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Can a Habit Formation Model really explain the Forward Premium Anomaly?*

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Abstract

Verdelhan (2009) shows that if one is to explain the foreign exchange forward premium behavior using Campbell and Cochrane (1999)'s habit formation model one must specify it in such a way to generate pro-cyclical short term risk free rates. At the calibration procedure, we show that this is only possible in Campbell and Cochrane's framework under implausible parameters specifications given that the price-consumption ratio diverges in almost all parameters sets. We, then, adopt Verdelhan's shortcut of fixing the sensivity function $\lambda(s_t)$ at its steady state level to attain a finite value for the price-consumption ratio and release it in the simulation stage to ensure pro-cyclical risk free rates. Beyond the potential inconsistencies that such procedure may generate, as suggested by Wachter (2006), with pro-cyclical risk free rates the model generates a downward sloped real yield curve, which is at odds with the data. *Keywords: Forward Premium Puzzle, Equity Premium Puzzle, Habit formation, Asset Pricing; J.E.L. codes: G12, G15.*

Introduction

The forward premium anomaly or Forward Premium Puzzle – FPP, henceforth - is closely related to the failure of Uncovered Interest Parity – UIP henceforth – relation, i.e. if covered interest parity holds then the forward discount and the interest differential should be unbiased predictors of the ex-post change in the spot rate, assuming rational expectations. The UIP arises from a simple arbitrage argument in a risk neutral world. Higher interest rates imply currency devaluation so that the expected returns on domestic and foreign short-term bonds are equalized. Once we depart from risk neutrality, violations of UIP need not be puzzling, it is always possible to define a risk premium that accommodates the behavior of excess returns on foreign markets.

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The relevant questions are whether this risk premium is derived from a sensible model and whether it is compatible with the behavior of other asset prices. The main difficulty one faces in trying to accommodate foreign market facts with a risk model is the poor performance of most asset pricing models, as evidenced by the Equity Premium Puzzle—EPP.

It is in this context that the results in Verdelhan (2009) seem very heartening. There it is shown that the external habit model of Campbell and Cochrane (1999) may, at the same time, account for most stylized facts in both domestic and foreign markets.

Campbell and Cochrane (1999) external habit formation model delivers time-varying countercyclical risk premia. In bad times investor's risk aversion is higher, so he demands a larger risk premium. Hence, when consumption falls, expected returns, return volatility and the price of risk rise. Economic fluctuations generate in the model important aspects of asset prices behavior: (a) long-horizon predictability of excess returns from consumption-price ratio; (b) mean reversion in returns; (c) high stock price and return volatility despite smooth consumption growth. All these characteristics make the model a suitable candidate for the exercise conducted by Verdelhan (2009).

In this paper, we follow Verdelhan's lead and calibrate Campbell and Cochrane (1999) model for US post-war data and created endowment processes for fictitious countries based on US data. The process of each fictitious country is generated arbitrarily aiming to reach a target correlation with US consumption data. Thus, we can use the same model to generate the stochastic discount factor for each fictitious country, which allow us to evaluate the real exchange rate between US and its counterparts. So, we are able to evaluate an appropriate forward premia in currency markets which fits domestic markets asset behavior.

Interest rates generated by the model must be pro-cyclical to reproduce the forward discount anomaly, as shown by Verdelhan (2009). The problem with this imposing a parametrization that generates pro-cyclical risk-free rates is that the price-consumption ratio delivered by the model diverges for almost all calibration parameters sets. Moreover, not only a very narrow range of model parameters results in price-consumption ratio convergence, but also all parameters in that range are not capable of generating a reasonable behavior of other variables. How, one might ask, is this compatible with Verdelhan's claim of successful results? When we adopt a stricter assumption on the time series behavior of the surplus function (which relates consumption and habit in a tractable manner) at model calibration stage, we are able to match a finite price-consumption ratio, now with reasonable parameters. In such a way, we can calibrate the model under pro-cyclical interest rates and match the forward discount anomaly

simultaneously.

As we will show, this assumption has a significant impact on real yields. In contrast with empirical data, which usually exhibits an upward sloping real yield curve, we show that when risk free rates are pro-cyclical the habit formation model delivers a downward sloping real yield curve. This behavior is at odds with evidence in almost all data sets.

In a broad sense, although our results seem to indicate that the original Campbell and Cochrane consumption-based model is not able to produce the behavior of exchange markets we cannot state it for sure, given the inconsistency in the procedure. Indeed, this another drawback of this application of the model: the calibration shortcut used to reach a finite price-consumption ratio and all its potential inconsistencies. These potential inconsistencies should result given that stock market returns depend on price-consumption ratio.

The remainder of the paper is organized as follows. Section 2 explains the FPP puzzle and presents UIP null hypothesis in nominal and real terms. Section 3 describes the model and how the exchange rates are attained. Moreover, is derived a sufficient and necessary condition to reproduce the FPP evidence in simulated data. Section 4 exhibits the methodology and simulation results after having imposed a constant sensibility function, besides a critical analysis about that assumption. Section 5 concludes.

1 Literary Review: a glance at the FPP

1.1 The null hypothesis in nominal terms

Following the seminal work of Hodrick (1987), a large volume of research aiming at evaluating the efficiency of forward markets for foreign exchange has been produced. The focus of such agenda has been to explain the most puzzling aspect of this market's behavior: an apparent large conditional bias in the use of forward rates to forecast the future spot exchange rates.

If a market is said to be efficient, in equilibrium, all agents have access to all relevant information in the market and possibilities of excesses of returns by arbitrage are impossible.[1]

In an efficient market with rational expectations and that risk neutrality by the agents, the expected earnings in keeping a foreign currency more valued than the domestic will be compensated by the opportunity cost of maintaining foreign assets instead domestic assets.

[1] See Taylor (1995), for a very enlightening survey.

The following expression,

$$1 + i_t = (1 + i_t^*) \frac{E_t \mathcal{E}_{t+1}}{\mathcal{E}_t}, \tag{1}$$

for UIP holds, where i_t is the domestic nominal interest rate; i_t^* is the equivalent foreign nominal interest rate, $\mathcal{E}_t$ and $E_t \mathcal{E}_{t+1}$ are, respectively, the spot exchange rate at time t and the expected future spot exchange rate evaluated in units of domestic currency. In all that follows, E_t represents the mathematical expectation conditional on information available to the market at time t.

Thus, in logs, and if expectations are rational, $\mathsf{e}_{t+1} - \mathsf{e}_t = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + \epsilon_{t+1}$ with ϵ_{t+1} uncorrelated with time t variables representing the error on future spot exchange rate expectations. Using log-approximation $\log(1 + x) \sim x$, the expression (1) becomes

$$E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) = E_t \Delta \mathsf{e}_{t+1} = i_t - i_t^* \tag{2}$$

with $\mathsf{e}_t = \log(\mathcal{E}_t)$. Using the covered interest parity relation in logs $f_t = i_t - i_t^* + \mathsf{e}_t$, where f_t represents the log of exchange rate one period forward, the expression (2) can be rewritten as the forward discount version of UIP

$$E_t \Delta \mathsf{e}_{t+1} = \alpha + \beta(f_t - \mathsf{e}_t) + \varepsilon_{t+1} \tag{3}$$

This regression appears frequently in the literature. Generally, the null hypothesis tested is $\alpha = 0$ and $\beta = 1$ assuming that $E_t[\varepsilon_{t+1} \mid \Omega_t] = 0$ and $E_t[\varepsilon_{t+1}(f_t - \mathsf{e}_t) \mid \Omega_t] = 0$ where Ω_t is the available information set in period t. Under this null hypothesis, f_t is an unbiasedness predictor of spot exchange rate one period forward.

However, empirical studies like Hansen and Hodrick (1980) and many others,[2] have produced strong evidence against the unbiasedness hypothesis showing an apparent large conditional bias in models which use forward rates to forecast the future spot exchange rates. A large volume of studies have found that currency devaluation are negatively correlated with the cross-country interest differentials, i.e. currencies with higher interest rates tend to appreciate. This outcome is often referred to as the forward discount anomaly. Indeed, according to Froot (1990), the mean estimate of β over 75 empirical works is -0.88, challenging on empirical grounds the unbiasedness hypothesis.

Because this widespread evidence poses a serious challenge to our understanding of international financial markets, the nature of this anomaly has been the focus of an

[2] Hodrick (1987) has surveyed a large number of works on efficiency tests of foreign exchange forward markets.

enormous literature concerning the efficiency of these markets. We can mention three main lines of work aiming at explaining the FPP. First, the existence of a distortion on the component of rational expectations carried by exchange market participants turns the forward discount anomaly a rejection of the rational expectation hypothesis. Second, market imperfections as source of asymmetries in foreign exchange markets, which are supported by evidence of conditional-mean nonlinearities in risk premia. Third, the presence of a risk premium not appraised in the UIP expression.

