# Finance and Misallocation: Evidence from Plant-Level Data

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# Our goal

- Measure effect of finance frictions on resource (mis)allocation
  - TFP losses

# Mechanism we study

- An agent's entrepreneurial ability fluctuates over time
- Ask:
  - Do frictions distort re-allocation $K, L$ across entrepreneurs?
  - Do frictions distort entry entrepreneurship?
- Narrow focus
  - Can model *endogenously* generate large dispersion $MP_K$?
  - Abstract differences i-rates due to government etc.

# Our approach

- Model of establishment dynamics with borrowing constraints

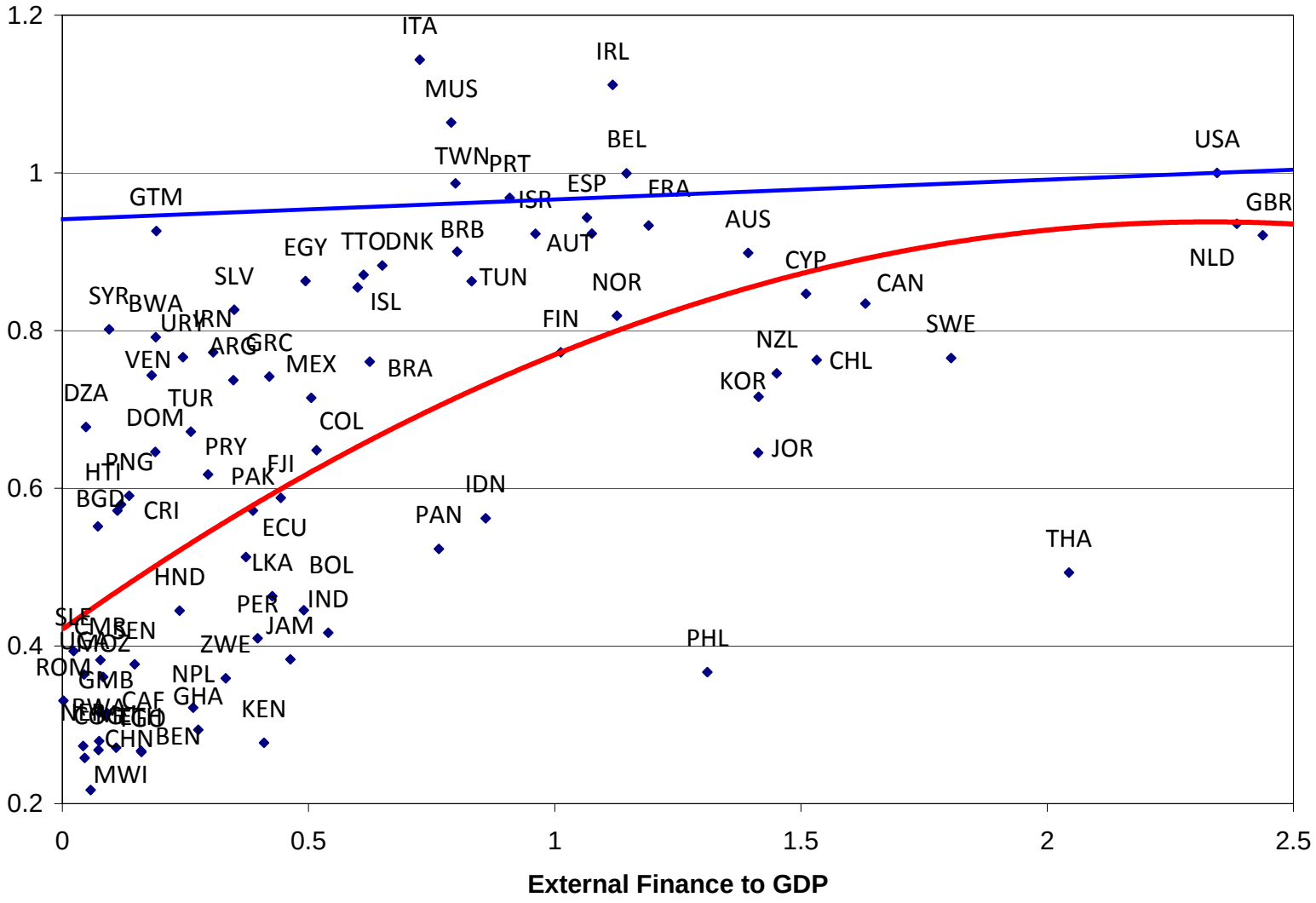
- Require model accounts plant-level facts (Korea, Colombia):

  1. Size distribution of establishments
  2. Variability and persistence of output
  3. Difference growth rates young vs. old plants
  4. Difference returns to capital young vs. old

# Our findings

- Find mechanism is weak:
  - 7% TFP losses in economy without external finance
  - Much weaker (up to 40% losses) than previously found:
    - Jeong-Townsend'06, Buera-Kaboski-Shin'09, Amaral-Quintin'05, Moll'09, Greenwood-Sanchez-Wang'09
- Plant-level facts key to this result
  - TFP losses much larger if ignore facts 2-4

# Finance vs. TFP our model



# Outline

- Model with no exit-entry
- Model with exit-entry and occupational choice

# Model Overview

- Small open economy
- Continuum of entrepreneurs differ in (time-varying) productivity
- Plant only source of income. Risk not diversifiable

- Finance [Evans-Jovanovic (1989)]:
  - Save risk-free asset, $r$: $\beta(1+r) < 1$
  - Must pay labor, capital before production.
  - Debt subject to collateral constraint

# Technology

- Production function:

$$Y_{it} = A_{it}^{1-\eta} \left( L_{it}^{\alpha} K_{it}^{1-\alpha} \right)^{\eta}$$

- $\eta < 1$, span of control
- Productivity : $\Phi(A_{it+1}|A_{it})$

# Problem of entrepreneur

$$\sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma}$$

- $B_{it}$: assets
- spend $WL_{it} + K_{it}$ before producing
  - borrow $D_{it} = WL_{it} + K_{it} - B_{it}$ from bank
  - collateral constraint: $D_{it} \leq (\lambda - 1) B_{it}$

$$C_{it} + B_{it+1} = Y_{it} + (1-\delta) K_{it} + (1+r) \left[ B_{it} - WL_{it} - K_{it} \right]$$

# Timing

$$A_t^{1-\eta}\left(L_t^{\alpha}K_t^{1-\alpha}\right)^{\eta}+(1-\delta)\,K_t$$

borrow

$$B_t \rightarrow \quad W_tL_t+K_t-B_t \quad \text{repay } (1+r)\,(W_tL_t+K_t-B_t) \quad \rightarrow B_{t+1}$$

consume $C_t$

$t$ $\qquad\qquad\qquad\qquad$ $t+1$

# Problem of entrepreneur

Reduces to

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma}$$

s.t.

$$C_{it} = (1+r)\, B_{it} + \pi\left(B_{it}, A_{it}\right) - B_{it+1}$$

where

$$\Pi\left(B_{it}, A_{it}\right) = \max_{K_{it}, L_{it}} A_{it}^{1-\eta} \left(L_{it}^{\alpha} K_{it}^{1-\alpha}\right)^{\eta} - (1+r)\, W_t L_{it} - (r+\delta)\, K_{it}$$

$$\text{s.t.} \;\; WL_{it} + K_{it} \leq \lambda B_{it}$$

# Solution to static problem

- Homogeneity: $b = B/A, k = K/A, \pi = \Pi/A$

$$\pi(b) = \max_{k,l} \left(l^{\alpha} k^{1-\alpha}\right)^{\eta} - (1+r)\, Wl - (r+\delta)\, k$$
$$\text{s.t.} \;\; Wl + k \leq \lambda b$$

