FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

VICTOR WONG

# COORDINATION FAILURES AND SLOW RECOVERIES IN MACROECONOMICS

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Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para obtenção do título de Mestre em Economia de Empresas

Campo de Conhecimento:
Macroeconomia

Orientador: Prof. Dr. Bernardo Guimarães

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Macroeconomia

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*"Work gives you meaning and purpose and life is empty without it."*

*(Stephen Hawking)*

**ABSTRACT**

Coordination failures are one of the possible reasons why economies suffer during a recession and why they take long time to recover. This work studies a macroeconomic model with endogenous hazard rates that captures in a simple way the dynamic coordination problem and assess it quantitatively. We could find that, for the US economy, coordination failures have very small effects but very persistent.

**Key-words**: coordination failures, business cycles, demand externalities

**RESUMO**

As falhas de coordenação são uma das possíveis razões pelas quais as economias sofrem durante uma recessão e por que demoram para se recuperar. Este trabalho estuda um modelo macroeconômico que capta de forma simples o problema da coordenação dinâmica e o avalia quantitativamente. Encontramos que, para a economia dos Estados Unidos, falhas de coordenação têm efeitos pequenos, mas bem persistentes.

**Palavras-chaves**: falhas de coordenação, ciclos econômicos, externalidades de demanda.

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# 1 Introduction

In 2014, six years after the financial crisis hit the world, international organizations such as IMF warned about the risks of stagnation and persistently weak activity as countries failed to recover strongly from the recessions. In its own words at that time: "With weaker-than-expected global growth for the first half of 2014 and increased downside risks, the projected pickup in growth may again fail to materialise or fall short of expectation" [1].

Today, despite enjoying a period of better economic growth, world economy is not growing as quickly as it were between 2003 and 2007. To a certain extent, the crises that took place recently still have effects in current growth. As often emphasized, this is a time of fragile recovery.[2]

When we focus on the US economy, this process is more evident. Figure 1 shows the US real GDP per capita. The US economy has returned to grow but, after the trough of the recession was reached in the second quarter of 2009, it never turned back to its trend level.

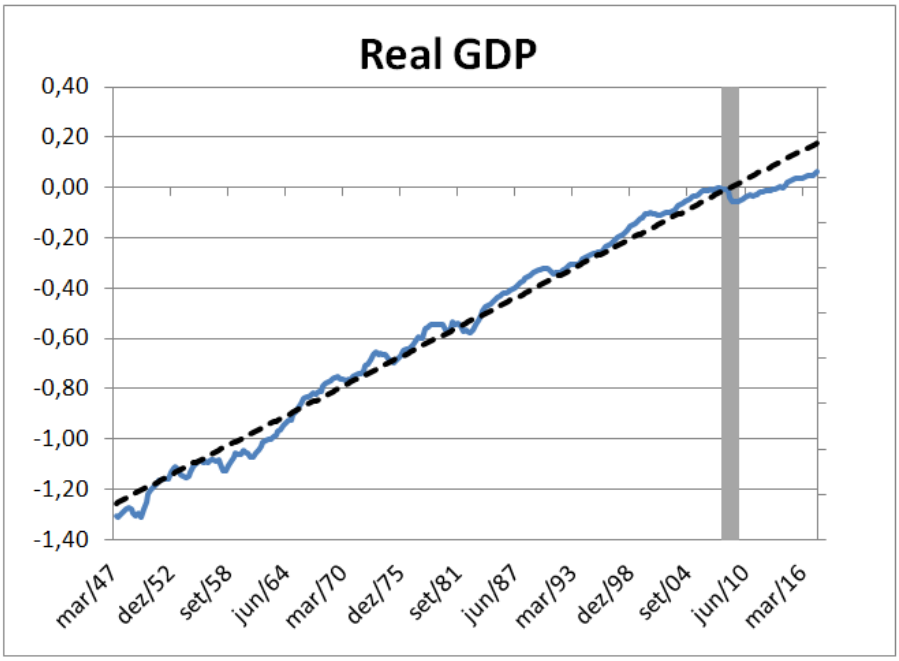



Figure 1 – US Real GDP per capita seasonally adjusted (log), undetrended, centered at 2007Q4. Linear trend 1947Q1-2007Q3

[1] https://www.theguardian.com/business/2014/oct/07/imf-economic-growth-forecasts-downgraded-crisis

[2] https://www.ft.com/content/532f1142-4163-11e8-93cf-67ac3a6482fd

Therefore, a valid question to ask is why do economies take long time to recovery from big recessions. To shed lights on it, we seek to investigate whether negative shocks may have a persistent/permanent effect on economic growth.

This phenomenon is quite intriguing and calls for attention. Hence, it has been studied recently for many economists that propose several kind of explanation for it: secular stagnation, the zero lower bound restriction and financial frictions are the most popular hypothesis.

However, we think coordination also plays a role in this type of circumstances. Thus, we suggest a simple model that capture in a straight way the coordination issue and assess it quantitatively. The idea of building a simple model here is to clean up some assumptions that most of DSGE models present nowadays and focus on the coordination configuration of the model, assessing it quantitatively. Traditional DSGE models have their advantage to use microfoundations to match data, but by a cost of too many structural assumptions, which might turn difficult to see the transmission channels in the model.

We explore the role of demand externalities that generate strategic complementarities in production. When negatives shocks hit the economy, the choice of a firm to slow down investment, and thereby production, affects other firms by reducing the demand for other firms' products. Hence one firm's reduction in investment decreases others' incentives to invest and produce. In a dynamic framework, this coordination feedback might be self-reinforcing, causing the economy to suffer the effects of these shocks for a longer period of time.

Our model follows closely Guimaraes e Machado (2017). This framework yields a unique equilibrium, which eases our quantitative work. Investment is a cost that agents pay to intensify the arrival of switching opportunities to increase production capacity. In the model, investment is the choice of the switching intensity. It captures an idea similar to R&D investment where innovation becomes a useful idea with a probability that in our case is represented by this hazard rate to switch to a higher production capacity.

Agents will make investment decisions based on two things: future demand and productivity. The former is linked with the assumption of strategic complementarity and will depend on how others are choosing the intensity to switch to a higher production regime. Then agents have to form expectations about others' future decisions when deciding about how much to invest. Thus they face a dynamic coordination problem in their investment decisions.

The latter captures supply side incentives. If the increase in production resulting from investing is large enough, then high investment is a dominant strategy. Likewise, if productivity is very low, low investment is a dominant strategy. In an intermediate range,

an agent's decision depends not only on productivity (supply side) but also on her beliefs about the actions of others (demand side).

There are shocks to the aggregate productivity. More specifically aggregate productivity will follow a stochastic process with mean reversion. Frankel e Burdzy (2005) show that a unique equilibrium arises in this class of the models[3]. Intuitively, the presence of shocks and the regions of dominance drive a contagion effect on all regions. Due to strategic complementarities, if an agent picks her action when productivity is near a region where high investment is strictly dominant, others have an incentive to pick that action as well. In this case, this action will be investing more, because while she is locked (she needs to wait a switching opportunity to arrive), productivity changes are likely to move the game into the strict dominance region of high investment, leading some of the other agents to invest more as well. Thus higher investment must be played when sufficiently close to this dominance region. Likewise, this process happens near the other dominance region of low investment: agents invest less. Then, these local interactions give rise to a contagion effect that begins in the dominance regions and spreads throughout the state space.

Schaal e Taschereau-Dumouchel (2015) explore a similar question. In a model with strategic complementarities, they develop a very rich quantitative macroeconomic theory of business cycles with coordination failures that stems a quasi-permanent recession after a large transitory shock. Shocks depending on their size might have persistent effects on the economy. In their model, there is a binary choice in the firm's decision about capacity utilization that is essential for the use of global games techniques and thereby equilibrium uniqueness. Here, on the other hand, we have a simpler model, but with firms having a richer set of (continuous) choice at investing, which causes our capacity utilization (or equivalent to) variable to move continuously. And still, effects of transitory shocks are very persistent.

This result, thereby, suggests that the persistent effect generated by strategic complementarities is not related to the simplifying modeling assumption, namely, the binary choice of capacity utilization that generates an aggregate binary capacity or any larger class of non-convexities.

The main insight that emerge from the theory is that a shock stems very persistent effects. With this in mind, we use the model to assess quantitatively the United States economy. When we calibrate for it, we found that a half life of $h_t$ (demand) is approximately twelve times bigger than a half life of $a_t$ (productivity). Despite the persistence, our calibration also suggests that shocks have small effects on economy. In order to quantify these effects, we propose an exercise with an impulse response of output to a

[3] For that, we need finite-lived mean reversion

1% productivity shock in two cases: (i) with our variable $h$ also responding to the shock and (ii) with $h$ fixed (without the demand channel). The difference between the two cases is the coordination failure effect. We found that the instantaneous effect is approximately 0.008%, but since the shocks have persistent effects, when we also consider the duration, coordination failures have a total effect of 0.55%, which is modest but significant.

The paper is organized as follows. In section 2, we present the model and deliver the main theoretic results. Section 3 presents the data and, in section 4, we calibrate the model for the United States economy and show the solution strategy to assess it quantitatively. In section 5, we simulate the economy and show the empirical results. Section 6 concludes.

## 1.1 Related literature

This paper is related to a branch in the literature that explores dynamic coordination, where the main contributions are in Frankel e Pauzner (2000) and Frankel e Burdzy (2005). They study the role of adding a Calvo-like friction in a dynamic game of regime change with time-varying fundamentals. This friction cause actions to be asynchronous, and by doing so, it helps discipline equilibrium selection and rationalize a unique equilibrium.

More closely related to our paper is the work of Guimaraes e Machado (2017) that uses the same framework as Frankel e Pauzner (2000) in a macroeconomic model. They examine the impact of investment subsidies in a context of monopolistic competition and staggered technology choice. In their model, firms receive endogenous opportunities to change their technology according to a Calvo-type Poisson process. Our paper uses this framework, but with a different goal. They are worried about social policies to improve welfare. Instead, we want to assess the model to contribute to debate on persistent effects of shocks, specially in times of recession.

Our paper also contributes to the business cycle literature that studies the amplification of the effects of shocks and their persistence. Schaal e Taschereau-Dumouchel (2015) build a real business cycle model with an aggregate demand externality and a nonconvex technology choice to explain quasi-permanent recessions. Coordination failures arise from a game where firms choose their capacity utilization at every period and open up space for multiple equilibria[4].

This amplification effect can also be explained by financial frictions: Machado (2016) relies on endogenous dynamic feedback between intermediaries' balance sheets and coordination to create a powerful effect of coordination failures on asset prices, in-

[4] Seminal works by Carlsson e Damme (1993) and Morris e Shin (1998) show how to discipline the equilibrium by use of global games.

vestment and welfare. In a more traditional DSGE fashion, Bernanke, Gertler e Gilchrist (1999) exhibits a “financial accelerator”, i.e., they propose credit market frictions that propagate and amplify both real and nominal shocks to the economy. The mechanism involves what they call "external finance premium" and the net worth of borrowers to finance entrepreneurs. This battle for scare resources, usual in models with asymmetric information and agency costs, enhance the swings in borrowing and thus in investment, spending, and production.