Concerning the first research line, one can mention as example Rogoff (1979) who suggested that, in a context where agents attribute a low probability for rare nature states (for instance, great changes in economy foundations) an asymmetry will appear in a prior event distribution, the well-known *Peso Problem*. Froot and Frankel (1989) attributed the anomaly to expectation errors. In a recent work, Bacchetta and Van Wincoop (2006) developed a model of rational inattention that produces sticky prices and it is consistent with the FPP. The role of market imperfections in FPP explanation, as the second research line, could be seen, for instance, in Coakley and Fuertes (2001) where authors considered an extension of the Dornbusch exchange rate overshooting theory as a special case of asymmetric or nonlinear behavior in foreign exchange markets that fits the FPP.

The third line of research, perhaps the most natural explanation for why the forward premium predicts the wrong direction of exchange rate movements, focuses on a wedge between expected changes and actual changes driven by a risk premium. How to model the risk premium is the challenge of this literature and the purpose of our work. Hodrick (1987) concluded: "We do not yet have a model of expected returns that fits the data" (p.157). Engel (1996) provides a survey.

Many researchers tried to reproduce models of asset pricing that taking into account the forward premium correctly in the cross-country exchange market. Those models reproduced a negative slope coefficient when the theoretical spot rate changes are regressed on the theoretical lagged forward premium. See e.g. Backus et al. (1995), Bekaert (1996), Bekaert et al. (1997) and Macklem (1991). Unfortunately, many of these models cannot explain most of already mentioned foreign exchange market stylized facts.

A specific subgroup of rational models that departs from risk neutrality is that formed by consumption-based models. Unfortunately most of these models cannot reproduce many stylized facts from domestic asset markets, the most recognized of these failures being the Equity Premium Puzzle – EPP. In few words, it is based on the observation that in order to reconcile the much higher return on equity stock compared to

US government bonds,[3] individuals must have implausible high risk aversion according to classical models. In fact, the consumption-based asset pricing model (CCAPM) with power utility fails to explain important facts about stock returns, including the high equity premium, the high volatility of returns and the countercyclical variation in the equity premium.[4]

However, consumption-based models have been resurrected for forward risk premia evaluations by appealing to more sophisticated preferences. Moore and Roche (2007) relies upon external habit preferences imbedding them into a monetary model. More recently, Verdelhan (2009) has also forwarded a model using external habit preferences, which led to quantitatively large risk premia and matched the variance of real exchange rates. The purpose of our work is very similar to the forthcoming work by Verdelhan. Contrary to him, our findings are not favorable to the model.

1.2 The null hypothesis in real terms

The condition for the absence of profit opportunities by a risk neutral domestic investor endowed with rational expectations from forward market speculation is

$$E_t\left(\frac{F_t - \mathcal{E}_{t+1}}{P_{t+1}}\right) = 0,$$

where P_{t+1} is the domestic price of goods.

For simplicity, assume that all variables above are conditionally log-normally distributed.[5] We may, in this case, rewrite the expression above as

$$E_t(\mathsf{e}_{t+1}) = f_t - 0.5Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p_{t+1}).$$

Note that the risk premium $rp_t = f_t - E_t(\mathsf{e}_{t+1})$ for the risk neutral investor is not zero because the term $0.5Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p_{t+1})$ is not zero. This term is the Jensen inequality term—JIT.[6] Thus, f_t is not necessarily a conditional predictor of e_{t+1} despite the very small absolute value of JIT. According to Engel (1996), most empirical works do not consider the JIT due to its very small size.

[3] This feature prevails in many other industrialized countries, as pointed out by Kocherlakota (1996).

[4] See Grossman and Shiller (1981), Shiller (1981) and Kandel and Stambaugh (1990). For a deeper discussion about EPP, see Kocherlakota (1996) and Mehra and Prescott (2003). Both reviews present a detailed analysis of these explanations in financial markets and conclude that the puzzle is real and remains unexplained. Subsequent reviews of the literature have similarly found no agreed upon resolution.

[5] An analytically convenient assumption, very common in finance literature, though not very successful on empirical grounds.

[6] This subsection is based on a survey by Engel (1996), where the reader can find a deeper discussion about the Jensen inequality term.

The true risk premium can, nonetheless, be calculated as follows

$$trp_t \equiv f_t - E_t(\mathsf{e}_{t+1}) - 0.5Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p_{t+1}) = rp_t - JIT.$$

By the same token, a foreign risk neutral investor would require a risk premium given by

$$trp_t = f_t - E_t(\mathsf{e}_{t+1}) - 0.5Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p^*_{t+1} + \mathsf{e}_{t+1}),$$

where p^*_{t+1} is the log of the foreign price in foreign currency. The two expressions above will be the same when the purchasing power parity—PPP—condition holds, i.e.,

$$p_{t+1} = p^*_{t+1} + \mathsf{e}_{t+1}.$$

If PPP does not hold, then domestic and foreign investors evaluate real returns differently, so that there would be no equilibrium with risk neutral investors in each country.

The null hypothesis described in the latter subsection is equivalent to

$$rp_t = f_t - E_t(\mathsf{e}_{t+1}) = 0.$$

Considering the covered interest parity again and risk neutrality of domestic investor we can rewrite the null hypothesis as

$$i_t - i^*_t = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + 0.5Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p_{t+1}). \tag{4}$$

When a foreign investor is assumed to be risk neutral, the null is

$$i_t - i^*_t = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + 0.5Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p^*_{t+1} + \mathsf{e}_{t+1})) \tag{5}$$

So, we can express each the nulls (4), (5) as

$$i_t - E_t(p_{t+1} - p_t) = \left[i^*_t - E_t(p^*_{t+1} - p^*_t)\right] + E_t\left[(p^*_{t+1} + \mathsf{e}_{t+1} - p^*_t - \mathsf{e}_t) - (p_{t+1} - p_t)\right] + JIT \tag{6}$$

where the second term in right side of (6) could be referred as PPP deviation term. Note that the JIT terms will be different between foreign and domestic investors, unless PPP holds.

Using the Fisher relation between nominal and real interest rates, then

$$r_t - r^*_t = E_t\left[(p^*_{t+1} + \mathsf{e}_{t+1} - p^*_t - \mathsf{e}_t) - (p_{t+1} - p_t)\right] + JIT.$$

Assuming there are no arbitrage opportunities in forward exchange market, the law of one price holds, then $p_t = p_t^*$, $\forall t$. Besides, following a large number of papers in this literature, we will assume that $JIT \approx 0$. Thus,

$$E_t \Delta \mathsf{e}_{t+1} = r_t - r_t^*.$$

Further in section 3, under complete markets assumption we will be able to rewrite the UIP expression in real terms

$$E_t \Delta q_{t+1} = r_t - r_t^* \tag{7}$$

where q_{t+1} denotes the log of real exchange rate express in domestic goods.

2 The model

To depart from risk neutrality, one would like to have a model for the risk premium. We have chosen to use Campbell and Cochrane (1999) external habit model. The model is, in some sense, a reverse engineering procedure aimed at reproducing many of the stylized facts in domestic markets. Our idea is to calibrate the model as in Campbell and Cochrane (1999) to handle the behavior of US domestic asset markets, then, to assume complete foreign markets in order to make predictions about foreign exchange.

We start by presenting Campbell and Cochrane (1999)'s model.

2.1 An external habits pricing model

Campbell and Cochrane (1999) is a variant of presented by Abel (1990) external habit model. Identical investors have preferences over consumption that depend on a reference point X_t, the habit. Preferences are represented by a CRRA utility function,

$$U(C_t, X_t) = E\left[\sum_{t=0}^{\infty} \delta^t . \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}\right],$$

where $\delta > 0$ is the subject discount parameter of time preferences and $\gamma > 0$ is the utility curvature parameter.

It is convenient to model the behavior of habits indirectly through another variable called *consumption surplus ratio*, S_t, defined as

$$S_t \equiv \frac{C_t^a - X_t}{C_t^a},$$

where C_t^a is the aggregate consumption.

In equilibrium, given identical agents, $C_t^a = C_t$. So, the agent's coefficient of relative risk aversion rate is simply

$$-\frac{C_t u_{cc}}{u_c} = \frac{\gamma}{S_t}.$$

The expression above shows that S_t works as a proxy variable for economic recession periods, in which the risk aversion raises because S_t becomes low and vice-versa. This feature provides a time-variant risk aversion for investors which, as we are going to see soon, results in countercyclical risk premia for asset returns. So, the agent's main concern is the decline of consumption relative to the external habits. This latter feature characterizes this model as a *Catching Up with the Joneses* model, to use the terminology of Abel (1990). This model differs, however, from Abel's model in two aspects. First, the agent's risk aversion varies with the level of consumption relative to habit, whereas risk aversion is constant in Abel's model. And, second, consumption must always be above habit for utility to be well defined, whereas this is not required in Abel's model.

