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Entrepreneurship, Financial Frictions and the Welfare Gains of Business Cycles

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Entrepreneurship, Financial Frictions and the Welfare Gains of Business Cycles ^{*}

Giovanni Tondin Merlin [†]

Abstract

I propose a simple method to solve a heterogeneous agent model with entrepreneurship, financial frictions and aggregate uncertainty. Aggregate shocks, in this context, induce potential entrepreneurs to save in order to slack their financial constraints, leading to a significant increase in aggregate savings, pushing up wages and lowering interest rates. This channel can be strong enough to more than offset the effect of fluctuations on consumption, leading to welfare gains of business cycles.

Keywords: Heterogeneous Agents, Entrepreneurship, Financial Frictions, Business Cycles, Aggregate Uncertainty.

JEL Classification:D58,E44,E32

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1 Introduction

”...it is unrealistic to hope for gains larger than a tenth of a percent from better counter-cyclical policies.” - [Lucas \(2003\)](#)

As any article containing ”welfare” and ”business cycles” in the title, I should start citing Robert Lucas.¹ After publishing the well known formula to estimate the welfare costs of business cycle in a representative-agent economy with endowment, $1/2\gamma\sigma_C^2$, [Lucas \(1987\)](#) gave the shout that generated an avalanche of papers about this intriguing result. Even after 16 years of developments in the literature, [Lucas \(2003\)](#) maintained his conclusion. In this work, I not only agree with him, that large gains are not likely, but also show that there are forces acting in the opposite direction, enhancing welfare.

The model considered in [Lucas \(1987\)](#) was very stylized. His calculations are based on exogenous fluctuations on consumption within a representative agent framework. As expected, the subsequent literature added new important elements to evaluate the welfare costs of business cycles. In this paper, I develop a heterogeneous agent model with entrepreneurship, financial frictions and aggregate uncertainty to address this problem. The inclusion of heterogeneity become almost standard in this literature, since it is more realistic and allows us to understand the distribution of the potential welfare costs. Adding elements from the literature on entrepreneurship and financial frictions in this framework, I find that business cycles may increase welfare, measured as the equally distributed equivalent consumption, by around 0.05% for the US economy.

The key element behind the results is the inclusion of financial frictions (credit spreads and collateral requirements). Given that some entrepreneurs (and potential ones) may be constrained, aggregate shocks induce them to increase their savings, in a magnitude much higher than the usual precautionary savings channel. This excess of savings reduces the interest rate and is mainly absorbed by a frictionless corporate sector, what leads to an increase in average wages, benefiting poor workers (the majority of the population). Entrepreneurs and rich agents, on the other hand, would be better in a stabilized economy.

With the model in hands, I am also able to tell how entrepreneurship and capital misallocation reacts to aggregate shocks. In particular, I show that the result found by [Moll \(2014\)](#), about the negative relationship between the persistence of idiosyncratic productivity shocks and misallocation, is also valid for aggregate productivity shocks. Interestingly, it may be the case that misallocation is smaller under an economy with business cycles, thanks to the increase in savings and to changes in the extensive margin

¹As a curiosity, a subset of papers mentioned in this work, such as [Alvarez and Jermann \(2004\)](#); [Atkeson and Phelan \(1994\)](#); [Chatterjee and Corbae \(2003\)](#); [Cho et al. \(2015\)](#); [Ellison et al. \(2015\)](#); [Hairault et al. \(2010\)](#); [Imrohoroglu \(1989\)](#); [Krebs \(2006\)](#); [Krusell et al. \(2009\)](#); [Krusell and Smith \(1999\)](#); [Mukoyama and Sahin \(2006\)](#); [Wolfers \(2003\)](#) cite Robert Lucas in the first paragraph. Some authors are more anxious and do it in the abstract: [Chauvin et al. \(2011\)](#); [Jung and Kuester \(2011\)](#); [Otrok \(2001\)](#); [Schulhofer-Wohl \(2008\)](#); [Storesletten et al. \(2001\)](#).

of entrepreneurship.

I also contribute to the literature on heterogeneous agent models and aggregate uncertainty, by showing that the solution method proposed by [Krusell and Smith \(1998\)](#), can be easily adapted to solve a model with entrepreneurship accurately, and without rely on perturbation methods.² If extended and improved, it can allow further developments in the literature of occupational choice and financial frictions, since, so far, most of the papers analyse the effects of MIT shocks and transitions between steady-states.

The remaining of the paper is organized as follows. Section 2 provides a brief review of literature about the welfare costs of business cycles and also about entrepreneurship and financial frictions in heterogeneous agent models. In Section 3, I discuss the model and the solution method. The results of the model calibrated for USA are presented in Section 4. Finally, some concluding remarks are made in Section 5.

2 Related Literature

Maybe surprisingly, most of the works that followed Lucas' seminal paper have also found small effects of business cycles. Furthermore, virtually all authors would agree that the cost of business cycles are small relatively to microeconomic distortions.

One big strand of the literature tried to focus on the effects of business cycles through the unemployment channel. Since unemployment is a traditional source of consumption fluctuation, this channel is a natural choice to be considered. However, most of these works do not find large effects. [Imrohoroglu \(1989\)](#) was the first to use a heterogeneous agent model to investigate the welfare effects of business cycles, and found that liquidity constraints increase the cost of business cycle, but if an intermediation technology is included, unemployed agents can partially smooth consumption and the welfare costs turn out to be low, around 0.05%. [Atkeson and Phelan \(1994\)](#) consider a countercyclical policy that eliminate the correlation in individual unemployment risk, and find that the effects are very small, 0.02%, in fact smaller than in an economy with complete markets. [Krusell and Smith \(1999\)](#) consider heterogeneity on employment and discount factors, and find costs between 0.07-0.14%, on average, depending on the number of unemployment states. Ten years later, [Krusell et al. \(2009\)](#) revised their results, and concluded that the welfare gains from stabilization can achieve 1%, and can be much higher for long-term unemployed agents. [Mukoyama and Şahin \(2006\)](#) find heterogeneous costs between unskilled and skilled workers.

²Some other authors have been working to include aggregate uncertainty in heterogeneous agent models with entrepreneurship. [Pugh \(2016\)](#), for instance, develops a model with a few different characteristics and a different type of shock, and shows that additional moments should be considered to improve the forecasting of the interest rate. Efforts to solve this type of models in continuous time are also starting to show up. See, for instance, the project being developed by Benjamin Moll and co-authors: <http://www.princeton.edu/~moll/HACTproject.htm>.

They obtain that welfare gains from the elimination of business cycles during a recession can achieve up to 0.6% for restricted unemployed agents, and 0.12%, on average, for unskilled workers. [Hairault et al. \(2010\)](#) consider matching frictions, that generates a non-linear effect on unemployment, and find a welfare cost of 0.4% in their baseline calibration. [Jung and Kuester \(2011\)](#) also consider a matching model, and show that unemployment fluctuations have minor effects on the welfare costs of business cycles, but aggregate shocks increase the average unemployment, and generate a welfare cost around 0.15%. [Acedański \(2016\)](#) analyze the costs of youth unemployment in Poland. He finds an average gain around 0.05% from a stabilized unemployment, with stronger effects for younger workers, since they face higher unemployment risk and have low ability to self-insure against it.

Some other authors tried to play with assumptions on preferences, but also did not find substantial effects. [Otrok \(2001\)](#) shows that, by considering time-non-separabilities in preferences, the welfare cost of business cycles is only 0.004%. [Alvarez and Jermann \(2004\)](#), without specify preferences, and using assets prices, estimate a cost between 0.08 and 0.49 percent. [Schulhofer-Wohl \(2008\)](#) considers a complete markets economy, but with heterogeneous risk preferences. The magnitude of the costs for the average household is very similar to a representative agent model, less than 0.1%. Moreover, since a less risk-averse agent can sell insurance for a more risk-averse, they can actually be better off with business cycles.

