7% of the expected procurement cost, so there are reasonable stakes involved in the decision. This essay proposes that the regulator uses the robust value function as a template to select among technologies, as it minimizes the maximum potential loss from misspecifying \( F(\beta) \).

**Robust value function.** The min-max expected procurement cost is obtained from inserting the robust mechanism \( C^*(\beta) \) into problem \((P)\)'s objective function. Under Assumption 3.1, the robust value function has a mean-variance structure:

\[
V^* (\mu, \sigma, \alpha) = \mu - k + \alpha \sigma^2 \left( \frac{3}{2} R^* - R^* \right)
\]  

Having computed the optimal parameter \( R^* \), it becomes straightforward to analyze the value function’s sensitivity to the distribution moments \( \mu \) and \( \sigma \), and to the cost-reduction technology’s curvature parameter \( \alpha \). Such analysis is formalized in Proposition 3.6, and illustrated in Figure 3.7.

**Proposition 3.6 (comparative statics).** The robust value function \( V^* (\mu, \sigma, \alpha) \) exhibits the following behavior with respect to model primitives:

\[
\begin{align*}
\frac{d}{d\mu} V^* (\mu, \sigma, \alpha) &= 1 \\
\frac{d}{d\sigma} V^* (\mu, \sigma, \alpha) &\geq 0 \\
\frac{d}{d\alpha} V^* (\mu, \sigma, \alpha) &\geq 0 \quad \text{iff} \quad R^* \geq 2
\end{align*}
\]

![Graphs showing the response of robust expected value to \( \sigma \) and \( \alpha \); \( \mu = 100 \)](image)

(a) \( \alpha \) fixed, \( \sigma \) ranging from 0 to 50

(b) \( \alpha \) ranging from 0 to 0.05, \( \sigma \) fixed

Figure 3.7: Response of robust expected value to \( \sigma \) and \( \alpha \); \( \mu = 100 \)
Notice that the severity of asymmetric information (i.e. the dispersion of intrinsic costs \( \sigma \)) harms the regulator as it worsens the rent-efficiency tradeoff. In contrast, the effect of curvature is ambiguous, as it depends on the equilibrium value of \( R^* \). For low values of \( \alpha \), the regulator benefits from the increased scope for screening project types as the technology moves away from the linear shape. However, as \( \alpha \) becomes large, the curvature becomes too punitive, making it more expensive to provide incentives for effort.

The robust value function in (3.13) guides the regulator’s project selection criterion. Put another way, the min-max expected value serves to rank technological alternatives in a way that is robust to the regulator’s ignorance of the full cost distribution, in addition to optimally trading off efficiency and information rent. Hence, the regulator may robustly rank two projects \( X \) and \( Y \), with mean-variance profiles \( \{ \mu_X, \sigma_X \} \) and \( \{ \mu_Y, \sigma_Y \} \), according to:

\[
X \succeq Y \text{ if, and only if } V(\mu_X, \sigma_X, \alpha) \geq V(\mu_Y, \sigma_Y, \alpha)
\]

**Regulator’s indifference map.** Back to the example in the beginning of this section, we construct the regulator’s robust indifference map from (3.13) and plot the robust expected costs of technologies \( A–D \) in Figure 3.8. As it turns out, the robust criterion favors distributions with low variance in this example.

![Figure 3.8: Regulator’s indifference map on the mean-variance space, with \( \alpha = 0.025 \)](image-url)
The robust project ranking is as follows (expected cost in brackets):

\[ A[70.43] \succ B[72.38] \succ C[75.80] \succ D[75.96] \]

Comparing to the data in Table 3.3, it becomes apparent that the robust value exceeds the expected procurement costs under some of the assumptions on \( F(\beta) \). However, the maximum cost increment from the robust mechanism is 2.7% in this example (average 1.4%), which is a lower sacrifice than risking high potential losses from misspecifying \( F(\beta) \) in Bayesian contract design.

**Mean-variance substitution.** A related problem is how to allocate a limited budget to acquire sharper estimates of \( \mu \) and \( \sigma \). Suppose the regulator’s information is even more limited; she only knows the first two moments of \( F(\beta) \) belong to confidence intervals:

\[
\mu \in [\mu, \overline{\mu}] \quad \text{and} \quad \sigma \in [\sigma, \overline{\sigma}]
\]

The robustness problem remains largely unchanged: an adversarial nature would choose a worst-case distribution with the moments most detrimental to the regulator, i.e. \( \overline{\mu} \) and \( \overline{\sigma} \) (Kos and Messner, 2015).\(^{14}\) However, suppose the regulator has access to an estimation technology which allows her to increase the accuracy of each confidence interval for a cost.

Specifically, suppose it costs \( \phi_\mu \) and \( \phi_\sigma \), respectively, to reduce intervals’ upper boundaries \( \overline{\mu} \) and \( \overline{\sigma} \) by a unit. Then, the regulator’s robust value function (3.13) can be used to compute the marginal rate of substitution between mean and variance of the \( \beta \)-distribution:

\[
MRS_{\sigma,\mu}(\sigma,\mu) = -\frac{\partial}{\partial \sigma} V(\mu, \sigma, \alpha) \quad \frac{\partial}{\partial \mu} V(\mu, \sigma, \alpha)
\]

Hence, the regulator optimally allocates her forecasting budget according to the rule:

\[
MRS_{\sigma,\mu}(\overline{\sigma}, \overline{\mu}) = \frac{\phi_\sigma}{\phi_\mu}
\]

which produces a different relative desirability for sharper moment estimates, depending on the current mean-variance profile of the project.

For example, consider the robustly selected project \( A \), with profile \((\mu, \sigma) = (82.0, 5.0)\). Suppose the regulator has a forecasting budget of \$5 and that accuracy prices are \( \phi_\mu = \phi_\sigma = 1 \). Then, she optimally allocates \$4.85 to reduce \( \sigma \) and \$0.15 to reduce \( \overline{\mu} \).\(^{15}\)

\(^{14}\) Though, in an environment with curvature, nature could choose an interior point of \([\sigma, \overline{\sigma}]\) if it contained the maximum admissible variance \( \sigma^* \), which would amount to a slackness of the second moment condition.

\(^{15}\) The allocation is obtained as an interior optimum of \( \max_{\mu, \sigma} V(\mu, \sigma, \alpha) \) subject to \( \mu + \sigma = 5 \).
3.5 Conclusion

This chapter studied the problem of designing a procurement mechanism under limited knowledge about the contracting environment. The principal was assumed to have partial knowledge about the shape of asymmetric information, restricted to the first two moments of the distribution of project intrinsic costs. In that sense, the principal was said to be non-Bayesian, in that the design criterion searched for mechanisms that perform well under worst-case scenarios for the type-distribution.

Similarly to the Bayesian procurement model, the robust contract optimally trades off cost efficiency and information rent. Likewise, it is implemented with a menu of linear remuneration policies inducing more efficient firms to self-select into higher-powered contracts. However, when designing the mechanism, the principal considers a worst-case distribution that respects moment conditions. As a result, the optimal mechanism depends solely on the distribution’s mean and variance, and in this sense, is robust to the principal’s ignorance.

This essay argued that the principal’s min-max value function, which depends on the cost distribution’s moments as well as on the preference (curvature) parameter, is useful as a robust criterion to select projects. Faced with technological alternatives that differ in terms of average cost and cost dispersion, the principal may define a preference ranking that is robust to Bayesian assumptions on project costs distribution. Consequently, the regulator protects her payoff from potentially large losses due to prior misspecification.

The relevance of the present chapter’s analysis extends beyond concession design. As the pace of technical progress accelerates in infrastructure industries, historical databases that underlie planning parameters could become rapidly obsolete. Hence, policy choices that are subject to limited prior information tend to increase in frequency, broadening the demand for robust approaches to incentives design.

In future research, this chapter’s model could be combined with Garrett’s (2014) setup to provide a technique for designing procurement policies that are robust both to the distribution of project costs and to the shape of the agent’s preferences. In such environment, it would suffice that the principal knows the type-distribution’s first two moments and parameter $k$, the maximum cost savings from incentives. Hence, we would be interested in the following problem:

$$\inf_{C \in C} \sup_{(F, \psi) \in F \times \psi(k)} \mathbb{E}_F [C(\theta) + \psi(\theta - C(\theta)) + U(\theta)]$$

where $\psi(k) = \{ \psi(\cdot) : e^{fb} - \psi(e^{fb}) \geq k \}$

This initiative could bridge the gap between the normative theory of Bayesian optimal cost-based procurement, which prescribes an infinite menu of linear contracts, and the simple contracts observed in practice.
3.A Robust nonlinear pricing with moment conditions

This appendix presents a model of robust mechanism design with two moment conditions, considering payoffs curvature. The purpose is to discuss, with an illustrative quadratic case, the emergence of a maximum admissible variance $\sigma^*$ for the Nash equilibrium that satisfies moment conditions with equality.