To ensure that consumption is always above habit, the model specifies a non-linear process by which habit adjusts to consumption, remaining below consumption level at all times. We specify this process next.

Consumption growth is an i.i.d. log-normal process $\Delta c_{t+1} = g + v_{t+1}$, where $\Delta c_{t+1} = \log(C_{t+1}) - \log(C_t)$, g is the mean of the process and $v_{t+1} \sim i.i.d.\mathcal{N}(0, \sigma_v^2)$. The process for the log surplus consumption ratio, $s_t = \log(S_t)$, is assumed to be a heteroskedastic AR(1) model,

$$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \lambda(s_t) v_{t+1}, \tag{8}$$

where $\bar{s}$ represents steady-state log consumption surplus, ϕ the habits persistence parameter and $\lambda(s_t)$ a sensitivity function to innovations in consumption growth v_{t+1}.

Notice that the process of s_t is heteroskedastic and perfectly conditionally correlated with innovations in consumption process. The sensitivity function is originally built to completely offset the intertemporal substitution and precautionary savings effects in Campbell and Cochrane (1999), thus making the real risk-free rate constant. However, we will specify $\lambda(s_t)$ so that the real risk-free rate is linear in s_t and $x_{t+1} \equiv \log X_{t+1}$ is a deterministic function of past consumption on a neighborhood of $\overline{s}$, $s_t \approx \overline{s}$.[7]

Taking a linear approximation around the steady state in (8) the model can be shown to be a traditional habit formation model in which log habit responds slowly to

[7]This specification is originally due to Campbell and Cochrane (1995).

log consumption

$$x_{t+1} \approx \left[(1-\phi)\ln(1-\overline{S})+g\right]+\phi x_t+(1-\phi)c_t$$
$$= \left[\ln(1-\overline{S})+\frac{g}{1-\phi}\right]+(1-\phi)\sum_{i=0}^{\infty}\phi^i c_{t-i},$$

where $\ln(1-\overline{S})$ is the steady-state value of $(x_t - c_t)$.

These features imply that

$$\lambda(s_t) = \begin{cases} \frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})}-1 & ; \quad s_t \leq s_{\max} \\ 0 & ; \quad s_t > s_{\max} \end{cases}, \tag{9}$$

$$\overline{S} = \sigma_v\sqrt{\frac{\gamma}{1-\phi-\frac{b}{\gamma}}}, \tag{10}$$

and

$$s_{\max} = \overline{s}+\frac{1}{2}(1-\overline{S}^2), \tag{11}$$

where b is a preference parameter that determines the behavior of interest rates and has an important economic interpretation that we shall explained soon.

Given that the s_{t+1} process is driven by (8), using (9), and knowing that habits are external, the marginal utility of consumption is given by $u'(c_t) = (C_t - X_t)^{-\gamma} = (S_t C_t)^{-\gamma}$. We, thus, obtain the pricing kernel

$$M_{t+1} = \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \delta \mathsf{e}^{-\gamma[g-(1-\phi)(s_t-\overline{s})+(1+\lambda(s_t))v_{t+1}]}. \tag{12}$$

The reverse engineering nature of the sensitivity function is easily understood by taking into account that it was built to satisfy four properties: i) the domain of the pricing kernel is $\mathbb{R}_+$; ii) the natural logarithm of the risk-free rate, r_t^f, is linear in s_t; iii) the derivative of x_t with respect to c_t is zero at $\overline{s}$, and; iv) the second derivative of x_t with respect to c_t is zero at $\overline{s}$. Note that the last two properties implies that habits are predetermined at the steady-state and near it, so it moves non-negatively with consumption everywhere.

Using the fundamental pricing equation for any asset returns R_{t+1}

$$E_t\left[M_{t+1}R_{t+1}\right] = 1$$

and denoting $r_{t+1}^f \equiv \ln(R_{t+1}^f)$ as the one-period real risk-free rate, taking logs and using

(12) and (8) the model delivers

$$
\begin{aligned}
r_t^f &= \ln(1/E_t\left[M_{t+1}\right]) \\
&= -\ln\delta + \gamma g - \gamma(1-\phi)(s_t - \overline{s}) - \frac{(\gamma\sigma_v)^2}{2}\left(1+\lambda(s_t)\right)^2
\end{aligned} \tag{13}
$$

The first two terms are familiar from the power utility. The third term reflects intertemporal substitution, or mean-reversion in marginal utility. If the surplus consumption ratio is low, the marginal utility of consumption is high. However, the surplus consumption ratio is expected to revert to its mean, so marginal utility is expected to fall in the future. Therefore, the consumer would like to borrow and this drives up the equilibrium risk-free rate. The fourth term reflects precautionary savings. As uncertainty increases, consumers become more willing to save and this drives down the equilibrium risk-free rate.

Although (13) is, in fact, an approximation, for it is derived under the assumption that there is zero probability that s_t exceeds $s_{\max}$, the approximation is highly accurate, as discussed by Wachter (2005).

Substituting $\lambda(s_t)$ from (9) in (13) we obtain

$$
r_t^f = -\ln\delta + \gamma g - \frac{\gamma(1-\phi)-b}{2} - b(s_t - \overline{s}). \tag{14}
$$

As mentioned before, b ascribes important economic interpretations to the model. If $b > 0$, the intertemporal smoothing effect dominates the precautionary savings effect and an increase in the surplus consumption ratio, s_t, drives down the interest rate, so that interest rates are anti-cyclical. If $b < 0$, the precautionary savings effect dominates and an increase in the surplus consumption ratio, s_t, reduces the sensitivity function and drives up the interest rate so that interest rates are pro-cyclical. Setting $b = 0$ results in constant real interest rates because the two effects cancel each other and drives the results presented in Campbell and Cochrane (1999).

While the functional form of $\lambda(s_t)$ is chosen to match the behavior of the risk-free rate, it has important implications for returns on risky assets too. It follows from Euler equation that the Sharpe ratio of any asset return must obey

$$
\frac{E_t(R_{t+1}^e)}{\sigma_t(R_{t+1}^e)} = -\rho_t(M_{t+1}, R_{t+1}^e)\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}
$$

where $R_{t+1}^e = R_{t+1} - R_{t+1}^f$ and ρ_t denotes conditional correlation.

As M_{t+1} is lognormally distributed, we have the following approximation using

(12),[8]

$$\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} \approx -\gamma\rho_t(M_{t+1}, R^e_{t+1})\sigma_v\left(1+\lambda(s_t)\right). \tag{15}$$

Because $\lambda(s_t)$ is decreasing in s_t, the ratio of the volatility of the stochastic discount factor with respect to its mean varies countercyclically. This provides a mechanism by which Sharpe ratios and risk premia vary countercyclically over time.

Campbell and Cochrane models stocks as a claim to the consumption stream, taking stocks to represent the wealth portfolio.[9] Using the Euler equation, one can verify that the price-consumption ratio for a consumption claim satisfies

$$\frac{P_t}{C_t}(s_t) = E_t\left[M_{t+1}\frac{C_{t+1}}{C_t}\left[1+\frac{P_{t+1}}{C_{t+1}}(s_{t+1})\right]\right], \tag{16}$$

where P_t denotes the ex-dividend price of this claim.

Note that s_t is the only state variable in expression above. The model is solved by substituting the stochastic discount factor expression (12) and endowment process $\exp(\Delta c_{t+1}) = \exp(g + v_{t+1})$ into (16) and solving it by numerical integration for a grid of s_t over the normally distributed shock v_{t+1}. This allows one to evaluate conditional expectations and to determine the price-consumption ratio fixed-point. Then, we must calculate expected and conditional standard deviations of returns to match calibration parameters over the real data.

Evaluation of simulated stochastic discount factors and, consequently, risk-free rates, are easier. We simulate the model by drawing consumption shocks v_{t+1} and feeding (8) with these draws. In this fashion, we obtain draws for s_t and use them to attain the simulated risk-free rate and stochastic discount factor.

