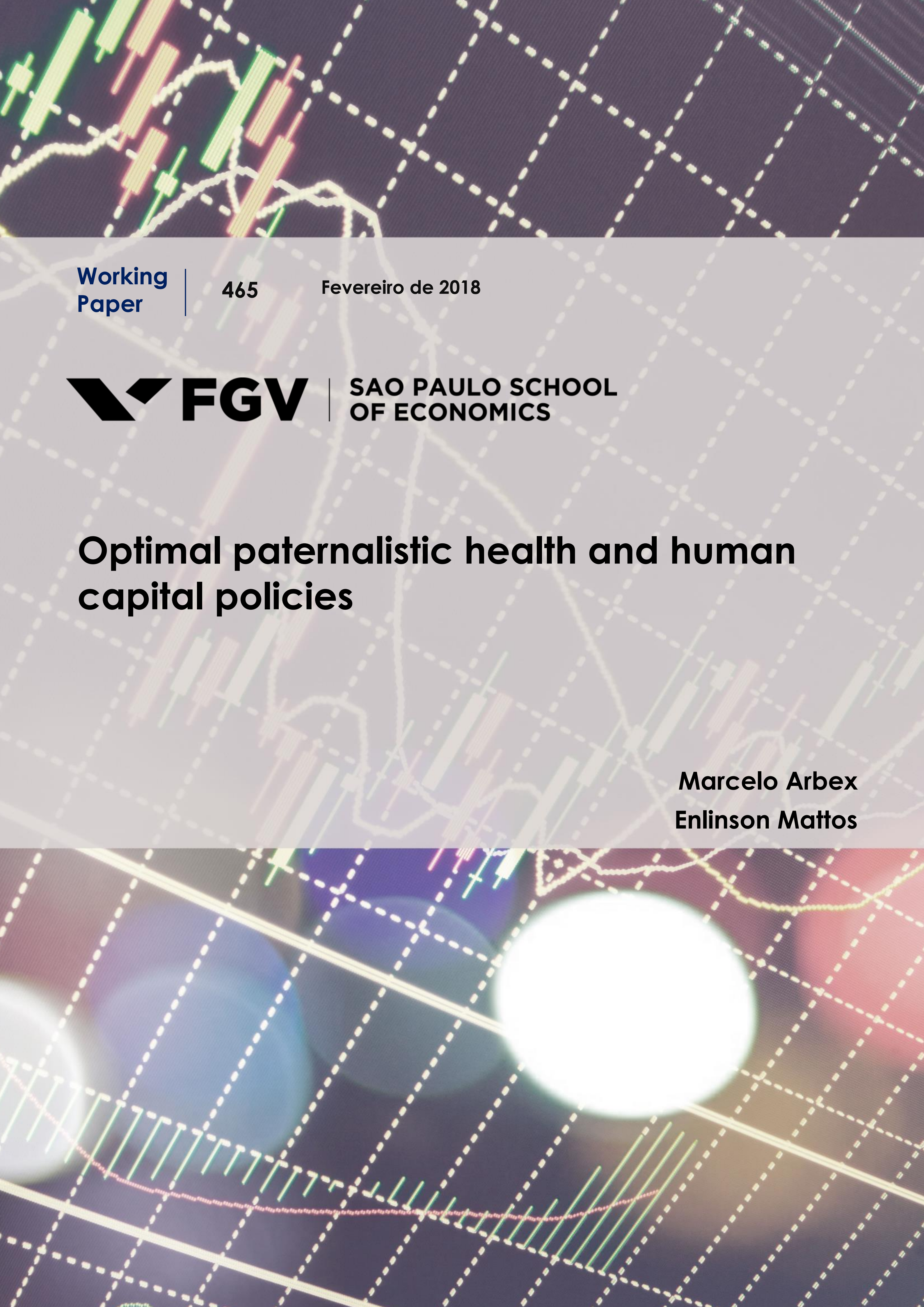

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Optimal Paternalistic Health and Human Capital Policies

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Abstract

We study optimal human and health linear policies when there is a paternalistic motive to overcome present bias problems of agents with heterogeneous cognitive skills. The paternalistic intervention rewards individuals for physical capital accumulation and the combined effect of health and human capital on future earnings. Our results highlight a novel effect of paternalistic policies due to the interaction between present-biased preferences and cognitive skills. We illustrate numerically that this policy package is the most effective and we analyze the relevance of agent's cognitive skills and present-biased preferences for the determination of first-best and constrained first-best optimal policies.

Keywords: Paternalism; Optimal Taxation; Education; Health.

JEL Classification: D61, D91, H21, I18, I28.

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1 Introduction

Human capital formation is arguably the most important investment decision individuals make during their lifetimes. And an individual's human capital is strongly correlated to good health. However, people either underestimate the effect of today's time allocation decision on future human capital or postpone human capital investments to a later date. Moreover, today's consumption of unhealthy food can have detrimental effects on health. In other words, health and human capital decisions might be poised by self-control, time-inconsistency problems. We study optimal human and health linear policies when there is a paternalistic motive to overcome individuals' present bias problems with heterogeneous cognitive skills. The paternalistic intervention is meant to reward individuals for the combined effect of health and human capital on their future earnings and physical capital accumulation. We further explore how paternalistic policies must also take into account potential interactions of present bias and cognitive skills.

We consider an economy consisting of agents who differ in their present-biased preferences and cognitive skills. Agents have a time-inconsistent preference for immediate gratification, i.e., the agent is naïve in the sense of not recognizing that the preference for immediate gratification is present also when the future arrives (O'Donoghue and Rabin (2003)). In our model, these preferences are associated with future decisions that include consumption of unhealthy food, savings and labor-school-leisure choice and follows an extensive literature on present bias and quasi-hyperbolic discounting (Laibson (1997), O'Donoghue and Rabin (1999) and Gruber and Koszegi (2004)). Cognitive skills involve conscious intellectual effort and, in our model, they are associated with agent's skills to accumulate more human capital at a low leisure cost. For each unit of time allocated to the accumulation of human capital, an individual with less cognitive skills sacrifices more time at schooling (Mejia and St-Pierre (2008); Koch et al. (2015)).

Agents work and value consumption of ordinary and unhealthy goods and leisure. They also derive utility from health stock or quality of health, which is negatively (positively) affected by the consumption of unhealthy goods (health care services), as in O'Donoghue and Rabin (2003, 2006); Aronsson and Thunstrom (2008); Cremer et al. (2012). In our model human capital and health decisions affect an individual's labor earnings. Although the human capital stock does not affect agents' instantaneous utility, her current and past decisions regarding schooling affect human capital accumulation and, consequently, time allocation choices.

The externality that the individual's current self imposes on her future selves is a two-dimension stock-externality, in line with the human capital and health economics literature (e.g., Grossman (1972, 2000)). Time-inconsistent individuals underestimate the correct shadow prices of physical, human and health capital, as well as the shadow price of their labor. Hence, there is a paternalistic motive for optimal taxation when self-control problems caused by present-biased discounting may lead to excessive consumption of unhealthy food (health capital), low savings (physical capital) and less time allocated to education (human capital).

We study an earnings and physical capital stock subsidies, which when implemented in the future take into account three behavioral responses of the individual. First, future earnings subsidies, just like future health and human capital, are valued less by the individual. Second, the individual can change the behavior of her future self by increasing future income. These effects are specific to present-biased preferences and often called the discounting and instrumental effects of future subsidies, respectively. When an individual's cognitive skills are considered in the context of present-bias preferences, a novel third effect emerge. An individual can change the behavior of her future self by correcting her misperception of her own future ability to accumulate human capital - the cognitive effect. Future subsidies enrich the instrumental effect by allowing the current self to recognize that her future self will have a biased perception of her (own) cognitive skills prompting her to shift future self human capital decision towards the allocation pattern which an unbiased individual would choose.

We also study two alternative packages following (i) O'Donoghue and Rabin (2003, 2006); Cremer et al. (2012) and Cremer et al. (2012), and (ii) Aronsson and Thunstrom (2008). Although these policy packages also implement the first-best optimal allocations, we show that the timing-target distinction are relevant both for the determination of the optimal subsidy and tax rates and for how cognitive skills and present-biased preferences interact in the optimal policies. This distinction also speaks to the subsidy's effectiveness, measured by the tax revenues required to overcome the present-bias. In fact, our numerical exercise illustrates that a policy package consisting of earnings and physical capital stock subsidies is the most effective, requiring tax revenues in the amount of 110.52 (versus 128.13 and 114.36 for packages (i) and (ii), respectively).

We define the constrained first-best outcome as the first-best outcome given that type-specific policies are not allowed or possible. In a constrained first-best equilibrium, we show that even if there is an individual with no present-bias she still faces non-zero taxes/subsidies. Evidently, in this constrained first-best setup, the resulting optimal equilibrium is clearly sub-optimal when compared to the (unconstrained) first-best equilibrium. We illustrate numerically the relevance of agent's cognitive skills and present bias for the determination of first-best and constrained first-best optimal policies.

Our paper is closely related to tax policies in the context of present-bias and self-control problems (Gruber and Koszegi (2004); Salanie and Treich (2006); Cremer and Pestieau (2011); Aronsson and Granlund (2011); Farhi and Gabaix (2015); Lockwood (2016); Moser and de Souza e Silva (2017), among others). Policies of this kind are an example of paternalism, and their purpose is to protect individuals when they act against their own best self-interest. This literature considers, for instance, how linear taxes can be used to either prevent over consumption of some goods (e.g., fossil fuels, drugs) or to foster consumption of other goods (e.g., retirement savings). For instance, O'Donoghue and Rabin (2003) model an economy where individuals have hyperbolic preferences and differ both in their taste for the sin good and in their degree of time inconsistency. The authors show how (heterogeneity in) time inconsistency affects the optimal (Ramsey) consumption tax policy. Aronsson and Thunstrom (2008) show that subsidies on wealth and health capital can be used to implement a socially optimal resource allocation. And in Cremer et al. (2012), individuals are myopic and underestimate the effect of the sinful consumption on health, but they may acknowledge, in their second period, their mistake or persist in their error. They characterize and compare the first-best and the (linear) second-best taxes when sin good consumption and health care interact in the health production technology.

