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Wages, Efficiency and Labor Market Regulation
In an Inflationary Environment

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I. Introduction

This paper deals with regulations that affect the relative cost of labor according to the time workers remain with a firm: severance pay and wage indexation. Particularly, it examines how these regulations affect labor turn-over. With this in mind, the paper explores the consequences of a particular technology, using specific human capital acquired by workers after some experience within a firm and useless at other firms.

The introduction of firms’ decision on the rate of labor turn-over brings other consequences in wage rigidities than the ones usually analyzed in the literature. In this sense some of the macroeconomic implications of wage rigidities are revised, especially those concerning some popular propositions on wage indexation and acceleration of inflation in Latin America.

In section II some propositions on the Latin America experience with indexation are revised. In section III the particular technology allowing normal turn-over is introduced as well as the one-period decisions of the firms subject to this technology. A macroeconomic model exploring
the consequences of these decisions is set up and the comparative static results as well as the dynamic properties of the model are offered.

Section IV presents an intertemporal generalization of the decision of firms, the macroeconomic consequences and, finally, the implications of frictional unemployment are analyzed. Section V summarizes the main findings and establishes a basis for empirical research.

II. The Peak-Average Wage Hypothesis

A popular proposition about the behavior of real wages in the presence of indexation and high rates of inflation has been formulated by several Latin American authors, a hypothesis that will be referred to as the Peak-Average wage hypothesis. Maybe the clearest formulation is due to Simonsen:

'Workers get in each wage adjustment a peak of real wages, at prices of the previous period, a peak that is higher than the real wage that the economy is able to pay. Inflation is the mechanism through which real wages are reduced to the level that the economy can pay effectively.' Simonsen (1987), p.10.1

An enormous amount of literature in Brazil and other countries of Latin America has been based on this proposition. Most of it, however has developed formulas that explain the behavior of real wages when different

1. Another description can be found on Kiguel and Liviatan (1991) p.207: "Broadly speaking this approach argues that because of wage indexation there is significant short-term inertia (or persistence) in inflation, and that increases in inflation are usually associated with attempts to erode the average real wage".
frequencies of wage readjustments or different intensities of adjustment of wages to past inflation are applied. However, no attention has been given to the microeconomic consequences of mandatory indexation.

For instance the proposition that real wage behavior, within a period of mandatory readjustment, will be of higher wages initially and lower wages near the date of the mandatory readjustment is easily questioned by its optimality. The optimality of such behavior has received little attention.\(^2\) The mandatory wage policy is not apt to be followed if workers are risk averse, since it will be cheaper for risk neutral firms to offer a lower average real wage with less variance within a period of wage readjustment than the one offered by the mandatory wage policy: an implicit contract argument. In fact, observed wage readjustments were higher than the obligatory indexation during the 1985-1990 period in Brazil.

However, the optimality of the constant-implicit-contract-wage can be questioned when workers have access to indexed financial assets: an argument of substitution between financial indexation and wage indexation.\(^3\)

Nonetheless, access to indexed assets has been relatively

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2. One skeptical exemption in the brazilian literature is Simonsen(1983).

3. See for instance Liviatan (1983) and the references contained on it
restricted for workers even though the financial system has been widely indexed in Brazil.

A common view against the implicit contracts argument is that there is no way to index to a perfect price index, since there will be a period of delay to collect information. Nonetheless, there are always pay schemes to compensate workers at the moment information is released.

The implicit contract argument is criticizable not on these grounds but because the opportunity cost, or participating condition of workers, is not linked to market wages in any sensible way. This is true if it is assumed that the labor market is widely subject to the same kinds of shocks such that the reservation wage will be invariant or unaffected by labor market conditions. Otherwise the reservation wage will be affected by current labor market conditions and in this case the positive marginal utility of income should imply a contract wage conditional to the market wage. The same criticism can be applied to the Peak-Average wage hypothesis, that defends a constant real average wage sought by workers.\footnote{One formulation that includes the effect of market on the targeted real wage by unions is the one formulated Modiano(1988), but this formulation is not derived from any optimizing behavior. The most interesting formulation is the one in Simonsen (1987) and it assumes an exogenous targeted real wage. Outside the Brazilian literature, a paper criticising the "peak-average" hypothesis comes from Helpman and Leiderman (1988). In this paper the wage setting is similar to Simonsen’s except that they assume that wages are}
wages of employed workers, measured by industrial sector wages in Sao Paulo (FIESP), are more closely associated with admission wages, or market wages, measured by the Labor Ministry, than with any other variable, such as official readjustments or past inflation rates.

In addition, indexation has been just one of the regulations in the labor market. When it is considered together with other regulations, important microeconomic effects on turn-over rates and real wages arise. One surprising fact is that when the regulations in labor markets, such as severance payments and indexation, are considered in the analysis, the behavioral equations obtained, like the demand for labor and cost functions, become functions of future expectations on interest rates and inflation instead of backward-looking or "inercial" (in the Latin American jargon) positions, depending on past inflation.¹

¹ fixed not only by past inflation but by expected inflation. Nonetheless, they assume a constant or 'targeted' real wage arbitrarily in the same vein as the Brazilian literature.

5. See for instance Dornbusch(1992): "Indexation is a mechanism that creates inertia and also preserves inertia. Reintroducing half-yearly indexation may therefore be a key step in establishing the expectations of low inflation. Once the wage is locked away, a very rapid resumption of inflation will not be expected. As a result horizons can lengthen far more effectively than under threshold provisions or in the absence of any kind of formal indexation". In this quotation it is not clear if the argument on expectations for inflation is based on the PA hypothesis or on the fact that the real wage might become more predictable under perfect indexation. The second
The next section attempts to set up some theoretical foundations for turn-over and wage determination in the presence of indexation and seeks testable implications in labor market regulation and in the behavior of real wages, turn-over and inflation. Turn-over has been quickly rejected by proponents of the PA hypothesis as a cost reducing device. Nonetheless, the table below shows labor turnover as a percentage of employment. It can be seen that it represents an enormous share of employment that don't have a fixed job what implies a social cost of training and an inefficient allocation of labor as discussed below. In this sense the main objective of this work is to see if this turn-over rate is associate in any way to the policies discussed.

Interpretation is confirmed under the conditions discussed below.

As a policy issue, this problem is important because widely accepted ideas in Latin America tend to emphasize distributive issues against price stability and Public Finance equilibrium and wage policies are the object of political pressure. Second, it is important because there is a wide belief that nominal rigidities can only be surpassed by higher inflation (as the of Simonsen’s reference above suggests). In fact, this rigidities can be solved, in some measure, by turnover. Taking into account the costs arising due to technical limitations to turnover and institutional costs, one probably will observe a little bit of both as empirical evidence shows.

The belief that nominal rigidities can only be solved by higher inflation is specially important when economic stabilization plans are constructed in Latin America, since

an stabilization may lead to real wage reductions, advocates of income policies and price controls suggest their use in order to facilitate the transition to a lower inflation equilibrium. This idea is deeply rooted in the assumption that the government determines the nominal wage in the labor market and that any rise in labor costs will be transmitted to prices. We wish to argue that this is not the only possibility and it could be interesting to consider the consequences of allowing for turnover and see how the market works subject to this regulation.

III. Microeconomic Implications of Labor Market Regulations.

It is assumed that the technology that a firm uses is one that allows for specific human capital. Outside the firm workers are all alike, when they are hired by a firm a technological restriction operates that can be expressed in a production function where workers that remain for more than one period are a different factor of production, i.e., it takes one period to acquire the specific human capital.

On empirical grounds Kiguel and Liviatan (1991) argue: "Two features observed during the cycles [price freezes/acceleration of inflation] cast doubts on the validity of this approach. First, inflation was accelerating during the upward part of the cycle rather than staying stable at past inflation levels. Second, in contrast to the predictions of this approach, in some cycles (especially in Brazil) the accelerations in inflation coincided with increases in real wages", p 207. In other words, wage reductions has not been observed as consequence of accelerating inflation, what agrees with the empirical evidence discussed below.
Both factors are needed to perform the production process. Graph 1 shows the whole story of how this technology operates.