2.2 Exchange rates

Some extra notation is needed to introduce foreign markets. We following the presentation of Lustig and Verdelhan (2006) and Lustig and Verdelhan (2007). Let $\mathcal{E}^i_t$ be the nominal exchange rate defined at amount of domestic currency that one must pay for a unit of country i currency as described in Section 2, and $R^{f,i}_{t,t+1}$, the one-period return of the risk-free rate in country i currency units. We shall associate this latter with the nominal interest rate that drives the returns of a one-year discount bond. Let

[8] Assume $\log M \sim iid(\mu, \sigma^2)$. Then,

$$\frac{\sigma(M)}{E(M)} = \sqrt{\frac{E(M^2)-E(M)^2}{E(M)^2}} = \sqrt{\frac{e^{2\mu+2\sigma^2}-e^{2\mu+\sigma^2}}{e^{2\mu+2\sigma^2}}} = \sqrt{e^{\sigma^2}-1} \approx \sigma$$

[9] This strong assumption represents an extra challenge to the model since, in their callibration exercise, stock prices are generated by the model.

finally $R^{i,\$}_{t,t+1}$ denotes the domestic investor exchange risk return from buying a foreign one-year discount bond in country i, selling the payoff - one unit of foreign currency - after one year and converting the proceeds back into domestic currency. Using this notation the following expression obtains

$$1 + R^{i,\$}_{t,t+1} = 1 + R^{f,i}_{t,t+1} \left(\frac{\mathcal{E}^i_{t+1}}{\mathcal{E}^i_t} \right).$$

Assuming no-arbitrage in bond markets, absence of frictions (e.g. *bid-ask spreads* and *short-sale constraints*), and market completeness, we have:

(i) Domestic investors who acquire a foreign one-year discount bond in country i have the following Euler equation:

$$E_t \left[M_{t+1} R^{i,\$}_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right) \right] = E_t \left[M_{t+1} R^{f,i}_{t,t+1} \left(\frac{\mathcal{E}^i_{t+1}}{\mathcal{E}^i_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{17}$$

where

$$R^{i,\$}_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right)$$

denotes the return in domestic consumption units from foregoing investment and P_t the price of a domestic consumption unit in period t. By the same token, the Euler equation for a foreign investor's who made the same investment in country i bonds is

$$E_t \left[M^i_{t+1} R^{f,i}_{t,t+1} \left(\frac{P^i_t}{P^i_{t+1}} \right) \right] = 1, \tag{18}$$

where P^i_t denotes the price of a country's i consumption unit in period t.

(ii) If no-arbitrage assumption holds it implies the law of One Price. Consequently,

$$\left(\frac{P_t}{P_{t+1}} \right) = \left(\frac{P^i_t}{P^i_{t+1}} \right)$$

and we obtain the equivalence between nominal and real exchange rates, i.e.:

$$\left(\frac{\mathcal{E}^i_{t+1}}{\mathcal{E}^i_t} \right) = \left(\frac{Q^i_{t+1}}{Q^i_t} \right),$$

where Q^i_t denotes the real exchange rate express in domestic goods.

(iii) Because markets are complete, there is only one stochastic discount factor in each

country,[10] which allows us to match (17) and (18). Hence,

$$\frac{Q^i_{t+1}}{Q^i_t} = \frac{M^i_{t+1}}{M_{t+1}}.$$

Taking logs in expression above we obtain

$$\Delta q^i_{t+1} = m^i_{t+1} - m_{t+1}. \tag{19}$$

Interpreting $log(1 + R^{i,\$}_{t,t+1}) = r^i_t$ - the country's i one-year discount bond log-return - as the abroad interest rate, substituting (7) above and considering that $r^i_t = -E_t m^i_{t+1}$, $\forall i$, we attain

$$E_t \Delta q^i_{t+1} = r_t - r^i_t \tag{20}$$

the expression that represents UIP for real terms.

2.3 A necessary and sufficient condition to reproduce the FPP

Here, we will show conditions under which the Campbell and Cochrane external habits model is able to reproduce a negative slope coefficient in regression

$$\Delta q^i_{t+1} = (m^i_{t+1} - m_{t+1}) = \alpha + \beta^{UIP}(r_t - r^i_t) + \epsilon_{t+1} \tag{21}$$

where ϵ_{t+1} is the regression error.

Finding $\beta^{UIP} < 0$ in simulated data would be an evidence that the FPP is a phenomenon, at least in part, explained by a model that fits coherently the risk premia. In our environment it is necessary to force interest rates to be pro-cyclical. That is we set the preference parameter that determines the interest rates behavior, b, to be negative. To show this dependence, we begin explaining how the model generates exchange risk premia.

The exchange risk premium is the excess returns obtained by an investor that makes the following operation: i) borrows resources in domestic currency to purchase foreign bonds; ii) converts these resources into foreign currency; iii) buy bonds and receives the yields at foreign risk-free rates; iv) resells the bond after a reference period, converting it again into domestic currency.

Excess returns can be described by three components: the real exchange rate de/appreciation during the income period; the yield rate from foreign country; and

[10] See Cochrane (2001), chapter 4.

the opportunity cost of not having acquired a domestic bond. In logs we obtain:

$$E_t r^e_{t+1} = E_t \Delta q^i_{t+1} + r^i_t - r_t. \tag{22}$$

Note that if there is no risk premium, the expression (22) is given by the UIP real version (20). So, given the log-normality of stochastic discount factor (12) we obtain

$$\log R^{f,i}_{t,t+1} = r^i_t = -\log E_t M^i_{t+1} = -E_t m^i_{t+1} - \frac{1}{2} Var_t(m^i_{t+1}).$$

Using the previous expression to evaluate domestic interest rate r_t, we can express $r_t - r^i_t$ as

$$r_t - r^i_t = E_t m^i_{t+1} - E_t m_{t+1} + \frac{1}{2}\left[Var_t(m^i_{t+1}) - Var_t(m_{t+1})\right].$$

In section 2.2, we have shown that $\Delta q^i_{t+1} = m^i_{t+1} - m_{t+1}$. Using this expression, we obtain the expectations form of (22),

$$E_t \Delta q^i_{t+1} = (r_t - r^i_t) - \frac{1}{2}\left[Var_t(m^i_{t+1}) - Var_t(m_{t+1})\right],$$

or

$$E_t r^e_{t+1} = \frac{1}{2}\left[Var_t(m_{t+1}) - Var_t(m^i_{t+1})\right]. \tag{23}$$

In Campbell and Cochrane's model,

$$Var_t(m_{t+1}) = (\gamma\sigma_v)^2 \left[1 + \lambda(s_t)\right]^2 = \frac{(\gamma\sigma_v)^2}{\overline{S}^2}\left[1 - 2(s_t - \overline{s})\right]. \tag{24}$$

Henceforth, we will adopt a simplifying assumption by imposing the same model parameters (g, σ_v, γ, ϕ, $\overline{S}$) for home and foreign countries. Given that s_t comes from stochastic process, it allows for each country to have its own s_t. Substituting (22), (24) in (23), the exchange risk premia may be found as

$$E_t \Delta q^i_{t+1} = \left(r_t - r^i_t\right) + \frac{(\gamma\sigma_v)^2}{\overline{S}^2}(s^i_t - s_t) \tag{25}$$

As we have seen in (14), the model delivers linear interest rates and is straightforward to see that $r_t - r^i_t = b(s^i_t - s_t)$. So, the expression above could be rewritten as

$$E_t \Delta q^i_{t+1} = \left[1 + \frac{(\gamma\sigma_v)^2}{b\overline{S}^2}\right](r_t - r^i_t).$$

Finally, replacing $\overline{S}$ with (10), we obtain

$$E_t \Delta q^i_{t+1} = \frac{\gamma(1-\phi)}{b}(r_t - r^i_t).$$

This calculation is found in Verdelhan (2009).

Note that the coefficient

$$\beta^{UIP} = \frac{\gamma(1-\phi)}{b}$$

must be negative to reproduce empirical findings in UIP estimation. This can be accomplished setting $b < 0$. Verdelhan gives a nice intuition to why pro-cyclical interest rates causes $\beta^{UIP} < 0$ in Campbell and Cochrane model.

In periods in which economy does poorly, domestic investors are more risk averse than foreign investor, so $s_t < s^i_t$. This delivers a higher conditional variance of stochastic discount factor at home than abroad as it can be seen in (24), due to $\lambda'(s_t) < 0$. So, the domestic consumption shocks effects on the real exchange rate dominate their foreign counterparts shocks. This relation can be seen by merging (12) and (19),

$$\Delta q^i_{t+1} = \gamma(1-\phi)(s^i_t - s_t) + \gamma[1 + \lambda(s_t)](\Delta c_{t+1} - g) - \gamma[1 + \lambda(s^i_t)](\Delta c^i_{t+1} - g).$$

The exchange rate decreases in response to a domestic negative consumption growth shock and *vice-versa*. Foreign currency is riskier the more risk averse the domestic investor relative to his foreign counterpart. Then, a greater risk premium is required by domestic investors for them to hold foreign bonds. As a result, the domestic investor gets a positive excess return if he is more risk averse than his foreign counterpart. Further, times of high risk aversion correspond to low interest rates. In other words, when s_t is low enough and is imposed pro-cyclical interest rates ($b < 0$) the equation (14) provides low interest rates.