Building up on the ideas of Summers (2013) and Krugman (2013) - that slow recoveries may be an outcome of low real interest rates, low inflation, and the zero lower bound that prevent authorities from maintaining the economy at its full-employment growth potential, Benhabib, Schmitt-Grohé e Uribe (2001) work on multiple equilibria resulting from Taylor rules and zero lower bound constraint. Evans e Honkapohja (2005) weaken the perfect foresight/rational expectations hypothesis to a adaptive learning assumption and explore the implications under different interactions of monetary and fiscal policies. Benigno, Fornaro et al. (2016) use a Keynesian growth framework with zero lower bound to show that interactions between endogenous growth and aggregate demand result in very persistent liquidity traps (stagnation traps). Schmitt-Grohé e Uribe (2017) shows that the interplay among nominal wage rigidity, Taylor rules, the zero lower bound and confidence shock explains the occurrence of liquidity traps that give rise to jobless growth recoveries. Mertens e Ravn (2014) investigate that fiscal policy interventions might work stimulating output depending on the monetary situation: if liquidity trap is driven by fundamental it succeeds, but it fails when it is driven by confidence.

Finally, our paper also belongs to the literature that views recessions as episodes of coordination failures due to demand externalities. Seminal contributions of Kiyotaki (1988), Cooper e John (1988) and Murphy, Shleifer e Vishny (1989) are some examples. Kiyotaki (1988) builds a model with monopolistic competition and increasing returns to scale to generate multiple equilibria. Cooper e John (1988) provide a general framework that can be used to show how complementarities in payoffs can generate coordination failures (multiple equilibria) in static models. Lastly, Murphy, Shleifer e Vishny (1989) model an imperfectly competitive economy with aggregate demand spillovers. The idea of the paper is that an economy can escape a no-industrialization trap if various sectors are simultaneously industrialized (Big Push).

# 2 Model

We propose to use Guimaraes e Machado (2017) model where hazard rates are endogenous.

## 2.1 Environment

Time is continuous. The economy consists of a representative household, a final good sector and an intermediate good sector. The final good is used for both consumption and investment. The intermediate goods consist of a continuum of varieties only used for the production of the final good.

## 2.2 Final Good producers

A composite good is produced by a perfectly competitive representative firm. It uses intermediate goods, indexed by $i \in [0, 1]$, to produce the final good $Y_t$, at time $t$, according to the technology:

$$Y_t = \left( \int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{2.1}$$

where $y_{i,t}$ is the quantity of good $i$ used as input and $\theta > 1$ is the elasticity of substitution. Profit maximization, taking output price $P_t$ and input prices $p_{i,t}$ as given, implies the standard demand for each variety:

$$p_{i,t} = \left( \frac{Y_t}{y_{i,t}} \right)^{\frac{1}{\theta}} P_t \tag{2.2}$$

where $P_t$ is the price of final good, and can be normalized to one without loss of generality.

## 2.3 Households and preferences

There is a measure-one continuum of agents who discount utility at rate $\rho$. An agent's instantaneous utility at time $t$ is given by

$$U_t = C_t, \tag{2.3}$$

where $C_t$ is her instantaneous consumption of the final good.

Agent $i \in [0, 1]$, which is the owner of the intermediate firm $i$, produces intermediate good $i$. Since $y_{i,t}$ is the quantity produced by agent $i$ at time $t$, her budget constraint is given by

$$C_t \leq p_{i,t} y_{i,t} \equiv w_{i,t} \tag{2.4}$$

Prices are flexible and each price $p_{i,t}$ is optimally set by agent $i$ at every time. Goods are non storable, then supply must equal demand at any time $t$.

## 2.4 Intermediate good producers

The technology is modeled as two production regimes: a High-capacity regime and a Low-capacity regime. An agent in the *Low* regime can produce up to $y_{L,t}$ units at zero marginal cost at every time $t$, and an agent in the $High$ regime can produce up to $y_{H,t}$ units at zero marginal cost as well, with

$$y_{i,t} \leq \begin{cases} y_{H,t} = A_t x_H & \text{if } High \\ y_{L,t} = A_t x_L & \text{if } Low \end{cases} \tag{2.5}$$

where $x_H > x_L$ are constants and $A_t$ is a time-varying productivity parameter.

On top of the production decisions, at each moment, an agent in the *Low* regime chooses the hazard rate of switching opportunities $\alpha_t^L \in [\underline{\alpha}^L, \bar{\alpha}^L]$, with $\bar{\alpha}^L < \infty$, subject to a cost $c(\alpha_t^L)$. The cost function is increasing, continuous and convex. Similarly, an agent in the $High$ regime switches to the *Low* regime, however, with an exogenous hazard rate $\alpha^H$. Moreover, after deciding the level of switching rate, if an agent receive the chance to switch, she must take it and she will be locked in this regime until she is selected again.[1]

**Interpretation:** the $High$ regime can be interpreted as the use of frontier technology, while the *Low* regime corresponds to a less productive technology. In one interpretation, the cost $c(\alpha_t^L)$ is the amount of resources a firm spends in R&D and some of the new ideas become useful at rate $\alpha_t^L$. In another possible interpretation, equipments break or become obsolete at rate $\alpha^H$ and are immediately replaced, but the time until a new equipment can be used in production is stochastic and depends on the amount of resources allocated to this end, $c(\alpha_t^L)$.

The assumptions on technology aim at modelling staggered investment decisions in a simple and tractable way, but similar to the idea of investment adjustment cost in the traditional macroeconomic models with investment. In those models, this adjustment

[1] In Frankel e Pauzner (2000), a player costlessly receives switching opportunities at some fixed rate d. She does not have to switch. Nevertheless, the switching this mechanism can be captured in our model by constraining $\alpha^H$ and $\alpha_t^L$ to be in [0, d] and letting $c(\alpha_t^L) = 0$. Switching with probability p when an opportunity arises is equivalent to choosing a switching rate of pd in our model.

cost is important because some authors found that a potential discontinuity in investment demand can occur when this cost is not present and because it allows to better calibrate investment volatility that, without this adjustment cost, can be too high. Recently, models incorporate this adjustment cost as a function of the change in investment rather than its level. The reason is that it introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to shocks (Smets e Wouters (2007)).

**Productivity:** Let $a_t = \log(A_t)$ follows a mean reverting Ornstein-Uhlenbeck process:

$$da_t = \nu(\mu - a_t)dt + \sigma dZ_t$$

where $\nu$ represents the strength of mean reversion on the process for $a_t$, $\mu$ represents the mean it reverts to, $\sigma > 0$ is the standard deviation of $a_t$ and $Z_t$ follows a standard Brownian Motion.

Furthermore, we assume dominance regions, i.e., for extreme values of the productivity parameter, agents have a strictly dominant switching rate. There are constants $\bar{a} > \underline{a}$ such that:

1. If $a > \bar{a}$, it is strictly dominant for agents in regime *Low* to switch at their maximum switching rate $\bar{\alpha}^L$.

2. If $a < \underline{a}$, it is strictly dominant for agents in regime *Low* to switch at their minimum switching rate $\underline{\alpha}^L$.

We model productivity this way once shocks and dominance regions are necessary to discipline the equilibrium and mean reversion is a suitable property for business cycle models.

## 2.5 The agent's problem

Since we have zero marginal cost, an agent will always produce as much as possible, i.e., in the *Low* regime she will produce $y_{L,t}$, and in the *High* regime she will produce $y_{H,t}$. As consequence of (2.2), at any time $t$ there is a price ($p_{L,t}$ and $p_{H,t}$) associated with each production level ($y_{L,t}$ and $y_{H,t}$, respectively). Hence the instantaneous income available to individuals in each regime is given by

$$w_{i,t} = \begin{cases} w_{H,t} = p_{H,t}y_{H,t} = y_{H,t}^{\frac{\theta-1}{\theta}} Y_t^{\frac{1}{\theta}} & \text{if } High \\ w_{L,t} = p_{L,t}y_{L,t} = y_{L,t}^{\frac{\theta-1}{\theta}} Y_t^{\frac{1}{\theta}} & \text{if } Low \end{cases} \tag{2.6}$$

Furthermore, using (2.1), we have

$$Y_t = \left(h_t y_{Ht}^{\frac{\theta-1}{\theta}} + (1-h_t) y_{Lt}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{2.7}$$

where $h_t$ is the measure of agents locked in $High$ regime.

Other aggregate variables are investment and consumption:

$$I_t = (1-h_t)c(\alpha_t^L) \tag{2.8}$$

$$C_t = Y_t - I_t \tag{2.9}$$

Investment is the amount of resources agents in regime $Low$ spend in costs to switch regimes. Thus, the aggregate investment is the measure of agents in regime $Low$ times the cost that each agent spend choosing $\alpha^L$. Aggregate consumption, thereby, is the difference between aggregate output and aggregate investment.

The flow payoff of agents locked in the $Low$ regime is given by $w_{L,t} - c(\alpha_t^L)$. Similarly, agents locked in the $High$ regime have flow payoff is given by $w_{H,t}$. Let $D(a_t, h_t, \alpha_t^L)$ be the relative gain of being in regime $High$ when the economy is at $(a_t,\ h_t)$. Combining (2.6) and (2.7):

$$D(a_t, h_t, \alpha_t^L) = e^{a_t}\left(h_t x_H^{\frac{\theta-1}{\theta}} + (1-h_t) x_L^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}}\left(x_H^{\frac{\theta-1}{\theta}} - x_L^{\frac{\theta-1}{\theta}}\right) + c(\alpha_t^L) \tag{2.10}$$

Function $D$ is strictly increasing in $a_t$ and $h_t$. On the one hand, the effect of $a_t$ captures the supply-side incentives to invest that consist of higher productivity differential between agents. On the other hand, the effect of $h_t$ captures the demand-side incentives to invest, given by the strategic complementarity of intermediate goods ($\theta > 1$), i.e., a larger $h_t$ means a higher demand for a given variety, which changes the relative price of it and hence the agent benefits from others producing $y_{H,t}$ regardless of how much she is producing.

As in Guimaraes e Machado (2017), one key implication of (2.10) is that investment decisions are strategic complements: the higher the production level of others, the higher the incentives for a given agent to increase her investment and therefore her production level.

## 2.6 Equilibrium

Frankel e Burdzy (2005) show that under certain assumptions, a unique rationalizable equilibrium arises and agents choose an optimal $\alpha^{L*}$ which depends on state ($a$,

$h$). Unfortunately, the model do not really satisfy all results for it. To apply their results, we need a model that mean reversion would have to die out eventually. In fact, we need to make two changes in the model:

$$\eta_t = \begin{cases} \eta & \text{if } t < T \\ 0 & \text{otherwise} \end{cases} \tag{2.11}$$

where $T$ is a large number. And, the difference between the instantaneous utility of agents locked in each regime is given by $\hat{D}$ instead of $D$, where

$$\hat{D}(a, h, \alpha^L) = \begin{cases} D(a, h, \alpha^L) & \text{if } a < M \\ D(M, h, \alpha^L) & \text{otherwise} \end{cases} \tag{2.12}$$

where $M$ is a large number.

As $M$ and $T$ approach infinity, this model converges to yours. The model can be seen as a limiting case of a sequence of models that have a unique rationalizable equilibrium. Therefore, we can apply Frankel e Burdzy (2005) results (specifically, Theorems 4 and 5). Still, numerical results show that if there is any region of multiple equilibria, it is negligible. Thus, we can overcome this technical issue without further concerns.