Model environment

Consider a principal selling one unit of a quality-differentiated good to a buyer (agent). The cost of elaborating the good to quality level $q$ is given by $c(q)$. The function $u(q, \theta)$ denotes the agent’s willingness to pay for quality, where $\theta$ is private information.

The principal knows the agent’s quality valuation $\theta$ is positive, i.e. $\theta \in \Theta = [0, \infty]$, and that the true distribution has mean equal to $k_1$ and variance bounded above by $k_2 - k_1^2$. The set of possible distributions satisfying these constraints is:

$$F = \left\{ F \in \Delta(\Theta) : \int \theta dF(\theta) = k_1; \int \theta^2 dF(\theta) \leq k_2 \right\}$$

In order to guarantee non-emptiness of $F$, the moments have to respect the restriction posed by the truncated Stieltjes problem. In the two-moment case, this amounts to $k_2 - k_1^2 \geq 0$ (i.e. positive variance).

Let $t$ denote the payment from the agent to the principal. The players’ payoffs are given by:

- Principal: $t - c(q)$, with $c', c'' > 0$
- Agent: $u(q, \theta) - t$, with $u_q, u_\theta, u_{\theta q} > 0$ and $u_{qq} < 0$
- Social surplus: $s(q, \theta) = u(q, \theta) - c(q)$

First best. Under complete information, the principal would assign quality $q^{fb}(\theta)$ to an agent with valuation $\theta$ such that:

$$s_q(q^{fb}(\theta), \theta) = 0$$

A direct mechanism $(t(\theta), q(\theta))$ is said to be incentive compatible if $\theta \in \arg\max_{\hat{\theta}} v(\hat{\theta}|\theta) \equiv u(q(\hat{\theta}), \theta) - t(\hat{\theta})$, or, equivalently:

$$\begin{cases} v'(\theta) = u_\theta(q(\theta), \theta) \\ q(\theta) \in Q = \{ q : \Theta \rightarrow \mathbb{R}_+ : q(.) \text{ is monotonic} \} \end{cases}$$

16 The model is set up in the nonlinear pricing environment of Mussa and Rosen (1978). It extends the work of Carrasco, Luz, Monteiro, and Moreira (2018) to the two-moment case.
Therefore, the IC transfer function can be expressed as:

\[ t^q(\theta) = u(q(\theta), \theta) - \int_0^\theta u_\theta(q(z), z) \, dz \]

The principal’s problem is formulated as the maximization of the revenue guarantee, considering the worst-case distribution in family \( F \), as follows:

\[
\sup_{q \in Q} \inf_{F \in F} \int_{\Theta} \left[ s(q(\theta), \theta) - \int_0^\theta s_\theta(q(z), z) \, dz \right] \, dF(\theta)
\]

The principal’s ex-post payoff can be stated as the virtual surplus from the trade:

\[ t^q(\theta) - c(q(\theta)) = s(q(\theta), \theta) - \int_0^\theta s_\theta(q(\tau), \tau) \, d\tau \]

**Characterization of max-min solution**

We conjecture the principal’s optimal revenue guarantee coincides with the positive monotonic hull of a second-degree polynomial on some interval \([\underline{\theta}, \bar{\theta}]\) (see Figure 3.9):

\[ \pi(\theta) = \lambda_0 + \lambda_1 \theta + \lambda_2 \theta^2, \quad \text{with } \lambda_2 < 0 \]

such that \( \pi'(\bar{\theta}) = 0 \) and \( \pi(\underline{\theta}) = 0 \)  \hspace{1cm} (3.14)

![Figure 3.9: Ex-post revenue vs. positive monotonic hull of quadratic polynomial](image-url)
Remark. The present problem features a central theme of nonlinear pricing problems, namely the rent-distortion tradeoff. Since $\theta$ is the highest possible valuation in the equilibrium support, the principal has no incentive to distort this type’s allocation as there is no one higher to leave rents to. Thus, the principal optimally assigns the efficient quality $q^{fb}(\theta)$ to the most profitable type considered in the robust mechanism.

Denote $\lambda \equiv \lambda_2$. In what follows, we will search for a Nash equilibrium of a the zero-sum game between principal and nature, considering the free parameters $\lambda < 0$ and $\bar{\theta} \leq k_1 \leq \bar{\theta}$. Notice that, under (3.14):

$$
\lambda_1 = -2\lambda\bar{\theta} \quad \text{and} \quad \lambda_0 = \lambda (\bar{\theta}^2 - 2\bar{\theta}\theta)
$$

Hence, the conjecture for the ex-post revenue can be equivalently restated as:

$$
\tilde{\pi}(\theta) = \begin{cases} 
0, & \text{if } \theta \leq \bar{\theta} \\
\lambda \left[ \theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta}) \right], & \text{if } \theta \in (\bar{\theta}, \bar{\theta}) \\
-\lambda (\bar{\theta} - \theta)^2, & \text{if } \theta \geq \bar{\theta}
\end{cases}
$$

(3.15)

Differentiating the ex-post revenue function with respect to $\theta$:

$$
\pi'(\theta) = s_q(q(\theta), \theta)q'(\theta) = 2\lambda(\theta - \bar{\theta})
$$

The conjecture implicitly defines the candidate mechanism $\hat{q}(\theta)$ as the solution to a first-order differential equation:

$$
\hat{q}'(\theta) = \frac{2\lambda(\theta - \bar{\theta})}{s_q(\hat{q}(\theta), \theta)}
$$

(3.16)

Lemma 3.3.\textsuperscript{17} For each pair $(\lambda, \bar{\theta})$, there exists a solution $\hat{q} = q^{\lambda, \bar{\theta}} : [\hat{\theta}^{\lambda, \bar{\theta}}, \bar{\theta}] \rightarrow \mathbb{R}_+$ for some $\hat{\theta}^{\lambda, \bar{\theta}} \geq 0$, satisfying the following properties:

1. Zero quality at the bottom: $\hat{q}(\hat{\theta}) = 0$
2. Strict monotonicity: $\hat{q}'(\theta) > 0$ for all $\theta \in (\hat{\theta}, \bar{\theta})$
3. Parameter dependence: $q^{\lambda, \bar{\theta}}(.)$ is decreasing and continuous in $\lambda$, $\forall \bar{\theta}$
4. Limiting behavior I: for each $\bar{\theta}$, $\lim_{\lambda \to 0} q^{\lambda, \bar{\theta}}(\theta) = q^{fb}(\theta)$ for almost all $\theta$
5. Limiting behavior II: for each $\bar{\theta}$, $\lim_{\lambda \to -\infty} \theta^{\lambda, \bar{\theta}} = \bar{\theta}$

\textsuperscript{17}Extension of Carrasco et al. (2018), Lemma 2, to the two-moment case.
Proof of Lemma 3.3. Define \( \varphi(q, \theta; \lambda, \overline{\theta}) = \frac{2\lambda(\theta - \overline{\theta})}{s_q(q, \theta)} \). Note this function is continuously differentiable on its domain. Therefore, the existence of a solution to ODE (3.16) on a neighborhood of \((\overline{\theta}, q^{l_b}(\overline{\theta}))\) is obtained from Hale (1969), Theorem 3.1 (p. 18). Let \((\theta^*, \overline{\theta})\) denote the left maximal interval of definition of the solution \(\tilde{q}(\theta)\).

**Property 1.** Define \(\hat{\theta} = \inf\{\theta : \tilde{q}(\theta) > 0\}\). Thus, \(\tilde{q}'(\hat{\theta}) = \lim_{\theta \to \hat{\theta}} 2\lambda(\theta - \overline{\theta})/s_q(q(\theta^*), \theta^*)\). Since by assumption \(s_q(0,0) \leq 0\), the maximality of the interval \((\theta^*, \overline{\theta})\) now ensures that \(\tilde{q}(\hat{\theta}) = 0\).

