Seller-optimal learning and monopsony pricing
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Abstract

This paper studies incentives for information gathering in a monopsonist pricing setting. Our motivation stems from public procurement contracts where the government is the single buyer, and the true cost of providing the good is \textit{ex ante} uncertain to potential suppliers. We develop a simple bilateral, monopsonistic trade model, based on Roesler and Szentes (2017), where the seller only observes a signal about actual production cost. The model is intended to highlight how information about seller’s cost affects resource allocation, price, and buyer welfare. More specifically, for any given continuous cost distribution, we characterize the seller-optimal learning. Taking the uniform prior as our benchmark case, we illustrate that seller’s equilibrium strategy induces less information acquisition than would be desired by the buyer. We also show that efficient trade may not happen with probability one under some seller-optimal signal structures.

KEYWORDS: Monoposony pricing, Optimal learning, Information design.

JEL CODES: D11, D43, D61, D82, D83, L13.
1 Introduction

In many public procurement settings, the government is the single buyer, and the true cost of providing a good is *ex ante* uncertain to potential suppliers. This feature underscores the relevant role of information gathering on both trade efficiency and social welfare. In fact, *ceteris paribus*, a single supplier is better off with more information because it would allow him to forecast more precisely whether or not the transaction at a given price is profitable. On the other hand, with more information available, the government will face suppliers with a different (and perhaps lower) distribution of cost. Thus, the government is likely to respond by decreasing its posted price. As a consequence, new information sources – even when freely available – might worsen the seller’s outcome if being better informed results in lower prices and, hence, lower rents.

Under this rent-efficiency trade-off, a natural question arises: How does an information source affect resource allocation, prices and government welfare? Based on Roesler and Szentes (2017), our paper contributes to this problem by identifying information structures which are best for the seller in a monopsonistic market. This provides a lower bound for government’s payoff and casts light on the incentives that drive supplier’s information gathering in public procurement settings. Understanding seller’s optimal-learning strategy is a first step towards modelling real-world situations where government may affect (yet not completely control) the endogenous information structure.

We consider a stylized model where the buyer (he) has full bargaining power, and the seller (she) receives a costless signal about her true cost. The buyer sets a price knowing the joint distribution of the seller’s cost and the signal but not their realizations. We assume that the buyer values the object at least as much as the largest possible cost of production, such that efficiency requires trade with probability one. In equilibrium, the seller-optimal distribution generates a unit-elastic supply and the buyer is indifferent between charging any price on its support. Taking the uniform prior as our benchmark case, we illustrate that the seller-optimal strategy involves minimal learning and induces less information gathering than would be desired by the buyer. We also show that efficient trade may not happen with probability one under some seller-optimal signal structures.
Our results should be seen as partial conclusions in a broader research agenda on information design in public procurement. In that sense, some extensions to our basic model are discussed. The first one keeps our original allocation mechanism (i.e., posted price) but allows the buyer to more directly affect how information flows to the seller. In this case, the buyer can disclose more information if seller’s learning is not satisfactory. The second extension introduces competition between potential sellers. We intend to model this competition using a second-price auction with reserve price and two bidders. Those extensions would render our model closer to observed market institutions which often allow for more flexibility either on the timing of interactions (e.g., prior communication, repeated talkings) or on the allocation mechanism (e.g., bargaining, auctions).

Related Literature

The observation that, in a contracting environment, an imperfectly informed agent can be better off than a perfectly informed one is not new. Kessler (1998) considers a principal-agent model where, prior to contracting, an agent observes a payoff-relevant state with a certain probability. The principal’s contract depends on this probability. The author shows that the agent’s payoff is not maximized when this probability is one: that is, an agent might benefit from having less than perfect information. We not only confirm this result, but also characterize seller-optimal learning without imposing any constraints on possible signal structures.

Several papers examine profit-maximizing contracts for related settings in which an agent can acquire information before signing. The main question in this literature is whether optimal contracts induce information acquisition. In a seminal work, Crémer and Khalil (1992) demonstrate that if precontractual information would already remove all uncertainty, the principal will design the contract such that the agent accepts without acquiring information. Terstiege (2016) consider that same environment but with imperfect information gathering. The author assumes a particular learning technology based on first-order stochastic dominance of posterior distributions. He shows that the principal deters the acquisition if and only if the agent’s investigation costs exceed some cutoff.
Various papers analyse buyers’ incentives to acquire costly information about their valuations before participating in auctions. The buyers’ learning strategies depend on the selling mechanism announced by the seller. Persico (2000) shows that if the buyers’ signals are affiliated then they acquire more information in a first-price auction than in a second-price one. Compte and Jehiel (2008) show that dynamic auctions tend to generate higher revenue than simultaneous ones. Shi (2012) also analyses models where it is costly for the buyers to learn about their valuations and identifies the revenue-maximizing auction in private-value environments. In all of these setups, the seller is able to commit to a selling mechanism before the buyers decide how much information to acquire. In contrast, we characterize seller-optimal learning in environments where the monopsonist best-responds to the seller’s signal structure.

Our paper is also related to the literature on robust mechanism design. For an in-depth review see Bergemann and Morris (2013). Carrasco et al. (2016) study the revenue maximization problem of a seller who is partially informed about the distribution of buyer’s valuations, only knowing its first $N$ moments. Carrasco et al. (2017) analyses a similar problem in a nonlinear environment. Carroll (2013) provides an interesting interpretation for robust monopoly pricing, which arises from the seller’s uncertainty about the information acquisition technology available to the buyer. All these works are built on a zero-sum, simultaneous game between the principal and Nature, who chooses a feasible distribution to minimize expected revenue. Conversely, our paper is based on a sequential game where the agent moves first.

