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PAULA KARINE RIBAS MAGALHÃES

IMF SENIORITY AS A COMPROMISE FOR AFFORDABLE DEBT

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Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para obtenção do título de Mestre em Economia de Empresas

Campo de Conhecimento:
Macroeconomia – Dívida Soberana

Orientador: Prof. Dr. Bernardo de Vasconcellos Guimarães

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Banca examinadora:

____
Prof. Dr. Bernardo de Vasconcellos Guimarães
(Orientador)
FGV-EESP

____
Profa. Dr. Luis Fernando de Oliveira Araújo
FGV-EESP

____
Prof. Dr. Marcos Yamada Nakaguma
FEA-USP

To my parents for their support in every way. To my husband for helping me in hard times. To my little nephews for putting a smile on my face.

“Anyone whose goal is ’something higher’ must expect someday to suffer vertigo. What is vertigo? Fear of falling? No, Vertigo is something other than fear of falling. It is the voice of the emptiness below us which tempts and lures us, it is the desire to fall, against which, terrified, we defend ourselves.”

Milan Kundera, The Unbearable Lightness of Being

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Nos últimos anos, também tive o privilégio de ser aceita em uma nova família, do meu marido Fernando. A constante afeição deles foi essencial. Mesmo que eu tenha que admitir qu quando eles me perguntavam aquela questão que nenhum economista jamais conseguirá escapar - 'qual o melhor investimento para mim agora?'- eu simplesmente falava algo sobre Tesouro Direto e seguia em frente. Me perdoem por isso, Rê, Lellis, Vó Jô, Bé e Bruno.

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Enquanto fiz cursinho para entrar na EESP no Insper, conheci uma pessoa maravilhosa chamada Sabrina - Sassá ou Vida. Nós dividimos nosso sentimento de não se encaixar com as pessoas do Insper, nosso desejo de terminar o mestrado e entrar no mercado financeiro, e nosso gosto pelas artes que não conseguimos entender. Ela é a melhor companhia no museu ou no bar. Vida, você é ótima.

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While studying to get into EESP at Insper, I have met a wonderful person named Sabrina - Sassá or Vida. We have shared our mutual feeling of not fitting in with the people at Insper, our desire to finish the masters and going into financial market and also our *posh* interest in the arts we cannot understand. She is the best company in either museums or bars. Vida, you're great.

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ABSTRACT

This paper addresses the role of the International Monetary Fund in the international monetary economy as a senior agent, as observed empirically. A sovereign agent subject to a stochastic shock must borrow to smooth out consumption. The international financial market offers fund, however it charges a premium for sharing the risk over the shock with the government. The IMF, however senior, lends at a lower rate. Hence, the sovereign government must choose its borrower. We find conditions under which the IMF presence in such market is relevant and positive to the borrowing agent. Such conditions will depend on the size of the risk premium charged, which in our analysis will be exogenously given.

Key-words: Sovereign Debt, Contingent Claim, Risk Premium, IMF, Default Risk, Debt, Seniority.

RESUMO

Este artigo trata do papel do Fundo Monetário Internacional como um agente sênior, fato observado empiricamente. Um agente soberano sujeito a um choque estocástico deve tomar emprestado para suavizar seu consumo. O mercado financeiro internacional oferece seus fundos, contudo cobra um prêmio por dividir o risco sobre o choque com o governo. O FMI, embora sênior, empresta a uma taxa menor. Encontramos as condições sob as quais a presença do FMI em tal mercado é relevante e positiva ao agente emprestador. Tais condições dependerão do tamanho do prêmio de risco cobrado, que em nossa análise é dado exogenamente.

Palavras-chaves: Dívida Soberana, Dívida Contingente, Prêmio de Risco, FMI, Risco de Default, Dívida, Senioridade.

List of Figures

List of Tables

Contents

1 The IMF as a senior agent: discussion and empirical evidence

1.1 Introduction

Default episodes on sovereign borrowing are fairly frequent in today's international monetary system and usually succeed economic distress by the indebted country. What we have observed in the historical evidence is that the IMF is rarely impaired by such default and at times it acts as counselor to the sovereign on how its finances should be run. It has become apparent that the IMF is always repaid, and that the international markets for sovereign borrowing have accepted the IMF as a senior agent.