In graph 1 workers just hired by firms are measured on the vertical axis, labelled H and experienced workers are measured in the horizontal axis, labelled N, given a level of output, Y, and a relative cost of workers already hired VB workers to be hired in the present period, r, the firm's optimal composition of workers A, is obtained by hiring $N_{t-1}$ and retaining $N_{t-1}$ workers hired in some previous period. Next period, by the assumption of specific human capital, the firm will have $N_{t-1}+H_{t-1}$ experienced workers, as projected in the graph by a 45°-line starting at $N_t=N_{t-1}$. If the relative cost of labor does not change this period the firm will have to fired $D_t=H_{t-1}$ workers in order to retain the same labor composition A, with $N_t=N_{t-1}$ and $H_t=H_{t-1}$ new hiring.

Given this technology a firm is supposed to hire workers for the current production period. The firm is assumed to exist just one previous period. In this period the firm decides if it will retain the workers hired during the first period or hire new workers. If it decides to hire new workers the lay-off decision for the old ones has a cost $\Theta W_{t-1}(1+\pi)$ in severance pay, where $\Theta$ is the fraction of the wage rate that is paid as a fine and $\pi$ is the mandatory
wage indexation. This roughly describes the Brazilian regulation.

The severance payment in Brazil has two components. The first one is a transference of 40% of the 'Fundo de Garantia'. The 'Fundo de Garantia' is a bank account, managed by the government, where 8% of the wage paid to the worker is deposited by the firm each month, that becomes available to the worker when she is laid off. An additional 0.5% is paid by the firm, as a risk premium, to protect the system against any failures among contributing firms. The second component is the 'aviso previo', firms have to retain workers for a month or else pay one month salary, this last alternative is what usually happens. Therefore, according to Brazilian regulation, the severance pay will be proportional to the wage rate but not to the time the worker has been employed in a firm; with no discounting it will cost the same in severance pay to dismiss one worker with two years seniority than to dismiss two workers of one year within a firm within the two years. With positive discounting turnover becomes more expensive, since the firm will have to pay half of the cost of dismissal at end of the first year; workers with greater seniority are protected of turnover only by discounting. Without discounting our formula of severance pay $\theta W_{t-1}(1+\pi)$ is exact.

The firm minimizes costs. Therefore, the problem of the firm is:
\( \min (a_{t}, b_{t}) \left( W_{t}H_{t} + W_{t-1}(1+\pi)(N_{t-1} - D_{t}) + \theta W_{t-1}(1+\pi) D_{t} \right) \| \gamma \\
= f(N_{t-1} - D_{t}, H_{t}) \)

\( H_{t} \) are the workers hired at period \( t \), \( D_{t} \) the workers laid off at time \( t \), \( W_{t} \) the wage at which a new worker is hired at period \( t \) given to the firm.

The first order condition for the firm is:

\[(1) \quad f_{1}/f_{2} = \frac{(1+\pi)(1-\theta)}{(W_{t}/W_{t-1})} = r_{t} \]

In the absence of lay off costs the condition reduces to

\[ f_{1}/f_{2} = (1+\pi)/(W_{t}/W_{t-1}) = r_{t} ' \]

This expression gives us a first insight. If \( r_{t} \) is equal to 1, then the firm will hire a mix of first period workers and second period workers that is referred as natural, i.e. one that is not compelled by a price differential. This

9. Why will a firm hire new workers in the absence of wage differentials? is a difficult question. To assume the contrary will imply an expansion path, in the isoquants map, that runs through the experienced workers axis with an slope equal to 1, what will yield corner solutions. Common sense is inclined toward search arguments. Firms should look for better workers constantly, this should imply a minimum of turnover rate. The reverse is also difficult to justify: Why will not a firm dispense all the experienced workers for some positive relative wage? One possibility is the one discussed in Blanchard and Fischer (1989 p. 452) the technology is such that you need at least to have one experienced worker to pass the knowledge to the new hires.
solution is described on graph 2, as the tangency between
the isoquant and the $45^\circ$ isocost line. If $r_*$ is more than
1 it means that the rate $\pi$ at which wages of workers already
hired are increasing is higher than the one at which new
workers can be hired in the labor market. Therefore, even
though less productive it compensates to hire new workers.
Since $dH_{t-1} = r_* dN_t$ and $r_* > 1$ the new hires will be, in
general, more than the number of laid off workers. The level
of employment even increases holding output constant, with
production becoming more inefficient, since it will be using
more labor for the same amount of production. 10

The presence of severance pay, of course, reduces the
feasibility that the mandatory indexing policy will affect
the layoffs in the economy or increase workers turnover
rate. 11 In fact, if $\Theta = 1 - (1 + \pi) / (W_t / W_{t-1})$, for a given level
of output, the mandatory policy would not have any effect on
a firms' employment or unemployment decisions: it will
always pay to retain your workers.

Blanchard and Fischer quote two papers one of Lindbeck and
Snower (1986) and one by Dickens I've had access to the
first one, but not to the one of Dickens that, it seems to
me, that it must discuss a technology similar to the one
used here.

10. The complete characterization of the conditional demand
for aggregate labor is offered below.
11. Here there is an asymmetry, if workers are hired for the
current period, their dismissal costs are not considered
for this period. This problem is solved in the intertemporal
model considered below.
If demand for the final product increases, layoffs will decrease and admissions will increase in relation to the original situation, if both types of labor are normal inputs.

Another proposition can be raised in relation to the firm's average nominal wage: If the indexation policy is binding, i.e. \( r^e > 1 \), then the average wage is greater than the market wage \( W_e \). Instead of proving this in a general setting, it is proved for a Cobb-Douglas production function. The average wage can be expressed as follows:

\[
W = \frac{W_e H_e + W_{e-1}(1+\pi)(N_{e-1} - D_e)}{(H_e + N_{e-1} - D_e)}
\]

For the production function:

\[
Y_e = (N_{e-1} - D_e) = H_e^n,
\]

the first order condition lead us to:

\[
H_e = (\beta r^e / \alpha)(N_{e-1} - D_e)
\]

and substituting this into the formula of the average wage you get:

\[
(2) \quad W = \frac{[(\beta/\alpha+1)/(\beta/\alpha+1/r^e)]W_e}{(\beta/\alpha+1)}
\]
This last expression proves our proposition. If the indexation policy is binding on the limit, then \( r = 1 \) in this case \( N = W_n \). On the other hand, if \( r > 1 \) then:
\[ N > W_n. \]
Since it is costly to dismiss old workers in terms of productivity, if some old workers remain employed, then obviously they will earn a higher wage, increasing the average wage for a given level of output. This proposition becomes important for empirical tests since we should expect convergency of \( N \) and \( W_n \) without exact equality.\(^{12}\)

The proportion of each type of worker in the Cobb Douglas case is also very simple:

\[ H_n / (N_n - D_n) = \beta / \alpha r_n, \]

the higher the \( r_n \), the higher the proportion of new workers in each firm.

Some aggregate results can be derived out of this simple setting. Suppose the economy faces a fixed labor supply as depicted in figure 3. In this figure a C-shaped demand for labor, conditional on output, is depicted. This demand is a result of the assumption that workers are

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12. That is, if \( N \) and \( W_n \) co-integrate, then \( N - a - b W_n = u_n u_n \sim I(0) \). There is no need to impose the restrictions \( a=0 \) and \( b=1 \); since not necessarily the technology allows for full substitution of experienced workers, as previously discussed.
alike when they are in the labor market but the firm in choosing a particular mix of each type of worker will yield different levels of aggregate demand for labor, therefore, when there is no relative cost distortion for both types of workers the firm will use a minimum amount of labor $L'$. As distortions arise, increasing the cost of experienced workers, when indexation is binding; or decreasing their marginal cost, as severance pays get larger the economy will use larger amounts of labor for the same output. This can be seen by tracing a $45^\circ$-line trough the minimum cost choice of both types of labor $A$ and $A'$ in the isoquant, the intercept of this line on any of the axes will measure the firms' total amount of labor demanded, that's why this $45^\circ$-line will be called Iso-Employment Line. The minimum amount of employment will coincide with the minimum cost for $r^*=1$.