In a symmetric setting, Roesler and Szentes (2017) inspired our work. That paper presents a bilateral trade model where the buyer’s valuation for the object is uncertain and she observes only a signal about her valuation. The seller gives a take-it-or-leave-it offer to the buyer. The authors characterize those signal structures which maximize buyer’s expected payoff. They conclude that a buyer-optimal signal structure generates efficient trade. In our model, however, inefficient outcomes may arise since trade does not always occur with probability one, as we show by a numerical example.
The paper proceeds as follows. Section 2 presents the model. Section 3 states our general results, which allow us to completely characterize seller-optimal distributions for any given continuous prior. Section 4 provides some examples and comparative statics exercises. Section 5 summarizes our main findings and indicates possible research extensions. All proofs are left to the Appendix.

2 The Model

A monopsonist buyer intends to buy an object from a single seller. The buyer’s valuation for the object is $S \geq 1$. Seller’s cost $\theta$ to produce the object is distributed according to the continuous CDF $F$ supported on $[0, 1]$. Let $\mu$ denote the expected cost, that is, $\int_0^1 \theta dF(\theta) = \mu$. The seller observes a signal $s$ about $\theta$. The joint distribution of $\theta$ and $s$ is common knowledge. The buyer then gives a take-it-or-leave-it price offer to the seller, $p$. Finally, the seller trades if and only if his expected cost conditional on his signal does not exceed $p$. If trade occurs, the payoff of the buyer is $S - p$ and the payoff of the seller is $p - \theta$; otherwise, both have a payoff of zero. Both the buyer and the seller are von Neumann-Morgenstern expected payoff maximizers.

Since the seller’s trading decision only depends on $E\{\theta|s\}$, we may assume without loss of generality that each signal $s$ provides the seller with an unbiased estimate about her cost $\theta$, that is, $E\{\theta|s\} = s$. In what follows, we restrict attention to such signals and refer to them as unbiased signals.

3 Results

First, we argue that the payoffs of both the buyer and the seller are determined by the marginal distribution of the signal. To this end, let $D(G, p)$ denote the supply at price $p$ if the signal’s distribution is $G$, that is, $D(G, p)$ is the probability of trade at $p$. Note that $D(G, p) = G(p)$. The buyer’s optimal price, $p$, solves $\max_s (S - s)G(s)$ and the seller’s payoff is $\int_0^p (p - s)dG(s)$. Therefore, the problem of designing a seller-optimal signal structure can be reduced to identifying the marginal signal distribution which maximizes the seller’s expected payoff subject to monopsony pricing.
Of course, not every CDF corresponds to a signal distribution. In what follows, we characterize the set of distributions that do. The notion of mean-preserving spreads plays a critical role in that regard (See Rothschild and Stiglitz (1970)).

**Definition 1.** A random variable $Y$ is a mean-preserving spread of a random variable $X$ if $Y \overset{d}{=} X + Z$, where “$\overset{d}{=}”$ means “has the same distribution as” and $Z$ is a random variable such that $\mathbb{E}\{Z|X\} = 0$ for all $X$.

For each unbiased signal structure, $\theta$ can be expressed as $s + \epsilon$ for a random variable $\epsilon$ with $\mathbb{E}\{\epsilon|s\} = 0$. This means that $G$ is the distribution of some unbiased signal $s$ about $\theta$ if and only if $F$ is a mean-preserving spread of $G$. Intuitively, the actual cost is just the observed signal plus a noise. We say that the signal structure $G$ is more informative the more it spreads probability mass around the mean (i.e., the greater its variance is). Having thicker tails increases the likelihood of a unbiased signal $s$ being away from its mean $\mu$. Since $\mathbb{E}\{\theta|s\} = s$, a signal realization $\tilde{s}$ conveys more information the greater $|\mu - \tilde{s}|$ is.

Let $\mathcal{G}_F$ denote the set of CDFs of which $F$ is a mean-preserving spread. The following proposition yields a useful characterization of $\mathcal{G}_F$.

**Proposition 1.** Consider two random variables $A$ and $B$ with the same mean and distributions $H_A$ and $H_B$ respectively, both supported on intervals contained in $[0, \bar{x}]$, $\bar{x} \geq 0$. Then $A$ is a mean-preserving spread of $B$ if and only if

$$\int_0^x H_A(z)dz \geq \int_0^x H_B(z)dz \text{ for all } x \in \mathbb{R}_+$$

**Proof.** See Appendix. \qed

Using Proposition (1), we have that

$$\mathcal{G}_F = \left\{ G \in \Delta([0,1]) \mid \int_0^x F(\theta)d\theta \geq \int_0^x G(s)ds \text{ for all } x \in [0,1], \int_0^1 s \, dG(s) = \mu \right\}$$
The problem of designing a seller-optimal signal structure can be stated as follows:

\[
\max_{G(\cdot) \in \mathcal{G}_F} \int_0^P (p - s) dG(s)
\]

\[
s.t. \quad p \in \underset{s \geq 0}{\arg\max} (S - s)G(s)
\]

(1)

To solve Problem (1),\(^1\) we follow the approach developed by Roesler and Szentes (2017). We call a pair \((G, p)\) an outcome if \(G \in \mathcal{G}_F\) and \(p\) is the maximum value of the set \(\{\underset{s \geq 0}{\arg\max} (S - s)G(s)\}\).\(^2\) In other words, the pair \((G, p)\) is an outcome if there exists an unbiased signal \(s\) about \(\theta\) which is distributed according to the CDF \(G\) and it induces the buyer to set the highest optimal price \(p\). Next, we define a set of distributions and prove that a seller-optimal signal distribution lies in this set.

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1 A solution to Problem (1) – that is, a seller-optimal signal structure – always exists. Although this could be stated as an existence result at this point, it follows naturally from our characterization results. These results simplify our initial infinite-dimensional problem to a finite-dimensional one, where the Weierstrass theorem easily applies.

