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GABRIEL SINGLE TOLEDO

THE ROLE OF INFORMATION IN CONFLICTS

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Dissertação apresentada à Escola de Economia de São Paulo como pré-requisito à obtenção de título de mestre em Economia de Empresas.

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Resumo

Estudamos um modelo dinâmico de comunicação em conflitos, investigando como a divulgação de informação pode induzir equilibrios de paz mais frequentemente. Comunicação com múltiplos emissores sob Cheap talk pode ser informativa e previnir conflitos, induzindo ciclos endógenos de paz. Com *commitment* na divulgação de informação, emissores *short-lived* experimentam ganhos esperados maiores, já que paz pode ser induzida com maior probabilidade. No entanto, a distorção de mensagens pelos *designers* torna paz instável, fazendo-a durar apenas um período. O *trade-off* quando comparamos os dois protocolos de comunicação é que Cheap talk consegue induzir paz por longos períodos de tempo, enquanto que Information design o faz mais frequentemente. Quando os agentes são *short-lived*, por vezes será preferível, em termos de paz no longo prazo, que sociedades pós-conflito não sejam dotadas com poder de *commiment* na comunicação.

Palavras-chave: conflito, comunicação, cheap talk, desenho de informação.

Abstract

We study a model of dynamic communication in conflicts, investigating how information disclosure can induce peaceful equilibria more often. Multi-sender communication under Cheap talk can be informative and prevent conflicts, inducing endogenous cycles of peace. With commitment in information disclosure, short-lived senders experiment higher expected payoffs, as peace can be induced with higher probability. However, the distortion of messages by the designers makes peace unstable, lasting for only one period after the good message is sent. The trade-off when we compare both protocols is that Cheap talk communication can induce peace for *longer* periods of time, while Information design does it *more often*. When agents are short-lived, sometimes it will be preferable, in terms of long-run peace, that post-conflict societies are not endowed with commitment power in communication.

Keywords: conflict, communication, cheap talk, information design.

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1 Introduction

Is information transmission relevant in conflict onset and offset? Or even, how does the different protocols or forms of communication shapes current incentives for war? There exist some evidence that countries, groups or tribes seems to care about the informational content of post-conflict communication. A branch o literature from History, Psychology and Social Sciences works on documenting the role of information transmission in conflicts (Cairns and Roe (2002)). Through case-study evidence, these works can detect strategies of selective omission, fabrication, exaggeration and embellishment, implication of causal linkages, blaming the enemy or extraneous circumstances, and re-framing contextual factors; from different groups in different post-conflict scenarios.

Part of this literature focus on studying the impact of different *history textbook* design in post-conflict societies. Bentrovato (2017) and Bentrovato et al. (2016) surveys a extensive case-study literature that points to some evidence of concern and strategy of governments and other institutions, in trying to develop reconciling textbook material. This kind of policy, according to theses authors, have functioned as central instruments of nation building and citizenship formation. Combined with other sources of information, the language and content of textbooks seems to affect peace-building and future conflict onsets.

To illustrate and motivate we present some case-study examples, from the surveys, of societies that had adopted different policies of post-conflict communication. For instance, the 1989 Taif peace agreement, which ended civil war in Lebanon after fifteen years of conflict, explicitly urged the revision of school curricula, in order to strengthen national belonging, fusion, spiritual and cultural openness, and in a way that unifies textbooks on the subjects of history and national education, being a clear evidence of government concern about the informational content in conflict aftermath. Some other groups have opted to more evasive strategies, decreting *moratoria*, "the temporary suspension of history education or its recent history segment including its textbooks”. This measure was officially implemented, for instance, in Afghanistan, Bosnia and Herzegovina, Cambodia, Croatia, Guatemala, Lebanon, Libya, Rwanda and South Africa. Evasive strategies towards history textbooks have been dictated by concerns that, in the following moments of a violent conflict, legacies of violence pervasive, confronting the painful past may be too sensitive and may provoke controversy and commotion that could hamper inter-group reconciliation. Immediate Post-WWII, Allies banned militaristic and ultra-nationalist content from textbooks in Axis countries. Similar purge occurred in 1995 Dayton agreement, settled Bosnian conflict, by promoting the removal of “offensive or misleading” content from the largely ethnonationalist textbooks. More recently in Afghanistan and Iraq, post-

US invasion, the established regimes intervened by erasing propagandistic and militant textbook content, including Jihadist teachings.

Some groups made use of conciliatory strategies, for instance the Franco-German and German Polish textbook commissions that were established in the years that followed the WW-II. Their aim was to transcend narrow nationalists perspectives, which tend to perpetuate conflict. Single Narrative strategies were also adopted, for instance in Rwanda post-1994, when a hegemonic narrative from the Hutu group , dismissing ethnic differences and silencing memories of victimization from the Tutsi minority that were massacred, took place. This approach appears problematic as a conciliatory strategy, if one considers that the public dominance of particular narratives in society has frequently been itself an object of grievances that have adversely factored into the conflict. Also, with a single source of information, this official version of the history is often discredited by other groups, since there are incentives to distort the events toward a preferred view, as documented by the literature in the case of Rwanda.

In order to better understand the underlying incentives embedded in these case-studies examples, we develop a political economy model to study the role of information disclosure in conflicts. Rather than look just to *cross-group* communication, a more explored topic in economic literature, we investigate within group communication, mainly how intergenerational transmission of information, affects the presence or persistence of conflicts. Intuitively, the way that information disclosed, in the sense of how often the truth is told, may affect present beliefs about some conflict-relevant uncertain state of the world, shaping nowadays outcomes. Considering different frameworks we try to shed some light into the case-studies above, by assessing how the informational environment can affect the presence of conflict. We consider set ups of both Cheap talk (Crawford and Sobel (1982), Sobel (2013)) and information design (Kamenica and Gentzkow (2011), Bergemann and Morris (2017)) to provide contributions to the questions raised before.

In our model, a conflict game is to be played repeatedly by two groups, with short-lived players do not observing the costs of conflict, which is the state of the world that will pin down if peace can or cannot occur. The states are not permanent and will change over time following a Markov Chain. After the conflict stage game is played, if war has occurred, players will observe the state of the world, *before* it changes again; while if peace happened nothing is observed. This ingredient tries to capture the idea that a generation only learns the true costs of war if they lived through conflict. If in their lifetime, a society has not experienced war they cannot asses exactly the costs of it and will have to rely on past histories to construct their beliefs. We allow for players, after observing the state, to send a public message to the next generation that will born to play the conflict game. The short-lived agents will care only for gains in the two periods they are alive, in the first when they actually play the conflict game, and in the second when they only send messages and earn payoffs aligned to the next generation from the same group.

Within this framework we will try to answer two main questions. The first would be how can intergenerational communication, in either Cheap talk or Information design fashion, of short-lived player affects the existence of peace in equilibrium. The second point of this paper is to compare both types of protocols and try to determine which of them would produce more peace in the long-run.

We find out that Cheap talk with senders from both groups can implement a truth-telling strategy in equilibrium, one that fully reveals the states when war takes place. In this equilibrium, war follows the revelation of the bad states, while peace can be induced following a good state. We also have an *externality* effect, since a peace cycle may occur for several generations after the good state is once revealed. On the other hand, short-lived designers will split the priors they hold, as in the Kamenica and Gentzkow (2011), and will be able to always induce peace in the good states, but also sometimes when the bad states are shows up. However, incentives to lead to peace may be too strong for short-lived players, which will try to guarantee peace happening in the very next period. This incentives makes the information conveyed too noisy, so that peace becomes an unstable state, happening only once after the good message is sent.

So there it seems to exist a trade-off here. While Cheap talk with full-revelation induces peaces for *longer* periods of time, commitment under information design does induce peace *more often*, so that the comparison of these two outcomes might depend on the parameters of the model. We find out that if the persistence is sufficiently low, when even truth-telling would generate just one period of peace, information design would dominate. However, as the state becomes more persistent the *how long* effect would dominate the *how often* one, and Cheap talk would produce more peace in the long-run.

This article goes as follows. In Section 2 we review the recent literature of information and conflicts and the theoretical works on communication games; in Section 3 we present the primitives model, in Section 4 we present the findings of the model, with benchmarks and all it variations; and in Section 5 we conclude and pose further research steps. We leave some proofs and other extension results to the appendix.

2 Literature

We briefly review the economic literature of information and conflict, where our model most closely fits[1]. Our stage conflict game will resemble those from Baliga and Sjöström (2004), Baliga and Sjöström (2008), Baliga and Sjöström (2012), where an uncertainty in the costs of going to war may shift, enabling different static Nash equilibrium, for different values of the parameter. The focus of those models were to study cross group communication, understanding when mediated or unmediated talks between parties can avoid or propel conflict. Hörner et al. (2015) uses a mechanism design approach to study when parties may truthfully disclose information to a mediator, who seeks to design a mechanism that screens types in conflict. On a dynamic environment but with complete information Anderlini and Lagunoff (2005), Anderlini et al. (2010) construct *dynastic* games, with inter-generational transmission. In a slight different fashion, Acemoglu and Wolitzky (2014) study a dynamic model, when bounded information on the history of the game, may fuel cycles of conflict, even when both groups would benefit from peace.

We model information with costless transmission, that in principle could be dissociated with true states of nature. Our model is in a broad class of sender-receiver games, that we can divide in two branches of literature: Cheap talk, that follows from Crawford and Sobel (1982) and Information Design following Kamenica and Gentzkow (2011), Bergemann and Morris (2017)). The later abstracts from incentive compatibility issues, since senders can pre-commit to a certain rule, sending messages conditionally on states of the world; while the former, accounts for incentive compatibility, and equilibrium in transmission must have senders willing to follow the strategy, given the revealed state. You may find comprehensive and recent reviews for both topics in Sobel (2013) and Kamenica (2018), respectively.

A key difficulty in our scenario is that we have both multiple-senders and multiple receivers in our communication game. We believe that a valid way to model information transmission in conflict is to consider that, not only the receivers act strategically in the conflict game, but also the senders must take into account what kind of message the other group is sending. A father strategically telling his children about a past history may be worried about what information the other group is discolouring about the same event. That's why we need to consider this interaction in our model, which cannot be captured by single-sender models.

For Cheap talk with multiple-senders we have the classical works of Krishna and Morgan (2001) and Battaglini (2002). The first one shows that is beneficial for full-revealing

[1] For a comprehensive review on the topic, see Kimbrough et al. (2017).

to consult senders with opposite bias, while the second shows that full transmission is possible in general for multiple-senders in a multi-dimensional space. More recently, Takahashi and Ambrus (2008) extends the full-revealing results, under some conditions, for when the state space is bounded. McGee and Yang (2013) studies multiple-senders with some private information, and finds that if one sender is transmitting more information about his type, the others will have incentives to act complementary, and also send more. Finally, Schmidbauer (2017) studies multiple-senders with private information in a dynamic game, where senders must choose how much to say about their type in every period, while considering that this type might change. Even in this impermanent world, in general, an informative equilibria does exist.

There exists a very recent literature of Information Design with multiple-senders with Gentzkow and Kamenica (2016) and Gentzkow and Kamenica (2017), that characterize the set of equilibrium (under strong assumptions), and asses the impact of competition in information disclosure. Li and Norman (2015) and Li and Norman (2018), violates some assumptions from the previous ones, to investigate the impact in the set of equilibria, while Albrecht (2017) and Au and Kawai (2017) study zero-sum games, when senders have a private state of the world (quality), and want to persuade a decision-maker to vote for them. We believe that our work can contribute to this growing literature, as we develop a model of strategic interaction of senders, that may want to change the informational environment of a stage game.

A recent body of literature addresses information design in a dynamic environment. Kremer et al. (2014) analyses a long-lived social planner that wants to optimally disclose information to short-lived agents. The planner wants agents to experiment and produce new information, that could be useful to future players. With a similar motivation Che and Horner (2015) construct an optimal recommendation system, in which spamming occurs in equilibrium in order to players to experiment. Ely (2017) construct a time continuous model where an informed sender knows how a state variable evolve and wants to optimally persuade a receiver. Ely and Szydlowski (2017) develops a model of dynamic effort, where an informed designer optimally disclose information through time regarding the difficulty of a task, in order to induce optimal effort by an uninformed receiver.

In our work, both senders and receivers will be short-lived, but their actions might affect the informational environment of future generations, potentially affecting strategies and outcomes of the next players. Although the solutions of the sender-receiver games will have a static characterization, their decisions will echoes in other periods of time, making us want to analyze the chain of events that will follow a message being sent by a short-lived sender.

3 Model

We propose a model where two groups, A and B, interact in time, in an infinite horizon. At each time $t \geq 1$ a generation is born in each group and lives for two periods, dying at the end of $t+1$. We denote the generation born at t from group $i \in \{A, B\}$ as player i_t. Each generation will play the conflict stage game at the time t they are born, choosing whether a peaceful action P, or to engage in war W . At $t+1$, i_t do not play the stage game, only earn the payoff from the actions of both i_{t+1}. At the end of t, after the game is played, player i_t can send a public message $m_i \in M_i$ to the players at $t+1$, where M_i is a finite set of abstract messages for group $i \in \{A, B\}$. Notice the payoffs are perfect aligned within groups and across generations. The stage game is as follows

		Player B_t	
		W	P
Player A_t	W	$(-c^t, -c^t)$	$(b - c^t, -d)$
	P	$(-d, b - c^t)$	$(0, 0)$

In the model, $c^t > 0$ stands for the cost of war (resources, institutional costs) at time t, $b > 0$ is a "first"-mover advantage, $d > 0$ is the loss of being attacked without defending yourself. We adapt the stage game Baliga and Sjöström (2004), as the size of c^t will pin-down the stage Nash Equilibria (NE), illustrating the incentives to go to war for different costs of it.

If $c^t < d$ and $b < c^t$ then both (W, W) and (P, P) are (pure strategy) NE,and we are in a *Stag Hunt* game. We let c_h be a cost that satisfies, both conditions. Conversely, if $c^t < d$ and $b > c^t$, the unique stage-game NE is (W, W), and we are in a *Prisoner's Dilemma* game. We let c_ℓ be the cost of war satisfying both conditions, being easy to check that $c_h > c_\ell$. Here the intuition is quite straightforward, that when the costs of war are low enough the temptation to gain the first-move advanced together with the fear that the other player is doing the same prevents any kind of coordination on the Pareto Superior outcome of (P, P). When the costs of war rises the attacker advantage is reduced, allowing for coordination on peace. However the fear of being the sole loser of the conflict and gain $-d$ remains, making W a best response to W in this case as well.