**Theorem 1.** *(Frankel and Burdzy, 2005) The relative value of being in regime High, $\Delta V_\tau \equiv V_\tau^H - V_\tau^L$, equals the expected integral of the discounted relative payoff flow in regime High:*

$$\Delta V_\tau = \mathbb{E}_\tau \Big[ \int_{t=\tau}^{\infty} e^{\left(-\int_{s=\tau}^{t} [\rho + \alpha^H + \alpha_s^L] ds\right)} D(a_t, h_t, \alpha_t^L) dt \Big] \tag{2.13}$$

This relative value is increasing in the current fundamental parameter, $a_t$, and in the measure of agents locked in the $High$ regime, $h_t$.

Note that the discount rate is the sum of the rate of time preference ($\rho$) and the two switching rates ($\alpha^H$ and $\alpha_t^L$). The reason the switching rates are added is that being in regime $High$ rather than $Low$ at time $\tau$ has an effect on an agent's payoff flow until the first time at which either (i) she moves to regime $Low$ (which occurs at the rate $\alpha^H$) or (ii) if she were in regime $Low$, she would have moved to $High$ (which would have occurred at the rate $\alpha_\tau^L$). The hazard rate for one or the other of these events to occur is the sum of the two events' hazard rates, $\alpha_\tau^L + \alpha^H$.[2]

**Theorem 2.** *(Frankel and Burdzy, 2005) Agents locked into regime Low choose a switching rate $\alpha_\tau^L$ that maximizes*

$$\alpha_\tau^L \Delta V_\tau - c(\alpha_\tau^L) \tag{2.14}$$

[2] The probability of their occurring simultaneously in continuous time is zero.

The expression in (2.14) shows that agents' optimal choice of hazard rates ($\alpha_\tau^L$) depends on the expected gains from switching ($\Delta V_\tau$). The relative value of being in the $High$ regime is increasing in expected values of $a$ and $h$, which depend positively on their current values. Therefore, larger $a$ and larger $h$ will induce higher hazard rates. Important to mention that, in equilibrium, agents take as given their own switching rate in the future and the switching rate of others and simply maximize the instantaneous benefit of doing it.

Moreover, it implies that an agent chooses a switching rate at which the marginal cost of switching faster equals the marginal benefit, which is the relative value of being in the other mode (if solution is interior).

One result from their Theorems is that together they imply the rate of change of the proportion of agents in regime $High$ is determined by its current state. Since there are $1 - h$ agents in regime $Low$, who switches to regime $High$ at same rate $\alpha^L$, and $h$ agents in regime $High$, who leave to regime $Low$ at the common rate $\alpha^H$, we have:

$$\dot{h}_\tau = \alpha_\tau^L(1 - h_\tau) - \alpha^H h_\tau \tag{2.15}$$

Another consequence that will be important to us is that equation (2.15) defines a locus where $\dot{h} = 0$:

$$\dot{h}_\tau = 0 \iff h_\tau^* = \frac{\alpha_\tau^L}{\alpha_\tau^L + \alpha^H} \tag{2.16}$$

where $h^*$ is a function of $\alpha^L$ and defines locus $\dot{h} = 0$.

**Equilibrium selection**: Agents play a coordination game with multiple equilibria. In some regions of the state space ($a$, $h$), agents may choose different switching rates depending on their beliefs. A central contribution of the global games and dynamic coordination literature is that multiplicity doesn't survive under strategic uncertainty. More specifically, uncertainty about the path of $a$ opens the door to uncertainty about the actions of others, which plays key role in determining the equilibrium.

The proof is similar to the standard models in the literature. As in Frankel e Pauzner (2000) and Frankel e Burdzy (2005), the model generates a unique set of rationalizable beliefs about others' actions. The main idea is to consider a slightly perturbation of $a$ near the regions of dominance[3]. The strategic uncertainty stems a contagion effect (iterative elimination of dominated strategies) that spreads from the regions of dominance throughout all areas.

[3] A technical assumption for this proof to work is to assume their existence.

Figure 2 and 3 try to illustrate the intuition of this process. Since we have only two dimensions, we depict the relationship between the productivity parameter $a$ and the measure $h$ of agents for some fixed switching rate $\alpha_t^L$, i.e., for each color, there is a isorate curve. So the yellow color represents the lowest rate $\underline{\alpha}^L$ and orange color, by its turn, the highest rate $\bar{\alpha}^L$. The colors between them represent a gradual transition from $\underline{\alpha}^L$ to $\bar{\alpha}^L$. In other words, colors closer to yellow mean values closer to $\underline{\alpha}^L$ and colors closer to orange mean values closer to $\bar{\alpha}^L$.

Figure 2 – Dominance regions: strict dominant strategy $\underline{\alpha}^L$ (yellow) and $\bar{\alpha}^L$ (orange)

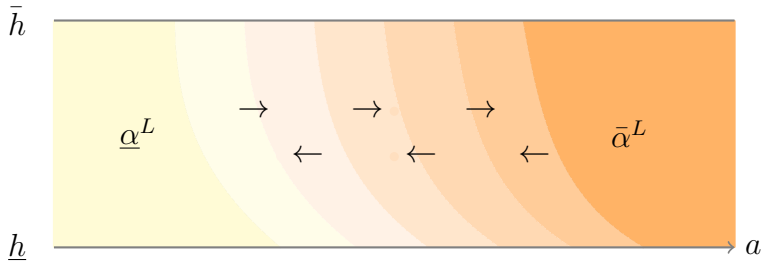



Figure 3 – Contagion effect: dominance regions affect with more intensity areas closer to them

The process of contagion originates someway like this: consider a situation where productivity is relatively low just to the right of region of dominance of low investment. It is represented by the black circle just to the right of yellow area in Figure 2. With shocks in productivity, the economy might cross to the region where low investment is a dominant strategy. Entering the region of dominance eliminates some rationalizable strategies, which initiates a process of iterative elimination of dominated strategies that yields a unique equilibrium.

This equilibrium is characterized by the isocurves in Figure 3. For a given investment ($\alpha^L$), larger $h$ implies that agents are willing to invest the same amount for lower values of $a$.

We now proceed showing the data, the strategy used to calibrate the parameters and the algorithm to solve the model numerically.

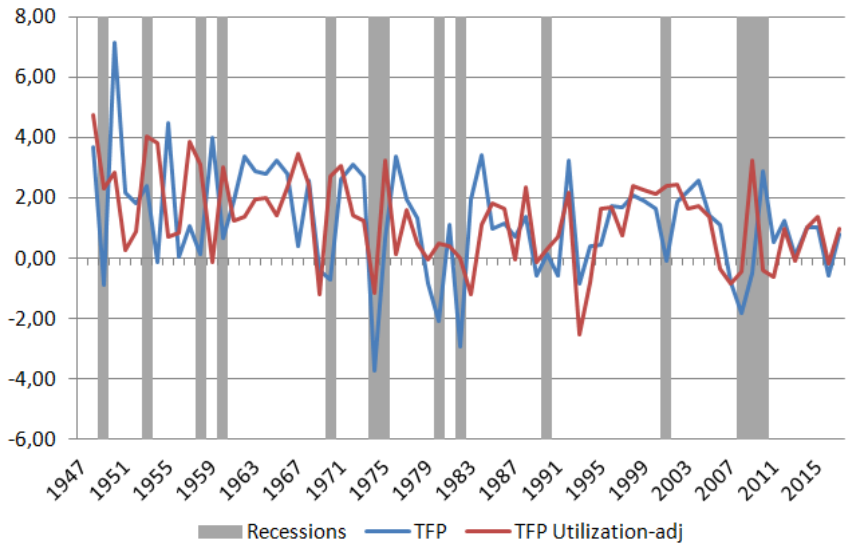
# 3 Data

Table 1 describes the data sources. All time series are quarterly from 1947Q1 to 2017Q3. Output, investment and consumption are seasonally adjusted. For productivity, we use series adjusted for variations in factor utilization - labor effort and capital's workweek.[1]

| Variable | Source |
|---|---|
| Output | Fed of St. Louis - Real Gross Domestic Product |
| Investment | Fed of St. Louis - Real Gross Private Domestic Investment |
| Consumption | Fed of St. Louis - Real Personal Consumption Expenditures |
| Productivity | Fed of San Francisco - Total Factor Productivity |

Table 1 – Data sources

**TFP**: Figure 4 compares two measures: raw TFP data and TFP adjusted for utilization of capacity. The adjustment follows Basu, Fernald e Kimball (2006). In their study, they argue that the utilization-adjusted quarterly series is an improvement over more "naive" measures of TFP as a high-frequency indicator of technological change. Notwithstanding with our model being different from traditional ones with utilization of capacity, we think utilization-adjusted TFP is more suitable for our model and hence this is the measure we use.



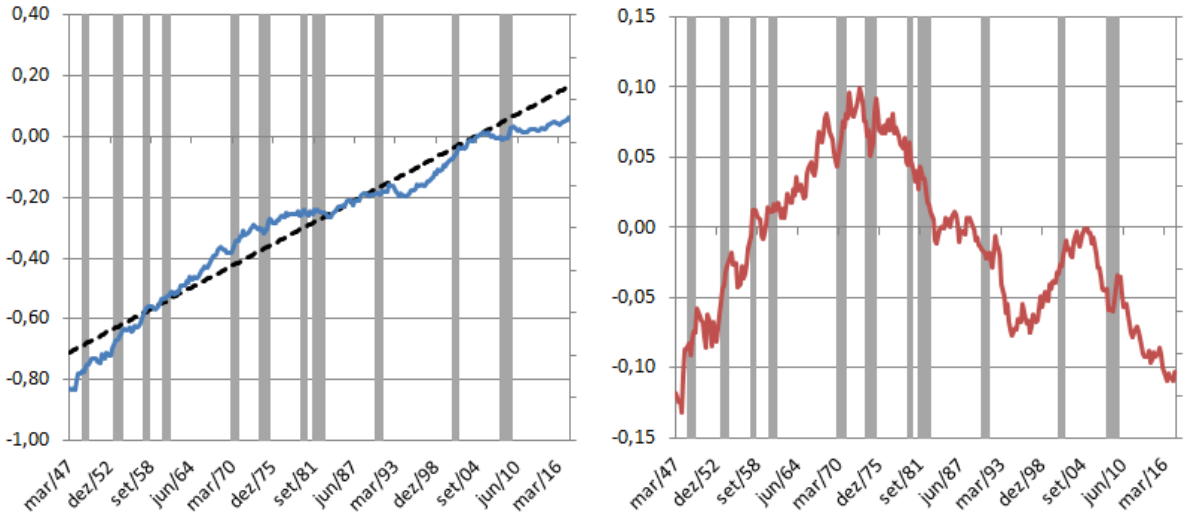

*Notes*: Series shown in annual change (%). Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald. Shaded areas correspond to NBER recessions.

Figure 4 – TFP and TFP adjusted for labor quality, provided by Fernald's website

[1] Basu, Fernald e Kimball (2006)

Following Schaal e Taschereau-Dumouchel (2015), TFP data are set in logs, detrended by a linear trend from 1947Q1 to 2017Q3 and centered at 2007Q4 (normalized).