**Property 2.** Suppose \(\tilde{q}'(\theta) = 0\) for some \(\theta \in (\hat{\theta}, \overline{\theta})\). Thus,

\[
\tilde{q}''(\theta) = \frac{2\lambda}{s_q(\tilde{q}(\theta), \theta)} - \frac{s_{\theta q}(\tilde{q}(\theta), \theta)\tilde{q}'(\theta) + s_q(q(\theta), \theta)\tilde{q}'(\theta)^2}{s_q(\tilde{q}(\theta), \theta)^2} < 0
\]

since \(\lambda < 0\) and \(s_q(\tilde{q}(\theta), \theta) > 0\), for all \(\theta < \overline{\theta}\). Hence, if \(\tilde{q}\) attains an extreme point in \((\hat{\theta}, \overline{\theta})\), it must be a local maximum.

Let \(x \in (\hat{\theta}, \overline{\theta})\) and \(y = \min_{\theta \in [x, \overline{\theta}]} \tilde{q}'(\theta)\). Then it must be that \(y \notin (x, \overline{\theta})\). Indeed, \(\tilde{q}'(y) = 0\) and \(\tilde{q}''(y) < 0\) imply that \(\tilde{q}(y)\) decreases to the right of \(y\). The minimum cannot be \(\overline{\theta}\) because \(\tilde{q}(\theta)\) increases to the left of \(\overline{\theta}\). Thus, the minimum has to be \(y = x\). Therefore, if for any sub-interval of \((\hat{\theta}, \overline{\theta})\) the minimum is the extreme left, \(\tilde{q}(\cdot)\) can only be increasing.

**Property 3.** The continuity of \(q^\lambda\) on \((-\infty, 0)\) is obtained from the argument in Hale (1969), Theorem 3.2 (p.20). Further, since the mapping \(\lambda \mapsto \lambda(\theta - \overline{\theta})/s_q(q, \theta)\) is differentiable for any \(\lambda < 0\) and \((\theta, q) \in [0, \overline{\theta}] \times \mathbb{R}_+\), by Theorem 3.3 (p.21) the solution \(q^\lambda(\theta)\) is differentiable in \(\lambda\).

Define \(k(\theta) = \frac{d}{d\lambda} q^\lambda(\theta)\). Note that \(k(\overline{\theta}) = 0\), since \(q^\lambda(\overline{\theta}) = q^{l_b}(\overline{\theta})\) for every \(\lambda\). Hence:

\[
k'(\theta) = \frac{d}{d\lambda} \left[ \frac{2\lambda(\theta - \overline{\theta})}{s_q(q^\lambda(\theta), \theta)} \right] = \frac{2(\theta - \overline{\theta})}{s_q(q^\lambda(\theta), \theta)} - \frac{2(\theta - \overline{\theta})s_{\theta q}(q^\lambda(\theta), \theta)}{s_q(q^\lambda(\theta), \theta)^2} \lambda k(\theta)
\]

\[
= a(\theta) - b(\theta)\lambda k(\theta)
\]

Define \(P(\theta) = \exp \left( \int_0^\theta b(z)dz \right) \geq 0\), for all \(\theta \leq \overline{\theta}\). It follows that:

\[
(Pk)' = P'k + PK' = PK' + Pk\lambda b(\theta) = P(k' + \lambda b(\theta)k) = Pa(\theta) \leq 0
\]

\(\text{The function } \varphi(q, \theta; \lambda, \overline{\theta}) \text{ is not Lipchitz-continuous on the first argument, thus there may be multiple solutions.}\)
Hence, \( P \cdot k \) is a decreasing function of \( \theta \). Therefore, for every \( \theta \leq \theta_{\overline{\theta}} \):

\[
P(\theta)k(\theta) \geq P(\theta_{\overline{\theta}})k(\theta_{\overline{\theta}}) = 0
\]

\[
\Rightarrow \quad k(\theta) \geq 0
\]

**Property 4.** Fix \( \theta_{\overline{\theta}} \) and let \( q^0(\theta) = \lim_{\lambda \to 0} q^\lambda(\theta) \). Take any \( \theta_1, \theta_2 \in [\theta, \theta_{\overline{\theta}}] \) with \( \theta_1 < \theta_2 \). Integrating the ODE (3.16) over the range \([\theta_1, \theta_2]\) yields:

\[
0 < q^\lambda(\theta_2) - q^\lambda(\theta_1) = \int_{\theta_1}^{\theta_2} \frac{2\lambda(\theta - \theta_{\overline{\theta}})}{s_q(q^\lambda(\theta), \theta)} d\theta
\]

where the first inequality follows from the strict monotonicity of \( q^\lambda \).

Taking the limit when \( \lambda \to 0 \), the only way to ensure strict positivity of the integral is the ratio \( \frac{\lambda(\theta - \theta_{\overline{\theta}})}{s_q(q^\lambda(\theta), \theta)} \) converging to an indeterminate form, i.e. \( s_q(q^\lambda(\theta), \theta) = 0 \). Hence, \( s_q(q^\lambda(\theta), \theta) \to s_q(q^0(\theta), \theta) = 0 \) for almost every \( \theta \). Since \( s_{0q} > 0 \), it follows that \( q^\lambda(\theta) \to q^{1b}(\theta) \), for almost every \( \theta \).

**Property 5.** Fix \( \theta_{\overline{\theta}} \). Note that, for all \( \lambda < 0 \) and \( \theta \in (0, \theta_{\overline{\theta}}) \), \( s_q(q^\lambda(\theta), \theta) > 0 \). Therefore, when \( \lambda \to -\infty \) we have:

\[
\frac{d}{d\theta}q^\lambda(\theta) = \frac{2\lambda(\theta - \theta_{\overline{\theta}})}{s_q(q^\lambda(\theta), \theta)} \quad \text{unif.} \to \infty
\]

i.e. \( q^\lambda(\theta) \) converges to a vertical line passing on \( \theta_{\overline{\theta}} \). Hence, as \( \theta_{\overline{\lambda}} \) is defined as \( q_{\lambda}(\theta_{\overline{\lambda}}) = 0 \), the result follows.

In parallel to the main text, the following lemma rules out singularity at the boundary condition of ODE (3.16).

**Lemma 3.4.** The first derivative of the mechanism allocation is bounded at the upper limit of the equilibrium support, i.e.

\[
q'(\theta_{\overline{\theta}}) = L < \infty
\]

Moreover, let \( \bar{s}_{qq} = s_{qq}(q^{1b}(\theta_{\overline{\theta}}), \theta_{\overline{\theta}}) \) and \( \bar{s}_{0q} = s_{0q}(q^{1b}(\theta_{\overline{\theta}}), \theta_{\overline{\theta}}) \). Then:

\[
L = -\frac{1}{2} \bar{s}_{0q} + \left( \frac{1}{4} \bar{s}_{qq}^2 + \frac{2\lambda}{\bar{s}_{qq}} \right)^{\frac{1}{2}} > -\frac{\bar{s}_{0q}}{\bar{s}_{qq}} > 0
\]

**Proof of Lemma 3.4.** First, suppose that \( q'(\theta) \) diverges as \( \theta \to \theta_{\overline{\theta}} \). Applying L’Hôpital’s rule to the indeterminate form in \( \frac{\lambda(\theta - \theta_{\overline{\theta}})}{s_q(q^\lambda(\theta), \theta)} \) as \( \theta \to \theta_{\overline{\theta}} \), it obtains that

\[
\lim_{\theta \to \theta_{\overline{\theta}}} q'(\theta) = \frac{2\lambda}{s_{0q} + s_{qq}q'(\theta)} \xrightarrow{\theta \to \theta_{\overline{\theta}}} 0
\]
which is a contradiction. Second, note that:

\[
L = \frac{2\lambda}{s_{\theta q} + s_{qq}L} \\
\iff s_{qq}L^2 + s_{\theta q}L - 2\lambda = 0
\]

As the coefficient \(s_{qq} < 0\) and the intercept \(-2\lambda > 0\), there are two real, opposite-signed solutions to the quadratic equation. Since \(\tilde{q} \in Q\) implies \(\tilde{q}'(\theta) > 0\), we focus on the positive solution and the result follows.

The candidate mechanism \(\tilde{q}\) is defined as the solution to the ODE (3.16) over the interval \((\theta, \bar{\theta})\), and by \(\tilde{q}(\theta) = 0\) for \(\theta \leq \theta\) and \(\tilde{q}(\theta) = q^{f_0}(\theta)\) for \(\theta \geq \bar{\theta}\).

**Best replies and Nash equilibrium**

The following proposition characterizes nature’s best reply to the principal’s playing a mechanism that induces a quadratic ex-post revenue consistent with conjecture (3.15).

