Long-Run Valuation Risk and Asset Pricing

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Correlation puzzle

• Standard asset pricing models
  – Expected return for holding an asset reflects covariance between asset’s payoff and agent’s stochastic discount factor.

• Important challenge facing these models:
  – Covariance and correlation between stock returns and measurable fundamentals, especially consumption, is weak at 1, 5, and 10 year horizons.

  – Lettau and Ludvigson (2011): shock that accounts for vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.
Weak correlation between stock returns, ‘fundamentals’

- Hansen and Cochrane (1992) and Cochrane and Campbell (1999) call this phenomenon the “correlation puzzle.”

- This fact underlies virtually all modern asset-pricing puzzles.
  - The equity premium puzzle, Hansen-Singleton-style rejection of asset pricing models, Shiller’s excess volatility of stock prices, etc..
Asset prices and economic fundamentals

• Classic asset pricing models load all uncertainty onto supply-side of the economy.
  – Stochastic process for the endowment in Lucas-tree models.
  – Stochastic process for productivity in production economies.

• Abstract from shocks to the demand for assets.

• Not surprising that these models can’t simultaneously account for the equity premium and correlation puzzles.
Fundamental shocks

- What’s the other shock?

- We explore the possibility that it’s a shock to the demand for assets.
Shocks to the demand for assets

- Demand shocks arise from stochastic changes in agents’ rate of time preference.

- ZLB literature suggests these shocks are useful way to model changes in household savings behavior.
  - e.g. Eggertsson and Woodford (2003) and Huo and Rios-Rull (2013).

- These shocks also capture effects of changes in demographics of stock market participants or other institutional changes that affect savings behavior.
Preference Shocks and the Risk-Free Rate

- Preference shocks are measurable because they map directly into the level of the one-period ahead risk free rate.

- We parameterize law of motion for preference shocks to be consistent with time series properties of risk-free rate.
Model

- Representative agent model with Epstein-Zin recursive preferences.

- Estimate model using data over sample period 1929 to 2011.

- Model accounts for key asset pricing moments
  - Equity premium, volatility of stock and bond returns, etc.
  - Correlation between stock returns and market fundamentals at short, medium and long horizons.
What drives asset prices?

- *Valuation risk*: risk associated with changes in the way future cash flows are discounted.

- In our model this risk is entirely due to demand (preference) shocks.

- According to our estimates, valuation risk is *much* more important source of variation in asset prices than conventional covariance risk.
A Natural Test

• Valuation risk is an increasing function of asset’s maturity.

• Can our model account for return on stocks relative to long-term bonds and the term structure of bonds?

• Yes.
The correlation puzzle: U.S. data

- We compute correlations between realized stock returns and the growth rates of consumption, dividends, output, and earnings.

- We use U.S. data covering the period 1929-2011.
  - Kenneth French’s website for nominal stock returns data.
  - Interest rate data from Ibbotson Associates.
  - We converted nominal returns to real returns using CPI inflation.
  - Shiller for real S&P500 earnings and dividends.
  - Barro and Ursúa (2008) for consumption expenditures and real per capita GDP, updated to 2011 using NIPA data

The correlation puzzle: U.S. data, 1929-2011

Correlation between stock returns and per capita growth rates of fundamentals

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Consumption</th>
<th>Output</th>
<th>Dividends</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>−0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>5 years</td>
<td>0.002</td>
<td>0.00</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>10 years</td>
<td>−0.11</td>
<td>−0.09</td>
<td>0.59</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.11)</td>
</tr>
</tbody>
</table>
The correlation puzzle: U.S. data, 1871-2006

Correlation between stock returns and per capita growth rates of fundamentals

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Consumption</th>
<th>Output</th>
<th>Dividends</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.090</td>
<td>0.136</td>
<td>-0.039</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.101)</td>
<td>(0.0956)</td>
<td>(0.1038)</td>
</tr>
<tr>
<td>5 years</td>
<td>0.397</td>
<td>0.249</td>
<td>0.382</td>
<td>0.436</td>
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<tr>
<td></td>
<td>(0.177)</td>
<td>(0.137)</td>
<td>(0.148)</td>
<td>(0.1790)</td>
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<tr>
<td>10 years</td>
<td>0.248</td>
<td>-0.001</td>
<td>0.642</td>
<td>0.406</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
<td>(0.113)</td>
<td>(0.173)</td>
<td>(0.125)</td>
</tr>
</tbody>
</table>
Robustness: NIPA measures of consumption, 1929-2011