2 Since \(G\) is a distribution function, it is upper semicontinuous. Therefore, the solution set to the buyer’s problem for a given \(G\) – that is, the set \(\{\underset{s \geq 0}{\arg\max} (S - s)G(s)\}\) \(\subseteq [0, S]\) – is nonempty and closed. So \(p\) is well defined for any signal distribution \(G\).
Proposition 2. Let \((G, p)\) be an outcome and \(\pi := (S - p)G(p)\). Then there exists a unique \(q \in [0, p]\) such that

\[
G^*(s) = \begin{cases} 
G^\pi_{qp}(s) & \text{if } s < p \\
G(s) & \text{if } s \geq p
\end{cases}
\]

where \(G^\pi_{qp}(s) = \begin{cases} 
0 & \text{if } s < q \\
\frac{\pi}{S - s} & \text{if } q \leq s < p
\end{cases}\)

satisfies

(i) \(G\) is a mean-preserving spread of \(G^*\);

(ii) \((G^*, p)\) is an outcome;

(iii) \(\int_0^p (p - s)dG^*(s) = \int_0^p (p - s)dG(s)\).

Proof. See Appendix.

Proposition (2), parts (ii) and (iii), establishes that for each outcome, \((G, p)\), there is another outcome \((G^*, p)\) which makes the seller equally well off while generating the same profit to the buyer. Part (i) states that the signal distributed according to \(G^*\) is weakly less informative than the one distributed according to \(G\).

The figure below illustrates graphically the shape of this distribution \(G^*\) for a given (arbitrary) outcome \((G, p)\).

![Figure 1: \(G^*\) is a concatenation of \(G^\pi_{qp}\) and \(G\).](image-url)
It is worth noting that Proposition (2) is sufficient to completely characterize a seller-optimal distribution whenever the probability of trade is equal to one (that is, when \( G(p) = 1 \)). This full characterization is obtained as a solution to the following problem:

\[
\max_{q,p} \int_0^p (p - s)d\left(\frac{S - p}{S - s}\right)
\]

s.t. (i) \[ \int_0^1 sdF(s) = \int_0^p sd\left(\frac{S - p}{S - s}\right) \]

(ii) \[ \int_0^x F(s)ds \geq \int_q^x \frac{S - p}{S - s}ds \quad \forall x \in (q,p] \]

(iii) \[ \int_0^x F(s)ds \geq \int_q^p \frac{S - p}{S - s}ds + x - p \quad \forall x \in (p,1] \]

(iv) \[ 0 \leq q \leq p \leq 1 \]

Constraint (i) is simply the equal-mean condition. Constraints (ii) and (iii) are the inequalities required by the mean-preserving-spread condition (restricted to support’s points where it does not obviously hold). Constraint (iv) is the natural domain for the choice variables.

Since trade’s probability is endogenous, we still need a characterization result for the seller-optimal signal structure when \( G(p) < 1 \) in equilibrium. To address this issue, we have the following proposition.

**Proposition 3.** Let \((G,p)\) be an outcome. If \( G(p) < 1 \) and \( G \) is seller-optimal, then there exists \( \hat{p} \geq p \) such that

(i) \[ \int_0^{\hat{p}} F(s)ds = \int_0^{\hat{p}} G(s)ds; \]

(ii) If \( p > 0 \), then \( G(\hat{p}) = F(\hat{p}) = G(p) \).

**Proof.** See Appendix.
Proposition (3) provides necessary conditions for a signal structure $G$ associated with an outcome $(G, p)$ to be seller-optimal under the assumption that $G(p) < 1$. Without loss of generality, these conditions allow us to assume that $G(s) = F(s)$ for all $s > \hat{p}$. The figure below illustrates graphically the shape of this seller-optimal signal distribution $G$ for a given (arbitrary) continuous prior $F$.

![Figure 2: Shape of a seller-optimal distribution $G$ when $G(p) < 1$ and the prior is $F$.](image)

Using Proposition (3), we can find the seller-optimal distribution under which trade happens with probability strictly less than one by solving the following problem:

$$
\max_{q, p, \hat{p}, \pi} \int_0^p (p - s) d\left(\frac{\pi}{S - s}\right)
$$

s.t.  

(i) $\int_0^p s dF(s) = \int_0^p s d\left(\frac{\pi}{S - s}\right)$

(ii) $\int_0^x F(s) ds \geq \int_q^x \frac{\pi}{S - s} ds$ \quad $\forall x \in (q, p]$  

(iii) $\int_0^x F(s) ds \geq \int_q^p \frac{\pi}{S - s} ds + (x - p) \frac{\pi}{S - p}$ \quad $\forall x \in (p, \hat{p}]$

(iv) $\int_0^{\hat{p}} F(s) ds = \int_q^p \frac{\pi}{S - s} ds + (\hat{p} - p) \frac{\pi}{S - p}$

(v) $F(\hat{p}) = \frac{\pi}{S - p}$

(vi) $0 \leq q \leq p \leq \hat{p} \leq 1$ \quad and \quad $0 \leq \pi < S - p$
Constraint (i) is simply the equal-mean condition. Constraints (ii) and (iii) are the inequalities required by the mean-preserving-spread condition (restricted to support’s points where it does not obviously hold). Constraints (iv) and (v) come from Proposition (3). Finally, constraint (vi) is the natural domain for the choice variables.

By Propositions (2) and (3), the problem of designing a seller-optimal signal structure can be described as a binary choice between the best distribution that ensures trade with probability one and the best distribution under which trade happens with probability strictly less than one. Both distributions, in turn, can be obtained from Problems (2) and (3), which depend only on the exogenous parameter $S$ and the prior distribution $F$.

Despite being clearly stated, Problems (2) and (3) can be difficult to solve without further structure being assumed on the prior distribution $F$. More specifically, both problems present a continuum of constraints that can potentially bind depending on $F$’s shape. In the next section, we illustratively assume some specific prior distributions (as well as $S$ values), and conduct a few comparative-statics exercises.