We let $c^t \in \{c_h, c_\ell\} := \Theta$ be the unknown state of the world at time t. Also we assume the state evolve in time following a Markov chain with a "persistence" parameter $q >\in (1/2, 1)$, this is:

$$Prob(c^{t+1} = c | c^t = c) = q, \quad \forall c \in \Theta$$

Although the state is unknown for players i_t, they might learn it after the stage game is played. We assume that if some of the groups play W at t, then both groups will observe c^t, before it changes to c^{t+1}. However, if both groups play P in the stage game, c^t is not revealed, and players do not get any other information. We say that the observed state is $\tilde{c} \in \Theta \cup \{\emptyset\} := \mathcal{C}$, where $\emptyset$ denotes the case when players do not observe the state. The intuition behind this assumption is that when a generation goes to war they actually suffer the costs of it, learning which was the state of the world when conflict took place. Instead if a generation spends its lifetime in peace, nobody will know exactly how costly war was at that point. So a peaceful generation will be left "in the dark" regarding costs of conflict, and may want to rely on past history to asses the possibilities of different state of the world. This ingredient of only observing the state when war takes places is crucial for our model's dynamic, but also, it is a valid way of modeling intergenerational learning in conflict scenarios.

At the beginning of time t each group sees the history of past actions from both players. So history a of actions of length $t-1$, is a list $\alpha^{t-1} = \{a^\tau\}_{\tau=1}^{t-1}$, where $a^\tau = (a_A^\tau, a_B^\tau) \in \{W, P\}^2$ is the pair of actions that both players chose at time τ. We let $\mathcal{A}^{t-1}$ be the set of all possible action histories of length $t-1$, and $\mathcal{A}$, the set of all possible action histories of any length. Also, at each t both players sees the history of public messages $m^{t-1} \in M^{t-1} := (M_A \times M_B)^t$, a list of past messages of length $t-1$. A message $m_t = (m_{At}, m_{Bt})$ is a combination of individual messages from both groups that arrives at t.

We denote the history of the game at t as $h^{t-1} = (\alpha^{t-1}, m^{t-1})$, this is, a sequence of messages and actions. As before we denote H^{t-1} the set of all histories of length $t-1$, and H the set off all histories of any length. Sometimes it will be useful to talk about the history of the game within a period t, after the stage game is played, but before the message is sent. We refer to this *interim history* at t as a sequence of pairs of actions of length t, but a sequence of past messages with length $t-1$, with an usual history being $\tilde{h} = (\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t := \mathcal{A}^t \times M^{t-1}$.

3.1 Strategies

To define the strategies of each player i_t we must specify both what he chooses to *play* at the stage game in the beginning of t and what he *says* at the end of the period to the following generation. Therefore, we define an *action strategy* (in pure strategies) for i_t as a map:

$$a_i^t : H^{t-1} \longrightarrow \{W, P\}$$

This is, for every possible past history of actions at t and every possible history of messages, specify an action to be taken. We say that a profile of strategies at t is some $a^t = (a^t_A, a^t_B)$; an strategy for each group at t. In the same fashion a *profile of strategies* $a = (a^t)_{t\geq 1}$ specifies an action strategy for each i_t, $i \in \{A, B\}$ and $t \geq 1$.

Also, a *message strategy* for i_t is a map:

$$\pi^t_i : \tilde{H}^t \times \mathcal{C} \longrightarrow \Delta(M_i)$$

Specifying, for every history of actions of length t, every list of past messages until $t-1$, and every observed state (including $\emptyset$) a distribution probability over the set of messages for group i. We denote $\pi^t_i(m_i|c, (\alpha, \tilde{m}))$ as the probability of i_t saying $m_i \in M_i$ given that the interim history is $(\alpha, \tilde{m}) \in \tilde{H}^t$ and the observed state was $c \in \mathcal{C}$. Also, we let $\pi^t((m_A, m_B)|c, \tilde{h}) := \pi^t_A(m_A|c, \tilde{h})\pi^t_B(m_b|c, \tilde{h})$, be the joint probability of players saying the combined message $m = (m_A, m_B) \in M_A \times M_B$. And we let Π be the set of all possible joint message strategies. We define a *profile of message strategies* as $\pi = (\pi^t)_{t\geq 1}$, this is a message strategy for every i_t, $i \in \{A, B\}$ and $t \geq 1$.

3.2 Beliefs

We now define the belief functions of the state of the world. Since in our model the state is binary, one single probability measure will define our beliefs. We normalize it with respect to the probability of $c^t = c_\ell$, for every time t. One important aspect however, is to distinguish *ex-ante* beliefs, form *ex-post* beliefs for each i_t. The *ex-ante* belief at t is the probability of $c^t = c_\ell$, at the beginning of t. We denote this probability, for a given $h \in H^{t-1}$, as $\mu^t(h)$. So we let the belief that player i_t have at the beginning of the period depend on both past history of actions and past messages.

The *ex-post* belief at t is the one players have at the end of the period, regarding $c^{t+1} = c_\ell$, after the stage game is played and they observe $\tilde{c} \in \mathcal{C}$. We write this probability, for any $\tilde{c} \in \mathcal{C}$, $\tilde{h} \in \tilde{H}^t$, as $\tilde{\mu}^t(\tilde{c}, \tilde{h})$. Notice that we let the argument of function $\tilde{\mu}^t$ be of actions histories of length t, since it is calculated after the the stage game is played, taking into account the most recent pair of actions chosen. It is straightforward to use Bayes Rule to compute these *ex-post* beliefs, taken as given the *ex-ante* ones. We know for any $\tilde{h} \in \tilde{H}^t$:

$$\tilde{\mu}^t(\emptyset, \tilde{h}) = \mu^t(\tilde{h}_{-1})q + (1 - \mu^t(\tilde{h}_{-1}))(1 - q) \tag{3.1}$$

$$\tilde{\mu}^t(c_\ell, \tilde{h}) = q \tag{3.2}$$

$$\tilde{\mu}^t(c_h, \tilde{h}) = 1 - q \tag{3.3}$$

Where $\tilde{h}_{-1}$ denote the interim history $\tilde{h} \in \tilde{H}^t$ without the most recent entry of pair of actions and messages, this is, a history of length $t-1$ that is observed in the beginning of

t. The first one, is the one-step-ahead belief when no information is added, and we only use the parameter of the Markov chain to update it, considering the belief held at the beginning of t, $\mu^t(\tilde{h}_{-1})$. The second and third ones are also using the parameter of the Markov chain, but also with the fact that the true state c^t was revealed. In equilibrium, we will use Bayes rule to compute the beliefs as above.

To capture the ingredient that the revelation of the true state of the world depends on the action played in the stage game, we introduce an auxiliary function $\rho^t(c|\alpha, m)$ that gives us the probability of $c \in \mathcal{C}$ been *observed* by the players t, given any history of actions $\alpha \in \mathcal{A}^t$ (including the immediate past action), and past messages until $t-1$, $m \in M^{t-1}$. We can compute these probabilities given the *ex-ante* beliefs as below. For any $\alpha = (\tilde{\alpha}, a) \in \mathcal{A}^t$, with last pair of actions a, and $\tilde{m} \in M^{t-1}$:

$$
\begin{aligned}
\rho^t(\emptyset|(\tilde{\alpha}, (P,P)), \tilde{m}) &= 1 \\
\rho^t(c_h|(\tilde{\alpha}, a), \tilde{m}) &= 1 - \mu^t(\tilde{\alpha}, \tilde{m}), \quad \forall a \neq (P,P) \\
\rho^t(c_\ell|(\tilde{\alpha}, a), \tilde{m}) &= \mu^t(\tilde{\alpha}, \tilde{m}), \quad \forall a \neq \quad (P,P)
\end{aligned}
$$

This is, players at t know that if (P,P) is to be played, they will for sure observe $\emptyset$. However, if $a \neq (P,P)$, then players at t believe they will see c_ℓ with their *ex-ante* beliefs, this is, the probability they hold of the state being c_ℓ in the beginning of the period t.

We can now use Bayes Rule to get expressions for the *ex-ante* beliefs at the beginning of any $t+1$. For any $\pi^t \in \Pi$, $\alpha \in \mathcal{A}^t$ and $m = (\tilde{m}, m') \in M^t$, with $\tilde{m} \in M^{t-1}$ and $m' \in M$:

$$
\mu^{t+1}(\alpha, m) = \frac{\sum_{c \in \mathcal{C}} \rho^t(c|\alpha, \tilde{m}) \pi^t(m'|c, \alpha, \tilde{m}) \tilde{\mu}^t(c, \alpha, \tilde{m})}{\sum_{\tilde{c} \in \mathcal{C}} \rho^t(\tilde{c}|\alpha, \tilde{m}) \pi^t(m'|\tilde{c}, \alpha, \tilde{m})} \tag{3.4}
$$

This posterior belief will be an weighted average of the *ex-post* beliefs held by the previous generation. The weights take into account the probability of predecessors seeing each state and how often do they send the specified message at each state. The posterior takes this recursive format, depending on past beliefs. Given any profiles (a, π) and any prior belief in the beginning of the game $\mu^1(\alpha^0, m^0) := \mu_0 > 0$, where α^0, m^0 denote the initial empty history of actions and messages, one should be able to calculate all posteriors, using from (3.1) to (3.4).

It is useful to simplify (3.4) to two cases: One in that (P,P) was the last pair of actions, and the other where at least one player played W. For the first case, the belief using Bayes rule simplify to:

$$
\mu^{t+1}(\alpha, m) = \tilde{\mu}^t(\emptyset, \alpha, \tilde{m}) \tag{3.5}
$$

for Bayesian players, the belief at $t+1$ after no real state being reveled at t, must be the same that i_t holds at the end of the period. This in turn, is the one-step-ahead belief of i_t from the initial belief $\mu^t(\alpha_{-1}, \tilde{m})$ after no other information is obtained. Now, considering histories that end with actions other than (P, P), (3.4) can be written as:

$$\mu^{t+1}(\alpha, m) = \frac{\mu^t(\alpha_{-1}, \tilde{m})\pi^t(m'|c_\ell, \alpha, \tilde{m})q + (1 - \mu^t(\alpha_{-1}, \tilde{m}))\pi^t(m'|c_{h,}\alpha, \tilde{m})(1-q)}{\mu^t(\alpha_{-1}, \tilde{m})\pi^t(m'|c_\ell, \alpha, \tilde{m}) + (1 - \mu^t(\alpha_{-1}, \tilde{m}))\pi^t(m'|c_{h,}\alpha, \tilde{m})} \tag{3.6}$$

Notice that using Bayes rule to calculate the beliefs, players try to infer from the message received which was the *ex-post* belief held by their predecessor, since it is the belief with the most recent piece of information available. This posterior calculation leads us to our very first result, that communication of any form cannot be *informative* for stages where the past observed state was $\emptyset$. We use the notion at Sobel (2013) of not informative messages as the ones that cannot affect beliefs. The lemma follows immediately from beliefs calculation of (3.1) to (3.5).

Lemma 3.1. *At every t, we have that messages are not informative for Bayesian players when the last observed state $c^{t-1} = \emptyset$. Formally, for any interim history $(\tilde{\alpha}, \tilde{m}) \in \tilde{H}^{t-1}$ where $a^{t-1} = (P, P)$, we must have that, $\forall m \in M$, $\mu^t(\tilde{\alpha}, (\tilde{m}, m)) = \tilde{\mu}^{t-1}(\emptyset, \tilde{\alpha}, \tilde{m})$.*

The above lemma implies that messages cannot contain any information about the state of the world when senders do not have any new private information. Since all players observe past actions, players know whether or not the past generation has learned the state at their time. In the case of (P, P) nothing is discovered, so Bayesian players knows that messages cannot contain anything new that will affect the beliefs, and will only use the Markov chain parameter to compute the one-step-ahead posterior. This result will simplify our analysis, since we will only need to specify the message strategies for players that are capable of communication, this is after histories where the last observed state was different than $\emptyset$.

3.3 Payoffs and Equilibrium Concepts

We now describe the expected payoff of each player i_t, both for his own period where he plays the stage game and for the next, where the next generation will play. In this article we assume that players are myopic in the sense that when they are deciding what to play at the stage game they do not take into account how the actions will affect the messages he will send at the end of the period. We are with this assumption separating the stage-conflict game from the sender-receiver game. This will simplify our analysis, while we will have only to account for deviations separately in stage game and in the communication game.

We add some convenient notation. We let $u_i(a;c)$ be the stage game utility for $i \in \{A, B\}$ at any time t where the pair $a \in \{W, P\}^2$ is played and the true state of the world is $c \in \Theta$. We define, for a generic belief held $\mu \in [0,1]$ the function:

$$\mathrm{E}u_i(a;\mu) = \mu.u_i(a;c_\ell) + (1-\mu)u_i(a;c_h)$$

So we can write player i_t expected payoff at the stage game t, given a history $h =\in H^{t-1}$, a profile of action strategies a and a profile of message strategies π as $\mathrm{E}u_i\left(a^t(h);\mu^t(h)\right)$.

The payoffs regarding the sender-receiver game will be presented in to ways. The first one is closely related with a Cheap talk environment, and is the expected payoff for a sender at $t \geq 1$ *conditional* on having observed the state $c \in \mathcal{C}$. So for any interim history $\tilde{h} = (\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t$, and profile of actions a and a profile of message strategies π we have:

$$\sum_{m \in M} \pi^t\left(m|c,\tilde{\alpha},\tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha},(\tilde{m},m)\right);\tilde{\mu}^t\left(c,\tilde{\alpha},\tilde{m}\right)\right) \tag{3.7}$$

Thus, conditional on a specific observed state, sender expected utility will take into account the probability of each message being sent and the action the $t+1$ receiver will choose after observing the history with the new message. Note, however, that the belief that weights the gain from the $t+1$ stage game is the *ex-post* belief at t regarding c^{t+1}, since it is the posterior held by the sender given what he last observed at t.