Given this particularity, this paper presents a model where the IMF is a senior agent and it is always paid. Specifically, while the international investors are risk neutral and share risk of the stochastic shock with the borrowing country, the IMF is a senior agent that charges the risk-free rate. We show that the IMF's presence in such circumstances can actually induce increase in welfare to the sovereign government.

Our key assumption is that as long as the private investors are bearing risk, they will charge an interest associated to it for lending. The intuition behind this is that the market has coordinated and accepted that the IMF is a senior agent, however, since it shares the risk with the sovereign government, it will need to charge a higher risk to do so. An important insight is that if risk premium is large enough, the country can borrow from the international market, but choose to bear the risk alone.

We propose a model based on Grossman & Huyck (1988)'s model for sovereign debt, where a country borrows from the international market and the IMF to smooth out consumption by investing in a risk-free technology, and its proceedings are consumed. The sovereign is subject to stochastic shocks, where there could be a continuum of states of the world. Debt can be considered a contingent claim on the sovereign government. The IMF is presented as a senior agent, while the international lenders must have expectations on the sovereign's ability and desire to repay its debt. Punishment for default is the expulsion of the international financial market, leaving the sovereign unable to attain risk sharing.

IMF's function in the international monetary system has been a heated discussion, considering its policy implications. This paper contributes to this discussion as it starts from the empirical evidence of the seniority the IMF enjoys to show that such presence - and seniority - is positive to borrowing countries. This result is derived from the fact that IMF is a source of affordable proceedings which are always available, and it is specially useful when the risk premium is sufficiently high. We believe this sheds a light on the IMF's role in the financial monetary system, as its cheaper lending can help countries dig themselves out of crises.

This paper is organized as follows: Section 1.2 describes the evidence on IMF seniority, Section 1.3 discusses the literature, Section 2.1 presents our model, Section 2.2 presents our results and Section 2.3 concludes. Proofs are found on the appendices.

1.2 Evidence on IMF seniority

The International Monetary Fund (IMF) has provided financial help to countries in distress since its creation in 1944. Conceived through cooperation among multiple nations, it has been vigilant of the financial system ever since. Through its decades of agreements, the IMF has rarely faced default from a borrowing country. Twenty years ago, the IMF had seven members with delayed repayment (Afghanistan, Democratic Republic of Congo, Iraq, Liberia, Serbia, Somalia and Sudan)[1]. Along the years, only two members haven't repaid and only one has been added to the list - Zimbabwe. What it most striking about this is the fact that these countries represent only a small portion of the Fund's borrowers. Members have consistently repaid the IMF even in times of economic turmoil - that have also resulted on default to foreign lenders - often appealing to renegotiation. The end result is an unambiguous seniority held by the IMF.

Perhaps the most surprising evidence on it is Argentina's case. Following its recession and a default on its bonds December 2001, the country spend years renegotiating debt with its foreign creditors, eventually settling an agreement of 35 cents on the dollar[2]. However, Argentina has paid the IMF in its full amount in 2006, and before it was supposed to[3].

Over the years, the IMF has had few of its members on the Protracted Arrears list - its registry for nations who delayed over 6 months repayments. The most unusual case reported of arrears with the fund was of Cuba, which stopped repaying the Fund in 1959. When its Communist Revolution took place, Fidel Castro withdraw the country from the IMF, but eventually repaid all of its obligations with the Fund along the 60's[4].

In addition to Argentina and Cuba, other nations have discontinued payments temporarily, but have also normalized their situations with the Fund. Most notably, over the last 17 years, nations have cleared arrears with the IMF have gone through atypical situations such as civil wars, which would make difficult to keep economies afloat. Countries in this category are former Yugoslavia (in arrears from 1992 to 2000)[5], Democratic Republic of Congo (1990-2002)[6] and Liberia (1984-2008)[7].

