The Brazilian Education Quality Index (Ideb): Measurement and Incentives Upgrades

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The Brazilian Education Quality Index (Ideb): Measurement and Incentives Upgrades

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Abstract

The increasing availability of social statistics in Latin America opens new possibilities in terms of accountability and incentive mechanisms for policy makers. This paper addresses these issues within the institutional context of the Brazilian educational system. We build a theoretical model based on the theory of incentives to analyze the role of the recently launched Basic Education Development Index (Ideb) in the provision of incentives at the sub-national level. The first result is to demonstrate that an education target system has the potential to improve the allocation of resources to education through conditional transfers to municipalities and schools. Second, we analyze the local government’s decision about how to allocate its education budget when seeking to accomplish the different objectives contemplated by the index, which involves the interaction between its two components, average proficiency and the passing rate. We discuss as well policy issues concerning the implementation of the synthetic education index in the light of this model arguing that there is room for improving the Ideb’s methodology itself. In addition, we analyze the desirable properties of an ideal education index and we argue in favor of an ex-post relative learning evaluation system for different municipalities (schools) based on the value added across different grades.

Keywords: Education; Education Finance; Theory of Incentives; Target-based system

JEL Classification: I2; I22; I28

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1. Introduction

The aim of this paper is twofold. First we build a theoretical model based on the theory of incentives to analyze the role of the Basic Education Development Index (Ideb), the main feature of the new Education Plan just launched in Brazil, in providing incentives to public managers. Then we discuss policy issues concerning both measurement and evaluation issues related to the implementation of the synthetic index within the framework of a target system.

In March 2007 the Brazilian federal government announced an Education Development Plan (PDE), a set of proposals aiming to improve the quality of education in the country. The plan’s main innovation was the creation of a synthetic indicator of education quality, the Basic Education Development Index (Ideb), based on the academic passing rate and the results of Prova Brasil (and Saeb) for each municipality in the country. The federal government will determine targets for the evolution of the Ideb and then condition part of its education-related transfers to the accomplishment of these targets. The one thousand municipalities with the lowest Ideb will receive extra resources and the remaining ones, only technical support.

The creation of a target system in education set an important and historical precedent in Brazil, not only in the field of education but also in the national social policy scenario as a whole, and provides a unique opportunity for the country to recover its educational delay. Despite all its virtues, however, there is still room for improvement concerning methodological issues and the design of incentive mechanisms related to the Ideb. In the last part of this paper we will then focus on methodological and policy issues concerning the implementation of the synthetic index within the framework of a target system.

On the next sections included in the introduction we will described the Ideb and its methodology and then analyze the rationale behind the implementation of social targets. In section 2 we build a model in two stages and derive some conclusions from it. In section 3 we first suggest some ways to improve the index methodology and then discuss evaluation issues. Then we conclude.

1.1 The Ideb

The Ideb is analytically expressed by the following formula

\[ Ideb = Q \times F \]

in which Q is a proficiency measure, that can be the students’ average performance in the Prova Brasil or in the Saeb, and F is a measure related to the school flow, corresponding to the students’ average passing rate.

The table that follows shows the initial Ideb values for each educational sector and their respective targets for 2021.
Ideb ranges on a scale from 0 to 10 and the index value in Brazil was 3.8 in 2005 – which was adopted as the baseline by the Educational Development Plan (PDE). A target of 6.0 was set for 2021, just before the celebration of the 200th anniversary of the Brazilian independence. This strategy aims to bring the educational results of the whole country in 2021 to the same level observed today in Brazilian private schools.

The methodology behind the Ideb is as follows. On one hand, from (i) the average passing rate of a specific schooling level one calculates (ii) the average time $T$ needed to complete one school year. (iii) Its inverse gives $P$, the average passing rate, which corresponds to the probability that a student passes on to the following school year. On the other hand, (i) one calculates the standard grade in both Math and Portuguese tests taken in Saeb (or in Prova Brasil), and (ii) from their average, one obtains $N$. From the simple multiplication of $P$ times $N$ we reach the value of the index.

The table below illustrates the Ideb methodology, showing how it was calculated for each Brazilian state in 2005, the year corresponding to the last edition of Prova Brazil.
As we can see, the passing rates are very low in Brazil, ranging from 64% to 80% across states in the secondary level and reaching values as low as 50% for certain grades. Proficiency scores, in turn, are also very reduced, ranging from 3.3 to 5.0 within a range that goes up to 10. Since generally the minimum average score for approval conventionally in Brazil is 5.0, we could say that all states would have failed if they were students.

### 1.2 Rationale for Educational Targets

The management of the Brazilian educational policy has become more complex and challenging. The decentralization of education as a result of the 1988 Constitution allied to the growing involvement of other actors (such as private firms, NGOs and the civil society in general) creates a diversity of simultaneous actions. The question that interests us here is: how should we increase the returns experienced by the society from this myriad of educational actions? It is up to the federal government to set goals to the different actors so that they will act simultaneously towards the same goal. These goals involve the coordination of diffused efforts through the establishment of targets and the design of mechanisms providing the incentives to achieve them.

The proposal is that specific locations - in particular, those at the sub-national level - commit to the educational targets as they have been specified, and challenge their respective populace to reach the proposed auspicious targets. The recent Brazilian experience with inflationary targets and electrical energy rationing targets reinforces the importance of setting tangible objectives.