We can take a step back to compute the *ex-ante* expected belief, the one obtained before the state is observed but after the stage game at t happens. We write, for any interim history $\tilde{h} = (\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t$, and profile of a and a profile of message strategies π:

$$\sum_{c \in \mathcal{C}} \rho^t\left(c|\tilde{\alpha},\tilde{m}\right)\left[\sum_{m \in M} \pi^t\left(m|c,\tilde{\alpha},\tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha},(\tilde{m},m)\right);\tilde{\mu}^t\left(c,\tilde{\alpha},\tilde{m}\right)\right)\right]$$

It is obtained by weighting the expected payoff at each state, given by (3.7), by the probability of observing each state conditional on the interim history. We are now ready to state our equilibrium concepts, which will depend mainly on the assumption we make on how communication is made in our model. First, we consider communication as in Cheap talk models (Crawford and Sobel (1982)), where senders do not have any commitment power, so that in equilibrium we must account for incentive compatible strategy of messages. In our model, it means that all players i_t, *after* observing the revealed state $c \in \mathcal{C}$ must have incentives to follow his own strategy, taking as given the strategy from the other group's sender. This together with i_t do not having incentives to deviate in the stage game at t and beliefs following Bayes rule gives us the notion of Perfect Bayesian Equilibrium (PBE) for our Cheap talk game. We formalize below.

[Cheap talk Equilibrium] A pair of profiles (a, π) is a Cheap talk Equilibrium (CTE) if:

1. For every $t \geq 1$, and every i_t, for any initial $h \in H^{t-1}$, and $\forall a' \in \{W, P\}$

$$\mathrm{E}u_i\left(a^t(h); \mu^t(h)\right) \geq \mathrm{E}u_i\left(a', a^t_{-i}(h); \mu^t(h)\right)$$

2. For every $t \geq 1$ and every i_t, for all observed $c \in \mathcal{C}$, for any initial $\tilde{h} = (\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t$ and $\forall\ \tilde{\pi}^t_i \in \Pi_i$, with $\tilde{\pi}^t = (\tilde{\pi}^t_i, \pi_{-i})$ as the joint signal:

$$\sum_{m \in M} \pi^t\left(m|c, \tilde{\alpha}, \tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha}, (\tilde{m}, m)\right); \tilde{\mu}^t\left(c, \tilde{\alpha}, \tilde{m}\right)\right) \geq$$

$$\sum_{m \in M} \tilde{\pi}^t\left(m|c, \tilde{\alpha}, \tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha}, (\tilde{m}, m)\right); \tilde{\mu}^t\left(c, \tilde{\alpha}, \tilde{m}\right)\right)$$

3. For every, $t \geq 1$, $h \in H^{t-1}$, $\tilde{h} \in \tilde{H}^t$ beliefs $\mu^t(h)$ and $\tilde{\mu}^t(c, \tilde{h})$ are computed following Bayes Rule as in (3.1) to (3.4), on equilibrium path.

Our second way to model communication in our game is to follow the Bayesian Persuasion literature, so as in Kamenica and Gentzkow (2011) and Bergemann and Morris (2017). Now we abstract from incentive compatibility for senders in every period, and act as if players i_t could commit themselves to communicate in a certain way after the stage game is played but *before* the state at t is observed. This is, they choose π^t_i before any $c \in \mathcal{C}$ is revealed.

We define an Information Design Equilibrium (IDE) using the commitment power of senders.

(Information Design Equilibrium) A pair of profiles (a, π) is an IDE if:

1. For every $t \geq 1$ and every i_t, for any initial $h \in H^{t-1}$, and $\forall a' \in \{W, P\}$

$$\mathrm{E}u_i\left(a^t(h); \mu^t(h)\right) \geq \mathrm{E}u_i\left(a', a^t_{-i}(h); \mu^t(h)\right)$$

2. For every $t \geq 1$ and every i_t, for any initial $(\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t$ and $\forall \tilde{\pi}^t_i \in \Pi_i$, with $\tilde{\pi}^t = (\tilde{\pi}^t_i, \pi_{-i})$ as the joint signal:

$$\sum_{c \in \mathcal{C}} \rho^t\left(c|\tilde{\alpha}, \tilde{m}\right)\left[\sum_{m \in M} \pi^t\left(m|c, \tilde{\alpha}, \tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha}, (\tilde{m}, m)\right); \tilde{\mu}^t\left(c, \tilde{\alpha}, \tilde{m}\right)\right)\right] \geq$$

$$\sum_{c \in \mathcal{C}} \rho^t\left(c|\tilde{\alpha}, \tilde{m}\right)\left[\sum_{m \in M} \tilde{\pi}^t\left(m|c, \tilde{\alpha}, \tilde{m}\right) \mathrm{E}u_i\left(a^{t+1}\left(\tilde{\alpha}, (\tilde{m}, m)\right); \tilde{\mu}^t\left(c, (\tilde{\alpha}, \tilde{m}\right)\right)\right]$$

3. For every, $t \geq 1$, $h \in H^{t-1}$, $\tilde{h} \in \tilde{H}^t$ beliefs $\mu^t(h)$ and $\tilde{\mu}^t(c, \tilde{h})$ are calculate following Bayes Rule as in (3.1) to (3.4).

Notice that in the second point of the definition, the incentive compatibility constrains of senders do not condition on every observed state $c \in \mathcal{C}$. This is due to the commitment power of the senders, where they design their communication rule before the state is revealed.

4 Results

4.1 One-shot Model

As a first simple benchmark, let's suppose the game is played only once, with no communication and with players do not observing the initial state, only holding a prior $\mu_0 > 0$. We can now find the set of all possible Bayes Nash Equilibria (BNE) from this game, for all possible values of the prior. It is straightforward to see that, if the other player is playing W, the best response is to play W, for any $\mu_0 \in [0, 1]$, since $(W.W)$ is a stage game NE in both states. So we will have that $(W.W)$ is a BNE for any value $\mu_0 \in [0, 1]$. Now, if the other player is playing P, there will be a cutoff belief, that induces others equilibrium. We can see that the player $i \in \{A_1, B_1\}$ indifference condition, for an arbitrary belief $\tilde{\mu}$, when the other is playing P is:

$$\tilde{\mu}(b - c_\ell) + (1 - \tilde{\mu})(b - c_h) = 0$$

$$\mu^* := \frac{c_h - b}{c_h - c_\ell} \in (0, 1) \tag{4.1}$$

So when the belief held is smaller than $\mu^* \in (0, 1)$, players can coordinate to achieve (P, P) in equilibrium, earning the higher payoff of 0. We can now describe the set of (pure strategy) BNE as given by:

$$\Big\{((W, W), \mu) : \mu \in [0, 1]\Big\} \bigcup \Big\{((P, P), \mu) : \mu \leq \mu^*\Big\}$$

The stage game already gives us the flavour the we are always going to be dealing with multiple equilibria, since the trivial equilibrium of war, for any belief, will always exists. If we consider the game without communication, it will depend on the prior probability μ_0. It is straightforward to see that if $\mu_0 > \mu^*$, there does not exist an equilibrium profile of strategies in that both groups play P. The senders expected payoffs are then:

$$\mu_0(-c_\ell) + (1 - \mu_0)(-c_h) < 0 \tag{4.2}$$

If $\mu_0 \leq \mu^*$ it's possible to induce (P, P), since playing P is a best response to P for both players. In this case, the expected gain for both senders is 0. Also, in this case, (W, W) can also be induced in equilibrium, since a best response to the other player playing W is also W. Then again, the expected gain is given by (4.2). It the following sections we are going to look for conditions where, even if the prior is not favorable, $\mu_0 > \mu^*$, communication may effectively lower the beliefs, possibly inducing peace on equilibrium path.

4.2 First Results and Benchmarks

We first present a straightforward result that greatly simplifies our analysis, when we focus in pure strategies of myopic agents. Since histories are public, both in terms of past actions and past messages, beliefs will be public, so both Bayesian players will hold the same belief after any history. This, together with the assumption of myopic agents, will force players to play BNE in every stage game, making them coordinate in either peace or war in every period. This simplify our game, because now all that matters are payoffs from (P, P), which are state independent, and from (W, W). Also, we know that any deviation from the senders do not allow for a unilateral deviation of the receivers from either group, since they are all holding the same belief. We thus, have the following lemma. All omitted proofs are in appendix.

Lemma 4.1 (Coordination among Receivers)**.** *Let (a, π) be a (pure-strategy) IDE or CTE with belief function $\mu^t(h)$, for every, $t \geq 1$ and every possible history $h \in H^{t-1}$. Then we must have that $\Big(a^t(h), \mu^t(h)\Big)$ is a Bayes Nash Equilibrium for all $h \in H$ on equilibrium path, which in turn implies that $a^t_A(h) = a^t_B(h)$.*

This simple result greatly simplify our analysis, since now a public signal will always guarantee that receivers are coordinating around either (W, W) or (P, P) when we focus on pure strategies. It implies that on equilibrium we don't have to worry with a sender trying to induce his respective receiver to deviate from the play of the opposite receiver. The public message throws both receivers at the same information set, with a common posterior, making this public belief a coordination device around a symmetrical pure-strategy Nash-equilibrium.

Before we proceed one word on the existence of equilibrium. As in the vast Cheap talk literature, existence is not an issue in our model, since we can always find the babbling equilibrium, one in which senders say non informative messages (say the same arbitrary message at each state) and receivers just ignore the message, turning to the history of actions to compute beliefs. A goal of this article is to characterize equilibrium beyond the babbling, outcomes in which messages can be sometimes informative and change how receivers play after some histories. In a fully babbling equilibrium the outcome of the game is identical to one of *no-communication* presented below.

4.2.1 No Communication

The first benchmark we describe in the full dynamic environment is the one without any communication, so that players i_t will only observe past history of plays when computing beliefs. This benchmark it useful to understand some assumptions on parameters that will make the model more interesting for the next sections. First, recall that the threshold μ^* is key to pin down whether peace can be played in equilibrium. Recall that we are assuming that agents are myopic and play the stage game only considering the belief they held at the time. Also, since we are looking for upper-bounds of peace, let's look at the most optimistic scenario, that whenever both groups have a belief $\tilde{\mu} \leq \mu^*$ they will both coordinate on P. Given this assumptions, how this benchmark plays out?

First, let's pin down how the beliefs evolve without any information being transmitted. At $t = 1$, we know the belief is the primitive prior of the model μ_0. At $t = 2$, after no information, we use the Markov matrix to compute the belief $\mu^2 = q\mu_0 + (1-q)(1-\mu_0)$ which is the one-step-ahead belief after no information. actually, we can find an expression for the posterior at any time t. We do this by finding the eigenvalues of our transition matrix Q, and decompose it as below

$$Q := \begin{pmatrix} q & 1-q \\ 1-q & q \end{pmatrix} = U \begin{pmatrix} 1 & 0 \\ 0 & (2q-1) \end{pmatrix} U^{-1}$$

Where U is the eigenvector matrix. It is easy to see that the n-step transition of Q will be:

$$Q^n = U \begin{pmatrix} 1 & 0 \\ 0 & (2q-1)^n \end{pmatrix} U^{-1}$$

So the belief at any time $t \geq 1$ will be given by:

$$\mu^t = A + B(2q-1)^t$$

Where A and B are constants that can be found as function of the initial condition μ_0. Substituting we have:

$$\mu^t = \frac{1}{2} - \frac{1-2\mu_0}{2(2q-1)}(2q-1)^t \tag{4.3}$$

Notice that as $t \to \infty$, $\mu^t \to 1/2$ for any $q \in (1/2, 1)$. Whether it goes decreasing or increasing to this limit depend if the prior is above or below $1/2$. Now it gets straightforward to analyze some cases.

First suppose that the prior is favorable for peace: $\mu_0 \leq \mu^*$. So in the first period (P, P) will be played. Now if $\mu_0 > 1/2$ from (4.3) we know the belief decreases over time, still being lower than μ^*, so peace will go on forever. Otherwise if $\mu_0 < 1/2$ belief will increase over the periods, and we now depend on the size of μ^*. If it is also smaller than $1/2$, there will exist a finite time in which the posterior will get above the threshold and henceforth (W, W) will be played forever. In the case that the threshold is above $1/2$, the posterior will always stay below it, and peace will endure in all periods. None of theses cases seems to be the most interesting, since either peace will happen forever, so that communication cannot improve it, or the game will start in peace for some periods, making communication useless at the beginning of the game. If we are interested in assessing how can communication affect peace from the start of the game onward, theses assumptions may not bet the best way to describe the model.

Now if the prior is unfavorable $\mu_0 > \mu^*$, we can decompose in similar cases. If $\mu_0 > 1/2$ and $\mu^* < 1/2$, the belief decreases but never goes below the threshold, so war will occur in every period without communication, while if $\mu^* > 1/2$ peace will onset in finite time and will last forever. The former case seems to be the most interesting if we are focusing on understanding how much can communication lead to peace-building. It is a worst case scenario, where the prior is high and the threshold is not achieved by the decreasing posteriors, so that with out any further communication, two groups are at war in every period of time. We will keep this assumption in the remainder of the article[1].

Assumption 4.1. (Unfavorable Prior and Low Threshold)

We assume that the prior is unfavorable for peace, $\mu_0 > \mu^*$ and that the threshold is low enough, $\mu^* < 1/2$.

4.2.2 Full-revealing

We now proceed to another useful benchmark of full revelation of past states under war. Here we assume that both players at t perfectly observes the past true states of the world if at least one player played W in that period. So at t players see as sequence of $\{c^\tau\}_{\tau=1}^{t}$, where $c^\tau \in \mathcal{C}$, which may include some $\emptyset$ for periods when (P, P) has been played. It is easy to see that under this benchmark communication is not informative, since any sender at t, after the observed state $c^t \in \mathcal{C}$ is revealed has the same information as the $t+1$ players. The past states are common knowledge and any message strategy will not affect how beliefs are computed, since they do not add any piece of information. So messages can be disregarded and the history of the game can be summarized by the sequence of observed states.

With slight abuse of notation, we denote the history at t as $h^t = \{c^\tau\}_{\tau=1}^{t-1}$ and $\mu^t(h^t)$ as the belief held at t after history h^t. Again, we focus on equilibrium where at any time t all agents i_t are using a cutoff strategy μ^*, for any belief $\mu^t(h^t)$ they hold. Note, also, that the fact that $q \in (1/2, 1)$, together with Assumption 4.1 guarantees that $q > \mu^*$. Let's see how the equilibrium unfolds.

At $t = 1$, given the prior μ_0, (W, W) will be played, so the real state will be revealed. Then at $t = 2$, if $c^1 = c_\ell$ the posterior will be $\mu^2(c_\ell) = q > \mu^*$, so war will be played again. However, if $c^1 = c_h$ the posterior will be $\mu^2(c_h) = 1 - q$, which we do not know how it compares with μ^*. If we assume $1 - q > \mu^*$ then war will follow, even after the good

[1] One case the was left aside is when $\mu_0 = 1/2$. It is clearly a not interesting, since beliefs to not change overtime and either peace or war will occur in every period depending on the size of μ^*

state is revealed, so peace will never happen in equilibrium. More interesting would be to assume $1-q<\mu^*$, so peace would occur following a good signal. We use this assumption in the reminder of the paper.