[Schulhofer-Wohl \(2008\)](#) result about the possible gains from business cycles is not unique. Some recent works also indicate this possibility. [Lester et al. \(2014\)](#) show that if factor supply is sufficiently elastic, increases on the variance of shocks may be welfare enhancing. [de Carvalho Guillén et al. \(2014\)](#) empirically find that, decomposing aggregate shocks into transitory and permanent effects, the welfare effect of business cycles fluctuations are small and positive. [Cho et al. \(2015\)](#) show that representative-agent economies with multiplicative shocks may enjoy higher welfare than their steady-state counterparts, if the risk aversion parameter is low enough. In these cases, the mean effect can be higher than the fluctuation effect. They estimate the welfare implications of business cycles for different models, and find that usually the gains/losses are around 0.15%, depending on the parametrization.

While most papers find the magnitude of the welfare costs/gains of business cycles in line with [Lucas \(1987\)](#) predictions, some authors have found significant effects. In an empirical work, [Wolfers \(2003\)](#), using data from surveys of subjective well-being, estimates that the elimination of business cycle would be equivalent to an average reduction on unemployment around a quarter of a percentage point. His measure is not comparable with the usual measure of changes in equivalent consumption, but the magnitude seems to be relevant. In the theoretical field, papers that have found strong effect of fluctuations usually relies on different assumptions about risk, convexities or any kind of market failure.

De Santis (2007) finds larger effects when includes idiosyncratic consumption risk, around 5% in his baseline calibration. Ellison et al. (2015) also consider consumption risk, and show that this feature, interacted with a fear of a model misspecification, can lead to relevant welfare costs, about 0.8-2%, even if risk aversion parameters are relatively small. Krebs (2006) considers multiple sources of labor income risk and, under his baseline calibration, finds a substantial welfare cost, around 4%. Storesletten et al. (2001) consider an OLG economy in which, besides aggregate shocks, they add countercyclical variation in idiosyncratic risk, what amplifies the effects of shocks, and find a welfare cost around 2.5% in their baseline calibration. Turnovsky and Bianconi (2005), considering the fact that a share of idiosyncratic risk arises from aggregate shocks, eliminate the aggregate shock and reduce the idiosyncratic risk at the same time, based on empirical estimates, implying in a significant welfare cost of business cycle, around 2-3%.

Chauvin et al. (2011) model in such way that the costs of asset bubbles are convex, and obtain a welfare cost of 3% in their benchmark calibration. Due to the convexity, they argue that is not important to eliminate asset bubbles, just reduce their size is enough to alleviate the costs substantially. For Den Haan and Sedlacek (2014) business cycles are very costly, around 2-12% of GDP. They consider a model with job creation costs and agency problem, in which business cycles make some jobs with positive surplus temporarily or permanently impossible. Finally, Chatterjee and Corbae (2003) consider a small likelihood of crisis, and obtain welfare costs around 2%, even considering that a depression comes only about every 83 years.

This work is also related with the increasing literature on entrepreneurship and financial frictions in heterogeneous agent models. This literature usually considers the effect of MIT shocks and evaluate the transition between two steady-states. Bassetto et al. (2015), for instance, using the same ingredients that I consider here, show that credit or TFP shocks have persistent effects, since it erode entrepreneur wealth. Brumm (2015) considers that collateral constraints interacts with idiosyncratic risk, and the effects are stronger. Once a negative shock hits the economy, allocation deteriorates (wealth of constrained agents falls, and their number rises), implying in recessions sharper than booms. Price of capital plays an important role in his model, since it affects the collateral value. Moll (2014) shows that, if idiosyncratic productivity shocks are persistent, self-financing is powerful to reduce misallocation and steady-state losses are small, whereas transitions are costly.

3 Model

The model here presented is quite simple and relies on standard assumptions. It is represented by a unit mass of infinitely lived agents, which differ each other by their actual business “idea”, i.e., their productivity while managing a firm. This approach is similar to studies in which “managerial skill” dictates productivity, à la [Lucas \(1978\)](#), but since agents are infinitely lived, shocks in idiosyncratic productivity are necessary in order to generate capital misallocation. At each period, based on their wealth and idea, agents will choose between being workers or entrepreneurs. On the production side, besides the entrepreneurs, there is a representative firm operating with constant returns to scale, representing the corporate sector. Savings can be used as equity, by entrepreneurs, invested directly into the corporate sector, or are deposited in a financial institution, which lends to the corporate sector and to entrepreneurs (charging a spread, in the latter case). For tractability, I abstract from unemployment and wage risk and do not include a government sector.

3.1 Households

Households inelastically supplies one unit of labor and maximize their intertemporal utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_{it}^{1-\gamma} - 1}{1-\gamma} \right) \right] \quad (1)$$

where β is the intertemporal discount rate, $c_{it} \in C \subset \mathbb{R}_+$ is the consumption of agent i at time t , and γ is the relative risk aversion coefficient.

Households’ budget constraint is given by

$$c_{it} + s_{it+1} \leq \omega_{it} + (1 + r_t)s_{it} \quad (2)$$

where $s_{it} \in A \subset \mathbb{R}_+$ is savings/wealth, which can be used as equity, $e_{it} \in A \subset \mathbb{R}_+$, if they choose to be entrepreneurs; r_t is the real interest rate; and ω_{it} is the labor/business related income. More precisely,

$$\omega_{it} = \max \{ w_t(\Gamma_t, \bar{A}_t), \Pi(z_{it}, s_{it}, \Gamma_t, \bar{A}_t) \} \quad (3)$$

where w_t is the equilibrium wage and $\Pi(z_{it}, s_{it}, \Gamma_t, \bar{A}_t)$ is the optimal profit of an entrepreneur, which is a function of the idea and savings of agent i , besides the aggregate state of the economy, characterized by the distribution of agents over asset holdings and ideas, Γ_t , and the aggregate productivity, \bar{A}_t . The individual idea, $z \in Z$, follows a Markov process with log-normal distribution, $\ln z_{it} \sim N(0, \sigma_z^2/(1-\rho^2))$,

with autocorrelation ρ .

3.2 Banking Sector

Households' unused savings are distributed between banks and the corporate sector firms (described in section 3.4), which remuneration is r_t . Lend to entrepreneurs is costly, thus banks charge a constant credit spread, χ . Hence, the lending interest rate to entrepreneurs is $r_t^L = r_t + \chi$. Loans are intra-temporal and the intermediation cost is assumed to be discarded.

3.3 Entrepreneurs

Each period, entrepreneur's problem is to decide the total physical capital and labor demand. Agents will use their own resources or borrow from banks, in order to acquire physical capital. To obtain a loan, they are subject to a credit constraint, proportional to their equity. The production technology is a traditional Cobb-Douglas production function with decreasing returns to scale. Thus, the entrepreneur i in the period t solves the following problem:

$$\max_{l_{it}, d_{it}, e_{it}} \bar{A}_t z_{it} (k_{it}^\alpha l_{it}^{1-\alpha})^{1-\nu} - w_t l_{it} - r_t e_{it} - r_t^L d_{it} - \delta k_{it} \quad (4)$$

subject to

$$\begin{aligned} k_{it} &= e_{it} + d_{it} \\ e_{it}, d_{it} &\geq 0 \\ d_{it}(1 + r_t^L) &\leq e_{it}\phi \\ e_{it} &\leq s_{it} \end{aligned} \quad (5)$$

where d_{it} is the total debt of the entrepreneur; $e_{it} \in [0, s_{it}]$ is the equity, which opportunity cost is r_t ; δ is the depreciation rate of the physical capital; l_{it} is the quantity of labor hired; ν is the span of control parameter; α is the capital-share; $\phi \geq 0$ imposes a collateral constraint (if $\phi = 0$ there is no credit in the economy and if $\phi = \infty$, entrepreneurs are not financially constrained for any $s_{it} > 0$). \bar{A}_t is the aggregate productivity, and its dynamics is detailed in section 3.5.