4 Examples and Analysis

a The information gathering trade-off: the uniform case

In our first exercise, we numerically solve for an optimal information structure when seller’s cost is uniformly distributed, and compare the outcome to that realized in the full information environment. This example illustrates the trade-off faced by any seller in a monopsonist market: on the one hand, better information improves trade efficiency in the extensive margin; yet, by inducing lower equilibrium prices, it also reduces the seller’s expected gains from trade (conditionally on the exchange taking place). The seller-optimal signal structure involves some (yet not perfect) learning. From the buyer’s perspective, a better informed seller would be preferred.

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3 For computational purposes, Problem (2) can be embedded into Problem (3) by allowing the strict inequality present in constraint (vi) to be weak. Thus, Problem (1), which was originally infinite-dimensional, is replaced by a finite-dimensional problem of maximizing a continuous function on a compact set.
Suppose $\theta \sim U[0, 1]$ so that $F(\theta) = \theta$ for all $\theta \in [0, 1]$ and $\mu = 0.5$. Assume initially that $S = 1$. In this case, our seller-optimal distribution has support on the interval $[0.1272, 0.7964]$. Under this information structure, trade occurs with probability one at price $p = 0.7964$. The buyer’s payoff is $\pi = 0.2036$, whereas the seller’s payoff is $0.2964$. The figure below illustrates graphically the shape of this seller-optimal signal distribution for the uniform prior.

![Seller-optimal distribution for uniform prior](image)

Figure 3: Seller-optimal distribution for uniform prior; $q = 0.1272, p = 0.7964, \hat{p} = 1, \pi = 0.2036$.

It is insightful to compare the outcome resulting from a seller-optimal signal structure to that realized in the full information case, i.e., $s = \theta$. If the seller observes his cost prior to trade then he is willing to trade at price $p \in [0, 1]$ with probability $p$. Hence, the buyer’s optimal price solves $\max_s (1 - s)s$, so the equilibrium price is 0.5. In turn, the buyer’s payoff is 0.25 since the seller trades with probability one-half at price 0.5. The buyer is therefore better off if the seller receives a perfectly informative signal than if he receives a seller-optimal one. The seller’s payoff is $\int_0^{0.5} (0.5 - v)dv = 0.125$ which is less than one-half of his payoff under the optimal signal structure. The deadweight loss due to the seller’s perfectly informative signal is also 0.125.
b The role of the buyer’s stake $S$

Still under the uniform prior assumption, our second exercise investigates what happens to the seller-optimal distribution when the buyer’s stake $S$ increases strictly above 1. The figure below illustrates graphically the shape of this seller-optimal signal distribution for different $S$ values.

Figure 4: Seller-optimal signal structures for different $S$ values.

(a) $S = 1.5$; $q = 0.2160, p = \hat{p} = 1, \pi = 0.5$.

(b) $S = 2$; $q = 0.3513, p = \hat{p} = \pi = 1$.

(c) $S = 10$; $q = 0.4859, p = \hat{p} = 1, \pi = 9$.

(d) $S = 100$; $q = 0.4987, p = \hat{p} = 1, \pi = 99$. 
As $S$ increases, the seller-optimal signal structure converges to a $\delta$-dirac distribution that puts mass one at $\mu = 0.5$. The seller’s payoff is 0.5 while the buyer’s payoff is $S - 1$. Intuitively, the interpretation is that, as $S$ exceeds one, the buyer’s price response to different information structures becomes less sensitive to the amount of information gathered by the seller. The equilibrium price $p$ equals 1, which ensures ex post trade efficiency for any cost realization. In other words, trade becomes always profitable to the seller, regardless of her actual cost. The trade-off described in the previous section vanishes and incentives for information gathering disappear. Therefore, no information is acquired by the seller. Thus, her expected cost, conditional on the observed signal (0.5), is ex ante equal to the unconditional average cost (0.5) given by the prior distribution.

c A broader class of prior distributions

Our third exercise numerically solves for the seller-optimal signal structure in the class of prior distributions with shape given by $F(\theta) = \theta^\alpha, \alpha > 0$, and $S = 1$. If $\alpha < 1$, the prior distribution $F$ puts relatively more weight on its lower tail when compared to the uniform case ($\alpha = 1$) so that lower costs (“good news”) are more likely to occur. The opposite happens when $\alpha > 1$ and higher costs (“bad news”) become more frequent.

Figure 5: Seller-optimal signal structures for $F(\theta) = \theta^\alpha, \alpha > 0$. 

\begin{align*}
\text{(a) } & \alpha = 0.5; \quad q = 0.0394, p = 0.6846, \hat{p} = 1, \pi = 0.3154; \\
& \text{Seller’s payoff } = 0.3517.
\end{align*}

\begin{align*}
\text{(b) } & \alpha = 0.25; \quad q = 0.0036, p = 0.6389, \hat{p} = 1, \pi = 0.3611; \\
& \text{Seller’s payoff } = 0.3665.
\end{align*}
Notice that the equilibrium price and agent’s payoffs behave exactly as expected when \( \alpha \) varies. Under a favourable prior (i.e., \( \alpha < 1 \)), lower prices are just enough to ensure efficient trade with probability one. Additionally, both seller’s and buyer’s expected payoffs are higher since the social economic surplus resulting from trade \((S - \alpha/(1 + \alpha))\) is also greater. Under an unfavourable prior (i.e., \( \alpha > 1 \)), higher prices are required to ensure efficient trade with probability one. Moreover, both seller’s and buyer’s expected payoffs are lower since the social economic surplus resulting from trade is now smaller.