- Solution:

$$\begin{aligned} f_l(l,k) &= [1+\tilde{r}(b)]\, W \\ f_k(l,k) &= \tilde{r}(b) + \delta \end{aligned}$$

- $\tilde{r}(b)$: shadow cost of funds, $\tilde{r}(b) = r + \mu(b)$

# Dynamic program

$$V(b,a) = \max_{b'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp(a'-a)^{1-\gamma} V\left(\frac{b'}{\exp(a'-a)}, a'\right) d\Phi(a'|a)$$

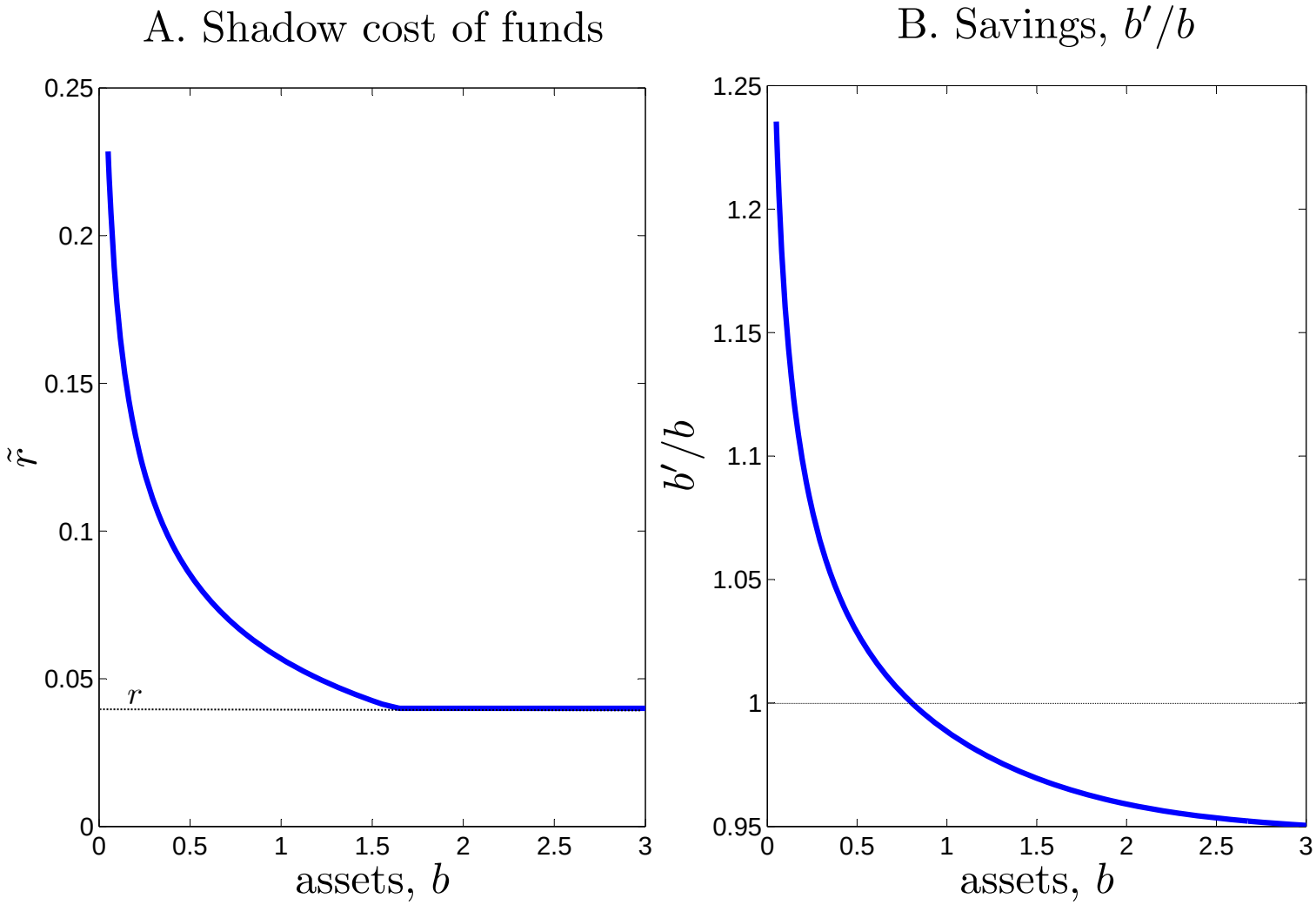
where

$$c = (1+r)\,b + \pi(b) - b'.$$

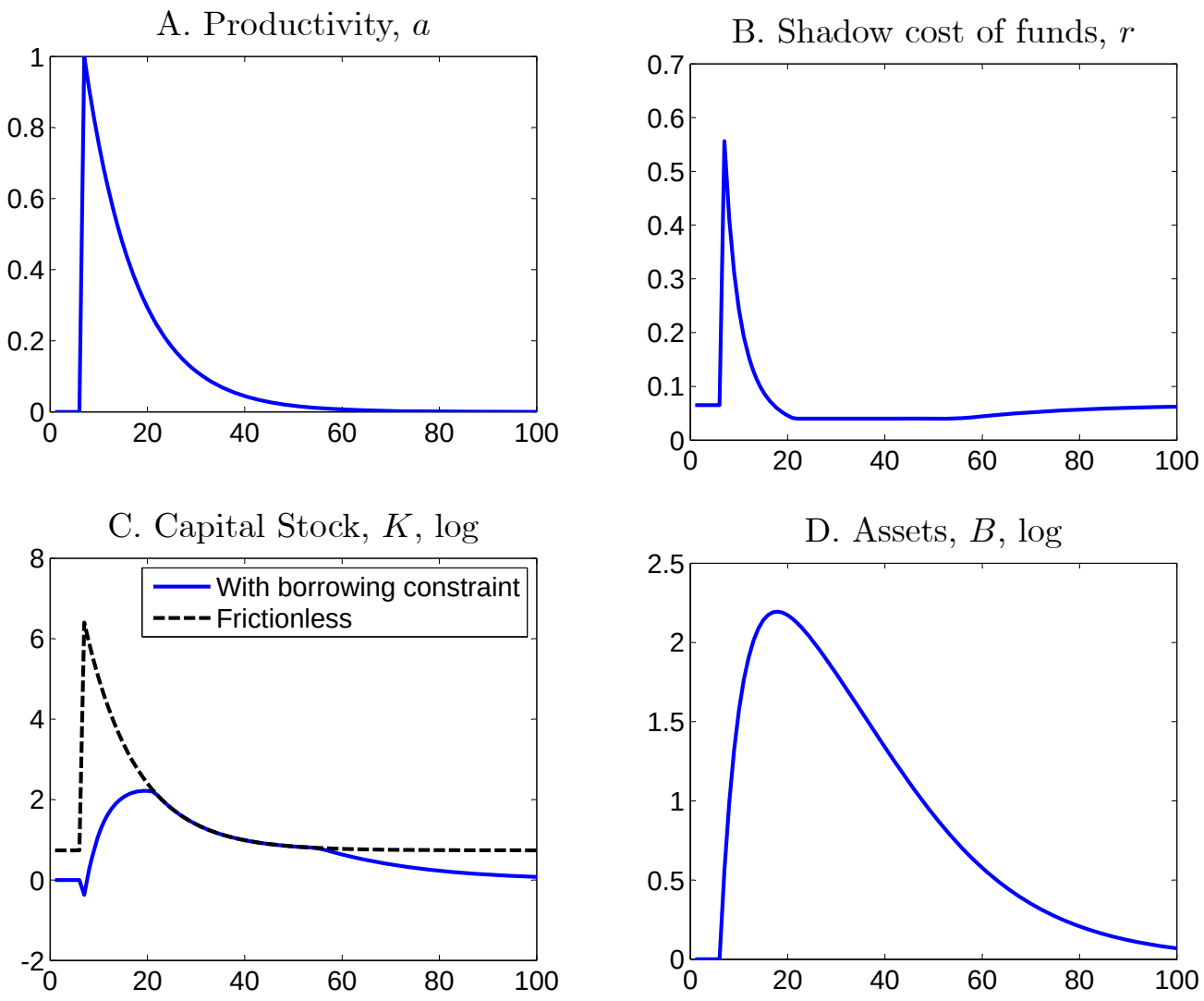
- Solution:

$$c^{-\gamma} = \beta \int (1+r+\mu') \exp(a'-a)^{-\gamma} c'^{-\gamma} d\Phi(a'|a)$$

# Decision rules



# Response to productivity shock



# Summarize

- Absent $\Delta a$ ergodic distribution $b$ degenerate
  - $\tilde{r}$ equal across entrepreneurs
  - No misallocation
  - Banerjee-Moll '09
- Misallocation requires large dispersion $b$: large shocks to $a$

# TFP losses due to misallocation

- Efficient allocations:

$$\max_{K_i, L_i} Y = \int_0^1 A_i^{1-\eta} \left(L_i^{\alpha} K_i^{1-\alpha}\right)^{\eta} di$$

$$\text{s.t.} \int_0^1 K_i di = K, \quad \int_0^1 L_i di = L$$

- Solution:

$$L_i = \frac{A_i}{\int_0^1 A_i di} L, \qquad K_i = \frac{A_i}{\int_0^1 A_i di} K$$

$$Y = A\left(L^{\alpha} K^{1-\alpha}\right)^{\eta}, \qquad A = \int_0^1 A_i di$$

# TFP losses due to misallocation

- Allocations with frictions:

$$L_i = \omega_i^l \frac{A_i}{\int_0^1 A_i di} L, \qquad K_i = \omega_i^k \frac{A_i}{\int_0^1 A_i di} K$$

$$Y = A\left(L^\alpha K^{1-\alpha}\right)^\eta, \qquad A = \int_0^1 \omega_i A_i^{1-\eta} di$$

- 'Worst-case': $\omega_i = 1$ $(L_i = L, K_i = K)$.
- Efficient: $\omega_i = A_i^\eta$

# Data: Korea ('91-'96) and Colombia ('81-'91)

- Manufacturing plants
- All establishments 5+ (Korea), 10+ (Colombia) workers
- Balanced panel, 31500 plants Korea, 5000 Colombia
- Revenue, labor, intermediate inputs, investment, capital
- $Y$ = value added = revenue - intermediate inputs

# Fact 1: size distribution of establishments

- Output (value-added) concentrated largest establishments

| | Korea | Colombia |
|---|---|---|
| fraction $Y$ largest 1% | 0.57 | 0.30 |
| fraction $Y$ largest 5% | 0.77 | 0.61 |
| fraction $Y$ largest 10% | 0.84 | 0.75 |
| fraction $Y$ largest 20% | 0.91 | 0.88 |

# Fact 2: distribution of output growth rates

- $\Delta y_{it}$ volatile, fat-tailed

| | Korea | Colombia |
|---|---|---|
| $\sigma(\Delta y_{it})$ | 0.54 | 0.49 |
| $kurtosis(\Delta y_{it})$ | 12.9 | 20.8 |
| $iqr(\Delta y_{it})$ | 0.49 | 0.36 |

# Fact 3: persistence of output

- $y_{it}$ persistent, autocorrelation decays slowly

| | Korea | Colombia |
|---|---|---|
| $corr(y_{it}, y_{it-1})$ | 0.93 | 0.96 |
| $corr(y_{it}, y_{it-3})$ | 0.89 | 0.93 |
| $corr(y_{it}, y_{it-5})$ | 0.86 | 0.90 |

# Fact 4: Debt-to-GDP

- Korea: Bank of Korea Financial Statement Analysis Survey. Manufacturing, 1991-1996
- Colombia: Beck, Demirguc-Kunt, Levine (2000)

| | Korea | Colombia |
|---|---|---|
| Debt-to-GDP | 1.2 | 0.3 |

# Parameterization

- Assigned parameters
  - $\gamma = 1$ (CRRA)
  - $\beta = 0.92$ (discount factor)
  - $r = 0.04$ (risk-free rate)
  - $\delta = 0.06$ (depreciation rate)
  - $\eta = 0.85$ (span of control)
  - $\alpha = 0.67$ (labor share)
- Calibrate rest to minimize distance moments model-data

# Calibration

- Productivity:

$$\ln(A_{it}) = a_{it} = Z_i + \tilde{a}_{it}$$

  - $Z_i$: permanent component. Bounded Pareto.

$$\Pr\left[\exp\left(Z_i\right) \leq x\right] = \frac{1 - x^{-\mu}}{1 - H^{-\mu}}.$$

  - $\tilde{a}_{it}$: variable component. Fat-tailed shocks.

$$\tilde{a}_{it} = \rho \tilde{a}_{it-1} + \varepsilon_{it}$$

$$\varepsilon_{it} \sim \begin{cases} N\left(0, \sigma_{1,\varepsilon}^2\right) & \text{with prob. } 1 - \kappa \\ N\left(0, \sigma_{2,\varepsilon}^2\right) & \text{with prob. } \kappa \end{cases}$$

# Calibration

- Calibrate $\theta = \{\lambda, \rho, \sigma_{1,\varepsilon}, \sigma_{2,\varepsilon}, \kappa, \mu, H\}$
- Match plant-level moments and debt-to-GDP for Korea

$$\min_{\theta} \left[\Gamma(\theta) - \Gamma^d\right]' W \left[\Gamma(\theta) - \Gamma^d\right]$$

  - $W = var(\Gamma^d)^{-1}$, boostrap.