Correlation between stock returns and per capita growth rates of fundamentals

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Durables</th>
<th>Non-durables</th>
<th>Non-durables, Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>-0.04 (0.13)</td>
<td>0.05 (0.14)</td>
<td>0.27 (0.12)</td>
</tr>
<tr>
<td>5 years</td>
<td>0.07 (0.11)</td>
<td>-0.08 (0.09)</td>
<td>0.18 (0.10)</td>
</tr>
<tr>
<td>10 years</td>
<td>0.21 (0.15)</td>
<td>-0.20 (0.13)</td>
<td>0.06 (0.13)</td>
</tr>
</tbody>
</table>
Covariance versus Correlation

  - Needs a risk aversion coefficient of 379 to account for equity premium.

- There’s a larger covariance between current stock returns, cumulative consumption growth over next 12 quarters.

- He also uses this larger covariance in his calculations.
  - Still needs a risk aversion coefficient of 38 to rationalize equity premium.
Correlation puzzle: a challenge for ‘all supply-side’ models

• Lucas-style CRRA or standard Epstein-Zin type models.

• Habit-formation model (internal or external).

• Long-run risk models.

• Rare-disaster models: all shocks, disaster or not, are to supply side of the model.

• In principle, model with time-varying disaster probability could account for correlation puzzle as small sample phenomenon.
  
  – But correlation puzzle holds even in long sample 1870 - 2006.
A model with time-preference shocks

- Epstein-Zin preferences
  - Life-time utility is a CES of utility today and the certainty equivalent of future utility, $U_{t+1}^*$.
  
  $$U_t = \max_{C_t} \left[ \lambda_t C_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- $\lambda_{t+1}/\lambda_t$ determines how agents trade off current versus future utility, isomorphic to a time-preference shock.

- $\psi$ is the elasticity of intertemporal substitution.
A model with time-preference shocks

\[ U_t = \max_{C_t} \left[ \lambda_t C_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)} \]

- The certainty equivalent of future utility is the sure value of \( t + 1 \) lifetime utility, \( U_{t+1}^* \) such that:

\[ (U_{t+1}^*)^{1-\gamma} = E_t \left( U_{t+1}^{1-\gamma} \right) \]

\[ U_{t+1}^* = \left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)} \]

- \( \gamma \) is the coefficient of relative risk aversion.
Special case: CRRA

• When $\gamma = 1/\psi$, preferences reduce to CRRA with a time-varying rate of time preference.

\[ V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma}, \]

where $V_t = U_t^{1-\gamma}$.

• This case was the one considered by Garber and King, and Cambell.
Stochastic processes

- Consumption follows a random walk

\[
\log(C_{t+1}) = \log(C_t) + \mu + \sigma_c \varepsilon_{t+1}^c \\
\varepsilon_{t+1}^c \sim N(0, 1)
\]

- Process for dividends:

\[
\log(D_{t+1}) = \log(D_t) + \mu + \pi_{dc} \varepsilon_{t+1}^c + \sigma_d \varepsilon_{t+1}^d \\
\varepsilon_{t+1}^d \sim N(0, 1)
\]

- \(\varepsilon_{t+1}^c\) and \(\varepsilon_{t+1}^d\) are uncorrelated.

- Our assumptions imply \(\log(D_{t+1}/C_{t+1})\) follows a random walk with no drift.
Stochastic processes

- Time-preference shock:
  \[
  \log \left( \frac{\lambda_{t+1}}{\lambda_t} \right) = \rho \log \left( \frac{\lambda_t}{\lambda_{t-1}} \right) + \sigma \varepsilon_{t+1}^\lambda \\
  \varepsilon_{t+1}^\lambda \sim N(0,1)
  \]

- Agents know \( \lambda_{t+1} \) at time \( t \).

- We assume \( \lambda_{t+1}/\lambda_t \) is highly persistent but stationary (\( \rho \) is very close to one).

- The idea is to capture, in a parsimonious way, persistent changes in agents’ attitudes towards savings.
Stochastic processes

• We assume that $\varepsilon^\lambda_{t+1}$ is uncorrelated with $\varepsilon^c_{t+1}$ and $\varepsilon^d_{t+1}$.

• This assumption is reasonable for an endowment economy but not for a production economy.
CRRA Case

• Suppose \( \gamma = 1/\psi \).