Aside from the coordination and mobilization characteristics of determining educational target, conditioning the provision of public finance to the achievement of social outcomes is an interesting practice to
be adopted by the system. The same spirit of conditional cash transfers such as *Bolsa-Familia*, that reward poor families whose children attend school, can be applied to the annual re-allocation of the educational budget at numerous administrative levels. The process of rewarding, with additional resources, those units progressing swiftly, may be applied from upper towards lower levels of the government: from the federal to the state realm, from the state to their respective municipalities and from the latter to their respective administrative regions and schools. The Ministry of Education (MEC) and the Brazilian Institute of Geography and Statistics (IBGE) provide increasing levels of information, which constitutes the stepping-stone for such as system to function in various geographical levels. There is no doubt that the core of social action should focus on the poorest and least educated segments, and we show in this paper that way to produce the best results is by rewarding those moving towards the emancipation of their needs. The main comparative advantage of being poor – in this case poorly educated - is the larger relative capacity of prospering. Future success should also be rewarded, instead of compensating for past failures.

The educational target’s main problem may occur particularly in the short term, given the presence of shocks. The result obtained by the social actor depends on factors beyond his reach, as the outcome does not depend solely on his efforts or skills. Therefore, we argue in favor of using relative evaluation schemes. In addition, the system of incentives should be announced a priori and the relative performance should be evaluated a posteriori. The advantage of the system, if well designed, is to attract better social actors and encourage them to follow the best practices.

2. The model

The main idea of this section is to build a theoretical model based on the literature of mechanism design and theory of incentives, in order to address issues related to the incentives provided by the index based on a target system. In other words, here we discuss what to do (and not to do) with an overall quality of education index to boost the quality of education itself through incentive mechanisms.

Here there are two dimensions involved. The first one concerns the role of Ideb in influencing decisions about public expenditures on education. The second issue concerns the local government’s decision about how to allocate its educational expenditures towards seeking the different objectives contemplated in the index.

Therefore, we will solve the agent’s problem in two stages, each stage corresponding to each of the dimensions described above.

2.1 First Stage - The role of the Ideb in a Target-Based System

In this first stage we assume that the federal government transfers funds to local governments, who are the ones in charge of implementing the educational policies. The federal government thus may be regarded as the principal, whereas the agents are the local governments, hereafter referred to as municipalities. We will have the policy-makers maximizing a political function involving the allocation of public resources between different departments, from where we derive the optimal investment in education.

We will concentrate on static models with complete information, when the principal knows the agent type.
The utility functions for the federal government, \( U_F \), and for the municipality, \( U_M \), are respectively given by:

\[
U_F = g(G_F, (f(E))) \\
U_M = m(G_M, E)
\]

where \( E \) corresponds to the municipality's expenditures on education, \( G_M \) to its expenditures with other departments - the available budget after the educational expenditures are carried out - and \( G_F \) is the budget available to the federal government for spending on everything else, after resources have been transferred to the municipalities for education expenses. As we can see, the municipalities' utility depends on how much they spend on education and on its other departments\(^1\); and the federal government's utility depends on a function of the amount spent by the municipalities on education - that can be interpreted as the education quality - as well as on its expenditures towards other purposes.

Under a target-based system, the federal government faces a problem about how to offer a contract to the agent under which there is a transfer \((T)\) conditioned to the achievement of a pre-determined educational target \((E)\). Hence, his objective is to define a contract \((E, T)\) so that a target and a transfer are established. Firstly, it has to ensure that, upon accepting the contract, the agent will obtain at least the same utility it would obtain in autarchy.

If we choose a Cobb-Douglas functional form for both federal government and the municipality's utility function, we will have the following problem to be solved by the federal government:

\[
\begin{align*}
\text{Max } & (G_F)^n (f(E))^n \\
\text{s.a} & \\
G_F & \leq Y_F - T \\
G_M + E & \leq Y_M + T \\
U_M & = (G_M)^a (E)^b \geq U_A
\end{align*}
\]

where, besides the variables already described, \( Y_F \) is the government's total budget, \( T \) is the part of this budget that may be transferred to the municipalities for education purposes and \( Y_M \) is the municipalities' revenue. Looking at the restrictions, we have that \( G_F \leq Y_F - T \) is the government's budget restriction; \( G_M + E \leq Y_M + T \) the municipality's budget restriction; and the last one is the well-known individual rationality (IR) constraint - restriction of participation – where \( U_A \) is the outside option of the municipality (autarchy).

Supposing that both budget restrictions are biding, and inserting them in the utility and in the IR constraint, the principal's problem corresponds to

\[
\begin{align*}
\text{Max } & (Y_F - T)^n (f(E))^n \\
\text{s.a} & \\
U_M & = (Y_M + T - E)^a (E)^b \geq U_A
\end{align*}
\]

---

\(^1\) Here we are assuming implicitly that the results achieved on each field by the public managers, which in turn would be their source of political capital, will depend on the resources invested by them.
The Restriction of Participation will be binding, since in equilibrium the principal will give the minimum necessary for the agent to participate in the contract. Thus, we have that \((Y_M + T - E)^a.(E)^b = U_A\), which gives
\[ T = U_A^{\frac{1}{a}}E^{-\frac{b}{a}} + E - Y_M. \]

The problem can then be restated as
\[
\max_{E} (Y_F - U_A^{\frac{1}{a}}E^{-\frac{b}{a}} + E - Y_M)^m(f(E))^n
\]
whose first order conditions imply that
\[
[1 + \frac{m}{n} \varepsilon_E E_F]E_{TS} - \left[\frac{na - mb}{na}U_A^{\frac{1}{a}}\right]E_{TS}^{-\frac{b}{a}} = Y_M + Y_F
\]
where \(\varepsilon_E\), defined by \(\varepsilon_E = \frac{\partial f(E)}{\partial E} \frac{E}{f(E)}\), is the quality of the education elasticity with respect to the resources invested in education.