**Proposition 3.7.** Suppose the principal plays \(\tilde{q}\), as defined in (3.16), in the auxiliary zero-sum game. Then nature optimally plays any \(\tilde{F} \in F\) such that \(\text{supp}(\tilde{F}) \subseteq [\theta, \bar{\theta}]\).

**Proof of Proposition 3.7.** Let \(F \in F\) and denote \(\tilde{\pi}\) the revenue guarantee function defined in the conjecture.

\[
\int \tilde{\pi}(\theta)dF(\theta) = \int_0^\theta dF(\theta) + \int_\theta^{\bar{\theta}} \lambda \left[\theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta})\right]dF(\theta) + \int_\bar{\theta}^\infty (-\lambda) (\bar{\theta} - \theta) dF(\theta)
\]

\[
= \int_0^\infty \lambda \left[\theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta})\right]dF(\theta) - \int_0^\theta \lambda \left[\theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta})\right]dF(\theta)
\]

\[
- \int_\theta^{\bar{\theta}} \lambda \left[\theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta}) + (\bar{\theta} - \theta)^2\right]dF(\theta)
\]

\[
= \lambda \left[k_2 - \bar{\theta}^2 - 2\bar{\theta}(k_1 - \bar{\theta})\right] - \int_0^\theta \lambda \left[\theta^2 - \bar{\theta}^2 - 2\bar{\theta}(\theta - \bar{\theta})\right]dF(\theta)
\]

\[
- \int_\theta^{\bar{\theta}} \lambda (\theta - \bar{\theta})^2 dF(\theta)
\]

Provided \(\theta \leq k_1 \leq \bar{\theta}\), it follows that:

\[
\lambda \left[k_2 - \theta^2 - 2\bar{\theta}(k_1 - \theta)\right] \geq \lambda \left[k_1^2 - \theta^2 - 2\bar{\theta}(k_1 - \theta)\right]
\]

\[
= \lambda (k_1 - \theta) [k_1 + \theta - 2\bar{\theta}]
\]

\[
\geq 0 \quad \text{iff} \quad k_1 + \theta \leq 2\bar{\theta}
\]

---

\[\text{Extension of Carrasco et al. (2018), Proposition 1, to the two-moment case.}\]
The second integrand denotes the negative region of the polynomial, for \( \theta \leq \theta \). As the integral itself is always positive, minimizing nature optimally sets \( F(\theta) = 0 \). Moreover, the last integrand is always positive, so a minimizing nature optimally sets \( F(\theta) = 1 \).

Therefore, nature’s best reply is to choose any distribution \( F \in \mathcal{F} \) with support contained in \([\theta, \theta]\).

The worst-case distribution that supports \( \tilde{q} \), as defined in conjecture (3.14), as the principal’s best reply, is constructed to induce a hazard rate coinciding with the function:

\[
\gamma(\theta) = \frac{s\tilde{\theta}q(\tilde{q}(\theta), \theta)}{s\tilde{q}(\tilde{q}(\theta), \theta)}
\]

Solving the simple differential equation given by \( \frac{F'}{1-F} = \gamma \), with boundary condition \( F(\theta) = 0 \), it obtains:

\[
\tilde{F}(\theta) = \begin{cases}
0, & \theta \leq \theta \\
1 - \exp\left(-\int_{\theta}^{\theta} \gamma(z)dz\right), & \theta \in (\theta, \theta) \\
1, & \theta \geq \theta
\end{cases}
\]

(3.17)

Note there might be positive mass assigned to \( \theta \), i.e. if \( \tilde{F}(\theta) - \tilde{F}(\theta) > 0 \).

Considering the worst-case distribution defined in (3.17), the following proposition establishes the principal’s best reply.

**Proposition 3.8 (Carrasco et al., 2018).** Suppose nature plays \( \tilde{F} \), as defined in (3.17), in the auxiliary zero-sum game. Then the principal optimally plays \( \tilde{q} \) as defined in (3.16).

**Proof of Proposition 3.8.** Follows directly from Carrasco, Luz, Monteiro, and Moreira (2018, Proposition 2).

The profile \((\tilde{q}^*, \tilde{F}^*, \theta^*)\) is a Nash equilibrium if there exists a combination of the free parameters \( \lambda < 0 \) and \( \theta > k_1 \) such that the following moment conditions are satisfied:

\[
M_1(\lambda, \theta) \equiv \int \theta \tilde{F}^*(\theta)d\theta = k_1
\]

(3.18a)

\[
M_2(\lambda, \theta) \equiv \int \theta^2 \tilde{F}^*(\theta)d\theta \leq k_2
\]

(3.18b)

**Proposition 3.9.** There exist equilibrium parameters \((\lambda^*, \theta^*) \in \mathbb{R}_- \times \mathbb{R}_+ \) such that the system of moment conditions (3.18) is satisfied.

\(^{20}\)Extension of Carrasco et al. (2018), Proposition 3, to the two-moment case.
Proof of Proposition 3.9. Fix \( \bar{\theta} > k_1 \). Focusing on the first moment condition, we mean to show that:

(i) \( M_1(\lambda, \bar{\theta}) \equiv \mathbb{E}^{\lambda, \bar{\theta}}[\theta] \) is continuous and strictly increasing in \( [0, \bar{\theta}] \)

The first moment expression can be equivalently restated as:

\[
M_1(\lambda, \bar{\theta}) = \bar{\theta}^{\lambda, \bar{\theta}} + \int_{\bar{\theta}}^{\bar{\theta}} \left(1 - F^{\lambda, \bar{\theta}}(\theta)\right) d\theta \tag{3.19}
\]

The parameter dependence of \( q^{\lambda, \bar{\theta}} \), from Lemma 3.3, carries over straightforwardly to the first moment expression in (3.19).

(ii) \( \lim_{\lambda \to 0} M_1(\lambda, \bar{\theta}) = 0 \)

Recall that \( 1 - F^{\lambda, \bar{\theta}}(\theta) = \exp \left( -\int_{\bar{\theta}}^{\theta} \gamma^{\lambda, \bar{\theta}}(z) dz \right) \). From Lemma 2.4, \( q^{\lambda, \bar{\theta}} \overset{\text{ase}}{\to} q^{f, \theta} \) as \( \lambda \to 0 \); hence, \( \gamma^{\lambda, \bar{\theta}}(\theta) \to \infty \), for all \( \theta \). In particular, as \( q^{f, \theta}(0) = 0 \), it follows that \( \theta^{\lambda, \bar{\theta}} \to 0 \). As \( 1 - F^{\lambda, \bar{\theta}}(\theta) \to 0 \), the integrand in (3.19) becomes null and the result follows.

(iii) \( \lim_{\lambda \to -\infty} M_1(\lambda, \bar{\theta}) = \bar{\theta} \)

From Lemma 3.3, recall that \( \theta^{\lambda, \bar{\theta}} \to \bar{\theta} \) as \( \lambda \to -\infty \). This implies the integral in (3.19) becomes null, and the result follows.

Therefore, by the Intermediate Value Theorem, for each \( \bar{\theta} \) there exists a unique \( \lambda^*(\bar{\theta}) \) such that \( M_1(\lambda^*(\bar{\theta}), \bar{\theta}) = k_1 \). Moreover, \( \lambda^*(\bar{\theta}) \) is continuous.

Consider now the second moment condition. We mean to show that:

(i) \( M_2(\bar{\theta}) \equiv \mathbb{E}^{\lambda^*(\bar{\theta}), \bar{\theta}}[\theta^2] \) is continuous in \( [k_1, \infty] \).

Substituting the solution to the first equation in the second, it obtains:

\[
M_2(\bar{\theta}) = \left(\theta^{\lambda^*, \bar{\theta}}\right)^2 + \int_{\theta^{\lambda^*, \bar{\theta}}}^{\bar{\theta}} 2\theta \left(1 - F^{\lambda^*, \bar{\theta}}(\theta)\right) d\theta \tag{3.20}
\]

The continuity and parameter dependence of \( q^{\lambda, \bar{\theta}} \), from Lemma 3.3, again carries over to the second moment expression in (3.20).