• Then in this CRRA case, unconditional equity premium is proportional to risk-free rate:

\[
E \left( R_{c,t+1} - R_{f,t+1} \right) = E \left( R_{f,t+1} \right) \left[ \exp \left( \gamma \sigma_c^2 \right) - 1 \right].
\]

• Average risk-free rate \( E \left( R_{f,t+1} \right) \) and volatility of consumption \( \sigma_c^2 \) are small in the data.

• Constant of proportionality \( \exp \left( \gamma \sigma_c^2 \right) - 1 \), is independent of \( \rho \) and \( \sigma_\lambda \).

• So time-preference shocks don’t help to resolve equity premium puzzle without having counter-factual implications for \( E \left( R_{f,t+1} \right) \).
Solving the model

• Define returns to the stock market as returns to claim on dividend process:
  – Standard assumption in asset-pricing literature (Abel (1999)).

• Realized gross stock-market return:

\[ R_{d,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}. \]

\[ r_{d,t+1} = \log(R_{d,t+1}), \]

• Realized gross return to a claim on the endowment process:

\[ R_{c,t+1} = \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}. \]

\[ r_{c,t+1} = \log(R_{c,t+1}), \]
Solving the model

- The log-SDF is:

\[ m_{t+1} = \theta \log (\delta) + \theta \log (\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \]

\[ \theta = \frac{1 - \gamma}{1 - 1/\psi}. \]

- Euler equation:

\[ E_t [\exp (m_{t+1} + r_{d,t+1})] = 1 \]


- Taylor series approximation of \( r_{ct}, r_{dt}, \) method of undetermined coefficients, log-normality.
Returns to equity

- Conditional expected return to equity is given by:

\[ E_t (r_{d,t+1}) = -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi + \left[ \frac{(1-\theta)}{\theta} (1-\gamma)^2 - \gamma^2 \right] \sigma_c^2 / 2 + \pi_d c (2\gamma \sigma_c - \pi_{dc}) / 2 - \sigma_d^2 / 2 + b(\lambda) \]

\[ b(\lambda) = \left[ (1-\theta) (\kappa_{d1} A_{c1}) [2 (\kappa_{d1} A_{d1}) - (\kappa_{c1} A_{c1})] - (\kappa_{d1} A_{d1})^2 \right] \sigma_\lambda^2 / 2 \]

- \( E_t (r_{d,t+1}) \) is a decreasing function of \( \log(\lambda_{t+1}/\lambda_t) \).
  - If agents value the future more, relative to the present, they want to save more. Since aggregate savings can’t increase, the returns to savings has to fall.
Returns to equity

- Compensation for *valuation risk*: part of one-period expected return due to volatility of the time preference shock, $\sigma^2_\lambda$.

- Compensation for *conventional risk*: part of expected return due to volatility of consumption and dividends.
Valuation risk for stocks

• Suppose stock market is a claim on consumption.

• Then valuation risk for stocks is

\[ v_d = -\theta (\kappa_{c1}A_{c1})^2 \sigma^2 / 2. \]

• \( v_d \) is positive as long as \( \theta \) is negative, i.e. as long as \( \gamma > 1 \) and \( \psi > 1 \) or \( \gamma < 1 \) and \( \psi < 1 \).
  - If agents have coefficient of risk aversion higher than one, condition requires that agents have a relatively high elasticity of intertemporal substitution.
  - If agents have coefficient of risk aversion lower than one, they must have a relatively low elasticity of intertemporal substitution.
The risk-free rate

\[
rf_{t+1} = - \log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1 - \theta) \kappa_{c1}^2 A_{c1}^2 \sigma^2_\lambda / 2 + \left[ \frac{(1 - \theta)}{\theta} (1 - \gamma)^2 - \gamma^2 \right] \sigma^2_c / 2.
\]

- Risk-free rate is a decreasing function of \( \log(\lambda_{t+1}/\lambda_t) \).
  - As with equity, if agents value the future more, relative to the present, they want to save more.
  - Since aggregate savings can’t increase, the risk-free rate has to fall.
The risk-free rate and preference shocks

\[ rf_{t+1} = - \log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1 - \theta) \kappa_c^2 A_c^2 \sigma_\lambda^2 / 2 \]

\[ + \left[ \frac{(1 - \theta)}{\theta} (1 - \gamma)^2 - \gamma^2 \right] \sigma_c^2 / 2, \]

- Risk-free rate is constant minus \( \log(\lambda_{t+1}/\lambda_t) \).