Summarizing, domestic investors expects positive foreign bonds excess returns when two situations happen at the same time: they are more risk averse than their foreign counterparts and domestic interest rates are low and foreign ones are high. This results in $\beta^{UIP} < 0$. As we have seen, the Campbell and Cochrane model guarantees that these two situations always happened simultaneously when interest rates runs pro-cyclically.

2.4 Business cycle behavior of real interest rates

Interest rates primarily depend on policy and expectations, thus the relationships with the business cycle depend on explicit decisions and subjective judgements of key players (Central Banks, Holders, etc...). At this moment, it is important to verify stylized facts about real interest rates to establish if the main hypothesis in last subsection is

reasonable, i.e., if real interest rates keep up with economic cycles.

Ang and Bekaert (2002) summarized a set of stylized facts about nominal and real interest rates and inflation that are seen commonly in empirical papers. We quote them:

> Interest rates are often associated with the business cycle. According to the conventional wisdom, interest rates are pro-cyclical and spreads counter-cyclical (see, for example, Fama (1990)). In fact, interest rates are overall larger during recessions. However, when we focus on real rates, the conventional story is right.

The same conclusion is found in Veronesi and Yared (1999).[11,12]

3 Methodology and Results

In what follows, we present our results and our interpretation of their meaning for the model's capacity of accommodating the forward premium puzzle.

3.1 Creating fictitious countries

As in subsection 3.3, we assume that each pair, US/fictitious country i, can be characterized by the same set of parameters (g, σ_v, γ, ϕ, b). We draw values for v_{t+1} to form the US endowment shock process Δc_{t+1} and use it to generate the other country's consumption processes by setting their correlation ρ^i in the following manner:

$$\Delta c^i_{t+1} = \rho^i \Delta c_{t+1} + \sqrt{1 - (\rho^i)^2} \Delta c_{t+1} \tag{26}$$

where Δc^i_{t+1} is the consumption process for fictitious foreign country "i".

There is a subtle difference in these two simulated variables Δc_{t+1} presented in (26). Both represents the same US consumption process but are generated by different *seeds* in their Normal random process.[13] This assures that the two process are instantaneously different from one another, which allows us to create Δc^i_{t+1} with a controlled correlation value ρ^i. We input 21 different correlation values equally spread between

[11] In the work quoted above, Fama (1990), shows that the one-year U.S. interest rate is lower at the business trough than at the preceding or following peak in every business cycle of the 1952-1988 period.

[12] Other policy rules would imply different behavior. For instance, if the target is mainly inflation, during a stagflation period (a depressed GDP with high inflation) the interest rates may be particularly high. Thus, a counter-cyclical pattern would emerge.

[13] **Seed:** an integer used to set the starting point for generating a series of random numbers. The seed sets the generator to a random starting point. A unique seed returns a unique random number sequence.

−1 and 1. So, we have 21 different countries to evaluate exchange rates. Note that (26) did not attribute correlation values exactly as those we have input but they become very alike as the number of v_{t+1} draws grows.

3.2 Calibration

The task here is to calibrate the external-habit asset pricing model to US post-war data.[14] Following Campbell and Cochrane (1999), Wachter (2005) and Verdelhan (2009), we fix the intertemporal elasticity, γ, to 2. For other model parameters we adjust them to fit the mean and volatility of US consumption process, 90-day T-Bill mean rate, as the US risk-free asset, and average equity excess returns. The parameter ϕ is the first-order autocorrelation of price-consumption ratio verified in the data. It determines the speed of mean reversion exhibited by the s_t stochastic process described in (8).

The preference's free parameter b is quite difficult to determine and has been shown to be very controversial. Its absolute value does not make much difference in equity returns, as Wachter (2006) has pointed out, but its sign determines the slope of the term structure of interest rates generated by the model. In order to generate an upward-sloping yield curve and anti-cyclical interest rates, it is necessary that $b > 0$, i.e., the risk-free rate loads negatively on b and, consequently, negatively correlated with surplus consumption ratio. Wachter (2006) estimate $b = 0.011$ for US data in the 1952-2004 quarterly sample.[15]

However, in contrast to what is claimed by Wachter, there is also large empirical evidence in favor of pro-cyclical interest rates, as we have already discussed. Indeed, Ma (2006) uses pro-cyclical interest rates to investigate the macroeconomic conditions that accompany high interest rates and currency appreciation by observing the cyclical behavior of cross-country interest differentials and exchange rate movements.

[14] We are avoiding the well acknowledged structural break between pre and post-war data.

[15] Wachter (2006) conducted a brief investigation of real risk-free rate time-series implications. She ran the following regression

$$r^f_{t+1} - \Delta\pi_{t+1} = a_0 + a_1 \sum_{k=1}^{40} \phi^k \Delta c_{t-k} + \varepsilon_{t+1},$$

where the regressed variable is the ex-post real interest rate and the regressor is a consumption surplus ratio proxy for quarterly simulation, which is approximately equal to s_t. Data is quarterly, beginning in the second quarter of 1952 and ending in the second quarter of 2004.

3.3 Data sources

Annual US population and consumption expenditures data are from Robert Shiller's database between 1947 and 2004.[16] We use the return of 90-day Treasury bill deflated by the CPI index as our measure of US real interest rate. Stock market excess returns, Treasury bill rates and CPI are from CRSP.

3.4 Simulation

We simulate 100,000 quarters of data. To check the calibration of US data, the price-consumption ratio and, consequently, stock market returns and real yields we use a numerical algorithm based on a fixed-point evaluation of Euler equation (16), as in Campbell and Cochrane (1999).[17] For numerical procedures, we have used a grid of state variable S_t with 24 grid points, where 15 of them are equally distributed between $(0, S_{\max}]$, 4 additional points were added at intervals of 0.01 slightly bellow $S_{\max}$ and 5 other discretional points closer to zero.[18]

3.5 Is it possible to impose $b < 0$?

We have imposed $b < 0$ in the calibration, which we have already shown, in section 3, to be necessary and sufficient to reproduce a negative value for β^{UIP}. However, we were not capable of finding a finite value for the price-consumption ratio. The price-consumption ratio diverges for any reasonable set of calibration parameters when b is negative. Thus, we did not get to match other real data such as mean and volatility of equity returns, risk-free rate, etc..

To show the robustness of our calculations and rule out the possibility of computational errors we ran our algorithm using the calibration parameters of two well-known works: Campbell and Cochrane (1999) and Wachter (2005). Given that we are capable to find the fixed point of (16) only when $b \geq 0$, these papers are useful benchmarks for a robustness test, because b is non-negative in both works. The annualized parameters for each paper are summarized in table bellow.

[16] http://www.econ.yale.edu/~shiller/data/chapt26.xls

[17] To check the robustness of our results, we have also computed the equilibrium price-consumption ratio through the series algorithm proposed by Wachter (2005). All results are similar to the results found by the method of fixed point used in this paper.

[18] By discretional points we mean [.0005 .0015 .0025 .0035 .0045], as in Wachter (2005). We added additional density on grid ends following Campbell and Cochrane's advice. They argue that this procedure will improve numerical fixed-point calculation in artificial data.

Parameter	Symbol	Cochrane	Wachter
Used:			
Consumption growth mean	g	1.89	2.20
Consumption growth volatility	σ_v	1.5	0.86
Risk-free rate	r^f	0.94	–
Habits persistence	ϕ	0.87	0.89
Utility curvature	γ	2	2
Interest behavior coeff.	b	0	0.011
Derived parameters:			
Intertemporal discount factor	δ	0.89	0.93
Steady consumption surplus	$\overline{S}$	0.057	0.04
Max. consumption surplus	$S_{\max}$	0.094	0.07

Tab. 1 - Calibration parameters from Campbell and Cochrane (1999) and Wachter (2005)

Feeding our program with these parameters we reach the following results.[19]

Campbell and Cochrane (1999) - C&C

Statistic	C&C	Simulated
$E(\Delta c_{t+1})$ (%)	1.89	1.90
$\sigma(\Delta c_{t+1})$ (%)	1.22	1.20
$E(r^f)$ (%)	0.94	0.94
$\sigma(r^f)$ (%)	0.00	0.00
$E(r^m)$ (%)	6.64	6.68
$\sigma(r^m)$ (%)	15.20	15.08
Sharpe ratio	0.44	0.44
$E(p-d)$	2.91	2.90
$\sigma(p-d)$	0.27	0.27
$Corr(p-d)$	0.87	0.87

Tab. 2 - Comparison between simulation results from Campbell and Cochrane (1999) and our computational calculations at same calibration parameters set.