(ii) \( \lim_{\bar{\theta} \to k_1} M_2(\bar{\theta}) = k_1^2 \)

Note that \( \lambda^*(k_1) = -\infty \) because, as \( \bar{\theta} \to k_1 \), the only way to preserve the mean is making \( \theta \to k_1 \). In turn, \( \theta^{\lambda, \bar{\theta}} \to k_1 \) is only possible if \( \lambda \to -\infty \), due to the continuous monotonicity of \( \theta^{\lambda, \bar{\theta}} \). Thus, \( \theta^{\lambda^*(k_1), k_1} \to \bar{\theta} = k_1 \). Since \( (1 - F^{\lambda^*(k_1), k_1}(\theta)) \leq 1, \forall \theta \), it follows that \( \int_{k_1}^{\bar{\theta}} 2\theta(1 - F(\theta))d\theta \to 0 \). Hence the result \( M_2(\bar{\theta}) \overset{k_1}{\to} \bar{\theta}^2 = k_1^2 \).
To complete the proof, there remains to show that the second moment condition (3.18b) is satisfied (with or without equality). For this purpose, we consider two cases.

**Case I)** \(\liminf_{\theta \to \infty} \lambda^*(\theta) > 0\)

First, note that \(\theta^{\lambda^*(\infty),\infty} < k_1\), otherwise it would contradict the definition of \(\lambda^*(\theta)\) solving (3.19) for all \(\theta\). Second, if \(0 < \gamma^{\lambda^*(\infty),\infty} < \infty\), it follows that, for each point \(\theta\), the integrand

\[
1 - F^{\lambda^*,\theta}(\theta) = \exp \left( - \int_{\theta}^{\theta^*} \gamma^{\lambda^*(\infty),\infty}(z) dz \right) > 0
\]

As the integration range \([\theta^{\lambda^*,\theta}, \theta]\) in (3.20) diverges, it follows that \(M_2(\theta) \to \infty\).

Therefore, by the Intermediate Value Theorem, there exists some value \(\theta^* \geq k_1\) such that \(M_2(\theta^*) = k_2\).

**Case II)** \(\liminf_{\theta \to \infty} \lambda^*(\theta) = 0\)

From Lemma 3.3, the allocation \(\tilde{q}(\theta) \xrightarrow{\text{a.e.}} q^H(\theta)\), and hence, \(\gamma(\theta) \to \infty\), for all \(\theta\). Then the integrand \(2\theta \exp \left( - \int_{\theta}^{\theta^*} \gamma^{\lambda^*(\infty),\infty}(z) dz \right)\) becomes arbitrarily close to zero, whereas the integration range \([\theta^{\lambda^*,\theta}, \theta]\) diverges, so the integral in (3.20) has an indeterminate form. Consequently, the second moment condition (3.18b) may not be satisfied with equality.

Let \(k_2^*\) denote the accumulation point such that \(\limsup_{\theta \to \infty} M_2(\theta) = k_2^*\). If \(k_2 < k_2^*\), the existence result obtains trivially.

Suppose \(k_2 > k_2^*\). Fix \(k_1\) and denote \(V^I(\tilde{\theta})\) and \(V^{II}(k_2)\) as the equilibrium max-min values of the design problems in the one-moment and two-moment cases, respectively.\(^{21}\)

Since the robust mechanism for the one-moment case exists for any \(\tilde{\theta}\), it follows that \(V^I(\tilde{\theta}) \geq 0\).

Let \(\tilde{\theta} \mapsto \tilde{k}_2(\tilde{\theta})\) denote the second moment of the equilibrium worst-case distribution in problem \(V^I\), as a function of the exogenous parameter \(\tilde{\theta}\). Denote \(V^{II}(\tilde{k}_2(\tilde{\theta}))\) as the value of the two-moment problem when the second moment is constrained to \(\tilde{k}_2(\tilde{\theta})\).

As the two-moment case features one additional constraint for nature, its value to the principal cannot be lower than the equivalent one-moment case, so \(V^{II}(k_2(\tilde{k}_2(\tilde{\theta}))) \geq V^I(\tilde{\theta})\).

Conversely, in the two-moment case with \(k_2 \leq \tilde{k}_2(\tilde{\theta})\), the worst-case distribution in the associated one-moment problem is feasible, so nature cannot perform worse than that. Hence, \(V^I(\tilde{\theta}) \geq V^{II}(\tilde{k}_2(\tilde{\theta}))\).

It follows that \(V^I(\tilde{\theta}) = V^{II}(\tilde{k}_2(\tilde{\theta}))\). Hence, for \(\tilde{\theta} \to \infty\), the robust value function exists and attains the value of \(V^I(\infty)\), but the robust mechanism is undefined. \(\square\)

\(^{21}\) The one-moment case is the object of analysis in Carrasco, Luz, Monteiro, and Moreira (2018).
The proof of Proposition 3.9 shows the second moment of the equilibrium worst-case distribution $F^*$ could attain an accumulation point $k_2^*$, which amounts to a maximum admissible variance compatible with condition (3.18b) being satisfied with equality. In that case, for $k_2 \geq k_2^*$, the robust mechanism would set the equilibrium support at $[0, \infty]$ with quadratic coefficient $\lambda = 0$, thus becoming undefined.\textsuperscript{22} Nature’s optimal choice, in turn, entails slackness of the second moment constraint.

This result stands in stark contrast to the robustness problem with linear payoffs, for which there exists a max-min solution with binding moment conditions for all values of $k_1 \geq 0$ and $k_2 \geq k_1^2$ (Carrasco, Luz, Kos, Messner, Monteiro, and Moreira, 2018). Furthermore, if $M_2(\theta)$ accumulates at $k_2^*$, the optimal value of the game converges to some positive value, which also contrasts with the linear case.\textsuperscript{23}

### The quadratic payoffs case

We illustrate the emergence of the maximum admissible variance with a special case, where $s(q, \theta)$ is specified as a quadratic function. Suppose $u(q, \theta) = \theta q$ and $c(q) = \alpha q^2/2$, so that $q^{\text{fb}}(\theta) = \theta/\alpha$.

The differential equation defining the robust mechanism becomes:

$$\tilde{q}'(\theta) = \frac{\lambda (\theta - \bar{\theta})}{\theta - \alpha \tilde{q}(\theta)} \quad \text{s.t.} \quad \tilde{q}(\bar{\theta}) = \bar{\theta}/\alpha \quad (3.21)$$

The differential equation (3.21) admits a linear solution:

$$q^{L,\bar{\theta}}(\theta) = L(\theta - \bar{\theta}) + \frac{\bar{\theta}}{\alpha} \quad (3.22)$$

with $L = \frac{1}{2\alpha} + \sqrt{\frac{1}{4\alpha^2} - \frac{2\lambda}{\alpha}} > \frac{1}{\alpha}$

and the lower limit of the endogenous support becomes:

$$\theta^{L,\bar{\theta}} = \frac{\alpha L - 1}{\alpha} \bar{\theta}$$

Similarly to the procurement model, the quadratic case also facilitates the explicit computation of the worst-case distribution:

\textsuperscript{22} An analogous situation occurs when one considers the random variable $X_n \sim U[-n, n]$. The limiting expected value $\lim_{n \to \infty} E[X_n]$ exists and equals zero, but the limiting distribution $\lim_{n \to \infty} U[-n, n]$ is undefined.

\textsuperscript{23} Carrasco, Luz, Kos, Messner, Monteiro, and Moreira (2018) show that there is no limit to how much nature could jeopardize the principal if there is no bound on $k_2$, so the optimal value of the game converges to zero.
\[
\gamma^{L, \bar{\theta}}(\theta) = \frac{1}{(\theta - \bar{\theta})(1 - \alpha L)}
\]

\[
F^{L, \bar{\theta}}(\theta) = 1 - \left(\frac{\theta - \bar{\theta}}{\theta^L, \bar{\theta} - \bar{\theta}}\right)^{\frac{1}{\alpha L - 1}}
\]

\[
f^{L, \bar{\theta}}(\theta) = \frac{1}{(\theta - \bar{\theta})(1 - \alpha L)} \left(\frac{\theta - \bar{\theta}}{\theta^L, \bar{\theta} - \bar{\theta}}\right)^{\frac{1}{\alpha L - 1}} = \gamma^{L, \bar{\theta}}(\theta) \left(1 - F^{L, \bar{\theta}}(\theta)\right)
\]

Fix \(\bar{\theta} > k_1\). The first moment condition becomes:

\[
M_1(L, \bar{\theta}) = \theta^L, \bar{\theta} + \int_{\theta^L, \bar{\theta}}^{\bar{\theta}} \left(1 - F^{L, \bar{\theta}}(\theta)\right) d\theta
\]

\[
= \theta^L, \bar{\theta} + \frac{\alpha L - 1}{\alpha L} (\bar{\theta} - \theta^L, \bar{\theta}) = \frac{\alpha^2 L^2 - 1}{\alpha^2 L^2} \bar{\theta}
\]