- So we can measure movements in \( \log(\lambda_{t+1}/\lambda_t) \) using the risk-free rate.
Equity premium

\[ E_t (r_{d,t+1}) - r_{f,t+1} = \pi_{dc} \sigma_c^2 (2 \gamma - \pi_{dc}) / 2 - \sigma_d^2 / 2 \]
\[ + \kappa_{d1} A_{d1} [2 (1 - \theta) A_{c1} \kappa_{c1} - \kappa_{d1} A_{d1}] \sigma^2_{x} / 2. \]

• For time-preference shocks to help explain equity premium, we need \( \theta < 1. \)
Valuation risk and the equity premium

• When stock is claim on consumption, component of the equity premium to valuation risk is:

\[(1 - 2\theta) \left( \frac{k_{c1}}{1 - \rho k_{c1}} \right)^2 \sigma^2_{\lambda}/2.\]

• This expression is positive as long as \(\theta < 1/2\).

• This condition is always satisfied in our empirical work.
Valuation Risk vs Conventional Risk

• Say there’s no risk associated with physical payoff of assets like stocks.
  – Standard models imply equity premium is zero.
  – In our model, there’s a positive equity premium because bonds, stocks have different exposure to valuation risk.

• Agents are uncertain about how much they’ll value future dividend payments.
  – Since $\lambda_{t+1}$ is known at time $t$, this valuation risk is irrelevant for one-period bonds.
  – Not irrelevant for stocks, because they have infinite maturity.

• The longer the maturity of an asset, the higher is its exposure to time-preference shocks and the larger is the valuation risk.
Valuation Risk vs Conventional Risk...

• Say there are supply-side shocks to the economy but agents are risk neutral ($\gamma = 0$).

• Component of equity premium due to valuation risk is positive as long as $\psi$ is less than one.

• Stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of the representative agent, $\lambda_t C_t^{1-1/\psi}$.

• Even when agents are risk neutral, they must be compensated for risk of this unwanted variation.
Relation to long run risk models

- Our model and long-run-risk model pioneered by Bansal and Yaron (2004) emphasize low-frequency shocks that induce large, persistent changes in SDF.

- Re-write representative agent’s utility function

  \[ U_t = \left( \tilde{C}_t^{1-1/\psi} + \delta (U^*_t + 1) \right)^{1/(1-1/\psi)} \]

  where

  \[ \tilde{C}_t = \lambda_t^{1/(1-1/\psi)} C_t. \]

- Taking logarithms of this expression we obtain:

  \[ \log(\tilde{C}_t) = 1/ (1 - 1/\psi) \log(\lambda_t) + \log(C_t) \]
Relation to long run risk models

\[ \log (\tilde{C}_t) = \frac{1}{1 - \frac{1}{\psi}} \log(\lambda_t) + \log(C_t) \]

- Bansal and Yaron (2004) introduce highly persistent component in \(\log(C_t)\), which is source of long-run risk.

- We introduce highly persistent component into \(\log(\tilde{C}_t)\) via our specification of time-preference shocks.

- Both specifications can induce large, persistent movements in \(m_{t+1}\).

- Two models are not observationally equivalent.
  - Different implications for correlation between \(\log(C_{t+1}/C_t)\) and asset returns.
  - Very different implications for average return to long-term bonds, and the term structure of interest rates.
Estimating the parameters of the model

- We estimate the model using GMM.

- We find the parameter vector $\hat{\Phi}$ that minimizes the distance between the empirical, $\Psi_D$, and model population moments, $\Psi(\hat{\Phi})$,

$$L(\hat{\Phi}) = \min_{\Phi} [\Psi(\Phi) - \Psi_D]' \Omega_D^{-1} [\Psi(\Phi) - \Psi_D].$$

- $\Omega_D$ is an estimate of the variance-covariance matrix of the empirical moments.
The moments used in GMM

- The vector $\Psi_D$ includes the following 16 moments:
  - Consumption growth: mean and standard deviation;
  - Dividend growth: mean, standard deviation;
  - Correlation between 1-year growth rate of dividends and 1-year growth rate of consumption;
  - Real stock returns: mean and standard deviation;
  - Real risk free rate: mean, standard deviation and serial correlation;
  - Correlation between stock returns and consumption growth (1, 5 and 10 years);
  - Correlation between stock returns and dividend growth (1, 5 and 10 years).

- We constrain the growth rate of consumption and dividends to be the same.
GMM estimation

• Assume agents make decisions at monthly frequency.

• Derive model’s implications for population moments computed at an annual frequency, $\Psi(\Phi)$.

• In estimating $\Psi_D$, we used a standard 2-step efficient GMM estimator.
  – We use a Newey-West weighting matrix with 10 lags.

