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The allocation of talent in teacher's occupation and Economic Growth

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"THE ALLOCATION OF TALENT IN TEACHER'S OCCUPATION AND ECONOMIC GROWTH".

Dissertação apresentado(a) ao Curso de Mestrado em Economia do(a) Escola de Pós-Graduação em Economia para obtenção do grau de Mestre(a) em Economia.

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Abstract

The quality of teachers is a determining factor in the formation of human capital, so it is also an object of study of economic development. We develop a general equilibrium model in which workers are heterogeneous and make decisions in a labor maket with frictions. Additionally, teacher quality is input to human capital formation. Empirically we use a cross country dataset to check the results. We also simulate the US's labour market frictions to all economies. We found that improving the eviroment where teacher's occupational choices are made, we increase the overall productivity by 110%.

Palavras-chave: Economic Growth; Misalocation; Education

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1 Introduction

Teacher quality is extremely relevant when human capital is discussed and it might be important to economic development. Teachers produce educational goods that increases individual's human capital, meanwhile the labour engaged in teaching is not used to the production of consumption goods. The individual decision that ends up in this trade off is endogenous to the model. Every individual makes their according to the information and environmental conditions available. The main goal of this thesis is to study how the allocation of talent in teaching positions impacts economic growth across different countries.

If we consider the idea the teacher's human capital is a key input for the formation the other individuals, we should expect that more developed areas present a body of teachers with higher quality than in areas less developed. Most of our knowledge was handed down to us by previous generations and a good part of it is due to teachers. Then, qualified teachers are essential in a economy where productivity depends on education. In the context of learning on the job, Jovanovic and Nyarko (1995) argue that the transfer of knowledge from the old to the young is therefore a cornerstone of productivity growth. If we extend this idea to a more general context, teacher's human capital could be a important determinant of other professionals' human capital, i.e., in a society where there are good teachers we expect to find productive workers in other occupations. Therefore, if teacher's human capital is important for development, we should expect that more developed areas should present relative teachers' human capital higher than in less developed areas.

We expect higher levels of development to be correlated to economies with higher average human capital in all occupations. However, it's not clear how the distribution of talent among occupations should occur throughout the growth path. Do more developed economies have relatively more human capital engaged teachers' occupation? Individuals with high quality human capital when engaged in teaching, efficiently produce educational goods. Nevertheless, it also represents a high opportunity cost for the production of consumption goods. As this individual's human capital grows, the opportunity cost of his labor growths along.

In literature there are many studies that relate teachers' quality and student's performance. In general, a value added methodology is used to evaluate teacher's quality. This methodology is based on reduced form models to deal with panel data to observe how much each teacher has added on average to the students in a class in terms of grades on standardized tests. Hanushek and Rivkin (2010) present a long summary of value-added studies that look at the influence of teachers on achievement gains to illustrate its importance. The magnitude of the differences presented were truly large, with some teachers producing 1,5 years of gain in achievement in an academic year while others with equivalent students produce only about 0,5 year of gain. This micro-evidence highlights the importance that teacher's quality might have. Hanushek (2010) uses a similar approach but in stead of looking at student's academic performance he looks at individual's earning. He finds a great impact on improving teacher quality and the economic performance of its students. Although these papers bring important insights, as they look only to student's performance it lacks formal modeling for the labours supply in education. In addition, they do not provide evidence for the impact of aggregated higher quality of teachers.

We model the labor market to consider the supply side of teacher's occupation. We take use a human capital function composed by two factors, the first is the innate skills for the pursuits of each profession, the second is the ability of these individuals to choose the amount of educational assets they want to consume to increase their human capital. The existence of heterogeneous agents culminate in a selection process in the labor market.

Roy (1951) provides the baseline of our analysis. He develops a model for the distribution of incomes based on an aprioristic distribution of skills. With this, it is possible to study the labour market impact of heterogeneous innate abilities in the population. Hsieh et al. (2013) use a version of this model from which we will be inspired to build the consumer's side of this model. They analyze the impact of gender and race based distortions in the US' labour market. These distortions are captured by a exogenous term that indicates a difference between the paid salary and the received salary. They represent the existence of non-observable part of the return of an occupation such as discrimination, relative power of trade unions, prestige of a carrier, labor market regulation, the difficulty of finding a job in an job, or any other form of expropriation.

We also consider the presence of intermediary goods. The notion of linkages between sectors and performance economy dates back to the first half of the last century. The study of output input economy is described more than 80 years in Leontief (1936). Hirschman (1958) emphasizes the impact of the existence of stages of the production chain in development. It focuses in particular on the importance of the complementarity of these stages in development. Subsequently to it, there is a vast literature within practically every line of economy thinking about linkages and complementarities.

Recently, Jones (2011) documents that once intermediate goods are used in the production of final goods, the quality of the former can explain part of economic development and determine the failure of a nation. We present a model of general equilibrium in which the intermediary goods represented two distinct forms. The first is consumer goods that enters directly into the production function of all other goods. The second is education goods that indirectly enters into the production function of all or other goods as it makes up the human capital used in the production of all other goods, including the the educational itself. It represents a economy where teacher's human capital is an key input for human capital of all other professionals. In this environment, we could expect that small changes in the quality of teachers should have a large aggregate impact in the output.

The most similar work to the one develop in this thesis is Delalibera (2018). He analysis the alocation/missallocation of workers in teacher's occupation Brazilian states and its consequences to the country's economy as well as the impact of the misallocation in the states' economy. He also uses as reference for the misallocation in labor market Hsieh et al. (2013). However there are large differences in the way we model education, we use different function and do not consider the time spent in school as cause of desutility, but only as source of future income. This brings consequences to the equilibrium results that will be further discussed in the next sections. We also differ in the production side. Delalibera (2018) uses human capital as the only input, while we use it together with consumption goods in a Cobb-Douglas function. Finally, the object of our studies are very different, he focuses on the Brazilian economy, using data from 7 occupations, while we propose a cross-country study of education and non-educational sector, where the educational sector is further divided into 4 sub-sectors.

In summary, we model an economy where: First, there is an occupational choice of multi-ability workers driven by labor market incentives (net of frictions wage) and the costs of investment in specific human capital (acquisition of education). Second, teacher's human capital is an input for the educational goods and so that for the formation of the human capital of whole workforce in an economy. Thus, when many people with lower idiosyncratic ability choose to be a teacher, the average of teachers' human capital drops and, therefore, the formation of human capital of all workers is compromised. We combine these ideas in a cross-country general equilibrium model where people are heterogeneous with respect to abilities for different occupations and there are market frictions distorting their occupational choice. How distortions in this occupation affect output? The mains questions we want o answer are: What is the impact of human capital in teacher's occupation in economy?

In the theoretical perspective we conclude about the distortions that: First, the greater the distortion, the lower the output. Second, the greater the distortion in a sector, the greater the price of the final good of this sector. Furthermore we find that the production function of convex consumer goods and that the function of production of goods education is concave with respect to the average human capital employed in each of these sectors.

We find that by simulating labor market frictions from the United States to other economies, there is an average expected growth of 110% in productivity per worker, reaching up to 1235% in optimistic outlooks and up to 274% in conservative calculations[1].

This work is organized as follows: Session 2 describes a general equilibrium model, consumer and firm's side, defines an equilibrium and calculates it. Session 3 builds a simplified model with which to study the impact of frictions on prices and on output. Session 4 gives an empirical analysis of the model, describing the data, the calibration strategy and the main results. Session 5 presents the final remarks.

[1]We use the terms conservative and optimistic about calibration errors. The conservative calculations relate to countries with excellent fitting of the model.

2 Model

In this session we describe our general equilibrium model. It is organized as follows: Sub-session 1 describes the behavior of consumers. Sub-session 2 describes the behavior of firms Sub-session 3 defines the equilibrium and describes how to calculate equilibrium prices.

2.1 Consumers

We consider a Roy model based on Hsieh et al. (2013). There is a continuum of people in the economy working in either goods sector or educational sector. Educational sector is split in pre-primary, primary, secondary and tertiary sector. Each person has idiosyncratic abilities toward each occupation. In regards to abilities, people have a different talent for the different occupations: some people can present a high talent for many occupations, some people have high talent for only one occupation and some people do not have great skills for any of the occupations is the economy. An individual values consumption and is endowed with one unit of time that is inelastically offered in the labor market. The utility of a person is given by

$$U(c) = log(c) \tag{1}$$

where c represents the consumption.

First, there is a distortion in the labor market. People working in occupation i is paid a wage of $(1 - \tau_i)w_i$ where w_i is the wage per efficiency unit of labor paid by a firm and τ_i is a distortion specific for occupation i.

The formation of human capital of a worker is given by:

$$h(e, \epsilon) = e^{\eta}\epsilon^{\phi}, \tag{2}$$

where e is the consumption of educational goods, ϵ is the ability of and individual, and η is the elasticity of human capital toward educational goods.

Please note that we are omitting the indices related to individuals to keep the notation clean. For example, c and $h(e, \epsilon)$ represent the amount of consumption and human capital per person.

Following McFadden (1974) and Eaton and Kortum (2002), we model abilities dispersion in accord to a multivariate Fréchet distribution. Let ϵ_i be the ability of an individual in occupation i, then the distribution of abilities is given by

$$F(\epsilon_1, \dots, \epsilon_N) = \exp\left[-\left(\sum_{i=1}^{N} \epsilon_i^{-\frac{\tilde{\theta}}{1-\rho}}\right)^{1-\rho}\right], \tag{3}$$

where $\tilde{\theta}$ governs the dispersion of skills and $\rho \in [0, 1]$ gives the correlation of individual's skills. A higher $\tilde{\theta}$ implies a smaller dispersion in abilities. Also, a $\rho = 1$ indicates that skills are perfect correlated, while a $\rho = 0$ means that individual's skills are uncorrelated across occupations. For convenience, let $\theta = \tilde{\theta}/(1 - \rho)$.

The worker's problem can be solved in two steps. First, given the occupational choice i, for which the individual has an idiosyncratic ability ϵ_i, and taking wage w_i as given, each worker chooses consumption c_i and e_i to solve the following problem:

$$\underset{c,e}{\text{Max}}\, log(c) \tag{4}$$

$$\text{s.t. } c = (1 - \tau_i)h(e_i, \epsilon_i)w_i - P_E e$$

Note that in this step we omitted individual and sector-specific variables to make the notation cleaner. Solving this problem above, we find the amount of goods and time spent in education and in consumption for each i occupation:

$$c_i^*(\epsilon_i) = \quad (1-\eta)\bar{\eta}\left(\frac{(1-\tau_i)w_i}{P_E^{\eta}}\right)^{\frac{1}{1-\eta}} \epsilon_i^{\frac{\phi}{1-\eta}}, \tag{5}$$

$$e_i^*(\epsilon_i) = \quad \tilde{\eta}\left(\frac{(1-\tau_i)w_i}{P_E}\right)^{\frac{1}{1-\eta}} \epsilon_i^{\frac{\phi}{1-\eta}}. \tag{6}$$

where $\bar{\eta} = \eta^{\frac{\eta}{1-\eta}}$ and $\tilde{\eta} = \eta^{\frac{1}{1-\eta}}$.

We can substitute the expressions in equations (5) into the utility function and get directly the expression for indirect utility function for occupation i, U_i. Therefore, the occupational choice problem reduces to picking the occupation that delivers the highest value of U_i. Since talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution (McFadden, 1974). The overall occupational share can then be obtained by aggregating the optimal choice across people, as we show in the next proposition.