Hence, the equation \(M_1(L, \bar{\theta}) = k_1\) is solved by:

\[
L^*(\bar{\theta}) = \frac{1}{\alpha} \sqrt{\frac{\bar{\theta}}{\bar{\theta} - k_1}}
\]

with limiting behavior \(L^*(\bar{\theta}) \xrightarrow{\bar{\theta} \to k_1} \infty\) and \(L^*(\bar{\theta}) \xrightarrow{\bar{\theta} \to \infty} 1/\alpha\). Substituting in the second moment condition yields:

\[
M_2(\bar{\theta}) = \left(\bar{\theta} L^*, \bar{\theta}\right)^2 + \int_{\bar{\theta} L^*, \bar{\theta}}^{\bar{\theta}} 2\theta \left(1 - F^{L^*, \bar{\theta}}(\theta)\right) d\theta
\]

\[
= \left(\bar{\theta} L^*, \bar{\theta}\right)^2 - 2(\alpha L^*) \left[\frac{(\bar{\theta} - \bar{\theta})^2}{2\alpha L^* - 1} - \bar{\theta} (\bar{\theta} - \bar{\theta})\right]
\]

\[
= \frac{1}{\alpha^2 L^2(2\alpha L^* - 1)} \left[\bar{\theta} (\alpha L^* - 1)\right]^2 (2\alpha L^* + 3)
\]

Taking the relevant limits, it obtains:

\[
\lim_{\bar{\theta} \to k_1} M_2(\bar{\theta}) = k_1^2 \quad \text{and} \quad \lim_{\bar{\theta} \to \infty} M_2(\bar{\theta}) = \frac{5}{4} k_1^2
\]

Note the second moment has an accumulation point at \(k_2^* = 1.25 k_1^2\), which is independent of the curvature parameter \(\alpha\).\(^{24}\) Therefore, the linear conjecture in (3.22) is only supported as a robust mechanism for values \(k_2 \in [k_1^2, k_2^*]\). This corresponds to the distribution having a maximum coefficient of variation at 0.5.\(^{25}\)

\(^{24}\) It can be shown that the accumulation point \(k_2^*\) depends on \(\alpha\) only if \(s_{\theta q}(\bar{q}(\bar{\theta}), \bar{\theta})\) is a function of \(\alpha\).\(^{25}\) Recall \(cv = \mu/\sigma\) is a scale-invariant measure of dispersion of a distribution.
3.B Proofs of Chapter 3

Proof of Lemma 3.1. See the analogous proof of Lemma 3.3. Let \( \lambda = -\xi \) and \( \bar{\beta} = \beta \). Redefine types as \( \theta = (\bar{\beta} - \beta) / \Delta \beta \), where \( \Delta \beta = \bar{\beta} - \beta \). The mechanism allocation becomes \( q = \bar{C} - C \), where \( \bar{C} = \tilde{C}(\bar{\beta}) \). Instead of \( \tilde{q}(\theta) = 0 \), define \( \bar{C} \) is the fixed point \( \tilde{C}(\beta) = \beta \). The social surplus function is equivalently restated as:

\[
s(q, \theta) = q - \psi (\bar{\beta} - \Delta \beta \cdot \theta + q - C) + \psi (\bar{\beta} - \Delta \beta \cdot \theta - C)
\]

and thus first-best is defined as \( s_q(q^{fb}(\theta), \theta) = 1 - \psi' (\bar{\beta} - \Delta \beta \cdot \theta + q^{fb}(\theta) - C) \), where \( q^{fb}(\theta) = C - \beta + \psi' - 1 (1) \).

Proof of Lemma 3.2. Suppose \( \tilde{C}'(\beta) \xrightarrow{\beta \rightarrow \infty} 0 \). Applying L’Hopital’s rule to the indeterminate form in \( \frac{2\xi (\theta - \beta)}{1 - \psi'(\beta - \tilde{C}(\beta))} \) as \( \beta \rightarrow \bar{\beta} \), it obtains that

\[
\lim_{\beta \rightarrow \bar{\beta}} \tilde{C}'(\theta) = \frac{2\xi}{-\psi''(e^{fb}) (1 - \tilde{C}'(\bar{\beta}))} \tilde{C}'(\beta) \xrightarrow{\beta \rightarrow \bar{\beta}} 0
\]

which is a contradiction. Moreover, note that:

\[
R = \frac{2\xi}{-\psi''(e^{fb}) (1 - R)} \iff R^2 - R - \frac{2\xi}{\psi''(e^{fb})} = 0 \tag{3.23}
\]

The convex quadratic form in (3.23) has a strictly negative intercept \( -2\xi / \psi''(e^{fb}) < 0 \), so it admits two real, opposite-signed solutions. \( \bar{C} \in \mathcal{C} \) implies \( \tilde{C}'(\beta) > 0 \), hence the positive solution yields the result. \( \square \)

Proof of Proposition 3.1. Suppose the regulator chooses a cost recommendation inducing the payment function \( T^C(\beta) \) as in (3.3), and let \( F \in \mathcal{F} \). Then, we have

\[
\int_k^\infty T^C(\beta) dF(\beta) = \int_k^\beta \pi(\beta) dF(\beta) + \int_\beta^{\bar{\beta}} \pi(\beta) dF(\beta) + \int_\bar{\beta}^\infty \beta dF(\beta)
\]

\[
= \int_k^\infty \pi(\beta) dF(\beta) - \left[ \int_k^\beta \pi(\beta) dF(\beta) - \int_\beta^{\bar{\beta}} \pi(\beta) dF(\beta) \right] - \int_\bar{\beta}^\infty (\pi(\beta) - \hat{\pi}(\beta)) dF(\beta)
\]

A sufficient condition for a maximizing nature to choose \( F \in \mathcal{F} \) contained in support \([\beta, \bar{\beta}]\) is that the expression in (1) is constant and strictly positive, and that the expressions in (2) and (3) are positive.

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1. Let $\mu_2 = \sigma^2 + \mu^2$ and recall $\underline{\beta} < \mu < \overline{\beta}$. Note that the expected payoff under quadratic payments is:

$$
\int_{\underline{\beta}}^{\overline{\beta}} \pi(\beta)dF(\beta) = \int_{\underline{\beta}}^{\overline{\beta}} \xi \left( \beta^2 - \overline{\beta}^2 - 2\overline{\beta}(\beta - \overline{\beta}) \right) + \overline{\beta}dF(\beta)
$$

$$
= \xi \left( \mu_2 - \overline{\beta}^2 - 2\overline{\beta}(\mu - \overline{\beta}) \right) + \overline{\beta}
$$

$$
\geq \xi \left( \mu_2 - \overline{\beta}^2 - 2\overline{\beta}(\mu - \overline{\beta}) \right) + \overline{\beta}
$$

$$
> \xi \left( \mu_2 - \beta^2 - 2\beta(\mu - \overline{\beta}) \right) + \xi(\beta^2 - 2\beta\overline{\beta} + \overline{\beta}^2)
$$

$$
= \xi(\mu - \overline{\beta})^2 \geq 0
$$

2. Follows from the definition of $\pi(\beta) = \min_{\beta} \pi(\beta) \leq \pi(\beta), \forall \beta$. Hence, nature sets its distribution so that $F(\overline{\beta}) = 0$.