The equation above gives an implicit solution for \(E_{TS}\), the municipality's optimal investment in education under a target-based system.

If we make the assumption that the municipalities and the federal government give the same weight to education, that is, if we suppose that \(a=m\) and \(b=n\), we find as a result a close solution, which is
\[
E_{TS}^* = \left(\frac{n \varepsilon_E}{m + n \varepsilon_E}\right)[Y_M + Y_F] = \left(\frac{b \varepsilon_E}{a + b \varepsilon_E}\right)[Y_M + Y_F]
\]
This assumption means only that the managers in both political levels give the same relative importance to education, which is apparently not a strong assumption, since there is no reason to believe they do not.

If, in addition, we assume that the educational quality depends linearly on the resources invested in it by the municipalities, such as that \(f(E) = k E\), the municipality's optimal investment in education becomes
\[
E_{TS}^* = \left(\frac{n}{m + n}\right)[Y_M + Y_F] = \left(\frac{b}{a + b}\right)[Y_M + Y_F].
\]
This second assumption, in spite of being rather strong, is made for the sake of allowing for comparisons between the performance of a target-based system vis-à-vis other possible systems of education finance.

In order to assess the consequences of adopting a target-based system, we have to compare it with other alternatives. We will analyze the outcomes under the following regimes: (i) Autarchy, the basic situation in which the federal government does not carry out any transfer to the municipality; (ii) Unconditional Transfers, in which the federal government chooses to invest in determined places, transferring a fixed fund for the municipality to invest in the education area, without establishing any condition based on the accomplishment of results by the municipality; and a regime we will call (iii) The-Worst-The-Best, where we assume that the government decides to transfer more resources to the municipalities where the educational situation is worst, so that the lower the educational level, the greater is the per capita transfer carried out by the government to the municipality.
In Autarchy, for example, the municipality’s problem is:

$$\text{Max } (G_M)^a, (E)^b$$

$$\text{s.a.: } G_M + E \leq Y_M$$

From the first order conditions, supposing interior solution, we find as a solution

$$E_A^* = \frac{b}{a + b} Y_M$$

As we can see, the educational expenditures are a fraction of the total revenue, and depend proportionally on the relative weight given by the municipalities to education on their objective function. We see clearly that the investment in education is larger within a target system than it would be without any system of federal transfers. But this can also be explained by an income effect, since in autarchy the municipality has fewer funds to invest. Let’s examine now the financial arrangements that involve a transfer from the federal government.

Under an Unconditional Transfer regime, in turn, the problem of the municipality is given by:

$$\text{Max } (G_M)^a, (E)^b$$

$$\text{s.a.: } G_M + E \leq Y_M + T$$

From the first order conditions, supposing interior solution, we find as a solution

$$E_{UT}^* = \left(\frac{b}{a + b}\right) [Y_M + T]$$

The result is analogous to the one in autarchy, since the educational expenditures are a fraction of the total revenue available for the local manager - fraction that once again depends on the relative weight given by the municipalities to education on their objective function - but which includes here the federal transfers as well. Since $Y_F \geq T$, it becomes clear that public investment in education will be larger under a financial regime involving conditional transfers than when based on unconditional transfers, unless the federal government transfers all its revenue to the municipalities, a quite unrealistic scenario.

A The-Worst-The-Best transfer system would be one in which the federal government decides to transfer more resources to the places with the lowest level of educational quality, without any conditionality. In such case, the municipality’s problem becomes

$$\text{Max } (G_M)^a, (E)^b$$

$$\text{s.a.: } G_M + E \leq Y_M + T$$

$$T = K - f(E)$$

which means that the transfers depend on the difference between the municipality’s quality of education and a baseline value $K$ previously determined, which can be interpreted as a education poverty line.

The first order conditions, supposing interior solution imply that

$$\left(\frac{a E_E + b}{b}\right) f(E_{WB}) + \left(\frac{a + b}{b}\right) E_{WB} = Y_M + K$$

which defines the optimal value $E_{WE}^*$ implicitly. If we assume, as in the case of educational target system, that the educational quality depends linearly on the resources invested on it by the municipalities, assuming the form $f(E) = kE$, the municipality’s optimal investment in education then becomes
\[ E^{WB^*} = \left( \frac{b}{a + b} \right) \frac{1}{1 + k} [Y_M + K] \]

We see that this system as well generates less investment in public education than a target-based one. As long as the transfers depend on the difference between an ideal and a real value, rewarding the places with the worst performance, it provides a perverse incentive in the direction of keeping the educational quality as low as possible – since the larger the improvements, the less are the resources to be transferred in the future.

From these results we can state the following proposition, which summarizes the findings above.

**Proposition**

* A target-based transfer system generates more investment in public education by the local government than a system based on unconditional transfers, which in turn provides even more public expenditures on education than a system that transfers more resources to the places in worst situation.

*Proof*  Since \( K \leq T \leq Y_F \) and \( k > 0 \), we have that

\[
\left( \frac{b \epsilon_F}{a + b \epsilon_E} \right) [Y_M + Y_F] \geq \left( \frac{b}{a + b} \right) [Y_M + T] \geq \left( \frac{b}{a + b} \right) \frac{1}{1 + k} [Y_M + K]
\]

If we add the realistic assumptions that \( K < T < Y_F \), we have as a consequence that

\[ E^{TS^*} > E^{UT^*} > E^{WB^*} \]

The main conclusion so far is that a target-based system, by conditioning the federal government transfers to the educational performance, provides an incentive that leads the local governments to invest more resources in education than other alternative transfer systems. In an unconditional transfer regime, as well as in autarchy, the municipalities invest on education a fixed fraction of its total revenue that depends on the relative weight given by the municipalities to education on their objective function. In the *The-Worst-The-Best* regime, it spends a fraction of the revenue that is smaller than the relative importance given to education, due to a perverse incentive it creates. With a target-based system, in turn, the municipality will direct towards education a part of its revenue which is more than proportional to the importance it gives to education, due to the positive incentive it generates. Thus, we see that is best to reward future achievements than to compensate for past failures, or not to condition on anything.