(a) log utilization-adjusted TFP (b) detrended log utilization-adjusted TFP

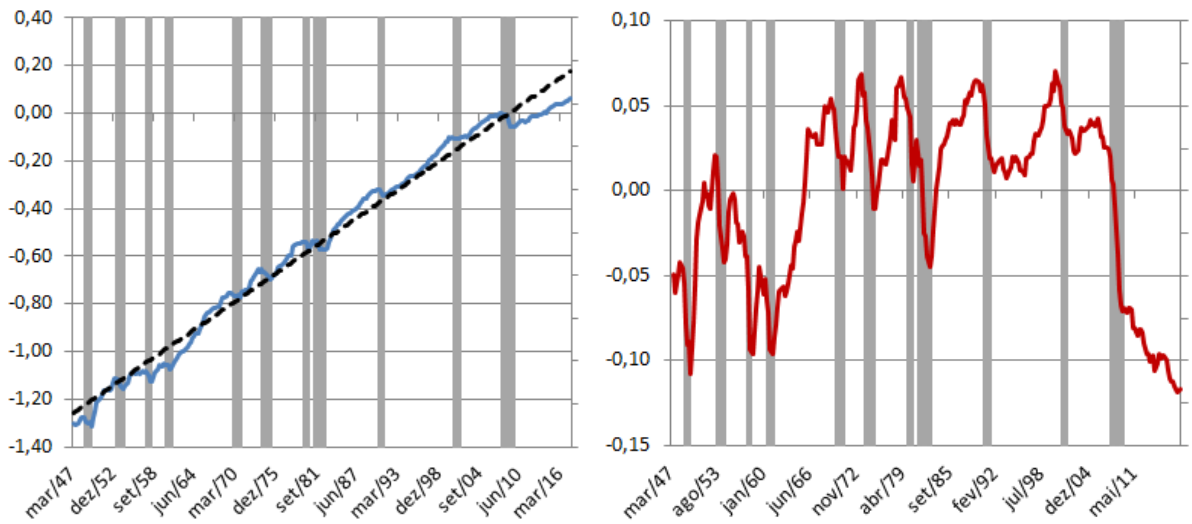



*Notes*: TFP series shown in logs, centered at 2007Q4. Linear trend for TFP computed over the period 1947Q1-2017Q3. Shaded areas correspond to NBER recessions.

Figure 5 – Evolution of US productivity over 1947-2017

**Output**: Panel (a) of Figure 6 shows output in logs and its linear trend from 1947Q1-2017Q3. It moves accordingly to this trend until Great Recession in 2008, when output collapsed and never turned back to its trend level. Moreover, this gap is lasting longer than expected.

(a) log output (b) detrended log output

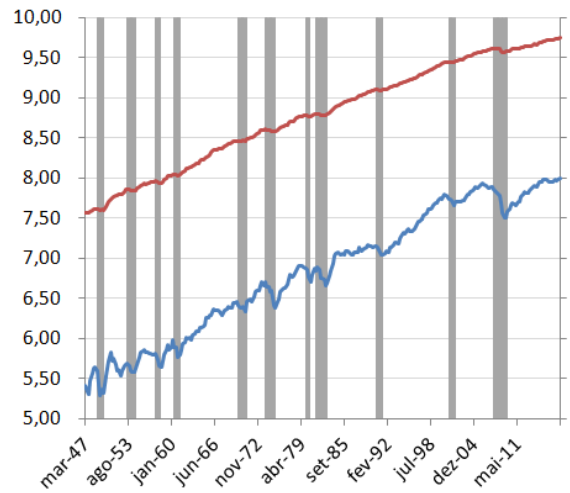



*Notes*: Output series shown in logs, undetrended, centered at 2007Q4. Linear trend computed over the period 1947Q1-2017Q3. Shaded areas correspond to NBER recessions.

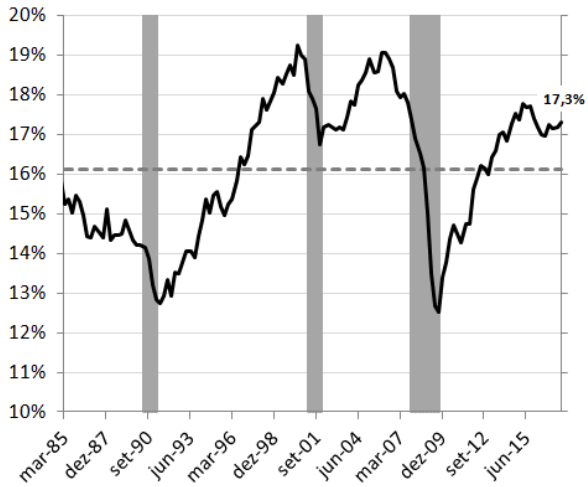
Figure 6 – Evolution of US real GDP over 1947-2017

**Investment**: Panel (a) of Figure 7 shows that investment (blue line) also seems to follow a linear trend, but is more volatile than output (red line). Specifically, for purposes of the model, we focus on investment/output ratio data. Panel (b) shows that, from 1985Q1 and 2017Q4, investment/output ratio varied between 12% and 19% approximately. The average value in this period was 16.1%. This will be one target of the calibration as we will see later.

(a) log investment (blue) and log output (red)




(b) investment/output ratio

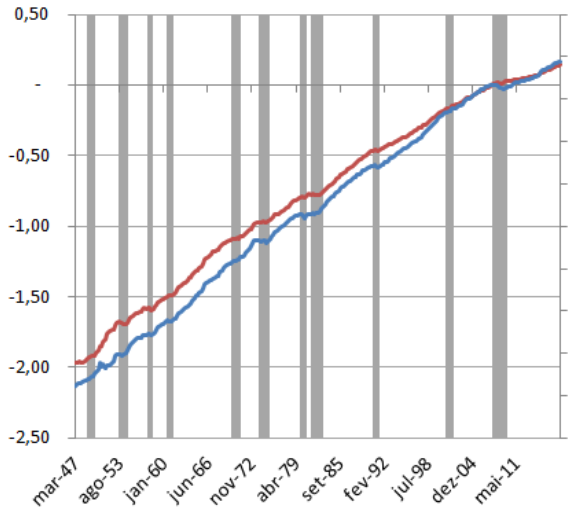



*Notes*: Output and investment series shown in logs, undetrended, centered at 2007Q4. Investment/Output ratio is computed from 1985Q1-2017Q3. Shaded areas correspond to NBER recessions.

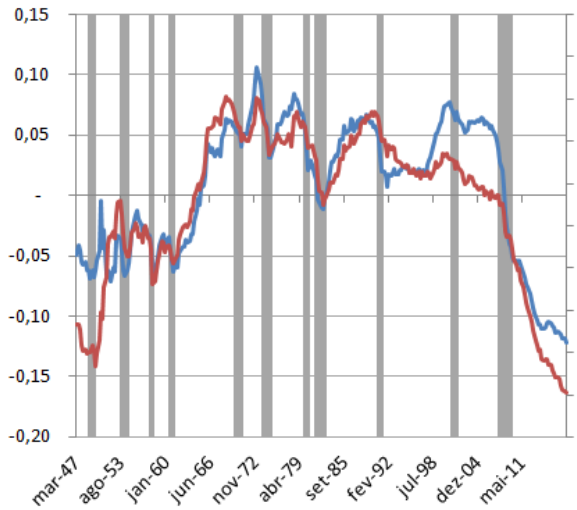
Figure 7 – Evolution of US real investment over 1947-2017 and Investment/Output ratio

**Consumption**: Panel (a) of Figure 8 compares two measures of consumption: real personal consumption expenditures (blue line) and the difference between real output and real investment (red line). The latter is shown since we model consumption in this way. It seems that both measures of consumption have similar patterns: after detrending by their respective linear trends, both suffered during the Great Recession and continued to fall.

(a) $\log C$ (blue) and $\log(Y - I)$ (red)




(b) detrended: $\log C$ (blue) and $\log(Y - I)$ (red)




*Notes*: Consumption series shown in logs, centered at 2007Q4. Shaded areas correspond to NBER recessions.

Figure 8 – Evolution of US real personal consumption expenditures over 1947-2017

# 4 Calibration

## 4.1 Productivity parameters ($\nu$, $\sigma$ and $\mu$)

### 4.1.1 $\nu$ and $\sigma$ calibration

To calibrate $\nu$ and $\sigma$, we approximate an Ornstein–Uhlenbeck process to an AR(1) process accordingly to match the data. In order to do that, we (i) regress TFP data to get AR(1) coefficient; (ii) simulate an Ornstein–Uhlenbeck process to calibrate parameters; (iii) make the correspondence of continuous time to discrete time in periods of quarters and (iv) regress simulated data to verify if parameters are calibrated.

**(i) AR(1) Regression**

Table 2 shows estimated AR(1) coefficient. It turns out that the coefficient is 0.9871. Moreover, we have that standard deviation of TFP data is 0.055. These two informations are set as targets to calibrate the parameters $\nu$ and $\sigma$.

| Variable | Coefficient | Std. Error |
|---|---|---|
| c | -0.0001 | 0.0005 |
| a(-1) | 0.9871 | 0.0089 |

Table 2 – AR(1) regression

**(ii) Simulation**

We simulate an Ornstein–Uhlenbeck process for a long period: $T = 3000$ years, using $dt = 0.01$. To match the targets we set $\nu = 0.09$ and $\sigma = 0.0235$.

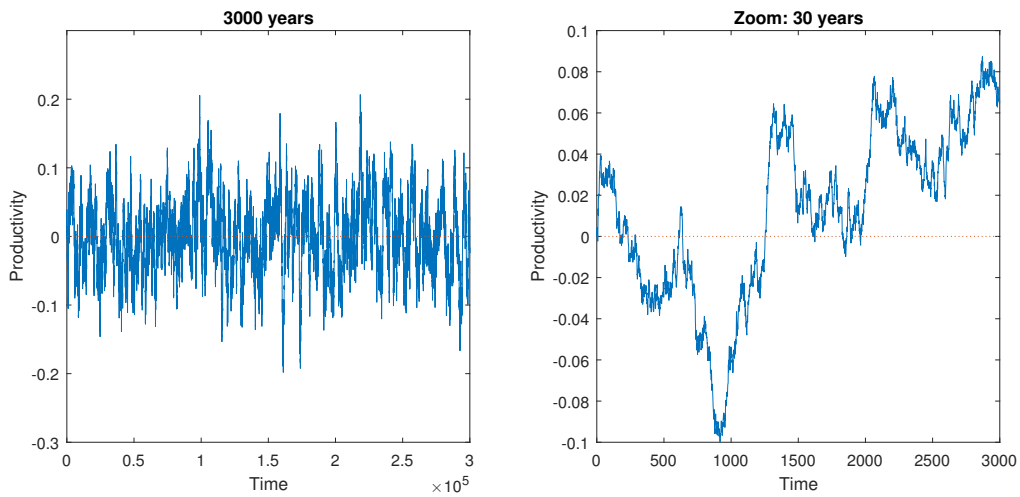



Figure 9 – Ornstein–Uhlenbeck process in continuous time with $\nu = 0.09$ and $\sigma = 0.0235$

### (iii) Time period correspondence

Since the Ornstein–Uhlenbeck process is continuous and AR(1) process is discrete, a correspondence of continuous time to discrete time in quarters is needed. There are two ways to do that: (1) match the final value of each quarter in the process, or (2) get the average value of each period that matches a quarter.