Proposition 1 *Aggregating across people, the solution to the individual's occupational choice problem allows us to write*

$$s_i = \frac{\tilde{w}_i^{\theta}}{\sum_{j=1}^{N} \tilde{w}_i^{\theta}}, \tag{7}$$

where s_i is the fraction of people that work in occupation i and

$$\tilde{w}_i = ((1 - \tau_i) w_i)^{\frac{1}{\phi}} \; .$$

See appendix A.1.

We can interpret $\tilde{w}_i$ as a liquid reward for a person with mean ability working in occupation i. It is just composed by post-friction wage per efficiency unit in the occupation w_i. The occupational sorting depends on $\tilde{w}_i$. Moreover, it depends on the relative returns and not absolute returns.

Also note that, the fraction of people employed in a given profession also depends on the ratio $\frac{\theta}{\phi}$. If this ratio is small, there is little difference between the participation of the sectors. This can happen because θ is large or because ϕ is small. In these cases we could imagine: (1) there is a great correlation between idiosyncratic abilities. (2) There is little variance between the different skills. (3) The impact of innate ability on human capital is very small.

Additionally, we can write an expression for the average human capital of worker in occupation i.

Proposition 2 *The average human capital of workers in occupation i, including both* *educational choice and idiosyncratic abilities, is*

$$\mathbb{E}[h(e_i,\epsilon_i)|person\ chooses\ i] = \gamma\bar{\eta}\left[\frac{(1-\tau_i)w_i}{P_E}\right]^{\frac{\eta}{1-\eta}} s_i^{-\frac{\phi}{\theta(1-\eta)}}, \tag{8}$$

where $\gamma = \Gamma\left(1 - \frac{\phi}{\theta(1-\eta)(1-\rho)}\right)$ *is related to the mean of the Fréchet distribution for abilities.*

See appendix A.1

This result show that there is a selection effect in the economy. The average quality is inversely related to the share of workers in occupation s_i. If the distortion is high for a occupation i, than only the most qualified people will work in that occupation. For example, in a country where it is very easy to become a teacher, the average quality of a teacher will be low.

Proposition 3 *The average demand on consumption and educational goods in occupation i is*

$$\mathbb{E}[c_i(\epsilon_i^*)|person\ chooses\ i] = \gamma\bar{\eta}(1-\eta)\left[\frac{(1-\tau_i)w_i}{P_E^{\eta}}\right]^{\frac{1}{1-\eta}} s_i^{-\frac{\phi}{\theta(1-\eta)}}, \tag{9}$$

$$\mathbb{E}[e_i(\epsilon_i^*)|person\ chooses\ i] = \gamma\tilde{\eta}\left[\frac{(1-\tau_i)w_i}{P_E}\right]^{\frac{1}{1-\eta}} s_i^{-\frac{\phi}{\theta(1-\eta)}}. \tag{10}$$

It is analogue to preposition 2.

The aggregation of consumption and educational goods through individuals is

$$C_C^D = \sum_i s_i\mathbb{E}[c_i(\epsilon_i)|\text{person chooses } i] = (1-\eta)\bar{\eta}\gamma\frac{\left[\sum_i \tilde{w}_i^{\theta}\right]^{\frac{\phi}{\theta(1-\eta)}}}{p_E^{\frac{\eta}{1-\eta}}}, \tag{11}$$

$$C_E^D = \sum_i s_i\mathbb{E}[e_i(\epsilon_i)|\text{person chooses } i] = \tilde{\eta}\gamma\frac{\left[\sum_i \tilde{w}_i^{\theta}\right]^{\frac{\phi}{\theta(1-\eta)}}}{p_E^{\frac{1}{1-\eta}}}. \tag{12}$$

Furthermore, the aggregate supply of human capital for each occupation is:

$$H_i^o = s_i \mathbb{E}[h(e_i, \epsilon_i) | \text{person chooses } i] \tag{13}$$

$$= \gamma \bar{\eta} \frac{\tilde{w}_i^{\frac{\theta-\phi}{\phi}}}{P_E^{\frac{\eta}{1-\eta}} \left(\sum_i \tilde{w}_i^{\frac{\theta}{\phi}} \right)^{\frac{(1-\eta)\theta-\phi}{(1-\eta)\theta}}} \tag{14}$$

We can also aggregate the goods to get the aggregated demand. From $Y = P_C + P_E C_E$:

$$Y^D = \frac{\gamma}{P_E^{\frac{\eta}{1-\eta}}} \bar{\eta} \left[\sum_i \tilde{w}_i^{\theta} \right]^{\frac{\phi}{\theta(1-\eta)}} \tag{15}$$

2.2 Firms

The side of firms is based on Jones (2011). There are five types of goods that are produced here using the following technology:

$$Q_i = A_i X_{i,c}^{1-\sigma_i} H_i^{\sigma_i}, i \in \{c, E_1, E_2, E_3, E_4\} \tag{16}$$

where Q_c is the production of consumption goods, Q_{E_i} is the production from pre-primary, primary, secondary and tertiary education, respectively, $X_{i,c}$ is the amount of consumption good used in the production of i, H_i is the human capital used to produced variety i, $(1 - \sigma_i)$ is the elasticity of production to intermediate consumption good and σ_i is the elasticity of production to intermediate human capital. We assume that $\sigma_i = \sigma_E, \forall i \in \{E_i\}_{i=1}^4$.

Consumption good can be used as either a final good (c_i) or as an intermediate input $(X_{i,c})$. Moreover, we assume that education is not used directly as an intermediate good for consumer goods. By the way we structure the problem education is a key input in production because it enters into the formation of human capital used in the production of all goods. Thus, even if education is not directly used as an input for the good of consumption, it also remains indirectly as an intermediate good. So,

$$Q_i = \begin{cases} c_c + \sum_j X_{j,c}, j \in \{c, \{E_i\}_{i=1}^4\} & \text{if } i = c \\ c_{Ei} & \text{if } i \neq E_i \end{cases}. \tag{17}$$

Educational good for final consumption is produced combining each kind of educational good as given by:

$$C_E = \prod_{j=E_1}^{E_4} c_j^{\alpha_j} \tag{18}$$

where $\sum_{j=E_1}^{E_4} \alpha_i = 1$.

The educational goods E_j, $i \in \{E_i\}_{i=1}^4$ can be used to produce a final educational good C_E. To emulate this process we use a Cobb-Douglas technology in a competitive firm. The firm's problem is then:

$$\underset{\{c_i\}_{i\in\{\{E_j\}_{j=1}^4\}}}{\text{Max}} \; p_E C_E - \sum_{i\in\{\{E_j\}_{j=1}^4\}} p_i c_i \tag{19}$$

2.3 Equilibrium

2.3.1 Equilibrium Definition

Let $\mathbb{I}$ and $\mathbb{J}$ be two sets, such that $\mathbb{I} = \{c, \{E_j\}_{j=1}^4\}$ and $\mathbb{J} = \{\{E_j\}_{j=1}^4\}$. A competitive equilibrium to this model consists of quantities Y, C_E, $\{Q_i, H_i, c_i, s_i, X_{i,c}\}_{i\in\mathbb{I}}$ and prices $\{P_c = 1, P_E, \{p_j\}_{j\in\mathbb{J}}\}$ and $\{w_i\}_{i\in\mathbb{I}}$ and market friction $\{\tau_i\}_{i\in\mathbb{I}}$ such that:

1. Given an occupational choice i, the occupational wage w_i, educational price P_E, idiosyncratic ability ϵ_i, and occupational market friction $\{\tau_i\}$; $\{c^*, e^*\}$ solve the consumer problem.

2. Given $\{P_E, \epsilon_i, \tau_i, w_i\}$, each individual chooses the occupation i^*, which maximizes his/her utility.

3. Given $\{P_c = 1, P_E, \{p_j\}_{j \in \mathbb{J}}\}$, $\{X_i^*, H_i^*\}$ solve the Variety i Problem for $i \in \mathbb{I}$.

4. Markets clear:

   (a) $\forall i \in \mathbb{I}$; w_i clears labor market i;

   (b) P_E clears educational market for final consumption;

$$Q_c = c_c + \sum_{i \in \mathbb{I}} X_{i,c}$$

   (c) $\forall j \in \mathbb{J}$; p_j clears market j:

$$Q_j = c_j$$

5. The output per worker is given by

$$Y = P_c C_c + P_E C_E \tag{20}$$

2.3.2 Equilibrium Solution

To solve the equilibrium we first show that: $\forall i \in \mathbb{I}$ there is a biunivocal relationship between P_i and W_i that allows us to solve the system only in terms of prices or in terms of wages. We then use the first order conditions associated with the condition of market clearing to obtain the system of equations that defines equilibrium prices.

In order to simplify the presentation of the results, we now assume two standardizations. The first is the default: $P_c = 1$. The second follows the literature, especially Hsieh et al. (2013) and we assume that $\phi = 1$.

Proposition 4 $\forall i \in \{c, E_1, E_2, E_3, E_4\}$, *there is biunivocal relation between prices and salaries such that:*

$$w_i = \left(\tilde{\sigma}_i A_i P_i\right)^{\frac{1}{\sigma_i}} \tag{21}$$

where $\tilde{\sigma}_i = (1 - \sigma_i)^{1-\sigma_i} \sigma_i^{\sigma_i}$

See appendix A.2

Let us now have a look at the educational sector. The First Order Conditions of problem (19) yields that, for all $j \in \mathbb{J}$:

$$p_{E_j} c_j = \alpha_j P_E C_E \tag{22}$$

Using the conditions above and, the market clearing for the educational goods. We conclude that $\tilde{w}_j^\theta$ is a linear combination of all $\tilde{w}_i^\theta$.

Proposition 5 *Considering the index $j \in \{c, \{E_j\}_{i=j}^4\}$, and index $i \in \{c, E_1, E_2, E_3, E_4\}$, there is a linear relation between $\tilde{w}_j^\theta$ and $\tilde{w}_i^\theta$ salaries such that:*

$$\tilde{w}_i^\theta = (1-\tau_i)\alpha_i \eta \sigma_i \sum_i \tilde{w}_i^\theta, \forall i \neq c \tag{23}$$

See appendix A.2

Let us now define:

$$F = \begin{bmatrix} F_1 & 0 & ... & 0 \\ 0 & F_2 & ... & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & ... & F_N \end{bmatrix} \tag{24}$$

$$F_i = \frac{(1-\tau_i)^\theta}{(1-\tau_i)\alpha_i \eta \sigma_i}, \forall i \neq c \tag{25}$$

$$T^\theta = \begin{bmatrix} (1-\tau_1)^\theta & (1-\tau_2)^\theta & ... & (1-\tau_N)^\theta \\ (1-\tau_1)^\theta & (1-\tau_2)^\theta & ... & (1-\tau_N)^\theta \\ \vdots & \vdots & \ddots & \vdots \\ (1-\tau_1)^\theta & (1-\tau_2)^\theta & ... & (1-\tau_N)^\theta \\ (1-\tau_1)^\theta & (1-\tau_2)^\theta & ... & (1-\tau_N)^\theta \end{bmatrix} \tag{26}$$

$$W^{\theta} = \begin{bmatrix} W_1^{\theta} \\ W_2^{\theta} \\ ... \\ W_N^{\theta} \end{bmatrix} \tilde{W}_c^{\theta} = \begin{bmatrix} \tilde{W}_c^{\theta} \\ \tilde{W}_c^{\theta} \\ ... \\ \tilde{W}_c^{\theta} \end{bmatrix} \tag{27}$$

From these definitions we can reorganize (62) the into matrix:

$$FW^{\theta} = T^{\theta}W^{\theta} + \tilde{W}_c^{\theta}$$

From this system of linear equations it is possible to obtain an equilibrium value for the wages of the education sectors, in terms of and $\tilde{w}_c$:

$$W^{*\theta} = [F - T^{\theta}]^{-1}\tilde{W}_c^{\theta} \tag{28}$$

According to Proposition 4 this also gives equilibrium prices $p_i, i \in \{\{E_i\}_{i=1}^4\}$ to all the educational sectors. Furthermore, the first order condition from (19) yields:$P_{Ei}c_i = \alpha_i P_E C_E$. Together with (18) we get an expression for pE:

$$pE = \prod_{i \in \{E_i\}_{i=1}^4} \left(\frac{P_{Ei}}{\alpha_i}\right)^{\alpha_i} \tag{29}$$

Using the equilibrium prices and taking the frictions as given, we are able to compute all other equilibrium objects.