Wachter (2005)

[19]In Campbell and Cochrane (1999), the authors have simulated 500,000 months of artificial data. We adopted the same procedure. Later, 400,000 quarters of artificial data were generated. Wachter (2005) has also used many different grid sets. Here, we used as benchmark the second grid set suggested in her work.

Statistic	Wachter	Simulated
$E(\Delta c_{t+1})$ (%)	2.20	2.20
$\sigma(\Delta c_{t+1})$ (%)	~ 0.86	0.71
$E(r^f)$ (%)	1.47	1.46
$\sigma(r^f)$ (%)	–	1.91
$E(r^m - r^f)$ (%)	5.43	5.25
$\sigma(r^m - r^f)$ (%)	16.07	15.53
Sharpe ratio	0.34	0.34
$E(p - d)$	3.10	3.15
$\sigma(p - d)$	0.31	0.32
$Corr(p - d)$	0.89	0.89

Tab. 3 - Comparison between simulation results from Wachter (2005).

Taking into account the fact that in a computer simulation there are many variables that we cannot match exactly, for instance: virtual random machine used in normal v_{t+1} draws, decimal number approximation rules and so many others issues that make two computer simulations different from one another, we do not believe that program errors underlie the divergence that we found with $b < 0$. Our program seems appropriate as we can see for its ability to reproduce finely the results in Campbell and Cochrane (1999) and Wachter (2005).

Unfortunately, we do not have a reasonable explanation as of why we cannot reach finite values for the price-consumption ratio when interest rates are increasing in s_t. Technically, by focusing on (24) one can see that the absolute value of $\lambda(s_t)$ drives positively the pricing kernel volatility. We can see in expression (10) that a negative b decreases the steady-state consumption surplus ratio $\overline{S}$ and, therefore, reduces $S_{\max}$. Thus, the state-variable grid is filled with lower values. Given that λ is decreasing in s_t, the sensitivity function will be higher than when a positive b is set on consumption surplus grid points raising the stochastic discount factor volatility. How this increased volatility produces a divergence of our calibration procedure is of as this moment still not understood.

In another effort to understand the computational problems with $b < 0$, we have used an approach which is similar to the one proposed by Campbell and Cochrane (1995), footnote 10. When we impose $b < 0$, the risk free rate is pro-cyclical and a linear increasing function of consumption surplus. In this case, if we fix an upper bound for the risk-free rate,[20] we were able to find a closed expression for b in terms of γ, ϕ,

[20] *Here, we chose high values for upper bounds in way to guarantee that the model's curve of the risk free return has positive values in all grid points. For instance, we have used never reached values 5%, 10%, and 15% by the US real interest rate in our sample data.*

σ_v, r_0^f and the upper bound using (14) as follows:

$$r^f(s_{\max}) = r_0^f - b\left[s_{\max} - \bar{s}\right] \leq r_{upper\ bound}.$$

We then created three grids for $(\gamma,\, \phi,\, \sigma_v)$ [21] and used all possible combinations of these parameters to find which ones have implied negative values for b. Our outcomes point to the fact that there is a very narrow range of parameters (0,98%) that delivers $b < 0$. Moreover, all parameters combinations which yielded $b < 0$ displayed $\phi = 1$, implying that the consumption surplus is a random-walk. This feature creates many difficulties to accommodate asset pricing phenomena other than the one the model is aiming at matching.

These results reinforce our perception that Campbell and Cochrane's consumption-based model does not deal with pro-cyclical interest rates, at least in a well-behaved way, i.e. with $\phi < 1$. To overcome this difficulty, Verdelhan (2009) proposed an *ad hoc* approach to the original external habits model, which consists in making the sensitivity function, λ, described in (9), constant, at least during the price-consumption ratio computation from (16). The approach and its consequences will be the issue of our next subsection.

3.6 Verdelhan's assumption

How is it possible to reconcile our negative results with Verdelhan's well succeeded use of Campbell and Cochrane's approach? That is, how did Verdelhan succeed in finding a finite value for the consumption-price ratio while at same time imposing $b < 0$? The answer lies in the sensitivity function, $\lambda(s_t)$. To find a closed form expression for the risk-free volatility, Verdelhan (2009) assumes that $\lambda(s_t) \approx \lambda(\bar{s})$, i.e., $\lambda(s_t)$ is a constant. With a constant λ no divergence problem arises and we can calibrate the model.[22]

After finding an appropriate price-consumption ratio, we release the sensitivity function λ in the simulations to vary as the original model does. In this manner, we are ensuring that risk free rate is pro-cyclical and, consequently, the UIP slope coefficient in simulated data is negative. In contrast, with a fixed λ, the risk fee rate is driven by $r_t^f = -\gamma(1-\phi)(s_t - \bar{s}) + \kappa$, where $\kappa \equiv -\ln\delta + \gamma g - .5(\gamma\sigma_v)^2\left(1 + \lambda(\bar{s})\right)^2$. Because $\gamma(1-\phi) > 0$, r_t^f is still decreasing in s_t, the model would not generate a negative β^{UIP}. Also, fixing λ in simulation stage implies that risk premium no longer varies

[21] We estabilished grids for $(\gamma,\, \phi,\, \sigma_v)$ using 50 points equally-spaced, respecting each one of the following intervals $\sigma_v(\%) \in [0, 10]$; $\gamma \in (0, 20]$; $\phi \in [0, 1]$ and set r_0^f at 0,9%.

[22] Note,that, if one does not impose constancy of λ, $\lambda(.)$ exhibits significant variation in s_t. For instance, in the parameter set used in Campbell and Cochrane's calibration parameters, between the first and last s_t grid terms, we have a difference roughly of 38 in λ values.

counter-cyclically. This can be verified by examination of the Sharpe ratio expression for risky asset returns, (15).

To summarize, the procedure is potentially problematic because we are computing the price-consumption ratio under a stochastic discount factor that leads to anti-cyclical risk free rates although we intend to set interest rates pro-cyclically. We guess that this shortcut may generate inconsistencies for the model, mainly in realized equity returns that are driven by price-consumption's time series.[23]

Instead of focusing on this potential source of inconsistencies we shall focus on other aspects of the model's output. So, we pursue the strategy of fixing λ in the calibration part of the exercise, and letting it vary in the simulations.

By running the calibration parameters from table II of his working paper, our program has been successful in reproducing his simulation results. Verdelhan draws 10,000 endowment shocks to create artificial quarterly data. Unfortunately, we do not know which state variable grid format was assumed in his working paper, thus little differences between his results and ours could happen at his parameters set.[24]

Verdelhan (2009)*'s* [25]

Calibration parameters	Verdelhan
g	2.12
σ_v	1.02
r^f	1.36
ϕ	0.96
γ	2
b	-0.01
Implied parameters	
δ	1.00
$\overline{S}$	0.07
$S_{\max}$	0.12

Statistic	Verdelhan	Simulated
$E(\Delta c_{t+1})$ (%)	2.13	2.13
$\sigma(\Delta c_{t+1})$ (%)	1.04	0.84
$E(r^f)$ (%)	1.65	1.40
$\sigma(r^f)$ (%)	2.54	2.35
$E(r^m - r^f)$ (%)	3.98	3.93
$\sigma(r^m)$ (%)	8.72	13.62
Sharpe ratio	0.46	0.29
$E(p-d)$	3.44	3.60
$\sigma(p-d)$	0.49	0.49
$Corr(p-d)$	0.97	0.97

[23] Realized equity returns are computed as follows

$$R_t = \frac{C_{t+1}}{C_t}\left(\frac{\frac{P_{t+1}}{C_{T+1}}(s_{t+1})}{\frac{P_t}{C_t}(s_t)}\right),$$

so, any price-consumption inconsistency will affect realized returns.

[24] How much the use of different grids could change simulation results in this framework may be seen in Wachter (2005). The author uses 3 different grid types and summarizes her simulation results in tables 2 and 3 where great differences can be seen.

[25] Notice that the difference in simulated consumption growth deviations displayed here and in Wachter (2005)'s comparison exercise is due the Campbell and Cochrane's annualizing procedure. Put differently, if we would evaluate the deviation from quarterly artificial data and later multiply it by $\sqrt{4}$, we could match Wachter and Verdelhan's value for annualized $\sigma(\Delta c_{t+1})$. As evidence, in table 5 we performed this same comparison exercise but using the latter annualizing procedure.

Tab. 4 - Comparison between simulation results from Verdelhan (2008) working paper and our computational calculations at same annualized calibration parameters set.