- Standard errors:

$$V = \frac{1}{N}\left[\frac{\partial \Gamma(\theta)}{\partial \theta'} W \frac{\partial \Gamma(\theta)}{\partial \theta}\right]^{-1}$$

# Parameter values

| | Estimate | (s.e.) | |
|---|---|---|---|
| $\lambda$ | 2.58 | (0.01) | collateral constraint |
| $\rho$ | 0.74 | (0.01) | AR(1) productivity |
| $\sigma_{1,\varepsilon}$ | 0.09 | (0.00) | s.d. shocks |
| $\sigma_{2,\varepsilon}$ | 0.31 | (0.00) | s.d. shocks |
| $\kappa$ | 0.07 | (0.00) | probab. 2 |
| $\mu$ | 3.64 | (0.02) | Pareto exponent |
| $H$ | 4.91 | (0.02) | upper bound $Z_i$ |

- Implies $Z_i$ accounts 2/3 variance $a_i$

# Fit

| | Korea Data | Model |
|---|---|---|
| $\sigma(\Delta y_{it})$ | 0.54 | 0.51 |
| $kurtosis(\Delta y_{it})$ | 12.9 | 12.9 |
| $iqr(\Delta y_{it})$ | 0.49 | 0.47 |
| $corr(y_{it}, y_{it-1})$ | 0.93 | 0.95 |
| $corr(y_{it}, y_{it-3})$ | 0.89 | 0.89 |
| $corr(y_{it}, y_{it-5})$ | 0.86 | 0.85 |
| fraction $Y$ largest 1% | 0.57 | 0.59 |
| fraction $Y$ largest 5% | 0.77 | 0.83 |
| fraction $Y$ largest 10% | 0.84 | 0.90 |
| fraction $Y$ largest 20% | 0.91 | 0.95 |
| Debt-to-GDP | 1.2 | 1.2 |

# Role of permanent component

- Eliminate $Z_i$ and re-calibrate

| | Korea Data | No $Z_i$ |
|---|---|---|
| $\sigma(\Delta y_{it})$ | 0.54 | 0.51 |
| $kurtosis(\Delta y_{it})$ | 12.9 | 18.2 |
| $iqr(\Delta y_{it})$ | 0.49 | 0.43 |
| $corr(y_{it}, y_{it-1})$ | 0.93 | 0.95 |
| $corr(y_{it}, y_{it-3})$ | 0.89 | 0.87 |
| $corr(y_{it}, y_{it-5})$ | 0.86 | 0.78 |
| fraction $Y$ largest 1% | 0.57 | 0.29 |
| fraction $Y$ largest 5% | 0.77 | 0.53 |
| fraction $Y$ largest 10% | 0.84 | 0.66 |
| fraction $Y$ largest 20% | 0.91 | 0.79 |
| Debt-to-GDP | 1.2 | 1.2 |

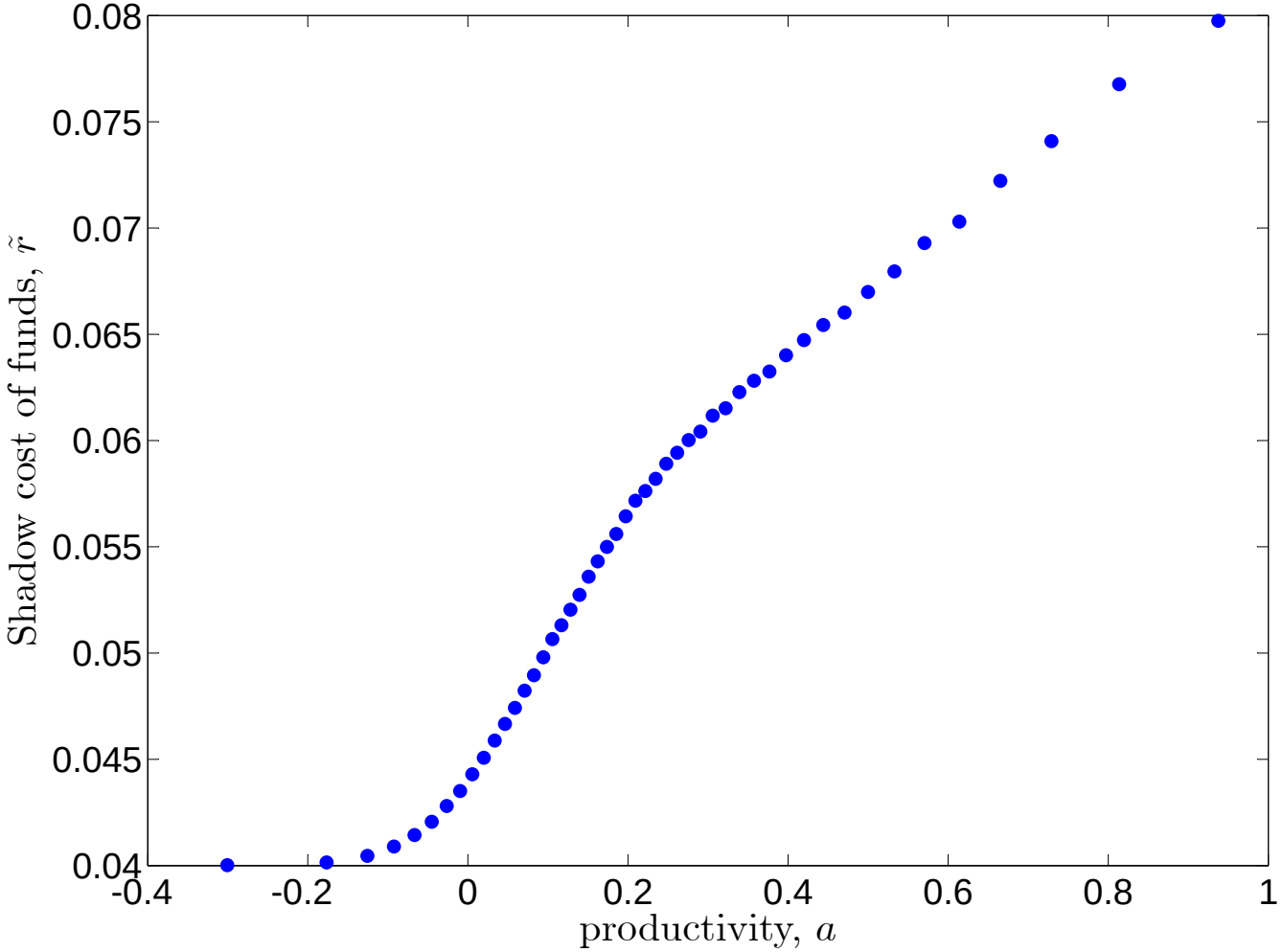
# Model predictions

- Report results for Korean calibration
- Also for US, Colombia, No external debt.
  - Same $a_i$ process. Vary $\lambda$ to match Debt-to-GDP.

# Model predictions

| | US | Korea | Colombia | No Debt |
|---|---|---|---|---|
| $\lambda$ | 50 | 2.6 | 1.2 | 1 |
| Debt-to-GDP | 2.3 | 1.2 | 0.3 | 0 |
| Fract. constrained | 0.04 | 0.54 | 0.80 | 0.86 |
| Median $\tilde{r} - r$ \|constr | 0.01 | 0.03 | 0.04 | 0.05 |
| iqr $\tilde{r} - r$\|constr | 0.02 | 0.03 | 0.04 | 0.05 |
| 99% $\tilde{r} - r$\|constr | 0.16 | 0.15 | 0.19 | 0.20 |
| $\sigma(\Delta y_{it})$ | 0.70 | 0.51 | 0.37 | 0.35 |