3. Note that the polynomial $p(x) = \xi \left( x^2 - \overline{\beta}^2 - 2\overline{\beta}(x - \overline{\beta}) \right) + \overline{\beta} - x$ has two real roots at $x_1 = 2\beta - \overline{\beta} + 1/\xi$ and $x_2 = \overline{\beta}$. Since $\xi > 0$, $p(x)$ is positive on the region $x \in (-\infty, x_1] \cup [x_2, \infty)$, and the result follows. Hence, nature sets its distribution so that $F(\overline{\beta}) = 1$. \qed

**Proof of Proposition 3.2.** Suppose nature chooses the distribution $\hat{F}$ defined by (3.6), and let $C \in C$ inducing the payment function $T_C^C(\beta)$. The regulator’s problem is to minimize the following expected total payment:

$$
\int T_C(\beta)d\hat{F}(\beta) = \int_{\beta}^{\overline{\beta}} \left[ C(\beta) + \psi(\beta - C(\beta)) + \int_{\beta}^{\overline{\beta}} \psi'(z - C(z))dz \right] \hat{f}(\beta)d\beta
$$

Integrating by parts the term representing information rent, it obtains that:

$$
\int_{\beta}^{\overline{\beta}} \int_{\beta}^{\overline{\beta}} \psi'(z - C(z))dz \hat{f}(\beta)d\beta = \int_{\beta}^{\overline{\beta}} \psi'(\beta - C(\beta)) \left[ \hat{F}(\beta) - \hat{F}(\beta) \right] d\beta
$$

Plugging into the expected payoff expression:

$$
\int T_C^C(\beta)d\hat{F}(\beta) = \int_{\beta}^{\overline{\beta}} \left[ C(\beta) + \psi(\beta - C(\beta)) + \psi'(\beta - C(\beta)) \frac{\hat{F}(\beta) - \hat{F}(\beta)}{\hat{f}(\beta)} \right] d\hat{F}(\beta)
$$

$$
+ \hat{F}(\beta) \left[ C(\beta) + \psi(\beta - C(\beta)) + \int_{\beta}^{\overline{\beta}} \psi'(z - C(z))dz \right]
$$

$$
= \int_{\beta}^{\overline{\beta}} \left[ C(\beta) + \psi(\beta - C(\beta)) + \psi'(\beta - C(\beta)) \frac{\hat{F}(\beta)}{\hat{f}(\beta)} \right] d\hat{F}(\beta)
$$

$$
+ \hat{F}(\beta) \left[ C(\beta) + \psi(\beta - C(\beta)) \right]
$$

(3.24)
1) As for the second term in expression (3.24), note the candidate mechanism \( \tilde{C} \) assigns the first-best cost recommendation to the most profitable type, hence:

\[
\beta - e^b + \psi(e^b) = \tilde{C}(\beta) + \psi(\beta - \tilde{C}(\beta)) \\
\leq C(\beta) + \psi(\beta - C(\beta)) \text{ for any } C \in C
\]

2) As for the first term in expression (3.24), note the integrand is pointwise minimized if the following first-order condition is satisfied:

\[
1 - \psi'(\beta - C(\beta)) - \psi''(\beta - C(\beta)) \frac{\tilde{E}(\beta)}{\tilde{f}(\beta)} = 0 \\
\Leftrightarrow 1 - \psi'(\beta - C(\beta)) = \psi''(\beta - C(\beta)) \frac{1}{\gamma(\beta)}
\]

Therefore, for each \( \beta \in (\underline{\beta}, \overline{\beta}] \) the candidate mechanism \( \tilde{C} \) exactly satisfies the first-order condition for pointwise minimization of the regulator’s ex-post payment.

3) The regulator is indifferent to any allocation outside the equilibrium support, since nature assigns zero probability to project types in this region.

Therefore, the candidate mechanism \( \tilde{C}(\beta) \) qualifies as the principal’s best reply. \( \square \)

**Proof of Proposition 3.3.** Substituting (3.11c) into (3.11a) obtains:\( ^{26} \)

\[
\beta = \mu - \frac{1}{\alpha R(R - 1)}
\]

Back to equation (3.11c), it follows that:

\[
\overline{\beta} = \mu + \frac{1}{\alpha R}
\]

Substituting both expressions in equation (3.11b), and rearranging, yields:

\[
2R^3 - R^2 - \frac{1}{\alpha^2 \sigma^2} = 0 \quad (*)
\]

The last expression denotes a cubic polynomial \( h(x) = 2x^3 - x^2 - c \), with \( c = (\alpha^2 \sigma^2)^{-1} > 0 \), whose roots define the equilibrium coefficient \( R^* \) (Figure 3.10). Taking a closer look at the polynomial, note the following properties:

\(^{26}\)This equation does not consider corner solutions where the lower boundary of the equilibrium range \( \beta \) hits the lowest possible value of \( \beta \), which is \( k \). Note the fraction on the right-hand side grows without bound as \( R \to 1 \).
• \( h''(\frac{1}{6}) = 0 \) and \( h''(x) \) increases, meaning \( h(x) \) is convex for \( x > 1/6 \) and concave elsewhere;

• \( h'(x) = 0 \) for \( x \in \{0, \frac{1}{3}\} \), so \( x_1 = 0 \) and \( x_2 = \frac{1}{3} \) are, respectively, the maximum and minimum points;

• From inspection of \( h'(x) \), the polynomial \( h(x) \) is increasing for \( x < 0 \) and for \( x > 1/3 \), and decreasing for \( x \in [0, \frac{1}{3}] \);

• Analyzing the signs of the local extremes, note that \( h(0) = -c < 0 \) and \( h(\frac{1}{3}) = -\frac{1}{27} - c < 0 \), implying the polynomial has exactly one real root \( R^* \);

• At the limit when \( c \to 0 \), the polynomial \( h(x) \) has two real roots \( \{0, \frac{1}{3}\} \), implying \( R^* > \frac{1}{2} \) when \( c > 0 \).

![Figure 3.10: Behavior of equation (*)](image)

As it turns out, a sufficient condition for the solution to \((*)\) to satisfy \( R^* > 1 \) is that \( c > 1 \), or, equivalently:\[\sigma < \frac{1}{\alpha}\]

Therefore, in the quadratic case, a Nash equilibrium satisfying moment conditions with equality does not exist for large enough values of \( \sigma \). Conversely, if the curvature parameter is too punishing (large \( \alpha \)), there may not exist a robust mechanism with such shape.

\[\text{Recall the constant } (R - 1) \text{ denotes the slope of incentives in the robust optimal mechanism. Therefore, } R^* \leq 1 \text{ becomes inconsistent with a well-behaved mechanism allocation.}\]

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Proof of Proposition 3.4. First, notice that equation (\(\ast\)) defines the equilibrium coefficient \(R^*\) independently of the mean \(\mu\). Moreover, equation (\(\ast\)) implicitly defines \(R\) as a function of \((\alpha, \sigma)\), with derivatives:

\[
\frac{d}{d\alpha} \left[ 2R^3 - R^2 \right] = (6R^2 - 2R) \frac{dR}{d\alpha} = \frac{-2}{\alpha^3\sigma^2} \Rightarrow \frac{dR}{d\alpha} < 0
\]

\[
\frac{d}{d\sigma} \left[ 2R^3 - R^2 \right] = (6R^2 - 2R) \frac{dR}{d\sigma^2} = \frac{-1}{\alpha^2(\sigma^2)^2} \Rightarrow \frac{dR}{d\sigma^2} < 0
\]

since \(6R^2 - 2R > 0\), for all \(R > \frac{1}{3}\).

Proof of Proposition 3.5. To show that \(T^*(C)\) is implemented by the menu of linear contracts defined by parameters \((a, b)\) in (3.12), we must check that such menu induces a type-\(\beta\) firm to participate and to self-select into the contract designed for its type.

Check: firm’s self-selection
Anticipating that its optimal effort choice will coincide with the regulator’s recommendation \(e^* = \hat{e}(\beta)\), a type-\(\beta\) firm selects its contract according to:

\[
\max_{\beta} \ a(\beta) - b(\beta)(\beta - \hat{e}(\beta)) - \psi(\hat{e}(\beta))
\]

The associated first-order condition is:

\[
\tilde{a}'(\hat{\beta}) - \tilde{b}'(\hat{\beta})(\beta - \hat{e}(\beta)) + \left( \tilde{b}(\hat{\beta}) - \psi'(\hat{e}(\hat{\beta})) \right) \hat{e}'(\hat{\beta}) = 0
\]

\[
\Leftrightarrow -\alpha(R-1)\hat{\beta} + 2\beta(\hat{\beta} - \hat{\beta}) + \alpha(R-1) \left( \beta - (R-1)(\hat{\beta} - \hat{\beta}) \right) = 0
\]

\[
\Leftrightarrow \hat{\beta} = \beta
\]

Check: firm’s participation
To ensure every type of self-selecting firm actually picks a contract, the firm’s profit from the mechanism must exceed its opportunity cost (normalized to zero), i.e.

\[
\tilde{a}(\beta) - \tilde{b}(\beta)\tilde{C}(\beta) - \psi(\hat{e}(\beta)) \geq 0
\]

\[
\Leftrightarrow \alpha(R-1)(\beta - \beta)\bar{\beta} - \beta(\beta - \beta)^2
\]

\[
- \alpha(R-1)(\beta - \beta) [\bar{\beta} - R(\beta - \beta)] - \frac{\alpha}{2} [(R-1)(\beta - \beta)]^2 \geq 0
\]

\[
\Leftrightarrow \alpha(R-1)(\beta - \beta)^2 \geq 0
\]

and the last inequality is satisfied since \(R^* > 1\). \(\square\)
**Proof of Proposition 3.6.** Comparative statics for \( \mathbb{E}[\tilde{\pi}(\beta)] \) with respect to exogenous parameters \( \mu, \sigma \) and \( \alpha \) follow from total differentiation:

\[
\frac{d}{d\mu} \mathbb{E}[\tilde{\pi}(\beta)] = 1
\]

\[
\frac{d}{d\alpha} \mathbb{E}[\tilde{\pi}(\beta)] = \frac{1}{2\alpha^2} + \sigma^2\left(\frac{3}{2}R^2 - R\right) - \frac{1}{R\alpha^2} \geq 0
\]

\[\text{iff } R^2\alpha^2\sigma^2(3R^2 - 3R - 2) \geq 1\]

\[
\frac{d}{d\sigma} \mathbb{E}[\tilde{\pi}(\beta)] = \alpha\left(\frac{3}{2}R^2 - R\right) - \frac{1}{2R\alpha\sigma^2} \geq 0
\]

\[\text{iff } R^2\alpha^2\sigma^2(3R - 2) \geq 1\]

Given the regularity condition \( \alpha\sigma > 1 \) implies \( R^* > 1 \), the expected procurement cost rises with the mean and variance of the project’s intrinsic cost.