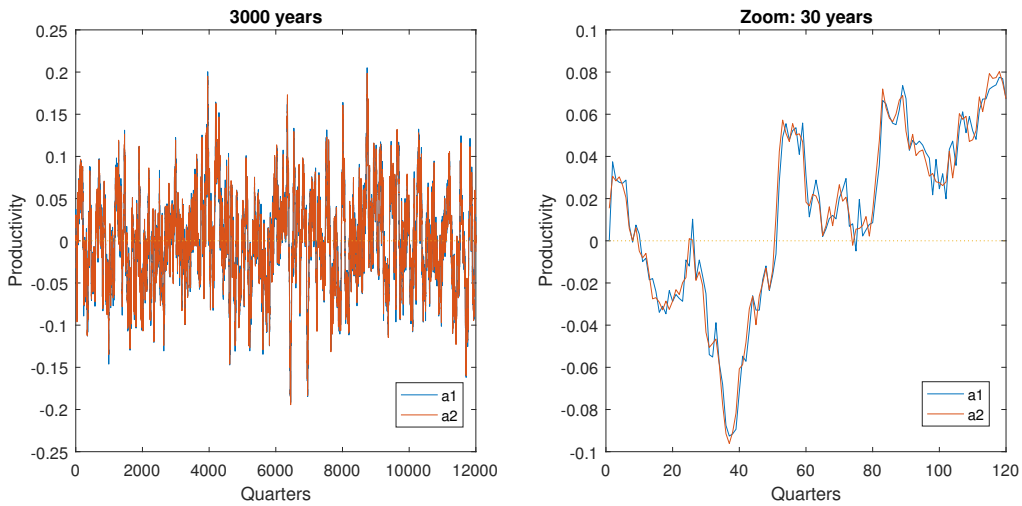



Figure 10 – Ornstein–Uhlenbeck process in quarters: final value (a1) and average value (a2)

Since the difference is small and it does not change the results at all, we continue the analysis with the second method which uses the average value (a2).

**(iv) Verify calibrated parameters**

To conclude the calibration, we regress an AR(1) with the simulated data (a2) and verify if targets are matched. [1]

For these calibrated parameters, if we consider a cycle every time productivity process reverts to its mean from the bottom (it could be from the top), we have an average cycle length of 45 quarters (approximately 11 years) and median cycle length of 25 quarters (approximately 6 years), which is consistent with our data (average cycle length of 45,8 quarters).[2]

### 4.1.2 $\mu$ calibration

Since $a_t$ affects both $y_t$ and $\alpha_t^L$, the productivity mean parameter $\mu$ is essential when computing investment/output ratio[3]. Thus, we calibrate $\mu$ to match data, which is the ratio of 16.1%. Unfortunately, due to the simplifications of the model, this number is never reached. The maximum value is 0.84% under the parameters that we show in the next section.

| | $\mu = 0$ | $\mu = 0.9$ | $\mu = 1$ | $\mu = 1.1$ |
|---|---|---|---|---|
| I/Y | 0.81% | 0.83% | 0.84% | 0.82% |

Table 3 – $\mu$ calibration: sensibility analysis

## 4.2 Other parameters

| Parameter | Value | Source/Target |
|---|---|---|
| Time period | one year | |
| Production at L | $x_L = 1$ | 2009Q3 Data - trough of recession (Normalization) |
| Production at H | $x_H = 1.1$ | 2007Q3 Data - peak before recession |
| Cost parameters | $\kappa_1 = 2$ | Machado (2016) |
| | $\kappa_2 = 1.5$ | Machado (2016) |
| Elasticity of substitution | $\theta = 3$ | Schaal e Taschereau-Dumouchel (2015) |
| Discount factor | $\rho = 0.05$ | 0.95 annual |
| Depreciation | $\alpha^H = 0.1$ | 10% annual |
| Mean $a_t$ | $\mu = 1$ | as close as possible to 16.1% Investment/Output |
| Standard Deviation $a_t$ | $\sigma = 0.0235$ | Standard deviation of 0.055 in TFP quarterly data |
| Mean reversion $a_t$ | $\nu = 0.09$ | Persistence of 0.987 in TFP quarterly data |

Table 4 – Parameters

[1] Notice that $\mu = 0$. However it doesn't change our estimation since we include a constant term.
[2] cycles with less than 8 quarters were not considered.
[3] Notice that aggregate investment is $I = (1 - h)c(\alpha^L)$.

The baseline scenario is shown in Table 4. Production at regime *Low* $x_L$ is normalized to one and it represents the trough of recession in 2009. On the other hand, production at regime *High* $x_H$ is defined as the output level produced using the maximum capacity, i.e., $h = 1$. Output in the third quarter of 2007 was 1.057, the peak of economy before recession. We decided to set $x_H = 1.1$. The remaining parameters are calibrated according to the literature. There is little debate about discount factor ($\rho$) and depreciation ($\alpha^H$). Nevertheless, for elasticity of substitution ($\theta$) and cost parameters ($\kappa_1$ and $\kappa_2$) we give a further discussion.

**Elasticity of substitution $\theta$:** The literature uses a wide range of values for the elasticity of substitution $\theta$. We use $\theta = 3$, according to Hsieh e Klenow (2014) and Broda e Weinstein (2006). The first study uses the same value to study the life cycle of plants in India and Mexico and the second estimates elasticities of substitution among goods at various levels of aggregation and finds that the median estimate is consistent with an elasticity of 3. In DSGE literature, this value is also common. For instance, Christiano, Eichenbaum e Trabandt (2015) estimate a New-Keynesian model with financial friction with an estimated elasticity of 3.78. Other papers closely related to ours that also use $\theta = 3$ are Schaal e Taschereau-Dumouchel (2015) and Machado (2016).

**Cost function parameters ($\kappa_1$ and $\kappa_2$):** cost to switch to regime *High* in our model capture a similar idea to capital adjustment cost. Literature usually uses a quadratic function, which is increasing and convex. It suggests that $\kappa_1 = 2$.

For $\kappa_2$, there is a wider range of possible values. Machado (2016) assume quadratic adjustment costs on net investment ($g$) with depreciation ($\delta$): $\iota(g) = (g+\delta)+\frac{\kappa}{2}g^2$ and use $\kappa = 3$. Smets e Wouters (2007) estimates $\kappa$ using a bayesian approach with prior equals 4 in a same fashion equation. They found an estimated value of 5.48 in their baseline scenario. Since there is no consensus about this parameter, we chose to set $\kappa_2 = 1.5$ which is equivalent to $\kappa = 3$ of Machado (2016).

## 4.3 Solution Strategy - Discrete Time Model

In order to solve the model numerically we work with an approximation of the model presented in section 2, where we make time discrete and each period has length $dt \approx 0$: time $t$ equals 0, $dt$, $2dt$, $3dt$ and so on.

### Algorithm to compute the equilibrium

1. Set values to parameters: $x_H$, $\theta$, $\rho$, $\alpha^H$, $\kappa_1$, $\kappa_2$, $\mu$, $\sigma$ and $\nu$

| **Parameter** | |
|---|---|
| $x_H$ | Production at H |
| $\theta$ | Elasticity of substitution |
| $\rho$ | Discount factor |
| $\alpha^H$ | Depreciation |
| $c = \kappa_2(\alpha^L)^{\kappa_1}$ | Cost |
| $\kappa_1$, $\kappa_2$ | Productivity |
| $\mu$ | Mean |
| $\sigma$ | Standard Deviation |
| $\nu$ | Mean reversion |

Table 5 – Parameters

2. Set values to grids:

| **Grids** | |
|---|---|
| $\underline{a}$ | Lower bound for $a$ |
| $\bar{a}$ | Upper bound for $a$ |
| $na$ | Number of grid points for $a$ |
| $\underline{h}$ | Lower bound for $h$ |
| $\bar{h}$ | Upper bound for $h$ |
| $nh$ | Number of grid points for $h$ |
| $\underline{\alpha}^L$ | Lower bound for $\alpha^L$ |
| $\bar{\alpha}^L$ | Upper bound for $\alpha^L$ |
| $nn$ | Number of simulations[4] |

Table 6 – Grids

3. Fix a tiny value for $dt$ and a big value for $T$ ($T$ were suppose to be infinity): time is discrete and each period has length $dt$

$$t \in \{dt, 2dt, \ldots, Tdt\}$$

4. Guess a initial strategy, i.e., values of $\alpha^L$ for each point of the grid $nh \times na$: we set values of $\underline{\alpha}^L$ to values of $\underline{a}$ and values of $\bar{\alpha}^L$ to values of $\bar{a}$ in the grid. For the rest, we make a linear combination of $\underline{\alpha}^L$ and $\bar{\alpha}^L$.

5. Loop: for each point in the grid $nh \times na$:

   - For $t \in [1, T/dt]$:

- Simulate a stochastic process for $a_t$:

$$a_t = a_{t-1} + \nu(\mu - a_{t-1})dt + \sigma\sqrt{dt}\epsilon_t$$

  where $\epsilon_t$ are iid normal shocks with zero mean and unit variance.
- $h_t$ follows the law of motion:

$$h_t = h_{t-1} + (1 - h_{t-1})\alpha^L_{t-1}dt + h_{t-1}\alpha^H dt$$

- Given paths of $a$ and $h$ and strategy $\alpha^L(a, h)$: Compute the path for $\alpha^L_t$ Interpolate for values of $(a, h)$ between the points in the grid.

- Compute $\Delta V$:
  - Discount factor matrix:

$$\begin{bmatrix} \exp(-[\rho + \alpha^H + \alpha^L_1]1) \\ \exp(-[\rho + \alpha^H + \alpha^L_2]2) \\ \vdots \\ \exp(-[\rho + \alpha^H + \alpha^L_{T/dt}]T/dt) \end{bmatrix}$$

  - Payoff matrix:

$$\begin{bmatrix} e^{a_1}g(h_1) + c(\alpha^L_1)1 \\ [e^{a_2}g(h_2) + c(\alpha^L_2])]2 \\ \vdots \\ [e^{a_{Tdt}}g(h_{Tdt}) + c(\alpha^L_{T/dt})]T/dt \end{bmatrix}$$

  where $g(h_t) = \left(h_t x_H^{\frac{\theta-1}{\theta}} + (1 - h_t)x_L^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}}\left(x_H^{\frac{\theta-1}{\theta}} - x_L^{\frac{\theta-1}{\theta}}\right) + c(\alpha^L_t)$
- Simulate this process $nn$ times
- Compute the expected value ($\mathbb{E}\Delta V$): take the mean of $\Delta V$ matrix
- Set the new strategy profile:
  - If $\mathbb{E}\Delta V < c'(\underline{\alpha}^L)$:

$$\alpha^L = \underline{\alpha}^L$$

  - If $\mathbb{E}\Delta V > c'(\bar{\alpha}^L)$:

$$\alpha^L = \bar{\alpha}^L$$

  - If $\mathbb{E}\Delta V \in [c'(\underline{\alpha}^L), c'(\bar{\alpha}^L)]$:

$$c'(\alpha^L) = \mathbb{E}\Delta V$$

6. If new strategy profile is very close to the previous one (difference in each point in the grid is almost zero), stop. Otherwise, update strategy profile and go back to step 5

# 5 Results

Our simulation of $T = 1000$ years have the following results[1]:

First, in order to understand the results, it is important to have in mind the timing of facts, noticing that we made an approximation of the model, by making time discrete to solve it numerically. In each period $dt$, (i) nature chooses measure $(1-h)\alpha^L$ of agents to switch to regime $High$ and measure $h\alpha^H$ to switch to regime $Low$; (ii) nature also draws the new productivity level $a_t$ from the stochastic process; (iii) investment decisions $(\alpha^L_{t+1})$ are made according to the state of economy $(a_t, h_t)$, specifically, investment is increasing in both arguments; (iv) then, production decisions take place and all markets clear.