Assumption 4.2. (Peace following the Good State) We assume, $1-q<\mu^*$, which implies that peace may occur for at least one period if c_h was revealed in the previous play.

Now, at $t=3$, if the history is $h^3=(c_h,\emptyset)$ the belief can be computed, using the Markov chain parameter, q, and the fact that two periods ago c_h was revealed. So $\mu^3(h^3)=2q(1-q)$, which is the one-step-ahead belief after no information at $t=2$. Since $q\in(1/2,1)$ we know that the belief increases after no information, so $2q(1-q)>(1-q)$. Then, if $\mu^3(h^3)>\mu^*$ war occurs at $t=3$, and peace only lasted one period. Otherwise, if it is smaller, then peace will occur for two periods in a row. We now turn to compute how long peace will last after the good state was revealed. It will last as long as the posteriors, remain below the threshold μ^*. Using a similar reasoning as to obtain equation (4.3), but now with initial belief $(1-q)$, we can write those posteriors at t following any history $h_{\tau-1}$ of the form:

$$h_{\tau-1}=(\cdots,c_h,\underbrace{\emptyset,\cdots,\emptyset}_{\tau-1\quad times})$$

As:

$$\mu^t(h_{\tau-1})=\frac{1}{2}-\frac{1}{2}(2q-1)^\tau,\forall\ \ \tau\in\mathbb{N}^* \tag{4.4}$$

Now we can find, given the parameters q and μ^*, for how many periods can peace occur in a row. First we find the real number n such that:

$$\frac{1}{2}-\frac{1}{2}(2p-1)^n=\mu^*$$

Isolating n we get:

$$n=\frac{\ln(1-2\mu^*)}{\ln(2q-1)}>1 \tag{4.5}$$

So, since time is discrete we must round down n, using a floor function, and find that the length of periods playing peace will last for τ^*, given by

$$\tau^*=\lfloor n\rfloor=\left\lfloor\frac{\ln(1-2\mu^*)}{\ln(2q-1)}\right\rfloor \tag{4.6}$$

So players that observe a history such as h_{τ^*} will hold a belief $\mu^t(h_{\tau^*})>\mu^*$, by construction, and will therefore play W. In a similar fashion as in Acemoglu and Wolitzky

(2014), our model presents endogenous cycles of war and peace. Their cycles were driven by bounded memory of players whch made made the evolution of the beliefs to depend heavily on time calendar. In our model all that matters is the last time the good state is revealed. It lowers the posterior below the threshold, staying in that region for some finite period of time. Given our assumption over the parameters, the lower initial belief will increase over time, when information is not revealed, towards 1/2. This means that the longer the period of peace, which are periods of no information, more probable it is that the bad state is back, increasing the incentives to play war, up to the point in which the posterior surpasses μ^*, an war becomes a dominant action in the stage game once again.

We summarize the results in the following proposition.

Proposition 4.1 (Full-revealing Equilibrium)**.** *In the game with full-revealing, there exists an equilibrium a profile of action strategies a (disregarding messages) with the following properties:*

1. *For every $t \geq 1$, and every history $h^{t-1} \in H^{t-1}$, myopic agents $i \in \{A, B\}$ will play the following strategy;*

$$a_i^t(h^{t-1}) = \begin{cases} W & \text{if} \quad \mu^t(h^{t-1}) > \mu^* \\ P & \text{if} \quad \mu^t(h^{t-1}) \leq \mu^* \end{cases}$$

2. *At every $t \geq 1$, if the observed state $c^{t-1} = c_\ell$ then war occurs in t;*

3. *At every $t \geq 1$, if the observed state $c^{t-1} = c_h$ then peace occurs in t and for the $\tau^* - 1$ following periods;*

4. *At every $t \geq 1$, with a history of the form h_τ, $\tau \in \mathbb{N}$, players updates beliefs as in (4.4), and play accordingly to the strategy in item (i).*

4.3 Cheap talk

With our benchmarks defined, we can explore now how does Cheap talk communication compares with them. Here we are interested in protocols of communication that are incentive compatible, in the sense that, in equilibrium, senders, after observing an state, do not want to deviate from the proposed strategy of messages.

4.3.1 Single-Sender Benchmark

Following Lemma 4.1, we can construct a *single-sender* benchmark, assuming that there is only one sender, with 'monopoly' of the information transmission. This unique sender will derive utility from an arbitrary receiver, say A. Since the game is symmetrical, focusing on just one group is without loss of generality. Actually, in our set up, the single-sender analysis can be seen as a social planner analysis. By Lemma 4.1 we know that receivers are always coordinating either around (W, W) or (P, P), and a sender with public messages cannot induce unilateral deviations by any receivers. So we can achieve the same results if the single sender is from A or B or even if he is a mediator that cares about the sum of payoffs. In this interpretation, the sender is trying to move from (W, W) to a (P, P) outcome, that would be Pareto improving, by credibly convening information about the state of the world.

Formally, at every t the sender chooses a message strategy π_S^t from Π_S, following the same notation and timing from previous sections. To check for equilibria, we just need to check incentive compatibility for the sender, and receivers best responding to each other. Babbling is still an equilibrium, with receivers always playing W. In fact, what we can show is that a single sender cannot be informative about the state of the world at any time, so they are not improving upon a babbling outcome. The result is in the proposition below.

Proposition 4.2 (Single Sender cannot induce peace)**.** *Let (π_S, a) be a CTE with a single sender. Then, we must have that for each history on equilibrium path $h \in H$, $a(h) = (W, W)$.*

The intuition behind this result is that, the lack of *any* commitment device prevents the sender from inducing peace, even when the state is good c_ℓ. The best sender can do, is just the same as the *no-communication* outcome. If there were a message that would induce peace for the receivers, sender would have incentives to send it with probability one, at each state. This in turn, cannot be an equilibrium, since the messages would not be changing receivers beliefs after any history, making (P, P) not an equilibrium outcome.

The lack of commitment makes the single sender want to *inflate* any message that would induce peace, discrediting, therefore, any attempt of communication by the eyes of the receivers. Tough it is a quite simple result, it is somewhat counter intuitive. Even the payoffs being perfectly aligned between the sender and receivers, one cannot, in this Cheap talk model, expect any kind of communication, since sender would always have incentives to exaggerate, and promote peace more often then what would be credible to the receivers, making them disregard the message and update the belief as if no information was added.

4.3.2 Two Senders Cheap talk

We now go back to a scenario with two senders, and ask ourselves if the disappointing results from last subsection still holds. By adding a sender, representing the other group we might open the possibility for divergent messages being sent at a given equilibrium, maybe contributing to a result of no communication. On the other hand, the additional source of information will make the receivers strategy *richer*, since they can now observe deviations from senders, and maybe create strategies to avoid divergent messages.

This discussion leads to the first positive result, that a *truth-telling* strategy is an equilibrium, with two senders in the game. This kind of strategy is one that produce messages that fully reveal the state of the world when at least one player has played war in the past period.

We assume that $|M_i| \geq 3$ for both $i \in \{A, B\}$

Proposition 4.3. *Let (a, π) be such that:*

1. *For every $t \geq 1$ with any history $\tilde{h} \in \tilde{H}^t$, and for both $i_t \in \{A_t, B_t\}$, $\pi_i^t(m_i'|c_\ell, \tilde{h}) = \pi_i^t(\tilde{m}_i|c_h, \tilde{h}) = \pi_i^t(\bar{m}_i|\emptyset, \tilde{h}) = 1$, for some $m_i' \neq \tilde{m} \neq \bar{m}$. We label $m_i' = w$, $\tilde{m}_i = p$ and $\bar{m}_i = m_\emptyset$.*

2. *For both $i_t \in \{A_t, B_t\}$, for any history $h \in H^{t-1}$;*

$$a_i^t(h) = \begin{cases} P & \text{if} \quad \mu^t(h) \leq \mu^* \\ W & \text{otherwise.} \end{cases} \tag{4.7}$$

3. *For every $t \geq 1$ and every history on equilibrium path $h \in H^{t-1}$, the belief function $\mu^t(h)$, follows Bayes rule. For histories h' out of equilibrium path, we set belief function $\mu^t(h') = \mu_0$.*

We claim that (a, π) is a CTE with belief function μ^t.

Proof. First we check if the receivers want to follow their strategy at t. Notice taht only (p, p), (w, w), $(m_\emptyset, m_\emptyset)$ are equilibrium messages. Applying Bayes' Rule we get $\mu^{t+1}(\tilde{h}, (w, w))) = q$ and $\mu^{t+1}(\tilde{h}, (p, p)) = 1 - q$. So at (w, w) given the posterior and that B_t is playing W, the best response is to play W. Similarly at (p, p) given the posterior, which is lower than the threshold μ^*, and that B_t is playing P the best response is to play P as well. If receivers get $(m_\emptyset, m_\emptyset)$ and observe that (P, P) was played, they proceed to look at the past history $\tilde{h}$, to compute the belief after a sequence of peaces. In the fashion on subsection 4.2.2, we can compute the beliefs after a finite sequence of (P, P), following (4.4), and finally asses whether is is larger or smaller that μ^*. With any out of equilibrium message, we would have an imposed posterior of μ_0, and given that B_t is playing W, A_t best response is to play W as well, since (W, W) is a BNE for any belief.

Now we proceed to check for sender's incentives. By symmetry we just need to check if, for some t the sender A_t want to deviate, conditional on the observed state of world. Fix a $t \geq 1$ and an interim history $\tilde{h} \in \tilde{H}^t$. As before if the observed state is $c^t = \emptyset$, no message can be informative, since senders do not have any additional information, so there is no incentive to deviate, since all messages produce same posteriors, as in Lemma 3.1. If $c^t = c_h$ the expected payoff for A_t to say p will be 0, since B_t is saying p with probability 1. This is strictly greater than the expected payoff from deviating to any $m_A \in M_A$ with $m_A \neq p$, which is $q(-c_h) + (1-q)(c_\ell) < 0$. Notice that any deviation of the form (m_A, p) would lead to receivers getting to out-of equilibrium histories, where they both play W, given the specified strategy. So A_t would prefer to say p with probability 1 at $c_h \Rightarrow \pi^t_A(p|c_h, \tilde{h}) = 1$

At $c^t = c_\ell$, a similar reasoning applies. The expected payoff of saying w is $q(-c_\ell) + (1-q)(-c_\ell)$. But then, he cannot gain by deviating, to any $m_B \in M_B$, since then receivers would play W regardless the deviation. So he is indifferent between any sending message, when B is playing $\pi^t_B(w|c_\ell, \tilde{h}) = 1$. In particular, when, both are playing in this way neither wants to deviate. □

The intuition behind this result it that, with more than one source of information, receivers are able to construct more sophisticated strategies, that allows the cross-check of messages from both sender. Since messages are public and coordination is guaranteed in equilibrium, receivers would want to hold into account, not only the rival sender, but also the one from his own group, punishing deviations from any of them. When both receivers are acting like this, in a form of *collusion toward punishment*, we should expect that both senders do not want to deviate from telling the truth, since any deviation would be detected, resulting in payoff loss. This additional source of information may, therefore, act as a commitment device, that align incentives to tell the true, a result that was not achieved with just one source, in the single-sender benchmark.

Also, this result implies that with multiple-senders we can implement the result in Proposition 4.1 in equilibrium. So war will follow whenever the bad state is observed, while revealing the good state c_h, allows for several periods of peace in a row, which is a clear improvement over the single sender benchmark, where war happened every period of time. In this multiple sender equilibrium, we get a flavour of a *good externality*, since revealing the good state do not only guarantees a higher payoff for the short-lived senders, but also does it for maybe several other generations, that will ride in the peace cycle as long as it lasts. In this kind of equilibrium, it may be hard to enter in the cycle of peace, since the prior μ_0 is high and the state tends to persist, as q is high too. But, once a good state is observed, the cycle is onset, and longer it will last the higher q is, as can be seen in τ^* expression (4.6). The higher the persistence in the model, the harder it will be to enter in the cycle of peace, since c_ℓ may last for many periods. However it benefits the length of the cycle of peace, since the higher q, the lower is the belief which the agents enter in the cycle, $(1-q)$. Bayesian players will infer, given the high persistence that the revealed good state is expect to persist for some time, such that in expected terms they can coordinate on peace for several periods.

This kind of equilibrium, where there is full revelation with multiple-senders, is a well documented feature in Cheap talk models (Sobel (2013)). In our case, this result would provide payoffs that are an upper bound to equilibrium *ex-ante* expected payoffs at each t, when $\mu_0 > \mu^*$. The following proposition, addresses this.

Proposition 4.4 (Limits to Cheap talk Communication)**.** *Let (a, π) be an (pure-strategy) CTE with belief function μ^t. We must have that, for any $t \geq 1$ and history of actions $\alpha \in \mathcal{A}^t$ and past messages $\tilde{m} \in M^{t-1}$, if there exists a message $m = (m_A, m_B) \in M$ such that $a^{t+1}(\alpha, (\tilde{m}, m)) = (P, P)$, then we must have that $\pi_i^t(m_i|c_\ell, \alpha, \tilde{m}) = 0$ for both $i_t \in \{A_t, B_t\}$.*

This result implies that if any sender says any message with positive probability when c_ℓ is revealed at a certain time t, it cannot induce (P, P) in the receiver's game in $t + 1$. Therefore, when the state is bad, Cheap talk communication can never induce peace, implying that there exists some bounds to what one may expect to gain at the communication game with multi-sender Cheap talk. The following corollary is immediate.

Corollary 4.1. *In any CTE, at any t with interim history $\tilde{h} \in \tilde{H}^t$ with last pair of actions $a^t \neq (P, P)$, senders ex-ante expected payoffs are bounded from above by $\mu^t(\tilde{h}_{-1})(q(-c_\ell + (1 - q)(-c_h))$.*

So, to wrap-up, the Cheap talk communication may improve the outcome when compared to no communication, with unfavorable beliefs $\mu_0 > \mu^*$. There exists a full-revealing equilibrium, where peace is achieved when the state of the world is good, increasing the gain relative to the babbling outcome. But there is an upper-bound to this gain, in the sense that in equilibrium we cannot induce both receivers to play peace (the Pareto-Superior outcome) when the state is bad (c_ℓ). The senders cannot *collude* to sometimes induce their offsprings in playing peace, when the state of the world is in fact a prisoners dilemma. This gives us the idea that might be quite hard to induce peace since, senders must rely on the good state being revealed. However, as presented above, this full revelation strategy will induce cycles of peace once the good state is observed, so that several following generations will benefit from peace, without having observed any new information, just gaining from the externality that full revelation promotes.