**2.2 Second Stage – Interaction between Proficiency and Progression within the Ideb**

The second stage involves the local government’s decision about how to allocate its education budget – defined on the previous stage - as seeking to accomplish the different objectives contemplated by the index. The local government maximizes a function that represents the net benefit it derives from education, which in turn depend in some dimensions on the Ideb - at least that is the idea behind this target-based system. In order to simplify the problem, we will suppose the municipality maximizes the index itself, since its objective is to increase the index. The manager of the municipality’s problem is then how much of the budget he will invest in educational policies aiming to enhance each of the two components of the index, and can be formally stated as follows.
Max $\text{Ideb}$

\[
\begin{align*}
\text{s.a.: } C_Q(Q) + C_F(F) \leq E^* &= E
\end{align*}
\]

where $Q$ represents the variation in proficiency and $F$ the variation in the passing rate. The functions $C_Q(Q)$ and $C_F(F)$ correspond to the costs associated with an improvement in the proficiency and passing rates, respectively. By solving it, we find the optimal allocation of local resources between the two components of the index, namely $Q^*$ and $F^*$. In order to solve this problem, we have to choose specific functional formulas both for the objective function and for the budget restriction.

With respect to the objective function, we choose to model the educational index in a Cobb Douglas fashion for different reasons. Firstly, it is the functional form of the Ideb, in its simplest form, with both coefficients equal to one. Besides, the use of an index in a Cobb Douglas fashion has some advantages. One of them is that its exponents somehow express the degree of substitutability or complementarity between the index components, an issue that will also be analyzed. Another advantage is that it allows us to break down the growth rate of the education index into the growth rates of each of these components in an additive fashion, as follows.

\[
\text{Ideb} = (Q)\alpha \cdot (F)\beta \rightarrow \ln \text{Ideb} = \alpha \ln(Q) + \beta \ln(F) \rightarrow \gamma(\text{Ideb}) = \alpha \gamma(Q) + \beta \gamma(F)
\]

The option of choosing weights different than a unity in the index, differing from the original one, is explained on the next section.

In what concerns the budget restriction, we have to analyze how an improvement in proficiency and in the passing rate can be achieved, as well as the interaction between them. If proficiency and the passing rate were independent, the problem the local government would have to solve would be as simple as the following.

\[
\begin{align*}
\text{Max } (Q)\alpha \cdot (F)\beta.
\end{align*}
\]

\[
\begin{align*}
\text{s.a.: } p_qQ + p_fF &= E
\end{align*}
\]

whose first order conditions, assuming interior solutions, would give us

\[
\begin{align*}
Q^* &= \left( \frac{\alpha}{\alpha + \beta} \right) \frac{E}{p_q} \quad \text{and} \quad F^* = \left( \frac{\beta}{\alpha + \beta} \right) \frac{E}{p_f}
\end{align*}
\]

which would be the optimal values for proficiency and the passing rate that would lead to the largest evolution of the Ideb as possible. However, proficiency and the passing rate interact, as the following analysis suggests.

Increasing the student’s average proficiency has costs, which are related to the investment in the improvement of educational inputs, among which the most important are those related to labor - hiring more teachers and others involved in the educational process, increasing their salaries, providing additional training or creating monetary rewarding mechanisms - and to the schools’ infrastructure. Hence, it concerns a direct monetary cost. We will then define $p_q$, which can be understood as the price of each unity of additional proficiency.

The costs of increasing the passing rate, in turn, depend on the way it will be achieved. There are two main possibilities to attain an improvement in the passing rates. The first one concerns the adoption of artificial progression mechanisms, that is, exogenous changes in the rules that make it easier for the students to pass on to the next school year. This alternative involves a cost related to a reduction in quality, and no additional monetary costs. The second one concerns the improvement of the passing rate by increasing the students’
human capital, through an enhancement in the quality of the education. This alternative, in turn, involves a
direct monetary cost, analogous to the first one just described. We will model the two cases separately.

Model “passing by changing the passing regime”

Formally, in the first case, the municipality’s problem could be stated as

$$\max_{\{Q,F\}} (Q)^\alpha. (F)^\beta.$$

s.a. : \( p_q q \leq E \)

\( Q = q - h(F) \)

where the variation in the student’s average proficiency is divided into two components, in which \( q \) represents
the dimension of proficiency the local government can increase by investing in more education inputs, and \( h(F) \)
is a term that captures the effect of a potential reduction in proficiency due to an exogenous increase in the
passing rate, which can be a consequence, for example, of a loosening in the passing criteria or to a law that
determines automatic progression without changing the educational system as a whole\(^2\).