Knowledge about human behavior from psychology and sociology has enhanced the field of economics of education and health. Grossman and Kaestner (1997) and Grossman (2000, 2005), among others, have provided detailed evidence suggesting that years of formal schooling completed is the most important correlate of good health. Moreover, cognitive skills and soft skills are equally important drivers of later economic outcomes (Shoda et al. (1990), Golsteyn et al. (2014), Koch et al. (2015), Courtemanche et al. (2015)). Recently, a rising literature shows the growing importance of social skills versus cognitive skills on earnings (Deming (2017); Edin et al. (2017)) and the multidimensionality of learning at school (Kraft (2017); Petek and Pope (2016)). More related to our work, Stantcheva (2017) and Stark and Wang (2002) characterize optimal policies associated with human capital investment.

In practice, there are many programs that aim to induce individuals to invest on their human and health capital. Following the tradition of conditional cash transfers (CCT) these programs aim to subsidize poor families as long as their children attend school and visit a health center regularly (Fiszbein et al. (2009)). These are policies that aim to induce current investment on education and health. Studies have also investigated the impact of such programs on health stocks (for instance, Attanasio et al. (2005) for Colombia, Morris et al. (2004) for Brazil) and on cognitive development (Schady (2007) for Ecuador, Macours et al. (2012) for Nicaragua). Alternatively, some redistributive programs award individuals with good levels of human and health stocks. In Brazil two educational programs provide cash transfers when students complete high school (Renda Melhor Jovem - Rio de Janeiro State and Poupanca Jovem - Minas Gerais State).

Our paper adds to previous research which has put more emphasis on health-related interventions. In particular, to the best of our knowledge, human capital decisions and their relationship with health outcomes, which are at the core of our approach, together with the role of cognitive skills and presentbias have not yet been analyzed in the context of optimal paternalistic policies. The paper is divided as follows. Section 2 presents our model economy. In Section 3, we characterize the first-best and constrained first-best optimal policy packages that include an earnings subsidy and a subsidy to an individual's stock of physical capital. An illustrative example is provided. In Section 4 two alternative policy packages are analyzed. Section 5 concludes the paper.

2 The Model

We consider an economy consisting of $I \times J$ types of individuals indexed by superscript ij, for $i \in [1, I]$, $j \in [1, J]$. Agents are different regarding their cognitive (i) skills and present-bias (j) discounting. Agents have a time-inconsistent preference for immediate gratification denoted by a discount factor $\beta^j < 1$. We follow the present-biased preferences literature by using an approach developed by Phelps and Pollak (1968) and later used by e.g. Laibson (1997) and O'Donoghue and Rabin (2003). In our model, these preferences are related to the consumption of unhealthy food (accumulation of health capital), savings (physical capital accumulation) and whether to work or enjoy leisure instead of studying (investment in human capital).

The effective time cost (in terms of leisure) per unit of time devoted to human capital formation is denoted by $\zeta^i \in (0, 1)$. This term captures an individual's cognitive skills, i.e., an agent's ability to convert units of studying time (thinking and reasoning) in productive human capital with less effort. In other words, cognitive skills refer to an agent's exogenously given endowment of the complementary factors to the schooling process - skills associated with agent's ability to accumulate more human capital at a low leisure cost. For each unit of time that ij-type individual allocates to the accumulation of human capital, she sacrifices a fraction of leisure time equal to $\zeta^i s_t^{ij}$ as she decides to spend s_t^{ij} hours building human capital (studying, training). An agent with high cognitive skills (low ζ^i) experiences a lower leisure cost of studying and she can accomplish more for each unit of time dedicated to study. This assumption that captures the fact that different individuals face different costs of acquiring human capital (Mejia and St-Pierre (2008); Koch et al. (2015)).

The instantaneous utility function facing the ij-type agent is

$$u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) \quad + \quad v\left(z_t^{ij}\right) \tag{1}$$

where c_t^{ij} is the consumption of an ordinary (not unhealthy) good, x_t^{ij} the consumption of the unhealthy good, m_t^{ij} the stock of health capital. An individual's leisure is given by $z_t^{ij} = 1 - \zeta^i s_t^{ij} - l_t^{ij}$, where l_t^{ij} is the time in market work. We assume that functions $u(\cdot)$ and $v(\cdot)$ are increasing in each argument and strictly concave.

An individual chooses among non-mutually exclusive education and labor market options in order to maximize lifetime utility, knowing that current education, consumption habits and labor market decisions affect future earnings and her health and human capital stocks. The intertemporal objective at time t is given by

$$U_t^{ij} \;=\; \left[u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right)\right] + \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[u\left(c_s^{ij}, x_s^{ij}, m_s^{ij}\right) + v\left(z_s^{ij}\right)\right] \tag{2}$$

where $\Theta^t = 1/(1+\theta)^t$ is a conventional utility discount factor with utility discount rate θ.

Following O'Donoghue and Rabin (2003), we assume that the agent is naïve in the sense of not recognizing that the preference for immediate gratification is present also when the future arrives. Notice that since a time-inconsistent individual consists of multiple selves, she is not able to commit to a particular future consumption behavior. Every self has a tendency to pursue immediate gratification in a way that their future selves do not appreciate. She will therefore choose allocations that maximizes her current utility plus a biased version of future utilities, expression (2), and not the individual's long-run utility as expressed by U_t^{ij} when $\beta^j = 1$.[1]

The agent's human and health capital stocks evolve as follows

$$h_{t+1}^{ij} - (1-\delta_h)\, h_t^{ij} \;=\; B\left(s_t^{ij}\right) \tag{3}$$

$$m_{t+1}^{ij} - (1-\delta_m)\, m_t^{ij} \;=\; g\left(x_t^{ij}, e_t^{ij}\right) \tag{4}$$

where δ_h, δ_m are the human and health capital stock depreciation rates, respectively, $B\left(s_t^{ij}\right)$ is an increasing and concave function of the fraction of time invested in human capital formation, s_t^{ij} (i.e., $\partial B\left(s_t^{ij}\right)/\partial s_t^{ij} > 0$), e_t^{ij} denotes health care services and $g(\cdot)$ is a health production function with the properties $\partial g\left(x_t^{ij}, e_t^{ij}\right)/\partial x_t^{ij} < 0$ and $\partial g\left(x_t^{ij}, e_t^{ij}\right)/\partial e_t^{ij} > 0$.

The household budget constraint is

$$c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij} \;=\; (1 + R_t - \delta_k)\, k_t^{ij} + W_t A_t^{ij} l_t^{ij} \tag{5}$$

where $A_t^{ij} = m_t^{ij} h_t^{ij}$ and the household holds an asset in the form of physical capital k_t^{ij}, which depreciates at rate δ_k.[2] The prices of the two consumption goods $\left(c_t^{ij}, x_t^{ij}\right)$ and health care services $\left(e_t^{ij}\right)$ are set equal to one. We assume that the agent takes the wage and the interest rates as exogenous given, W_t and R_t, respectively.

A representative firm produces a single good (Y_t) with capital $K_t = \sum_{i,j} \gamma^{ij} k_t^{ij}$, where γ^{ij} is the share of ij-type in the population $\left(\sum_{i,j} \gamma^{ij} = 1\right)$ and the quality-adjusted labor input, $L_t = \sum_{i,j} \gamma^{ij} L_t^{ij} = \sum_{i,j} \gamma^{ij} m_t^{ij} h_t^{ij} l_t^{ij}$, which takes into account the worker's health and human

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[1] We do not consider a mechanism through which individuals can regulate their self-control, presente bias problems. See, for instance, Koch and Nafziger (2011)).

[2] In a previous version of this paper, we allowed the individual's wage to depend on the total health and human capital of the economy ($A_t = \sum A_t^{ij}$). Adding such (social) externality term, this feature would enhance the size of the optimal policies and potentially justifying government intervention alone. Results available upon request.

capital, i.e., $Y_t = F(K_t, L_t)$. The firm operates under perfect competition and maximize profits. Factors of production are paid their marginal products, implying that $\partial F(K_t, L_t)/\partial K_t = R_t$ and $\partial F(K_t, L_t)/\partial L_t = W_t$.

The economy resource constraint for period t is as follows

$$F(K_t, L_t) + (1-\delta_k)\sum_{i,j}\gamma^{ij}k_t^{ij} = \sum_{i,j}\gamma^{ij}\left(c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij}\right) \tag{6}$$

In period t, the household chooses allocations $\{c_t^{ij}, x_t^{ij}, e_t^{ij}, s_t^{ij}, l_t^{ij}, k_{t+1}^{ij}, m_{t+1}^{ij}, h_{t+1}^{ij}\}$ to maximize the utility function (2) subject to equations (3), (4), and (5), treating the initial physical, health and human capital stocks, k_0^{ij}, m_0^{ij} and h_0^{ij}, as exogenously given. A ij-type agent problem in Lagrangian form is as follows:

$$\begin{aligned}
\mathscr{L}_{ij} &= u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right) \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[u\left(c_s^{ij}, x_s^{ij}, m_s^{ij}\right) + v\left(z_s^{ij}\right)\right] \\
&+ \lambda_t^{ij}\left[W_t\left(m_t^{ij}h_t^{ij}\right)l_t^{ij} + (1+R_t-\delta_k)k_t^{ij} - c_t^{ij} - x_t^{ij} - e_t^{ij} - k_{t+1}^{ij}\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\lambda_s^{ij}\left[W_s m_s^{ij}h_s^{ij}l_s^{ij} + (1+R_s-\delta_k)k_s^{ij} - c_s^{ij} - x_s^{ij} - e_s^{ij} - k_{s+1}^{ij}\right] \\
&+ \mu_t^{ij}\left[m_{t+1}^{ij} - (1-\delta_m)m_t^{ij} - g\left(x_t^{ij}, e_t^{ij}\right)\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\mu_s^{ij}\left[m_{s+1}^{ij} - (1-\delta_m)m_s^{ij} - g\left(x_s^{ij}, e_s^{ij}\right)\right] \\
&+ \xi_t^{ij}\left[h_{t+1}^{ij} - (1-\delta_h)h_t^{ij} - B\left(s_t^{ij}\right)\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[h_{s+1}^{ij} - (1-\delta_h)h_s^{ij} - B\left(s_s^{ij}\right)\right]
\end{aligned} \tag{7}$$