If both senders could commit themselves to, just some times, try to induce peace when the observed $c^t = c_\ell$, maybe one could expected higher payoffs being achieved in the sender-receiver game, since peace will induced more often. This kind of commitment issues will be addressed in the following section.

4.4 Information Design

We present now a modified model following the literature of Bayesian persuasion and information design (Bergemann and Morris (2017) and Kamenica (2018) for complete reviews), being closely related to Gentzkow and Kamenica (2017) and Li and Norman (2018) on games with multiple-senders. We introduce in the model commitment power to the senders, in the sense they can, after the stage game is played but previous to the revelation of the state of the world, commit themselves to the strategy they are using to send the messages forward. Senders now are able to pre-design how the information will be transmitted, when incentive compatibility do not need to hold at each state of the world, but just an *ex ante* condition on incentives.

First we motivate on why to study this model as an Information Design game. One way could be following a *literal interpretation* of the model, seeing pre-commitment to a rule of transmission as a cultural pattern that the older generation chooses, or has rooted from past times. This cultural pattern, is a language, that may sometimes distort some states of the world. It works out as if, a group could, *ex-ante* choose a certain way to write the history books, even when some narratives, do not fully reflect the true state of the world. Also, it can be represented as the design of a language that is used to pass information to their children.

Since pre-commitment and abstention of incentives are quite problematic, we can also borrow the *metaphorical interpretation* from Bergemann and Morris (2017), that uses Information Design games to study *boundaries* that players can achieve, when we vary across information structures, trying to understand the limits of the game through different forms of information disclosure. In our set up we may also look for bounds taking as given how the other group is disclosing information, asking ourselves how far can a player unilaterally changes the information of the game at a given time t.

Again, we begin with a single designer benchmark[2], in which only one group, say A, has the power to design the public messages. As before, this analysis also resemble a short-lived social planner analysis, since sender cannot induce unilateral deviations from receivers. It's also the same analysis if we assume a short-lived mediator with state independent preferences, that prefer always peace to war, and can commit himself before he observes the state.

With only one designer at each t, the solution of the model will be similar to the classical Bayesian Persuasion model of Kamenica and Gentzkow (2011). At any t, the sender maximize the probability of (P, P) happening in $t + 1$, choosing $\{\pi_S^t(p|c_\ell, \alpha, m), \pi_S^t(p|c_h, \alpha, m)\}$, for any history of actions $\alpha \in \mathcal{A}^t$ and past messages

[2] We leave the multi-sender analysis to the appendix. There we show that with senders from both groups we can always replicate the single-sender preferred strategy in equilibrium. However there does exist other equilibria, for instance one that replicates the full-revealing.

$m \in M^{t-1}$, where p is the message that will induce peace. Notice that we are already disregarding the case when the observed state is $\emptyset$, since communication cannot be effective. Also, as in Kamenica and Gentzkow (2011) we just need two messages to solve for the designer's optimal, and we will denote by w the message that induces (W, W).

Let's see how the game unfolds, when sender behave as above, and again receivers are using a cutoff strategy around μ^*. At $t = 1$, given the unfavorable prior, $\mu_0 > \mu^*$, (W, W) will occur. So, before the state at t is revealed, sender knows with probability μ_0 he will observe $c^1 = c_\ell$ and $c^1 = c_h$ with remainder probability. Also he knows that if he commit to the following strategies $\pi_S^1(p|c^1 = c_j) := \varepsilon_j$, for $j \in \{\ell, h\}$, the posterior the receiver will hold at the beginning of $t = 2$ is:

$$\mu^2((W, W), p) = \frac{\mu_0 \varepsilon_\ell q + (1 - \mu_0)\varepsilon_h(1 - q)}{\mu_0 \varepsilon_\ell + (1 - \mu_0)\varepsilon_h} \tag{4.8}$$

Notice, that the sender wants to set ε_ℓ and ε_h as high as possible (at most 1), since they increase the probability of getting the higher payoff from peace. Their choices are restricted by $\mu^2((W, W), p) \leq \mu^*$ and $\mu^2((W, W), w) > \mu^*$, so that the receivers want to follow the recommendations. One can show that, since $q \in (1/2, 1)$, (4.8) is decreasing in ε_h, it is optimal to set it equal to one. It is easy to see that the solution will be to increase ε_ℓ up to the point that the restriction after p binds, and we get:

$$\varepsilon_\ell = \left(\frac{1 - \mu_0}{\mu_0}\right)\left(\frac{\mu^* - (1 - q)}{q - \mu^*}\right) \quad \in (0, 1) \tag{4.9}$$

So using this *split* the designer is either throwing the receiver at a posterior equals to μ^*, after p or to q, after w, which reveals that c_ℓ has happened. So it is incentive compatible for receivers to play (P, P) after observing p, and (W, W) otherwise. Notice that the fact that ε_ℓ is interior it is due $q > 1/2$ and $\mu_0 > 1/2$ so $\mu_0 q + (1 - \mu_0)(1 - q) \geq 1/2 > \mu^*$.

Now suppose the game is at $t = 3$ and (P, P) was played last period, so no information is disclosed. Then, the posterior held at $t = 3$ will be the one-step-ahead of the belief held by those in $t = 2$. So μ^3 after this history will be equal to $\mu^* q + (1 - \mu^*)(1 - q) > \mu^*$ for any $q \in (1/2, 1)$, so war will take place in period 3. Here we can see that, differently from the full-revealing strategy, where peace lasts for possibly more than one period; with information design implementing sender's preferred outcome peace only lasts for one period. The reason for that is because the designer is inducing peace sometimes when the bad state comes out, elevating the posterior to μ^*, which is just enough to guarantee one period of peace. Driving this result is also the fact that designer is short-lived, and cares only to see peace happening in his sender-receiver game. So given the prior his is holding μ_0, he is maximizing the probability of peace in $t = 2$, but by doing that war will happen for sure, following (P, P), making peace a quite unstable state of the world.

Back to $t = 2$, suppose now that w was observed and players engage in war at this period. Now, the designer can again commit his strategies, holding the belief q, since he knows in $t = 1$, $c^1 = c_\ell$ was the true state of the world. He will commit to the following strategies $\pi_S^2(p|c_j, (W, W), w)) := \hat{\varepsilon}_j$ for $j \in \{\ell, h\}$, the posterior the receiver will hold at the beginning of $t = 2$ is:

$$\mu^2((W, W), p) = \frac{q\hat{\varepsilon}_\ell q + (1-q)\hat{\varepsilon}_h(1-q)}{q\hat{\varepsilon}_\ell + (1-q)\hat{\varepsilon}_h} \tag{4.10}$$

Once again it is optimal to set, $\hat{\varepsilon}_h = 1$ and let the restriction bind, so we again can find:

$$\hat{\varepsilon}_\ell = \left(\frac{1-q}{q}\right)\left(\frac{\mu^* - (1-q)}{q - \mu^*}\right) \in (0, 1) \tag{4.11}$$

Notice that once again, the short lived designer is splitting between μ^* and q, with different probabilities, since the prior they hold is different. But then again, what follows the realization of message p is peace in the next period followed necessary by war, as reasoned above.

It only remains to specify what designers do following any history $\tilde{h} \in \tilde{H}^t$ that ends up with (P, P) as last play. Once again, the belief following such history will be $\tilde{\mu} := \mu^* q + (1-\mu^*)(1-q) > \mu^*$ so that war will take place. Then the posterior $\tilde{\mu}$ will be the one used in the splitting in a similar way as above. Designers will choose at t, commitment strategies $\pi_S^t(p|c_j, h) := \tilde{\varepsilon}_j$ for $j \in \{\ell, h\}$, so the posterior will be:

$$\mu^{t+1}(h, p) = \frac{\tilde{\mu}\tilde{\varepsilon}_\ell q + (1-\tilde{\mu})\tilde{\varepsilon}_h(1-q)}{\tilde{\mu}\tilde{\varepsilon}_\ell + (1-\tilde{\mu})\tilde{\varepsilon}_h} \tag{4.12}$$

The solution will be to set $\tilde{\varepsilon}_h = 1$ and $\tilde{\varepsilon}_\ell \in (0, 1)$ to throw in the indifference:

$$\tilde{\varepsilon}_\ell = \left(\frac{1-\tilde{\mu}}{\tilde{\mu}}\right)\left(\frac{\mu^* - (1-q)}{q - \mu^*}\right) \in (0, 1) \tag{4.13}$$

We summary the results in the proposition below.

Proposition 4.5. *In the information design game with a short-lived single sender, we have the equilibrium with the following properties:*

1. *At any $t \geq 1$ following war, it is always optimal for the sender to induce peace after the good state is revealed $\Rightarrow \varepsilon_h = \hat{\varepsilon}_h = \tilde{\varepsilon}_h = 1$;*

2. *At $t = 1$, after (W, W), sender commit to induce peace after $c^1 = c_\ell$ with probability $\varepsilon_\ell \in (0, 1)$ given by (4.9);*

3. *At any $t > 1$ after any history with p as last message, players will play (P, P), and no informative messages will be sent to $t + 1$;*

4. *At any* $t > 1$*, with any* $h = (\cdots, (W, W), w) \in H^{t-1}$*, players will choose* (W, W) *at* t*, and designers will commit to induce peace after* $c^t = c_\ell$ *with probability* $\hat{\varepsilon}_\ell \in (0, 1)$ *given by (4.11);*

5. *At any* $t > 1$*, with any* $h = (\cdots, (P, P), m) \in H^{t-1}$*, for any* $m \in M$*, players will choose* (W, W) *at* t*, and designers will commit to induce peace after* $c^t = c_\ell$ *with probability* $\tilde{\varepsilon}_\ell \in (0, 1)$ *given by (4.13);*

6. *On equilibrium path, peace cannot occur in two subsequent periods.*

As depicted previously, short lived designers are only interested in maximizing the probability of seeing peace in their life time. So, when given the opportunity to credibly communicate, they will induce peace the most they can, by throwing his receivers in the indifference μ^*. While it does guarantee peace more often, since now we are sometimes avoiding war under a bad state, this kind of strategy makes peace an unstable state, since war will follow up for sure. In our model, if we give power to one group to commit on how communication will be done and if the agents are short-lived, the might want to persuade the next generation at its limit, by garbling information just enough for them to still be willing to coordinate on peace. But then, in the very next period, players, knowing that the information previously disclosed was highly noisy, would not hold favorable beliefs towards peace, and war will becomes inevitable once again.

Here it is a good time to compare this results with Cheap talk results, in particular with multi-senders when a full 'war'-revealing is an equilibrium[3]. Previously, we had that senders were only able to induce peace when good states were observed, which can happen with a quite low probability, since the prior is not favorable and states are persistent. Under information design we are able to induce peace more often, because senders send peaceful messages not only at c_h, but sometimes at c_ℓ too. So, if the game were to be played in just one shot, there is no doubt that commitment is a tool to improve gains and to avoid conflicts.

However, we are in a dynamic environment and short-run strategies may have impacts that echoes to other periods. A key state variable in the model is the belief players hold at the beginning of each stage, that will determine whether or not peace will occur. Under information design, senders have incentive to push this belief as far as possible to increase the probability of entering in peace at the very next period. This inflated probability makes peace not stable, and it will last for just one period. On the other hand, under full 'war'-revealing, the belief after the good states is revealed is low, and

[3] At first glace, it seems not right to compare multiple-senders Cheap talk with one sender Information design. However, in the appendix we describe the model with multi-senders and commitment and find that there exists an equilibrium that replicates the single-sender preferred one, which is alos the most beneficial for short-lived designers. So when we are comparing Cheap talk and ID, one may think we are comparing them both with two-senders.

it will increase over time as Bayesian players goes through periods without information. However, this growing posterior may stay low enough for some time, so peace can happen during several stage games. Thus, there seems be a trade-off here between *how often* do we want to enter in peace and *how long* do we expect it to last. At first glance this dynamic trade-off is not obvious, and makes us want to know in which scenario there exists more peace in the long-run? And how does this kind of comparison depend on the parameters of the model? We address this questions in the following subsection.

4.5 Comparing Full-revealing and Short-Lived Information Design

In this section we turn to compare the two main results so far: *i)* With multiple-senders Cheap talk, we can implement a truth-telling strategy that fully reveal the previously observed state of the world, producing endogenous cycles of peace; *ii)* With information design (single or multiple-senders), we have that designers have incentives to split the beliefs leading receivers to their indifference, making peace more probable, but also more unstable. We try to compare both of them in terms of *how frequent* peace is achieved in equilibrium in the long-run.

The idea here is to write both equilibrium outcomes in an automata representation, so we can define a Markov chain over states of the automata. Then we can proceed to compute the stationary distribution of the chain, to extract the *long run fraction of time* in which the automata is in states where peace is being played. With both fractions calculated we can compare both outcomes and asses which of them would be more peaceful in the long run.

It must be said that this cannot be viewed as a proper welfare analysis, since we are just considering stationary patterns that emerge from each equilibrium, do not allowing for discounting over the time, as would occur with a standard long-lived agent. The reason why we choose this stationary analysis is because it is intuitive and straightforward to compute. A complication that would arise under a standard welfare analysis is that a long-lived agent, who observes past realizations of states when war occurs, may at some histories have different beliefs than the ones agents have, simply because the messages received may not perfectly reveal the states, as in the Information design outcome. To avoid this issue of private and public beliefs, we develop this comparison of stationary distribution to asses which communication protocol is more peaceful in a long-run sense.

We proceed to construct the automatas, starting with the full revealing one coming from multiple-sender Cheap talk. In order to build the states of our automata, one must acknowledge that we are dealing with a *hidden Markov*, since the underlying state of the world is changing, so that at every t one never knows the state before the conflict game is played. So what will define the states in our automata are both the underlying state

$\{c_h, c_\ell\}$ and the action specified for players to play, either (P, P) or (W, W). So players do not observe in which state of the automata they are but we are able to compute the transitions between any states, which is sufficient to get the stationary distributions.

We depicted below the automata for the full "war'-revealing.