Other noticeable examples are Afghanistan (1995-2003)[8], a nation under war for decades

1 IMF Protracted Arrears List
2 See Argentina Echange Offer of bonds published on the SEC
3 Evidence in IMF Report
4 (HORSEFIELD, 1969)
5 IMF Press Release on Yugoslavia, 2000
6 IMF Press Release on Congo, 2002
7 IMF Press Release on Liberia, 2008
8 IMF Press Release on Afghanistan, 2003

and Iraq (1990-2004)[9], which was able to restore relations with the IMF following the fall of Saddam Hussein - and enters a Post-Conflict Loan agreement that would try to bring about growth.

More recently, Zimbabwe has been the latest country to arrange payment of its obligations with the IMF. Zimbabwe had halted the remuneration in 2001, however while going through difficulties finding funding in the market and following long years of negotiation, it decided to settle its debts in October 2016. The amount outstanding was close to 107.9 million dollars[10].

Currently only two members remain in arrears with the IMF: Somalia and Sudan. Both countries have been in civil wars for over two decades, however, Somalia has been in talks with the IMF in order to enter another reform monitoring program in order to reconstruct a war struck economy[11].

Protracted Arrears as of March, 2017 (USD mn)				
Member	Total	Under 1 year	1-3 years	Over 3 years
Somalia	324.3	1.6	3.0	319.8
Sudan	1,320.3	2.8	5.8	1,311.7
Source: IMF				

Table 1 – IMF members still in debt with the Fund

The evidence above points to a case for the seniority of the IMF. What is most startling in countries with arrears with the IMF striving to strike other deals is that the Fund's conditions to do so is to restructure debt with other creditors - all the countries mentioned above (except Cuba which has withdraw from financial markets) were required to settle other financial disputes. This would add to the argument that the international financial market would accept the IMF as the only senior agent. In our model, we consider such evidence to pose an IMF as senior agent which also lends at a lower rate - which is reasonable, as the market bears risk of default.

1.3 Literature Review

This paper contributes to the discussion of IMF's role as a senior creditor in a framework where the sovereign's reputation as a credible borrower is what drives equilibrium. The literature on reputation relies on general equilibrium models where a benevolent sovereign borrows to increase its consumption. We apply the Grossman & Huyck (1988) model, which considers a market of risk neutral investors whose expectations are influenced by the sovereigns' reputation as a trustworthy borrower. In such model, sovereign debt as a contingent claim finance the government's investments, smooth out consumption and allow for risk shifting. The authors'

9 IMF Press Release on Iraq, 2004
10 Statement by the IMF on Zimbabwe, October 21, 2016
11 IMF Press Release on Somalia

main finding is that if the sovereign acts according to the lenders' expectation, excusable default is possible in bad states of the world, and does not result in expulsion from markets.

One simplifying assumption in our model is that the sovereign is not able to save, and whether this is important to sustain reputational equilibrium. Bulow & Rogoff (1989) state that reputational equilibrium is reliant on the sovereign's inability to save: its only possibility to secure itself from stochastic surprises is by leaning on foreign investors. However, Hellwig & Lorenzoni (2009) apply endogenous debt constrains and allow for sovereign savings, and still reach the conclusion that positive levels of debt will be possible in an equilibrium sustained by the sovereign's reputation. The authors believe that the interest rate at which borrowing occurs is a key variable in sustaining such equilibrium. In our model, although saving is not possible, the interest rate borrows is takes a central position.

An important feature of this paper is the role that the IMF plays as a senior lender, and a creditor whose interest charged is lower than other credit suppliers. Other works have cited the many functions the IMF can wield. Sachs (1995) builds his theory through examples in which IMF's role as a lender of last resort (LOLR) should address market failure such as some institutions do to private investment. Rochet & Vives (2004) set up a model of liquidity crisis where there's a probability that a liquid bank cannot find a lender in the market. Hence there would be a gap to be filled by an LOLR. The implication for the sovereign debt models is that an IFI could address such liquidity problem by a solvent country.