3 Analytical Investigations

In this section we develop a analytical investigation over the properties of our model. For the sake of simplifying the calculations we assume that there is just one educational good E. The value of the parameter α_1 becomes 1; $P_E = p_{E_1}$ and the market clearing conditions are:

$$Q_i = \begin{cases} c_c + X_{c,c} + X_{E,c} & \text{if } i = c \\ c_E & \text{if } i = E \end{cases}. \tag{30}$$

Letting the consumption good be the numeraire, the maximization for firm i's problem is given by:

$$\underset{\{X_i,H_i\}}{\text{Max}}\, p_i Q_i - p_c X_i - w_i H_i \tag{31}$$

where $p_c = 1$.

All the other equilibrium prositions remain the same, using just one educational sector with $\alpha_E = 1$. Considering $\phi = 1$ Thus, it is straightforward that we can write the output as

$$Y = \bar{\eta}\gamma \frac{\left[\tilde{w}_C^\theta + \tilde{w}_E^\theta\right]^{\frac{1}{\theta(1-\eta)}}}{p_E^{\frac{\eta}{1-\eta}}} \tag{32}$$

where

$$p_E = \frac{A_c^{\frac{\sigma_e}{\sigma_c}}}{A_E} \left[\frac{\eta\sigma_E(1-\tau_E)}{1-\eta\sigma_E(1-\tau_E)}\right]^{\frac{\sigma_E}{\theta}} \left[\frac{(1-\tau_c)}{(1-\tau_E)}\right]^{\sigma_E} \left[\sigma_E^{-\sigma_E}(1-\sigma_E)^{\sigma_E-1}\tilde{\sigma}_c^{\frac{\sigma_E}{\sigma_c}}\right] \tag{33}$$

Proposition 6 *Assume that $\tau_i > 0$ and $\theta > 1$. So that,*

1. The derivative of educational price toward distortion in the educational occu-

pation is positive.

2. The derivative of educational price toward distortion in the goods occupations is negative.

3. The derivative of output toward distortion in the educational occupation is negative

4. $\frac{\partial Y}{\partial \tau_c} < 0 \Leftrightarrow \left(2 - \tau_E - \frac{1}{\eta \sigma_E}\right) < 0;$

See appendix B.

The proposition indicates the variation of the price of education and the product with respect to variations in frictions. The result is as expected by theory, the greater the friction in the education sector, the higher the price of education relative to consumer goods. That is, the greater the difference between the value paid by employers and the value received by employees in the higher education sector is the price of that education for the economy as a whole.

At the same time, the greater the friction in the consumer goods market, the lower the relative price of these goods.

Note that this proposition is expressed in absolute terms, however, in order to solve the problem we use the price of the consumption good as numeraire. In doing so, we assume that the price of educational good is always relative to the prices of consumer goods that is equal to one. Then an increase in the frictional parameter τ_E that lead to an increase in the price of the good of education actually means an increase in the price of that good. Whereas a increase in the frictional parameter τ_C of consumer goods that is suppoused to lead to an increase in the price of this good actually means an increase in its relative price or a reduction in the value of the educational good.

Once the prices put in relative terms, the first two parts of this proposition are also related to what has already been developed by this model mainly as the decisions of the consumers in the choice of the labor market, based on the relative salary of each sector to the detriment of their absolute salary.

So far, one could think that the distortion in the education market could compensate the distortion in the consumer market and vice-versa. Naturally, this compensation happens in terms of price, however it does not happen in terms of output.

Proposition 6.3 states that the greater the distortion of education in relation to the less distortion the product.

We can not be this much straight forward with respect to distortion related to consumer's goods. Variations in consumer distortion may have ambiguous effect on output. Specifically, proposition 6.4 states that the effect of variations in consumption distortion depends on the parameters η, σ_e and τ_E.

The higher the η parameter, the lower the term in parentheses, and therefore the greater the negative impact of changing the distortion of consumer goods over the output. This means that the more sensitive the human capital function to education the lower the impact of consumer's good distortions in the output. Equivalently, the less sensitive the human capital function to education the higher the impact of consumer's good distortions in the output. Intuitively, in economies where there is a great capacity to increase the amount of human capital from small variations in the amount of consumed educational goods, a increase of distortions in the sector of consumer goods causes a reduction in the relative price of education. This reduction increases vastly the individual's human capital and the output increases.

The discussion of the σ_E parameter is analogous to the discussion made in the previous paragraph regarding the parameter η. Only that this time the σ_E is bounded to 1 and represents the elasticity of the production function of the final goods in relation to the aggregate human capital. The greater the σ_E the greater the ability of the economy to use aggregate human capital accumulated in production of educational good.

From a quantitative analysis notice that $\tau_e < 1$, otherwise we would have salaries less than or equal to zero for the occupation of education, incompatible with the model. Let us now suppose an extreme scenario with a intense distortion in education, τ_E is close to 1. In this situation we would have that the derivative is negative only if $\eta\sigma_E < 1$. The parameter σ_E is bounded to one, what means that in a extreme scenario, η must be higher than on to break this condition. According to the literature reviewed so far, specially from Hsieh et al. (2013) used as reference here η is lower than 1. So we conclude indirectly that in the context of the exercises performed in this work $\frac{\partial Y}{\partial \tau_c} < 0$.

4 Empirical Investigation

In this session we describe our empirical exercise. In subsection 1, we describe the data and its retrieval process. The dataset we use is fully available in the appendix D.2. In subsection 2 we describe the calibration strategy and show that we have been able to fit the data. Subscript 3 shows the results of the model. In subsection 4 we perform the counterfactual exercise by simulating for the economies of our database the effects of distortions similar to those found for the US economy.

4.1 Data

We analyse the relation between the allocation of talented workers to teachers' career and its impact in the economic development. We select a sample of 43 countries[2] for 2010.

For each country k, we select GDP per worker from Penn World Table 9.0 (PWT). We calculated the share of teachers in the educational sector j as the ratio:

$$\text{Share of teacher}_j^k = \frac{\text{Teachers staff}_j^k}{\text{Workers}_j^k}$$

Where the $Workers_j^k$ are people aged 15 years and over, who worked ate the reference week, or had a job from wich they were temporarily absent. This variable is defined in the PWT as Engaged Works. Aditionally, $Teachersstaff_j^k$ include professional personal involved in teaching student. This variable is defined by the same name in the OECD database[3].

Table 1 provides an overview of the data, while the full dataset in provided in the appendix D.

[2]United States; Aruba; Austria; Bhutan; Brazil; Bulgaria; Burundi; Cabo Verde; Chile; China; China, Macao SAR; Croatia; Cyprus; Czech Republic; Finland; Hungary; Indonesia; Japan; Kyrgyzstan; Lao Peoples DR; Latvia; Lebanon; Lithuania; Luxembourg; Malaysia; Mauritius; Mexico; Mongolia; Niger; Panama; Poland; Portugal; Republic of Moldova; Romania; Saint Lucia; Serbia; Slovakia; Tajikistan; TFYR of Macedonia; Thailand; Uganda; Uruguay; Uzbekistan.

[3]The variables definitions according to OECD and PWT can be found at the appendix D.1

Table 1: Data Overview

Country	*GDP*	*Pre − primary*	*Primary*	*Secondary*	*Tertirary*	*Consumption*
USA	1	0,38%	1,27%	1,24%	1,02%	96,09%
# Countries above USA	2	20	9	28	4	24
# Countries bellow USA	40	22	33	14	38	18
Mean	0,40	0,40%	1,41%	1,69%	0,59%	95,91%
Vairance	0,09	0,0000	0,0007	0,0001	0,0000	0,0013
Max	1,31	1,09%	18,48%	5,70%	1,74%	99,92%
Min	0,01	0,01%	0,02%	0,04%	0,01%	74,10%

Among the 43 countries available in the database only two countries have a per worker GDP higher than the United States'. Nominally these countries are a Macao and Luxembourg. The former is 31% and the latter is 7% higher. All the other 40 countries in the sample present lower GDP per worker. The lowst GDP in the sample are from Niger (0,02) and Burundi (0,01)[4].

Analyzing our benchmark in relation other countries in the sample, It is possible to perceive that the US's share of workers engaged in pre-primary education is slightly below the sample mean. However, it's share of workers in primary and tertiary education is far above the median, while the share engaged in secondary education is below. Also the share of workers in consumption is slightly above the median (0.18%).There is a relatively large variance in the sample, that allows us to perform our exercises.

The United States does not employ a much higher share of workers in the education sector than the sample median. However, it surely has a notable GDP. It indicates that not only the share of workers in Education sector as a whole matters, but also the distribution of these workers within this sector.

4.2 Calibration

For the calibration we first define a set of parameters that is kept constant for the whole sample. These parameter are listed in the table below:

The parameters ϕ and η are the elasticity of human capital with respect to idiossincratic skills and elasticity of elasticity of human capital with respect to education

[4]The values are expressed in terms of the GDP per worker in the United States. E.g. Niger's GDP per worker 0,02 represents 2% of US's GDP per worker

Table 2: Fixed Parameters

Parameter	*Value*	*Description*	*Source*
ϕ	1	Elasticity of Idiossincratic skills in the human capital function	Hsieh et al. (2013)
η	0,25	Elasticity of education in the human capital function	Hsieh et al. (2013)
$\tilde{\theta}$	3,44	Dispersion of Skills	Hsieh et al. (2013)

spending. The former is a normalization where we assume that $\phi = 1$. The later is a parameter long discussed in the literature. Similar parameters have been discussed in the literature, as in Erosa et al. (2010) Manuelli and Seshadri (2014). Our model follows Hsieh et al. (2013), η will equal the share of output spent on human capital accumulation. There is no solid evidence on the value this parameter should assume. However, according to the literature, specially Delalibera (2018) and other robustness tests, we set $\eta = 1/4$. We tested this parameter to values between 1/10 and 1/2 and our results do not change.

The $\tilde{\theta}$[5] parameter refers to the distribution of comparative advantages between different occupations. The dispersion of wages across people within an occupational group also obeys a Fréchet distribution that depends strongly on the parameters $\tilde{\theta}(1-\eta)$.The lower this combination, the higher the wage dispersion there is within an occupation. Hsieh et al. (2013) estimates this parameter for the US's economy. This estimation has the baseline value of $\tilde{\theta} = 3.44$.