TT_Automata.jpg

Figure 1 – Full 'war'-revealing automata

We denote a state by K^j, for $K \in \{W, P\}$, and $j \in \{\ell, h\}$, a state in which players play (K, K) in the period and the underling state world is j. The numbers subscripts are just to represent states with the same action, but with possible different transitions, since the belief held at these states might differ. So notice that we only goes to either P_1^ℓ or P_1^h, from war states that are in the state h, because in the truth-telling equilibrium, only after c_h is observed and revealed to the following generation we can induce peace. Also, we have represented in the automata the fact that peace lasts for τ^* periods, with the state of the world possible changing during the cycle.

We let Γ^r denote the transition matrix over the states of the full-revealing automata and let:

$$\gamma^r = (\gamma(W_0^\ell), \gamma(W_0^h), \gamma(P_1^\ell), \gamma(P_1^h), \cdots, \gamma(P_{\tau^*}^\ell), \gamma(P_{\tau^*}^h), \gamma(W_1^\ell), \gamma(W_1^h), \gamma(W_2^\ell), \gamma(W_2^h))$$

Be the unique stationary distribution of Γ^r [4], where $\gamma(K^j)$ is the stationary probability of being at state K^j, that can be interpreted as the long-run frequency of time in which state K^j is visited [5]. We want to compute the total fraction of time in which the automata is in peace, for any state $\{\ell, h\}$:

$$\gamma_P^r := \sum_{i=1}^{\tau^*} (\gamma(P_i^\ell) + \gamma(P_i^h)) \tag{4.14}$$

We know that, since the system starts in either W_0^ℓ or W_0^h and never comes back, we have that the long-run probability of being in each of these states will be zero. Thus,

[4] The stationary distribution exists and it unique because we have a finite Markov Chain with only one closed communication class. See Stokey and Lucas (1989) for details.

[5] For some rigorous foundation Acemoglu and Wolitzky (2014) and Liu and Skrzypacz (2014)

we must solve the following linear system, considering Γ^r already without W_0^ℓ and W_0^h:

$$\tilde{\Gamma}^r := \begin{bmatrix} q & (1-q) & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-q) & q & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q & (1-q) & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-q) & q & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & q & (1-q) \\ q & (1-q) & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & (1-q) & q \\ 0 & 0 & 0 & (1-q) & q & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.15}$$

And:

$$\tilde{\gamma}^r = \left(\gamma(W_1^\ell), \gamma(W_1^h), \gamma(P_1^\ell), \gamma(P_1^h), \gamma(P_2^\ell), \gamma(P_2^h), \cdots, \gamma(P_{\tau^*}^\ell), \gamma(P_{\tau^*}^h), \gamma(W_2^\ell), \gamma(W_2^h)\right) \tag{4.16}$$

To solve:

$$\tilde{\gamma}^r \tilde{\Gamma}^r = \tilde{\gamma}^r \tag{4.17}$$

With the condition that, the fractions sum up to 1:

$$\gamma(W_1^\ell) + \gamma(W_1^h) + \gamma_P^r + \gamma(W_2^\ell) + \gamma(W_2^h) = 1 \tag{4.18}$$

Solving the system, we first conclude that the sum of fractions for a given period of peace for both states must be the same for all periods, $\gamma(P_1^\ell) + \gamma(P_1^h) = \gamma(P_2^\ell) + \gamma(P_2^h) = \cdots = \gamma(P_{\tau*}^\ell) + \gamma(P_{\tau*}^h)$, simplifying (4.14) to:

$$\gamma_P^r = \tau^*(\gamma(P_1^\ell) + \gamma(P_1^h)) \tag{4.19}$$

After some cumbersome algebra, we can find the fraction of peace only as a function of the parameters:

$$\gamma_P^r = \frac{\tau^*(1-q)}{(1+\tau^*)(1-q) + \hat{\mu}} \tag{4.20}$$

Where τ^* is given as is (4.6) and $\hat{\mu}$ is the belief held at the beginning of any period in which peace was played was the last τ^* periods, so that is the first one after a sequence of peace which is strictly greater than μ^*. We calculate this belief as in (4.4):

$$\hat{\mu} = \frac{1}{2} - \frac{1}{2}(2q-1)^{\tau^*+1} > \mu^* \tag{4.21}$$

Notice the fraction of peace depend only on the Markov chain parameter q and on μ^* through τ^*. However, this is not a well-behaved function due the discontinuities imposed by the floor on τ^*. There are intervals of both q, μ^* such that the number of periods of peace are same, but the parameters still affect γ_P^r through the probability of entering in the cycle of peace. It will sometimes be convenient to write $\gamma_P^r(q)$ as a function of the persistence parameter. Later on, we plot this function to compare with the fraction of peace that comes from the designer's solution.

We proceed to do the same analysis with the automata generated by the short-lived information design environment. Here we will have fewer states, since peace cannot happen two times in a row. However our transitions will be somewhat more complicated, because now sometimes when the bad states are observed, players are induced to play (P, P) in the following period. We present the automata below, in Figure 2, where we ommit the initial states for better presentation:

ID_AutomataV3.jpg

Figure 2 – Short-lived Information Design automata

At states of war, transitions will now depend on the probabilities of sending the peaceful message after some history and c_ℓ being revealed, namely $\varepsilon_\ell, \hat{\varepsilon}_\ell$ and $\tilde{\varepsilon}_\ell$. Once again the fractions regarding the initial periods will be equal to 0 in the long-run. In order to find the stationary probability of peace under information design, $\gamma_P^d = \gamma(P^\ell) + \gamma(P^h)$ we solve the following problem:

$$\tilde{\gamma}^d \tilde{\Gamma}^d = \tilde{\gamma}^d \tag{4.22}$$

With the condition that, the fractions um up to 1:

$$\gamma(W_1^\ell) + \gamma(W_1^h) + \gamma_P^d + \gamma(W_2^\ell) + \gamma(W_2^h) = 1 \tag{4.23}$$

With:

$$\tilde{\Gamma}^d := \begin{bmatrix} q(1-\hat{\varepsilon}_\ell) & (1-q)(1-\hat{\varepsilon}_\ell) & q\hat{\varepsilon}_\ell & (1-q)\hat{\varepsilon}_\ell & 0 & 0 \\ 0 & 0 & (1-q) & q & 0 & 0 \\ 0 & 0 & 0 & 0 & q & (1-q) \\ 0 & 0 & 0 & 0 & (1-q) & q \\ q(1-\tilde{\varepsilon}_\ell) & (1-q)(1-\tilde{\varepsilon}_\ell) & q\tilde{\varepsilon}_\ell & (1-q)\tilde{\varepsilon}_\ell & & \\ 0 & 0 & (1-q) & q & 0 & 0 \end{bmatrix} \tag{4.24}$$

And:

$$\tilde{\gamma}^d = \Big(\gamma(W_1^\ell), \gamma(W_1^h), \gamma(P^\ell), \gamma(P^h), \gamma(W_2^\ell), \gamma(W_2^h)\Big) \tag{4.25}$$

Solving the linear system of equations[6], we can find γ_P^d as a function of only q and μ^*:

$$\gamma_P^d = \frac{1}{2} - \frac{q(1-\tilde{\varepsilon}_\ell)}{2(4\tilde{\varepsilon}_\ell q^2 - 4\hat{\varepsilon}_\ell q^2 + 3\tilde{\varepsilon}_\ell q - \tilde{\varepsilon}_\ell + q + 1)} \tag{4.26}$$

One result that follows from (4.26) is that, under information design with short-lived senders, one cannot expect to have peace in equilibrium more than half of the times, since the fraction on peace is bounded from above by $1/2$, for any values of q, μ^*. This is quite intuitive, since peace only lasts one period, every time we visit in peace states we must necessarily follow into a war state. So, $\gamma_P^d = \gamma(W_2^\ell) + \gamma(W_2^h)$, and the proportion of peace cannot be larger than half. Thus, if we find parameter values for which the full-revealing fraction of peace is grater than $1/2$ we may conclude that full-revealing dominates persuading in terms of long run peace. Let's plot both γ_P^d, γ_P^r, as a function of q to get a sense of how this objects behave, to then extract some final results.

Comparing_gamma_03.jpg

Figure 3 – Both fractions as function of q, with $\mu^* = 0.3$

In Figure 3 we have γ_P^r with the full line, and γ_P^d with the dashed one. First note, that the domains of functions are not the whole $(1/2, 1)$. This is because we are assuming that $1 - q < \mu^*$ in order to generate at least one period of peace following a good state. So the domain is restricted from $(1-\mu^*, 1)$.

We can note the unusual shape of γ_P^r, where the discontinues arises from the *floor* function in τ^*. Given μ^*, we have intervals of q for which the τ^* is the same. When q gets larger, eventually τ^* will jump to next integer, causing the discontinuities displayed. Notice however, that γ_P^r is decreasing in each "segment", which means that if any $q, q' \in (1-\mu^*, 1)$ have the same $\tau^* = \tau^*(q) = \tau^*(q')$, the smallest one of them would induce peace more frequently. This is quite intuitive, since the periods of peace are fixed, one would prefer that the state is *less permanent* in order to not remain for so long time under c_ℓ. So, since the segments jump discontinuously but are decreasing within the segment interval, γ_P^r presents some non-monotonicities that make some analysis harder. We summarize the results in the Lemma below:

Lemma 4.2. *For any $q, q' \in (1-\mu^*, 1)$ with $q > q'$ and with lengths of peace cycle $\tau^*(q) = \tau^*(q') = \tau$, we must have that $\gamma_P^r(q') > \gamma_P^r(q)$.*

On the other hand γ_P^d is more conventional, since it a continuous strictly increasing function of q. However, the increasing property is quite surprising, since one should expect the higher the persistence of the state the harder it is to go to peace, leaving a states of c_ℓ, even under information design.

[6] Hat tip to *sympy* Python package

This intuition is only partially correct, because it focus only on transitions regarding W_1^ℓ and W_1^h states, where c_ℓ is known to be the last state revealed. However, there exists another effect that dominates this one, and it is in relation with transition probabilities at W_2^ℓ and W_2^h. First notice, the higher q the smaller it is the one-step-ahead belief of μ^*, namely $\tilde{\mu}$, meaning that in the period of war that follows the peaceful one, designers would prefer to have higher q, since the belief they are splitting would be more favorable to return to peace. The higher the persistence of the state the world, the lower is the belief after peace, making the splitting more favorable and increasing the transition probability to any state of peace.

We can conclude that the second effect dominates the first one and thus γ_P^d will be strictly increasing in the interval of interest.

Lemma 4.3. *For any $q \in (1-\mu^*, 1)$, we have that $\gamma_P^d(q)$ is strictly increasing in q.*

Figure 3 leaves us with the flavour that γ_P^d may dominate when parameters are such that $\tau^* = 1$ and γ_P^r seems to be higher when $\tau^* \geq 2$. But, while the first one it is easy to check to hold true, the second one must be looked carefully. To address the first issue we have the proposition below:

Proposition 4.6. *Fix a threshold $\mu^* \in (0, 1/2)$. Then for any $q \in (1-\mu^*, 1)$ such that $\tau^*(q) = 1$ we must have $\gamma_P^d(q) > \gamma_P^r(q)$.*

Proof. The result will follow from past Lemmas 4.2 and 4.3. First, notice that when $q \to (1-\mu^*)^+$ we have $\tau^* \to 1$ and $\tilde{\varepsilon}_\ell, \hat{\varepsilon}_\ell$ both going to zero. Then, if we compare the following limits:

$$\lim_{q \to (1-\mu^*)^+} \gamma_P^d(q) = \lim_{q \to (1-\mu^*)^+} \gamma_P^r(q) = \frac{1}{2(2-\mu^*)} \tag{4.27}$$

But we know that $\gamma_P^d(q)$ is strictly increasing in the interval $(1-\mu^*, 1)$ and that $\gamma_P^r(q)$ is strictly decreasing for $\tilde{q}$ such that $\tau^*(\tilde{q}) = 1$. Then for any $q_\xi := (1-\mu^*) + \xi$, with $\xi > 0$ such that $\tau^*(q_\xi) = 1$, we know $\gamma_P^d(q_\xi) > \gamma_P^r(q_\xi)$, and that concludes the proof. □

Now to discuss the second issue presented, we plot another example when $\mu^* = 0.475$. Then again information design does dominate in the region where $\tau^*(q) = 1$. But, we can see there there exists some non-monotonicity of dominance, since there is a region of q in which $\tau^*(q) = 2$ and for some low values of q full-revelation dominates, while for some other higher values in the same region, Information design dominates. This kind of situation unable us to get some threshold result, a $\bar{q}$ in which for values below this point, the designer would induce peace more frequently, while for higher values the Cheap talk equilibrium would dominate.

□Comparing_gamma_0475.jpg

Figure 4 – Both fractions as function of q, with $\mu^* = 0.475$

What we can say is regarding the limits of both functions when q approaches 1, in which the fraction of peace under full-revealing would be strictly greater than the one under short-lived information designers. The higher the persistence of the state the longer it will last the cycles of peace, and this *how long* effect dominate the *how often* effect of information design in the limit scenario.