Hence, the problem can be restated as

$$\max_{\{F\}} (q - h(F))^\alpha. (F)^\beta.$$

s.a. : \( p_q q \leq E \)

It is straightforward to see that the municipality will spend the entire budget in quality-related actions
that will increase students’ proficiency, choosing thereby \( q = \frac{E}{p_q} \), and the problem thus becomes
choosing the optimal increase in the passing rate. For this the government will take into account that increasing
the passing rate has a twofold consequence: on one hand it increases the index through the passing rate
component itself, but on the other hand it decreases the index through reducing the proficiency component,
since it will reduce student’s incentive to produce effort, among other consequences. Formally, the local
government’s problem becomes

$$\max_{\{F\}} \left( \frac{E}{p_q} - h(F) \right)^\alpha. (F)^\beta.$$

The first order conditions for this problem, considering interior solution, give

\( F. h'(F) + \frac{\beta}{\alpha} h(F) = \left( \frac{\beta}{\alpha} \right) \frac{E}{p_q} \)

which defines \( F^* \), the optimal passing rate chosen by the local government, implicitly.

Let’s define \( \varepsilon_F^h = \frac{\partial h(F)}{\partial F} F = \frac{\partial Q}{\partial F} \frac{F}{h(F)} \) as the proficiency elasticity with respect to the passing rate,
corresponding intuitively to the rate according to which an exogenous increase in the passing rate impacts
negatively proficiency.

\(^2\) Pedagogical studies suggest that the implementation of an automatic progression system demand a deep reformulation in
the educational system - with a radical changing in the organization of school, the adoption of different strategies of teaching
and evaluation, investments in the training of teachers and a increase in the ratio of teachers per pupil – in order not to
produce a decrease in student’s performance.
The equation above then becomes

\[ h(F^*) = \left( \frac{\beta}{\alpha e_F^* + \beta} \right) \frac{E}{p_q} \]

which defines the optimal value of the approval rate chosen by the local government. To find a close value for \( F^* \) is just a matter of defining a closed functional form for \( h(F^*) \).

By using the fact that \( Q = q - h(F) \) and that \( q = \frac{E}{p_q} \), we find the optimal value for the proficiency evolution, which is

\[ Q^* = \left( \frac{\alpha e_F^*}{\alpha e_F^* + \beta} \right) \frac{E}{p_q}. \]

If we assume a linear form for the function \( h \) such as \( h(F^*) = \psi F^* \) - just for the sake of comparability with the following models – we find

\[ F^* = \left( \frac{\beta}{\alpha + \beta} \right) \left( \frac{1}{\psi} \right) \frac{E}{p_q} \quad \text{and} \quad Q^* = \left( \frac{\alpha}{\alpha + \beta} \right) \frac{E}{p_q} \]

From these results, we can infer that

(i) Both the optimal proficiency and passing rate improvements are positively related to the total investment in education \( E \) and negatively related to the cost of improving proficiency \( p_q \), due to an income effect and a price effect, respectively.

(ii) The larger the weight given to the approval rate vis-à-vis to the students’ proficiency in the index \( \left( \frac{\beta}{\alpha} \right) \), the larger it will be the improvement in the passing rate aimed by the municipality, and the smaller the increase in students’ average proficiency, and vice-versa.

(iii) The proficiency elasticity with respect to the passing rate \( \psi \) is unrelated to the variation in proficiency and negatively related to the variation in the passing rate. The larger the cost in terms of quality induced by an increase in the passing rate, the less the municipality will be willing to induce students to progress through artificial mechanisms.

Model “passing by learning”

If, in turn, the second strategy is adopted, the one of improving the passing rate by means of investing in school quality, the agent’s problem will be rather different. It can be stated as

\[
\begin{align*}
\text{Max} & \quad (Q)\alpha (F)^\beta \\
\text{s.a : } & \quad p_q Q \leq E \\
F = & \quad F(Q)
\end{align*}
\]

where the last restriction expresses the fact that in this case the increase in the passing rate will depend on the increase in the quality of education.

Since improvements in both proficiency and the passing rate will be achieved through investments to enhance the quality of education, hereafter we can adopt only one interest variable to be chosen by the local manager. Therefore, he will use the entire budget towards quality-related policies in order to increase
proficiency. Formally speaking, we can use only the budget restriction to find the optimal $Q^*$ he will attain, which gives. Thus we will have $q^* = \frac{E}{p_q}$.

Finding the optimal $F^*$ is then only a matter of inserting $Q^*$ into $F(Q)$. To illustrate the point, let's suppose that the impact of a variation in proficiency in the passing rates assumes a linear functional form, so that $F(Q) = \eta Q^*$ In this case we will have a closed solution to $Q^*$ and $F^*$, which are

$Q^* = \frac{E}{p_q}$ and $F^* = \frac{\eta E}{p_q}$

As we can see from these results

(i) Both the optimal proficiency and passing rate improvements are positively related to the total investment in education ($\bar{E}$) and negatively related to the costs of improving the quality of education provided ($p_q$), the first expressing a pure income effect and the second that the income effect out weights the substitution effect.

(ii) The optimal passing rate improvement is positive related to the sensibility of the passing rate to variations in proficiency ($\eta$), whose intuition is straightforward.

(iii) The resulting proficiency and passing rate improvements are not related to the index’s weights.

General Model

We have considered so far two polar cases. In the first one, local governments increase the passing rates only by means of slackening the passing criteria, which means that more students are progressing through the schooling levels because it was made easier for them to progress. In the other case, the increase in the passing rates is only achieved through an improvement in school quality, which means that more students are progressing because they are learning more. However, the local governments may use both strategies at the same time. That is the reason why our next exercise will be to model both strategies in an integrated framework, in which part of the improvements in the passing rates can be achieved through a change in the passing regime and part through an improvement on the quality of education.