3.4 Corporate Sector

In addition to entrepreneurs, there is a corporate sector, characterized by the absence of financial frictions (either credit spreads or collateral constraints) and by a constant returns to scale production function,

$\bar{A}_t A^C (K_t^C)^\alpha (L_t^C)^{1-\alpha}$. Without loss, I can assume a representative firm in this sector, and the first order conditions are:

$$w_t = (1 - \alpha) \bar{A}_t A^C \left(\frac{K_t^C}{L_t^C} \right)^\alpha \quad (6)$$

$$r_t = \alpha \bar{A}_t A^C \left(\frac{K_t^C}{L_t^C} \right)^{\alpha-1} - \delta \quad (7)$$

where A^C is the relative productivity of the corporate sector, and $K^C(L^C)$ is the capital (labor) employed by this sector. K^C can be obtained directly from households' savings (e.g., debt securities, stocks), or via banking sector (loans), without intermediation costs.

3.5 Aggregate Shocks

The exogenous aggregate shocks affect only \bar{A}_t directly. For tractability, I assume that \bar{A}_t can assume only two values, $1 - \Delta^L$ and $1 + \Delta^H$, like in [Krusell and Smith \(1998\)](#), henceforth KS. Furthermore, it is assumed to be a first order Markov-process, with transition probability matrix given by $\pi^A = \begin{bmatrix} P_{LL'} & 1 - P_{LL'} & 1 - P_{HH'} & P_{HH'} \end{bmatrix}$, where $P_{ii'}$, $\forall i \in (L, H)$, is the probability of staying in the same aggregate state in the next period.

3.6 Equilibrium

Definition: A recursive competitive equilibrium consists of a value function $V : A \times Z \times \Gamma \rightarrow \mathbb{R}$; a decision sequence of the agents $S : A \times Z \times \Gamma \rightarrow \mathbb{R}_+$, $C : A \times Z \times \Gamma \rightarrow \mathbb{R}_+$; choices of entrepreneurs e, d, l ; prices, r and w ; such that:

1. Subject to (2), households maximize their individual problem, given by:

$$V(s, z; \Gamma, \bar{A}) = \max_{c, s'} \left\{ \frac{c^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E} [V(s', z'; \Gamma', \bar{A}') | z, \Gamma, \bar{A}] \right\} \quad (8)$$

2. Entrepreneurs maximize (4), subject to (5), and the first order conditions of the corporate sector, (6) and (7), are satisfied.
3. The goods, labor and asset markets are in equilibrium:

$$\int c(s, z) d\Gamma(s, z) + K' + \chi \int d(s, z) d\Gamma(s, z) = \int y(s, z) d\Gamma(s, z) + Y^C + (1 - \delta)K \quad (9)$$

$$\int \mathbb{1}_W d\Gamma(s, z) = \int l(s, z) d\Gamma(s, z) + L^C \quad (10)$$

$$\int (s - e(s, z)) d\Gamma(s, z) = \int d(s, z) d\Gamma(s, z) + K^C \quad (11)$$

4. The distribution of agents over asset holdings and ideas evolve as $\Gamma' = H(\Gamma, \bar{A})$.

3.7 Approximate Solution

In order to solve their problem, agents do not need to know the evolution of all elements of Γ , but only the price dynamics (w and r). It turns out that, by assuming a constant returns to scale production function for the corporate sector, one price can be pinned down given the other price value and the aggregate productivity, $w = A^C \bar{A}(1 - \alpha)[(r + \delta)/(A^C \bar{A}\alpha)]^{(\alpha/(\alpha-1))}$, e.g.. Therefore, the problem is reduced to know the future values of the interest rate, given an exogenous process for the productivity shocks. However, the interest rate is an unknown function of the infinite-dimensional aggregate state, $r' = \mathcal{F}(\Gamma', \bar{A}')$. To deal with this problem numerically, I show that r' can be satisfactorily well predicted by using only the current value of the interest rate and the current and future aggregate productivity, $r' \approx F(r, \bar{A}, \bar{A}')$. Using this approximation I solve the problem stated in (8) by replacing the value function with $V(s, z; r, \bar{A})$ and, given that the exogenous stochastic process is known, the expected value function in the next period is approximated by $\mathbb{E}[V(s', z'; F(r, \bar{A}, \bar{A}'), \bar{A}') | z, r, \bar{A}]$. The described procedure drastically reduces the state size and computational burden³, and the idea is very similar to the solution of the traditional KS model, in which authors find that we only need to forecast the aggregate capital in order to predict prices accurately, since the labor supply is exogenously given.

Another issue is that, differently from the KS model, in which aggregate capital time series can be easily obtained by integrating over the distribution of assets, here I have to solve for the equilibrium prices (r , and consequently w) every period, which is not straightforward like just integrating the distribution. Interpolation schemes and a minimization routine to find prices that clear markets are required, then increasing the computational burden. Appendix A.1 describe in details the algorithm used to solve the model.

³In the numerical part of this paper, I set the number of grid points for assets equal to 300, for r equal to 20 and for idea equal to 9. Since I assume only 2 aggregate states, the total number of grid points is 108,000(300x20x9x2), which is still large, but solvable.

4 Results

4.1 Calibration

I calibrate the model with US data and it is assumed that a period of time is equivalent to a year. The calibrated values are shown in Table 1 and described below.

Table 1: Calibration

Parameter	Value	Target
β	0.96	Real interest rate - 2%
A^C	0.923	Share of capital in the corporate sector - 60%
ϕ	6	Credit to firms/GDP - 70%
σ^z	0.083	Gini - Entrepreneurs - 0.45
ν	0.2	Share of Entrepreneurs - 7.5%
$\Delta^H = \Delta^L$	0.019	GDP per capita growth S.D. - 2%
Source		
γ	2	Standard
α	0.36	Standard
χ	0.04	Buera et al. (2013)
ρ	0.95	Monacelli et al. (2011); Ranasinghe et al. (2016)
$P_{LL} = P_{HH}$	2/3	Average Duration of a Cycle - 6 years - NBER
δ	0.1	Standard

Sources: Share of capital in corporate sector (see [Quadrini \(2000\)](#)); Entrepreneurs Gini (see [Quadrini \(1999\)](#)); Share of Entrepreneurs (see [Cagetti and De Nardi \(2009\)](#)); GDP per capita growth data is from World Bank, covering 1961-2016; Credit to non-financial firms/GDP data is from Bank for International Settlements (series Q:US:N:A:M:770:A); Real interest rate data is calculated by the difference between the 5-year treasury rate and the CPI inflation, obtained on FRED, covering 1960 to 2017.

β , A^C , ϕ , σ^z , ν , Δ^H and Δ^L are endogenously calibrated to match some chosen moments.⁴⁵ The remaining parameters values are obtained in other studies or in data. The autocorrelation of the log-normal idea process is set at 0.95, as in [Monacelli et al. \(2011\)](#); [Ranasinghe et al. \(2016\)](#). I set $\chi = 0.04$, in line with the information of net interest margin from Fed⁶ and [Buera et al. \(2013\)](#). Using expansion and recession data from NBER, I find that the average duration of a cycle is around 6 years, what implies in a probability of staying in the same state of 2/3, if we consider symmetric cycles. Finally, following the literature, I set $\gamma = 2$, $\alpha = 0.36$, and $\delta = 0.1$.

Besides aggregate variables, a good track of the wealth distribution is important when we look at the distributional effects of business cycles. In addition to the targeted parameters, the model is also

⁴In the calibration for ϕ , in order to match the Credit to Firms/GDP ratio, I also consider, using data from US Financial Accounts, that the ratio $\frac{Loans}{Loans+Debt\ Securities} = 0.24$ for the non-financial corporate sector, during 1990 to 2016.