Proposition 4.7. *For any value of the threshold $\mu^* \in (0, 1/2)$, we have that:*

$$\lim_{q \to 1^-} \gamma_P^r(q) > \frac{1}{2} > \lim_{q \to 1^-} \gamma_P^d(q)$$

Proof. First, let we compute those limits. For $\gamma_P^d(q)$ it is straightforward, since it is a continuous function and we have the simplification used in Lemma 4.3 to conclude:

$$\lim_{q \to 1^-} \gamma_P^d(q) = \lim_{q \to 1^-} \frac{-1}{2(2\mu^* q - 3q + 1)} = \frac{1}{4(1-\mu^*)} < \frac{1}{2}$$

To compute the limit of $\gamma_P^r(q)$ we must take a detour, due the indeterminacy that arises when $\lim_{q \to 1^-} \tau^* = +\infty$. We first define two auxiliary functions that will bound $\gamma_P^r(q)$. Pose $\tilde{\tau}(q) = ln(1-2\mu^*)/ln((2q-1) > 1$, which is the *smooth* time function, without the floor and let:

$$\gamma_+(q) = \frac{\tilde{\tau}(q)(1-q)}{(1+\tilde{\tau}(q))(1-q) + \mu^*}$$

$$\gamma_-(q) = \frac{(\tilde{\tau}(q)-1)(1-q)}{\tilde{\tau}(q)(1-q) + \tilde{\mu}}$$

Then, it easy to see that any function of the form:

$$\frac{t(1-p)}{(t+1)(1-p)+k}$$

Is increasing in t for any $k, p \in (0,1)$. All three gammas, $\gamma_+(q), \gamma_i(q), \gamma_P^r(q)$ fit in the above function if we are only changing τ^* or $\tilde{\tau}$, holding q constant. Knowing this property, the fact that $\tilde{\mu}(q) \geq \hat{\mu}(q) > \mu^*$ and that $\tilde{\tau}(q) \geq \tau^*(q)$ it is easy to check the following inequalities:

$$\gamma_+(q) > \gamma_P^r(q) > \gamma_-(q), \quad \forall q \in (1-\mu^*, 1) \tag{4.28}$$

Since the bounds are differentiable functions we can use L'Hôpital rule to get rid of the indeterminacy. We can compute then the limit when q goes to 1.

$$\lim_{q \to 1^-} \gamma_+(q) = \lim_{q \to 1^-} \gamma_-(q) = -\frac{ln(1-2\mu^*)}{2\mu^* - ln(1-2\mu^*)} > 0$$

So we can apply Sandwich Theorem to conclude that:

$$\lim_{q \to 1^-} \gamma_P^r(q) = -\frac{ln(1-2\mu^*)}{2\mu^* - ln(1-2\mu^*)} > 0$$

We turn then to show that this limit is greater than the one related to $\gamma_P^d(q)$. Actually, it is easier to show that it is an upper bound to 1/2 which is a bound to $\lim_{q \to 1^-} \gamma_P^d(q)$. We want to show that:

$$-\frac{ln(1-2\mu^*)}{2\mu^* - ln(1-2\mu^*)} > \frac{1}{2} \Leftrightarrow$$
$$2ln(1-2\mu^*) > 2\mu^* - ln(1-2\mu^*) \Leftrightarrow$$
$$ln(1-2\mu^*) > 2\mu^*$$

Note that both sides from the last inequality are strictly increasing in μ^*, and if we would evaluate the expression at $\mu^* = 0$ they would be the same. So if we proof that the LHS increases faster than the RHS, we are done for any $\mu^* \in (0, 1/2)$. Taking the first derivative from both side we have:

$$\frac{2}{1-2\mu^*} > 2$$

which will hold for any $\mu^* \in (0, 1/2)$. Therefore, we can conclude that $ln(1-2\mu^*) > 2\mu^*$ for any $\mu^* \in (1-p, 1/2)$, and that concludes the proof.

□

The message we can get from these final results is that commitment power from myopic players are not always good for peace in the long-run. If the state of the world is persistent enough, a society that is just revealing the past states, which can be implemented by multi-sender Cheap talk communication, will experiment peace more frequently in the long-run, since the peace cycles will last for several periods. The intuition is very clear, since the more persistent the state is, more periods are expected to remain under the good state after its revelation. This in turn, would allow for Bayesian players to keep their beliefs low enough for a quite long period of time in which peace will occur. This long-run fraction of peace dominates the one produced by short-lived designers, in which peace occurs with higher probability in the stage game, but only lasts for one period. Under Information design, one cannot expect to be more than half of the time in peace in the very long-run, since every single period of peace is followed by conflict.

5 Conclusion

In this article we develop a dynamic model of intergenerational communication in conflicts. In order to shed some light into the case studies from history literature, our model tries to capture how groups can disclose information in order to avoid war after some histories of play. One key ingredient of the article is that player will only learn states of the world if war has occurred in the last period, and will not receive any new information in the peaceful periods.

We want to compare two types of protocols of communication, one *a lá* Cheap talk communication, where incentive compatibility must hold at each revealed state; and the second modeled as Information design, where we abstract from incentive compatibility, as sender can commit themselves to an specific strategy.

Our main results point that multiple-sender Cheap talk communication can improve upon the no-communication outcome, since we can implement in equilibrium a full-revealing strategy. War will follow the revelation of bad sates, and peace will follow for a finite number of periods after the good state is observed. With short-lived senders playing this truth-telling strategy, we get a positive *externality* result, since several generation will benefit from one good state being revealed. Under information design, short-lived senders will try to maximize the probability of peace in their communication game, by splitting the belief they hold at the time in a way that leaves the receiver in the indifference of war and peace. With this highly distorted messages, the following generation will barely belief the state is favorable. This in turn will make peace an unstable state, that will only last for one period.

This dynamic result might guide some peace keeping policies. For instance, if in post-conflict societies decision-makers are seen to be short-lived, and only cares for war or peace in a short future horizon, then, giving the power to institutions to design memory and textbook policies may not be the best choice for long-run peace. These agents may want to avoid war by distorting too much the real state of the world making the incentives to coordinate on peace to not last for much time. If instead parts of post-conflict society are disclosing truthful information, it may be quite hard to stop the conflict as long as war states are enduring. But once the world changes and the state is truly conveyed to the society, we may see cooperation and peace for several periods in a row.

When we compare both communication protocols we have an explicit trade-off, where in one hand we have Cheap talk communication producing peace for longer periods of time, and in the other hand Information design inducing peace more often in the stage game. When we compare both outcomes in terms of how frequent peace occur in the long-run we find out that if the state is very infrequent, the *how often* effect of Information design dominates the Cheap talk, for values of the persistence parameter where only one period of peace would occur under full-revealing. However, as the state becomes sufficient persistent, the *how long* effect would dominate, making peace more frequent in the Cheap talk set up.

One natural extension of our model, left for future works, is to pin down an optimal information disclosure, one that would, for instance maximize the long-run fraction of peace in the model. Such an optimal mechanism would probably be a combination of both protocols: one in which we can enter in peace more often (under the revelation of the bad state), but not with so noisy information, allowing for peace to last for several periods.

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Appendix

APPENDIX A – Omitted Proofs

Lemma 4.1 (Coordination among Receivers). *Let (a, π) be a (pure-strategy) IDE or CTE with belief function $\mu^t(h)$, for every, $t \geq 1$ and every possible history $h \in H^{t-1}$. Then we must have that $\Big(a^t(h), \mu^t(h)\Big)$ is a Bayes Nash Equilibrium for all $h \in H$ on equilibrium path, which in turn implies that $a^t_A(h) = a^t_B(h)$.*

Proof. It follows from, the analysis of BNE. Suppose, by way of contradiction,that there exist, t and $\tilde{h} \in H$ such that $a^t_A(\tilde{h}) \neq a^t_B(\tilde{h})$. By symmetry we just need to check two cases.

1. If $a^t_B(\tilde{h}) = W$
   Expected gain for A is $-d$, which is strict less than he would gain by deviating to W, namely:
   $$\mu^t(\tilde{h})(-c_\ell) + (1 - \mu^t(\tilde{h}))(-c_h) > -d, \quad \forall \mu^t(\tilde{h}) \in [0, 1]$$
   So a cannot be an equilibrium profile of strategies.
2. If $a^t_B(\tilde{h}) = P$
   Expected gain for A is:
   $$\mu^t(\tilde{h})(b - c_\ell) + (1 - \mu^t(\tilde{h}))(b - c_h)$$
   Then if $\mu^t(\tilde{h}) > \mu^*$, A strictly prefers W to P. But then, by the previous item B would like to deviate to W. If $\mu^t(\tilde{h}) \leq \mu^*$, A would like to deviate to P. More specifically, at $\mu^t(\tilde{h}) = \mu^*$, A is indifferent, given that B is playing P. But then, if A would choose to remain with W, it would not be an equilibrium as before. So, a cannot be an equilibrium profile of strategies.

□

Proposition 4.2 (Single Sender cannot induce peace). *Let (π_S, a) be a CTE with a single sender. Then, we must have that for each history on equilibrium path $h \in H$, $a(h) = (W, W)$.*

Proof. We want to show that a single sender can not send informative messages about the observed state of the world at any time t We obtain the result as follows. At $t = 1$ since the prior in unfavorable only (W, W) is played in equilibrium. Now suppose by way of contradiction the sender could send a message $m_S \in M_S$, such that players at $t = 2$ plays P with positive probability. We assume without loss that m_S is the message that induces (P, P) with highest probability. But then, the single sender would have incentive to set $\pi_S^2(m_S|c,(h,a)) = 1$ for every $c \in \mathcal{C}$, since (P, P) dominates (W, W) at every state of the world.

So, the *ex-ante* posteriors at $t = 2$ for Bayesian players, after m_S are just the one-step-ahead of the belief in $t = 1$, using the Markov parameter, namely $\mu_0 q + (1-\mu_0)(1-q) < \mu_0$. We know that this posterior will decrease, but never below μ^*. The same reasoning will apply in subsequent periods, allowing us to conclude that the single sender is not being informative and the beliefs will evolve as in the no-communication benchmark. This in turn, with our assumptions over the parameters, will imply that (W, W) will be played at every period.

□

Proposition 4.4 (Limits to Cheap talk Communication)**.** *Let (a, π) be an (pure-strategy) CTE with belief function μ^t. We must have that, for any $t \geq 1$ and history of actions $\alpha \in \mathcal{A}^t$ and past messages $\tilde{m} \in M^{t-1}$, if there exists a message $m = (m_A, m_B) \in M$ such that $a^{t+1}(\alpha, (\tilde{m}, m)) = (P, P)$, then we must have that $\pi_i^t(m_i|c_\ell, \alpha, \tilde{m}) = 0$ for both $i_t \in \{A_t, B_t\}$.*

Proof. Fix $t \geq 1$ and $\tilde{h} = (\tilde{\alpha}, \tilde{m}) \in \tilde{H}^t$, with $\mu^t(\tilde{h}_{-1}) > \mu^*$ so that war will be played, the state will be revealed and communication can be informative.

Suppose by way of contradiction that there exist $m' = (m'_A, m'_b)$ with $a^{t+1}((\tilde{\alpha}, (\tilde{m}, m'))) = (P, P))$ and there exists $i_t \in \{A_t, B_t\}$, say A_t with $\pi_A^t(m'_A|c_\ell, \tilde{h}) > 0$. But then, B_t at state $c^t = c_\ell$ would have incentives to match m'_A, and play m'_B with probability 1, since the gain under (P, P) is greater than expected gain at (W, W), which is $q(-c_\ell) + (1-q)(-c_h) < 0$. However, the same incentives hold for A_t, and ends up saying m'_A with probability one at c_ℓ. Then, applying Bayes rule after receiving m' would lead to $\mu^{t+1}(\tilde{h}, m') = \mu^t(\tilde{h}_{-1})q + (1-\mu^t(\tilde{h}_{-1}))(1-q) > \mu^*$ which is not incentive compatible for receivers to play P, so they both end up playing (W, W). We have shown that it cannot be an equilibrium, which concludes the proof. □

Lemma 4.2. *For any $q, q' \in (1-\mu^*, 1)$ with $q > q'$ and with lengths of peace cycle $\tau^*(q) = \tau^*(q') = \tau$, we must have that $\gamma_P^r(q') > \gamma_P^r(q)$.*

Proof. Fix $q, q' \in (1-\mu^*, 1)$ with $q > q'$ and with lengths of peace cycle $\tau^*(q) = \tau^*(q') = \tau$ and lets show that:

$$\frac{\tau(1-q)}{(1+\tau)(1-q) + \hat{\mu}(q)} < \frac{\tau(1-q')}{(1+\tau)(1-q') + \hat{\mu}(q')}$$

Where $\hat{\mu}(\tilde{q})$ stands as $\hat{\mu}$ as in (4.21) when the parameter of persistence is $\tilde{q}$, but the number of periods of peace is fixed at τ. Rearranging the above expression is equivalent to show:

$$\frac{\hat{\mu}(q')}{1-q'} < \frac{\hat{\mu}(q)}{1-q}$$

Or that the function $f(q) := \frac{\hat{\mu}(q)}{1-q}$ is increasing in the interval $(1-\mu^*, 1)$. Since it is a continuous differentiable function we take the first derivative with respect to q:

$$f'(q) = \frac{-2(\tau+1)(2q-1)^{\tau}2(1-q) + 2(1-(2q-1)^{\tau+1})}{(2(1-q))^2}$$

We need to show that the numerator is strictly greater than 0, which in turn implies that, the following must hold:

$$1 > (2q-1)^{\tau}[2\tau(1-q)+1] \tag{A.1}$$

Let's evaluate the RHS of the above expression. Notice that if we would evaluate it at $q = 1$ both sides would be equal. Then if we show that the RHS is an strictly increasing function of q in the interval $(1-\mu^*, 1)$, we would conclude that the condition (A.1) holds. So we proceed taking the derivative of the RHS, to get:

$$2\tau(2q-1)^{\tau-1}[2\tau(1-q)+1] - 2\tau(2q-1)^{\tau} > 0$$

Which holds because $\tau(2q-1)^{\tau-1} > \tau(2q-1)^{\tau}$ for any $q \in (1-\mu^*, 1)$. Notice that it equals to zero at $q = 1$, so the RHS attains a local maximum at this point. We can conclude that $f(q)$ is strictly increasing in the interval $q \in (1-\mu^*, 1)$, and that's conclude the proof. □

Lemma 4.3. *For any $q \in (1-\mu^*, 1)$, we have that $\gamma_P^d(q)$ is strictly increasing in q.*

Proof. The proof is very standard, since we can apply calculus to conclude the frist derivative with respect to q is positive for any $q \in (1-\mu^*, 1)$ and given our assumptions over the parameters. First let's rewrite $\gamma_P^d(q)$ only as a function of q and μ^*, opening expressions for $\hat{\varepsilon}_\ell$ and $\tilde{\varepsilon}_\ell$. We obtain:

$$\gamma_P^d(q) = \frac{(1-q)(2q-1)}{2(1-q)(2q-1) + (q\mu^* + (1-q)(1-\mu^*))(2q-1) - \mu^* + 1 - q}$$

We can work to factor out the denominator, to be able to cancel some terms. We can get:

$$\gamma_P^d(q) = \frac{(1-q)(2q-1)}{2(q-1)(2\mu^* q - 3q + 1)}$$

Differentiating and rearranging we get:

$$(\gamma_P^d)'(q) = \frac{1 - 2\mu^*}{2(2\mu^* q + 3q + 1)^2} > 0$$

Which holds because the denominator is clearly positive being a square, and the numerator is also positive because we assume $\mu^* \in (0, 1/2)$. □