Taking the US as baseline its TFP is normalized to one,$A^{US} = 1$. We also assume absence of labour market frictions in all i sectors, $\tau_i^{US} = 0$. As a simplification, we calibrate $\sigma_e = \sigma_c$ to fit the share of consumption.

The alphas are calibrated to fit the US educational market according to a closed formula described in Proposition 7:

Proposition 7 *$\forall i \in \{E_1, E_2, E_3, E_4\}$,we can calibrate α_i such that:*

$$\hat{\alpha}_i = \frac{s_i^{US}}{\sum_j^{US} s_j}; j \in \{E_1, E_2, E_3, E_4\} \tag{34}$$

[5]Remember from section 2 that: $\tilde{\theta} = \theta(1 - \rho)$

See appendix C

Proposition 7 uses the previous assumption that the elasticity of production function with respect to aggregated human capital is the same for all the production functions in this economy. This proposition asserts that the final educational good is produced using the shares of each education sector within the educational sector as a whole. It says that the production of the educational good is ideally done as in the baseline economy. This is a strong assumption, yet it is reasonable in the context of using the US as reference.

Finally, ρ is calibrated to fit the output level. It's a deep parameter that directly fits the output. This parameter relates to the out since it is directly connected to γ, that sets the output level. The found value differs from the one found in the literature, specially due to the way we have chosen to the calibration. Delalibera (2018) uses 0,19 as reference, while we use 0,4795. The results of our calibration for the baseline are shown in the table bellow:

Table 3: Calibrated Parameters

Parameter	*Description*	*Value*
$\hat{\sigma_e}$	Elasticity of agregated human capital in the production function of c	0,3516
$\hat{\sigma_c}$	Elasticity of agregated human capital in the production function of e	0,3516
$\hat{\alpha_1}$	Elasticity of final educational good with respect to C_{E_1}	0,0977
$\hat{\alpha_2}$	Elasticity of final educational good with respect to C_{E_2}	0,3244
$\hat{\alpha_3}$	Elasticity of final educational good with respect to C_{E_3}	0,3178
$\hat{\alpha_4}$	Elasticity of final educational good with respect to C_{E_4}	0,2601
$\hat{\rho}$	Frechét parameter, correlation between skills	0,4795

Parameters $\{\tau_i, A_i\}_{i\in\{c,E_1,E_2,E_3,E_4\}}$ are idiosyncratic for each country. To calculate then to all other countries, we first assume that $\tau_c = 0$ and $A = A_i$ for $i \in \{c, E_1, E_2, E_3, E_4\}$. Thus, $\{\tau_i\}_{i\in\{E_1,E_2,E_3,E_4\}}$ and A target the proportion of teachers and GDP per worker, respectively. Please notice that the normalization made changes in the way we should interpret the numerical results. By making $\tau_c = 0$, we calculate the distortions for each sector of the economy in relation to the goods sector. As all salaries, gross and net of distortions are positive, $(1-\tau_i)$ is greater than zero . It implies that τs must be lower than one.

Table 4 shows the calibration results. In this table, in all entries whose values are zero for the friction parameters actually have very small values, rounded to zero to the fourth decimal place. We could consider it a consequence of the calibration algorithm that sets a lower bounds the parameters. There are two possible scenarios: Either the economies present distortions that similar to the baseline economy or the parameters were supposed to be negative and the algorithm has led us to a miscalculated result. This second scenario would lead to high Calibration Error. However, out of the 13 countries where our frictions are all equal to zero, only in 4 of the then (Burindi, Niger, Unganda and Kyrgyztan) present Calibration Errors are higher than 0,001.

The algorithm used could also lead to error since it limits the maximum value to 0.99999999999. Looking at the results this imitation is found in only three cases: China Macao secondary sector and tertiary sector besides Luxembourg secondary sector. This result would represent the extreme case of disincentives to education. Notably Luxembourg and Macao are countries with a high level of education and income, and are very small. An alternative explanation that could be better explored is the migratory movements for studies, as wealthy and well educated students are able to study abroad there is no need to develop a strong high educational sector. There are strong evidences suggesting it, according to the government of Luxembourg in 2018 an average of 160000 people commute daily from France, Belgium, and Germany[6]. It represents 29% of the population. Whereas in Macao it's possible to do the same with Chine mainland of Hong Kong, although we could not find any official source reporting any commutation statistics. It is only an initial hypothesis that needs further study. Regardless from the fact that these indicator have reached the problem's upper bound, the fitting was very good, and the Calibration Error for any of theses countries is rounded to zero to the sixth decimal place.

[6] http://www.luxembourg.public.lu/en/se-deplacer-au-luxembourg/en-commun/transport-transfrontalier/index.html

Table 4: Calibration Results

Country	*Abreviation*	A	τ_{E_1}	τ_{E_2}	τ_{E_3}	τ_{E_4}	*CalibrationError*
United States	USA	1	0	0		0	0,000007
Aruba	ABW	0,85	0,168	0,205	0,190	0,098	0
Austria	AUT	0,86	0,289	0,134	0,342	0,262	0
Bhutan	BTN	0,36	0	0	0	0	0,000252
Brazil	BRA	0,5	0,001	0,001	0,001	0	0
Bulgaria	BGR	0,53	0,002	0,001	0,002	0,001	0
Burundi	BDI	0,17	0	0	0	0	3,137853
Cabo Verde	CPV	0,38	0	0,001	0	0	0,000001
Chile	CHL	0,65	0,026	0,01	0,01	0,013	0
China	CHN	0,39	0	0	0	0	0
China, Macao SAR	MAC	1,05	0,753	0,627	0,958	0,999	0
Croatia	HRV	0,69	0,029	0,014	0,056	0,023	0
Cyprus	CYP	0,89	0,332	0,316	0,528	0,253	0
Czech Republic	CZE	0,7	0,002	0,002	0,003	0,001	0
Finland	FIN	0,85	0,304	0,16	0,284	0,124	0
Hungary	HUN	0,71	0,063	0,023	0,057	0,019	0
Indonesia	IDN	0,4	0	0	0	0	0,000001
Japan	JPN	0,78	0,037	0,04	0,063	0,067	0
Kyrgyzstan	KGZ	0,27	0	0	0	0	0,002287
Lao Peoples DR	LAO	0,28	0	0	0	0	0,000784
Latvia	LVA	0,66	0,031	0,014	0,025	0,013	0
Lebanon	LBN	0,68	0,039	0,04	0,053	0,038	0
Lithuania	LTU	0,72	0,116	0,028	0,115	0,051	0
Luxembourg	LUX	0,99	0,919	0,743	0,999	0,246	0
Malaysia	MYS	0,65	0,013	0,022	0,016	0,008	0
Mauritius	MUS	0,58	0,005	0,003	0,005	0,001	0
Mexico	MEX	0,57	0,004	0,003	0,004	0,002	0
Mongolia	MNG	0,41	0	0	0	0	0
Niger	NER	0,19	0	0	0	0	2,135483
Panama	PAN	0,56	0,002	0,003	0,003	0,003	0
Poland	POL	0,69	0,024	0,03	0,035	0,016	0
Portugal	PRT	0,74	0,046	0,054	0,079	0,036	0
Republic of Moldova	MDA	0,35	0	0	0	0	0,00003
Romania	ROU	0,6	0,007	0,003	0,009	0,002	0
Saint Lucia	LCA	0,47	0,001	0,001	0,001	0	0
Serbia	SRB	0,63	0,014	0,006	0,023	0,007	0
Slovakia	SVK	0,75	0,079	0,029	0,096	0,034	0
Tajikistan	TJK	0,29	0	0	0	0	0,000399
TFYR of Macedonia	MKD	0,56	0,003	0,003	0,006	0,002	0
Thailand	THA	0,44	0	0	0	0	0,000037
Uganda	UGA	0,23	0	0	0	0	0,072623
Uruguay	URY	0,57	0,003	0,004	0,005	0,004	0
Uzbekistan	UZB	0,39	0	0	0	0	0

From the results found for calibration and the discussion in the previous two paragraphs about possible limitations of calculation of taxes by the algorithm, we can conclude that there was a good adjustment of the model to the data. The figures bellow illustrate our calibration results. Figure 1 shows that calibration fit well into the product data. This is not a as result since the calibration targets GDP. Figure 2 shows a positive relation between the calculated TFP and GDP per worker. The result is trivial because the calibration targets the GDP, that is correlated to the TFP. Although trivial both figures reinforces the good fitting of the model.

Figure 1: Fitting GDP

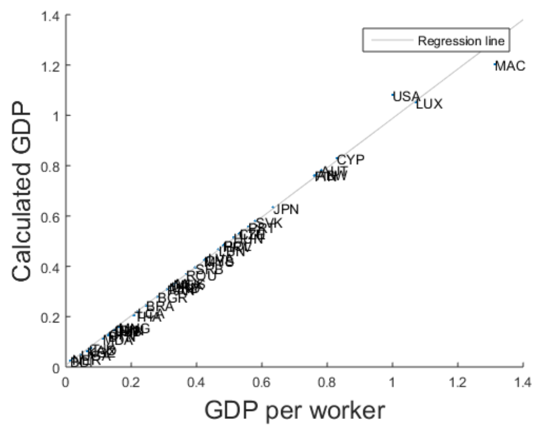
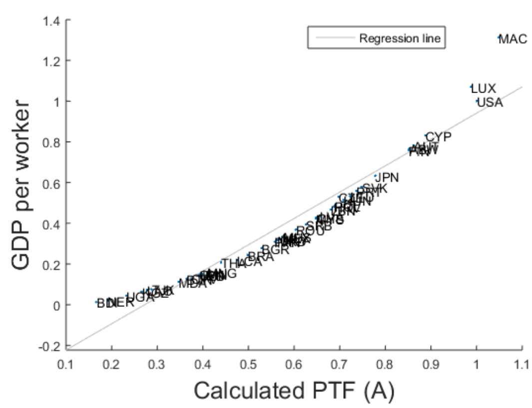


Figure 2: GDP vs PTF



4.3 Results

Our model brings several insights on dynamic economic development of several countries. It highlights different aspect from different stages of development. In this section we will present the main results this model has so far pointed out to. The first result relates to crescent importance of the TFP along the growth path. The second result relates to the impact of the human capital in the production of consumption goods. Finally, we discuss the importance of human capital in teachers' occupation to economics growth.

Figure 2 suggests that total factor productivity has a greater impact at more advanced levels of development. The amount of inputs in the production function have a very large impact on output levels at the initial phases of development. Whereas the growth part now well explained by productions inputs has a greater impact on more advanced economies. This Impact is mainly explained by the shape of the human capital function chosen to this models. At very low levels of output, this function yields a high marginal return to education. Moreover, the structure of intermediary goods reinforces this aspect.

The figures 3 shows two results. First, the GDP is crescent with regard to the average human capital of the employees in the sector. Second, the product per worker is increasing convexly in relation to the average human capital of the employees in the sector. These results are consequences of the production function is crescent in all inputs and also consequence of the presence of intermediary goods.

Figures 4, 5, 6, 7 all illustrate the same behavior. In the early stages of the growth trajectory the role of teachers' human capital in product development is greater than in more develop phrases.

This result is the same for all levels of education: pre-primary, primary, secondary and tertiary. The figures below represent distinct educational levels and lastly a weighted average by each education sector's share of the human capital of all teachers for each country. These figures are similar but are not identical and the higher the alpha calibrated for each sector the more concave are these figures are.