• Our procedure yields an estimate of $\Omega_D$. 
Data

• We use realized real stock returns.

• As in Mehra and Prescott (1985) and the associated literature, we measure the risk free rate using realized real returns on nominal, one-year Treasury Bills.

• This measure is far from perfect because there’s inflation risk, which can be substantial.
## Parameter estimates, benchmark model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimates</th>
<th>Parameter</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.068 (5.555)</td>
<td>$\sigma_d$</td>
<td>0.0158 (0.0005)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.028 (2.274)</td>
<td>$\pi_{dc}$</td>
<td>0.000003 (0.0005)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.998 (0.0007)</td>
<td>$\rho$</td>
<td>0.9936 (0.0026)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.0067 (0.0002)</td>
<td>$\sigma_\lambda$</td>
<td>0.0004 (0.00008)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0015 (0.00006)</td>
<td>$\theta$ (implied)</td>
<td>$-2.56$</td>
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</table>
Moments (annual), data and benchmark model

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_{d,t})$</td>
<td>7.55</td>
<td>6.20</td>
<td>6.27</td>
</tr>
<tr>
<td></td>
<td>(1.74)</td>
<td>(1.87)</td>
<td></td>
</tr>
<tr>
<td>$E(r_{f,t})$</td>
<td>0.36</td>
<td>0.06</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.81)</td>
<td>(0.83)</td>
<td></td>
</tr>
<tr>
<td>$E(r_{d,t}) - E(r_{f,t})$</td>
<td>7.19</td>
<td>6.13</td>
<td>5.47</td>
</tr>
<tr>
<td></td>
<td>(1.77)</td>
<td>(1.84)</td>
<td></td>
</tr>
</tbody>
</table>
Moments (annual), data and benchmark model

<table>
<thead>
<tr>
<th>Moments</th>
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<th>Data Unconstrained</th>
<th>Model Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std($r_{d,t}$)</td>
<td>17.22 (1.31)</td>
<td>17.49 (1.39)</td>
<td>14.1</td>
</tr>
<tr>
<td>Std($r_{f,t}$)</td>
<td>3.19 (0.80)</td>
<td>3.47 (0.80)</td>
<td>4.21</td>
</tr>
<tr>
<td>Corr($r_{f,t}, r_{d,t}$)</td>
<td>0.20 (0.10)</td>
<td>0.26 (0.09)</td>
<td>0.13</td>
</tr>
<tr>
<td>AR 1 ($r_{f,t}$)</td>
<td>0.61 (0.11)</td>
<td>0.60 (0.08)</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Equity premium: intuition

• Model generates a high equity premium despite a low coefficient of relative risk aversion because stocks are longer duration, more exposed to valuation risk.

• $\gamma$ is close to one so traditional covariance effect is very small.

• Paper decomposes equity premium into valuation risk premium and conventional risk premium.
  
  – Conventional risk premium is always roughly zero (consistent with Kocherlakota’s (1996)).
  
  – Valuation risk premium and the equity premium are increasing in $\rho$. 
Correlations between consumption growth and real stock returns

<table>
<thead>
<tr>
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<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>−0.03 (0.12)</td>
<td>−0.05 (0.12)</td>
<td>0.00007</td>
</tr>
<tr>
<td>5 year</td>
<td>0.07 (0.17)</td>
<td>0.002 (0.14)</td>
<td>0.00007</td>
</tr>
<tr>
<td>10 year</td>
<td>−0.02 (0.30)</td>
<td>−0.11 (0.20)</td>
<td>0.00008</td>
</tr>
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</table>
Correlations between dividend growth and real stock returns

<table>
<thead>
<tr>
<th>Dividends</th>
<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.08 (0.12)</td>
<td>0.05 (0.11)</td>
<td>0.39</td>
</tr>
<tr>
<td>5 year</td>
<td>0.27 (0.14)</td>
<td>0.30 (0.13)</td>
<td>0.39</td>
</tr>
<tr>
<td>10 year</td>
<td>0.51 (0.22)</td>
<td>0.59 (0.14)</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Special case: Epstein-Zin but no time-preference shocks

- The model doesn’t generate an equity premium, even with a risk aversion of 11 (Kothcerlakota, 1996).

- It also can’t account for the correlation puzzle
  \[ \text{corr}(\Delta d_t, r_{d,t}) = 1, \text{corr}(\Delta c_t, r_{d,t}) = 0.42. \]
Shortcomings of the benchmark model

- We made the benchmark model as simple as possible to provide intuition.