APPENDIX B – Multiple Sender Information Design

The aim of this section is to show that when we introduce two senders, one from each group, with commitment power to design messages, they can achieve many equilibria, including one that replicates the full-revealing and one that replicates the single sender preferred outcome. To avoid going throughout all the cases as we did in the main body of the article, here we will only show that, at any given time, when both designers hold a belief $\mu > \mu^*$ prior to the revelation of the state, they can split them either in a full-revealing fashion of in a single-sender preferred one. With the analysis of a single arbitrary belief μ we can get all the cases in the main text cover by analogy. The idea is that μ is the posterior calculated after some history of play, so that when we arrive at t war will take place and further the state will be revealed, enabling designers to send informative messages. As in the CTE, we must now account for senders deviation, taking as given the message strategy of other group sender. First we show that at any t, with senders holding a belief of $\mu > \mu^*$, fully-revealing the state of the world is an IDE. It convenient to denote $c_\ell^q = q(-c_\ell) + (1-q)(-c_h)$

Proposition B.1. *Fix $t \geq 1$ and assume the belief hold by senders before the state is revealed is $\mu > \mu^*$. Let $\pi_i^t(p|c_h) = \xi_h = 1$ and $\pi_i^t(p|c_\ell) = \xi_\ell = 0$ for both $i_t \in \{A_t, B_t\}$. We show that senders have no incentives to deviate when receivers are playing a μ^* cutoff strategy.*

Proof. Holding μ the posteriors following the messages $(p,p),(w,w)$ taken as given the specified message strategies would be:

$$\mu^{t+1}(p,p) = \frac{\mu\xi_\ell q + (1-\mu)\xi_h(1-q)}{\mu\xi_\ell + (1-\mu)\xi_h} = (1-q) < \mu^*$$

$$\mu^{t+1}(w,w) = \frac{\mu(1-\xi_\ell)q + (1-\mu)(1-\xi_h)(1-q)}{\mu(1-\xi_\ell) + (1-\mu)(1-\xi_h)} = q > \mu^*$$

So, it is straightforward that receivers want to follow the recommendations. The expected gains for senders would be μc_ℓ^q. Let's check if any sender wants to deviate. By symmetry we just need to check for one group, say A. The deviations are any $(\pi')_A^t(m|c_h) = \xi(m)$ and $(\pi')_A^t(m|c_\ell) = \kappa(m)$ for all $m \in M_A$. With the deviations we would have the following posteriors after messages, with any $m \in M_A$.

$$\mu^{t+1}(m,p) = (1-q) < \mu^*$$

$$\mu^{t+1}(m,w) = q > \mu^*$$

So receivers are just ignoring the deviation, and following the one that reveals the state. Thus, A do not have any incentives to deviate, so it is an equilibrium.

□

The intuition to this is the following, when a single player is committed to fully revealing the state of the world, nothing the other player says has any power to affect beliefs of a Bayesian receiver. Therefore, receivers simply ignore the deviating sender, and play accordingly to the state revealed by the truth-telling sender.

The above intuition can lead us to a second result, which states that *truth-telling* equilibrium, provides us with a lower bound to equilibrium sender-receiver game payoffs, with player holding a prior μ.

Corollary B.1. *Fix $t \geq 1$ and assume the belief hold by senders before the state is revealed is $\mu > \mu^*$. We can show that $\mu c_\ell^q < 0$ is a lower bound to any expected gain in the sender-receiver game that begins at t.*

Proof. It follows from previous proposition. If at any t with belief $\mu > \mu^*$ before the state is revealed, senders are getting a sender-receiver game payoff lower than μc_ℓ^q, a unilateral deviation to a full-revealing strategy would increase expected gains. So, fix B playing any $\pi_B^t(p|c_h) = \xi_h$ and $\pi_B^t(p|c_\ell) = \xi_\ell$ and once again show that receivers follow the deviation to truth-telling for A with $\pi_A^t(p|c_h) = 1$ and $\pi_i^t(p|c_\ell) = 0$. We would have the posteriors:

$$\mu^{t+1}(p,p) = \mu^{t+1}(p,w) = (1-q) < \mu^*$$

$$\mu^{t+1}(w,p) = \mu^{t+1}(w,w) = q < \mu^*$$

And the expected gain would increase to μc_ℓ^q. □

So, unilaterally any sender can deviate to attain the *truth-telling* payoff. The intuition is as before, it is enough one of the senders to be fully revealing the state of the world, to both receivers follow the public belief, ignoring the other sender's signal. Also, we recall that in the Cheap talk model, the truth-telling outcome constituted an upper bound to any sender equilibrium expected gain, while when we introduces commitment it becomes the minimum value the sender can expected to receive in equilibrium from the communication game.

We now, turn to look for equilibria with higher payoffs than the full-revealing one. In particular we show that we can replicate the single sender preferred payoff when receivers are playing a cutoff strategy around μ^*.

Proposition B.2. *Fix $t \geq 1$ and assume the belief hold by senders before the state is revealed is $\mu > \mu^*$. Let senders have the following strategies for both $i_t \in \{A_t, B_t\}$, $\pi_i^t(p|c_\ell) = (\varepsilon(\mu))^{\frac{1}{2}}$ and $\pi_i^t(p|c_h) = 1$, where:*

$$\varepsilon(\mu) = \frac{(1-\mu)}{\mu} \cdot \frac{(\mu^* - (1-q))}{q - \mu^*}$$

We claim that these are equilibrium strategies in the sender-receiver game with receivers following a cutoff-strategy around μ^*.

Proof. First, notice that $\varepsilon(\mu) \in (0,1)$ for any $\mu > \mu^*$. We have the following posteriors with positive probability following the messages

$$\mu^{t+1}(w,w) = \frac{\mu(1-\varepsilon(\mu))q + (1-\mu)(0)(1-q)}{\mu(1-\varepsilon(\mu)) + (1-\mu)(0)} = q \tag{B.1}$$

$$\mu^{t+1}(p,w) = \mu^{t+1}(w,p) = \frac{\mu(1-\varepsilon(\mu))^{\frac{1}{2}})\varepsilon(\mu)^{\frac{1}{2}}}{\mu(1-\varepsilon(\mu))^{\frac{1}{2}}\varepsilon(\mu)^{\frac{1}{2}}) + (1-\mu).0} = q \tag{B.2}$$

$$\mu^{t+1}(c_\ell|p,p) = \mu(c_\ell|w,p) = \frac{\mu\varepsilon(\mu)q + (1-\mu)(1-q)}{\mu(\varepsilon(\mu)) + (1-\mu)} = \mu^* \tag{B.3}$$

So the payoffs for senders would be $\mu(1-\varepsilon(\mu))(c_\ell^q)$.

Now we must check for deviations, and by symmetry, just need to check if A want to deviate. Let $(\pi')_A^t$ be any deviation. To ease notation, we pose for each $m \in M_A$

$$(\pi')_A^t(m|c_\ell) = \kappa(m) \tag{B.4}$$

$$(\pi')_A^t(m|c_h) = \xi(m) \tag{B.5}$$

Also let us write to simplify, $\pi_B^t(p|c_\ell) = \varepsilon(\mu)^{1/2} = \sigma(\mu)$. With the deviations we have the following posteriors, for all $\in M_A$:

$$\mu^{t+1}(m,w) = q \tag{B.6}$$

$$\mu^{t+1}(m,p) = \frac{\mu\kappa(m)\sigma(\mu)q + (1-\mu)\xi(m)(1-q)}{\mu\kappa(m)\sigma(\mu) + (1-\mu)\xi(m)} \tag{B.7}$$

First we show that we cannot put all posteriors of the form (B.7) below the threshold μ^*. For any message $m \in M_A$ a necessary condition for $\mu^{t+1}(m,p) \leq \mu^*$ is:

$$\xi(m) \geq \frac{\kappa(m)\sigma(\mu)}{\varepsilon(\mu)} \tag{B.8}$$

Then, summing up for each $m \in M_A$:

$$\sum_{m \in M_A} \xi(m) = 1 \geq \frac{\sum_{m \in M_A} \kappa(m)\sigma(\mu)}{\varepsilon(\mu)} \tag{B.9}$$

$$\sum_{m \in M_A} \xi(m) = 1 \geq \frac{\sigma(\mu)}{\varepsilon(\mu)} \tag{B.10}$$

Which cannot hold because $\varepsilon(\mu) \in (0,1)$ so $\sigma(\mu) > \varepsilon(\mu)$. So we can only put $|M_A| - 1$ posteriors below μ^*. Suppose we exclude $\tilde{m} \in M_A$, and set the others to be smaller than the threshold.We show that there does not exists any profitable deviation.

First since $\mu^{t+1}(\tilde{m},p) > \mu^*$ A_t expected gain would be

$$\mu[1 - \sigma(\mu) + \kappa(\tilde{m})\sigma(\mu)](c_\ell^q) + (1-\mu)\xi(\tilde{m})(q(-c_h) + (1-q)(-c_\ell))$$

So it would be optimal for A_t to set $\xi(\tilde{m}) = 0$ since it does not affect any restriction. Now, summing (B.8) for every $m \neq \tilde{m}$:

$$\sum_{m \in M_A \setminus \{\tilde{m}\}} \xi(m) = 1 \geq \frac{\sum_{m \in M_A \setminus \{\tilde{m}\}} (\kappa(m)\sigma(\mu))}{\varepsilon(\mu)} \tag{B.11}$$

$$\frac{\varepsilon(\mu)}{\sigma(\mu)} \geq \sum_{m \in M_A \setminus \{\tilde{m}\}} \kappa(m) = 1 - \kappa(\tilde{m}) \tag{B.12}$$

$$\kappa(\tilde{m}) \geq \frac{\sigma(\mu) - \varepsilon(\mu)}{\sigma(\mu)} \tag{B.13}$$

So the best A_t can do is to decrease $\kappa(\tilde{m})$ while (B.13) still holds, until it binds. So senders expected gain would be at most:

$$\mu\left[1 - \sigma(\mu) + \left(\frac{\sigma(\mu) - \varepsilon(\mu)}{\sigma(\mu)}\right)\sigma(\mu))\right](c_\ell^q) = \mu(1 - \varepsilon(\mu))(c_\ell^q) \tag{B.14}$$

So there do not exist a profitable deviation, because even in the best case scenario of $\xi(\tilde{m}) = 0$ and $\kappa(\tilde{m})$ to bind the constraint, the best they can do is to equal the gains form the original strategy. A similar reasoning can be applied if the sender want to put any of $|M_A| - 1$ beliefs below the threshold, so we conclude that the proposed strategy in an IDE. □

Now we can show that this equilibrium achieves an upper-bound on payoffs to the sender-receiver game when receivers are playing a cutoff strategy.

Lemma B.1. *Fix $t \geq 1$ and assume the belief hold by senders before the state is revealed is $\mu > \mu^*$. We claim that $\mu(1-\varepsilon(\mu))(c_\ell^q)$ is an upper-bound to expected gains in the sender-receiver game*

Proof. Fix $t \geq 1$ and assume the belief hold by senders before the state is revealed is $\mu > \mu^*$. The proof follows from the single-sender benchmark. Suppose, by way of contradiction, that $\exists \tilde{\pi}^t = (\tilde{\pi}_A^t, \tilde{\pi}_B^t)$ such that the expected gain under this joint strategy is $V > \mu(1-\varepsilon(\mu))(c_\ell^q)$. We will find a contradiction by stating that if the above condition holds, the single sender could also had achieved it. Consider the sets:

$$M_1 := \{m \in M, : \mu^{t+1}(m) \leq \mu^*\}$$

$$M_2 := \{m \in M, : \mu^{t+1}(m) > \mu^*\}$$

With μ^{t+1} being the posterior obtained with the joint strategy $\tilde{\pi}^t$, while we simplify the assumption and just consider the belief held μ and the last message m:

$$\mu^{t+1}(m) = \frac{\mu\pi^t(m|c_\ell)q + (1-\mu)\pi^t(m|c_h)(1-q)}{\mu\pi^t(m|c_\ell) + (1-\mu)\pi^t(m|c_h)} \tag{B.15}$$

We let $\pi_S : \Theta \longrightarrow \Delta(\mathbb{S})$ be such that:

$$\pi_S(w|\theta) = \sum_{(m_A,m_B)\in M_2} \tilde{\pi}_A(m_A|\theta)\tilde{\pi}_B(m_B|\theta) \tag{B.16}$$

$$\pi_S(p|\theta) = \sum_{(m_A,m_B)\in M_1} \tilde{\pi}_A(m_A|\theta)\tilde{\pi}_B(m_B|\theta) \tag{B.17}$$

For $\theta \in \{c_\ell, c_h\}$. Clearly we have:

$$\pi_S(p|\theta) + \pi_S(w|\theta) = \sum_{m_A\in M_A} \pi_A(m_A|\theta)\Big[\sum_{m_B\in M_B} \pi_B(m_B|\theta)\Big] = 1 \tag{B.18}$$

So it is a feasible signal for single sender S. Under π_S the posteriors are;

$$\mu^{t+1}(w) = \frac{\mu\pi_S(w|c_\ell)q + (1-\mu)\pi_S(w|c_h)(1-q)}{\mu\pi_S(w|c_\ell) + (1-\mu)\pi_S(w|c_h)} = \sum_{m\in M_2} \frac{Prob(m)}{Prob(w)}\mu^{t+1}(m) > \mu* \tag{B.19}$$

$$\mu^{t+1}(p) = \frac{\mu\pi_S(p|c_\ell)q + (1-\mu)\pi_S(p|c_h)(1-q)}{\mu\pi_S(p|c_\ell) + (1-\mu)pi_S(p|c_h)} \sum_{m\in M_1} \frac{Prob(m)}{Prob(p)}\mu^{t+1}(m) \leq \mu* \tag{B.20}$$

So we have receivers would play (P, P) after the single-sneder message p, and (W, W) after w.Thus, the single sender payoff will be:

$$\mu\pi_S(w|c_\ell)(c_\ell^q) + (1-\mu)\pi_S(w|c_h)(c_h^q) = V \tag{B.21}$$

So, we conclude that the single sender could choose π_S which will get a strictly greater payoff than his optimal choice, which is a contradiction. □

We've showed then, that under a cutoff strategy μ^*, the game with multiple sender achieve the upper-bound equilibrium payoff with a pair of strategies that replicate the single sender preferred outcome. Under this result, we've shown that both sender could be both *committed* and in *collusion* to achieve the maximum expected payoff of equilibrium. Each one of them is designing an information structure, that combined with the other sender, achieves the upper bound.

Also,it must be stated that there exists many other equilibria that replicate this outcome. For instance, one can show a sender committing always inducing peace after the the good state c_h, and inducing peace at c_ℓ with probability $\varepsilon(\mu)$, while the other is babbling is also an equilibrium with the maximum gain. In summary, any pair of strategies from senders that induces peace at c_ℓ with *joint probability* of $\varepsilon(\mu)$, while always induces peace at the good state, will be one such that the single sender preferred outcome is reached.