⁵In the baseline calibration, the obtained Credit to Firms/GDP ratio is 0.68, and the Gini Index of Entrepreneurs' Income is 0.45. Values of the remaining targeted variables are shown in Table 7.

⁶See Figure 24 in Federal Reserve Bulletin: "Profits and Balance Sheet Developments at U.S. Commercial Banks in 2008", available at <https://www.federalreserve.gov/pubs/bulletin/2009/pdf/bankprofits09.pdf>

able to follow some wealth inequality indicators. This is a well-known characteristic of models with entrepreneurship, since the willingness to save by entrepreneurs generate high wealth inequality without impose preference shocks. Table 2 presents common moments discussed in the literature. The reader can note that, although the model does not generate sufficiently high income inequality (due to the absence of unemployment and wage dispersion), wealth inequality measures generated by the model are extremely high, and relatively close to the values observed in data.

Table 2: Wealth and Income Inequality

Variable	Baseline	Data
Financial Wealth Gini	0.94	~0.90
Top 10% Financial Wealth Share	0.95	~0.80
Top 1% Financial Wealth Share	0.39	0.40-0.45
Fraction of Constrained Agents ($s = 0$)	0.34	0.25-0.30
Gini Income	0.15	~0.50
Top 10% Income Share	0.24	~0.40
Top 1% Income Share	0.08	~0.15

Source: Wolff (2017), Table 2. Approximate values between 1962 and 2016.

4.2 Accuracy of the Law of Motion

The estimated law of motion for interest rate, $\hat{F}(r, \bar{A}, \bar{A}')$, obtained in a 10,000 period simulation under the baseline calibration was:

$$r' = \begin{bmatrix} \alpha_{HH'} \\ \alpha_{HL'} \\ \alpha_{LH'} \\ \alpha_{LL'} \end{bmatrix}' \begin{bmatrix} \mathbb{1}_{HH'} \\ \mathbb{1}_{HL'} \\ \mathbb{1}_{LH'} \\ \mathbb{1}_{LL'} \end{bmatrix} + \begin{bmatrix} \beta_{HH'} \\ \beta_{HL'} \\ \beta_{LH'} \\ \beta_{LL'} \end{bmatrix}' \begin{bmatrix} \mathbb{1}_{HH'} \\ \mathbb{1}_{HL'} \\ \mathbb{1}_{LH'} \\ \mathbb{1}_{LL'} \end{bmatrix} r = \begin{bmatrix} 0.0025 \\ -0.0007 \\ 0.0067 \\ 0.0026 \end{bmatrix}' \begin{bmatrix} \mathbb{1}_{HH'} \\ \mathbb{1}_{HL'} \\ \mathbb{1}_{LH'} \\ \mathbb{1}_{LL'} \end{bmatrix} + \begin{bmatrix} 0.8766 \\ 0.8130 \\ 0.9051 \\ 0.8724 \end{bmatrix}' \begin{bmatrix} \mathbb{1}_{HH'} \\ \mathbb{1}_{HL'} \\ \mathbb{1}_{LH'} \\ \mathbb{1}_{LL'} \end{bmatrix} r \quad (12)$$

$R^2 = 0.9994 \quad \hat{\sigma} = 0.000037 \quad \text{Max residual} = 0.00027$

At first sight, the LOM seems to be accurate, as the R^2 is very high. However, as pointed by Den Haan (2010), this is not necessarily an indicative that the approximate solution is good. Furthermore, the R^2 of a regression is a measure of the fit of the model that uses the actual values as predictors every period. Since agents have to forecast not only the next period prices, but prices for all the following periods as well, it is important to look at the accuracy of the LOM replacing the actual values by the predicted ones, using only the first observation of the interest rate as given, $\hat{F}(\hat{r}, \bar{A}, \bar{A}'|r_0)$. Figure 1 shows the actual and predicted values in the last 100 periods of the simulation, considering both the cases where we use the actual and the predicted values in the RHS of the estimated equation. It is possible to note in the upper panel that the fit of the model is, indeed, very good. The bottom panel corroborates this, and shows that,

even not updating the interest rate with the actual value every period, there are no systematic or substantial errors after 10,000 periods. The R^2 using only the predicted values instead of actual values is 0.9987.

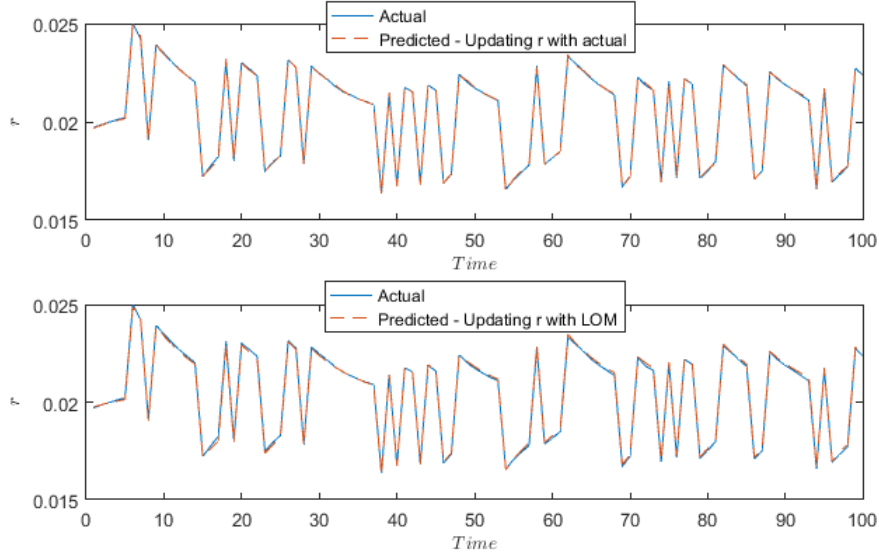


Figure 1: Actual and predicted interest rate.

Notes: The plots show the last 100 periods of the actual and predicted interest rates. In the upper graph, the actual and fitted values ($\hat{r}' = \hat{F}(r, \bar{A}, \bar{A}')$) are plotted, while in the bottom graph the predicted values are obtained using only the law of motion ($\hat{r}' = \hat{F}(\hat{r}, \bar{A}, \bar{A}'|r_0)$).

Is this apparently good fit in both graphs of Figure 1 reflected in accurate approximated policy functions? To answer that, I calculate the difference between the implied individual consumption, for each pair (s, z) , considering the predicted interest rates with the equivalent measure using the actual interest rate, i.e., $\varepsilon = |c(\hat{r})/c(r) - 1|$. The results in Table 3 shows that, averaging across the triple (s, z, t) , the absolute error is small, around 0.11% when we update r with the actual value in the LOM, and 0.17% when we do not. The maximum error across the triple (s, z, t) is around 3%, and averaging across time this value falls to 0.4% (0.6% if we update r with the predicted value).

Table 3: Difference in individual consumption choices

	Updating r with Actual	Updating r with Predicted
Average error	0.11%	0.17%
Maximum error during simulation	2.75%	3.12%
Average maximum error	0.40%	0.61%

It is also important to mention that a number of periods of 10,000 seems to be more than enough to generate stable results. Different simulations of 10,000 periods lead to virtually the same results.

4.3 The welfare gains of business cycles

When comparing an economy with aggregate shocks with an economy without it, two main welfare effects are expected. The first one is the negative effect of consumption volatility, considered by Lucas (1987), the so called fluctuation effect. In order to reduce this volatility, households increase their precautionary savings, leading, in general equilibrium, to changes in the average level of production, which is the mean effect. However, in a heterogeneous agent setting, other effects are also acting. The first is related to the fact that different agents react differently, what may lead to winners and losers, generating a distribution effect. With the inclusion of entrepreneurship and financial frictions, a second effect emerge, related to changes in the production efficiency. Financial frictions also generate incentives to save in order to slack credit constraints, possibly intensifying the mean effect. The overall welfare impact will, then, depend on the intensity of each effect.