- Two important shortcomings
  - Persistence of the risk-free rate.
  - The correlation between preference shocks and consumption.
Measuring the preference shocks

- According to the model

\[ r_{f,t+1} = -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1 - \theta) \kappa c_1 A c_1 \sigma^2 \]

\[ + \left[ \frac{(1 - \theta)}{\theta} (1 - \gamma)^2 - \gamma^2 \right] \sigma^c / 2, \]

- So

\[ \log(\lambda_{t+1}/\lambda_t) = \chi - r_{f,t+1} \]

\[ \chi = -\log(\delta) + \mu/\psi - (1 - \theta) \kappa c_1 A c_1 \sigma^2 / 2 \]

\[ + \left[ \frac{(1 - \theta)}{\theta} (1 - \gamma)^2 - \gamma^2 \right] \sigma^c / 2, \]

- So, up to a constant, we can measure the preference shock as minus the risk-free rate.
Persistence in the risk-free rate

- The previous observations imply that $\log(\lambda_{t+1}/\lambda_t)$ should be as persistent as the risk-free rate.

- In our estimated model,

$$
\log(\lambda_{t+1}/\lambda_t) = 0.9936 \log(\lambda_t/\lambda_{t-1}) + \sigma_\lambda \epsilon_{t+1}
$$

- If we regress the demeaned risk-free rate on one lag we obtain an AR coefficient of 0.61, with a standard error of 0.11.
Getting persistence right

• Fixing the persistence problem is straightforward, doesn’t have major effect on other aspects of models’ performance.

• Suppose that:

\[ \log(\lambda_{t+1}/\lambda_t) = x_{t+1} + \sigma \eta_{t+1}. \]

\[ x_{t+1} = \rho x_t + \sigma \lambda \varepsilon_{t+1} \]

where \( \varepsilon_{t+1} \) and \( \eta_{t+1} \) are uncorrelated, i.i.d. standard normal shocks.

• So time preference shock is sum of a persistent shock and an i.i.d. shock.
A quasi-production model

• In a production economy time-preference shocks would induce changes in aggregate output and consumption.

• Our Lucas-tree specification doesn’t allow for such co-movements.

• Modify model to mimic a production economy by allowing for correlation between dividends, consumption with the time-preference shock.
Incorporating the two model extensions

\[ \log(\lambda_{t+1}/\lambda_t) = x_{t+1} + \sigma_\eta \eta_{t+1}. \]

\[ x_{t+1} = \rho x_t + \sigma_\lambda \epsilon_{t+1}^\lambda \]
\[ \eta_{t+1} \sim N(0,1), \epsilon_{t+1}^\lambda \sim N(0,1) \]

\[ \log(C_{t+1}) = \log(C_t) + \mu + \sigma_c \epsilon_{t+1}^c + \pi_{c\lambda} \epsilon_{t+2}^\lambda \]
\[ \epsilon_{t+1}^c \sim N(0,1) \]

\[ \log(D_{t+1}) = \log(D_t) + \mu + \pi_{dc} \epsilon_{t+1}^c + \sigma_d \epsilon_{t+1}^d + \pi_{d\lambda} \epsilon_{t+2}^\lambda \]
\[ \epsilon_{t+1}^d \sim N(0,1) \]

• \( \epsilon_{t+1}^c, \epsilon_{t+1}^d, \epsilon_{t+1}^\lambda, \) and \( \eta_{t+1} \) are mutually uncorrelated.
Augmented Model

- Adding $\sigma_\eta$ to vector $\Phi$ and AR(1) coefficient of risk-free rate, $\tau$, to $\Psi_D$.

- With exception of $\sigma_\eta$, estimated structural parameters are very similar across the two models.

- Models’ implications for data moments are also very similar, taking sampling uncertainty into account.

- One exception: serial correlation of risk-free rate falls to 0.62 in the augmented model.
### Parameter estimates, quasi-production model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimates</th>
<th>Parameter</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.93</td>
<td>$\sigma_d$</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(7.67)</td>
<td></td>
<td>(0.0008)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.92</td>
<td>$\pi_{dc}$</td>
<td>0.00002</td>
</tr>
<tr>
<td></td>
<td>(8.49)</td>
<td></td>
<td>(0.0008)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.99951</td>
<td>$\sigma_\eta$</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td></td>
<td>(0.0005)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.008</td>
<td>$\rho$</td>
<td>0.9988</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td></td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.001</td>
<td>$\sigma_\lambda$</td>
<td>0.00015</td>
</tr>
<tr>
<td></td>
<td>(0.00007)</td>
<td></td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$\pi_{c\lambda}$</td>
<td>0.00003</td>
<td>$\pi_{d\lambda}$</td>
<td>$-0.009$</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td></td>
<td>(0.001)</td>
</tr>
</tbody>
</table>
### Moments (annual), data and models