Others address a problem in lending to sovereign government which there's a perverse incentive to default as the IMF is an institution of rescue. Unlike markets for private citizens, lack of enforcement on country debt repayment generates moral hazard. Fafchamps (1996) argues that conditions demanded by the IMF work as a commitment mechanism and show that it backs repayment. In our model, repaying the IMF is important to maintain is reputation as a credible borrower. Following on IMF obligations has the same weight as complying with the market's expectations.

Another possible commitment a nation would want to make is to take policy measures which would provide growth in the long run. As such measures are costly, the IMF conditions are a method of guaranteeing that the sovereign stays on track. This is the conclusion Vreeland (2003) arrives at after studying specific cases in which IMF lending came along with harsh adjustment. This view is shared by Drazen (2002) in a political economy framework. The IMF would also fix information asymmetry as is shown in Marchesi & Thomas (1999), where the IMF acts as a signaling device that a government is willing to reform and hence would be a productive and trustworthy borrower, addressing Sach's concerts of moral hazard.

In a global games model, Corsetti, Guimaraes & Roubini (2006) and Morris & Shin (2006) develop frameworks in which catalytic finance is the underlying outcome of IMF borrowing to a country. Both papers stress that under some circumstances, entering an IMF agreement provide incentives for politically costly reforms with long term implications, which would be even economically costly if the government was facing liquidity difficulties. This signaling framed as reform would spur other creditors to rollover debt or to borrow to the sovereign.

As mentioned before, we highlight that the most relevant function the IMF has on international markets is of a senior creditor. The body of work of seniority on sovereign debt usually addresses informational issues. In a corporate finance literature Berkovitch & Kim (1990) compare effects of symmetric and asymmetric information and find that seniority may cause over-investment on the latter case. Detragiache (1994) highlights that sovereign debt is usually not prioritized, and equal sharing risk by creditors may cause over-investment. In our model, risk sharing is suited only to the risk neutral investors, while the IMF bares no risk on stochastic shocks. In a framework where national crises can lead to repudiating debt, Dooley (2000) argues that creditors and debtors have incentives to design and accept contracts that are costly to renegotiate in order to avoid strategic default. In our repeated game equilibrium, seniority does not directly play a role in avoiding repudiation, however a default on the IMF induces loss of credibility.

Saravia (2010) addresses the case where the IMF is a senior creditor in an investment model with three periods. He finds that the presence of a senior agent - the IMF - could be positive to the sovereign government because of the cheaper funds it can provide. He highlights the fact that a senior agent can provide the socially optimal solution - although it might need a commitment device to it - in coping with a liquidity shock. Our model abstracts from IMF to help the sovereign follow on its promises. We're also not concerned with socially optimal solutions - our analysis focuses on whether the IFI's presence increases welfare.

2.1 The Model

This paper extends the sovereign debt model from Grossman & Huyck (1988). We suppose a benevolent government willing to maximize national consumption. The analysis assumes that the sovereign has no endowment or wealth, so it must borrow to achieve its desired consumption. The sovereign is also subject to a stochastic distress. The amount borrowed will serve two purposes: (i) to invest in a technology which yields the risk free rate; (ii) to shift underlying stochastic risk to the lenders.

$$max_{\{c_t\}} \quad E_{c,t}U_t = u(c_t) + \sum_{i=1}^{\infty} E_{c,t}\Big[\beta^i u(c_{t+i})\Big] \tag{2.1}$$

$$\begin{aligned} for \quad & c_t = F(b_{t-1}) + z_t - s_t \\ subject\ to \quad & z_t \sim U(z_L,\, z_H), \quad E(z) = \bar{z} \\ and \quad & s_t \geq 0 \end{aligned} \tag{2.2}$$

Where $u(.)$ is a strictly increasing function, and $u''(.) < 0$ and $\lim_{c \to 0} u(c) = \infty$, and $0 < \beta < 1$. The concavity of the utility function indicates that the sovereign will desire consumption smoothing. Consumption is composed of the return from last period's borrowings, however subject to a stochastic component of income (z_t) and the debt service (s_t).

The supposition that the government cannot save to provide self financing in the future is simplifying, although should not change the results in our analysis. Even if the sovereign could accumulate wealth, it would also want to issue contingent debt claims to insure itself against adverse states of the world. Hence we resume the assumption that the sovereign cannot save whether from the amount borrowed or from its shocks.