In order to better understand the mechanism behind the results, I start presenting the effects of aggregate shocks in a representative agent model, which can be easily incorporated in the model by setting the productivity of entrepreneurs to zero, $z_{it} = 0, \forall i, t$. After, I discuss the results of an economy with entrepreneurship, but without financial frictions ($\phi = \infty$ and $\chi = 0$). These two approaches lead to very similar welfare costs of business cycles. Finally, the results of the full model are discussed.

Starting from the usual representative-agent economy, Table 4 summarizes the results. In the last column, the moments from the same economy without aggregate shocks are shown and normalized to 100 (when relevant), and in the previous columns, the average of the same moments under the whole simulation with cycles of different durations are considered. Under the baseline cycle duration, in bold, the Equally Distributed Equivalent Consumption (EDEC), the measure of welfare adopted here, is virtually the same (-0.003%) as in an economy without shocks, on average. This result may contradict Lucas (1987) approximate formula to calculate the costs of business cycles, $1/2\gamma\sigma_C^2$, which would imply in a welfare cost of 0.013%⁷, but remember that he does not consider the mean effect. In order to smooth their consumption, agents save more, what implies in more capital to the corporate sector (the only production sector in this setting), leading to a slightly lower interest rate (-0.01p.p) and higher wages (+0.02%). This increase in average income partially offset the welfare costs of the consumption fluctuation.

Another thing that we can observe is that as the expected duration of the cycles increases, the higher is the GDP volatility, since the economy can stay a very long period in good and bad times and it allows capital to accumulate or shrink over time. Therefore, agents will save more, on average, when cycles are longer, in order to insure themselves against long periods of bad realizations. Furthermore, the convexity

⁷Considering the case with a expected duration of each state of 3 years, $\sigma_C = 1.15\%$, which would imply in a welfare cost of 0.013%.

Table 4: Average effects of aggregate shocks in a representative-agent economy.

	Expected Duration of each State					
	2 years	3 years	4 years	5 years	8 years	w/o shocks
r	4.17%	4.16%	4.17%	4.17%	4.16%	4.17%
w	100.01	100.02	100.01	100.02	100.03	100.00
K	100.03	100.06	100.05	100.06	100.08	100.00
GDP	100.01	100.02	100.01	100.02	100.03	100.00
Std. Dev. GDP	1.97%	2.10%	2.30%	2.37%	2.58%	0.00%
EDEC	99.997	99.997	99.991	99.989	99.985	100.00

channel, explored by [Cho et al. \(2015\)](#)⁸, is also acting for cycles with expected duration of each state above 2 years, since the correlation between \bar{A} and K is positive. Nevertheless, the fluctuation effect is stronger than the mean effect, implying in a marginal welfare loss which is increasing in the duration of the cycle.

The results obtained with the representative-agent model do not change significantly when we add heterogeneity and entrepreneurship in the model. Table 5 shows the results of the heterogeneous-agent economy with entrepreneurship, but without financial frictions ($\phi = \infty$ and $\chi = 0$). As in any heterogeneous-agent model, uncertainty about the idiosyncratic process leads to an interest rate below $1/\beta - 1$. Agents save a lot when they have good ideas (and become entrepreneurs), in order to smooth consumption when their ideas become poor and they have to rely on wages. However, it does not change substantially the effects of aggregate shocks in the economy. Since we do not have credit frictions, collateral is irrelevant, and the only determinant for occupational choice is the individual idea z_{it} ⁹, then we do not have any problem with capital misallocation, i.e., all entrepreneurs and the corporate sector operate with the same marginal return on capital, $(r_t + \delta)$.

When we add aggregate shocks in this environment, additional precautionary savings motive arises, increasing aggregate capital. Differently from the representative agent case, aggregates respond less to the duration of cycle. The main reason is that the corporate sector, responsible for the convexities in production with respect to the shocks, is shrunk ($K^C/K \approx 16.5\%$) by entrepreneurship and its decreasing returns to scale. Given this, the GDP volatility is lower than in the representative agent model. However, even with lower volatility the negative effects on welfare are a little bit stronger than in the representative agent case. In the baseline cycle duration, aggregate shocks are deleterious, and their welfare costs are 0.012%. What explains this marginal difference is the distribution effect, since most of the increase in

⁸[Cho et al. \(2015\)](#) show that we can obtain welfare gains of business cycles for $\gamma = 1$, considering an appropriate second-order perturbation method that does not ignore the convexity of the production function with respect to the TFP shock (in his case, an AR(1) with $\rho = 0.95$).

⁹Furthermore, since the capital-share, α , is the same for the corporate sector and entrepreneurs, the aggregate distribution also does not matter for profits, only for wage determination. Then, $\omega_{it} = \max \{w_t(\Gamma_t, \bar{A}_t), \Pi(z_{it}, s_{it}, \Gamma_t, \bar{A}_t)\}$ becomes $\omega_{it} = \max \{w_t(\Gamma_t, \bar{A}_t), \Pi(z_{it}, \bar{A}_t)\}$.

Table 5: Average effects of aggregate shocks in a heterogeneous agent economy with entrepreneurship, but without financial frictions.

	Expected Duration of each State					
	2 years	3 years	4 years	5 years	8 years	w/o shocks
r	1.73%	1.73%	1.73%	1.73%	1.73%	1.74%
w	100.02	100.02	100.03	100.03	100.03	100.00
K	100.07	100.07	100.08	100.08	100.08	100.00
K^C/K	16.52%	16.52%	16.52%	16.52%	16.52%	16.50%
GDP	100.02	100.02	100.02	100.02	100.03	100.00
Std. Dev. GDP	1.91%	1.98%	2.04%	2.10%	2.21%	0.00%
Entrepreneurs (%)	11.11%	11.11%	11.11%	11.11%	11.11%	11.11%
EDEC	99.999	99.988	99.989	99.987	99.982	100.00

savings comes from talented agents.

Finally, I consider the full model. The inclusion of financial frictions has strong effects on the economy. Table 6 compares the economies with and without frictions in the case without aggregate uncertainty. Credit spreads and collateral requirements add a powerful source of inefficiency in the economy, generating capital misallocation. Two entrepreneurs with the same idea may have different returns on investment if their wealth are not the same. While unrestricted (wealthy) entrepreneurs operate with the MgRK equal to $r_t + \delta$, firms belonging to agents with very low savings have a MgRK much higher. Financial frictions generate a standard deviation in the MgRK of 4.84%, and reduce the entrepreneurship from 11.1%, in the case without frictions, to 7.5%. The (frictionless) corporate sector absorbs a share of the fall in capital and labor demand, avoiding stronger effects. Aggregate capital falls by 9% and GDP by 5.6%. Although wages fall 1.5%, the welfare effect is much stronger, -4.4%, since it is heterogeneous and can be up to 80% lower for some entrepreneurs.

Table 6: Model with and without financial frictions in the absence of aggregate shocks.

	without frictions	with frictions
r	1.74%	2.06%
w	100.00	98.46
K	100.00	91.02
K^C/K	16.50%	59.83%
GDP	100.00	94.38
Entrepreneurs (%)	11.11%	7.50%
Std. Dev. MgRK	-	4.84%
EDEC	100.00	95.62

When we add aggregate shocks, the presence of financial frictions leads to results qualitatively different, and quantitatively more relevant, when compared with the previous cases. Results are shown in Table 7. In the absence of frictions, workers and entrepreneurs save during good states (idiosyncratic

or aggregate), and spend in bad states. This is the traditional precautionary savings motive, and usually is stronger when duration of the cycles is larger, i.e., there is more volatility. With financial frictions, however, aggregate capital is higher in shorter cycles.