\section{d An inefficient equilibrium allocation}

In all examples presented so far, the equilibrium allocation resulting from the seller-optimal signal structure is \textit{ex post} efficient: trade occurs with probability one. This raises a natural question: is there any prior that results in an inefficient equilibrium allocation? Is efficiency a general property of this model? \cite{RoeslerSzentes2017} prove that, in their settings, a buyer-optimal signal structure always generates efficient trade.
In contrast, we now present a numerical example to show that, in our model, inefficient outcomes may arise in equilibrium for some prior distributions and specific parameters values. The example is constructed in a sequence of steps. First, we take a prior distribution $G$ with mean $\mu$ within the class described by Proposition (3). Second, we show that, if a distribution $\tilde{G}$ with mean $\mu$ implies efficient trade, then $\tilde{G}$ is strictly dominated by $G$ from the seller’s perspective. It suffices to illustrate our claim that inefficient outcomes may arise in equilibrium (since there is always a seller-optimal distribution – see footnote 3).

Let $G$ be our prior distribution within the class described by Proposition (3). Recall that such a $G$ is fully characterized by $q, p, \hat{p}, \pi \in [0, 1]$, $S \geq 1$, and $F : [\hat{p}, 1] \to [0, 1]$, continuous, non-decreasing with $F(\hat{p}) = \pi/(S - \hat{p}) < 1$. Thus, $G$ is defined by:

$$G(s) = \begin{cases} G_{\pi}^{\hat{p}}(s) & \text{if } s < \hat{p} \\ F(s) & \text{if } s \geq \hat{p} \end{cases}$$

where

$$G_{\pi}^{\hat{p}}(s) = \begin{cases} 0 & \text{if } s < q \\ \frac{\pi}{S - s} & \text{if } q \leq s < p \\ \frac{\pi}{S - p} & \text{if } p \leq s < \hat{p} \end{cases}$$

For computational simplicity, we fix $q = 0, p = \hat{p}, S = 1$ and let $F(s) = \pi/(S - p)$, for all $s \in [p, 1)$, with $F(1) = 1$. This construction creates a flat segment on $F$’s graph that contradicts our initial assumptions on the prior $F$ (more specifically, it violates the hypothesis of $F$ being continuous and having full support on $[0, 1]$). However, our approach is still consistent since this flat segment may be arbitrarily approximated by continuous, strictly increasing functions defined on $[p, 1]$.

The figure below presents two instances when the prior distribution $G$ strictly dominates the seller-optimal distribution that induces efficient trade.
In Figure (6a), trade happens with probability 0.91 under the prior, which yields 0.2208 as seller’s payoff. Under the efficient-trade, seller-optimal distribution, her payoff is 0.2191. Therefore, no feasible signal distribution is simultaneously seller-optimal and trade-efficient.

In Figure (6b), trade happens with probability 0.95, which yields 0.2251 as seller’s payoff. Under the efficient-trade, seller-optimal distribution, her payoff is 0.1978. Again, no feasible signal distribution is simultaneously seller-optimal and trade-efficient.
5 Concluding Remarks and Future Extensions

The goal of this paper is to analyze seller’s optimal learning when facing a monopsonist. We characterize the optimal signal distribution which solves the rent-efficiency trade-off from the seller’s perspective. The supply generated by this signal is unit-elastic and makes the buyer indifferent between setting any price on its support. Taking the uniform prior as our benchmark case, we illustrate that the seller-optimal strategy involves minimal learning and induces less information gathering than would be desired by the buyer. We also show that efficient trade may not happen with probability one under some seller-optimal signal structures.

A key ingredient in our model is buyer’s limitation to move, which is restricted in timing and manner. He can only react to signal structures through posted prices. This setup, although quite simplified, reveals the underlying incentives that drive information acquisition beyond buyer’s command. We believe this is particularly relevant to public procurement settings where government cannot fully control the optimal learning strategy developed by potential suppliers. However, real-world market institutions often allow for more flexibility either on the timing of interactions (e.g., prior communication, repeated talkings) or on the allocation mechanism (e.g., bargaining, auctions). Based on this observation, two possible extensions of our model are considered for future research.

In the first extension, we keep our original allocation mechanism (i.e., posted price) but allow the buyer to more directly affect game’s information structure. This is done by altering the timing of the game. We drop the common knowledge assumption about the prior distribution $F$, and assume it is now buyer’s private information. We then introduce an initial stage where the buyer chooses which signal distribution $H$ he will disclose to the seller: $H$ needs to be a mean-preserving spread of $F$. After that, the game proceeds as before: a seller-optimal signal structure $G$ is obtained by maximizing seller’s expected payoff subject to feasibility constraints ($H$ should be a mean-preserving spread of $G$) and monopsonist pricing. Our intuition suggests $G$ to always induce efficient trade and to be more informative than the signal structure that arises when no initial move by the buyer exists. Thus, a higher payoff to the buyer is expected.
In the second extension, we keep our original timing of interactions (i.e., buyer moves second), but change the allocation mechanism so as to permit competition between potential sellers. We model this competition using a second-price auction with reserve price and two bidders. A seller-optimal signal structure $G^C$ is obtained by maximizing seller’s expected payoff subject to feasibility constraints ($F$ is a mean-preserving spread of $G^C$) and optimal reserve price fixed by the buyer. More specifically, we solve for a symmetric Nash equilibrium $(G^C, G^C)$ between the two potential suppliers. Given that seller 2 chooses $G^C$, the problem of designing an optimal signal structure for seller 1 can be stated as follows:

$$\max_{G(\cdot) \in F} \int_0^p \int_0^{s_2} (s_2 - s_1)dG(s_1)dG^C(s_2)$$

s.t. $p \in \arg\max_{s \geq 0} \left\{ \int_0^1 \int_0^1 R(S, s_1, s_2, s)dG(s_1)dG^C(s_2) \right\}$

where

$$R(S, s_1, s_2, s) = \begin{cases} 
0 & \text{if } s_1, s_2 > s \\
S - s & \text{if } s_1 > s \geq s_2 \\
S - s & \text{if } s_2 > s \geq s_1 \\
S - \max\{s_1, s_2\} & \text{if } s \geq s_1, s_2
\end{cases}$$