We follow the simplifying assumption from Grossman & Huyck (1988) that the technology at the sovereign's disposal is risk free. This assumption separates the government's decision to invest from its exposure to the stochastic shock. Their argument is that this useful considering that this separation allows us to define the efficient investing independently from the attainable risk shifting. Hence, the function of financial return is given by:

$$F(b) = b^{\alpha}, \quad 0 < \alpha < 1 \tag{2.3}$$

The conditions under α assures that $F'(.) > 0$, $F''(.) < 0$ and $\lim_{b \to 0} F(b) = \infty$.

The sovereign can borrow in period t from the international markets and from an International Financial Institution which has seniority over other lenders and the contract formed allows repayment one period after, in $t + 1$, to allow for investments to mature. Hence, once the government takes a loan from either source, its possible actions are:

1. Repaying its loans in $t + 1$ and remain in the financing market, harboring the possibility of borrowing again or;
2. Repudiating its loans in $t+1$, to which the consequence is expulsion from financial markets forever.

The IFI aforementioned can lend at the risk free rate of $\rho > 0$. This assumption could be obtained if the IFI's decision was modeled. This is because we assume that this lender is senior, and it must be repaid. Since the IMF will not share the stochastic risk with the borrower, it will charge no risk premium:

$$R_t^F = (1 + \rho)b_t^F \tag{2.4}$$

Where R^F denotes the repayment function to the IFI.

On the other hand, the market for borrowing from foreign investors is competitive, and such lenders are risk neutral, however, they don't enjoy the same certainty of repayment as the IMF. Hence, the investors will have expectations under what the sovereign's actions will be:

$$E_{z,t}\Big\{E_{s,t}[S_{t+1}(z_{t+1})]\Big\} = (1 + \rho + x)b_t^M \tag{2.5}$$

In equation (5), $E_{s,t}[S_{t+1}(z_{t+1})]$ denotes the expectation over the service function to the market dependent on the stochastic component of income, z_{t+1}, and b_t^M denotes the amount borrowed from the market in period t. Note also that foreign lenders charge an interest higher than the IFI, denoted by $x \geq 0$. The intuition behind this is that foreign lenders accept the seniority of the IFI and share the risk over z_t with the sovereign, but charge a higher interest to do so. Also there is a higher risk associated with the fact that there's a senior lender in this market, since the borrower will be paying the IMF before it can repay the investors, there's a chance there will be no resources left before foreigner investors can get their proceedings. We suppose that x is exogenously given, and we discuss it in more detail below.

2.1.1 Discussion

The findings on the literature on investing in the presence of uncertainty or rationality boundaries support the argument that external factors are important in determining the risk premium. Goldstein & Pauzner (2004) analyse the case where investors betting on two countries which might run into a self-fulfilling crisis suffer a wealth loss when one of the sovereigns faces such crisis. A wealth drop would increase their strategic risk aversion, as they come across the uncertainty about actions taken by the individuals of the other country they bet on. In this scenario, a self-fulfilling crisis on country A would inflict investments withdrawal on country B. This will make risk premia of A and B positively correlated.

Guimaraes & Morris (2007) apply the global games model to a currency crisis framework, where strategic uncertainty is an important feature: when investors decide to attack the currency, they don't know whether they'll be successful. The authors find that under short-selling constraints, an exogenous increase in wealth - derived from investments on the lower risk currency - increases agents' appetite for risk (they assume relative risk aversion constant), which would make agents more willing to hold the riskier currency. This result is analogous to Goldstein and Pauzner's.

An additional argument by Mondria (2010) constructed on a model of rational expectations and rationally inattentive agents shows that two uncorrelated assets may present co-movement in prices. The underlying assumption is that agents have information processing constraints which causes them to choose to observe a linear combination of the assets payoffs as a private signal. Since the agents use the information received to choose optimal asset holdings, changes in one asset are reflected both assets. Hence, if investors observe a high signal, they will attribute such good news to both assets, which results in co-movement in its prices.