Figure 12 compares shocks in productivity $(a_t)$ to shifts of output in logs $(\log y)$. There are small but persistent effects of shocks in $a_t$ on $\log y$. The persistence occurs because shocks in productivity affects not only directly the output, but also indirectly though variable $h$. For example, consider a negative shock of $a_t$. Agents will take into account this drop in productivity in their investment decisions ($\alpha^L_{t+1}$ falls) which thereby decrease the measure of agents in regime $High$ in the next period, $h_{t+1}$. This last effect makes output to drop further. Importantly, if this fall in $h$ is not offset by a positive shock in productivity, the persistent effect of the first negative shock continues though the following periods, making economy to drop even more. Therefore productivity shocks impact more than one-by-one output.

Moreover, notice that $h_t$ is less volatile than $a_t$ and follows productivity movements with lag. This lag effect is due to the fact agents react to productivity shocks after a period. More importantly, this reaction comes from agents' investment decision and this channel shuts down some volatility. It is the continuation of the process described above. A negative shock in $a_t$ makes $\alpha^L_{t+1}$ to fall, that by its turn, decreases $h_{t+1}$. In the next period, when a new shock hits productivity, say a positive one, agents, when choosing investment, have to ponder two things that are moving in different direction: $a_{t+1}$ that is increasing and $h_{t+1}$ that is falling. Thereafter, this investment decision will affect $h_{t+2}$. We might say the high frequency present in productivity is turned down by the momentum that $h_t$ has, so it is less volatile.

[1] Algorithm to simulate the economy is detailed in the appendix.

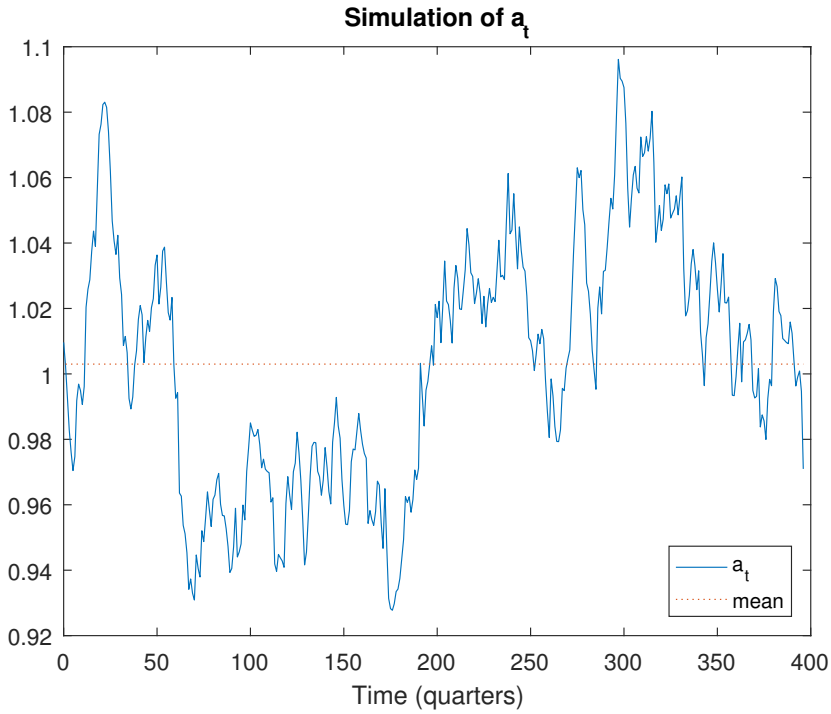



Figure 11 – Path for $a_t$: window of 100 years

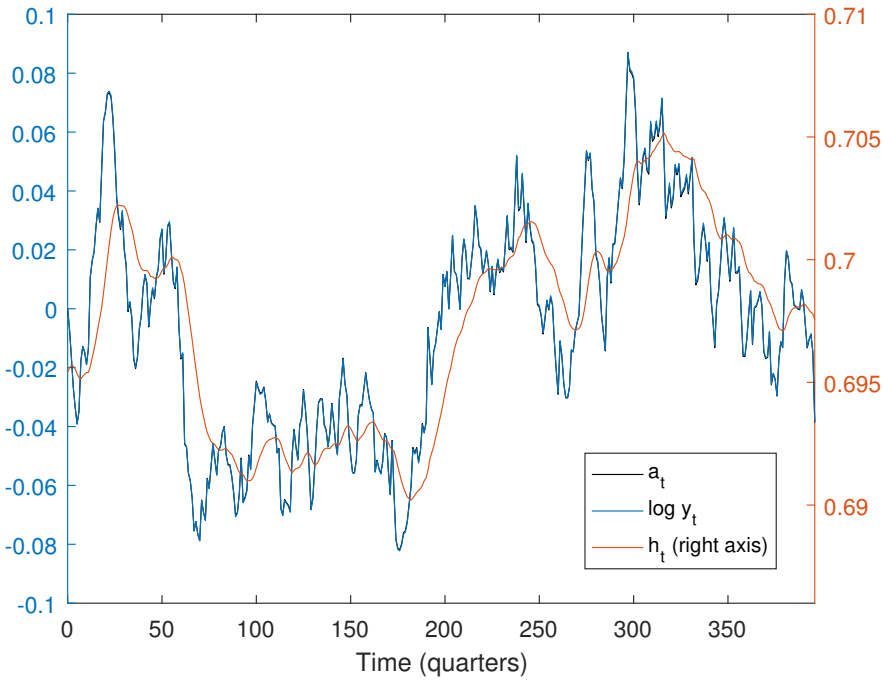



Figure 12 – Output in log ($logy$) follows the productivity $a_t$ movements. Series are decreased by their first value

However, coordination failures have small effects. It happens because $h_t$ moves only in a short range (from 0.6757 to 0.7211), which can be explained by two factors: (i) mean reversion of $a_t$ and (ii) an almost flat curve locus $\dot{h} = 0$ that is given by an upward curve between $h_t$ and $a_t$.

Figure 13 shows this curve locus $\dot{h} = 0$ with simulation path. The intuition is that, although a productivity shock has persistent effect, it fades away once state of economy reaches the locus $\dot{h} = 0$. For example, take a point beginning in this locus. When a negative shock hit $a_t$, economy moves to the left. By the process already explained above, $h_t$ goes down and continues to fall until it reaches the locus again. Once it happens, the effect of shock in $a_t$ ceases and $h_t$ stops to fall.

For our calibrated parameter set, curve locus $\dot{h} = 0$ is almost flat and we have that $a_t$ movements are limited because of mean reversion, i.e., the probability for values distant from the mean are very remote. Therefore, a locus $\dot{h} = 0$ almost flat and mean reversion of $a_t$ causes movements in $h_t$ to be limited as well.

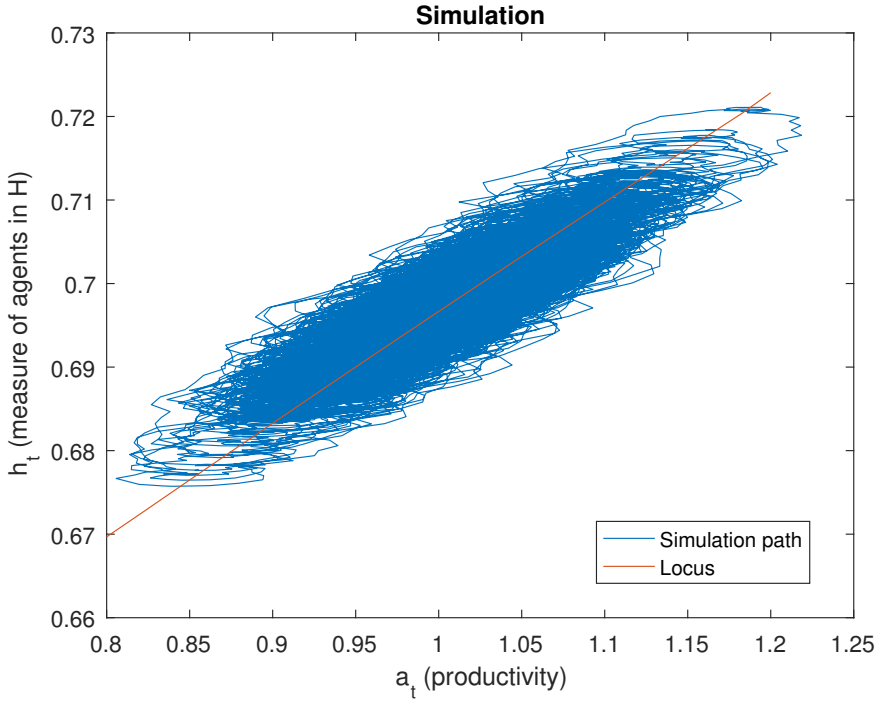



Figure 13 – Simulation path

## 5.1 Half life - persistence of shocks

In this section, we compare simulation of $a_t$ and $h_t$ as if they were an AR(1) process. Thus, we regress both simulations and get the estimated AR(1) coefficients and calculate de half life of $a_t$ and $h_t$ to have a notion of persistence.

| Parameter | Ornstein–Uhlenbeck | AR(1) |
|---|---|---|
| Mean $a_t$ | $\mu_a = 1$ | |
| Standard Deviation $a_t$ | $\sigma_a = 0.0235$ | Standard deviation of 0.055 |
| Mean reversion $a_t$ | $\nu_a = 0.09$ | $\rho_a = 0.987$ |
| Mean $h_t$ | $\mu_h = 0.70$ | |
| Standard Deviation $h_t$ | $\sigma_h = 0.0007$ | Standard deviation of 0.0062 |
| Mean reversion $h_t$ | $\nu_h = 0.003$ | $\rho_h = 0.9989$ |

Table 7 – Estimated parameters for quarterly data

Half life of $a$

$$n_a = \frac{\log 1/2}{\log \rho_a} = 53,4\text{quarters}$$

Half life of $h$

$$n_a = \frac{\log 1/2}{\log \rho_h} = 629,8\text{quarters}$$

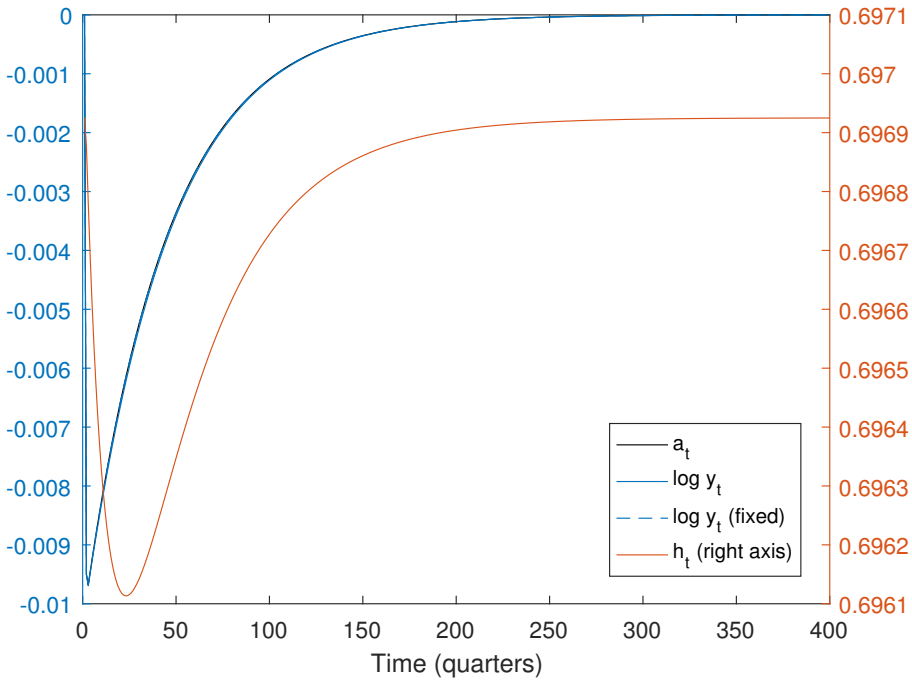
We found that a half life of $h_t$ is approximately twelve times bigger than a half life of $a_t$.