The result found here has an important impact on the proposal of public policies,

Figure 3: GDP vs Average Human Capital in Goods Sector

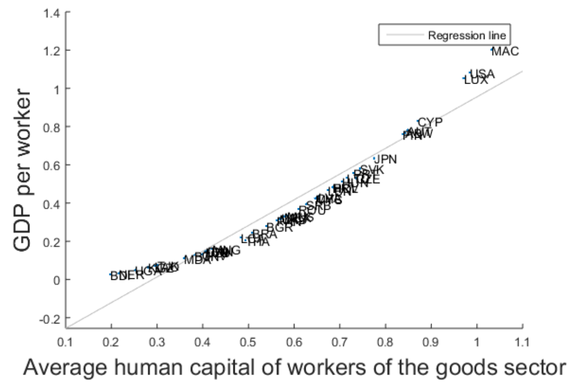


Figure 4: GDP vs Teachers average human capital

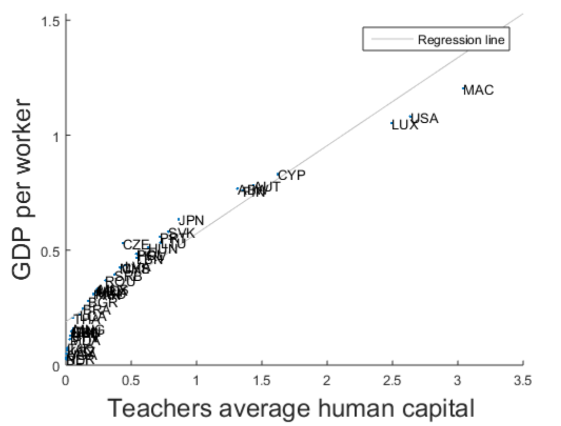


Figure 5: GDP vs Teachers average human capital of pre-school

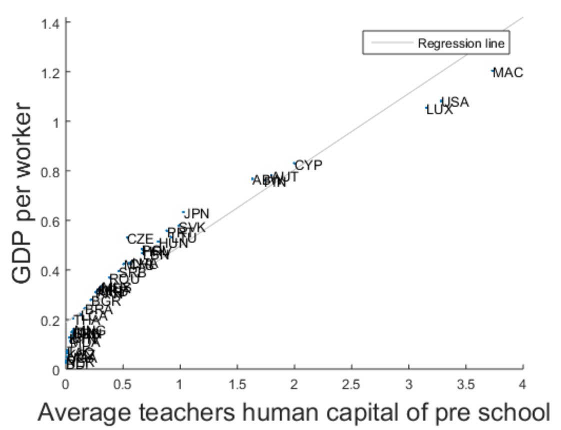


Figure 6: GDP vs Teachers average human capital of primary

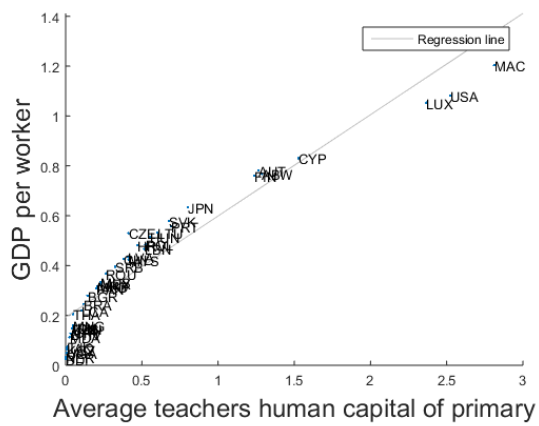
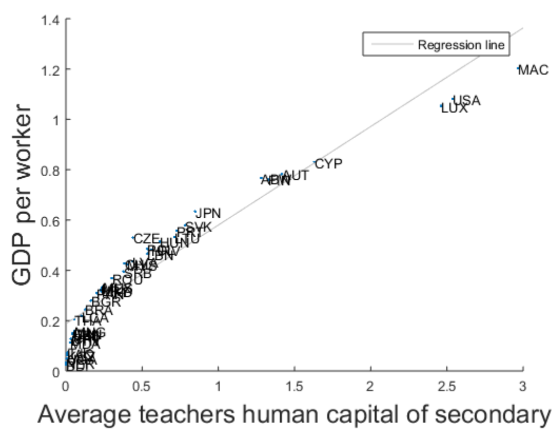


Figure 7: GDP vs Teachers average human capital of secondary



especially in the context of the value of teachers' careers. Discussions inside and outside the academy debate how important it is to bring the best students to teaching career.

The results of the model indicate that this discussion should take into account the level of development of the countries. On the one hand, if a person with high human capital is employed in the teaching profession, he produces human capital with great efficiency, but he also represents a high cost in the production final goods consumption. On the other hand, if that same individual is employed in the production of consumer goods, he ceases to produce educational goods so efficiently. In terms of maximizing well-being, this trade-off has no obvious solution, as the relative costs differ according to the stage of development that each country finds itself.

In poorer countries teacher career enhancement policies have a higher output than in richer countries. Concomitantly, attracting the best students or encouraging selection for those individuals with the highest human capital for Teaching Profession has the potential to increase the product significantly more than in richer countries.

This result is in line with initiatives to attract the best students to the teaching career in developing countries. However in more develop countries, initiatives such as teachers for america may have negative impacts on the worker's productivity. The discussion of the microeconomics perspective is more complex because it takes into account the possibility of other positive externalities, besides taking into account any regional aspects.

The model specifies that the occupational choice is done through observation of relative salaries. As so the way to attract people with greater human capital to a particular profession is through promotion of the best salaries for educators in relation to the salaries of the rest of the economy. This is a direct result of the utility maximization process described in section 2.

4.4 Counterfactual

First, we develop a short counterfactual exercise. We assume that the distortions presented in the US economy are the same as those presented for all other economies in our dataset. Through this exercise we can observe that from the 43 countries in our sample, only 5 would present a reduction in the output per worker. Nominally these countries are Aruba (-1%), Austria(-1%), Macau(-8%), Finland(-1%) and Luxembourg(-2%). Macao and Luxembourg are peculiar cases that have already been previously discussed.

The counterfactual results found that it is possible to have a reduction of the product in the situation of presenting a similar distortion to the baseline economy seems contradictory to that developed in Section 3 of this thesis. However Section 3 is developed in a model without normalization of the friction parameters and only illustrates theoretical results on the impact of the existence of distortions.

Table 5: Counter Factual

$Country$	GDP	$CounterfactualGDP$	$Variation$
United States	1,00	1,00	0%
Aruba	0,77	0,76	-1%
Austria	0,78	0,78	-1%
Bhutan	0,13	0,30	136%
Brazil	0,24	0,38	56%
Bulgaria	0,28	0,40	44%
Burundi	0,01	0,17	1235%
Cabo Verde	0,13	0,31	133%
Chile	0,43	0,48	13%
China	0,15	0,32	115%
China, Macao SAR	1,31	1,21	-8%
Croatia	0,48	0,52	8%
Cyprus	0,83	0,83	0%
Czech Republic	0,53	0,53	0%
Finland	0,76	0,75	-1%
Hungary	0,51	0,54	5%
Indonesia	0,15	0,32	113%
Japan	0,63	0,63	0%
Kyrgyzstan	0,06	0,24	274%
Lao Peoples DR	0,07	0,25	244%
Latvia	0,44	0,49	13%
Lebanon	0,47	0,51	10%
Lithuania	0,53	0,56	5%
Luxembourg	1,07	1,05	-2%
Malaysia	0,42	0,48	14%
Mauritius	0,33	0,43	29%
Mexico	0,33	0,43	30%
Mongolia	0,16	0,33	106%
Niger	0,02	0,19	866%
Panama	0,31	0,42	35%
Poland	0,48	0,52	8%
Portugal	0,56	0,58	3%
Republic of Moldova	0,11	0,29	159%
Romania	0,37	0,45	22%
Saint Lucia	0,22	0,37	68%
Serbia	0,40	0,47	18%
Slovakia	0,58	0,59	2%
Tajikistan	0,08	0,26	237%
TFYR of Macedonia	0,32	0,42	33%
Thailand	0,21	0,35	67%
Uganda	0,04	0,22	395%
Uruguay	0,32	0,43	32%
Uzbekistan	0,15	0,32	118%

For all other countries in our sample there is a significant increase in GDP per worker. Also note that the lower the GDP the higher the prediction of growth using the American distortions. The simulation is shown in the Table 5.

From the sample of 43 countries, 34 presented an increase in productivity while in 13 cases the expected growth was higher than 100%. The highest growth rates can be seen in Burundi (1235%), Niger (866%) and Uganda (395%). They are also the countries with the lowest productivity per worker in the sample and the highest calibration errors: 3,13; 2,13 and 0,07 respectively. This observation may lead to the conclusion that the larger the calibration error the greater the expected effect on the counterfactual. However, among economies with very small calibrations errors, namely less than 10^{-5}, the rates of change in the counterfactual range from -8% to 133%. Also the correlation coefficient between the errors and the counter factual variation is not statically different than zero leading us to discard the hypothesis that the errors are responsible for the found variation.

Let us now try to decompose the counterfactual results. This exercise proposal systematizes the question: What is the macroeconomic impact of reducing the distortions observed in a single sector? Since all the distortions are relative, this analysis would have a deeper meaning than just simulating a sector without distortion. As we simulate one sector's distortion to be equal to the one presented in the baseline all else equal we actually simulate an economy with incentives to this peculiar sector. It happens because the frictions matter in relative ways to the the occupational choices. For example, if we simulate an economy where the i sector does not have frictions ($\tau_{Ei} = 0$) while holding all other parameters constant, we do not simulate the case where the labour market for one specific sector is similar to the baseline's. In stead we would simulate an economy where this i educational sector has individually similar properties to the baseline's but in the context of a general equilibrium it represents a much less distorted sector the the one in the baseline. It opens spaces to results in which the counter factual simulation for a single sector without distortions actually presents a higher increases in GDP per worker than in the full simulation. In the context of a normalization to the friction parameters, it might be understood as a $\tau < 0$ that emulates incentives to a sector as $1 - \tau > 0$.

Table 6 and 7 show the results for the decomposed Counter Factual. We name decomposed counter factual 1, 2, 3 and 4 the exercises in which the countries present respectively similar pre-primary, primary, secondary and tertiary educational to the ones in the baseline's economy.

In the first decomposed counter-factual there are three countries that stand out: Burundi, Niger and Macao. They stand out specifically because they exhibit decomposed effects higher than those previously presented. While in the first exercise the results were respectively: 1235%, 866% and -8% now they are 1419%, 931% and -8,8%.

This very same situation is going to be found in all the remaining results for Burundi and Niger. These are also the economies by far with the lower GDP and worst fitting in the calibration, as we have already discussed. On average the variation in the first decomposed counter factual result correspond to 51,2% of the variation observed the exercise presented in Table 5.

The second decomposed exercise simulates an economy where the distortions toward primary education is the same as in the baseline, all else constant. In this context, additional to Burundi and Niger another five countries present a variation higher than the one seen in table 5. They are namely: Tajikistan, Thailand, Uganda, Lao Peoples DR and Kyrgyzstan. These five countries present similar results in the third and fourth decomposed CF. On average this CF presented a variation equivalent to 78,9% of the total variation presented in table 5.