If the inclusion of financial frictions changed qualitatively the results, the explanation is likely to rely on the behaviour of financially restricted entrepreneurs. With credit frictions, a new source of savings arises. Middle-talented workers will have more incentives to save, in order to have collateral to operate a firm, if they get better (enough to become an entrepreneur) ideas in the next period. In other words, they will save more because the expected marginal benefit of an extra unit of capital in the next period, $\mathbb{E} [V_{s'}(s', z'; \Gamma', \bar{A}') | z, \Gamma, \bar{A}]$, is higher, on average, since extra savings slack their credit constraint. This effect is stronger when cycles are shorter. The opposite is true in the scenario without frictions, in which agents do not need to have collateral to operate a firm, and will prefer to wait for a good idiosyncratic shock hits them to save, even with aggregate fluctuations.

In the baseline cycle scenario, aggregate savings increases by 0.52%, on average, compared with only 0.07% in the economy without frictions. Wages are 0.13% higher, and entrepreneurship 0.1p.p. lower. The welfare effect now turns to be positive, around 0.05%, on average, thanks to this strong increase in wages. If the expected duration of each state is 2 years, welfare gains of business cycles can achieve 0.07%.¹⁰

Table 7: Average effects of aggregate shocks in a heterogeneous agent economy with entrepreneurship and financial frictions - USA.

	Expected Duration of each State					
	2 years	3 years	4 years	5 years	8 years	w/o shocks
r	2.03%	2.04%	2.04%	2.04%	2.05%	2.06%
w	100.14	100.13	100.12	100.11	100.10	100.00
K	100.61	100.52	100.51	100.49	100.41	100.00
K^C/K	60.40%	60.31%	60.32%	60.31%	60.23%	59.83%
GDP	100.14	100.12	100.12	100.11	100.10	100.00
Std. Dev. GDP	1.94%	2.03%	2.08%	2.13%	2.27%	0.00%
Entrepreneurs (%)	7.38%	7.40%	7.40%	7.40%	7.42%	7.50%
Std. Dev. MgRK	4.90%	4.84%	4.79%	4.79%	4.76%	4.84%
EDEC	100.07	100.05	100.04	100.03	100.02	100.00

It is interesting to note that the standard deviation of MgRK, the measure of misallocation adopted here, is decreasing with the persistence of the cycle. This result is in line with [Moll \(2014\)](#), who finds that, the more persistent the shocks are (in his case, only idiosyncratic), more powerful is self-finance to undo capital misallocation, but in a smaller extent. To illustrate this point, Panel A of Figure 2, shows

¹⁰This positive effect depends on the calibrated parameters. In Appendix A.3, results for Brazil, an economy with tighter frictions and larger shocks, are presented. In this case, I cannot find welfare gains of business cycles.

how misallocation evolves over time, in each state (good or bad). After a shock that moves an economy from a bad state to a good state, entrepreneurs will try to grow their business, but since they face credit constraints, the misallocation tends to be high (above the case with no shocks, represented by the blue dashed line). As times passes, and the economy continues in the good state, entrepreneurs accumulate wealth (Panel B) and misallocation reduces. However, a composition effect is also acting: a wealthier economy pays higher wages, which forces some low productivity/restricted entrepreneurs (Panel C) to become workers, reducing the average misallocation in good states. A similar reasoning applies for bad states. Just after a transition from a good to a bad state, aggregate capital (and wages) is relatively high, but credit demand by entrepreneurs is weak, implying in a low misallocation. As the bad state continues, agents will spend their savings to smooth consumption and credit frictions will start to tight. After some years in a bad state, misallocation starts to reduce again, but not for a good reason: a larger share of entrepreneurs becomes restricted (the average MgRK, shown in Panel D, increases, decreasing the variance). The composition effect also acts here, but bringing more low productivity/restricted agents into entrepreneurship, since wages falls.

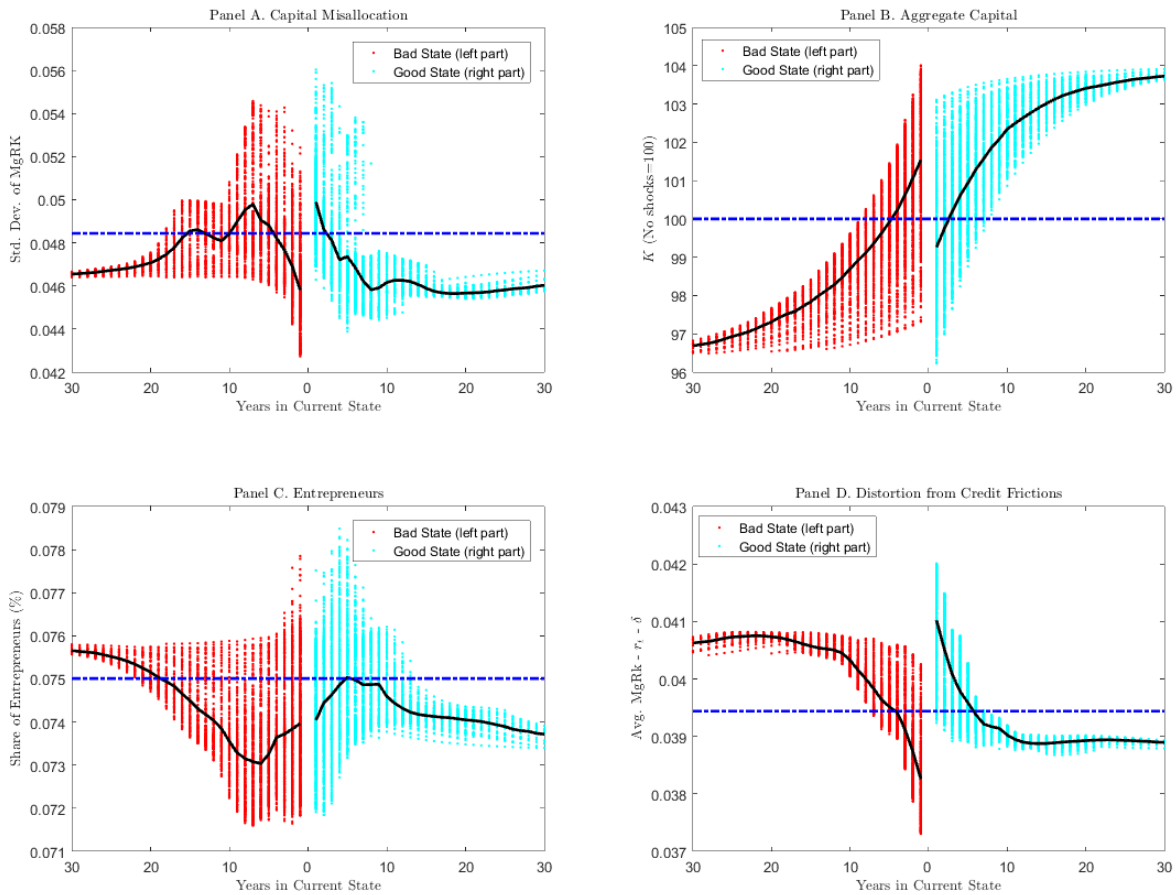


Figure 2: Dynamics in Good and Bad States.

Notes: Each dot represents the value of the variables in a period of time in the 10,000 period simulation. The solid lines are averages within each year. This Figure was constructed assuming that the expected duration in each state is 8 years.