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
<th>Model Quasi-production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_{d,t})$</td>
<td>7.55 (1.74)</td>
<td>6.20 (1.87)</td>
<td>6.27</td>
<td>2.81</td>
</tr>
<tr>
<td>$E(r_{f,t})$</td>
<td>0.36 (0.81)</td>
<td>0.06 (0.83)</td>
<td>0.80</td>
<td>$-0.92$</td>
</tr>
<tr>
<td>$E(r_{d,t}) - E(r_{f,t})$</td>
<td>7.19 (1.77)</td>
<td>6.13 (1.84)</td>
<td>5.47</td>
<td>3.73</td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td></td>
<td>$-2.56$</td>
<td>$-0.79$</td>
</tr>
</tbody>
</table>
## Moments (annual), data and models

<table>
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<tr>
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<th>Model Benchmark</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Std($r_{d,t}$)</td>
<td>17.22 (1.31)</td>
<td>17.49 (1.39)</td>
<td>14.1</td>
<td>18.24</td>
</tr>
<tr>
<td>Std($r_{f,t}$)</td>
<td>3.19 (0.80)</td>
<td>3.47 (0.80)</td>
<td>4.21</td>
<td>4.72</td>
</tr>
<tr>
<td>Corr($r_{f,t}, r_{d,t}$)</td>
<td>0.20 (0.10)</td>
<td>0.26 (0.09)</td>
<td>0.13</td>
<td>0.098</td>
</tr>
<tr>
<td>AR1($r_{f,t}$)</td>
<td>0.61 (0.11)</td>
<td>0.60 (0.08)</td>
<td>0.95</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Correlations between fundamentals and real stock returns

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
<th>Model Quasi-production</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>−0.03 (0.12)</td>
<td>−0.05 (0.12)</td>
<td>0.00007</td>
<td>0.004</td>
</tr>
<tr>
<td>5 year</td>
<td>0.07 (0.17)</td>
<td>0.002 (0.14)</td>
<td>0.00007</td>
<td>0.004</td>
</tr>
<tr>
<td>10 year</td>
<td>−0.02 (0.30)</td>
<td>−0.11 (0.20)</td>
<td>0.00008</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Correlations between fundamentals and real stock returns

<table>
<thead>
<tr>
<th>Dividends</th>
<th>Data Constrained</th>
<th>Data Unconstrained</th>
<th>Model Benchmark</th>
<th>Model Quasi-production</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.08 (0.12)</td>
<td>0.05 (0.11)</td>
<td>0.39</td>
<td>0.07</td>
</tr>
<tr>
<td>5 year</td>
<td>0.27 (0.14)</td>
<td>0.30 (0.13)</td>
<td>0.39</td>
<td>0.09</td>
</tr>
<tr>
<td>10 year</td>
<td>0.51 (0.22)</td>
<td>0.59 (0.14)</td>
<td>0.39</td>
<td>0.11</td>
</tr>
</tbody>
</table>
The importance of the correlation puzzle

- Model faces a tradeoff between matching $\text{corr}(\Delta d_t, r_{d,t})$ and $\text{corr}(\Delta c_t, r_{d,t})$ and matching the equity premium.

- Since $\text{corr}(\Delta d_t, r_{d,t})$ and $\text{corr}(\Delta c_t, r_{d,t})$ are estimated with more precision than average rates of return, the estimation criterion gives them more weight.

- To illustrate this point, we estimate model subject to constraint that it matches the equity premium.
## Matching the equity premium

<table>
<thead>
<tr>
<th></th>
<th>Data Constrained</th>
<th>Quasi-production Match equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>0.9302 (7.67)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>-</td>
<td>0.9184 (8.49)</td>
</tr>
<tr>
<td>$E(r_{d,t})$</td>
<td>7.55 (1.74)</td>
<td>2.81</td>
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<tr>
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<tr>
<td>$\theta$</td>
<td>−0.79</td>
<td>−6.65</td>
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## Matching the equity premium