Suppose a fixed supply at $L_0$ on graph 3. At $r_1$ there is full employment, if $1+\pi$ increases, $r$ will rise and so will the demand for labor but as the demand is bigger than supply, $W_s/W_{s-1}$ will increase up to the point where $r=r_1$. What is a solution for a single firm it is not for the market as a whole. At $r_0$ there is also full employment, but in this case an increase in the mandatory indexation $(1+\pi)$ will lead to a reduction of employment, wages should fall as a result of excess supply of labor and the labor market should shift to the $(r_1,L_0)$ equilibrium, therefore there are potential problems of instability. Notice that $(r_1,L_0)$ can not be a steady-state equilibrium. In steady state $r=(1-\theta)$,
otherwise real wages will be in continuous fall. Since neither $x$ nor $y$ will be independent in a general equilibrium, setting further analysis is delayed to the next section.

As output increases the demand for labor also increases, shifting the C-demand curve to the right, pressing market wages, from the stable point, and eventually reducing the distortion in the relative cost of labor. Therefore, there is a map from increase in wages due to the mandatory policy, output and inflation to the rate of growth of market wages. The mandatory wage policy is just one determinant of the rate of growth of market wages; therefore, it is right to take it into account in stabilization plans, but not as the only determinant of wages. These results lead to some aggregate macroeconomic implications that are explored systematically in the next section.

IV. Macroeconomic Consequences of Labor Market Regulations With Turn-Over.

IV.1 A Simple Macroeconomic Model

In this section a simple macroeconomic equilibrium is discussed that will help to understand the macroeconomic consequences of the assumptions adopted in the previous section.

Consider a Cobb-Douglas production function and assume specific human capital, as considered in the previous
section. Assume decreasing returns for labor, i.e. \( \theta + \alpha < 1 \).

Further, we assume a mandatory indexation \( \pi \) equal to the current inflation rate, \( 1 + \pi = P_t/P_{t-1} \), in this particular case
\[
r_t = (1-\theta)(W_{t-1}/P_{t-1})/(W_t/P_t) = (1-\theta)\mu_{t-1}/\mu_t,
\]
here \( \mu_t = W_t/P_t \), the real wage. Additionally, \( \Gamma = 1/(\alpha + \beta) \). Under this circumstances the conditional demands of both types of labor are the following:

\[
(3) N = y^F[(\beta/\alpha)]^{-\alpha'}(1-\theta)\mu_{t-1}/\mu_t \]

\[
H = y^F[(\beta/\alpha)]^{-\alpha'}(1-\theta)\mu_{t-1}/\mu_t \]

Additionally, full employment is assumed, this assumption has implications on the efficiency of the economy and on the output effects of several policies. Natural (or frictional) unemployment is discussed in the last section.

In addition, assuming a fixed supply of labor and equilibrium in this market leads to:

\[
(4) N + H = y^F [(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'} + [(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'} = L
\]

where \( L \) is the fixed labor supply, \( \alpha' = \alpha/(\beta + \alpha) \) and \( \beta' = \beta/(\beta + \alpha) \). This equation will be referred as the Full Employment Condition. The cost function can be expressed as:

\[
C = [(\alpha/\beta)^{-\alpha'} + (\alpha/\beta)^{-\alpha'}(1-\theta)](1-\theta)'(W_{t-1}P_t/P_{t-1}) = W_{t-1}'y^F + \theta W_{t-1}P_t/P_{t-1}N_{t-1}
\]

By profit maximization:

13. Notice that the existence of severance pay creates a fixed cost for firms that can be interpreted as a reserve for eventual dismissals. As profits are residual income in the short run, the law reduces profitability. A cost for being in business.
(5) \( \mu_{s}^{-1} = \Gamma \left[ (a/b)^{-\alpha'} + (a/b)^{-\beta'} \right] (1-\theta)^{\alpha'} \left( \mu_{s-1}/\mu_{s} \right)^{\alpha'} \gamma^{\alpha-1} \),

Since \( \alpha' = 1-\beta' \).

Consider first the steady state of this system. Equation (4) shows that the level of output is independent of the real wage, \( \mu \):

(4') \( \gamma^{*} = \left( L/\left( \left( \beta/a \right) \left( 1-\theta \right) \right)^{-\alpha'} + \left( \beta/a \right) \left( 1-\theta \right)^{-\alpha'} \right)^{-\alpha'^{-1}} \).

Given \( \gamma^{*} \), equation (5) determines the steady state real wage:

(5') \( \left( \mu^{*} \right)^{-1} = \Gamma \left[ (\beta/a)^{-\alpha'} + (\beta/a)^{-\beta'} \right] (1-\theta)^{-\alpha'} (\gamma^{*})^{\alpha-1} \).

The appendix develops the comparative statics of the steady state, nevertheless, the results are simple. The steady state output is directly related to labor availability, as \( L \) increases so does \( \gamma^{*} \) and the real wage should fall by (5').

The steady-state \( \gamma^{*} \) is inversely related to severance pay. As this cost increases, since \( r_{c} < 1 \) in steady-state, this economy must employ more labor (remember the C-shaped conditional demand for labor) for a given output. In order to balance the labor market, the level of output should fall. Now, the effect of an increase in \( \theta \) on the steady-state \( \mu^{*} \) is a little bit more complex. As the severance pay increases the level of output should fall, implying a higher productivity of the workers employed, this can be seeing in equation (5') as a necessity of \( \mu \) to rise to restore the equality. At the same time since the economy becomes more inefficient, demand for labor increases and
therefore this will push up real wages in the labor market. As a result of both of these effects the real wage should rise. This is all about the real sector of this economy.

The model is closed with a quantitative equation, or Clower constraint, as an aggregate demand which determines the price level or the rate of inflation:

\[ (6) \quad M/P = \gamma. \]

The quantitative equation (6) only determines the price level (or the rate of inflation) given the level of output determined by (4) and the quantity of money (or given a positive rate of growth of the money supply). A constant rate of growth of the money supply implies a rate of inflation that is constant in steady-state but not in the short run. The real quantity of money is equal to the output. Therefore, deviations of the level of output should lead to deviations of the rate of inflation in the short run.

The model (4) to (6) yields a graphical representation, depicting the endogenous variables. However, in terms of our observables, it is much more interesting to continue with the isoquant representation to see the effects of labor supply shocks and severance payments on hiring and output.

In graph 4 an increase in labor supply is analyzed. Suppose that the economy is in equilibrium A, with \( H_0 \) and \( N_0 \). Since this is an equilibrium, an Iso-Employment LL (defined in p.11 above) line goes through point A that depicts the combination of different qualities of labor that yield the
full employment level of demand for labor, L. As L increases to higher amount of labor availability for this economy the Iso-Employment function shifts to the right to L'L'. The new equilibrium should be along the same scale expansion ray R, at point A', since severance pay doesn't change with the experiment. Labor proportions do not change and output increases.

Next, consider an increase in severance pay. The effects are depicted in graph 5. Again starting from an equilibrium point A then an increase in θ shifts the scale-expansion ray to R', a new equilibrium should be found along the same iso-employment line, at A'. A lower level of turnover should be observed (defined as H/N), and a lower level of output as the economy becomes more inefficient. The reverse is true for an decrease in the severance pay. Output increases and turnover too. Now consider a wage law that increases workers' wages by a constant k of (P_e/P_{e-1}), if k is greater than one, more than the inflation rate is passed on to wages, the result will be equivalent to a reduction of θ. The steady-state relative cost of both types of labor is affected with k. This exercise approximates the issue of the frequency of wage adjustments, higher frequency in wage adjustments tend to increase the real average wage of experienced workers. A counter intuitive result arises. A more severe indexation increases the steady-state level of output and therefore reducing temporarily the rate of inflation, given a constant rate of growth of money. This
result is a consequence of severance pay that holds the economy at a certain level of inefficiency. Imposing a stricter mandatory wage indexation reduces this inefficiency to some degree; a second best type of argument. This is because higher rotation doesn't have any additional cost, the inclusion of frictional unemployment imposes such a cost and changes this counter-intuitive result.