Let $u^{ij}(t) = u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right)$ and $u_c^{ij}(t) = \partial u^{ij}(t)/\partial c_t^{ij}$, for a ij-type individual, and likewise for other allocations and functions. Combining the first order conditions for the household, while eliminating the Lagrange multipliers, the necessary conditions for an interior solution of the household's maximization problem are given by

$$
\begin{aligned}
u_x^{ij}(t)-u_c^{ij}(t)+u_c^{ij}(t)\frac{g_x^{ij}(t)}{g_e^{ij}(t)} &= 0 \qquad (8)\\
u_c^{ij}(t)-\beta^j u_c^{ij}(t+1)\left[1+R_t-\delta_k\right] &= 0 \qquad (9)\\
-\frac{u_c^{ij}(t)}{g_e^{ij}(t)}+\beta^j\Theta\left[u_m^{ij}(t+1)+u_c^{ij}(t+1)W_{t+1}h_{t+1}^{ij}l_{t+1}^{ij}+(1-\delta_m)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)}\right] &= 0 \qquad (10)\\
v_z^{ij}(t)-u_c^{ij}(t)W_t m_t^{ij}h_t^{ij} &= 0 \qquad (11)\\
-\zeta^i\frac{v_z^{ij}(t)}{B_s^{ij}(t)}+\beta^j\Theta\left[u_c^{ij}(t+1)W_{t+1}m_{t+1}^{ij}l_{t+1}^{ij}+(1-\delta_h)\zeta^i\frac{v_z^{ij}(t+1)}{B_s^{ij}(t+1)}\right] &= 0 \qquad (12)
\end{aligned}
$$

A ij-type agent's optimal behavior and conditions concerning the trade-off between consumption, time and capital stock allocations are represented by equations (8) - (12), which together with equations (3), (4) and (5), characterize the equilibrium in the decentralized market economy. Equation (8) represents the optimal choice of x_t^{ij}, in which the shadow price associated with health capital is equal to $(u_c^{ij}(t)/g_e^{ij}(t))$ at the equilibrium. Equation (11) is the condition for the optimal choice between schooling and hours of work. Similarly, equations (9), (10) and (12) refer to the optimal choices of k_{t+1}^{ij}, m_{t+1}^{ij} and h_{t+1}^{ij}, respectively. The conditions concerning the optimal choice of health and human capital take into account the effect of these choices on accumulation of capital stocks, as well as their effects on an agent's earnings (and the direct effect of health status on agent's utility). Notice that while ζ^i only appears in equation (12) affecting the marginal rate of substitution between study and leisure time, the present-biased discounting β^j affects all other choices.

3 Earnings and Physical Capital Stock Subsidies

We assume that the planner is paternalistic utilitarian and its objective consists of the sum of utilities where $\beta^j = 1$ following O'Donoghue and Rabin (2003); Aronsson and Thunstrom (2008); Cremer et al. (2012), among others. The reason for the difference between the planner's and the individuals' preferences resides in the (unrecognized) mistakes made by individuals. Time-inconsistent individuals underestimate the real (correct) shadow prices of physical, human and health capital, as well as the shadow price of their labor.

The planner's goal is to design policies that induce individuals to internalize the external effects of their time-inconsistent preference for immediate gratification and their misperception of their own cognitive skills. Future policies are to be announced in each period and they must be part of a "surprise policy". That is, since agents do not expect to be time-inconsistent in the future, policies are announced in a given period, to be implemented in the next period.[3]

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[3] Following the optimal paternalistic policy literature, for instance, Aronsson and Thunstrom (2008), a surprise

The planner's policy choice is constrained by human and health capital laws of motion and the aggregate resource constraint, equations (3), (4), and (6), respectively. The planner's problem in the Lagrangian form is as follows

$$\begin{aligned}
\mathscr{L}_P^{1st} &= \sum_{t=0}^{\infty} \Theta^t \left\{ \sum_{i,j} \gamma^{ij} \left[u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right) \right] \right. \qquad (13)\\
&+ \eta_t \left[F\left(K_t, L_t\right) + \left(1-\delta_k\right) \sum_{i,j} \gamma^{ij} k_t^{ij} - \left(\sum_{i,j} \gamma^{ij} \left(c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij} \right) \right) \right] \\
&+ \widehat{\eta}_t^{ij} \sum_{i,j} \gamma^{ij} \left[h_{t+1}^{ij} - \left(1-\delta_h\right) h_t^{ij} - B\left(s_t^{ij}\right) \right] \\
&+ \left. \widetilde{\eta}_t^{ij} \sum_{i,j} \gamma^{ij} \left[m_{t+1}^{ij} - \left(1-\delta_m\right) m_t^{ij} - g\left(x_t^{ij}, e_t^{ij}\right) \right] \right\}
\end{aligned}$$

The necessary conditions for an interior solution of the planner's maximization problem are similar to the household's ones, except for the fact that $\beta^j = 1$, for all ij-type agents. Denote the socially optimal (first-best) resource allocation, i.e., the solution of the planner's problem, as $\{c_t^{ij*}, x_t^{ij*}, e_t^{ij*}, s_t^{ij*}, l_t^{ij*}, k_{t+1}^{ij*}, m_{t+1}^{ij*}, h_{t+1}^{ij*}\}$ for all agents type ij and period t, and define $u^{ij*}(t) = u\left(c_t^{ij*}, x_t^{ij*}, m_t^{ij*}\right)$, $v^{ij*}(t) = v\left(z_t^{ij*}\right)$, $B^{ij*}(t) = B\left(s_t^{ij*}\right)$, $g^{ij*}(t) = g\left(x_t^{ij*}, e_t^{ij*}\right)$, and $F^*(t) = F\left(K_t^*, L_t^*\right)$.

3.1 Optimal First-Best Paternalistic Policies

In our economy an individual's earnings are determined by the health-quality of her human capital, i.e., the combination of her health and human capital. If the planner can identify each agent's cognitive skills and present-bias, it can design type-specific policies. We assume that the planner can commit to policies that subsidize the individual's physical capital stock and earnings, taking into account the interaction of her time-inconsistent preference for immediate gratification, i.e., agent's present-biased preferences and cognitive skills. The subsidies to an individual's earnings and stock of physical capital reward individuals for the combined effect of health and human capital decisions on their future earnings.

Consider a ij-type individual's problem similar to problem (7), except for the modified budget constraint

$$\begin{aligned}
c_{t+1}^{ij} + x_{t+1}^{ij} + e^i{}_{t+1} + k_{t+2}^{ij} &= \left(1 + R_{t+1} - \delta_k\right)\left(1 + P_{t+1}^{ij*}\right) k_{t+1}^{ij} \\
&+ \left(1 + O_{t+1}^{ij*}\right) W_{t+1} A_{t+1}^{ij} l_{t+1}^{ij} + T_{t+1}^{ij*}
\end{aligned}$$

---

policy is needed to achieve first-best in this economy because the planner has to impose a policy on agent's self today to provide the correct incentives for tomorrow's decisions. And this has to be done every period.

The first-order conditions of this problem are equivalent to equations (8) - (12), where P_{t+1}^{ij*} and O_{t+1}^{ij*} are the physical capital stock and earnings subsidies, respectively, to be implemented in period $t+1$. The lump-sum tax T_{t+1}^{ij*} is such that the government's budget constraint is satisfied for all ij-type agent, $\forall t$, i.e., $(1+R_{t+1}-\delta_k)\left(P_{t+1}^{ij*}\right)k_{t+1}^{ij}+\left(O_{t+1}^{ij*}\right)W_{t+1}A_{t+1}^{ij}l_{t+1}^{ij}=T_{t+1}^{ij*}$. The following proposition characterizes the optimal policies needed to implement the first-best allocations in our economy. For the ease of readability, all proofs are contained in the Appendix.

Proposition 1. *In each period t and for each agent ij, suppose the government announces a surprise policy package to be implemented in period $t+1$ that contains a subsidy to agent's physical capital stock, $(1+R_{t+1}-\delta_k)\left(1+P_{t+1}^{ij*}\right)k_{t+1}^{ij}$, and earnings, $\left(1+O_{t+1}^{ij*}\right)W_{t+1}A_{t+1}^{ij}l_{t+1}^{ij}$. With subsidies*

$$P_{t+1}^{ij*} = \frac{1-\beta^j}{\beta^j}, \tag{14}$$

$$O_{t+1}^{ij*} = \left(\frac{1-\beta^j}{\beta^j u_c^{ij*}(t+1)F_L^*(t+1)l_{t+1}^{ij}}\right)\left\{\begin{array}{l} u_A^{ij*}(t+1) \\ +u_c^{ij*}(t+1)F_L^*(t+1)l_{t+1}^{ij} \\ +\left(1-\delta_m\right)\frac{u_c^{ij*}(t+1)}{g_e^{ij*}(t+1)h_{t+1}^{ij}} \\ +\left(1-\delta_h\right)\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \end{array}\right\}, \tag{15}$$

where $A_{t+1}^{ij}=m_{t+1}^{ij}h_{t+1}^{ij}$, the equilibrium in the decentralized economy is equivalent to the social optimum.

The subsidy on the agent's physical capital stock P^{ij*}, equation (14), depends only on the agent's time-inconsistent preference for immediate gratification, i.e, the individual's present bias. That is, the subsidy is equal to the rate $\left(1-\beta^j\right)/\beta^j$ at which the j-type underestimate the future benefit of physical capital accumulation. This policy is equivalent to Aronsson and Thunstrom (2008)'s wealth policy (Proposition 1 in their paper), and the subsidy is higher, the more present bias an individual j is. Also, the physical capital subsidy is similar to Cremer et al. (2012)'s policy on health care services, equation (6). The planner subsidizes (unit) health care consumption at fixed rate given by the difference of the agent's present-bias discount rate with respect to the correct discount rate, i.e., β^j-1.

The optimal subsidy O^{ij*}, equation (15), balances the wedge between the biased and unbiased joint evaluation of health and human capital decisions. With the policy O^{ij*} the planner takes into account all possible consequences of health and human capital-related decisions a self t individual, with cognitive skills ζ^i and present-biased preferences β^j, make that her future self would not appreciate, thereby correcting for the bias. The first two terms in the curly bracket of equation (15) captures the policy bias correction of the direct effects of an individual's mistakes, namely the effects on her utility and her earnings. The first term gives the present value of the undervaluation of the marginal utility of better health capital $\left(u_A^{ij*}\right)$ while the second term captures the present value of higher earnings due to both better health and human capital stocks, i.e., the impact on future consumption due to an increase in earnings, $(u_c^{ij*}F_L^{*}l^{ij})$.