# Model predictions



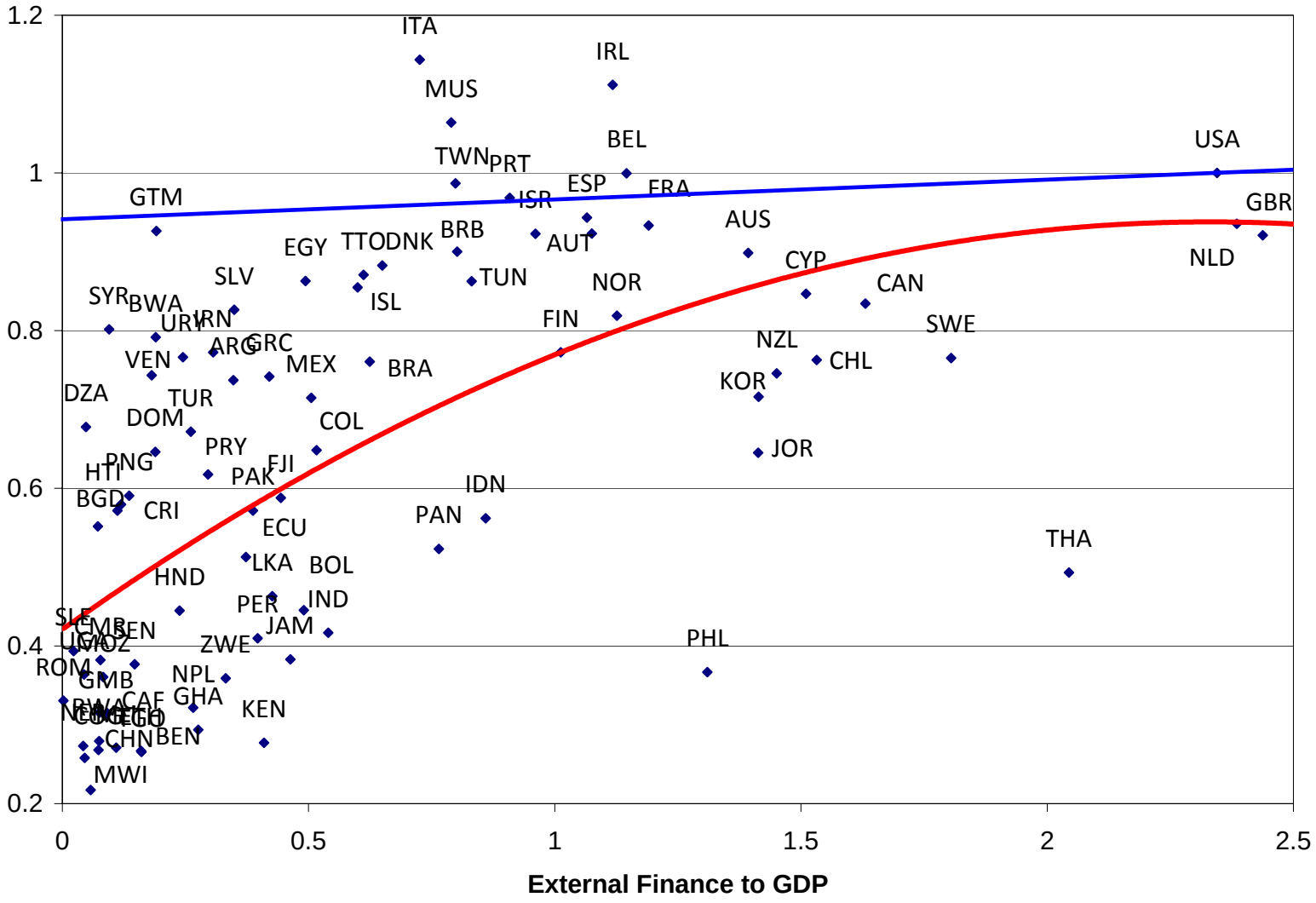
# TFP losses

- $\Delta TFP$ if eliminate finance frictions, $\lambda = \infty$:

| | US | Korea | Colombia | No Debt |
|---|---|---|---|---|
| TFP losses, % | 1.0 | 3.9 | 5.5 | 6.9 |

**Bottomline: TFP losses small**

# Finance vs. TFP



# Why are TFP losses small?

- Recall if $\ln(A_{it}) = Z_i + \tilde{a}_{it}$ constant: no TFP losses
- Too little time-series variation $\ln(A_{it})$
- TFP losses if $K, L$ do not move at all with $\tilde{a}_{it}$:
  - $A = \int_0^1 \exp(\tilde{a}_{it})\,di$ vs. $A = \int_0^1 \exp(\tilde{a}_{it})^{1-\eta}\,di$

| | US | Korea | Colombia | No Debt |
|---|---|---|---|---|
| TFP losses, % | 1.0 | 3.9 | 5.5 | 6.9 |
| 'Worst-case' losses, % | 8.6 | 8.6 | 8.6 | 8.6 |

# Summarize

- Finance frictions: small TFP losses from misallocation
- Too little time-series variability output (productivity)
- Losses small even if $K, L$ do not react at all $\Delta$ productivity
- Show next TFP losses much larger if ignore data $\Delta y_{it}$

# Counterfactual experiments

- Illustrate role of micro facts for TFP numbers
- Set $Z_i = 0$
- $a_{it} = \rho a_{it-1} + \epsilon_{it}, \epsilon_{it} \sim N(0, \sigma^2)$
- Three experiments
  1. Choose $\rho, \sigma^2$ to match $corr(y_{it}, y_{it-1})$, size distribution
  2. Lower $\rho = 0.80$ (Moll'09)
     - raise $\sigma^2$ to match size distribution
     - keep $\sigma^2$ constant

# Counterfactual experiments

| | Our | 1 | 2 | 3 |
|---|---|---|---|---|
| $\sigma(\Delta y_{it})$ | 0.51 | 1.05 | 2.17 | 1.03 |
| $corr(y_{it}, y_{it-1})$ | 0.95 | 0.93 | 0.80 | 0.77 |
| $corr(y_{it}, y_{it-3})$ | 0.89 | 0.80 | 0.52 | 0.47 |
| $corr(y_{it}, y_{it-5})$ | 0.85 | 0.69 | 0.34 | 0.30 |
| Fraction Y top 1% | 0.59 | 0.52 | 0.44 | 0.13 |
| Fraction Y top 10% | 0.90 | 0.88 | 0.88 | 0.48 |
| TFP loss Korea | 3.9 | 10.5 | 18.3 | 6.6 |
| TFP loss Colombia | 5.5 | 18.1 | 29.5 | 10.9 |
| 'Worst-case' loss | 8.6 | 54.3 | 69.9 | 20.2 |

# Summarize

- Model can easily generate much larger TFP losses
- But only if $\Delta y_{it}$ much more volatile than data
- Lower $\rho$ lowers TFP losses if hold $\sigma(\Delta y_{it})$ constant

# Alternative Parameterizations

- Lower $\beta = 0.85$: less internal accumulation
- Higher $\eta = 0.95$: greater losses misallocation
- Lower elast. subst $K, L$: $\theta = 0.5$

$$Y_i = A_i^{1-\eta} \left[ \alpha L_i^{\frac{\theta-1}{\theta}} + (1-\alpha) K_i^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}\eta}$$