IV.2 Dynamics

The model comprised in (4) to (6) is highly nonlinear and a nonlinear solution will not be attempted. Equations (7) to (9) below present a log-linear version that yields a simple solution.¹⁴

The equilibrium in the labor market can be expressed as:

(7) \[ l = k_0 + Iy - (a'a - b'b)/(a+b) (\mu_\ell - \mu_{\ell-1}) \]

The maximum profit condition can be expressed as:

(8) \[ \mu_\ell = -k_1 + (1-\Gamma)y_\ell + \alpha' (\mu_\ell - \mu_{\ell-1}) \]

Finally aggregate demand is given by:

(9) \[ p_\ell - p_{\ell-1} = m - (y_\ell - y_{\ell-1}) \]

Here \( k_0 \) and \( k_1 \) are functions of parameters of the production function and \( \theta \) with \( dk_0/d\theta > 0 \) and \( dk_1/d\theta < 0 \). In addition \( a = ((\beta/\alpha)(1-\theta))^{\gamma} \) and \( b = ((\beta/\alpha)(1-\theta))^{-\gamma} \).

¹⁴ All derivations of the log-linear approximation are in the appendix, as well as the reduced form and the conditions for stability.
In this section \( y \) and \( \mu \) are in logs, \( l = \log(L) \), \( p = \log(P) \) and \( m \) is the constant rate of growth of money. The reduced form for the real wage equation is:

\[
(10) \: \mu_t = \varphi \mu_{t-1} + D^{-1} [(1-I)(1-k_0) - \Gamma k_1]
\]

Where \( \varphi = (-\alpha' \Gamma + (1-\Gamma)(a/(a+b) - \beta'))/D \), and \( D = (\beta' - (1-\Gamma)(a/(a+b) - \beta')) \) is the determinant of the system (7) to (9). (10) has as a solution:

\[
(11) \: \mu_t = (\mu_0 - \mu^*) \varphi^t + \mu^*
\]

Where \( \mu^* = \frac{FK_1 + (F-1)(1-k_0)}{\beta' - F + (F-1)(a/(a+b))} \)

\( \mu^* \) is the steady state of the real wage in this linear version. The reduced form for the output equation is:

\[
(12) \: y_t = -D^{-1}(a/(a+b)-\beta')\mu_{t-1} + D^{-1}[k_1(a/(a+b)-\beta') - \beta'(1-k_0)]
\]

Real output depends on last period real wage. The coefficient is positive. The dynamics of real wages determine the dynamics of output and inflation. The coefficient \( \varphi \) will depend on the parameters of the production function, particularly on the proportionality of both types of labor, \( \beta/\alpha \). In general, \( \varphi \) will be negative and in some cases less than unity assuring stability. In the appendix a sample of parameters that assure stability is offered. Since \( \varphi \) is negative a dampening fluctuation any time \( \mu \) differs from \( \mu^* \) will be produced. For instance, suppose 1 increases this will imply, as was already shown, a fall in \( \mu^* \). How will this economy get to this new equilibrium? Starting at time 0 the real wage will be now higher than the new steady-state one. Therefore at period 1
the real wage will be below the new steady state, a temporary undershoot. Since currently employed workers will be employed with a higher wage, the firm will take advantage of the situation by hiring at above the regular hiring rate and dismissing more workers than usual, holding the level of output constant, since this is affected only with a lag. The fall in wages will initially provoke a reduction in output below the steady-state level of output jumping afterwards toward a higher level. The process will converge with fluctuations over and below the steady state real wage and the steady state real output. Inflation will follow counter-cyclically and will end up at the same steady state level, nonetheless in order to obtain a reduction in real wages, measured inflation along the path of adjustment should have grown faster than nominal wages. This is a behavior that resembles the Peak-Average-Wage hypothesis. Nonetheless, the economy is able to adjust to a shock in the labor market with a reduction in real wages even though the indexation law is supposed to avoid it. The cost is a period of instability and, as discussed later, of unemployment.

Now consider a more severe wage indexation, as argued above, that can be approximated as a reduction in \( \theta \). This will lead to an increase in the steady state real wage. Starting at \( t=0 \), the real wage will be below the new steady-state level, therefore, at period one the real wage will be above the steady-state level for the same level of output. After a while the economy will reach a new steady state
equilibrium, while measured inflation must have been lower than the rate of growth of wages in order to reach a higher real wage. Therefore, by way of contrast with the Peak-Average hypothesis, the inflation rate may even fall temporarily when mandatory indexation targets a higher real wage than the economy is "able to pay". This result changes drastically when frictional unemployment is taken into account below, and is less clear when a more appropriate intertemporal setting is developed.

One major critique to the setting presented is the one advanced recently by Lazear (1990) in which he proposes that severance payments might be neutral in as much as an arrangement could be made between contracting parties by which workers made an up-front payment to finance their eventual dismissal. These up-front payments are not observed in Brazil. Of course they may be implemented through a lower wage at hiring and a higher wage in the following periods. This pay scheme is usually observed, but is probably not only due to severance pay and will not affect the results presented here. However, the economy will be closer to r=1.

Evidence that supports the approach adopted here in terms of available observables is presented in another paper (Malaga 1992). The view that wages at hiring influence strongly the average wages at firms of the industrial sector of Sao Paulo is confirmed by empirical analysis. Also there is evidence that turnover rates react to this wage differentials.
IV. Intertemporal Considerations.

The simple model presented in the previous section explores only one aspect of wage policy: the mandatoriness of the wage adjustment for already hired workers. The lesson extracted is to make turnover adjustments for whatever distortions arises from the wage policy, at least in as much as technology allows for it. One of the most popular arguments about wage determination in an inflationary environment suggest that the inflationary environment allows the adjustment of real wages just by staggering wage readjustments. It is argued here that even if firms behave that way, there is an opportunity cost for workers that the firm can not violate and that this behavior will not reduce the labor turn-over eventually provoked by wage policy distortions.

To take this into account a small modification to the simple model is introduced. The modification allows a less restrictive wage policy, making wage adjustment mandatory only every two periods. This describes most of the policies adopted in Brazil since the 1960's, with the possible exception, of a short period of the 'gatilho' that was supposed to readjust wages by 20% every time the 

\[ \text{accumulated inflation} \text{ surpassed } 20\%. \]

During the intra

15. See appendix @@@.
period the firm may adjust voluntarily or not adjust the wage rate. The decision of the firm can be split in two sub-problems. One for the second period, when the firm will have to pay at least what the law enforces. The firm will hire workers for two periods at the end of the second period though it will have to dismiss all workers paying the severance pay $\theta$. It will have to minimize costs for a given output:

$$\text{Min } (N_2, H_2, N_3, i) \{ W_0iR_2 N_2 + W_3 H_2 + \theta W_0iR_2 D_2 + \sigma_3 W_3 (N_2 + H_2) \mid Y_2 = f(N_2, H_2), R_2 > W_2/W_0i, R_2 \geq 1, D_2 \geq 0 \}$$

Where $\sigma_3$ is a discount factor, $i$ is a mandatory wage adjustment that will adjust wages for a two period inflation. For instance, $i = P_2/P_0$ if the whole accumulated inflation is mandatory. Experienced workers all earn the same wage, even though they could have been hired at different dates. This is a simplifying assumption that has consequences on the conclusions as will be seen shortly.