[Fig. 1 - Price-consumption ratio with Verdelhan parameters]

Our numerical exercise seems to match all results from Verdelhan (2009), but equity returns volatility, and, consequently, the Sharpe ratio. However, we ought to highlight the low numbers of draws used by Verdelhan (10,000), which can make such results less reliable. To see this, we ran the model with the same calibration parameters used in latter comparison exercise but different seeds (See footnote 13). There are significant differences in such results as one can see in table 5. Whether one is matching or not the results becomes a more fuzzy matter.

Statistics	Verdelhan	seed1	seed2	seed3	seed4	seed5
$E(\Delta c_{t+1})$ (%)	2.13	2.10	2.06	2.10	2.12	2.15
$\sigma(\Delta c_{t+1})$ (%)	1.04	1.04	1.02	1.03	1.02	1.02
$E(r^f)$ (%)	1.65	1.13	0.72	0.99	1.24	1.93
$\sigma(r^f)$ (%)	2.54	2.07	2.51	2.51	2.32	1.47
$E(r^m - r^f)$ (%)	3.98	4.32	5.10	4.66	4.17	2.82
$\sigma(r^m)$ (%)	8.72	14.08	14.49	13.78	13.48	12.02
Sharpe ratio	0.46	0.31	0.34	0.34	0.31	0.25
$E(p-d)$	3.44	3.53	3.44	3.51	3.57	3.72
$\sigma(p-d)$	0.49	0.49	0.54	0.55	0.52	0.47

Tab. 5 - Comparison between simulation results from Verdelhan (2008) working paper using 10,000 draws for consumption artificial data with different random seeds.

Nevertheless, the equity returns volatility found in simulations are consistently higher than his. Apparently, this is the only difference between the results in exercises using our algorithm and those elsewhere, given that we have succeeded in replicating Campbell and Cochrane (1999) and Wachter (2005) results and almost all statistics from Verdelhan's paper.

Now, assuming that λ is constant and equal to its steady-state value, $\lambda(\bar{s})$, allows us to calibrate the model and match the US real data.[26] The parameter values and simulation results are described in tables 6, below.

[26]Setting $\lambda(s_t) = \lambda(\bar{s})$ is just an intuitive choice. Any other constant value, $\bar{\lambda}$, results in model convergence. However, the choice of $\bar{\lambda}$ affects dramatically the equity returns volatility. Indeed, for $\lambda(\bar{s})$ values above 30 the equilibrium price-consumption ratio seems to blow up despite still converging.

Calibration parameters		Statistic	Real data	Simulated
g	2.19	$E(\Delta c_{t+1})$ (%)*	2.21	2.21
σ_v	2.02	$\sigma(\Delta c_{t+1})$ (%)*	1.73	1.73
r^f	0.98	$E(r^f)$ (%)*	1.02	1.02
ϕ	0.931	$\sigma(r^f)$ (%)	2.96	2.16
γ	2	$E(r^m - r^f)$ (%)*	6.27	6.27
b	-0.01	$\sigma(r^m - r^f)$ (%)	15.15	17.32
Implied parameters		*Sharpe ratio*	0.41	0.36
δ	0.95	$E(p-d)$	3.33	3.11
$\overline{S}$	0.095	$\sigma(p-d)$	0.44	0.45
$S_{\max}$	0.156	$Corr(p-d)$	0.915	0.93

Tab. 6 - Annualized calibration parameters set and simulation results chosen to match 1947-2004 US consumption, price and equity real data. Statistics that calibration parameters were chosen to replicate.

The UIP coefficients we found confirms that the model delivers a consistent negative bias, whatever correlation between consumption processes of different nations is used. Therefore, the *ad hoc* Campbell and Cochrane model version proposed by Verdelhan is able to reproduce the FPP. Its values are summarized below.

UIP Coefficients			
ρ^i	β^{UIP}	ρ^i	β^{UIP}
-1.0	-3.2444	0.9	-4.4325
-0.9	-3.3499	0.8	-4.8277
-0.8	-3.4146	0.7	-5.0623
-0.7	-3.4911	0.6	-5.0707
-0.6	-3.5485	0.5	-4.9533
-0.5	-3.6016	0.4	-4.8047
-0.4	-3.6726	0.3	-4.6212
-0.3	-3.7593	0.2	-4.4344
-0.2	-3.8693	0.1	-4.2607
-0.1	-3.9948	0.0	-4.0883

Tab.7 - Respective UIP coefficients in relation to ρ^i. All values are significant under an 95% confidence interval. The level coefficient α was statistically null in every regression.

What consequences this assumption about the sensitivity function will have for other asset prices? Is it a reasonable shortcut?

In their original paper, Campbell and Cochrane (1999) built the sensitivity function λ to reach 3 main goals: *i)* to make risk-free rate linear in the state variable s_t, *ii)* to guarantee that habits are always lower than consumption, and; *iii)* to impose a non-negative co-movement between habits and consumption. The first objective is attained and can be seen in (14). For our calibration parameters, the latter two objectives are intact as can be seen in figure below. So, up to this moment, imposing a constant sensitivity function does not hurt the stated goals of Campbell and Cochrane (1999).

[Fig.2 - Consumption and Habits]

3.7 Real bond yields

Nevertheless, impose pro-cyclical interest rates to match the forward discount anomaly in habits formation framework results in a detachedness between the average slope of the real yield curve delivered by model's simulation and that observed in data. Wachter (2006), like us, used Campbell and Cochrane external habits model to fit main features of the US nominal and real term structure of interest rates. She pointed out that under a positive b, i.e. when intertemporal substitution effects dominates precautionary savings effects in investor preferences, this model generates a positive real bond premia that increases with maturity. Hence, an upward-sloped yield curve, a feature which Boudoukh et al. (1999) have found support in real data, is generated by the model when $b > 0$.

As mentioned in Andersen and Lund (1996), inspection of historical U.S. interest rates reveals that the real yield curve tends to be upward-sloping and quite steep at the short end (0 to 5 years), while it is relatively flat for maturities in excess of 5 years. To check this observation in more recent data, figure 3 displays the real yields at 2-yr and 20-yr maturities for the period that runs from Apr/99 to Jan/04.[27]

[Fig. 3 - Real Yields]

To understand the link between the sign of b and the slope of the real yield curve we follow Wachter (2006) and write the standing representative investor's Euler equation in the covariance form,

$$E(R_{n,t} - R_{1,t}) = -Cov(R_{n,t} - R_{1,t}, M_t)\frac{\sigma(M_t)}{E(M_t)}. \tag{27}$$

[27] You may find this US real yield data in FED's webpage at http://www.federalreserve.gov/econresdata/researchdata/feds200805.xls

Return at t on a bond maturing n periods in the future is $R_{n,t}$, the real short-term interest rate is the one-period maturity return $R_{1,t}$ and M_t is the pricing kernel.

The reason why this model generates a positively sloped yield curve is the combination of two facts. First, the fact that bonds returns move in the opposite direction from short term returns. Second, the fact that, with $b > 0$, short term returns is negatively related to s_t, as one can see in (14).

Adjusting a positive b and calibrating the model to match real data we were able to reproduce an upward-sloping real yield curves for 1-year and 5-year bonds coinciding with Wachter (2006) and Andersen and Lund (1996) findings. These curves are presented below.

[Fig. 4 - Bond yields with $b > 0$]

Turning back to our calibration parameters, i.e with $b < 0$, the model attain the following yield curves for same long-term bonds

[Fig. 5 - Bond yields with $b < 0$]

In figure 5 we have a downward-sloping real yield curve with negative values. Using the symmetrical argument discussed above in case $b > 0$, the pro-cyclical behavior of interest rates results in a positive covariance between economic cycles and short term interest rate. Since bond returns move in opposite direction from the short term interest rate, when economy goes badly bond returns are higher. This behavior brings real bonds closer to insurance assets. Then, investors demands smaller risk premia or even negative to hold them. Because long-term bonds have smaller expected returns than if there were no risk premia, they must have smaller yields.

This real yield curve behavior contradicts the well documented empirical statements discussed before. The reader can find out more about this in Mishkin (1990) and Piazzesi and Schneider (2006).

4 Conclusion

The presence of a forward premium anomaly in foreign exchange currency markets is tightly associated with the pro-cyclical behavior of equilibrium interest rates in Campbell and Cochrane (1999)'s habit formation model framework. The model seemed incapable to reproduce the US equity and risk-free rates in post-war data when pro-cyclical

interest rates were imposed in our calibration process. This outcome is strengthened by the lack of other works in literature that have used such model with pro-cyclical risk-free rates.

In a recent work, Verdelhan has had success using Campbell and Cochrane's model to reproduce FPP findings in exchange markets calibrating it for industrialized economies. In the same way, our paper could match real data for US economy and reply the stylized fact of a negative UIP slope coefficient. However, this triumph is attained at some costs.