- Recalibrate to match moments

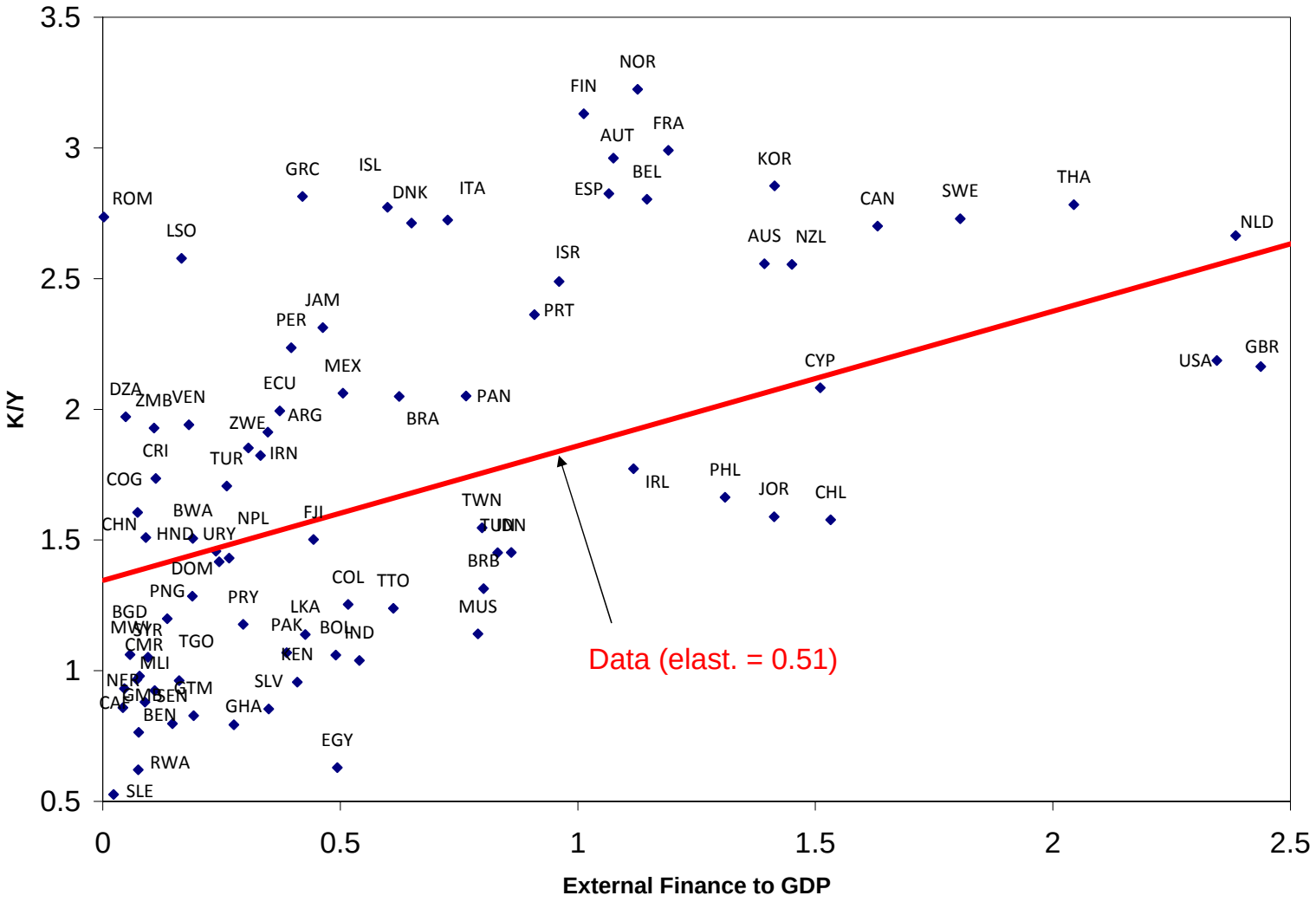
# Alternative Parameterizations

| | Benchmark | Low $\beta$ | High $\eta$ | Low $\theta$ |
|---|---|---|---|---|
| TFP loss US | 1.0 | 2.2 | 0.7 | 2.9 |
| TFP loss Korea | 3.9 | 6.5 | 3.2 | 5.6 |
| TFP loss Colombia | 5.4 | 8.8 | 5.4 | 6.9 |
| 'Worst-case' loss | 8.6 | 12.6 | 8.3 | 8.4 |

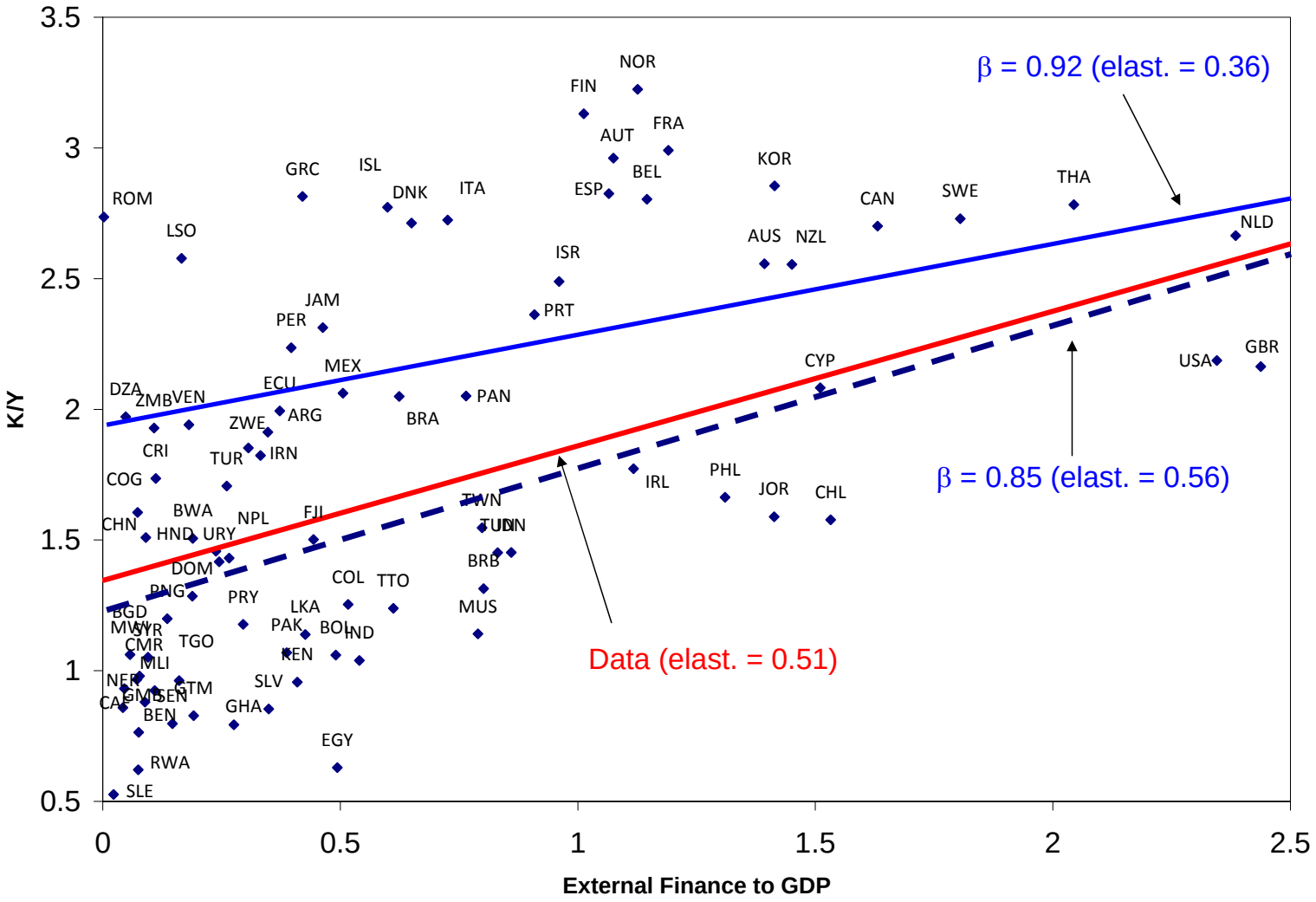
# Do entrepreneurs save?