Table 7 present the final results. On average the GDP variation seen in CF3 and CF4 was respectively 78,5% and 73,8% of the one presented in table 5. In summary, from our sample of 42 countries, 10 have shown in at least one of the decomposed counter factual results a variation higher than the one presented in the full counter factual. Two of these countries, Burundi and Niger have presented the pathological situation where all the partial CF are superior to the full CF. In this case, we need further studying, but initials observations indicates that there might be problems in the calibration as the calculated errors were by far the highest in the sample. For all the other 8 countries, the growth in GDP superior to the one displayed in table 5 is explained by the change of relative friction previously explained in this session.

Table 6: Decomposed Counter Factual 1 & 2

Country	GDP	Counter Factual 1	Variation CF1	Counter Factual 2	Variation CF2
Aruba	0,77	0,77	-0,2%	0,76	-0,6%
Austria	0,78	0,78	-0,1%	0,77	-1%
Bhutan	0,13	0,26	105,5%	0,31	137,6%
Brazil	0,24	0,32	29,4%	0,36	47,5%
Bulgaria	0,28	0,34	20,1%	0,38	34,5%
Burundi	0,01	0,2	1419,9%	0,22	1577,8%
Cabo Verde	0,13	0,25	91,5%	0,3	125%
Chile	0,43	0,44	3,3%	0,46	7,6%
China	0,15	0,28	84,9%	0,32	112,6%
China, Macao SAR	1,31	1,2	-8,5%	1,2	-8,8%
Croatia	0,48	0,49	1,5%	0,5	3,4%
Cyprus	0,83	0,83	-0,2%	0,83	-0,6%
Czech Republic	0,53	0,53	0,6%	0,54	1,1%
Finland	0,76	0,76	-0,1%	0,75	-0,7%
Hungary	0,51	0,52	1%	0,53	2,3%
Indonesia	0,15	0,28	80,7%	0,32	108,7%
Japan	0,63	0,63	-0,1%	0,63	-0,4%
Kyrgyzstan	0,06	0,22	246,9%	0,25	290,5%
Lao Peoples DR	0,07	0,23	221%	0,26	263,2%
Latvia	0,44	0,45	2,8%	0,46	6,8%
Lebanon	0,47	0,47	1,7%	0,49	4,7%
Lithuania	0,53	0,54	0,8%	0,54	1,4%
Luxembourg	1,07	1,05	-1,7%	1,05	-1,9%
Malaysia	0,42	0,44	3,1%	0,46	8,3%
Mauritius	0,33	0,37	10,4%	0,4	21,1%
Mexico	0,33	0,37	10,5%	0,4	21,5%
Mongolia	0,16	0,28	73,1%	0,32	98,9%
Niger	0,02	0,2	931,4%	0,23	1046,2%
Panama	0,31	0,35	13,5%	0,39	26,3%
Poland	0,48	0,49	1,4%	0,5	3,9%
Portugal	0,56	0,56	0,4%	0,56	1,2%
Republic of Moldova	0,11	0,26	125,7%	0,29	156,8%
Romania	0,37	0,39	6,8%	0,42	14,5%
Saint Lucia	0,22	0,3	38,6%	0,35	59,6%
Serbia	0,4	0,41	4,6%	0,44	10,5%
Slovakia	0,58	0,58	0,3%	0,58	0,3%
Tajikistan	0,08	0,23	205,3%	0,26	246,3%
TFYR of Macedonia	0,32	0,36	12,3%	0,39	24,3%
Thailand	0,21	0,32	52,5%	0,36	71,6%
Uganda	0,04	0,22	386,2%	0,24	446,4%
Uruguay	0,32	0,36	11,2%	0,4	23%
Uzbekistan	0,15	0,27	85,3%	0,31	112,7%

Table 7: Decomposed Counter Factual 3 & 4

Country	GDP	Counter Factual 3	Variation CF3	Counter Factual 4	Variation CF4
Aruba	0,77	0,76	-0,6%	0,76	-0,9%
Austria	0,78	0,78	-0,4%	0,78	-0,4%
Bhutan	0,13	0,3	136,4%	0,3	130,7%
Brazil	0,24	0,36	48%	0,35	43,8%
Bulgaria	0,28	0,38	35,5%	0,37	32,4%
Burundi	0,01	0,21	1541,2%	0,2	1475,3%
Cabo Verde	0,13	0,3	122,4%	0,29	115,4%
Chile	0,43	0,46	7,6%	0,46	6,8%
China	0,15	0,32	112,4%	0,31	106,8%
China, Macao SAR	1,31	1,2	-8,4%	1,2	-8,4%
Croatia	0,48	0,5	4,4%	0,5	3,3%
Cyprus	0,83	0,83	-0,3%	0,83	-0,6%
Czech Republic	0,53	0,54	1,5%	0,53	0,9%
Finland	0,76	0,76	-0,4%	0,75	-0,7%
Hungary	0,51	0,53	2,8%	0,52	1,8%
Indonesia	0,15	0,32	108,4%	0,31	102,1%
Japan	0,63	0,63	-0,1%	0,63	-0,1%
Kyrgyzstan	0,06	0,25	293,8%	0,25	286%
Lao Peoples DR	0,07	0,26	261,7%	0,26	254%
Latvia	0,44	0,47	7,1%	0,46	5,8%
Lebanon	0,47	0,49	4,8%	0,49	3,9%
Lithuania	0,53	0,55	2,2%	0,54	1,5%
Luxembourg	1,07	1,05	-1,6%	1,04	-2,5%
Malaysia	0,42	0,46	8%	0,45	6,5%
Mauritius	0,33	0,41	21,4%	0,39	17,9%
Mexico	0,33	0,4	21,5%	0,39	19,1%
Mongolia	0,16	0,32	99,8%	0,31	95,1%
Niger	0,02	0,22	1013,8%	0,22	1001,8%
Panama	0,31	0,39	26,1%	0,38	23,8%
Poland	0,48	0,5	3,9%	0,5	2,9%
Portugal	0,56	0,57	1,4%	0,56	0,8%
Republic of Moldova	0,11	0,29	158,9%	0,29	152,3%
Romania	0,37	0,42	15,3%	0,42	12,6%
Saint Lucia	0,22	0,35	59,3%	0,34	54,6%
Serbia	0,4	0,44	11,4%	0,43	9,2%
Slovakia	0,58	0,59	1,1%	0,58	0,4%
Tajikistan	0,08	0,26	248,2%	0,26	239,8%
TFYR of Macedonia	0,32	0,4	24,9%	0,38	21,6%
Thailand	0,21	0,36	72,6%	0,35	68,5%
Uganda	0,04	0,24	441,1%	0,23	426,6%
Uruguay	0,32	0,4	22,8%	0,39	20,5%
Uzbekistan	0,15	0,32	114,4%	0,3	106,5%

5 Final Remarks

We have developed a model for measuring the impact of the allocation of people on the occupation of teachers on economic growth. It is a general equilibrium model that features intermediary goods, heterogeneous agents and labour market frictions. These market frictions represent any reason why the salary received by employees is different from the salary paid by the employer. They exist for various reasons and are taken as exogenous. The frictions are given in relative terms, because in this model they mainly influence the occupational choices from the salaries perceived by the consumers in their process of occupational choice.

Analytically, we showed that the higher the frictions on the education sector are, the higher the price of the final educational good is. Analogously, the higher the friction in the consumption sector, the lower the educational price relative to the numeraire good. Moreover, a second result shows that under any condition, an increase in educational friction causes a reduction in output. A third result shows that under weak assumptions about the parameters an increase in frictions in the consumption parameter causes a reduction in output.

Empirically, we calibrated the data using the baseline of the US economy. As a result we find that the product is increasing in all types of human capital. However, this ratio seems to be marginally decreasing for human capital of employees in the education sector and marginally convex for human capital of employees in the capital goods sector.

Finally, by simulating the same baseline distortions for the other countries in the sample we forecast a growth for almost all economies. The predicted growth reached 1245% and was on average 110%. Not only that, the smaller the product, the greater the expected growth when subjected to the same distortions of the United States. In addition, it should be noted that the economies under investigation that have reduced their output are in general economies with high quality of education, high income and very small territory that allows easily for students and other professional to commute.

This work may be improved in several dimensions. For example: (i) Section 3 might be extended to include a investigation about the results after a normalization; (ii) The human capital function could be adapted to represent skills in stead of idiosyncratic skills; (iii) The same model could be applied to only 3 educational sectors in order to significantly expand the database.

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A Proofs of section 2

A.1 Proofs of section 2.1

Proposition 1 *Aggregating across people, the solution to the individual's occupational choice problem allows us to write*

$$s_i = \frac{\tilde{w}_i^{\theta}}{\sum_{j=1}^{N} \tilde{w}_i^{\theta}}, \tag{7}$$

where s_i is the fraction of people that work in occupation i and

$$\tilde{w}_i = ((1-\tau_i)w_i)^{\frac{1}{\phi}} .$$

Let

$$\tilde{w}_i = ((1-\tau_i)w_i)^{\frac{1}{\phi}} , \tag{35}$$

We can use equation (**??**) to rewrite the the utility function as the indirect utility. (**??**) as

$$U_i = log\left[\left(\frac{\eta}{p_E^{\eta}}\right)^{\frac{1}{1-\eta}} \tilde{w}_i{}^{\frac{\phi}{1-\eta}} \epsilon_i^{\frac{\phi}{1-\eta}}\right].$$

The solution of individual's problem involves picking the occupation with the highest value of U_i. This is analogous to picking up the highest value of $\tilde{w}_i\epsilon_i$.

Without loss of generality, consider the probability of an individual choose occupation 1

$$\begin{aligned} s_1 &= \Pr\left(\tilde{w}_1\epsilon_1 \geq \tilde{w}_i\epsilon_i\right)\ \forall i \neq 1 \\ &= \Pr\left(\epsilon_i \leq \frac{\tilde{w}_1}{\tilde{w}_i}\epsilon_1\right)\ \forall i \neq 1 \\ &= \int F_1(\mu_1\epsilon_1, \mu_2\epsilon_2, \ldots, \mu_N\epsilon_N)d\epsilon \end{aligned} \tag{36}$$

where F_1 represents the derivative of (3) with respect to its first argument and $\mu_i =$

$\tilde{w}_1/\tilde{w}_i$ for $i \in \{1, \ldots, N\}$. Taking the derivative of (3) with respect to ϵ_1 and evaluating at the appropriate arguments gives

$$F_1(\mu_1\epsilon, \mu_2\epsilon, \ldots, \mu_N\epsilon) = \hat{S}^{-\rho}\theta\epsilon^{-\theta(1-\rho)-1}\exp\left[-\left(\hat{S}\epsilon^{-\theta}\right)^{1-\rho}\right]$$

where $\hat{S} = \sum_{i=1}^{n} \alpha_i^{-\theta}$. Then, (36) can be written as

$$\begin{aligned} s_1 &= \int \frac{\hat{S}^{1-\rho}}{\hat{S}}\hat{S}^{-\rho}\theta\epsilon^{-\theta(1-\rho)-1}\exp\left[-\left(\hat{S}\epsilon^{-\theta}\right)^{1-\rho}\right]d\epsilon \\ &= \frac{1}{\hat{S}}\int \hat{S}^{1-\rho}\hat{S}^{-\rho}\theta\epsilon^{-\theta(1-\rho)-1}\exp\left[-\left(\hat{S}\epsilon^{-\theta}\right)^{1-\rho}\right]d\epsilon \\ &= \frac{1}{\hat{S}}\int dF(\epsilon) \\ &= \frac{1}{\hat{S}} \\ &= \frac{1}{\sum_i \left(\frac{\tilde{w}_1}{\tilde{w}_i}\right)^{-\theta}} \\ &= \frac{\tilde{w}_1^{\theta}}{\sum_{i=1}^{N}\tilde{w}_i^{\theta}}. \end{aligned} \tag{37}$$

This argument can be easily extended to occupation i.