Our intuition suggests that competition among sellers might induce more information gathering than our basic model. The underlying reason is that more informative signal structures, despite not directly affecting bidding strategies in a second-price auction, should increase the likelihood of winning. From seller’s perspective, this reinforces incentives to acquire information beyond trade’s efficiency concerns.
A Appendix

a Proof of Proposition 1


b Proof of Proposition 2

Proposition 2. Let \((G, p)\) be an outcome and \(\pi := (S - p)G(p)\). Then there exists a unique \(q \in [0, p]\) such that

\[
G^*(s) = \begin{cases} 
G^\pi_{q,p}(s) & \text{if } s < p \\
G(s) & \text{if } s \geq p
\end{cases}
\]

where \(G^\pi_{q,p}(s) = \begin{cases} 
0 & \text{if } s < q \\
\frac{\pi}{S - s} & \text{if } q \leq s < p
\end{cases}\)

satisfies

(i) \(G\) is a mean-preserving spread of \(G^*\);

(ii) \((G^*, p)\) is an outcome;

(iii) \(\int_0^p (p - s)dG^*(s) = \int_0^p (p - s)dG(s)\).

Proof. To prove (i), define the function \(\psi : [0, p] \rightarrow \mathbb{R}\) by

\[
\psi(x) := \int_0^p sdG^\pi_{x,p}(s) + \int_0^1 sdG(s) = p\frac{\pi}{S - p} - \int_x^p \frac{\pi}{S - s} ds + \int_p^1 sdG(s)
\]

which is clearly continuous and strictly increasing. Notice that \(\psi(0) \leq \mu \leq \psi(p)\).

On the one hand, by definition of \(p\), \((S - p)G(p) \geq (S - s)G(s), \ \forall s \in [0, 1]\). Thus,

\[
G^\pi_{0,p}(s) = \frac{\pi}{S - s} \geq G(s) \ \forall s \in [0, 1]
\]
This implies
\[ \psi(0) = \int_0^p sdG_{0,p}^\pi(s) + \int_p^1 sG(s) \leq \int_0^1 sdG(s) = \mu \]

On the other hand,
\[ \psi(p) = \int_0^p sdG_{p,p}^\pi(s) + \int_p^1 pG(s) = \pi\frac{p}{S-p} + \int_p^1 sdG(s) \]
\[ = \int_0^p pdG(s) + \int_p^1 sdG(s) \]
\[ \geq \int_p^1 sdG(s) = \mu \]

Therefore, there exists a unique \( q \in [0, p] \) such that
\[ \int_0^1 sdG^*(s) = \int_0^p sdG_{q,p}^\pi(s) + \int_p^1 sdG(s) = \mu \tag{4} \]

Now we show that for each \( x \in [0, p] \)
\[ f(x) := \int_0^x [G(s) - G_{q,p}^\pi(s)]ds \geq 0 \tag{5} \]

By (4),
\[ \int_0^p sdG(s) = q\frac{\pi}{S-q} + \int_q^p sdG_{q,p}^\pi(s) = pG(p) - \int_q^p pG_{q,p}(s)ds \]
So
\[ f(p) = \int_0^p [G(s) - G_{q,p}^\pi(s)]ds = 0 \tag{6} \]

Hence
\[ f(0) = f(p) = 0 \text{ and } f'(x) = G(x) - G_{q,p}^\pi(x) \begin{cases} \\ \geq 0 & \text{for } x \in (0, q) \\ \leq 0 & \text{for } x \in [q, p] \end{cases} \]

which implies (5) and, in turn, yields:
\[ \int_0^x [F(s) - G_{q,p}^\pi(s)]ds \geq 0 \quad \forall x \in [0, p] \tag{7} \]

Result (i) follows from (4) and (7).
To prove (ii), it suffices to show that
\[ p \in \arg\max_{s \geq 0} (S - s)G^*(s) \]
since, by (i), \( G^* \in G_F \) as \( G \in G_F \).

By construction,
\[ p := \max\left\{ \arg\max_{s \geq 0} (S - s)G(s) \right\} \]
and the buyer is indifferent among any price on \([q, p]\). Thus,
\[ [q, p] = \arg\max_{s \geq 0} (S - s)G^*(s) \]
This concludes the proof of (ii).

To prove (iii),
\[
\int_0^p (p - s)dG^*(s) = \int_0^p (p - s)dG^*_q(s) \\
= pG(p) - \int_q^p SdG^*_q(s) - q \frac{\pi}{S - q} \\
= \int_q^p G^*_q(s)ds \\
= \int_0^p G(s)ds \\
= \int_0^p (p - s)dG(s)
\]
where the fourth equality comes from (6). \( \Box \)
c Proof of Proposition 3

Proposition 3. Let \((G, p)\) be an outcome. If \(G\) solves the seller’s problem and \(G(p) < 1\), then there exists \(\hat{p} \geq p\) such that

\[(i) \quad \int_0^{\hat{p}} F(s)ds = \int_0^p G(s)ds;

(ii) If \(p > 0\), \(G(\hat{p}) = F(\hat{p}) \geq G(p)\) with equality whenever \(F\) is continuous at \(\hat{p}\).

Proof. We may restrict our analysis to the relevant class of distributions described in Proposition (2).

Given a outcome \((G, p)\) in that class such that \(G(p) < 1\), we break the proof of Proposition (3) into two possible cases:

\[
\begin{cases}
(\text{I}) & \exists p^+ > p; \ G(x) = G(p), \ \forall x \in [p, p^+) \\
(\text{II}) & \exists p^+ > p; \ G(x) = G(p), \ \forall x \in [p, p^+) 
\end{cases}
\]

- Case (I)

We claim that \(\hat{p} = p\). To obtain a contradiction, suppose \((i)\) does not hold at \(p\), that is:

\[\int_0^p [F(s) - G(s)]ds > 0\]

If it happens, there is a distribution \(\tilde{G}\) that strictly dominates \(G\) from the seller’s perspective.