The local government’s problem, in this case, can be formally stated as

$$\max_{(Q,F)} (Q)^{\alpha}, (F)^{\beta}$$

s.a.

$$Q = q - h(F)$$

$$F = F_1 + F_2$$

$$F_2 = F_2(Q)$$

$$p_q q \leq \bar{E}$$

The first restriction is the same as the one used before, and express the fact that the variation in the student’s average proficiency can be divided into two components, in which $q$ represents the increase in quality achieved by investing in more education inputs, and $h(F)$ is a term that captures the effect of a potential reduction in quality due to an exogenous increase in the passing rate. The second one refers to the two ways through
which improvements in the passing rate can be achieved: $F_1$ is the part of the variation in the average passing rate attained artificially through a slackening in the passing criteria and $F_2$ the one attained through an improvement in quality. That is precisely what the third restriction expresses. The last one is the local government’s budget constraint.

First of all, we have the local government spending its entire budget in quality related expenditures, which gives us $q^* = \frac{\bar{E}}{p_q}$.

Then, inserting the restrictions inside the objective function, it becomes

$$\max (q - h(F_1))^\alpha (F_1 + F_2(q - h(F_1)))^\beta$$

As we can see, it depends only on $F_1$, which will be the local manager’s only variable of choice. Solving this problem, assuming interior conditions, we find

$$\left(\frac{\alpha}{\beta} \frac{F_1 + F_2(q - h(F_1))}{q - h(F_1)} = \frac{1 - F_2'(q - h(F_1)) h'(F_1)}{h'(F_i)}\right)$$

which defines $F_1^*$ implicitly.

Let’s define explicit functional forms for the functions described above, such as $F_2(Q) = \eta Q$ and $h(F_1) = \psi F_1$, where $\eta$ and $\psi$ are constants that explicit the assumption that the passing rate and the function $h$ are increasing functions of the students’ proficiency and the exogenous change in the passing rate, respectively, and therefore satisfy $\psi > 0$ and $\eta > 0$.

This allows us to find a close solution for $F_1^*$, which is

$$F_1^* = \frac{(\frac{\beta}{\alpha + \beta} - \eta \psi)}{(1 - \eta \psi \psi)} \frac{\bar{E}}{p_q}$$

and then we can proceed to find all the variables of interest. Knowing $q^* = \frac{\bar{E}}{p_q}$ and $F_1^*$ we find $Q^*$, which is $Q^* = q^* - h(F_1^*)$ and therefore

$$Q^* = [(\frac{\alpha}{\alpha + \beta})(\frac{1}{1 - \eta \psi})] \frac{\bar{E}}{p_q}$$

$Q^*$, in turn, determines $F_2^*$, given by $F_2^* = [(\frac{\eta}{\alpha + \beta})(\frac{\eta}{1 - \eta \psi})] \frac{\bar{E}}{p_q}$

And finally, by summing up $F_1^*$ and $F_2^*$, we have $F^*$, which is

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\[1\] It may happen that the more loosen the passing criteria, the larger the sensibility of the passing rate to an variation in quality, which would mean assuming that the parameters of the function $F_2(.)$ to be a functions of $F_1$. However, we will assume that those parameters are constant and exogenous.

\[2\] There are some studies that show that when the quality of education increases, teachers may demand more from the students, and therefore we can possibly have the passing rates decreasing as a consequence. However we will not consider here this possibility, which would correspond to assume passing to be endogenous. We will only take it as exogenous.
\[ F^* = \left[ \frac{\beta}{\alpha + \beta} \cdot \frac{1}{\psi} \right] \frac{\bar{E}}{p_q} \]

The results describe above imply the following proposition.

**Proposition**

(i) Both the optimal proficiency and passing rate improvements are positively related to the total investment in education (\( \bar{E} \)) and negatively related to the costs of improving the quality of education provided (\( p_q \)), as it was expected. The first expresses a pure income effect and the second that the income effect outweighs the substitution effect.

(ii) The larger the weight given to the passing rate vis-à-vis to the students’ proficiency in the index (\( \frac{\beta}{\alpha} \)), the larger it will be the improvement in the passing rate aimed by the municipality, and the smaller the increase in students’ proficiency, and vice-versa.

(iii) The more the passing rate responds (\( \eta \)) to an increase in proficiency, the larger the resulting variation in proficiency, whose intuition is straightforward.

(iv) The larger the sensibility of the quality of education to an exogenous change in the passing regime (\( \psi \)), the smaller the variation in the passing rate and the larger then variation in proficiency. The larger the cost in terms of quality induced by an increase in the passing rate, the more the municipality will substitute into proficiency and out of the passing rate.

**Proof** We just have to compute the comparative statics

In this general model, in which the local government can freely choose the best way to improve the index, the following happens. The entire budget towards educational expenses will be spent in educational inputs aiming to improve the students’ average proficiency. Then the local government will choose the passing regime, considering the amount he will allow the passing criteria to be slackened in order to improve the passing rate, which involves also a choice over the amount of reduction in proficiency the manager will allow to happen as a consequence of it. This choice combined with the amount spent seeking to improve proficiency, will determine the total variation in students’ average proficiency. This variation in students’ average proficiency, in turn, will determine the second component of the variation of the passing rates. Summing up these two components, we will have the total variation in the average passing rate.

The conclusion that follows is that, in order to simultaneously increase proficiency and the passing rate, we must have either an increase in the amount invested in education or the local governments becoming more efficient in what concerns spending less to improve each unity of quality. In addition, if the federal governments wish the local governments either to give larger priority to proficiency or to the passing rates, it must tackles Ideb’s weights.

The next table summarizes the results stemming from the three models above.
### The case described in the first model actually will not be found in reality, since it implicitly assumes that students’ proficiency does not impact the passing rate. The case described in the model “passing by learning”, instead, can be considered as a particular case in which there is no possibility of improving the passing by changing the passing regime, that is, by artificially slackening the passing criteria. Comparing then this model with the general model is analogous to comparing an equilibrium with commitment to a discretionary one, as in Person and Tabelini (1996), where commitment in the present case would mean a situation in which the municipalities would not be allowed of changing the passing criteria in order to improve the passing rate.