Proposition 2 *The average human capital of workers in occupation i, including both educational choice and idiosyncratic abilities, is*

$$\mathbb{E}[h(e_i, \epsilon_i)|\textit{person chooses } i] = \gamma\bar{\eta}\left[\frac{(1-\tau_i)w_i}{P_E}\right]^{\frac{\eta}{1-\eta}} s_i^{-\frac{\phi}{\theta(1-\eta)}}, \tag{8}$$

where $\gamma = \Gamma\left(1 - \frac{\phi}{\theta(1-\eta)(1-\rho)}\right)$ *is related to the mean of the Fréchet distribution for abilities.*

First, notice that

$$h(e_i^*(\epsilon_i), \epsilon_i) = \left(\frac{\eta(1-\tau_i)w_i}{p_E}\right)^{\frac{\eta}{1-\eta}} \epsilon_i^{\frac{\phi}{1-\eta}}, \tag{38}$$

Then, the total efficiency units of labor supplied to occupation i is given by

$$
\begin{aligned}
h_i &= \mathbb{E}\left(h(e_i, \epsilon_i) | \text{person chooses } i\right) && (39)\\
&= \left(\frac{\eta(1-\tau_i)w_i}{p_E}\right)^{\frac{\eta}{1-\eta}} \left(\epsilon_i^{\frac{\phi}{1-\eta}} | \text{person chooses } i\right). && (40)
\end{aligned}
$$

To calculate this last conditional expectation, we use the Fréchet distribution. Let $y_i = \tilde{w}_i \epsilon_i$. Since y_i is the thing we are maximizing, it inherits the extreme value distribution:

$$
\begin{aligned}
\Pr\left(\max_i y_i \le z\right) &= \Pr\left(\epsilon_i \le z/\tilde{w}_i\right) \ \forall i && (41)\\
&= F\left(z/\tilde{w}_1, \ldots, z/\tilde{w}_N\right) && (42)\\
&= \exp\left[-\left(\sum_{i=1}^{N} (z/\tilde{w}_i)^{-\theta}\right)^{1-\rho}\right] && (43)\\
&= \exp\left[-\left(\hat{S}^* z^{-\theta}\right)^{1-\rho}\right] && (44)\\
& && (45)
\end{aligned}
$$

That is, the extreme value also has a Fréchet distribution.

Straightforward algebra then reveals that the distribution of ϵ^*, the ability of people in their chosen occupation, is also Fréchet:

$$
G(z) = \Pr\left(\epsilon^* < z\right) = \exp\left[-\left(\hat{S}^* z^{-\theta}\right)^{1-\rho}\right], \tag{46}
$$

where $\hat{S}^* = \sum_{i=1}^{N} (\tilde{w}_i / \tilde{w}^*)^{\theta}$.

Finally, one can then calculate the expectation we needed above back in equation (39). Let i denote the occupation that the individual chooses, and let λ be some positive exponent. Then,

$$E(\epsilon_i^\lambda) = \int_0^\infty \epsilon_i^\lambda dG(\epsilon) \tag{47}$$

$$= \int_0^\infty \theta(1-\rho)\hat{S}^{*(1-\rho)}\epsilon^{-\theta(1-\rho)-1+\lambda} \exp\left[-\left(\hat{S}^*\epsilon^{-\theta}\right)^{1-\rho}\right] d\epsilon \tag{48}$$

$$= \hat{S}^{*\lambda/\theta} \int_0^\infty x^{-\frac{\lambda}{\theta(1-\rho)}} \exp(-x)dx, \tag{49}$$

where $x = (\hat{S}^*\epsilon^{-\theta})^{1-\theta}$. The last part of (49) is a gamma function which amounts to $\Gamma(1-\lambda(\theta(1-\rho))^{-1})$. [7] Therefore, we have

$$\mathbb{E}\left(\epsilon_i | \text{person chooses } i\right) = s_i^{-\frac{\lambda}{\theta}} \Gamma\left(1 - \frac{1}{\theta(1-\rho)}\right). \tag{50}$$

Using this result in the equation (39) and considering $\lambda = \frac{\phi}{1-\eta}$ completes the proof.

[7] Remember that a gamma function is

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx.$$

A.2 Proof of section 2.3

A.2.1 Final goods

Proposition 4 $\forall i \in \{c, E_1, E_2, E_3, E_4\}$, *there is biunivocal relation between prices and salaries such that:*

$$w_i = \left(\tilde{\sigma}_i A_i P_i\right)^{\frac{1}{\sigma_i}} \tag{21}$$

where $\tilde{\sigma}_i = (1-\sigma_i)^{1-\sigma_i}\sigma_i^{\sigma_i}$

The production function of firm i is:

$$Q_i = A_i X_{i,c}^{1-\sigma_i} H_i^{\sigma_i}, i \in \{c, E_1, E_2, E_3, E_4\} \tag{51}$$

Yields the following first order conditions:

$$[X_{i,c}] : \frac{p_i(1-\sigma_i)Q_i}{x_{i,c}} = p_c \tag{52}$$

$$[H_i] : \frac{p_i \sigma_i Q_i}{H_i} = w_i \tag{53}$$

Using $P_c = 1$ and substituting we have:

$$(1-\sigma_i)p_i Q_i = X_{i,c} \Rightarrow X_{i,c} = \frac{1-\sigma_i}{\sigma_i} H_i w_i \tag{54}$$

From the production function:

$$
\begin{aligned}
Q_i &= A_i\Big[\frac{1-\sigma_i}{\sigma_i}H_i w_i\Big]^{1-\sigma_i}H_i^{\sigma_i} \\
&= A_i\Big[\frac{1-\sigma_i}{\sigma_i}w_i\Big]^{1-\sigma_i}H_i \\
&= A_i\Big[\frac{1-\sigma_i}{\sigma_i}w_i\Big]^{1-\sigma_i}\frac{p_i\sigma_i Q_i}{w_i} \\
&= A_i\Big[\frac{1-\sigma_i}{\sigma_i}\Big]^{1-\sigma_i}p_i\sigma_i Q_i w_i^{-\sigma_i}
\end{aligned} \tag{55}
$$

Then:

$$
w_i^{\sigma_i} = A_i\Big[\frac{1-\sigma_i}{\sigma_i}\Big]^{1-\sigma_i}\sigma_i p_i \tag{56}
$$

Equivalently:

$$
w_i = \Big(\tilde{\sigma}_i A_i P_i\Big)^{\frac{1}{\sigma_i}} \tag{57}
$$

where $\tilde{\sigma}_i = (1-\sigma_i)^{1-\sigma_i}\sigma_i^{\sigma_i}$

A.3 Educational Goods

Educational good for final consumption is produced combining other educational good as given by:

$$
C_E = \prod_{j=E_1}^{E_4} c_i^{\alpha_i} \tag{58}
$$

where $\sum_{i=E_1}^{E_4}\alpha_i = 1$.

The educational goods E_i $i \in \{E_i\}_{i=1}^4$ can be used to produce a final educational good C_E. To emulate this process we use a Cobb-Douglas technology in a competitive firm. The firm's problem is then:

$$\underset{\{c_i\}_{i\in\{\{E_j\}_{j=1}^4\}}}{\text{Max}} p_E C_E - \sum_{i\in\{\{E_j\}_{j=1}^4\}} p_i c_i \tag{59}$$

Furthermore, the first order condition from (19) yields:

$$P_{Ei} c_i^* = \alpha_i P_E C_E \tag{60}$$

Using (60), we can calculate all $p_i, i \in \{\{E_i\}_{i=1}^4\}$. From (19), we get P_E from the identity:

$$pE = \prod_{i\in\{E_i\}_{i=1}^4} \left(\frac{P_{Ei}}{\alpha_i}\right)^{\alpha_i} \tag{61}$$

Proposition 8 *Considering the index* $j \in \{c, \{E_j\}_{i=j}^4\}$*, and index* $i \in \{c, E_1, E_2, E_3, E_4\}$*, there is a linear relation between* $\tilde{w}_j^\theta$ *and* $\tilde{w}_i^\theta$ *salaries such that:*

$$\tilde{w}_i^\theta = (1-\tau_i)\alpha_i \eta \sigma_i \sum_i \tilde{w}_i^\theta, \forall i \neq c \tag{62}$$

Remember from the production function, the First Order Condition yields that:

$$\frac{p_i \sigma_i Q_i}{w_i} = H_i$$

Also, the market clearing condition $c_i = Q_i \forall i \neq c$ and the first order condition from the final educational good, all together states that:

$$H_i = \frac{p_E C_E \alpha_i \sigma_i}{w_i} = \frac{\alpha_i \sigma_i}{w_i} \tilde{\eta}\gamma \frac{\left[\sum_i \tilde{w}_i^\theta\right]^{\frac{\phi}{\theta(1-\eta)}}}{p_E^{\frac{\eta}{1-\eta}}}$$

The equation above is the aggregated demand of human capital. Using the equation of the agreggated supply of human capital, for the market clearing:

$$\frac{\alpha_i \sigma_i}{w_i} \tilde{\eta} \gamma \frac{\left[\sum_i \tilde{w}_i^{\theta}\right]^{\frac{\phi}{\theta(1-\eta)}}}{p_E^{\frac{\eta}{1-\eta}}} = \gamma \bar{\eta} \frac{\tilde{w}_i^{\frac{\theta-\phi}{\phi}}}{p_E^{\frac{\eta}{1-\eta}} \left(\sum_i \tilde{w}_i^{\frac{\theta}{\phi}}\right)^{\frac{(1-\eta)\theta-\phi}{(1-\eta)\theta}}}$$

Using the $\phi = 1$ and $\sigma_i = \sigma_E, \forall i \in \{E_i\}_{i=1}^4$:

$$s_i = \eta \alpha_i \sigma_e (1 - \tau_i) \tag{63}$$

$$\tilde{w}_i^{\theta} = (1 - \tau_i) \alpha_i \eta \sum_i \tilde{w}_i^{\theta}, \forall i \neq c \tag{64}$$

B Demonstrations Analytical Investigation

Proposition 9 *Assume that $\tau_i > 0$ and $\theta > 1$. So that,*

1. The derivative of educational price toward distortion in the educational occupation is positive.

2. The derivative of educational price toward distortion in the goods occupations is negative.

3. The derivative of output toward distortion in the educational occupation is negative

4. $\frac{\partial Y}{\partial \tau_c} < 0 \Leftrightarrow \left(2 - \tau_E - \frac{1}{\eta\sigma_E}\right) < 0;$