Indirectly, the third term relaxes the shadow price between future consumption (adjusted by the depreciation of health capital) and medical expenditures $(1-\delta_m)\, u_c^{ij*}/g_e^{ij*}h^{ij}$, weighted by the individual's education level. The last term (curly bracket, equation (15)) shows that this particular policy also affects the shadow price between leisure and human capital investment $(1-\delta_h)\, \zeta^i v_z^{ij*}/B_s^{ij*}m^{ij}$, in this case, weighted by the individual's health level. Notice that with these two last terms, the earnings subsidy contemplate the effects of an individual's health-related and time allocation decisions on her health and human capital accumulation, respectively. The additional utility a self at t acquires through the subsidy if she increases her health or human capital stock by one unit is measured by the term $(\beta^j u_c^{ij*}F_L^{*}l^{ij})\, O^{ij*}$.

The direct effect on earnings and utility convey the marginal effects of both health and education changes. With a single policy that takes into account the interactions between health and human capital decisions and consequences, the key difference resides on the fact that these allocations' effects on shadow prices - health versus future consumption and education and leisure - are weighted by each complementary input (health and human capital) in the production function, respectively. Furthermore, the effects these inputs have on current (biased) decisions are positive, which affects the optimal subsidy positively. *Ceteris paribus*, equation (15) also suggests that low cognitive skills and present bias individuals, i.e., high ζ^i and low β^j, respectively, should receive a higher earnings subsidy than their counterparts.

The earnings subsidy takes into account three behavioral responses of the individual to paternalistic policies. First, future earnings transfers, just like future health and human capital, are valued less by the individual at period t. The self t, who makes human and health capital decisions, evaluates period $t+1$ utility and earnings differently from her self $t+1$, who receives the subsidy. Since these additional benefits are received in the future, the self t individual disregards a fraction $(1-\beta^j)$ of them obtained by the marginal spending on both capital stocks. Second, the individual can change the behavior of her future self by increasing future income. Future subsidies allow self t to shift self $t+1$'s decisions in a way self t appreciates. From self t's perspective, there should be no additional discounting of health-human capital benefit from period $t+2$ to period $t+1$. Since self $t+1$ makes biased decisions, the current self anticipates that the future self, for instance, spends less on human capital accumulation (i.e., studying) and/or more on unhealthy consumption than what the current self considers optimal. These effects, often called the discounting and instrumental effects of future subsidies, respectively, are specific to present-biased preferences.

When an individual's cognitive skills are considered in the context of present-bias preferences, a novel third effect emerge. An individual can change the behavior of her future self by correcting the misperception of her own future cognitive skills (last term of equation (15)). We call this the cognitive effect of the future subsidy. Future subsidies enrich the instrumental effect by allowing self t to recognize that self $t+1$ will have a biased perception of her cognitive skills prompting her to shift self $t+1$'s human capital decision towards the allocation pattern which an unbiased individual would choose.

In the absence of self-control problems ($\beta^j = 1$) the right-hand sides of equations (14) and (15) are equal to zero and, therefore, the only solution for the optimal subsidies is $P^{ij*} = 0$ and $O^{ij*} = 0$. The reason is that the individual does not exhibits time inconsistency problems and maximizes the same lifetime utility as the social planner. Therefore, there is no need for an intervention.[4]

3.2 Optimal Constrained First-Best Paternalistic Policies

The planner, however, might not be able to identify each agent's cognitive skills and present-biased discount being constrained to use a single policy package for all agents. To investigate such a case, we define the constrained first-best outcome as the first-best outcome given that type-specific policies are not allowed or possible. Evidently, in this constrained first-best setup, the resulting optimal equilibrium is clearly sub-optimal when compared to the (unconstrained) first-best equilibrium (Section 3.1).

Combining the equilibrium equations of all ij-types with the planner's equilibrium conditions (solution of problems (7) and (13)), we obtain a single optimal policy package for all agents (Appendix 5). These constrained first-best policies follow directly from Proposition 1, the main difference being that they give different weights to allocations of those with heterogeneous cognitive skills and present bias, i.e., policies take into account the weighted average of all individuals' allocations $\left(\sum_{i,j} \gamma^{ij}\right)$. The following corollary summarizes our results.

Corollary 1. *In each period t and for all ij-types, suppose the government announces a surprise policy package to be implemented in period $t+1$ that contains a subsidy to agent's physical capital stock,* $(1 + R_{t+1} - \delta_k)\left(1 + \widehat{P}^*_{t+1}\right) k^{ij}_{t+1}$*, and earnings,* $\left(1 + \widehat{O}^*_{t+1}\right) W_{t+1} A^{ij}_{t+1} l^{ij}_{t+1}$*. Then the constrained first-best equilibrium can be decentralized if*

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[4] We have also studied second-best optimal policies for this economy. However, their analytical solution are not informative and intuition is not as clear as the first-best optimal policies presented. Second-best results are available upon request.

$$\widehat{P}^*_{t+1} = \frac{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{\beta^j u_c^{ij*}(t+1)} - \sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}}{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}} \tag{16}$$

$$\widehat{O}^*_{t+1} = \left(\frac{1}{\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(t+1)F_L^*(t+1)l_{t+1}^{ij}}\right)\left\{\begin{array}{l} \sum_{i,j}\gamma^{ij}\frac{u_A^{ij*}(t+1)}{h_{t+1}^{ij}} - \sum_{i,j}\gamma^{ij}\beta^j\frac{u_A^{ij*}(t+1)}{h_{t+1}^{ij}} \\ +\sum_{i,j}\gamma^{ij}F_L^*(t+1)l_{t+1}^{ij}u_c^{ij*}(t+1) \\ -\sum_{i,j}\gamma^{ij}\beta^j F_L^*(t+1)l_{t+1}^{ij}u_c^{ij*}(t+1) \\ +(1-\delta_m)\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t+1)}{g_e^{ij*}(t)h_t^{ij}} \\ -(1-\delta_m)\sum_{i,j}\gamma^{ij}\beta^j\frac{u_c^{ij*}(t)}{g_e^{ij*}(t)h_t^{ij}} \\ +(1-\delta_h)\sum_{i,j}\gamma^{ij}\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \\ -(1-\delta_h)\sum_{i,j}\gamma^{ij}\beta^j\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \end{array}\right\} \tag{17}$$

The optimal earnings subsidy calls for a (weighted average) correction of the marginal effects on the individuals' utility, $\sum_{i,j}\gamma^{ij}u_A^{ij*}/h_{t+1}^{ij} - \sum_{i,j}\gamma^{ij}\beta^j u_A^{ij*}/h_{t+1}^{ij}$, the marginal effects on earnings, $\sum_{i,j}\gamma^{ij}F_L^*l^{ij}u_c^{ij*} - \sum_{i,j}\gamma^{ij}\beta^j F_L^*l^{ij}u_c^{ij*}$, the marginal rate of substitution between consumption and medical expenditures (weighted by individuals' education level), $\sum_{i,j}\gamma^{ij}\left(u_c^{ij*}/g_e^{ij*}h^{ij}\right) - \sum_{i,j}\gamma^{ij}\beta^j\left(u_c^{ij*}/g_e^{ij*}h^{ij}\right)$, and the marginal rate of substitution between leisure and hours of study, $\sum_{i,j}\gamma^{ij}\zeta^i\left(v_z^{ij*}/B_s^{ij*}m^{ij}\right) - \sum_{i,j}\gamma^{ij}\beta^j\zeta^i\left(v_z^{ij*}/B_s^{ij*}m^{ij}\right)$.

An interesting feature of this equilibrium is the fact that the no intervention case, i.e., $\widehat{P}^*_{t+1} = \widehat{O}^*_{t+1} = 0$, is only possible if $\beta^j = 1$ for all agents in the economy. However, if at least one individual exhibits present bias problems, the optimal subsidies will not be equal to zero, affecting all individuals in this case. These constrained first-best policies might, one one hand, correct the time-inconsistency of some agents but, on the other hand, affect the welfare of those without self-control problems.

3.3 An Illustrative Example

In order to illustrate our main results numerically we consider an economy populated by four types who are heterogeneous with respect to their cognitive skills and present-biased preferences. That is, individuals are heterogeneous with respect to their leisure cost of education (cognitive skill, ζ) and their time-inconsistent preference for immediate gratification (β). Some agents discount the future more heavily and have greater present bias towards consumption and leisure ($\beta^H = 0.85$) than others ($\beta^L = 0.90$). We assume that agents have the same present bias towards consumption and leisure. To an agent with high cognitive skills we assign $\zeta^H = 0.5$, i.e. she can accomplish more for each unit of time dedicated to study and, hence experiences a lower leisure cost of studying. We set $\zeta^L = 0.8$ to a low cognitive skills individual. Hence, the four ij-types are labeled as LL,

LH, HL, and HH. For instance, the LL type is an individual with low cognitive skills and low present-biased preferences.

We assume the following functional forms. Preferences: $u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) = log\left(c_t^{ij}\right) + log\left(x_t^{ij}\right) + \phi_1 log\left(m_t^{ij}\right)$ and $v\left(z_t^{ij}\right) = \phi_2 \frac{(1-\zeta^i s_t^{ij} - l_t^{ij})^{1-\eta}}{(1-\eta)}$; Technology: $F(K_t, A_t) = K_t^{\alpha} A_t^{1-\alpha}$; Health Production Function: $g\left(x_t^{ij}, e_t^{ij}\right) = D_1\left(e_t^{ij}\right)^{\gamma} - D_2 x_t^{ij}$; Human Capital Function: $B\left(s_t^{ij}\right) = B_1\left(s_t^{ij}\right)^{\theta}$. The weights on health status and leisure are normalized to one, i.e., $\phi_1 = \phi_2 = 1$. The conventional utility discount factor is $\Theta^t = 1/(1+\theta)^t$, where we set $\Theta = 0.99$ which is consistent with a steady-state real interest rate of one percent (per quarter). For our present purposes, we assume $\eta = 2.0$ and $\alpha = 0.33$. We set $D_1 = D_2 = 0.25$, $\gamma = 0.50$, $B_1 = 0.25$, and $\theta = 0.85$. We assume that physical capital does not depreciates and the depreciation rates of health stock and human capital are $\delta_h = \delta_m = 0.10$.[5]

In this four-type economy, we study a steady state equilibrium in which some agents save and others don't (Becker (1980); Malin (2008); Bosi and Seegmuller (2010)). That is, agents with lower time-inconsistent preference for immediate gratification, i.e., patient individuals, save while those with larger present bias (impatient) don't. In this equilibrium, physical capital accumulation is determined by the discount factor of the patient agents. Imposing that impatient agents do not save in equilibrium leads them to consume and work more, as well as to accumulate more health and human capital. In the first-best equilibrium, relative to the decentralized equilibrium, agents with better cognitive skills consume more of both the ordinary (not unhealthy) good and the unhealthy good, as well as health care services. These agents spend more hours studying and, hence, accumulate more human capital. The health capital stock of agents with more cognitive skills is also larger. On the other hand, higher time-inconsistent preference for immediate gratification agents experience a (small) reduction in their health and human stocks in the first-best equilibrium, leading them to increase labor.