Within this framework, we can state the following proposition.

**Proposition**

The resulting variation on the components of the index in each case will depend on the parameters. If the relative weight of the passing rate vis-à-vis proficiency is above a certain threshold – which corresponds to the multiplication of the sensibility of each component with respect to the another one – then a higher variation in average proficiency and a lower variation in the average passing rate will be achieved under a regime that does not allow for artificial mechanisms of progression (equilibrium with a rule) than under a non-restrict regime (equilibrium with discretion).

**Proof** From the comparison of the results stemming from the two alternatives models - Model “passing by learning” and General Model- we arrive at the following conditions:

* If \( \frac{\beta}{\alpha + \beta} \leq \eta \psi \) then \( Q^*_{\text{gn}} \geq Q^*_{\text{pbl}} \) and \( F^*_{\text{gn}} \leq F^*_{\text{pbl}} \)

* If \( \frac{\beta}{\alpha + \beta} > \eta \psi \) then \( Q^*_{\text{gn}} < Q^*_{\text{pbl}} \) and \( F^*_{\text{gn}} > F^*_{\text{pbl}} \)
3 Measurement and Evaluation Issues

In this section of the paper, which is mainly a normative one, we argue that there is room for improving both Ideb’s methodology as well as the way the evaluation of its evolution will be assessed. In the first part we discuss the desirable properties of an ideal education index and in the second one we focus on evaluation issues.

3.1 Pursuing an ideal Educational Index

Index’s Weighting

One of the virtues of the Ideb as an indicator is its simplicity; and its advantage is combining two central dimensions to the question of educational quality in a synthetic index. Nevertheless, the equal weight of its two components is an arbitrary choice, in the sense that there is no other reasons besides simplicity for which they should have the same weight.

In order to address this question, we have to analyze the incentive it seeks to provide. On one hand, the larger the weight of the flux component vis-à-vis the weight of the proficiency component, the larger the incentive for the local administrator to accelerate artificially the promotion of the students - in a radical and compulsory manner - without the reformulations that are necessary to the implementation of such regime, which could lead to high costs in terms of quality of education. On the other hand, the larger the weight of the proficiency component vis-à-vis the flow component, the larger the incentive for local managers not to resist evasion, or even to increase retention or to motivate the worst students to evade, so that only the best students are submitted to standardized tests, improving thereby measured proficiency.

Perhaps it is not a coincidence that some policymakers have recently adopted automatic promotion regimes soon after the plan was launched. This could be interpreted as the first signal about the Ideb’s capacity of changing policymaker’s actions.

We should therefore seek to avoid these kinds of unbalanced behaviors. Mathematically speaking, we have to avoid that the local government choose corner solutions when trying to increase the index.

Formally we suggest an index in the form

\[ \text{Ideb} = Q^\alpha F^\beta \]

and suggest that it is very important to try to analyze and to estimate which would be an optimal weighting rationale for the index, that is, which should be the coefficients \( \hat{\alpha} \) e \( \hat{\beta} \).

Including out-of-school children in the Index

Another relevant proposal is the inclusion of the dimension of out-of-school children in the index, with the double aim of (a) making the local managers responsible for non-enrolled school age children, while also (b) focusing on the process of enrollment expansion according to the evolution of the index.

\[ \text{\footnote{\textsuperscript{5}} However, to proceed this estimation is out of the scope of the present study.} \]
We observe that in 2006 only 2.7% of children aged between 7 and 14 years old were not enrolled at school. Nevertheless, 18.3% of children aged between 15 to 17 years old were not at school. From this, we can infer that the majority of those students who are out of school had evaded, as opposed to children who have never been to school. Besides, the data shows that the expected conclusion rate of basic education was only 31.2% in 2004, in a decreasing trend—the rate in 2000 was 40.3%.

This is a problem that has to be tackled. Otherwise, there may be an incentive for mayors to leave a good deal of marginalized young people out of the school system in order to preserve the municipality’s Ideb. The index, as it is built, provides only an incentive for increasing the performance of students who are in school, concerning both their proficiency and their probability of progressing on to the next school year. It does not provide incentives for children to be brought back into school. Actually, it provides an incentive for preventing children from failing and abandoning or evading school, but giving up on them as soon as they abandon school, since their re-inclusion would potentially worsen the Ideb, for those who abandon school have on average a lower proficiency rate and a higher probability of not being approved.

Some feasible possibilities would be (i) to include children out of school in the computation of average proficiency by using some counter-factual imputation methodology, such as in Neri e Carvalho (2002), Franco et all (2003), Reynaldo e Natzenon (2003) and Alves (2007); (ii) to include this dimension in the index as a third multiplicative factor; or to include in the index some factor that rewards public managers for each student brought back to school, through a sort of bonus within the index, for instance, by adding an addictive factor to be summed upon the index.

3.2 Evaluation of the index’s evolution in a target-based system

In this section we will discuss the use of the Ideb within a target-based system framework, analyzing the best way through which the evolution of the index should be assessed.

Value added

Schools should be evaluated for their capacity of adding value to the knowledge of the students, since this is their essential function. Therefore, we argue that it is best that the evaluation of the municipalities and schools through the Ideb to be based on the value added by the schools to the students rather than on level.