In fact, since \(G(p) < 1\), the following function is still a probability distribution for \(\epsilon > 0\)
sufficiently close to zero:

\[
G_\varepsilon(s) = \begin{cases} 
G(s) & s \leq p \\
\frac{\pi}{S-s} & p < s \leq p + \varepsilon \\
\frac{\pi}{S-p-\varepsilon} & p + \varepsilon < s < z(\varepsilon) \\
G(s) & s \geq z(\varepsilon) 
\end{cases}
\]

where \( z(\varepsilon) \in (\bar{z}(\varepsilon), +\infty) \) is the unique solution to the following equation:

\[
f(z) := \int_{p}^{z} sdG(s) - \int_{p}^{p+\varepsilon} sd\left(\frac{\pi}{S-s}\right) - z \left( G(z) - \frac{\pi}{S-p-\varepsilon} \right) = 0 \tag{8}
\]

with

\[
\bar{z}(\varepsilon) > p + \varepsilon \text{ and } G(\bar{z}(\varepsilon)) = \frac{\pi}{S-p-\varepsilon}
\]

Notice that \( \bar{z}(\varepsilon) \) is well defined because \( G \) is right-continuous and, in case (I), \( G(x) > G(p), \forall x > p \).

The existence and uniqueness of \( z(\varepsilon) \) follow from the fact that

\[
f(z) = 0 \iff \int_{p}^{p+\varepsilon} \left( G(s) - \frac{\pi}{S-s} \right) ds + \int_{p+\varepsilon}^{z} \left( G(s) - \frac{\pi}{S-p-\varepsilon} \right) ds = 0
\]

For \( \varepsilon > 0 \) sufficiently small, \( f \) is continuous on \( [\bar{z}(\varepsilon), +\infty) \) and strictly increasing on \( (\bar{z}(\varepsilon), +\infty) \) since

\[
f'(z) = G(z) - \frac{\pi}{S-p-\varepsilon}
\]

We have that \( f(\bar{z}(\varepsilon)) < 0 \) since \( G(s) < \pi/(S-s) \) for all \( s > p \).

For \( z \) sufficiently large, it is easy to see that \( f(z) > 0 \).

By the Intermediate Value Theorem, \( z(\varepsilon) \) is well defined.
Equation (8) implies that
\[ \int_0^1 s dG_\epsilon(s) = \int_0^1 s dG(s) = \mu \]

We now choose \( \epsilon > 0 \) such that
\[
\int_p^{p+\epsilon} \frac{\pi}{s-S} ds + \int_{p+\epsilon}^{\bar{z}(\epsilon)} \frac{\pi}{s-S} ds < \int_0^p [F(s) - G(s)] ds
\]

Therefore, \( \forall x \in (p, \bar{z}(\epsilon)) \) we have:
\[
\int_0^x F(s) ds \geq \int_0^p F(s) ds
\]
\[
> \int_0^p G(s) ds + \int_p^{p+\epsilon} \frac{\pi}{s-S} ds + \int_{p+\epsilon}^{\bar{z}(\epsilon)} \frac{\pi}{s-p-\epsilon} ds
\]
\[
= \int_0^{\bar{z}(\epsilon)} G_\epsilon(s) ds
\]
\[
\geq \int_0^x G_\epsilon(s) ds
\]

Given that \( G_\epsilon(s) \leq G(s), \forall s \notin (p, \bar{z}(\epsilon)) \) and \( G \in \mathcal{G}_F \), the definition of \( G_\epsilon \) implies that
\[
\int_0^x F(s) ds \geq \int_0^x G_\epsilon(s) ds \ \forall x \in [0, 1]
\]
(9)

By (8) and (9), \( G_\epsilon \in \mathcal{G}_F \). Let \( \tilde{G} = G_\epsilon \).

Since \( (G, p) \) belongs to the class identified by Lemma 1, \( (\tilde{G}, p + \epsilon) \) is clearly an outcome.

From the seller’s perspective, \( \tilde{G} \) strictly dominates \( G \):
\[
\int_{0}^{p+\epsilon}(p + \epsilon - s)d\tilde{G}(s) - \int_{0}^{p}(p - s)dG(s) = \int_{p}^{p+\epsilon}(p + \epsilon - s)d\tilde{G}(s) + \int_{0}^{p}\epsilon dG(s)
\]
\[
= \pi \ln \left( \frac{S - p}{S - p - \epsilon} \right) > 0
\]

This concludes the proof of (i) for case (I).

As to (ii), notice that

\[
\phi(x) := \int_{0}^{x}[F(s) - G(s)]ds \geq 0 \quad \forall x \in [0, 1] \quad \text{and} \quad \phi(p) = 0
\]

Thus, \( p \) is a global minimum of \( \phi \).

If \( p \in (0, 1) \), then \( \phi'(p) = F(p) - G(p) = 0 \).

If \( p = 1 \), then \( \phi'(1) \leq 0 \), but \( \phi'(1) = F(1) - G(1) = 1 - G(1) \geq 0 \). Therefore, \( F(1) = G(1) \).

This concludes the proof of (ii) for case (I).

- Case (II)

Without loss of generality, \( p^+ = \max\{\hat{p} \in [0, 1] \mid G(x) = G(p) \ \forall x \in [p, \hat{p}]\} \).

Since \( \{\hat{p} \in [0, 1] \mid G(x) = G(p) \ \forall x \in [p, \hat{p}]\} \neq \emptyset \) is closed, \( p^+ \) is well defined.

We claim that \( \hat{p} \in [p, p^+] \).