Remember that:

$$p_E = \frac{A_c^{\frac{\sigma_e}{\sigma_c}}}{A_E}\left[\frac{\eta\sigma_E(1-\tau_E)}{1-\eta\sigma_E(1-\tau_E)}\right]^{\frac{\sigma_E}{\theta}}\left[\frac{(1-\tau_c)}{(1-\tau_E)}\right]^{\sigma_E}\left[\sigma_E^{-\sigma_E}(1-\sigma_E)^{\sigma_E-1}\tilde{\sigma}_c^{\frac{\sigma_E}{\sigma_c}}\right]$$

let

$$A = \frac{A_c^{\frac{\sigma_e}{\sigma_c}}}{A_E} \tag{65}$$

$$B(\tau_E) = \left[\frac{\eta\sigma_E(1-\tau_E)}{1-\eta\sigma_E(1-\tau_E)}\right]^{\frac{\sigma_E}{\theta}} \tag{66}$$

$$C(\tau_c, \tau_E) = \left[\frac{(1-\tau_c)}{(1-\tau_E)}\right]^{\sigma_E} \tag{67}$$

$$D = \left[\sigma_E^{-\sigma_E}(1-\sigma_E)^{\sigma_E-1}\tilde{\sigma}_c^{\frac{\sigma_E}{\sigma_c}}\right] \tag{68}$$

$$p_E = AB(\tau_E)C(\tau_c, \tau_E)D \tag{69}$$

1.
$$\frac{\partial p_E}{\partial \tau_c} = AB(\tau_E)D\frac{\partial C}{\partial \tau_c} = AB(\tau_E)C(\tau_c,\tau_E)D\sigma_E\frac{-1}{(1-\tau_c)} < 0 \tag{70}$$

2.

$$\begin{aligned}\frac{\partial p_E}{\partial \tau_E} &= AD\left[B(\tau_E)\frac{\partial C(\tau_c,\tau_E)}{\partial \tau_E} + C(\tau_c,\tau_E)\frac{\partial B(\tau_E))}{\partial \tau_E}\right] \quad (71)\\ &= AD\left[\frac{-B(\tau_E)C(\tau_c,\tau_E)\sigma_E}{\theta(1-\tau_E)[1-\eta\sigma_E(1-\tau_E)]} + \frac{-B(\tau_E)C(\tau_c,\tau_E)\sigma_E}{(1-\tau_E)}\right]\\ &= \frac{-p_E\sigma_E}{1-\tau_E}\left[\frac{1-\theta(1-\eta\sigma_E(1-\tau_e))}{\theta(1-\eta\sigma_E(1-\tau_e))}\right]\end{aligned}$$

Note that the signal of this expression depends only on $1-\theta-\theta\eta\sigma_E(1-\tau_e)$. But as the Gamma parameter is positive, then $\theta > 1$. Then:

$$\frac{\partial p_E}{\partial \tau_E} > 0 \tag{72}$$

3. Let

$$F(\tau_c,\tau_E) = p_E^{\frac{\eta}{\eta-1}}; \quad G(\tau_c,\tau_E) = [(1-\tau_c)^\theta w_c^\theta + (1-\tau_E)^\theta w_E^\theta]^{\frac{1}{\theta(1-\eta)}} \tag{73}$$

Then

$$Y = \bar{\eta}\gamma F(\tau_c,\tau_E)G(\tau_c,\tau_E) \tag{74}$$

$$\begin{aligned}\frac{\partial Y}{\partial \tau_E} &= \bar{\eta}\gamma\left[\frac{\partial F(\tau_c,\tau_E)}{\partial \tau_E}G(\tau_c,\tau_E) + \frac{\partial G(\tau_c,\tau_E)}{\partial \tau_E}F(\tau_c,\tau_E)\right] \quad (75)\\ &= \bar{\eta}\gamma F(\tau_c,\tau_E)G(\tau_c,\tau_E)\left[\frac{-\eta}{(1-\eta)p_E}\frac{\partial p_E}{\partial \tau_E} - \frac{s_E}{(1-\eta)(1-\tau_E)}\right]\\ \Rightarrow \frac{\partial Y}{\partial \tau_E} &< 0 \quad (76)\end{aligned}$$

4. Using again (73):

$$\begin{aligned}
\frac{\partial Y}{\partial \tau_c} &= \bar{\eta}\gamma \left[\frac{\partial F(\tau_c, \tau_E)}{\partial \tau_c} G(\tau_c, \tau_E) + \frac{\partial G(\tau_c, \tau_E)}{\partial \tau_c} F(\tau_c, \tau_E) \right] \\
&= \frac{\bar{\eta}\gamma F(\tau_c, \tau_E) G(\tau_c, \tau_E)}{(1-\eta)(1-\tau_c)} \left[\eta\sigma_E - s_c \right]]
\end{aligned} \tag{77}$$

The Maket clearing condition $c_E = Q_E$ in this case implies that $s_E = \sigma_E \eta (1 - \tau_E)$. As $s_c = 1 - s_E$, we have that:

$$\frac{\partial Y}{\partial \tau_c} = \frac{Y\eta\sigma_E}{(1-\eta)(1-\tau_c)} \left(2 - \tau_E - \frac{1}{\eta\sigma_E} \right)$$

So:

$$\frac{\partial Y}{\partial \tau_c} < 0 \Leftrightarrow \left(2 - \tau_E - \frac{1}{\eta\sigma_E} \right) < 0 \Leftrightarrow 2 - \frac{1}{\eta\sigma_E} < \tau_E$$

C Calibration Proofs

Proposition 7 $\forall i \in \{E_1, E_2, E_3, E_4\}$,*we can calibrate* α_i *such that:*

$$\hat{\alpha}_i = \frac{s_i^{US}}{\sum_j^{US} s_j}; j \in \{E_1, E_2, E_3, E_4\} \tag{34}$$

Remember that, using the $\phi = 1$ and $\sigma_i = \sigma_E, \forall i \in \{E_i\}_{i=1}^4$, we reach (63)

$$s_i = \eta \alpha_i \sigma_e (1 - \tau_i)$$

Let's assume that for the baseline that all the distortions in education are equal, equivalently: $\tau_i = \tau_e$. Then:

$$\sum s_i = \sum \alpha_i \eta \sigma_e (1 - \tau_e) = \eta \sigma_e (1 - \tau_e) \sum \alpha_i = \eta \sigma_e (1 - \tau_e) \tag{78}$$

From (63) we have that:

$$\hat{\alpha}_i = \frac{s_i}{\sum_j s_j}; j \in \{E_1, E_2, E_3, E_4\} \tag{79}$$

D Data

D.1 Data definitions

D.1.1 Engaged Works

Per person engaged is defined in the Penn World Table (PWT) to include all persons aged 15 years and over, who during the reference week performed work, even just for one hour a week, or were not at work but had a job or business from which they were temporarily absent.

D.1.2 Teaching Staff

Teaching staff include professional personnel directly involved in teaching students, including classroom teachers, special education teachers and other teachers who work with students as a whole class, in small groups, or in one-to-one teaching. Teaching staff also include department chairs of whose duties include some teaching, but it does not include non-professional personnel who support teachers in providing instruction to students, such as teachers' aides and other paraprofessional personnel. Academic staff include personnel whose primary assignment is instruction, research or public service, holding an academic rank with such titles as professor, associate professor, assistant professor, instructor, lecturer, or the equivalent of any of these academic ranks. The category includes personnel with other titles (e.g. dean, director, associate dean, assistant dean, chair or head of department), if their principal activity is instruction or research.

D.2 dataset

Table 8: Database

Country	*Abreviation*	*GDP*	*Pre − premary*	*Premary*	*Seconday*	*Tertiray*	*Consumption*
United States	USA	1,00	0,38%	1,27%	1,24%	1,02%	97,11%
Aruba	ABW	0,77	0,30%	1,23%	1,12%	0,47%	97,35%
Austria	AUT	0,78	0,46%	0,71%	1,78%	1,11%	97,05%
Bhutan	BTN	0,13	0,02%	1,26%	0,83%	0,24%	97,89%
Brazil	BRA	0,24	0,38%	0,76%	1,40%	0,34%	97,46%
Bulgaria	BGR	0,28	0,49%	0,40%	1,18%	0,56%	97,93%
Burundi	BDI	0,01	0,04%	0,85%	0,26%	0,04%	98,85%
Cabo Verde	CPV	0,13	0,82%	18,48%	5,70%	0,90%	75,00%
Chile	CHL	0,43	0,76%	0,95%	1,00%	1,00%	97,29%
China	CHN	0,15	0,14%	0,77%	0,82%	0,20%	98,27%
China, Macao SAR	MAC	1,31	0,18%	0,49%	0,73%	0,63%	98,60%
Croatia	HRV	0,48	0,47%	0,72%	2,92%	0,96%	95,89%
Cyprus	CYP	0,83	0,39%	1,24%	2,03%	0,79%	96,34%
Czech Republic	CZE	0,53	0,03%	0,08%	0,13%	0,05%	99,76%
Finland	FIN	0,76	0,56%	0,97%	1,70%	0,61%	96,77%
Hungary	HUN	0,51	0,75%	0,91%	2,21%	0,61%	96,13%
Indonesia	IDN	0,15	0,27%	1,47%	1,51%	0,25%	96,75%
Japan	JPN	0,63	0,17%	0,62%	0,95%	0,82%	98,26%
Kyrgyzstan	KGZ	0,06	0,13%	0,72%	2,16%	0,78%	96,99%
Lao Peoples DR	LAO	0,07	0,16%	1,06%	0,71%	0,17%	98,07%
Latvia	LVA	0,44	0,80%	1,20%	2,06%	0,87%	95,94%
Lebanon	LBN	0,47	0,67%	2,25%	2,96%	1,74%	94,12%
Lithuania	LTU	0,53	1,09%	0,87%	3,53%	1,29%	94,51%
Luxembourg	LUX	1,07	0,38%	1,03%	1,36%	0,27%	97,23%
Malaysia	MYS	0,42	0,37%	2,12%	1,58%	0,62%	95,93%
Mauritius	MUS	0,33	0,48%	1,04%	1,58%	0,21%	96,90%
Mexico	MEX	0,33	0,38%	1,10%	1,35%	0,64%	97,17%
Mongolia	MNG	0,16	0,39%	0,80%	1,68%	0,77%	97,13%
Niger	NER	0,02	0,03%	0,77%	0,17%	0,03%	99,03%
Panama	PAN	0,31	0,34%	1,19%	1,17%	0,96%	97,30%
Poland	POL	0,48	0,38%	1,56%	1,77%	0,67%	96,29%
Portugal	PRT	0,56	0,37%	1,41%	2,04%	0,75%	96,18%
Republic of Moldova	MDA	0,11	0,94%	0,74%	2,35%	0,62%	95,97%
Romania	ROU	0,37	0,45%	0,62%	1,73%	0,37%	97,20%
Saint Lucia	LCA	0,22	0,45%	1,41%	1,37%	0,29%	96,77%
Serbia	SRB	0,40	0,57%	0,88%	3,13%	0,75%	95,42%
Slovakia	SVK	0,58	0,54%	0,65%	2,13%	0,62%	96,68%
Tajikistan	TJK	0,08	0,21%	1,20%	2,67%	0,58%	95,92%
TFYR of Macedonia	MKD	0,32	0,34%	1,02%	2,42%	0,51%	96,22%
Thailand	THA	0,21	0,01%	0,02%	0,04%	0,01%	99,93%
Uganda	UGA	0,04	0,17%	1,48%	0,57%	0,05%	97,78%
Uruguay	URY	0,32	0,32%	1,56%	1,60%	1,05%	96,52%
Uzbekistan	UZB	0,15	0,52%	0,96%	2,95%	0,20%	95,57%