- TFP losses small because internal accumulation
- Question: do agents save in the data?
- Answer by studying how $K/Y$ varies with $Debt/Y$
  - $K$ = debt + internal funds
  - If do not save ($\beta$ low), $K/Y$ low in low $\lambda$ economies

# K/Y vs. Debt-to-GDP Data



# Model predictions



# Model with entry/exit

- Do finance frictions distort entry/exit decision?
  - Inefficient selection into entrepreneurship
- Do finance frictions distort entry?
  - $Z_i$ source of TFP losses now

# Model Overview

- Small open economy
- Continuum of agents. Each period decide whether
  - Work: supply 1 unit labor. Earn $W$
  - Entrepreneur: $Y_{it} = A_{it}^{1-\eta} \left(L^{\alpha} K^{1-\alpha}\right)^{\eta}$
- Entrepreneurs can borrow subject to collateral constraint
- Agents die with prob. $1 - p$
- Replaced $1 - p$ newly born.
  - Draw $Z_i$, $(a_i = 0)$, receive endowment $B_0(Z_i)$

# Dynamic program

$$\pi(b) = \max_{k,l} \left(l^{\alpha} k^{1-\alpha}\right)^{\eta} - (1+r)\, Wl - (r+\delta)\, k$$

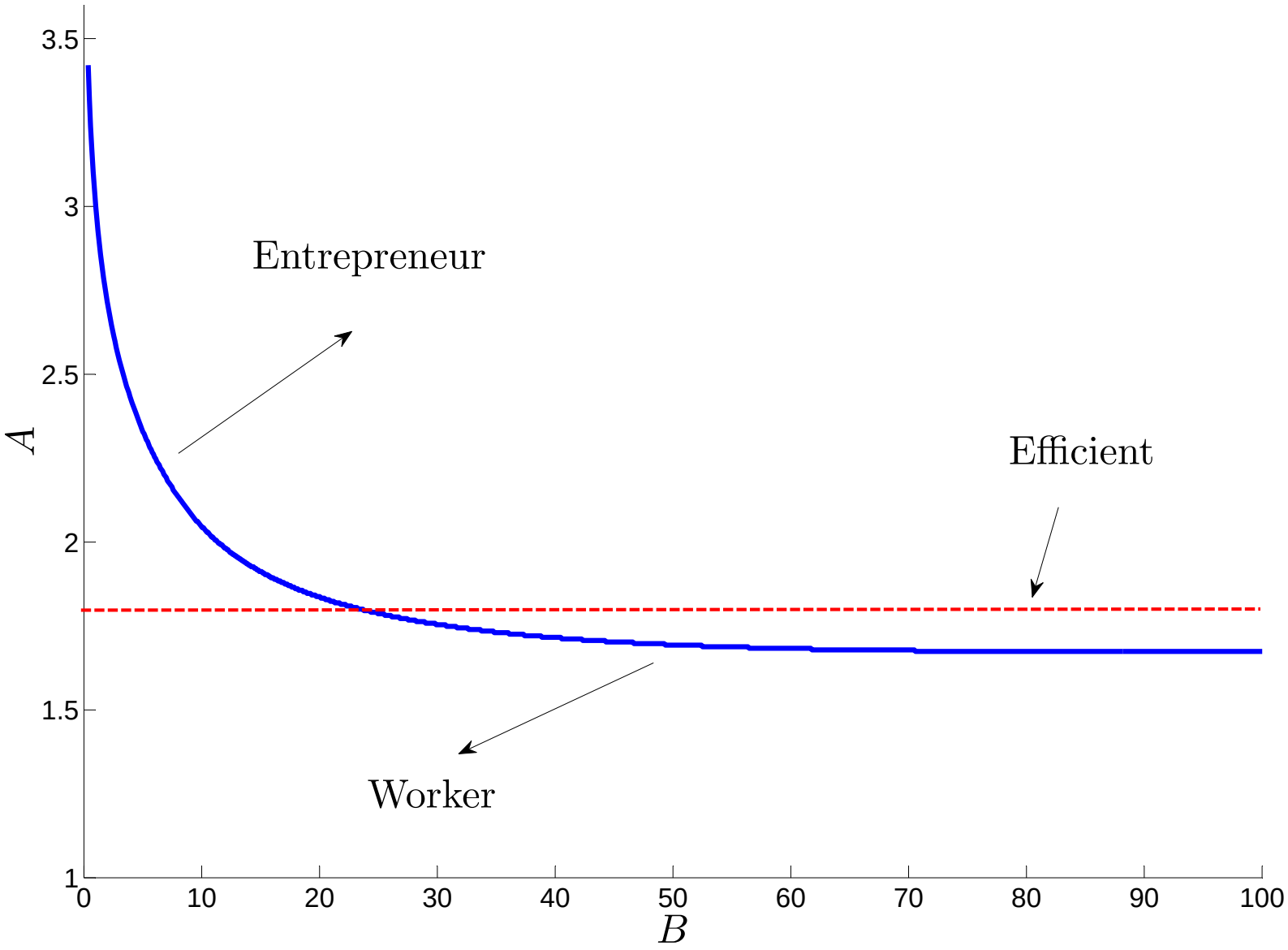
$$\text{s.t.} \;\; Wl + k \leq \lambda b$$

$$V(b,a) = \max_{b'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp(a'-a)^{1-\gamma}\, V\left(\frac{b'}{\exp(a'-a)}, a'\right) d\Phi(a'|a)$$

where

$$c = (1+r)\, b + \max\left[\pi(b), \frac{W}{\exp(a)}\right] - b'.$$

# Occupational choice



# Equilibrium

- $\mu(B, A)$: stationary distribution
- $I(B, A) = W > \Pi(B, A)$: work
- $W$ solves:

$$\int I\left(B, A\right) d\mu\left(B, A\right) = \int L\left(B, A\right)\left(1 - I\left(B, A\right)\right) d\mu\left(B, A\right)$$

# Parametrization

- Two economies:
  - No initial endowment: $B_0(Z_i) = 0$
  - With initial endowment: $B_0(Z_i) = \phi(WL(Z_i) + K(Z_i))$
- Additional parameters: $p$, $\phi$
  - Same set of moments as earlier (use entire panel)
  - Add exit hazards, by age, share output by exiting plants

# Economy with no initial endowment

| | Korea Data | Model |
|---|---|---|
| $\sigma(\Delta y_{it})$ | 0.56 | 0.57 |
| $corr(y_{it}, y_{it-1})$ | 0.92 | 0.93 |
| $corr(y_{it}, y_{it-5})$ | 0.86 | 0.74 |
| fraction $Y$ largest 5% | 0.72 | 0.66 |
| fraction $Y$ largest 20% | 0.87 | 0.89 |
| fraction age 1-5 | 0.51 | 0.62 |
| fraction age 6-10 | 0.26 | 0.16 |
| fraction age $> 10$ | 0.23 | 0.21 |
| exit hazard | 0.33 | 0.25 |
| output share if exit | 0.07 | 0.07 |
| Debt-to-GDP | 1.2 | 1.2 |