The notation is as follows: $R_2$ is the wage adjustment rate for period 2 voluntarily adopted by firms, $i$ the mandatory adjustment of wages, $N_2 = N_1 + H_1 - D_2$, is the number of experienced workers that the firm decides to retain which are $N_1 + H_1$ workers inherited from the previous period less $D_2$ workers dismissed in the current period. $H_2$ is the number of unexperienced workers hired for the second period. Production is subject to a given level of product and a given production function that presents the standard properties: $Y_2 = f(N_2, H_2)$. Additionally, during the second
period, wage adjustment will have to be large enough to make the internal wage competitive with the market alternatives for workers, \( W_2 / W_0 \). Also, by law \( R_2 \geq 1 \). Finally \( D_2 \geq 0 \), that is, you cannot hire experienced workers, because of the specific human capital assumption, you can only dismiss them.

The main difference of this setting, compared with the previous one, is that firms cannot pay less than the market wage, even if the law implies a lower than market wage. If this were the case, trained workers would leave the firm. This is captured by the restriction on \( R_2 \) assuring at least the market wage \( W_0 \). This too is a simplifying assumption. Alternatives: 1) The firm may have same market power over their experienced workers, especially if there exist unemployment and is costly for workers to change jobs; 2) Insider workers may use their bargaining power since they are privileged by the law, as in the Nordic literature, see

16. This setting is not the best to deal with this problem that is more properly dealt with in the contracts literature (see for instance Rosen(1985)). A Contract model with two kinds of labor and accumulation of specific human capital is a very difficult setting and was tried with not too much success. We are referring to the condition of wage adjustment at the market rate as "participating" condition as in the literature on contracts. One simple extension that was explored but not reported was to set the the participating condition equal to the present value of the market wage for two periods. The results are neat! The wage readjustment must equate the present value of income, for a worker that will remain for two periods at the firm. If the mandatory indexation is binding this determines, together with the present value of the market wage, the readjustment in the first period.
for instance Lindbeck and Snower (1986); 3) The firm may practice a differential wage adjustment over the workers that she doesn't want to retain forcing them to leave the firm voluntarily without incurring in the severance payment. 1) and 2) work in opposite direction therefore the alternative here proposed must work roughly right. 3) is more critical and it will have consequences. There is legislation against: 'isonomia salarial', something similar to 'equal pay' clause in the US, but in Brazil probably is very difficult to enforce. A practice of differential wages could also be counterproductive as a matter of incentives to maintain efficient production. On empirical grounds, it is observed in the Brazilian data a rate of 35% of voluntary separation against 65% of layoffs on average, during the sampling period analyzed on the next chapter: 1985-1991. One of the reasons to observe this unequal distributions on labor separations is the benefits obtained by the severance pay law. Most of the time even if the worker voluntarily wants to leave a firm she will force a layoff. Since part of the severance pay is already transferred to the governement's fund ('Fundo de Garantia'), sometimes it is even possible a mutual agreement for the worker to get at least what is deposited at the Fund. Nonetheless the assumption can only be considered as a simplifying device.

The first order conditions for this model will include:
\( f_1/f_2 = (W_0iR_2(1-θ)+σ_2θW_3)/(W_2 + D_2θW_3) \)

In this case the ratio of marginal labor costs will be an increasing function of \( i \), the mandatory wage policy. At the time of deciding the amount of labor to be hired the firm will compute a fund for eventual dismissals \( D_2θW_3 \) per worker. Once the worker is already hired, dismissal costs are a deterrent to firing experienced workers. As the wage policy becomes more restrictive (higher \( i \)) this advantage disappears.

This expression (13) is a bit more complicated than the equivalent first order condition (1), because now \( R_2 \) is an endogenous variable. However, since an inequality constraint has been imposed on it, its determination is easy. Consider the following cases: (i) Suppose that \( iW_2/W_0 \), if \( R_2 > W_2/W_0i \) the associated lagrange multiplier should be zero. This will imply \( N_2 = 0 \), since the first order condition for \( R_2 \) can be written as:

\( W_0i N_2 - δ = 0, \)

where \( δ \) is the associated Lagrange multiplier. But, \( N_2 \) cannot be zero since this will imply an infinite marginal productivity for \( N_2 \). All of these restrictions force only one solution. If internal wages dip below market wages, the firm cannot increase experienced workers and is forced to retain what it has at higher wages because of the participating restriction. Therefore \( R_2 = W_2/W_0i \) that in turn implies that (13) should be

\( f_1/f_2 = (1-θ+σ_2θW_3/W_2)/(1 + σ_2θW_3/W_3). \)
The marginal productivity of experienced workers will be below the one of the new workers. That is, if dismissal costs increase, more experienced workers will be needed due to decreasing marginal productivity.

(ii) Now suppose $i > W_2/W_0$, in this case $R_2=1$ and (14) becomes

$$W_0i N_2 - \tau = 0,$$

where $\tau$ is the multiplier associated with $R_2=1$, that will be positive if $R_2=1$, assuring less than total layoffs. Nonetheless in this case there is the possibility of dismissals induced by the wage policy. In this case (13) becomes:

$$(13'') \quad f_1/f_2 = [W_0i(1 - \theta) + \sigma_2\theta W_3]/(W_2 + \sigma_2\theta W_3)$$

All of these restrictions force only one solution. If internal wages fall below market wages, the firm cannot increase experienced workers and is forced to retain what it has at higher wages because of the participating restriction.

The total cost function for this firm will be in the second period:

$$c = c(W_0i(1-\theta) + \sigma_2\theta W_3, W_2 + \sigma_2\theta W_3, y) + \theta W_0i (N_1 + H_1)$$

The first component is the usual one, the second component is the dismissal cost, which makes the link between periods.
At the beginning of the first period the cost that should be minimized is:

\[(16) \text{Min } (d_1, h_1, w_1) \{ w_0 r_1 n_1 + w_1 h_1 + \theta_2 w_0 r_1 d_1 \\
+ \sigma_1[c(w_0 i (1-\theta) + \theta_3 w_3, w_3 + \theta_3 w_5, y) + \theta w_0 i (n_1 + h_1)] \}\]

In this case, the same restrictions that apply to the second period apply to this first period: \(r_1 \geq w_1 / w_0, \ r_1 \geq 1, \ d_1 \geq 0\). First attention is restricted to the case in which \(D_1 > 0\). In this case the first order conditions leads us to:

\[(17) f_1 / f_2 = (w_0 r_1 (1-\theta) + D_1 \theta w_0 i) / (w_1 + D_1 \theta w_0 i)\]

As in the second period, \(r_1\) is endogenous; nonetheless, the restrictions imposed upon it will imply \(r_1 = w_1 / w_0\), whenever \(w_0 < w_1\), as will usually be the case in an inflationary environment. Therefore (17) can be written as:

\[(17') f_1 / f_2 = [w_1 (1-\theta) + \theta w_0 i] / (w_1 + \sigma_1 \theta w_0 i)\]

Note particularly, that in case \(i = (P_2 / P_0)\), given that \(\sigma_i = (P_2 / P_0)(1+r)\) and defining \(\delta = (1+r)^{-1}\), constant, (17') can be written as \(f_1 / f_2 = [\mu_1 (1-\theta) + \delta \theta \mu_0] / (\mu_1 + \delta \theta \mu_0)\), that is, operating along the same lines as the relative cost of labor in demand for labor in the previous section model. Therefore, even though the mandatory wage adjustment is only in the second period, during the first period the firm will react with labor turnover. As before if \(\mu_0 / \mu_1\) increases, say
because the labor supply suffers a positive shock, then \( f_1 / f_2 \), should have to adapt to a higher level the experienced workers will have to increase their productivity relative to new entrants in the firm implying, conditional on output, an increase in \( H \) relative to \( N \). Of course, this effect will be less strong than before and will decrease if the interest rate rises; the cost reduction due to higher labor turnover is reduced when considered intertemporally.