This methodology has several advantages, among which we have: (i) such a system benefits students from disadvantaged backgrounds - since being less educated presents one advantage: that of being able to learn more; (ii) private and public resources will tend to migrate to places that offer higher returns, which will create a positive incentive for schools to improve the quality of their services, which would attract the best students, thus creating a virtuous cycle; and (iii) it also provides an incentive for schools to mix students with privileged and disadvantaged backgrounds, which can play a very important role in an unequal and diverse country like Brazil.

Evaluation of Comparative Performance

It is clearly better that it is best that the targets be based on the value added than on level. But, in a context of uncertainty, we have to go one step further. Evaluating the evolution of the index within each municipally and each school should be carried out using a standard methodology for evaluating social programs, namely the analysis of differences-in-differences. The idea is to compare the municipalities - and their schools - by the difference in the value added to the students through formal education by each of them.
A characteristic of contracts based on a simple value added evaluation is that there is a positive probability that, even if the municipality takes all the necessary procedures to reach the target, it fails in doing so due to unexpected negative shocks. Contracts based only on the variation, or value added, are usually procyclical, reducing the government transfers when they are more necessary, and increasing them when they are less necessary. Most of these aggregate shocks are exogenous, that is, are not under direct control of the public managers.

One way of tackling this issue is through the utilization of contracts based on comparison of performance across municipalities. This contract system involves conditioning transfers to the municipality performance vis-à-vis other municipalities. A transfer from the federal government, thereby, will depend only on the difference between the outcome resulted from the amount the public manager invested on education and the outcome obtained by other municipalities.

In Neri and Xerez (2007) it is shown that, when the social results do not depend only on the investment realized by the municipalities, but also on random factors, such as exogenous shocks, mechanisms based on performance comparison are the one capable of reaching the best results. Contracts based on the differences between the value added across municipalities avoid the pro-cyclical characters of pure value added based contracts. In such a system, a local government investing an optimal amount in education and doing it in an efficient way will receive an optimal transfer regardless of the shocks.

This robustness to aggregate shocks provided by this system based on relative measures of performance is especially important with respect to educational targets, since they consist, by their even nature, of long-term targets.

Integrated Evaluation System

Finally we propose also the integration of all standardized tests that are already being carried out in Brazil and their application at the end of each school level, with a convergence in their methodology and sample, so that they can be comparable. Hence, we would have a whole system of proficiency evaluation that would allow us to follow every student over his entire schooling trajectory, and the value added of each school level to his performance.

4. Conclusion

Some conclusions and suggestions concerning both policy and methodological issues emerge from this work. First we show, by means of building a principal agent model, that this target-based system with conditional transfers is an improvement in relation to other alternative educational finance systems. Since the transfer from the federal government are conditioned by an increasing in the index, the local governments will have a stronger incentive to invest more in education than they would if transfers were not conditional or if they went preferentially to the place with worst educational performance. Actually, it will spend in education a fraction of its revenue which is larger than the relative importance the local manager gives to education vis-à-vis other areas.

It is worth noting that the PDE educational plan implementing a target-based system in Brazil happens in a moment when different actors are converging around the need to determine educational targets as the basis of educational policies. In 2000 the Brazilian government signed the Dakar Goals - Education for All Commitment, with six main goals to be reached until 2015; while a very important
umbrella NGO involving many relevant actors of civil society, named *Compromisso Todos pela Educação* (All for Education Commitment) also set five targets to be reached by 2022, when the country completes two centuries of its political independence.

Second, we analyzed the strategies the municipalities can adopt to improve the components of the index and how they will allocate the resources while trying to accomplish this objective. We show that they will spend their resources in quality-related investment in order to enhance the student’s average proficiency and choose a passing regime based on how much they will allow the passing criteria to be slackened in order to improve the passing rate, which involves also a choice over the amount of reduction in proficiency the manager will allow to happen as a consequence of it. The resulting total improvement in the student’s average proficiency and in the passing rate will stem from this tension, and will depend also on the weights given to each component in the index as well as on the degree of interaction between them, which involves both the sensibility of the proficiency to an exogenous change in the passing regime and the sensibility of the passing rate to a variation in students’ proficiency.

Furthermore, we suggest that there is still room for improvement concerning methodological issues related to the construction of the index itself. We suggest that (i) an weight equal to the two components of the index - proficiency and the passing rate - is arbitrary and that an optimal weight should be pursued, to avoid an unbalanced behavior by the local managers, such as choosing corner solutions. We also propose that (ii) out-of-school children should also be incorporated in the index, with the double aim of making the local managers responsible for non-enrolled school age children, while also taking into account the process of expansion of enrollment in the evolution of the index.

Last, but not least, we analyzed how to best evaluate the evolution of the Ideb and the related incentives. We propose a methodology of value added evaluation, based on differences-in-differences and analysed its advantages vis-à-vis evaluation based on level are abundant. We argue in favor of a transfer system based on the comparative performance between municipalities, with conditionalities depending on the relative value added by each of them. Perhaps the main advantage of such a system is to eliminate the pro-cyclical effect that can result from aggregate shocks that cannot be diversified. Finally, we suggested that is essential to have a uniform and integrated system of evaluation for the success of a target-based system.

There are few dynamic extensions of the framework in various directions found in the literature that can be incorporated in the developed model as, for instance, the inclusion of the dichotomy between complete and incomplete contracts with and without renegotiation clauses and the incorporation in the model the incidence of measurement error in the variable targeted.

The main idea behind this study it the belief that, by improving the education quality indicator, which will provide the basis both for the education debate and the public transfers, we hope to improve the quality of education itself. By guaranteeing resources to areas with the fastest improvement rates, we move closer to fulfilling the promise of high quality education for all.
References


Franco et al. (2003) *Os resultados do SAEB em Perspectiva Longitudinal*