Uncertainty about political actions taken by sovereigns is another exogenous factor that would cause the risk premium to be volatile. Alesina & Tabellini (1989), Persson & Svensson (1989), and Ozler & Tabellini (1991) mention that if there's high polarization among a nation's political parties, and increasing political instability - captured by the risk of the sitting party losing office - capitalists (the agents with means to invest in domestic or foreign assets) reduce domestic investments. This would result in higher capital flight, and also add up to a fluctuating risk premium. The quantitative analysis in Cuadra & Sapriza (2008) relate to the Arellano (2008) model to construct a set up of political elements leading to short-sighted government, which increases default incentives. As the government cannot commit, this prompts higher interest rate spreads over any level of borrowing. Additionally, Pástor & Veronesi (2013) develop a general equilibrium model of government policy preference where stock prices react to political actions. They find that political uncertainty result in a risk premium, which is higher in frail economic circumstances.

Considering the evidence above, we find it reasonable to assume that external factors are significant to explain the volatility of the risk premium, which can fluctuate depending on the economic climate, agents' rationality bounds, or even risk preference by lenders. We address these issues by adding a x t o the rate charged by the foreign investors. We find that such variable summarizes in a simple form the exogenous factors that influence risk premia as it can float freely according to the market's spirits.

2.2 Results

2.2.1 Full commitment, full insurance

The sovereign is able to commit itself into paying $\tilde{R}_t^M(z_{t+1})$ to the foreign lenders, together with the already promised R^F to the IFI. In this case, from equation (5):

$$E_{s,t}[S_{t+1}^M(z_{t+1})] = \tilde{R}_t^M(z_{t+1}) \tag{2.6}$$

The equation above tells us that now the service function the sovereign has committed itself to will be a part of investors' expectations. We can substitute this into equation (5):

$$E_{z,t}[\tilde{R}_{t+1}^M(z_{t+1})] = (1+\rho+x)b_t^M \tag{2.7}$$

Hence, the government seeks to maximize (1) subject to (2) and (7). The sovereign can choose to borrow b_t^M from the market and b_t^F from the IFI. Such maximization will yield the following (proof in Appendix A):

$$\tilde{R}_t^M(z_t) = z_t - \bar{z} + (1+\rho+x)b_t^M \tag{2.8}$$

Of course, such servicing function will be followed if and only if:

$$\tilde{R}_t^M(z_t) \geq 0 \tag{2.9}$$

Which implies:

$$b_t^M \geq \frac{\bar{z} - z_L}{(1+\rho+x)} = \underline{b}^M \tag{2.10}$$

The lowest amount that the sovereign must borrow from the financial markets in order for the investors to be willing to provide full insurance is $\underline{b}^M$. This is because for the market to lend at full insurance, borrowing must be high enough so that the only case in which the lender receives nothing is if the shock is low. If borrowing weren't at such level, foreign investors would not desire to lend, as there would be times when zero payment would mean negative expected profit. Assuming that from the maximization (10) holds, consumption is given by:

$$\begin{aligned} \tilde{c}_t &= F(B_{t-1}) + z_t - R_{t-1}^F - \tilde{R}_{t-1}^M(z_t) \\ where: \; B_{t-1} &= b_{t-1}^M + b_{t-1}^F \end{aligned} \tag{2.11}$$

Substituting (4) and (8) into (11):

$$\tilde{c}_t = F(B_{t-1}) + \bar{z} - (1+\rho+x)b_{t-1}^M - (1+\rho)b_{t-1}^F \tag{2.12}$$

This consumption pattern is depicted by Figure 1.