In short, long periods in good states reduce misallocation and enhance efficiency, by selecting the best entrepreneurs and allowing them to slack their restrictions. On the other hand, long periods of low aggregate productivity binds entrepreneurs, eventually reducing misallocation, but only because the average inefficiency increases. The overall result is that misallocation can be lower, on average, in a economy with (long) cycles, since precautionary saving helps to slack entrepreneurs restrictions. For better visualisation, Figure 2 considers a long cycle, with a expected duration in each state of 8 years, but if it is not the case, and cycles are shorter, like in the baseline scenario, the precautionary savings alone is not enough to overcome the effects of uncertainty, and the economy remains stuck in a high misallocation environment. These insights can be seen in Figure 4, in Appendix A.4.

4.4 Distribution of gains from business cycles

Krusell et al. (2009) show that welfare effects can be very different across agents, and here it is not different. As exposed in Table 7, welfare gains of aggregate shocks can achieve 0.07%, but how these gains are distributed? By looking at prices only, we are able to partially answer this question. Since savings increase, pushing down interest rates, it is expected that rich agents are worse off. Furthermore, as wages increase, the gains are targeted to workers. Panel A of Figure 3, plots the average welfare gains of the business cycles in the baseline calibration for each idiosyncratic state (z, s) . As expected, rich agents lose with lower capital income (interest rate is 1.3% lower, on average), with a welfare up to 0.9% lower than in an economy without uncertainty. The winners, with gains close to 0.09% are the least fortunate, the poor and untalented workers. The same is not true for poor but talented agents, that will face more uncertainty and smaller expected future income, if they eventually become rich entrepreneurs. A top talented poor agent is around 0.17% worse in an economy with shocks.

In Panel B of Figure 3, we take a closer look at lower levels of wealth (normalized by the annual wage), in order to identify the threshold between winners and losers. Agents with bad ideas are better off in an economy with cycles if their wealth is below 11 annual wages¹¹. This threshold is decreasing with respect to the idea, but covers the largest share of the population.

This asymmetry in welfare effects is mainly generated by general equilibrium effects on prices, an element that Lucas (1987) and some subsequent works have ignored. If we do not take into consideration changes in prices, we would expect that poor agents would be those who are hurt the most by business cycles, while rich agents (well insured) would only be marginally affected.

¹¹For sake of comparison, the median annual wage in USA on 2017, according to BLS, was \$44,720. This would imply that untalented agents with wealth below \$500,000 are better living in an economy with shocks.

Table 8: Asymmetric Shocks - USA

	Expected Duration of each State					
Good	4	4	8	8	8	w/o shocks
Bad	2	4	2	4	8	
r	2.04%	2.04%	2.05%	2.05%	2.05%	2.06%
w	100.10	100.12	100.05	100.08	100.10	100.00
K	100.37	100.51	100.20	100.28	100.41	100.00
K^C/K	60.15%	60.32%	60.00%	60.07%	60.23%	59.83%
GDP	100.09	100.12	100.05	100.07	100.10	100.00
Std. Dev. GDP	2.01%	2.08%	2.06%	2.16%	2.27%	0.00%
Entrepreneurs (%)	7.45%	7.40%	7.47%	7.47%	7.42%	7.50%
Std. Dev. MgRK	4.80%	4.79%	4.74%	4.73%	4.76%	4.84%
EDEC	100.03	100.04	100.00	100.01	100.02	100.00

that can arises is: what happens if we shut down business cycles at a certain point of the time? When considering this problem, we need to look at the transition between the chosen point to the steady-state without shocks.

One may think that the gains from economic fluctuations found in previous simulations hold only on average, but may be the case that a stabilization policy should be desirable in some incredibly bad (or good) state. To account for it, I consider the transition dynamics¹² starting from different points. Table 9 summarizes the results.

Table 9: Welfare effects: business cycles vs economic stabilization.

Period of the Cycle	\bar{A}_t	Initial K	Initial r	$EDEC_t$ vs $EDEC_{SS}$	$EDEC_t$ vs Transition
Peak	$1 + \Delta^H$	103.52	2.04%	100.57	100.01
	$1 - \Delta^L$	103.58	1.60%	100.25	100.00
Middle	$1 + \Delta^H$	100.55	2.28%	100.19	100.02
	$1 - \Delta^L$	100.55	1.82%	99.86	100.03
Trough	$1 + \Delta^H$	96.89	2.53%	99.82	100.06
	$1 - \Delta^L$	96.96	2.06%	99.51	100.06

In the best state of the economy, aggregate capital is 3.5% higher than in the steady-state without aggregate shocks, and the EDEC is 0.57% above the respective measure in the economy without cycles. This difference in welfare is reduced by half if a negative shock hits the economy in the following period (0.25%). An opposite, but similar pattern is observed in the worst state of the economy.

Comparing with the transitions, results are qualitatively different and it is better to continue in an economy with cycles than a stabilized one, regardless of starting in the peak, middle or trough of the cycle. Stabilizing the economy when the initial capital is high has a small deleterious effect, around 0.01%. This makes sense, since agents are relatively insured (accumulated capital) and have expected higher wages in an economy with cycles. On the other hand, stabilizing the economy in the worst state of the economy, or just after a positive shock hits, has a substantial negative effect of welfare, 0.06%. Since

¹²Appendix A.2 describes the algorithm to solve for the transition, which actually turns out to be very simple, since idiosyncratic shocks hit idea process, not wages or employment state directly.

agents are poorly insured, they prefer to wait for a boom, that will allow them to accumulate assets faster, rather than slowly achieve a steady-state with lower wages.

5 Final Remarks

In this study, I contribute to the literature by proposing a simple method to solve a heterogeneous agent model with entrepreneurship, financial frictions and aggregate uncertainty. With the model in hands, I have shown that business cycles may improve welfare, on average. [Cho et al. \(2015\)](#) discuss some simple models in which it can be true, but the mechanism behind the results obtained here were unexplored in the literature. Trending ingredients in the heterogeneous agent literature are crucial: entrepreneurship and financial frictions. These elements generate two important features in the economy: wealth concentration and willingness to save in order to slack financial constraints. Aggregate shocks in this context, induce a strong increase in aggregate savings by potential entrepreneurs, what push wages up. This channel may be strong enough to more than offset the effect of fluctuations, but the magnitude is in line with [Lucas \(1987\)](#).

Although the model presented here abstracts from some aspects, it shows that with a better understanding of how different entrepreneurs reacts to aggregate shocks, and how their behaviour affects the whole economy, policy makers may be able to propose strategies to improve welfare. Even though aggregate shocks seem to do not harm the economy, financial frictions play an important role, and adding a government sector in the model, for instance, could allow us to study which type of policy focused in entrepreneurs can be used, or even design an optimal corporate or capital income taxation schedule.

The proposed model can, potentially, also be modified and expanded to include relevant aspects on the literature of credit frictions, like borrowing constraints depending on asset prices, à la [Kiyotaki and Moore \(1997\)](#), or even flow-based constraints. A richer occupational choice setting, with the inclusion of self-employment, as in [Allub and Erosa \(2014\)](#); [Gollin \(2008\)](#); [Merlin and Teles \(2014\)](#), may interact with financial frictions and generate non-trivial effects too.

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A Algorithm

A.1 Solving the model with aggregate uncertainty

1. Set grids for k , r and z , guess a value function (VF) over the grid, besides setting an initial forecasting rule for r .¹³
2. Given r' (from LOM), interpolate the VF (also over a finer grid of k , with 1500 grid points) to obtain the expected value function in the next period, and update the VF until convergence. In this step, I store the consumption, savings, labor demand, capital demand and occupational choice policy functions.
3. Given policy functions, simulate an economy for 11,000 periods, discarding the first 1000 observations. At each period, given the current distribution¹⁴ and labor/capital demand policy functions, an optimization routine interpolates the policy rules and finds the interest rate (and wages, consequently) that clear markets. Then, using the savings policy rule, I calculate the next period asset distribution.¹⁵
4. Estimate the parameters of equation (12), Θ . If $|\Theta_{NEW} - \Theta_{OLD}|$ is larger than a convergence criterion (I use 1e-4), update the LOM and repeat from step (2).