Table I illustrates the earnings and stock of physical capital subsidies for our four-type economy. With first-best paternalistic policies, low present bias agents accumulate much more physical capital which allow them to work less. The optimal subsidy of physical capital depends only on the present-biased discount and it is is smaller for those individuals with less time-inconsistency, i.e., less present bias. Our quantitative results suggest that to recover the first-best equilibrium, the planner should subsidize the physical capital accumulation of agents that are more (less) present bias, i.e., $\beta^H = 0.85$ ($\beta^L = 0.90$), at a rate of 18 percent (11%). The more time-inconsistent for immediate gratification agents are, the higher is the subsidy required to induce them to the unbiased (first-best) behavior.

On the other hand, the earnings subsidy is affected by both the individual's cognitive skills parameter and her present-biased discount factor. Agents who discount the future more heavily

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[5] Our main results are robust to reasonable variations around this benchmark parameterization.

(β^H) and have low cognitive skills (ζ^L) receives a higher earnings subsidy. They also pay relatively lower lump-sum taxes. For a given cognitive skill, the earnings subsidy is higher the greater is the time-inconsistency problem. And, for agents with the same present-biased preferences, those with high cognitive skills receive a lower subsidy, i.e., low cognitive skill Lj-types receive higher earnings subsidies relative to their high cognitive skill Hj-types counterparts.

Table I: First-Best and Constrained First-Best Optimal Policies

			$\beta^L = 0.90$		$\beta^H = 0.85$	
			$\zeta^L = 0.80$	$\zeta^H = 0.50$	$\zeta^L = 0.80$	$\zeta^H = 0.50$
Constrained First Best			First Best			
$\widehat{P}^*$	0.14	P^{ij*}	0.11	0.11	0.18	0.18
$\widehat{O}^*$	8.66	O^{ij*}	1.93	1.54	2.17	1.96
$\widehat{T}^*$	-45.11	T^{ij*}	-54.25	-54.59	-0.33	-1.35

For individuals with the same present-biased preferences, for instance, $\beta^L = 0.90$, low cognitive skill individuals ($\zeta^L = 0.8$) receive larger earnings subsidy at rate equals to 193, while their high cognitive skill counterparts ($\zeta^H = 0.5$) receive a lower subsidy (154%). Notice that the earnings subsidy less than compensate the heterogeneity in cognitive skills. That is, while a agents might differ in their cognitive skills by almost 40 percent, the difference in the subsidy receive amounts to only 20 percent. For the interaction between present bias and cognitive skills, consider agents with different cognitive skills and same but low present bias discount ($\beta^L = 0.90$). Compared to their high present-biased preference counterparts, i.e., ($\beta^H = 0.85$), even though their discount rate changes by only five percent, the optimal subsidy is different by about thirty percent (a 12 percent increase for low cognitive agents versus a 42 percent for high cognitive agents). These results highlight not only the discounting and instrumental effects, but also the cognitive effect - a novel effect due to the interaction between present-biased discounting and cognitive skills.

Our illustrative example also shed light on the subsidy's effectiveness, measured by the tax revenues required to overcome the present bias problem. Our results suggest the lump-sum tax agents have to pay is mainly determined by their discount rate (*vis-à-vis* their cognitive skills). Those agents that discount the future more heavily (β^H) pay lower taxes (0.33 and 1.35) compared to those with weaker present-bias preferences (54.25 and 54.59). This occurs because, in the equilibrium we have chosen to study, the latter agents(i.e., agents with low discount rate) are the only ones accumulating capital and consequently receiving a physical capital stock subsidy. Accordingly, in the first-best, to be able to provide them with these subsidies, besides an earnings subsidy, their lump-sum taxes must be also higher.

Constrained first-best policies are substantially different than first-best policies. While the physical capital subsidy (14%) falls in the first-best optimal rate range ($11 - 18\%$, Table I) the earnings subsidy is higher for all four types of agents. The total lump-sum taxes is lower (bigger) for agents that discount the future less (more) heavily. Overall, the results presented in Table I suggest that first-best optimal earnings and physical capital stock subsidies are cheaper to implement than their constrained first-best counterparts. Moreover, with constrained first-best policies the effect of subsidies and taxes is heterogeneous across different types. Agents with larger present-biased discount are required to pay higher taxes, which goes from 0.33 to 45.11 for $\zeta^H = 0.5$ agents and from1.35 to 45.11 for $\zeta^L = 0.8$ agents. This occurs essentially because the government averages out the physical stock (and taxes) in the economy. On one hand, a physical capital stock subsidy of 14 percent increases the return per unit of physical capital patient agents hold but, on the other hand, they can have the same income saving less at a higher rate. Subsidizing physical capital and earnings at a higher rates leads the government to reduce the lump-sum tax on those with weaker present-biased preferences (β^H), while increase taxation of agents with stronger time-inconsistency (β^L).

Finally, the individual's welfare will depend on the type of equilibrium and policies implemented. The decentralized equilibrium represents agents' allocations when no policies are in place. This is meant to represent the allocations and welfare when agents rely on their own skills and potentially make mistakes. First-best allocations are implemented through first-best type-specific optimal policies. Constrained first-best allocations and policies represent the case in which the planner can not identify each agent type and must design a single policy for all types. These results are presented in Table II.

As expected, welfare improves as we move from the decentralized equilibrium to the constrained equilibrium and, finally, to the first-best equilibrium. From the decentralized to the first-best equilibrium, we find a (approx.) 90% welfare improvement for the four types. For instance, lower present-biased preferences and low cognitive skill individuals (β^L, ζ^L) experience the highest welfare level (an improvement of 96%) with first-best optimal paternalistic policies. Our results suggest that constrained first-best policies improve the welfare of less present bias and high cognitive skill individuals (β^L, ζ^H) the most.

4 Two Alternative Policy Packages

In Section 3 we studied a policy package that includes an earnings subsidy and a subsidy to an individual's stock of physical capital. A paternalistic government may use alternative policy packages and intervene to counterbalance the intertemporal distortion of consumption and time allocation toward the present and hence improve agents health and human capital. In this section, we analyze two policy packages that either (i) immediately reward (or punish) an individual's health related decisions and proper (studying) time allocation (subsidies/taxes on current decisions regarding consumption of unhealthy good, health care services and studying time) or (ii) reward

Table II: Welfare: Decentralized Constrained First-Best and First-Best

	$\beta^L = 0.90$		$\beta^H = 0.85$	
	$\zeta^L = 0.80$	$\zeta^H = 0.50$	$\zeta^L = 0.80$	$\zeta^H = 0.50$
Welfare	Decentralized Equilibrium			
U^{ij*}	-8.65	-7.23	-10.45	-8.71
	Constrained First-Best Equilibrium			
U^{ij*}	-7.71	-2.59	-6.86	-5.98
	First-Best Equilibrium			
U^{ij*}	-0.26	-0.53	-1.99	-0.33

the individual's health and human capital outcome directly in the future (subsidies proportional to the stocks of physical capital, health capital and human capital). These policy instruments are, to some extent, similar to and closely resemble the policies studied by (i) O'Donoghue and Rabin (2006, 2003) and Cremer et al. (2012), and (ii) Aronsson and Thunstrom (2008). Although these policy packages also implement the first-best optimal allocations, we show that the timing-target distinction is relevant both for the determination of the optimal subsidy and tax rates and for how cognitive skills and present-biased preferences interact in the optimal policies. Moreover, such distinction also speaks to the subsidy's effectiveness, measured (numerically) by the tax revenues required to overcome the present bias problem.

4.1 O'Donoghue and Rabin (2006), Cremer et al. (2012) Policy Package: Unhealthy good, health care and studying time

Suppose that the planner were to introduce policies proportional to the agent's current consumption of unhealthy goods (X_t^{ij*}), health care services (E_t^{ij*}) and hours of study (S_t^{ij*}), as well as P_t^{ij*}on her physical capital stock. To the extent that current decisions affect future outcomes, in our economy, physical, human and health stock accumulation, and earnings, to implement the first-best optimal allocations the planner ought to design policies that induce individuals to internalize the external effects of their time-inconsistent preference for immediate gratification, as well as their (biased) misperception of their own cognitive skills.

The household problem is similar to problem (7), as well as the first-order conditions, except for the adjusted ij-type agent's budget constraint:

$$\begin{aligned} c_t^{ij} + x_t^{ij} + e^{i}{}_{t} + k_{t+1}^{ij} &= (1 + R_t - \delta_k)\left(1 + P_t^{ij*}\right) k_t^{ij} + \left(1 + E_t^{ij*}\right) e_t^{ij} + \left(1 + X_t^{ij*}\right) x_t^{ij} \\ &+ S_t^{ij*} s_t^{ij} + W_t m_t^{ij} h_t^{ij} l_t^{ij} + T_t^{ij*}. \end{aligned}$$

These policy instruments reward (or punish) an individual's health related decisions and proper time allocation to study (O'Donoghue and Rabin (2006, 2003); Cremer et al. (2012)), as they change the relative prices of goods consumed and decisions made today and to increase the incentives for individuals to make correct decisions. The optimal rate of these policies simply bridge the gap between the biased and the unbiased evaluation of health and human capital benefits, as summarized in the following proposition.