The sensitivity of \( \mathbb{E}[\tilde{\pi}] \) to the curvature parameter \( \alpha \) depends on which \( R^* \) arises in equilibrium: above or below the threshold \( R = 2 \) associated with a worst-case uniform distribution. It can be shown numerically that \( \frac{3R - 2}{R^2 - 2} > 1 \) whenever \( R > 2 \), and \( \frac{3R - 2}{R^2 - 2} < 1 \) for \( R \in (0, 2) \).
Final remarks

The use of concession contracts to deliver infrastructure assets and services has been increasing worldwide. According to the World Bank (2019), private investment in infrastructure concessions in low- and middle-income countries totaled USD 0.967 trillion in the past ten years (2009-18), with 3,349 new projects started over the period. The method’s popularity accrues from the possibility of expanding public investments without compromising fiscal budgets, as well as, and more importantly, the superior economic performance of concessions compared to conventional public provision.

The concession model enables the public sector to tap not only private finance, which tends to be more abundant, but also the private sector’s expertise and management flexibility, which makes it more likely that projects are delivered on time, to cost, and meeting quality standards. Moreover, the operation of infrastructure services under a concession contract reduces the probability of insufficient maintenance, which contributes to prolong asset quality-life, and consequently, results in higher service levels. However, the many instances in which concessions failed to deliver on its promise raise a warning flag: the mere granting of contracts to private enterprises does not guarantee success, unless there is a proper institutional framework in place to support contract enforcement, conflict resolution, and overall legal security. In addition, the economic incentives provided by the contract, especially regarding risk allocation and the remuneration scheme, must be carefully aligned with the public interest, if the efficiency benefits of concessions are to be realized.

A concession partnership may be thought of as a principal-agent relationship, first, because the public authority typically holds the power to set contract terms, whereas the private partner is free to accept the proposal or not. Second, the parties’ economic interests are at least partially in conflict, as the contractor’s profit maximization motive often clashes with users’ desire for service quality and affordability, habitually upheld by a regulatory agency. Lastly, there is inherent asymmetry of information between the parties, in the form of the firm’s superior knowledge of its capabilities and the project’s operating conditions, which hinder the government’s ability to implement Pareto-efficient allocations.

The present thesis inquired into the theme of concession design, focusing, across three independent essays, on the optimal design of the concessionaire’s remuneration mechanism. The latter corresponds to the main device in the concession contract to align the parties’ economic incentives, particularly in the presence of information frictions.
Hence, the analytical investigation on the properties of optimal incentive schemes, which arise from principal-agent optimization models, permitted this research project to attain applied insights that entail relevant policy recommendations. The main lessons are summarized as follows.

Chapter 1 focused on the consequences of information dynamics to the optimal design of the concessionaire’s remuneration mechanism. Specifically, it asked whether the regulator should base the contract on sequential private information, especially considering the firm’s projections about performance-relevant parameters like costs. The optimal remuneration mechanism consisted of a dynamic menu: first, the firm chose an award fee due at contract signature, thereby revealing its private forecast. Then, at the production stage, and in the light of definitive cost information, the firm chose from a contingent menu of linear reimbursement rules. The latter featured increasing incentive power the more optimistic were the firm’s forecast. This way, not only did the contract induce a truthful reporting of forecasts, but it also provided stronger incentives for cost efficiency compared to the optimal static contract. Such “tougher” incentives come at the cost of imposing heavy penalties on the firm for large (though honest) cost underestimations.

As it turns out, large penalties may not be credible in concession policy. Chapter 2 addressed that shortcoming by analyzing the effect of ex-post participation constraints to the design problem with dynamic private information. Motivated by the principle of “contractual equilibrium” in concession arrangements, it studied the optimal design of remuneration mechanisms in a dynamic environment when the firm is protected by ex-post limited liability. Such protection resulted in optimal pooling of first-period private information, i.e. the regulator offered the same incentive scheme regardless of cost forecasts. However, a numerical exercise suggested there is an optimal level of ex-post protection that maximizes social welfare, that being the sum of concessionaire’s and users’ payoffs. Such finding implies that virtue lies in the middle for concession policy: neither absolute contractual equilibrium protection, nor unlimited liability in case of adverse outcomes like (again, honest) cost overruns.

Chapter 3 examined the design of robust concession contracts in an environment that required less knowledge from the principal. The regulator was assumed to be non-Bayesian, in that she ignored the exact pattern of private information aside from the first two moments of the cost distribution. Hence, contract design considered a worst-case distribution, and the robust remuneration mechanism depended solely on the project’s average costs and cost dispersion. Furthermore, a robust selection criterion was constructed from the principal’s min-max value function, which allowed to rank project alternatives with different mean-variance profiles. Such criterion may be useful when there is little historical information for the planner to form a Bayesian prior over project costs. In stances with novel, or rapidly changing technology, such criterion protects the planner from potentially large losses due to prior misspecification.

28 For example, there is an ongoing debate in Brazil concerning the “friendly devolution” of concessions that ran into financial distress, with the imminent issue of detailed regulation on Law 13,448, of 2017. The debate became particularly relevant after several concessionaires filed for bankruptcy in the wake of Brazil’s 2015-17 economic recession.
The preceding analysis is rather homogeneous in its method, applying the theory of incentives to draw applied lessons to concession policy. Therefore, it may be subject to similar sorts of criticism as the broader branch of normative regulation theory. For one, there is little correspondence between the model-based optimal mechanisms, which consist of complex, highly nonlinear incentive schemes, such as the Laffont-Tirole infinite cost-reimbursement menu, and the simple contracts observed in the real world.\textsuperscript{29}

A second line of criticism concerns the lack of thorough empirical testing of normative theory, which casts doubt on the validity of optimal mechanisms outside the highly stylized modeling environments. In this respect, the difficulty in conducting empirical exercises with models featuring asymmetric information has so far functioned as the standard excuse. In recent years, however, there have been exemplary studies that creatively circumvented the latent-variable problem that plagues empirical analysis of endogenous (optimal) regulation.\textsuperscript{30} These works should pave the way to a surge in related empirical works in the next few years.

Nonetheless, the approach of drawing applied lessons from theoretical models has a long tradition in economics. Aside from the seminal work of Laffont and Tirole (1986; 1993), Averch and Johnson (1962), Rogerson (2003), Engel, Fischer, and Galeтовic (2009), and Iossia and Martimort (2014), to mention only a few, have also derived applied lessons from the results of theoretical models of optimal regulation. In this respect, the present thesis is not overly audacious in its approach and claims. Moreover, it benefits from the rigor of thought imposed by formal analysis, in that its findings are not only logically consistent, but also altogether compatible with rational behavior, supposing appropriateness of the simplifications assumed by the model.

To conclude, this thesis sets out a vision for the future of concession design. Ideally, there should be an integration of theory and practice with respect to the actual design of contractual incentives. In one side, theory has insights to contribute on how to optimally balance the relevant economic tradeoffs inherent to the concession format, which, however, would certainly benefit from empirical approaches to validation and calibration. On the other, the very design of concession contracts should accommodate experiments of different mechanism configurations, and other devices that facilitate the empirical assessment of economic performance.

\textsuperscript{29} Though the robustness literature somewhat addressed that sort of criticism, in its pursuit of simple mechanisms that optimally arise in environments with less principal omniscience.

Bibliography