# Model predictions (Korea)

| | No exit/entry | With exit/entry |
|---|---|---|
| Fract. constrained | 0.54 | 0.83 |
| Median $\tilde{r} - r$ \|constr | 0.03 | 0.11 |
| iqr $\tilde{r} - r$\|constr | 0.03 | 0.09 |
| 99% $\tilde{r} - r$\|constr | 0.15 | 0.30 |
| TFP losses | 3.9 | 10.6 |
| Due occup. choice | - | 0.2 |

**Bottomline: TFP losses larger**

# Why TFP losses larger?

- Equilibrium $W$ low.
  - Talented entrepreneurs enter almost immediately.
  - Initially very constrained.
- Question: are entering plants constrained data?
- Compare $\Delta y$, returns to capital young vs. old plants

# Young vs. Old plants

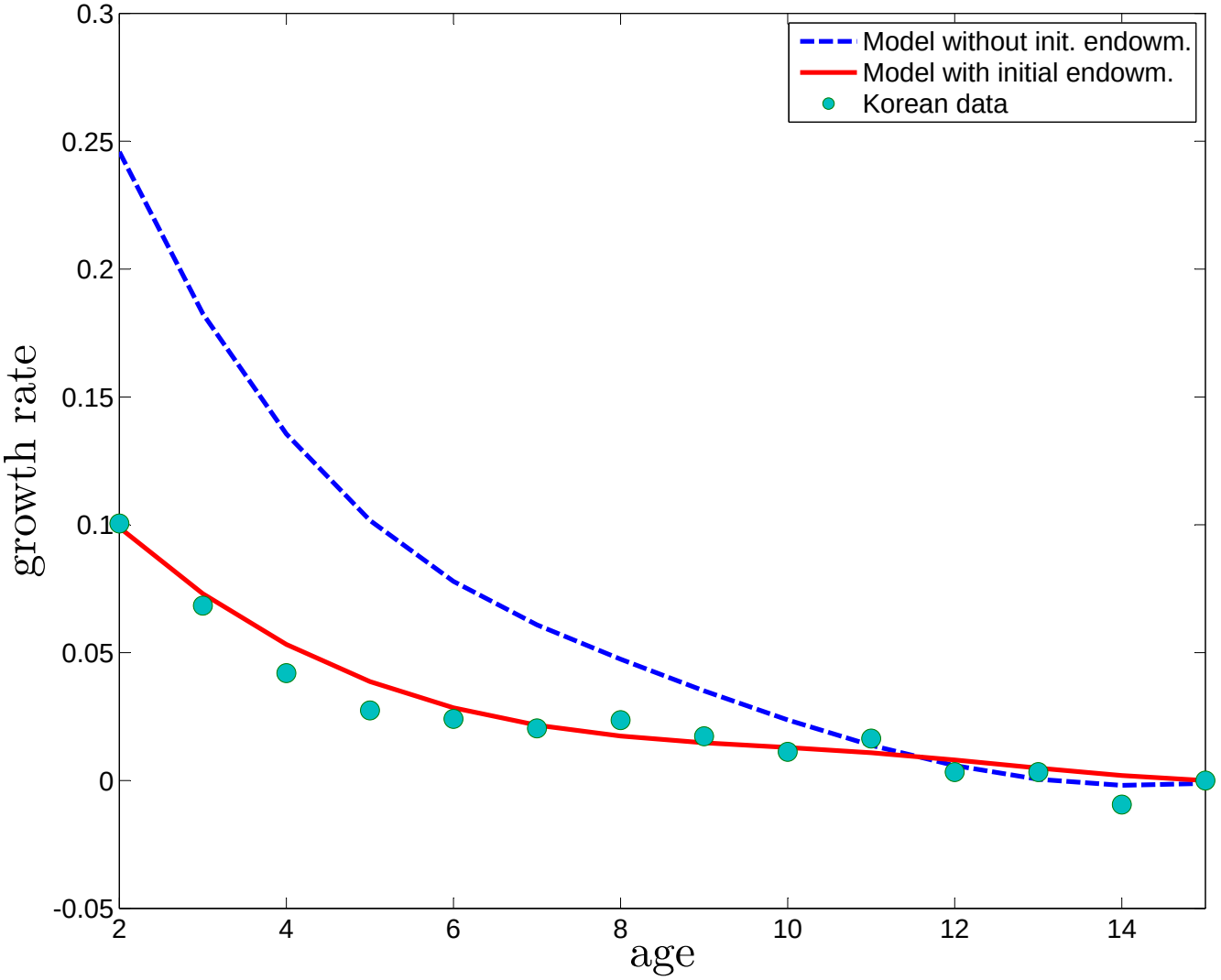
| | Korea Data | Model |
|---|---|---|
| $\Delta y$ ages 1-5 vs. 10+ | 0.05 | 0.20 |
| $\Delta y$ ages 6-10 vs. 10+ | 0.02 | 0.06 |
| $Y/K$ ages 1-5 vs. 10+ | 0.04 | 0.30 |
| $Y/K$ ages 1-5 vs. 10+ | 0.06 | 0.17 |

**Young much more constrained in the model**

# Economy with initial endowment

- At birth, all agents receive $B_0(Z_i) = \phi(WL(Z_i) + K(Z_i))$
- Choose $\phi$ to match relative growth rates, $Y/K$ young

# Growth rate by age



# Economy with initial endowment

| | Korea Data | Model |
|---|---|---|
| $\Delta y$ ages 1-5 vs. 10+ | 0.05 | 0.06 |
| $\Delta y$ ages 6-10 vs. 10+ | 0.02 | 0.01 |
| $Y/K$ ages 1-5 vs. 10+ | 0.04 | 0.10 |
| $Y/K$ ages 1-5 vs. 10+ | 0.06 | 0.05 |
| TFP losses US | - | 1.5 % |
| TFP losses Korea | - | 5.1 % |
| TFP losses Colombia | - | 6.7 % |

# Role of fixed costs

- Do fixed costs prevent internal accumulation?
- Assume $Y = A^{1-\eta}\left((L-\bar{L})^{\alpha}K^{1-\alpha}\right)^{\eta}$
- Choose $\bar{L}$ to match $L$ vs. $Y$ relationship data
  - Regress $\ln\left(\frac{Y}{L}\right)$ on $\ln(Y)$: 0.13 (vs. $\approx 0$ absent fixed costs)
- Need $\bar{L} = 0.06$ of aggregate labor used.

# Role of fixed costs

- Increase TFP losses, but not much:

| | No fixed cost | Fixed Cost |
|---|---|---|
| $\Delta y$ ages 1-5 vs. 10+ | 0.06 | 0.08 |
| $\Delta y$ ages 6-10 vs. 10+ | 0.01 | 0.03 |
| $Y/K$ ages 1-5 vs. 10+ | 0.10 | 0.15 |
| $Y/K$ ages 1-5 vs. 10+ | 0.05 | 0.10 |
| TFP losses US | 1.5% | 1.6 % |
| TFP losses Korea | 5.1% | 5.7 % |
| TFP losses Colombia | 6.7% | 7.1 % |

# Conclusions

- Model that accounts plant-level data:
  - Small TFP losses from misallocation
- Reason: productive entrepreneurs accumulate assets. Grow out of borrowing constraint.
- Need large shocks, constrained entrants to break correlation assets/productivity
  - Inconsistent with plant-level data