Proposition 2. *Suppose the government announces, in each period t, a surprise set of policies that contains subsidies proportional to the agent's physical capital stock and her decisions on health and human capital investment to be implemented in period t, i.e.,* $(1+R_t-\delta_k)\left(1+P_t^{ij*}\right)k_t^{ij}$, $\left(1+E_t^{ij*}\right)e_t^{ij}$, $\left(1+X_t^{ij*}\right)x_t^{ij}$ *and* $P_t^{ij*}s_t^{ij}$. *Then the equilibrium in the decentralized economy is equivalent to the social optimum if subsidies* P_t^{ij*} *are given by (14) and*

$$
\begin{aligned}
S_t^{ij*} &= \left(1-\beta^j\right)\zeta^i\left(\frac{v_z^{ij*}(t)}{u_c^{ij*}(t)}\right) && (18)\\
X_t^{ij*} &= \left(\beta^j-1\right)\left(\frac{g_x^{ij*}(t)}{g_e^{ij*}(t)}\right) && (19)\\
E_t^{ij*} &= \beta^j-1 && (20)
\end{aligned}
$$

With the introduction of human capital in the model, the novel policy S_t^{ij*} balances the wedge between the biased and unbiased evaluation of human capital, taking into account the individual's misperception of her own cognitive skill. The subsidy on hours of study depends on the relative impact on leisure time of that investment (v_z^{ij*}) versus the consumption of normal good associated with higher earnings in the future (u_c^{ij*}). In other words, when $\beta^j < 1$, the fraction $\zeta^i v_z^{ij*}(t)/u_c^{ij*}(t)$, equation (18), represents the present value of the marginal utility of leisure (or the disutility of hours of study) relative to the marginal utility of consumption today. This policy captures by how much the unbiased individual ($\beta^j = 1$) is willing to trade study time for leisure with her biased self ($\beta^j < 1$). *Ceteris paribus*, the study time subsidy is decreasing in the present-biased discount rate β^j and increasing in the agent's cognitive skill ζ^i.

To recover the first-best equilibrium it is optimal that the planner tax the individual consumption of the unhealthy good. This tax depends, however, on the relative effect of such consumption (g_x^{ij*}) *vis-à-vis* health services expenditures (g_e^{ij*}) on the agent's (next period) stock of health. The tax on unhealthy consumption forces the individual to internalize the full impact of consumption on her health today and it is proportional to the share of the marginal impact of unhealthy goods on health that she mistakenly internalize. It adjusts by how much the marginal willingness to pay for the unhealthy good differs between the unbiased and the biased agent. For $\beta^j < 1$, the numerator of equation (19) gives the present value of the undervaluation of the marginal harm of the unhealthy good on the health capital, while the denominator is the marginal benefit of health care expenditures, so that the policy $\left(\beta^{j}-1\right)\left(g_{x}^{ij*}(t)/g_{e}^{ij*}(t)\right)$ describes by how much the marginal (un)willingness to pay for the unhealthy good differs between the unbiased and the biased individual. Thus, the optimal current subsidy X_{t}^{ij*} balances the wedge between the individual's biased and unbiased evaluation of health capital. And finally, it is necessary to subsidize health care, as individuals underestimate its impact on health at a rate equal to $(\beta^{j}-1)$.

4.2 Aronsson and Thunstrom (2008) Policy Package: Savings, health capital and human capital stocks

The last policy package we study closely resembles the policies studied by Aronsson and Thunstrom (2008), in which human capital or cognitive skills are not considered. Suppose now that the planner were to announce future subsidies proportional to the agent's physical, human and health capital stocks to implement the social optimum in the decentralized economy.

Consider a ij-type agent decisions in period t when subsidies at the rates P_{t+1}^{ij*}, M_{t+1}^{ij*} and H_{t+1}^{ij*} reward the individual's health and human capital outcome directly and independently in the future. The modified budget constraint, equation (5), for $t+1$, is as follows

$$\begin{aligned} c_{t+1}^{ij}+x_{t+1}^{ij}+e^{i}{}_{t+1}+k_{t+2}^{ij} &= \left(1+R_{t+1}-\delta_{k}\right)\left(1+P_{t+1}^{ij*}\right)k_{t+1}^{ij}+H_{t+1}^{ij*}h_{t+1}^{ij} \\ &+ M_{t+1}^{ij*}m_{t+1}^{ij}+W_{t+1}\left(m_{t+1}^{ij}h_{t+1}^{ij}\right)l_{t+1}^{ij}+T_{t+1}^{ij*} \end{aligned}$$

The first-order conditions of this problem are similar to problem (7) and the lump-sum tax T_{t+1}^{ij*} satisfies the government's budget constraint, for all ij-type agent and for all $t>0$. Proposition 3 presents our results.

Proposition 3. *Suppose the government announces, in each period t and for each agent ij, a surprise set of policies to be implemented in period $t+1$ that contains subsidies to the agent's physical capital and his stocks of health and human capital, i.e., $\left(1+R_{t+1}-\delta_{k}\right)\left(1+P_{t+1}^{ij*}\right)k_{t+1}^{ij}$, $M_{t+1}^{ij*}m_{t+1}^{ij}$ and $H_{t+1}^{ij*}h_{t+1}^{ij}$. Then the equilibrium in the decentralized economy is equivalent to the social optimum if subsidies $P_{t+1}^{ij*}$are given by (14), and*

$$H_{t+1}^{ij*} = \left(\frac{1-\beta^{j}}{\beta^{j}u_{c}^{ij*}(t+1)}\right)\left\{\begin{array}{l} F_{L}^{*}(t+1)m_{t+1}^{ij}l_{t+1}^{ij}u_{c}^{ij*}(t+1) \\ +\left(1-\delta_{h}\right)\zeta^{i}\frac{v_{z}^{ij*}(t+1)}{B_{s}^{ij*}(t+1)} \end{array}\right\} \tag{21}$$

$$M_{t+1}^{ij*} = \left(\frac{1-\beta^{j}}{\beta^{j}u_{c}^{ij*}(t+1)}\right)\left\{\begin{array}{l} u_{m}^{ij*}(t+1) \\ +F_{L}^{*}(t+1)h_{t+1}^{ij}l_{t+1}^{ij}u_{c}^{ij*}(t+1) \\ +\left(1-\delta_{m}\right)\frac{u_{c}^{ij*}(t+1)}{g_{e}^{ij*}(t+1)} \end{array}\right\} \tag{22}$$

The human capital subsidy H^{ij*} is a novel policy in the optimal paternalistic taxation literature that has focused mainly on health-related interventions. This policy acts directly to increase future welfare through higher earnings, which self t does not fully take into account because of her present bias, and consequently larger consumption (first term in the bracket, equation (21)). Increasing human capital by one unity in period t increases the subsidy in period $t+1$ by $(1-\beta^j)\, F_L^* m^{ij} l^{ij} u_c^{ij*} / (\beta^j u_c^{ij*})$ units. Indirectly, the subsidy H^{ij*} also stimulates accumulation of human capital via changes in shadow prices of leisure *vis-à-vis* education, slaking the respective constraint (second term in the bracket). Similar to policy S_t^{ij*}, equation (18), this policy also balances the wedge between the biased and unbiased evaluation of human capital, taking into account the individual's misperception of her own cognitive skill.

The subsidy H^{ij*}also captures the effect of the individual's time allocation decision on her human capital accumulation, measured by the term $(1-\delta_h)\, \zeta^i v_z^{ij*} / B_s^{ij*}$, equation (21). It depends on the relative impact on future leisure time (v_z^{ij*}) versus human capital accumulation and the associated benefits via higher earnings in the future (B_s^{ij*}). When $\beta^j < 1$, this fraction represents the present value of the marginal utility of leisure (or the disutility of hours of study) relative to the marginal utility of consumption. Altogether, the terms in the curly bracket of equation (21) describe the (discounted) additional utility that self t acquires through the subsidy if she increases studying time (i.e., accumulate more human capital) by one unit. The optimal rate H^{ij*} is set such that this subsidy-induced utility gain equals the bias in the evaluation of future human capital benefits, thereby correcting for the bias.

The policy M^{ij*} has two direct effects namely (i) marginal increases in the utility of health (u_m^{ij*}) and (ii) an increase in earnings due to an increase on individuals' health status and its impact on future consumption (second and third terms in the curly bracket of equation (22)). This policy also affects the future set of the individual's choices, i.e., the marginal increase in consumption adjusted by the depreciation of the agent's health capital relative to the reduction in his private health expenditures in period $t+1$. This welfare gain is summarized by the shadow price of health capital, which is equal to $(1-\delta_m)\, u_c^{ij*} / g_e^{ij*} > 0$ at the equilibrium. An interpretation of this effect is that the increase in the stock of health capital leads the agent to reduce his private health expenditures, *ceteris paribus*, which increases resources available for private consumption. In other words, the agent's decision regarding future consumption *vis-à-vis* medical expenditures changes the corresponding shadow prices (third term). The right-hand-side of equation (22) describes the additional utility, measured by $(\beta^j u_c^{ij*})\, M^{ij*}$, that self t acquires through the subsidy if she increases her health capital by one unit. Similar to the subsidy H^{ij*}, the health capital subsidy M^{ij*} corrects for the present-bias by setting the subsidy-induced utility gain equal to the bias in the evaluation of future health benefits.

These subsidies entail direct and indirect effects on individuals decision. Individuals with high cognitive skills (low ζ) and low present bias (high β), *ceteris paribus*, should receive a lower human and health capital subsidy. Interestingly, the policy O^{ij*}, equation (15), is somewhat a combination of policies H^{ij*} and M^{ij*}, equations (21) and (22), respectively. If agents are not present-bias, as expected the first-best optimal policy is not to tax or subsidize any of the agent's physical, human or health capital stocks. If we ignore the effect of health on the production function, the subsidy M^{ij*} as in equation (22) is equivalent to Aronsson and Thunstrom (2008)'s policy on health status (Proposition 1 in their paper).

4.3 Optimal Constrained First-Best Paternalistic Policies

Recall that the constrained first-best problem is such that the planner's goal is to maximize agents' welfare subject to the economy feasibility constraint and to raising set revenues through non-type specific policies. Corollary (2) summarizes the results for the O'Donoghue and Rabin (2006), Cremer et al. (2012) policy package. These new optimal policies $\widehat{P}_t^*, \widehat{S}_t^*, \widehat{X}_t^*, \widehat{E}_t^*$ are the average of previous ones where the weight is determined by the size of each type in the population.