The two period problem can be extended easily to an arbitrary number of periods keeping the same structure of conditional demands for both types of labor, this is done below, where an extension is made to deal with a macroeconomic equilibrium. Specializing it further, to keep it tractable, it is assumed that every period the wage policy adjusts wages perfectly to their original level and a Cobb-Douglas technology is assumed.\(^{17}\) For any period \( t \) the demands for labor are:

\[
(18) \quad N = \gamma^{\mu_t} (\beta / \alpha)^{\gamma / (\gamma + \sigma)} \left( (1-\theta)(\mu_{t-1} / \mu_t + \delta) \right)^{-\gamma / (\gamma + \sigma)} (1+\delta)^{\gamma / (\gamma + \sigma)}
\]

\[
H = \gamma^{\mu_t} (\beta / \alpha)^{\gamma / (\gamma + \sigma)} [(1-\theta)(\mu_{t-1} / \mu_t + \delta)]^{-\gamma / (\gamma + \sigma)} (1+\delta)^{-\gamma / (\gamma + \sigma)}
\]

Equations in (18) are the basis for a macroeconomic model similar to the one expressed in equations (4) to (6) discussed previously. First the Full Employment Condition is defined:

\[^{17}\text{The detailed optimizing recursion that lead to this result is reported in the appendix.}\]
\[(19) \gamma^c((\theta/\alpha)\bar{\phi}^c[(1-\theta)(\mu_{t-1}/\mu_t) + 88] - (1+88)^{1-\theta} (1+88)^{1-\theta} = L \]

The maximum profit condition:
\[(20) \Gamma[(\theta/\alpha)^{1-\theta} + (\theta/\alpha)\bar{\phi}^c]y^c - \frac{1}{1-\theta}(\mu_{t-1}/\mu_t) + 88] - (1+88)^{1-\theta} \mu_{t-1} = 0 \]

Equations (19) and (20) define the real sector equilibrium for \(y\) and \(\mu\). It has the same structure as the model discussed above therefore it will not be discussed in detail. Particularly interesting are the stability properties that were too narrow in model (4) to (6).

The log-linear version of the model is developed in the appendix, and table A1 of the appendix shows a sample of values for parameters that either assure or do not assure stability. The determinant of the system now becomes:
\[D = [(1-\theta)/(\theta/(1-\theta) + 88)](\theta/88/(1-\theta)) \Gamma - (1-\Gamma)(a/(a+b)-\theta') \]

The reduced form for the wage equation can be written as:
\[(21) \mu_t = \phi \mu_{t-1} + k_2 \]

Where:
\[\phi = [-\Gamma \alpha' + (1-\Gamma)(\alpha' - b/(a+b))/[(\theta + 88/(1-\theta))\Gamma - (1-\Gamma)(a/(a+b)-\theta')]] \]

is an equivalent parameter to the one in equation (10) and differs from it only because of the first factor in the determinant in \(D\) that represents the factor of smoothness in the relative cost of both types of labor. \(k_2\) is a constant that depends on the parameters of the production function, \(\theta\) the cost of severance pay and the discount rate \(\delta\).

Again \(\phi\) is negative for the possible values of the parameters and is \(-1<\phi<0\) for some values of them. The important point here is that once the problem is correctly
formulated intertemporally, the equilibrium has more chances to be stable. The reason is that firms' reaction is no longer as violent, in that new hiring will entail a firing cost in the future too. The economy will also reach equilibrium quicker than in the previous case for a comparable set of parameters. The properties of the steady state equilibrium, though, do not change. However, the inclusion of frictional unemployment changes the properties, and therefore is the last extension that is dealt with.

V.1 Frictional Unemployment.

A counter-intuitive proposition was found on section IV.2. A more stricter rule of mandatory indexation resulted in an increase in output. This was a result of the fact that the rate of new hiring is below the natural one due to the existing severance pay. The stricter indexation then should result in a gain of efficiency, a second best argument.

Consider an equilibrium condition in the labor market such that any worker fired in any particular period will need a period to a job. The justification of this assumption could fit well in search models. In this case the equilibrium condition can be written as:

\[(22) \quad N_e + H_e + D_e = L\]

Since \(D_e = N_{e-1} + H_{e-1} - N_e\), (22) can be written as:

\[22') \quad H_e + N_{e-1} + H_{e-1} = L\]
The equilibrium in the labor market now becomes a second order difference equation in wages and first order in output and it can be written as:

\[ y_t = \left( \frac{\beta}{\alpha} \right)^{\gamma} \left[ (1-\theta)(\mu_{t-1}/\mu_t + \delta) \right]^{\gamma} (1+\delta)^{-\gamma} \]

\[ + y_{t-1} \left( \frac{\beta}{\alpha} \right)^{\gamma} \left[ (1-\theta)(\mu_{t-2}/\mu_{t-1}) + \delta \right]^{\gamma} (1+\delta)^{-\gamma} \]

\[ + \left( \frac{\beta}{\alpha} \right)^{\gamma} \left[ (1-\theta)(\mu_{t-2}/\mu_{t-1}) + \delta \right]^{\gamma} (1+\delta)^{-\gamma} = L \]

The maximum profit condition does not change and can be written as:

\[ \Gamma \left( \frac{\beta}{\alpha} \right)^{\gamma} + (\frac{\beta}{\alpha})^{\gamma} \gamma y_{t-1} \left[ (1-\theta)(\mu_{t-1}/\mu_t) + \delta \right]^{\gamma} (1+\delta)^{-\gamma} \mu_{t-1} = 1 \]

The system is rather complicated and so a graphical discussion of the steady state equilibrium is offered. Graph 6 uses the same representation as graph 5; the full employment locus LL, an isoquant yielding a level y of output and the scale expansion ray R. Point A is an equilibrium. It was obtained with a funny device: inscribing an square in the triangle formed by the ray R, the full employment locus LL and the horizontal axis. The justification is the following in steady-state \( H_0 = D_0 \), and therefore \( N_0 + H_0 = N_0 + D_0 \) is projected on the horizontal axis yielding a point on the full employment locus. Now consider a shift in the ray R to \( R' \) due for instance to a more aggressive indexation policy. The equilibrium shifts to A'. With the bundle of factors \((N,H)\) in A' it is no longer possible to attain the level y of output. As a result a higher rate of unemployment will be observed and a higher real wage. Therefore a more aggressive indexation might lead to a temporary rise in inflation due to equation (9) with
an effect similar to the one proposed by the Peak-Average hypothesis except that real wages will be rising instead of decreasing.

VI. Conclusions

Assuming a technology that allows for a normal or natural turn-over of labor the effects of policies that affect workers of different vintages differently in a firm the following propositions can be reached:

1) Binding indexation should result in higher rates of turn-over and higher severance pay will reduce it. In an intertemporal context these effects will be smoothed but not eliminated.

2) The greater differentials in costs created by both of these policies will result in a greater degree of inefficiency in the economy - more use of labor per unit of output.

3) Assuming a full employment macroeconomic equilibrium, the economy will adapt to different kinds of shocks in the real sector through a period of abnormal turn-over rates but that will accommodate the wage rigidity.

4) If the shock is created by one of these policies, second best effects may occur in output; a tighter indexation policy may reduce the inefficiency of the economy if severance pay is held fixed and vice versa.
5) Second best effects may not be observed with frictional unemployment, which imposes costs in output due to higher turnover rates.

6) Wages will mainly be determined by market forces instead of the simple application of wage policies.

Particular care must be taken with proposition 4 since it looks as if an appropriate combination of indexation and severance pay policy might lead to an efficient outcome. This is not a correct conclusion once that severance pay implies the creation of fixed costs, as discussed in the text, that reduce profitability in the economy. Both policies make labor more expensive relative to other factors reducing its use in longer runs.

However, no aspect that has not been considered is that severance pay might lead to a degree of inefficiency that will be the price paid in order to have an economy with less turn-over and less frictional unemployment.
APPENDIX

This appendix offers most of the derivations used in the text, to help the reader to follow the mathematical argument. It does not contain generalizations or rigorous proofs of the propositions as is common in mathematical appendices. It is offered just because the main text is based on the Cobb-Douglas case and in some points the algebra gets very messy.