To obtain a contradiction, suppose (i) does not hold for any \( \hat{p} \in [p, p^+] \), that is:

\[
\int_{0}^{\hat{p}} F(s)ds > \int_{0}^{\hat{p}} G(s)ds, \quad \forall \hat{p} \in [p, p^+]
\]

If it happens, there is a distribution \( \tilde{G} \) that strictly dominates \( G \) from the seller’s perspective.
In fact, since $G(p) < 1$, the following function is still a probability distribution for $\epsilon > 0$ sufficiently close to zero:

$$G_\epsilon(s) = \begin{cases} 
G(s) & s \leq p \\
\frac{\pi}{S - s} & p < s \leq p + \epsilon \\
\frac{\pi}{S - p - \epsilon} & p + \epsilon < s < z(\epsilon) \\
G(s) & s \geq z(\epsilon) 
\end{cases}$$

where $z(\epsilon) \in [\bar{z}(\epsilon), +\infty)$ is the unique solution to the following equation:

$$f_\epsilon(z) := p^+G(p) - z\frac{\pi}{S - p - \epsilon} + \int_p^{p^+} s d\left(\frac{\pi}{S - s}\right) + \int_p^z G(s) ds = 0 \tag{10}$$

with

$$\bar{z}(\epsilon) > p^+ \text{ and } G(\bar{z}(\epsilon)) = \frac{\pi}{S - p - \epsilon} \quad \text{if } G \text{ is continuous at } p^+$$

and

$$\bar{z}(\epsilon) = p^+ \quad \text{if } G \text{ is not continuous at } p^+$$

The existence and uniqueness of $z(\epsilon)$ follow from the fact that, for $\epsilon > 0$ sufficiently small, $p + \epsilon < p^+$, $f_\epsilon$ is continuous on $[p^+, +\infty)$, strictly decreasing on $(p^+, \bar{z}(\epsilon))$, and strictly increasing on $(\bar{z}(\epsilon), +\infty)$ since

$$f_\epsilon'(z) = G(z) - \frac{\pi}{S - p - \epsilon}$$

Given that $f_0(p^+) = 0$ and $\partial_\epsilon f_0(p^+) = -(p^+ - p)/(1 - p)^2 < 0$, $f_\epsilon(p^+) < 0$, for $\epsilon > 0$ small enough.

For $z$ sufficiently large, it is easy to see that $f_\epsilon(z) > 0$. 

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By the Intermediate Value Theorem, \( z(\epsilon) \) is well defined. Equation (10) implies that

\[
\int_0^1 s dG_\epsilon(s) = \int_0^1 s dG(s) = \mu
\]

To show that \( G_\epsilon \in \mathcal{G}_F \) it suffices to check that, for \( \epsilon > 0 \) sufficiently small:

\[
\psi(\epsilon)(x) := \int_x^0 [F(s) - G_\epsilon(s)] ds \geq 0 \quad \forall x \in [p, \bar{z}(\epsilon)]
\]

We do so by showing that the sequence of functions \( \{\psi_n\}_{n=1}^\infty \) converges to the function \( \psi \) in the sup norm, where:

\[
\psi_n(x) := \int_x^0 [F(s) - G_{1/n}(s)] ds \quad \text{for } x \in [p, p^+] 
\]

and

\[
\psi(x) := \int_x^0 [F(s) - G(s)] ds \quad \text{for } x \in [p, p^+] 
\]

In fact,

\[
||\psi_n - \psi|| = \sup_{x \in [p, p^+]} |\psi_n(x) - \psi(x)| = \sup_{x \in [p, p^+]} \left| \int_0^x [G_n(s) - G(s)] ds \right| \\
\leq \sup_{x \in [p, p^+]} \left( \int_0^x |G_n(s) - G(s)| ds \right) \\
= \int_0^{p^+} |G_n(s) - G(s)| ds \\
= \int_p^{p+1/n} \left| \frac{\pi}{S - s} - G(p) \right| ds + \int_{p+1/n}^{p^+} \frac{\pi}{S - p - (1/n)} - G(p) \right| ds \\
\to 0 \quad \text{as } n \to +\infty
\]

Therefore, \( \psi_n \) uniformly converges to \( \psi \) on \([p, p^+]\).
Since $\psi > 0$ by assumption, there exists $N \in \mathbb{N}$ such that $\psi_n(x) > 0$, $\forall n > N$, $\forall x \in [p, p^+]$.

The extension of this result to $[p^+, \tilde{z}(\epsilon)]$ when $G$ is continuous at $p^+$ is straightforward since $\tilde{z}(\epsilon) \rightarrow p^+$ as $\epsilon \rightarrow 0$.

Choosing $\epsilon \in (0, 1/N)$ to satisfy (10), we have $G_\epsilon \in G_F$. Let $\tilde{G} = G_\epsilon$.

Since $(G, p)$ belongs to the class identified by Proposition (2), $(\tilde{G}, p + \epsilon)$ is clearly an outcome.

From the seller’s perspective, $\tilde{G}$ strictly dominates $G$:

$$
\int_0^{p+\epsilon} (p + \epsilon - s) dG(s) - \int_0^p (p - s) dG(s) = \int_p^{p+\epsilon} (p + \epsilon - s) d\tilde{G}(s) + \int_0^p \epsilon dG(s)
$$

$$
= \pi \ln\left( \frac{S - p}{S - p - \epsilon} \right) > 0
$$

This concludes the proof of (i) for case (II).

As to (ii), notice that

$$
\phi(x) := \int_0^x [F(s) - G(s)] ds \geq 0 \quad \forall x \in [0, 1] \quad \text{and} \quad \phi(\hat{p}) = 0
$$

Thus, $\hat{p}$ is a global minimum of $\phi$. If $\hat{p} \in (0, 1)$, then $\phi'(\hat{p}) = F(\hat{p}) - G(\hat{p}) = 0$.

If $\hat{p} = 1$, then $\phi'(1) \leq 0$, but $\phi'(1) = F(1) - G(1) = 1 - G(1) \geq 0$. Therefore, $F(1) = G(1)$.

For $\hat{p} \in [p, p^+)$, $F(\hat{p}) = G(p)$. If $\hat{p} = p^+$, the equality holds whenever $F$ is continuous at $p^+$.

This concludes the proof of (ii) for case (II).
References