## 5.2 Impulse response functions

In order to quantify these effects, we propose an exercise with an impulse response of output to a 1% productivity shock in two cases: (i) with $h_t$ reacting shock in $a_t$ and (ii) with $h_t$ fixed, i.e., we fix $h_t$ to its "steady state" value which is when it is in the locus $\dot{h} = 0$ and $a_t = \mu$. The difference between the two cases is the coordination failure effect.

It seems that, for the US economy, coordination failures have a small instantaneous effect, approximately 0.008%. However, since the effect is persistent, if we take into account the duration of it, the aggregate effect of 1% drop in $a_t$ is $-0.55\%$ (Panel (b) of Figure 15), a modest but significant effect.

Figure 14 – Impulse response




(a) Difference between $\log y_t$ and $\log y_t$ (fixed) (b) Cumulative effect: impact and duration

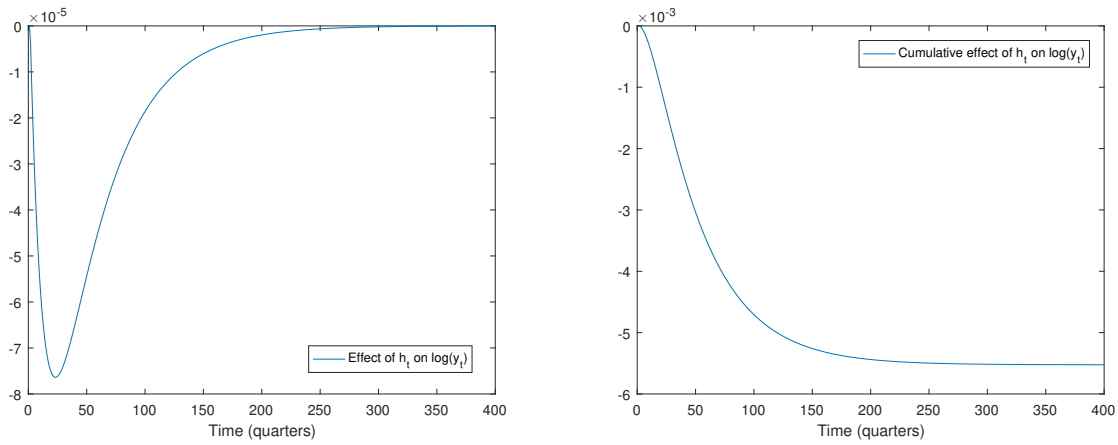



Figure 15 – Effect of 1% drop of $a_t$ on $\log y_t$

## 5.3 Sensitivity analysis

There are some parameters that alter the way $h_t$ behaves, in particular, parameters $x_H$ and $\kappa_1$. The former measures the incentives to produce in regime $High$ and the latter measures the way cost affects incentives to switch to regime $High$. Therefore changing the value of these parameters might increase the effect of coordination failures in the model. We select two alternative case: (a) with $x_H = 1.5$ and (b) with $\kappa_1 = 1.3$ (baseline scenario: $x_h = 1.1$ and $\kappa_1 = 2$)

For alternative case (a), the effect of coordination failures increased to 0.018% due to the fact $h_t$ now moves in a wider range: from 0.8569 to 0.8819. The aggregate effect raised to 1.1%

For alternative case (b), effect also increased, but to a higher value 0.026%. It has to do with the fact $h_t$ now moves from 0.4593 to 0.6553. Moreover, aggregate effect increased by two reasons: effect is higher and it also last longer. Combining both, aggregate effect is about 2.2%.

(a) Alternative case $x_H = 1.5$ (b) Alternative case $\kappa_1 = 1.3$

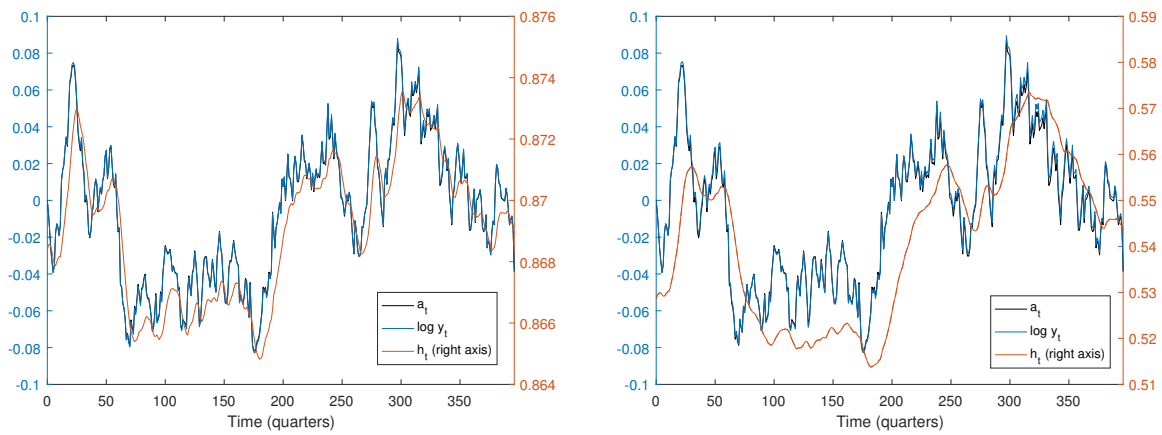



Figure 16 – Effect is still small: however both parameters increase impact

(a) Alternative scenario $x_H = 1.5$ (b) Alternative scenario $\kappa_1 = 1.3$

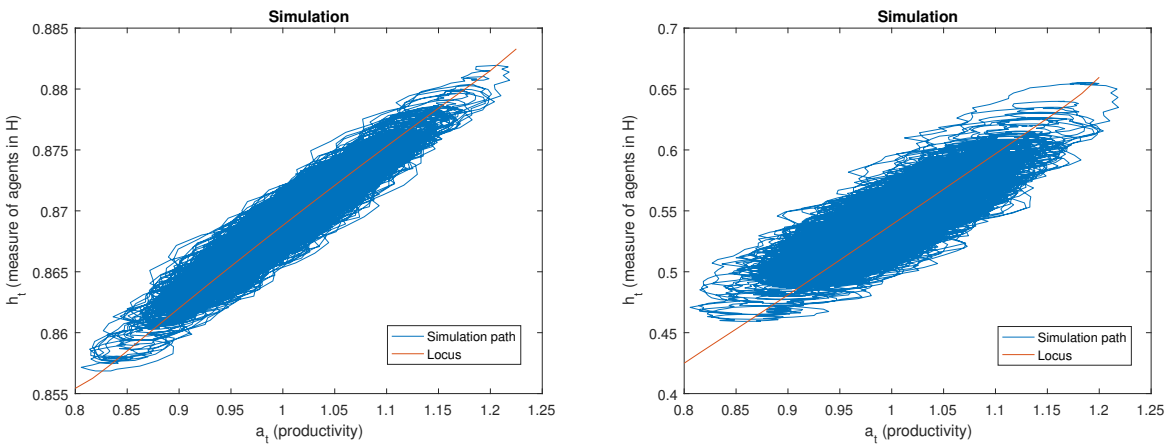
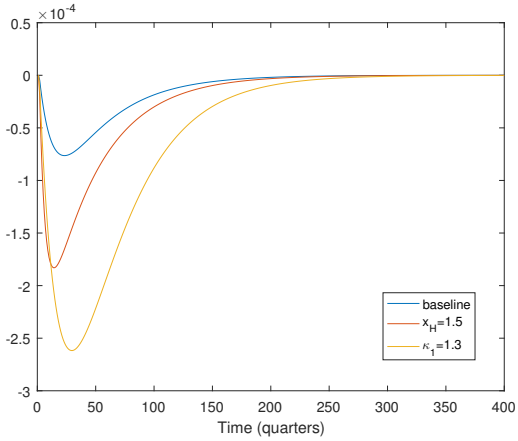



Figure 17 – Simulation path for alternative scenarios: in both, $h_t$ moves in a wider range

(a) Difference between $\log y_t$ and $\log y_t$ (fixed)




(b) Cumulative effect: impact and duration

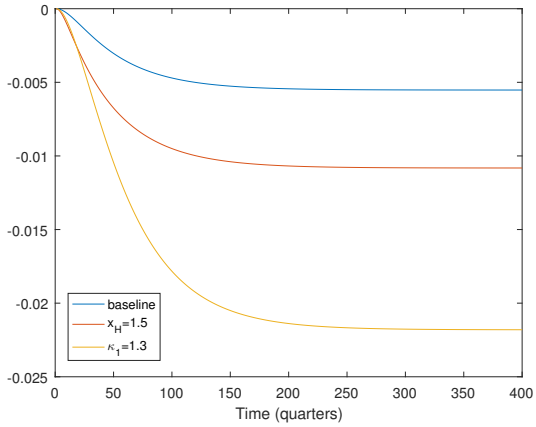



Figure 18 – Effect of 1% drop of $a_t$ on $\log y_t$

# 6 Conclusion

Based on Guimaraes e Machado (2017), we use a macroeconomic model with endogenous hazard rates that captures in a simple way the dynamic coordination problem to make a quantitative analysis of it.

With this framework, shocks have persistent effects. The persistence occurs because shocks in productivity affects not only directly the output, but also indirectly though variable $h$ (demand channel). Therefore, shocks on productivity impact more than one-by-one output. For our simulated economy, which mimics the US economy, we found that a half life of $h$ is approximately twelve times bigger than a half life of $a$.

Despite the persistence, our calibration also suggests that shocks have small effects on economy. The magnitude is approximately 0.01%. However when we consider impact and duration, coordination failures have an effect of $-0.55\%$, which is modest but significant.

Thus the paper offers a quantitative study of coordination failures that rely on demand externalities and could be useful as a first step to macroeconomists interested in modeling it, which is an alternative to traditional DSGE models.