¹³In the simulations I use 300 points for the capital grid, unevenly distributed with power 4 and with 0 as the lower bound and 2,500 as the upper bound, 20 points for the interest rate grid, and a 9-state Markov chain, using Tauchen (1986) procedure, for the idea process. I set a constant initial forecasting rule, i.e., $\alpha^{ij} = 0$ and $\beta^{ij} = 1, \forall i, j \in (H, L)$.

¹⁴To get more accurate results, I use a distribution with 1500 grid points in the simulations. It is important to note that using non-stochastic simulations provides more accurate results than using stochastic simulations with a finite number of agents, since the distribution has a fat right tail and small stochastic disturbances could lead to inaccurate results.

¹⁵In this step, I set that the gap in labor and asset markets (the difference between the LHS and RHS in equations (10) and (11)) can not exceed 0.001%.

A.2 Solving for the transition path

1. Solve the model as in section A.1, both with and without aggregate shocks.
2. The trajectory of the aggregate productivity during the transition is obtained using the conditional expectation of the economy with aggregate shocks.
3. Set T , the number of periods in which is imposed that the economy will achieve the new steady-state, and guess a path for the real interest rate, $\{r_t\}_{t=1}^T$.
4. Given path for prices, and using the steady-state value function of the economy without shocks, $V(s, z; \Gamma_{SS}, 1)$, iterate backwards to obtain the value and policy functions during the transition path.
5. Given the value function path obtained, iterate forwards to obtain $\{\Gamma_t\}_{t=1}^T$, as well as the implied equilibrium prices.
6. If the maximum absolute difference between the implied interest rate and guessed path is above a converge criteria, $1e-5$, update guess and repeat from 4, until convergence.
7. If the maximum absolute difference between Γ_t and Γ_{SS} is above a converge criteria, $1e-5$, increase T and repeat from 3.

A.3 Welfare implications in a more distorted economy

Table 10: Calibration for Brazil

Parameter	Value	Target
β	0.913	Real interest rate – 4.5%
A^C	0.795	Share of capital in the corporate sector - 30%
ϕ	1.4	Credit to firms/GDP - 24%
σ^z	0.034	Entrepreneurs Gini - 0.49
ν	0.076	Share of Entrepreneurs - 4.2%
$\Delta^U = \Delta^L$	0.035	GDP per capita growth S.D. - 3.87%
Source		
χ	0.11	Net interest spread

Source: Occupational choice data are from Monthly Employment Survey (PME - Pesquisa Mensal do Emprego); interest rate and spreads are from Central Bank of Brazil (BCB - Banco Central do Brasil).

The results for the Brazilian economy, in Table 11, are clearly different from those obtained for USA. The increase in aggregate savings is much smaller (around a half), even with a larger aggregate shock. This weaker effect is due to three main factors: high banking spreads, tighter borrowing constraints, and a larger difference between $1/\beta - 1$ and r . These features of the Brazilian economy makes restricted entrepreneurs less prone to save. Save to slack their constraints is costly (high $1/\beta - 1 - r$), and has small benefits (high χ and low ϕ). Therefore, the negative fluctuation effect prevails, and actually can be larger than in the representative agent economy. The actual

Table 11: Average effects of aggregate shocks in a heterogeneous agent economy with entrepreneurship and financial frictions - Brazil.

	Expected Duration of each State					
	2 years	3 years	4 years	5 years	8 years	w/o shocks
r	4.53%	4.54%	4.54%	4.54%	4.54%	4.54%
w	100.05	100.02	100.01	100.01	100.01	100.00
K	100.32	100.28	100.28	100.27	100.27	100.00
K^C / K	27.92%	27.89%	27.91%	27.91%	27.91%	27.60%
GDP	100.07	100.05	100.05	100.04	100.05	100.00
Std. Dev. GDP	3.77%	3.91%	4.07%	4.20%	4.48%	0.00%
Entrepreneurship	4.23%	4.22%	4.22%	4.22%	4.22%	4.28%
Std. Dev. MgRK	8.50%	8.48%	8.47%	8.47%	8.47%	8.35%
EDEC	99.96	99.87	99.85	99.84	99.83	100.00

duration of a cycle in Brazil is 4.15 years¹⁶, what would imply in an expected duration of each state of 2 years, and a welfare cost of business cycle around 0.04%. For longer cycles, however, the costs can hit 0.17%.

A.4 Additional Figures

¹⁶According to the Brazilian Business Cycle Dating Committee (IBRE- CODACE), with data covering 8 cycles since 1980.

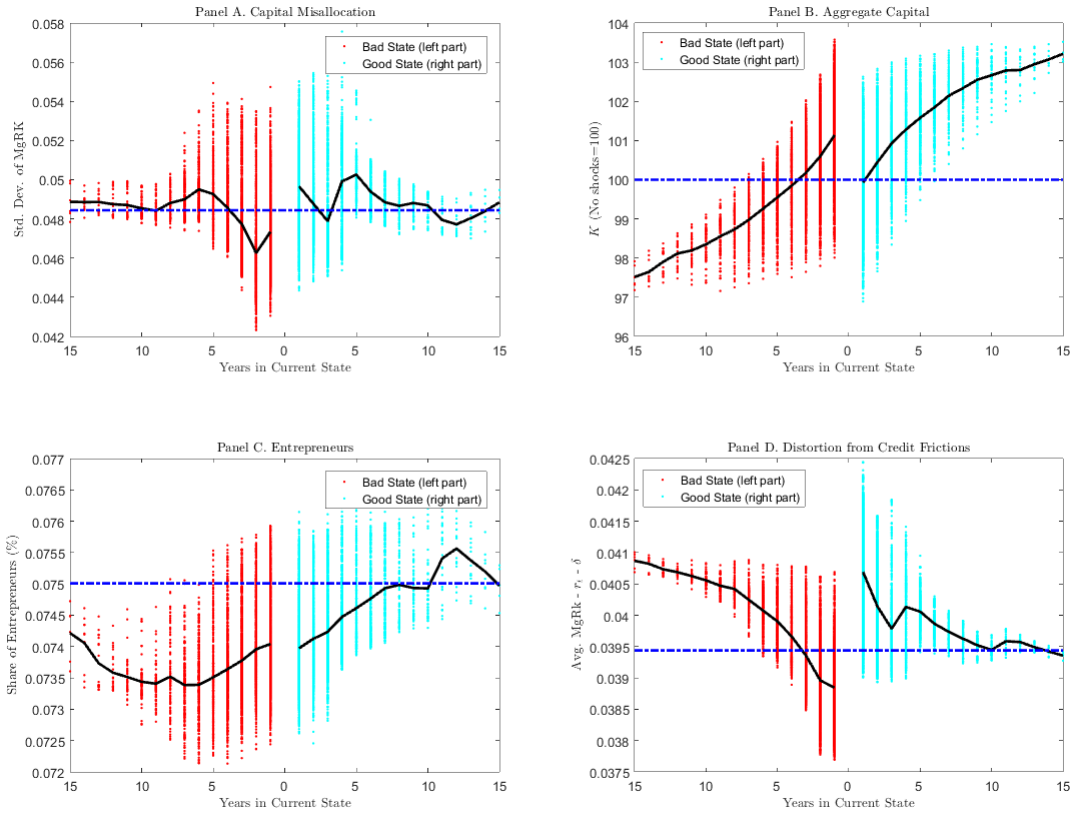


Figure 4: Dynamics in Good and Bad States - Baseline Model - 3 years in each state.

Notes: Each dot represents the value of the variables in a period of time in the 10,000 period simulation. The solid lines are averages within each year. This Figure was constructed assuming that the expected duration in each state is 3 years.