First, the shortcut proposed by Verdelhan to deal with the difficulties in the calibration stage may lead to inconsistencies on the model's time series, given that the price-consumption ratio is computed under an anti-cyclical environment. We did not explore this path so we do not provide any evidence of inconsistencies.

Another notable cost, not necessarily due Verdelhan's shortcut, is the real yield downward-sloped shape delivered by the model when pro-cyclical risk free rates are imposed. It is well-known in literature that real yields almost always display an upward-sloping curve or a humped curve, but only seldom a downward-sloped, whilst here this shape shows always under pro-cyclical interest rate assumption.

Campbell and Cochrane's consumption-based model seems not to be able to reproduce simultaneously stylized facts in bond markets and in exchange markets because pro-cyclical interest rates must be settled to match the forward discount anomaly. Our results are not very conclusive, however, since we used Verdelhan's shortcut. Unfortunately, as of this moment we do not have a satisfactory explanation as of why the price-consumption ratio lack of convergence during original model's solution when $b < 0$ is imposed. We know that there are definitely parameter values that can lead to non-convergence.

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Appendix - Figures

Figure 1 – Simulated price-consumption ratio with Verdelhan (2009)'s working paper parameters.

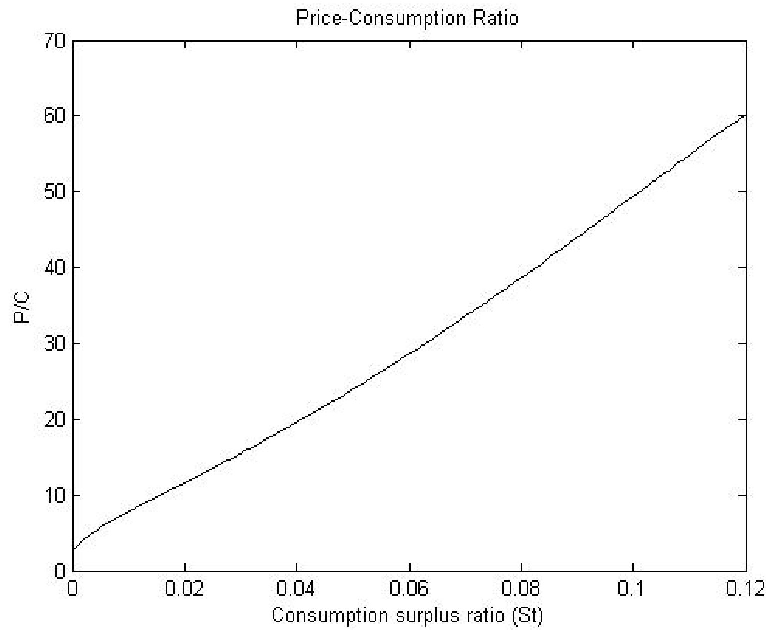

Figure 2 – Simulated consumption and habits levels and their growths when is imposed pro-cyclical interest rates with b < 0.

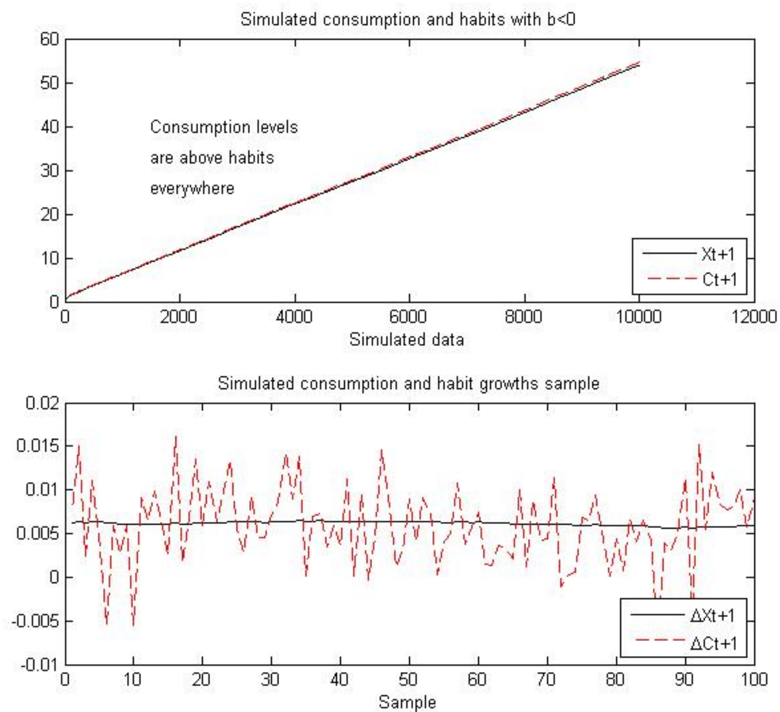

Figure 3 – The US Treasury real yield curve for maturities between 2-yr and 20-yr. Data are from US Federal Reserve daily yields between April/99 and Jan/04.

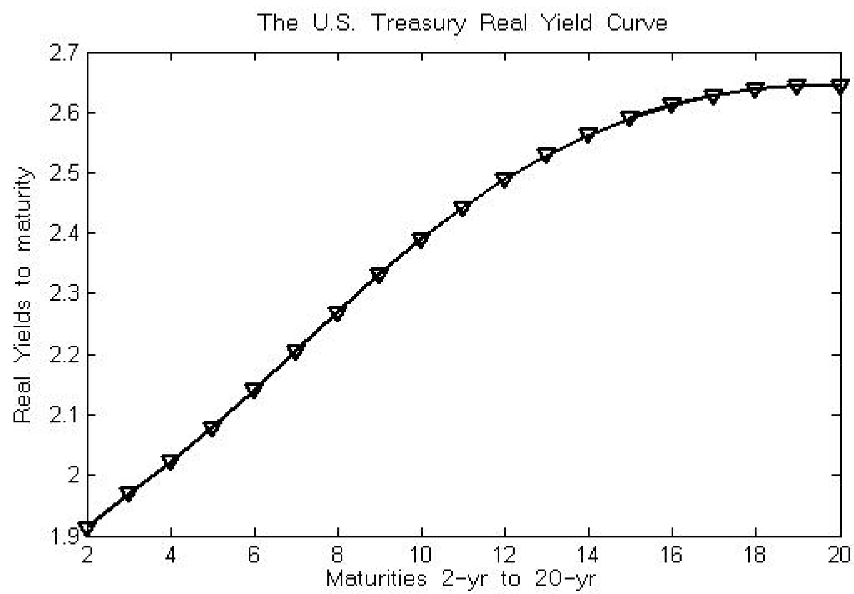

Figure 4 – Simulated 1-yr and 5-yr bond's yields when is imposed anti-cyclical interest rates with b > 0.

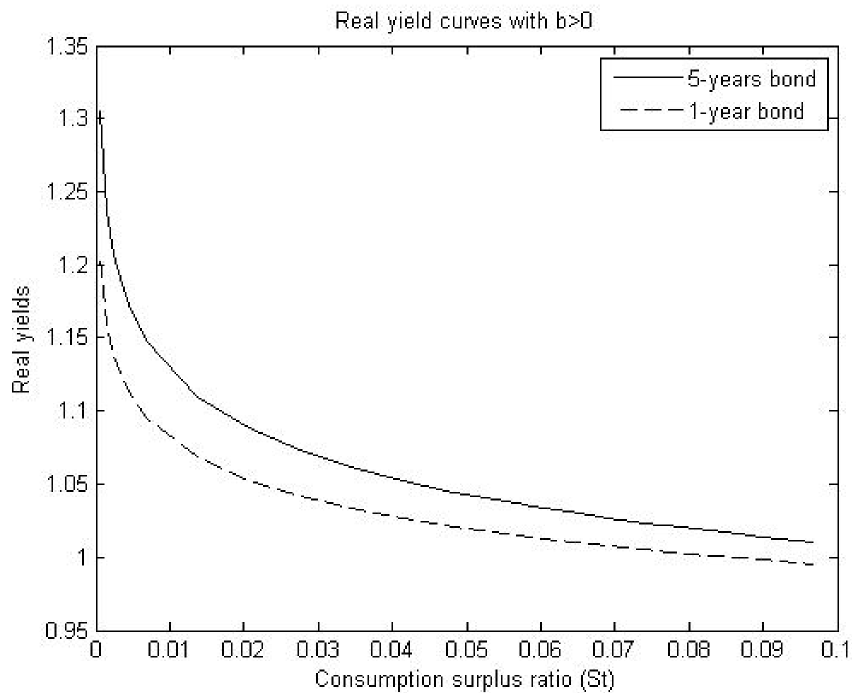

Figure 5 – Simulated 1-yr and 5-yr bond's yields when is imposed pro-cyclical interest rates with b < 0.