Further extensions: our model is quite stylized since firms cannot adjust their production in the intensive margin when economic conditions change. It would be interesting to extend the model to include labor and capital decision in firms' problem, which might contribute to match investment as proportion to output that our model couldn't achieve.

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# Appendix

# APPENDIX A – Equilibrium

We used this set of grids to compute the equilibrium:

| **Grids** | |
|---|---|
| $\underline{a} = 0.8$ | Lower bound for $a$ |
| $\bar{a} = 1.2$ | Upper bound for $a$ |
| $na = 41$ | Number of grid points for $a$ |
| $\underline{h} = 1$ | Lower bound for $h$ |
| $\bar{h} = 0$ | Upper bound for $h$ |
| $nh = 5$ | Number of grid points for $h$ |
| $\underline{\alpha}^L = 0.01$ | Lower bound for $\alpha^L$ |
| $\bar{\alpha}^L = 1$ | Upper bound for $\alpha^L$ |
| $nn = 100$ | Number of simulation[1] |

Table 8 – Grids

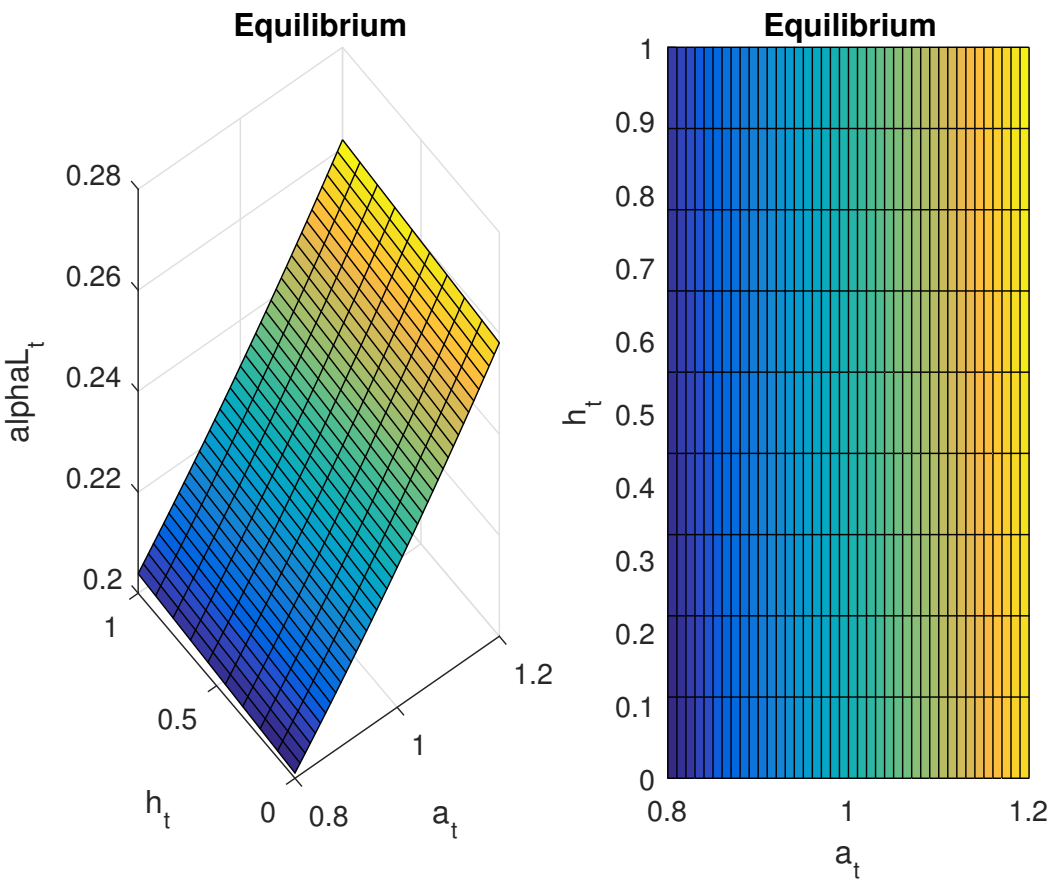


Figure 19 – Equilibrium: higher $a$ and $h$ increases $\alpha^L$

# APPENDIX B – Algorithms

## B.1 Algorithm to simulate the economy

1. Load data file with the equilibrium previously calculated.
2. Set number of years ($T$) desired to simulate and the burn in period (number of year that we drop in order to initial conditions do not affect our results).
3. Simulate economy.
   - Set initial conditions: $a_1$, $h_1$ and $\alpha_1^L$.
   - For $t \in [2, T/dt]$:
     - Simulate a stochastic process for $a_t$:

       $$a_t = a_{t-1} + \nu(\mu - a_{t-1})dt + \sigma\sqrt{dt}\epsilon_t$$

       where $\epsilon_t$ are iid normal shocks with zero mean and unit variance.
     - $h_t$ follows the law of motion:

       $$h_{t+1} = h_t + \alpha_t^L(1 - h_t)dt - \alpha^H h_t dt$$

     - Given paths of $a$ and $h$: Compute the path for $\alpha^L$ using the optimal values calculated in equilibrium before.
       When between the points in the grid, we interpolate.
4. Compute main variables output ($Y$), investment ($I$) and consumption ($C$).
5. Drop the burn in period and make the correspondence of continuous time to discrete time in quarters.
6. Compute statistics: 1st and 2nd moments.
7. Compute graphs.

## B.2 Algorithm to compute the locus $\dot{h} = 0$

1. Set values to parameters: $x_H$, $\theta$, $\rho$, $\alpha^H$, $\kappa_1$, $\kappa_2$, $\mu$, $\sigma$ and $\nu$
2. Set values to grids:

| **Parameter** | |
|---|---|
| $x_H$ | Production at H |
| $\theta$ | Elasticity of substitution |
| $\rho$ | Discount factor |
| $\alpha^H$ | Depreciation |
| $c = \kappa_2(\alpha^L)^{\kappa_1}$ | Cost |
| $\kappa_1$, $\kappa_2$ | |
| | Productivity |
| $\mu$ | Mean |
| $\sigma$ | Standard Deviation |
| $\nu$ | Mean reversion |

Table 9 – Parameters

| **Grids** | |
|---|---|
| $\underline{a}$ | Lower bound for $a$ |
| $\bar{a}$ | Upper bound for $a$ |
| $na$ | Number of grid points for $a$ |
| $\underline{\alpha}^L$ | Lower bound for $\alpha^L$ |
| $\bar{\alpha}^L$ | Upper bound for $\alpha^L$ |
| $nn$ | Number of simulation[1] |

Table 10 – Grids

3. Fix a tiny value for $dt$ and a big value for $T$ ($T$ were suppose to be infinity): time is discrete and each period has length $dt$, where $dt$ is a small number: $t \in \{dt, 2dt, \ldots, Tdt\}$

4. Guess a initial strategy, i.e., values of $\alpha^L$ for each point of the grid $na$:
we set values of $\underline{\alpha}^L$ to values of $\underline{a}$ and values of $\bar{\alpha}^L$ to values of $\bar{a}$ in the grid. For the rest, we make a linear combination of $\underline{\alpha}^L$ and $\bar{\alpha}^L$.

5. Loop: for each point in the grid $na$:

   - For $t \in [1, T/dt]$:
     - Simulate a stochastic process for $a_t$:

       $$a_t = a_{t-1} + \nu(\mu - a_{t-1})dt + \sigma\sqrt{dt}\epsilon_t$$

       where $\epsilon_t$ are iid normal shocks with zero mean and unit variance.
     - $h_t$ follows the law of motion:

       $$h_{t+1} = h_t + \alpha_t^L(1 - h_t)dt - \alpha^H h_t dt$$

       Then, to find locus $\dot{h} = 0$

       $$h_t = \frac{\alpha_t^L}{\alpha_t^L + \alpha^H}$$

       which makes h a function of $\alpha^L$

  - Given path of $a$ and strategy $\alpha^L(a)$: Compute the path for $\alpha_t^L$ and $h_t$ When between the points in the grid, we interpolate.

- Compute $\Delta V$:
  - Discount factor matrix:

$$\begin{bmatrix} \exp(-[\rho + \alpha^H + \alpha_1^L]1) \\ \exp(-[\rho + \alpha^H + \alpha_2^L]2) \\ \vdots \\ \exp(-[\rho + \alpha^H + \alpha_{T/dt}^L]T/dt) \end{bmatrix}$$

  - Payoff matrix:

$$\begin{bmatrix} e^{a_{dt}} g(h_{dt}) + c(\alpha_1^L)1 \\ [e^{a_{2dt}} g(h_{2dt}) + c(\alpha_2^L])]2 \\ \vdots \\ [e^{a_{Tdt}} g(h_{Tdt}) + c(\alpha_{T/dt}^L)]T/dt \end{bmatrix}$$

    where $g(h_t) = \left(h_t x_H^{\frac{\theta-1}{\theta}} + (1-h_t) x_L^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} \left(x_H^{\frac{\theta-1}{\theta}} - x_L^{\frac{\theta-1}{\theta}}\right) + c(\alpha_t^L)$ and $h = \frac{\alpha_t^L}{\alpha_t^L + \alpha^H}$

- Simulate this process $nn$ times
- Compute the expected value ($\mathbb{E}\Delta V$): take the mean of $\Delta V$ matrix
- Set the new strategy profile:
  - If $\mathbb{E}\Delta V < c'(\underline{\alpha}^L)$:

$$\alpha^L = \underline{\alpha}^L$$

  - If $\mathbb{E}\Delta V > c'(\bar{\alpha}^L)$:

$$\alpha^L = \bar{\alpha}^L$$

  - If $\mathbb{E}\Delta V \in [c'(\underline{\alpha}^L), c'(\bar{\alpha}^L)]$:

$$c'(\alpha^L) = \mathbb{E}\Delta V$$

6. If new strategy profile is very close to the previous one (error is almost zero), stop. Otherwise, update strategy profile and go back to step 5

# APPENDIX C – Business cycles moments

We also computed some business cycle moments from simulated time series and compare them to their empirical counterparts. The results are shown in Table 8: mean value for $a$, $Y$, $I$, $C$, $\alpha^L$, $h$ and their standard deviation.

We can see that mean value for $\alpha^L$ is 0.23. In the interpretation given before, it is as if the agents invested in R&D research and useful ideas come at every four years approximately. Moreover, average aggregate investment is 0.0241, which gives us a investment/output ratio of 0.84%. This number is really small compared to reality (16.1% in data).

Finally, when we compare to data, model poorly capture the second moments in data due to the fact we simplified it. In particular, investment volatility is very low, which causes other second moments to not match with data.

| Baseline | Productivity | Output | Investment | Consumption | $\alpha^L$ | $h$ |
|---|---|---|---|---|---|---|
| Panel A: Mean | | | | | | |
| Model | 1.0029 | 2.919 | 0.0241 | 2.8956 | 0.2305 | 0.6973 |
| Panel B: Std. deviations (%) | | | | | | |
| Model | 5.39 | 15.93 | 0.12 | 15.81 | 0.78 | 0.63 |
| Data | 5.5 | 5.11 | 13.32 | 5.67 | | |

Table 11 – Dynamic properties of the data