Equation (12) tells us that, if the sovereign borrows from both the market and the IMF, consumption will be constant over time (as the amount borrowed is given in Appendix B). The amount borrowed B and the portion of it gathered from the market or the IMF will depend on x in the following way:

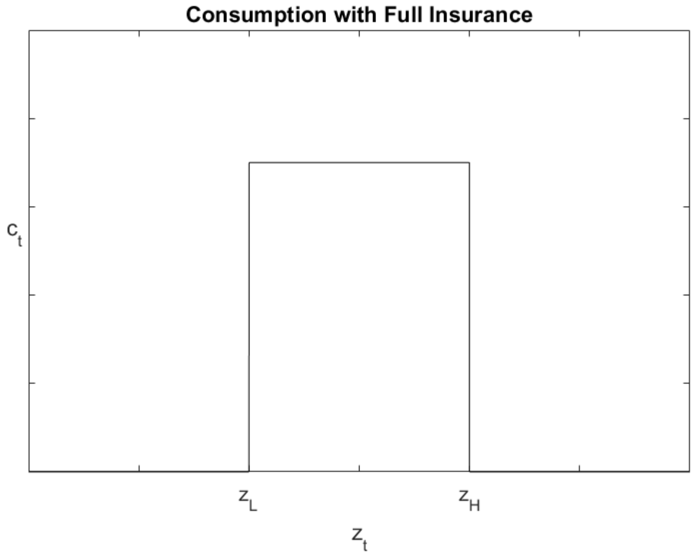

Figure 1 – Consumption under full commitment and full insurance

- If $x = 0$, equation (11) becomes $\tilde{c} = F(B) + \bar{z} - (1+\rho)B$. Let ϕ be the proportion of the debt desired the sovereign takes from the IMF. The sovereign government is indifferent between any $\phi \in [0, \underline{\phi}]$, where, $\underline{\phi}$ makes $(1-\underline{\phi})B = \underline{b}^M$. If the market is willing to lend without risk premium, it is in the sovereign's desire to borrow the desired amount from this source as it shares the risk over z_t. The presence of IFI is irrelevant in this case.

- If $x > 0$, the sovereign faces an opportunity cost as the foreign lenders offer full insurance at a cost. The sovereign government can maximize its consumption by posing $\phi = \underline{\phi}$, which means it borrows the lowest amount necessary from the market to still reap the benefits from full insurance and the rest of its needs is taken from the IFI, to take advantage of its interest. In this case, the IFI's presence is beneficial to the sovereign.

Note however that there will be a maximum x at which borrowing from the market is still an advantage. After such case, full insurance is too expensive to the sovereign, and it rather bear the risk over z_t. Such x prompts:

$$\begin{aligned} U(\tilde{c}_t) &\leq E_{c,t} U_t(c^F) \\ c^F &= F(B) + z_t - (1+\rho)B \end{aligned} \tag{2.13}$$

Figure 2 below shows the optimal ϕ as x changes.

2.2.2 Full commitment, partial insurance

We now turn to the case where equation (10) does not hold. Foreign investors won't be eager to offer full insurance to the government, as there would be shocks that would bring $\tilde{R}^M \leq 0$. This denotes that if investors provides full insurance when the amount lent is small, they would not be fully compensated for taking such risk, as there would be shocks greater than

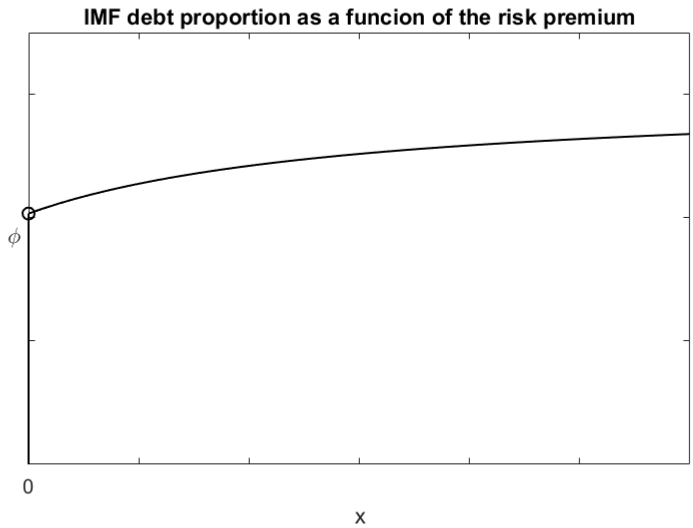

Figure 2 – Portion of debt taken from IMF according to the risk premium

z_L in which the sovereign does not remunerate. Risk neutrality condition (7) does not hold, and the market is not maximizing. Maximizing (1) subject to (2) and (7) will now bear (proofs are shown in Appendix C):

$$\tilde{R}^M_{t-1} = \begin{cases} 0, & z_t < \tilde{z} \\ z_t - \tilde{z}, & z_t \geq \tilde{z} \end{cases} \tag{2.14}$$
$$\tilde{z} = z_H - [2(z_H - z_L)(1 + \rho + x)b^M]^{\frac{1}{2}}$$