Corollary 2. *Suppose the government announces, in each period t, a surprise set of policies that contains subsidies proportional to the agent's private wealth and his stocks of health and human capital to be implemented in period t, i.e., $(1+R_t-\delta_k)\left(1+\widehat{P}_t^*\right)k_t^{ij}$, $\left(1+\widehat{E}_t^*\right)e_t^{ij}$, $\left(1+\widehat{X}_t^*\right)x_t^{ij}$ and $\widehat{S}_t^* s_t^{ij}$. Then the constrained first-best equilibrium can be decentralized if subsidies $\widehat{P}_t^*$ are given by (16), and*

$$\widehat{S}_t^* = \left(1-\sum_{i,j}\gamma^{ij}\beta^j\right)\left(\sum_{i,j}\gamma^{ij}\zeta^i\frac{v_z^{ij*}(t)}{u_c^{ij*}(t)}\right) \tag{23}$$

$$\widehat{X}_t^* = \left(\sum_{i,j}\gamma^{ij}\beta^j-1\right)\left(\sum_{i,j}\gamma^{ij}\frac{g_x^{ij*}(t)}{g_e^{ij*}(t)}\right) \tag{24}$$

$$\widehat{E}_t^* = \sum_{i,j}\gamma^{ij}\beta^j-1 \tag{25}$$

The constrained first-best Aronsson and Thunstrom (2008) policy package is presented in the corollary (3). Combining the equilibrium equations of all ij-types with the planner's equilibrium conditions (solution of problem (13)), we obtain a single optimal policy package for all agents, i.e., $\widehat{P}^*$, $\widehat{M}^*$ and $\widehat{H}^*$, for all ij-types. For instance, the optimal educational calls for a correction between (a weighted average) of (i) marginal effects on production, $\sum_{i,j}\gamma^{ij}F_L^* m^{ij} l^{ij} u_c^{ij*} - \sum_{i,j}\gamma^{ij}\beta^j F_L^* m^{ij} l^{ij} u_c^{ij*}$, and (ii) the marginal rate of substitution between leisure and hours of study, $\sum_{i,j}\gamma^{ij}\left(\zeta^i v_z^{ij*}/B_s^{ij*}\right) - \sum_{i,j}\gamma^{ij}\beta^j\left(\zeta^i v_z^{ij*}/B_s^{ij*}\right)$.

Corollary 3. *Suppose the government announces, in each period t and for all ij-types, a surprise set of policies to be implemented in period $t+1$ that contains subsidies to the agent's physical capital and his stocks of health and human capital, i.e., $(1+R_{t+1}-\delta_k)\left(1+\widehat{P}_{t+1}^*\right)k_{t+1}^{ij}$, $\widehat{M}_{t+1}^* m_{t+1}^{ij}$*

and $\widehat{H}^*_{t+1} h^{ij}_{t+1}$. *Then the first-best constrained equilibrium can be decentralized if subsidy* $\widehat{P}^*_{t+1}$ *is given by (16), and*

$$\widehat{H}^*_{t+1} = \left(\frac{1}{\sum_{i,j} \gamma^{ij} \beta^j u^{ij*}_c(t+1)} \right) \left\{ \begin{array}{l} \sum_{i,j} \gamma^{ij} F^*_L(t+1) m^{ij}_{t+1} l^{ij}_{t+1} u^{ij*}_c(t+1) \\ - \sum_{i,j} \gamma^{ij} \beta^j F^*_L(t+1) m^{ij}_{t+1} l^{ij}_{t+1} u^{ij*}_c(t+1) \\ + (1-\delta_h) \sum_{i,j} \gamma^{ij} \frac{\zeta^i v^{ij*}_z(t+1)}{B^{ij*}_s(t+1)} \\ - (1-\delta_h) \sum_{i,j} \gamma^{ij} \beta^j \frac{\zeta^i v^{ij*}_z(t+1)}{B^{ij*}_s(t+1)} \end{array} \right\} \tag{26}$$

$$\widehat{M}^*_{t+1} = \left(\frac{1}{\sum_{i,j} \gamma^{ij} \beta^j u^{ij*}_c(t+1)} \right) \left\{ \begin{array}{l} \sum_{i,j} \gamma^{ij} u^{ij*}_m(t+1) - \sum_{i,j} \gamma^{ij} \beta^j u^{ij*}_m(t+1) \\ + \sum_{i,j} \gamma^{ij} F^*_L(t+1) h^{ij}_{t+1} l^{ij}_{t+1} u^{ij*}_c(t+1) \\ - \sum_{i,j} \gamma^{ij} \beta^j F^*_L(t+1) h^{ij}_{t+1} l^{ij}_{t+1} u^{ij*}_c(t+1) \\ + (1-\delta_m) \sum_{i,j} \gamma^{ij} \frac{u^{ij*}_c(t+1)}{g^{ij*}_e(t+1)} \\ - (1-\delta_m) \sum_{i,j} \gamma^{ij} \beta^j \frac{u^{ij*}_c(t+1)}{g^{ij*}_e(t+1)} \end{array} \right\} \tag{27}$$

4.4 Back to our Illustrative Example

Table III presents the numerical results for the two policy packages studied in Section 4. For the first policy package, notice that the subsidy on the physical capital stock and the tax on consumption of unhealthy goods depend only on the magnitude of the present-biased discount rate. These policies correct for the misperception of current decisions on agents' future capital and health stock. A similar feature can also be observed in the first-best subsidies on health care expenditures and time allocated to human capital accumulation. However, in these policies the present bias problem interacts with the agent's cognitive skills, rendering different polices for agents with different skills. The difference between the optimal subsidy (tax) high and low skill individuals receive (pay) is smaller the larger the present-biased discount rate. In terms of magnitude, the hours of study subsidy is about two times bigger than the subsidy on health care expenditure (62% versus 35%). Even though both subsidies have a positive income effect, the difference in their magnitude might be explained by the fact that while the studying subsidy induces an increase in hours of study (positive effect), the health care subsidy might allow individuals to consume more of the unhealthy good (negative effect).

Regarding the second policy package, we first observe that the human capital subsidy is smaller than the health capital stock subsidy. Notice that although both capital stocks affect future earnings positively, individuals also derive utility from better health status. To recover the first-best equilibrium, the planner should subsidize the human and health capital stocks of agents with low present-biased preferences and low cognitive skills at rates of 8% and 20%, respectively. For a given present-biased discounting, high cognitive skill agents receive larger subsidies, but also pay more lump-sum taxes. In terms of the effectiveness of these policy packages, measured by the magnitude of tax revenues, the second package can be implemented at a lower (lump-sum tax) cost. This package is, however, more less effective that a package that includes only earnings and physical capital stock subsidies.

Table III: First-Best and Constrained First-Best Optimal Policies

			$\beta^L = 0.90$		$\beta^H = 0.85$	
			$\zeta^L = 0.80$	$\zeta^H = 0.50$	$\zeta^L = 0.80$	$\zeta^H = 0.50$
Constrained First Best		First Best				
$\widehat{S}_t^*$	0.14	S^{ij*}	0.11	0.11	0.18	0.18
$\widehat{E}_t^*$	-0.13	E^{ij*}	-0.10	-0.10	-0.15	-0.15
$\widehat{X}_t^*$	0.64	X^{ij*}	0.35	0.61	0.66	0.99
$\widehat{P}_t^*$	1.79	P^{ij*}	0.62	1.57	2.24	3.06
$\widehat{T}_t^*$	-39.01	T^{ij*}	-51.87	-58.54	-5.58	-12.14
$\widehat{H}_t^*$	0.51	H^{ij*}	0.08	0.15	0.12	0.16
$\widehat{M}_t^*$	1.46	M^{ij*}	0.20	0.33	0.27	0.40
$\widehat{T}_t^*$	-37.99	T^{ij*}	-54.25	-55.77	-1.37	-2.97

Our numerical exercise illustrates that a policy package consisting of earnings and physical capital stock subsidies is the most effective, requiring tax revenues in the amount of 110.52 (versus 128.13 and 114.36 for packages (i) and (ii), respectively). Since we consider a utilitarian planner, the redistributive motive is only due to the concavity of the utility function. A single policy on the agent's earnings, which combines the effects of both human and health (besides labor) decisions, is sufficient to achieve the first-best optimum at the lowest cost. Actually, the earnings subsidy captures and takes in to account in a single policy all the effects and corrections that would be required if the planner were to implement other policy instruments, for instance, subsidies targeting human and health capital separately or current, biased decisions.

5 Conclusions

We consider an economy consisting of agents who differ in their present-biased preferences and cognitive skills. Agents have a time-inconsistent preference for immediate gratification which may lead to excessive consumption of unhealthy food (health capital), low savings (physical capital) and less time allocated to education (human capital). The externality that the individual's current self imposes on her future selves is a two-dimension stock-externality, as health and human capital decisions might be poised by self-control, time-inconsistency problems. We study optimal human and health linear policies when there is a paternalistic motive to overcome individuals' present bias problems with heterogeneous cognitive skills. The paternalistic intervention is meant to reward individuals for the combined effect of health and human capital on their future earnings and physical capital accumulation. We further explore how paternalistic policies must also take into account potential interactions of present bias and cognitive skills.

We show that a single policy on the agent's earnings captures all the corrections that would be required if the planner were to implement other policy instruments, for instance, subsidies targeting human and health capital separately or current, biased decisions. In an economy where agents accumulate health and human capital, our results highlight a novel effect - the cognitive effect - of paternalistic policies due to the interaction between present-biased preferences and cognitive skills. The discounting and instrumental effects of future subsidies are also present in our model. We also study two alternative packages that implement the first-best equilibrium. A numerical exercise illustrates that a policy package consisting of earnings and physical capital stock subsidies is the most effective, requiring lower tax revenues to correct for present bias and agents misperception of their own cognitive skills problems. We also characterize and illustrate numerically constrained first-best optimal paternalistic policies, i.e., a single policy package for all agents.