I. Comparative Statics of the System of Equations expressed in Equations (4') and (5').

Starting with equations in the text:

(4') \( y^* = \frac{L}{[(\beta/\alpha)(1-\theta)]^{\sigma'} + [(\beta/\alpha)(1-\theta)]^{\theta'}} \)

(5') \( (\mu^*)^{-1} = \Gamma \left[ (\beta/\alpha)^{-\sigma'} + (\beta/\alpha)^{\theta'} \right] (1-\theta)^{1/(\sigma'-1)} \)

Differentiating the system (4') and (5') we obtain the following recursive system in the differentials:

\[
\begin{pmatrix}
\Delta y^{r-1} \\
\Delta \Gamma (1-\theta) y^{r-2} (1-\theta)^{1-r} \\
\Delta \Gamma' (1-\theta)^{1-r} y^{r-2}
\end{pmatrix}
= \begin{pmatrix}
0 \\
\Delta \theta \\
\mu
\end{pmatrix}
\]

(A.1)

The determinant of this system is:
(A.2) \( D = AB \Gamma = y^{x^{(x-1)}}(1-\theta)^{-1} > 0 \)

Where \( A = [\beta / \alpha(1-\theta)]^{-\alpha} + [\beta / \alpha(1-\theta)]^{\alpha} > 0 \)

\( A' = -\beta' [\beta / \alpha(1-\theta)]^{-\alpha} + \alpha' [\beta / \alpha(1-\theta)]^{\alpha} \)

\( = -\beta' (1-\theta)^{-\alpha-1} < 0 \)

\( B = [\beta / \alpha]^{-\alpha} + [\beta / \alpha]^{\alpha} > 0 \)

This constant will be reappear in several places in this appendix. \( A' \) is specially important to signing expressions since it measures the substitution effect.

The comparative static results described in the text are:

\[ \frac{dy}{dL} = (Ay^{x-1})^{-1} > 0 \]

\[ \frac{d\mu}{dL} = -(\Gamma - 1)\mu(\Gamma Ay)^{-1} < 0 \]

\[ \frac{dy}{d\theta} = A' [\Gamma \Gamma(1-\theta) y]^{-1} < 0 \]

\[ \frac{d\mu}{d\theta} = \alpha' (1-\theta)^{-1} - (\Gamma - 1) A' \mu [\Gamma \Gamma(1-\theta)]^{-1} > 0 \]

II. Log-Linear Approximation of System Formed by Equations (4) and (5)

Equations (4) and (5):

(4) \[ y^x \left( [(\beta / \alpha)(1-\theta)]^{-\alpha} (\mu_{x-1} / \mu_x)^{-\beta} \right. \]

\[ + \left. [(\beta / \alpha)(1-\theta)]^{\alpha} (\mu_{x-1} / \mu_x)^{\beta} \right) = L \]

(5) \[ \mu_{x-1} = \Gamma \left[ (\alpha / \beta)^{-\alpha} + (\alpha / \beta)^{\alpha} \right] (1-\theta)^{-\alpha} (\mu_{x-1} / \mu_x)^{\alpha} y^{x-1}, \]

(5) admits a simple approximation by taking logs on both sides of this expression:

\[ -\log(\mu_x) = \log(\Gamma) + \log(B) + \alpha' \log(1-\theta) + \]

\[ \alpha' (\log(\mu_{x-1}) - \log(\mu_x)) + (\Gamma - 1) \log(y_x). \]
(4) can only be linearized with two approximations. Taking logs on both sides of (4):

\[ \log(y_t) + \log[[(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'} + [(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'}]} \] = \log(L) \]

The expression inside brackets can be approximate by Taylor expansion around the steady state \( \mu_{t-1}/\mu_{t-1} = 1 \) as:

\[ \log[[(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'} + [(\beta/\alpha)(1-\theta)]^{-\alpha'}(\mu_{t-1}/\mu_t)^{-\alpha'}]} \approx \log(A) + (A'/A)(\mu_{t-1}/\mu_{t-1}) \]

Approximating again the rate of growth \( (\mu_{t-1}/\mu_{t-1}) = \log(\mu_{t-1}) - \log(\mu_t) \), we can express the system above around the steady state equilibrium as:

(A.3) \[ I = k_0 + \Gamma y + (A'/A)(\mu_{t-1} - \mu_t) \]

(A.4) \[ \mu_t = -k_1 + (1-\Gamma)y + \alpha'(\mu_t - \mu_{t-1}) \]

Where all the variables are now in logs and \( \log(L) = 1 \), \( k_0 = \log(A) \) and \( k_1 = \log(\Gamma) + \log(B) + \alpha'\log(1-\theta) \).

Or in matrix form:

\[
\begin{bmatrix}
\beta' & \Gamma^{-1} \\
-A'/A & \Gamma
\end{bmatrix}
\begin{bmatrix}
\mu_t \\
y_t
\end{bmatrix}
= 
\begin{bmatrix}
-a' \\
-A'/A
\end{bmatrix}
\begin{bmatrix}
\mu_{t-1} \\
1-k_2
\end{bmatrix}
\]

Since the system is a first difference system only in wages, the stability and the dynamic properties depend solely on the dynamics of \( \mu \). The determinant is:

\[ D = \beta'\Gamma + (\Gamma^{-1})A'/A \]
Since \((\Gamma-1) = (1-\alpha-\beta)/(\alpha+\beta) > 0\), once that we are assuming decreasing returns for labor. The determinant has to be worked a little to determine its sign. Defining 
\[a = [(\beta/\alpha)(1-\theta)]^{-1} \quad \text{and} \quad b = [(\beta/\alpha)(1-\theta)]^{-\lambda},\]
\(A'/A\) can be expressed as \([a/(a+b) - \beta']\). Therefore the determinant can be expressed as:
\[D = -A'/A + \Gamma a/(a+b) > 0\]

Reduced form equations for \(\mu\) and \(\gamma\) can be easily found as:
\[(A.5) \quad \mu_t = D^{-1}[(1-\Gamma)A'/A - \alpha' \Gamma] \mu_{t-1} - D^{-2}[\Gamma k_1 - (1-\Gamma)(1-k_0)]\]
\[(A.6) \quad \gamma_t = D^{-1}(A'/A) \mu_{t-1} - D^{-2}[k_1(A'/A) - \beta'(1-k_0)]\]

Since \(A' < 0\) the coefficient of the lagged real wage will be negative but not necessarily less than one in absolute terms. Table A.1 present some values for several parameter values and the coefficient \(\phi\). The steady state turnover in this case will be abnormally high. This is not the case with \(\phi'\), the equivalent coefficient of the intertemporal model.

Derivatives of the constants \(k_0\) and \(k_1\) are:
\[dk_0/d\theta = - (A'/A)(\beta/\alpha) > 0\]
\[dk_1/d\theta = -\alpha'/(1-\theta) < 0\]

The first measures the effect of an increase in severance pay on the steady-state demand for labor, the second the effect on steady-state output.
III. Dynamic Programming Solution for the Intertemporal Model
With Arbitrary Finite Horizon

In the last period T+1, without production, we assume that the firm dismiss all their workers, therefore:

\[(A.7) \quad C_{T+1} = \theta \mu_T (N_T + H_T)\]

In period T the optimizing problem of the firm becomes:

\[(A.8) \quad C_T = \min_{(N_T, H_T)} \{ \mu_{T-1} (1-\theta) N_T + \mu_T H_T + \theta \mu_{T-1} (N_{T-1} + H_{T-1}) + \delta C_{T+1} \}\]

Subject to \(y_T = N_T + H_T\)

Where \(C_{T+1}\) is given by (A.7). Problem (A.8) leads to the following labor demands:

\[N_T = y_T f(\beta/\alpha)^-\gamma(1+\theta)^\gamma \left[ (\mu_{T-1}/\mu_T) (1-\theta) + \delta \theta \right]^-\gamma\]

\[H_T = y_T f(\beta/\alpha)^-\gamma(1+\theta)^\gamma \left[ (\mu_{T-1}/\mu_T) (1-\theta) + \delta \theta \right]^-\gamma\]