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<tr>
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<th>Quasi-production Match equity premium</th>
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<tr>
<td>1 year</td>
<td>$-0.03 \ (0.12)$</td>
<td>0.00</td>
<td>$-0.01$</td>
</tr>
<tr>
<td>5 year</td>
<td>0.07 \ (0.17)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10 year</td>
<td>$-0.02 \ (0.30)$</td>
<td>0.00</td>
<td>0.00</td>
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## Matching the equity premium

<table>
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<th>Quasi-production</th>
<th>Quasi-production Match equity premium</th>
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<tbody>
<tr>
<td>1 year</td>
<td>0.08 (0.12)</td>
<td>0.07</td>
<td>0.56</td>
</tr>
<tr>
<td>5 year</td>
<td>0.27 (0.14)</td>
<td>0.09</td>
<td>0.48</td>
</tr>
<tr>
<td>10 year</td>
<td>0.51 (0.22)</td>
<td>0.11</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Trade-offs: the correlation puzzle vs the equity premium

- Model continues to produce low correlations between stock returns, consumption growth.

- But one-year correlation between stock returns, dividend growth implied by model is much higher than in data.

- One-year correlation between stock returns, dividend growth is estimated much more precisely than the equity premium.

- Estimation algorithm chooses parameters that imply lower equity premium to match one-year correlation between stock returns, dividend growth.
Implications for the bond term premium

• In models that stress long-run risk (e.g, Bansal, Kiku, and Yaron (2011)) long-term bonds command a negative risk premium.
  – This negative premium reflects fact that long-term bonds are a hedge against long-run risk.
  – BKY model implies a 10-year yield of -0.43 percent and 20-year yield of -0.88.

• Standard rare-disaster models also imply a downward sloping term structure for real bonds and a negative real yield on long-term bonds.

• Does our model account for the bond term premium?
Term premium

- Following table presents key statistics for ex-post real returns to short-term, intermediate-term long-term government bonds (1-month, 5 year, 20 year)

- Consistent with Alvarez and Jermann (2005), the term structure of real returns is upward sloping.

- Real yield on long-term bonds is positive.
  - Consistent with Campbell, Shiller and Viceira (2009): real yield on long-term TIPS has always been positive, usually above 2%
### Implications for the term premium

<table>
<thead>
<tr>
<th></th>
<th>Data Unconstrained Mean</th>
<th>Model Benchmark</th>
<th>Model Quasi-production</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long-term bond</strong></td>
<td>1.66 (0.85)</td>
<td>5.21</td>
<td>0.74</td>
</tr>
<tr>
<td><strong>Int.-term bond</strong></td>
<td>1.06 (0.90)</td>
<td>3.67</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>One-year bond</strong></td>
<td>0.36 (0.81)</td>
<td>0.80</td>
<td>−0.92</td>
</tr>
<tr>
<td>$r_{d,t} - \text{long-term yield}$</td>
<td>4.54 (1.84)</td>
<td>1.23</td>
<td>3.00</td>
</tr>
</tbody>
</table>

**Standard deviation**

<table>
<thead>
<tr>
<th></th>
<th>Long-term</th>
<th>Intermediate-term</th>
<th>One year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long-term</strong></td>
<td>3.54 (0.76)</td>
<td>2.22</td>
<td>2.72</td>
</tr>
<tr>
<td><strong>Intermediate-term</strong></td>
<td>1.06 (0.90)</td>
<td>3.04</td>
<td>2.93</td>
</tr>
<tr>
<td><strong>One year</strong></td>
<td>3.19 (0.80)</td>
<td>4.21</td>
<td>4.72</td>
</tr>
<tr>
<td>$r_{d,t} - \text{long-term yield}$</td>
<td>16.7 (1.48)</td>
<td>13.8</td>
<td>18.9</td>
</tr>
</tbody>
</table>
Stocks vs Bonds

- Quasi-production model implies difference between stock and long-term bond returns is roughly 3 percent.

- Positive premium that equity commands over long-term bonds reflects difference between asset of infinite and 20-year maturity.

- Binsbergen et al (2011) estimate that 90 (80) percent of value of the S&P 500 index corresponds to dividends that accrue after first 5 (10) year
Conclusion

• We propose a simple model of asset pricing with valuation risk that accounts for the level and volatility of the equity premium and of the risk free rate.

• The model is broadly consistent with the correlations between stock market returns and fundamentals, consumption and dividend growth.
  – The model accounts for these with low levels of risk aversion.

• Key features of the model
  – Consumption and dividends follow random walks; Epstein-Zin utility; stochastic rate of time preference.

• Valuation risk is by far the most important determinant of the equity premium and the bond term premia.

• A big future challenge is to produce an explicit production function.