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Appendix

Proofs: Earnings and Physical Capital Stock Subsidies

Proposition 1

Consider a ij-type individual's problem similar to problem (7), except for the modified budget constraint

$$\begin{aligned} c^{ij}_{t+1} + x^{ij}_{t+1} + e^{i}{}_{t+1} + k^{ij}_{t+2} &= \left(1 + R_{t+1} - \delta_k\right)\left(1 + P^{ij*}_{t+1}\right) k^{ij}_{t+1} \\ &+ \left(1 + O^{ij*}_{t+1}\right) W_{t+1} A^{ij}_{t+1} l^{ij}_{t+1} + T^{ij*}_{t+1} \end{aligned}$$

The first-order conditions of this problem are equivalent to equations (8) - (12), where P^{ij*}_{t+1} and O^{ij*}_{t+1} are the physical capital stock and earnings subsidies, respectively, to be implemented in period $t+1$. That is,

$$u^{ij}_x(t) - u^{ij}_c(t) + u^{ij}_c(t)\frac{g^{ij}_x(t)}{g^{ij}_e(t)} = 0 \tag{28}$$

$$u^{ij}_c(t) - \beta^j u^{ij}_c(t+1)\left(1 + R^*_{t+1} - \delta_k\right)\left(1 + P^{ij*}_{t+1}\right) = 0 \tag{29}$$

$$\begin{aligned} -\frac{u^{ij}_c(t)}{g^{ij}_e(t)h^{ij}_{t+1}} - \zeta^i \frac{v^{ij}_z(t)}{B^{ij}_s(t)m^{ij}_{t+1}} + \beta^j\Theta\left[\frac{u^{ij}_A(t+1)}{h^{ij}_{t+1}} + u^{ij}_c(t+1)\left(1 + O^{ij*}_{t+1}\right)W_{t+1}l^{ij}_{t+1}\right] & \\ +\beta^j\Theta\left[\left(1-\delta_m\right)\frac{u^{ij}_c(t+1)}{g^{ij}_e(t+1)h^{ij}_{t+1}} + \left(1-\delta_h\right)\zeta^i\frac{v^{ij}_z(t+1)}{B^{ij}_s(t+1)m^{ij}_{t+1}}\right] &= 0 \end{aligned} \tag{30}$$

$$v^{ij}_z(t) - u^{ij}_c(t)\left(1 + O^{ij*}_t\right)W_t A^{ij}_t = 0 \tag{31}$$

Notice that here we use the fact that $A^{ij}_t = m^{ij}_t h^{ij}_t$ and we rewrite the ij-type agent utility as $u\left(c^{ij}_t, x^{ij}_t, A^{ij}_t/h^{ij}_t\right) + v\left(z^{ij}_t\right)$ and the laws of motion for the agent's human and health capital stocks as follows: $A^{ij}_{t+1}/m^{ij}_{t+1} - \left(1-\delta_h\right)A^{ij}_t/m^{ij}_t = B\left(s^{ij}_t\right)$ and $A^{ij}_{t+1}/h^{ij}_{t+1} - \left(1-\delta_m\right)A^{ij}_t/h^{ij}_t = g\left(x^{ij}_t, e^{ij}_t\right)$.

Recall that the necessary conditions for an interior solution of the planner's maximization problem are similar to the household's ones, except for the fact that $\beta^j = 1$, for all ij-type agents.

Consider first the optimal policy P_{t+1}^{ij*}. At the first-best optimal allocations, the ij-type agent's and planner's equilibrium equations, respectively:

$$\left[\frac{u_c^{ij*}(t)}{\beta^j u_c^{ij*}(t+1)}\right] = \left(1+R_{t+1}^{*}-\delta_k\right)\left(1+P_{t+1}^{ij*}\right) \tag{32}$$

$$\left[\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}\right] = \left(1+R_{t+1}^{*}-\delta_k\right). \tag{33}$$

Combining equations (32) and (33), and solving for P_{t+1}^{ij*}, we obtain the physical capital stock subsidy, equation (14):

$$P_{t+1}^{ij*} = \frac{1-\beta^j}{\beta^j}$$

In order derive the earnings subsidy O_{t+1}^{ij*} we consider the ij-type agent's and planner's first-order conditions with respect to where A_t^{ij}, where $A_t^{ij} = m_t^{ij} h_t^{ij}$. The ij-type agent's and planner's equilibrium equations are respectively:

$$\begin{aligned} -\frac{u_c^{ij}(t)}{\beta^j g_e^{ij}(t) h_{t+1}^{ij}} - \zeta^i \frac{v_z^{ij}(t)}{\beta^j B_s^{ij}(t) m_{t+1}^{ij}} + \Theta\left[\frac{u_A^{ij}(t+1)}{h_{t+1}^{ij}} + u_c^{ij}(t+1)\left(1+O_{t+1}^{ij*}\right)F_L(t+1)l_{t+1}^{ij}\right] & \\ +\Theta\left[(1-\delta_m)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)h_{t+1}^{ij}} + (1-\delta_h)\zeta^i\frac{v_z^{ij}(t+1)}{B_s^{ij}(t+1)m_{t+1}^{ij}}\right] &= 0 \end{aligned} \tag{34}$$

$$\begin{aligned} -\frac{u_c^{ij}(t)}{g_e^{ij}(t) h_{t+1}^{ij}} - \zeta^i \frac{v_z^{ij}(t)}{B_s^{ij}(t) m_{t+1}^{ij}} + \Theta\left[\frac{u_A^{ij}(t+1)}{h_{t+1}^{ij}} + u_c^{ij}(t+1)F_L(t+1)l_{t+1}^{ij}\right] & \\ +\Theta\left[(1-\delta_m)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)h_{t+1}^{ij}} + (1-\delta_h)\zeta^i\frac{v_z^{ij}(t+1)}{B_s^{ij}(t+1)m_{t+1}^{ij}}\right] &= 0 \end{aligned} \tag{35}$$

Combining these two equations and solving for O_{t+1}^{ij*}, we obtain the earnings subsidy, equation (15), i.e.,

$$O_{t+1}^{ij*} = \left(\frac{1-\beta^j}{\beta^j u_c^{ij*}(t+1)F_L^{*}(t+1)l_{t+1}^{ij}}\right)\left\{\begin{array}{l} u_A^{ij*}(t+1) \\ +u_c^{ij*}(t+1)F_L^{*}(t+1)l_{t+1}^{ij} \\ +\left(1-\delta_m\right)\frac{u_c^{ij*}(t+1)}{g_e^{ij*}(t+1)h_{t+1}^{ij}} \\ +\left(1-\delta_h\right)\frac{\zeta^i v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \end{array}\right\}.$$

Corollary 1

In order to prove Proposition 1, we need to combine the equilibrium equations of all ij-types with the planner's equilibrium conditions, taking into account the weighted average of all individuals' allocations $\left(\sum_{i,j}\gamma^{ij}\right)$. Starting with the constrained first-best physical capital stock subsidy, we first multiply each equilibrium condition, equation (29) by its respective weight γ^{ij} and add them up. Hence, we obtain the following two expressions (for agents and planner):

$$\sum_{i,j}\gamma^{ij}\left[\frac{u_c^{ij*}(t)}{\beta^j u_c^{ij*}(t+1)}\right] = \sum_{i,j}\gamma^{ij}\left(1+R^*_{t+1}-\delta_k\right)\left(1+\widehat{S}^*_{t+1}\right) \tag{36}$$

$$\sum_{i,j}\gamma^{ij}\left[\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}\right] = \sum_{i,j}\gamma^{ij}\left(1+R^*_{t+1}-\delta_k\right). \tag{37}$$

where $\sum_{i,j}\gamma^{ij}=1$. Combining these two equations and solving for $\widehat{S}^*_{t+1}$, we obtain equation (16):

$$\widehat{S}^*_{t+1} = \frac{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{\beta^j u_c^{ij*}(t+1)} - \sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}}{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{u_c^{ij*}(t+1)}}$$

Following the same steps, using now equation (30), we have

$$\begin{aligned}
-\frac{u_c^{ij}(t)}{\beta^j g_e^{ij}(t)h_{t+1}^{ij}} - \zeta^i\frac{v_z^{ij}(t)}{\beta^j B_s^{ij}(t)m_{t+1}^{ij}} + \Theta\left[\frac{u_A^{ij}(t+1)}{h_{t+1}^{ij}} + u_c^{ij}(t+1)\left(1+O_{t+1}^{ij*}\right)F_L(t+1)l_{t+1}^{ij}\right]& \\
+\Theta\left[\left(1-\delta_m\right)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)h_{t+1}^{ij}} + \left(1-\delta_h\right)\zeta^i\frac{v_z^{ij}(t+1)}{B_s^{ij}(t+1)m_{t+1}^{ij}}\right] &= 0 \\
-\frac{u_c^{ij}(t)}{g_e^{ij}(t)h_{t+1}^{ij}} - \zeta^i\frac{v_z^{ij}(t)}{B_s^{ij}(t)m_{t+1}^{ij}} + \Theta\left[\frac{u_A^{ij}(t+1)}{h_{t+1}^{ij}} + u_c^{ij}(t+1)F_L(t+1)l_{t+1}^{ij}\right]& \\
+\Theta\left[\left(1-\delta_m\right)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)h_{t+1}^{ij}} + \left(1-\delta_h\right)\zeta^i\frac{v_z^{ij}(t+1)}{B_s^{ij}(t+1)m_{t+1}^{ij}}\right] &= 0
\end{aligned}$$

which combined give us the constrained first-best optimal earnings subsidy, equation (17)

$$\widehat{O}^*_{t+1} = \left(\frac{1}{\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(t+1)F_L^*(t+1)l_{t+1}^{ij}}\right)\left\{\begin{array}{l} \sum_{i,j}\gamma^{ij}\frac{u_A^{ij*}(t+1)}{h_{t+1}^{ij}} - \sum_{i,j}\gamma^{ij}\beta^j\frac{u_A^{ij*}(t+1)}{h_{t+1}^{ij}} \\ +\sum_{i,j}\gamma^{ij}F_L^*(t+1)l_{t+1}^{ij}u_c^{ij*}(t+1) \\ -\sum_{i,j}\gamma^{ij}\beta^j F_L^*(t+1)l_{t+1}^{ij}u_c^{ij*}(t+1) \\ +(1-\delta_m)\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t+1)}{g_e^{ij*}(t)h_t^{ij}} \\ -(1-\delta_m)\sum_{i,j}\gamma^{ij}\beta^j\frac{u_c^{ij*}(t)}{g_e^{ij*}(t)h_t^{ij}} \\ +(1-\delta_h)\sum_{i,j}\gamma^{ij}\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \\ -(1-\delta_h)\sum_{i,j}\gamma^{ij}\beta^j\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \end{array}\right\}$$

The proofs of Propositions 3 and 2 and Corollaries 3, 2 follow the same steps as the proof of Proposition 1 and Corollary 1. They are available upon request.