Computing with this solutions the cost-value function we obtain:

\[(A.9) \quad C_T = y_T f(\mu_{T-1} (1-\theta) + \delta \theta \mu_T)^-\gamma (\mu_T + \delta \theta \mu_T)^-\gamma + \theta \mu_{T-1} (N_{T-1} + H_{T-1})\]

Which depends again on the decisions on the previous period \((N_{T-1}, H_{T-1})\). The recursion repeats in period T-1:

\[(A.10) \quad C_{T-1} = \min_{(N_{T-1}, H_{T-1})} \{ \mu_{T-2} (1-\theta) N_{T-1} + \mu_{T-1} H_{T-1} + \theta \mu_{T-1} (N_{T-2} + H_{T-2}) + \delta C_T \}\]

Where \(C_T\) is given by (A.9). Problem (A.10) leads to the following demands for labor:

\[N_{T-1} = y_{T-1} f(\beta/\alpha)^-\gamma(1+\theta)^\gamma \left[ (\mu_{T-2}/\mu_{T-1}) (1-\theta) + \delta \theta \right]^-\gamma\]

\[H_{T-1} = y_{T-1} f(\beta/\alpha)^-\gamma(1+\theta)^\gamma \left[ (\mu_{T-2}/\mu_{T-1}) (1-\theta) + \delta \theta \right]^-\gamma\]
Computing with this solutions the cost–value function we obtain:

(A.9) \[ C_{t-1} = \gamma_{t-1} \varphi[\mu_{t-1}(1-\theta)+\theta\mu_{t-1}] = \gamma_{t-1} \varphi[\mu_{t-1} + \theta\mu_{t-1}] \]

\[ + \theta\mu_{t-1}(N_{t-1}+H_{t-1}) + \delta y_{t} \varphi[\mu_{t-1}(1-\theta)+\theta\mu_{t}] = \gamma_{t} \varphi[\mu_{t} + \theta\mu_{t}] \]

By induction the cost function from the first period is:

(A.10) \[ C_0 = \sum_{t=0}^{T} \delta y_{t} \varphi[\mu_{t}(1-\theta)+\theta\mu_{t}] = \gamma_{t} \varphi[\mu_{t} + \theta\mu_{t}] \]

\[ + \theta\mu_{0}(N_{0}+H_{0}) \]

And conditional demands for labor, for any period \( t \), can be expressed as:

(A.11) \[ N_{t} = y_{t} \varphi(\beta/\alpha) = \gamma_{t} \varphi(1+\delta) \cdot [(\mu_{t-1}/\mu_{t})(1-\theta)+\theta]^{-\beta} \]

(A.12) \[ H_{t} = y_{t} \varphi(\beta/\alpha) = \gamma_{t} \varphi(1+\delta) \cdot [(\mu_{t-1}/\mu_{t})(1-\theta)+\theta]^{-\beta} \]

Which are equation (18) in the text.

IV. Macroeconomic Model With Demand for Labor Derived from an Intertemporal Setting

Using The demand and cost equations derived in the previous section of this appendix we can set up a model with the same characteristics as the one discussed in section II. Although the cost function is derived from an intertemporal system, the profit maximization implies equating price to marginal cost at every period, and this
equation will not involve any output from previous periods or future periods.

We can express the full employment condition, using (A.11) and (A.12) as:

\[(A.13) \text{ } y^*[(\beta/\alpha)^{-\alpha'}(1+\theta)(\mu_{t-1}/\mu_t) + \delta \theta]^{-\alpha'}(1+\delta \theta)^{-\alpha'\theta} + (\beta/\alpha)^{-\alpha'}[(1-\theta)\mu_{t-1}/\mu_t + \delta \theta]^{-\alpha'}(1+\delta \theta)^{-\alpha'\theta} = L \]

Which is equation (19) in the text. The maximum profit condition can be obtained differentiating (A.10) in relation to the current level of output and equating to Price:

\[(A.14) \text{ } \Gamma[(\beta/\alpha)^{-\alpha'} + (\beta/\alpha)^{-\alpha'}]y^{*-1}[(1-\theta)(\mu_{t-1}/\mu_t) + \delta \theta]^{-\alpha'}(1+\delta \theta)^{-\alpha'\theta}\mu_{t-1} = 1 \]

Which is equation (20) in the text. (A.13) and (A.14) form a non-linear first order difference system and can be log-linearized as the model discussed in section II of this appendix.

\[(A.15) \text{ } l = k_0 + \Gamma y + (A'/A)[(1-\theta)/(1-\theta+\delta \theta)](\mu_t - \mu_{t-1}) \]

\[(A.16) \text{ } \mu_t = -k_1 + (1-\Gamma)y + \alpha'(1-\theta)/(1-\theta+\delta \theta)(\mu_t - \mu_{t-1}) \]

That look like (A.3) and (A.4) except for the term \([(1-\theta)/(1-\theta+\delta \theta)]\) that reduces the coefficients of the wage differential in (A.3) and (A.4). This is the effect of the intertemporal smoothing caused by the fact that eventually all workers will be dismissed and the firm will have to pay the severance pay. The parameters though all are contaminated by this effect therefore:

\[(A.16) \text{ } A = (\beta/\alpha)^{-\alpha'}[(1-\theta+\delta \theta)^{-\alpha'}(1+\delta \theta)^{-\alpha'}] \]
\[+ \left( \frac{\beta}{a} \right) = \left[ (1-\Theta)+8 \Theta \right] - \left( 1+8 \Theta \right) = \gamma > 0, \]

\[A = -\beta \left( \frac{\beta}{a} \right) - \gamma \left[ (1-\Theta)+8 \Theta \right] - \left( 1+8 \Theta \right). \]

\[a'(-\beta/a) = \left[ (1-\Theta)+8 \Theta \right] - \left( 1+8 \Theta \right) = \gamma < 0 \]

\[\kappa_0 = \log(A), \quad \kappa = \log(\Gamma)+\log(B)+\left( 1+8 \Theta \right) \gamma = \gamma \left( 1-8 \Theta \right). \]

In matrix notation we have:

\[
\begin{bmatrix}
B'C+8\Theta/(1-\Theta+8\Theta) & \Gamma-1 & \mu_\Theta \\
-(A'/A)C & \Gamma & \gamma_\Theta
\end{bmatrix}
= \begin{bmatrix}
-a'C & \mu_{\kappa-1} & -k_\kappa \\
-(A'/A)C & 1-k_\kappa
\end{bmatrix}
\]

Where \( C = (1-\Theta)/(1-\Theta+8\Theta) \)

The determinant is:

\[D = B'IC + (\Gamma-1)(A'/A)C = A'/A + \frac{C}{a+b} + \frac{\gamma\Theta}{(1-\Theta+8\Theta)} > 0 \]

The reduced forms are easily obtained:

\[\mu_\Theta = \frac{-1}{D} \left[ (1-\Gamma)A'/A - a' \Gamma \right] C + \mu_{\kappa-1} \]

\[- D^{-1} \left[ \Gamma k_\kappa - (1-\Gamma)(1-k_\kappa) \right] \]

\[\gamma_\Theta = \frac{-1}{D} (A'/A) C \mu_{\kappa-1} - D^{-1} \left[ k_\kappa (A'/A) - \beta' (1-k_\kappa) \right] \]

As can be seen by the coefficient of A.17 of \( \mu_{\kappa-1} \) the chances are higher of \( \phi \) being less than 1 in absolute value since it has been multiplied by \( C \) a positive number less than one. The table included below shows a sample of parameters that assure the stability of wages and, therefore, of the whole system.
PARAMETERS OF PRODUCTION FUNCTION AND SEVERANCE PAY AND THE STABILITY OF WAGE EQUATION

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Note: Discounting factor 0.005.
References


Kiguel, M.A. and Liviatan, N. The Inflation Stabilization Cycles in Argentina and Brazil, in Bruno et.al. (1991)


Simonsen M.H. "Inercia Inflacionaria e Inflacao Inercial", Fundacao UNESP, processed, Sao Paulo (1987).