Where $\tilde{z}$ maintains the constraint of risk neutrality:

$$E_z[z_t - \tilde{z}] = (1 + \rho + x)b^M_{t-1} \tag{2.15}$$

Note that equation (14) poses situations where the sovereign government pays nothing to the market. When the shock is lower than $\tilde{z}$, there's excusable default. The intuition behind this is that the investors accept that there will be situations in which the sovereign won't be able to repay its debt, given that the shock is small enough. Should the sovereign choose to borrow from both possible creditors, its consumption will be:

$$\tilde{c}_t = \begin{cases} F(b^M_{t-1} + b^F_{t-1}) + z_t - (1 + \rho)b^F_{t-1}, & z_t < \tilde{z} \\ F(b^M_{t-1} + b^F_{t-1}) + \tilde{z} - (1 + \rho)b^F_{t-1}, & z_t \geq \tilde{z} \end{cases} \tag{2.16}$$

Which is portrayed by:

The choice of the optimal amount borrowed will be given by the first order conditions also showed on Appendix C:

$$b^F_{t+i-1}: \quad F'(b^F_{t+i-1} + b^M_{t+i-1}) = (1 + \rho) \tag{2.17}$$

$$b^M_{t+i-1}: \quad E_z[u'(c_{t+i})] = u'[c_{t+i}(\tilde{z})]\frac{(1 + \rho + x)}{(1 + \rho)} \tag{2.18}$$

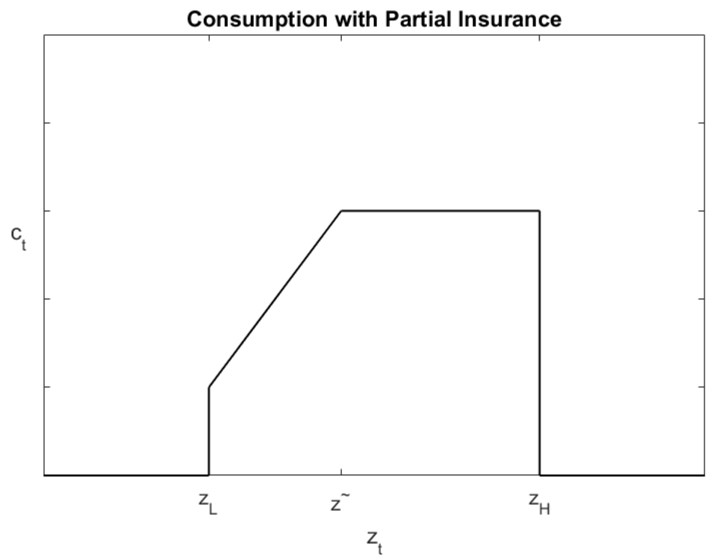

Figure 3 – Consumption under full commitment and partial insurance

First order condition (17) shows that the sovereign will be able to borrow the efficient amount according to its production function. Equation (18) suggests that two effects arise: first, a fluctuating x will cause a substitution effect, as borrowing from the risk sharing source becomes more expensive; also, a variation in the risk premium will affect the size of the insurance provided, namely, $\tilde{z}$. Note that equation (14) implies a negative $\frac{\partial \tilde{z}}{\partial x}$, however, a falling $\tilde{z}$ translates to increasing realizations of z_t that will be insured (as per Figure 3). We now pose the following:

Proposition 1: There exists $\underline{x}$ such that:

1. $\forall x \in [0, \underline{x}]$, $b^F = 0$
   For the sovereign government, it is better to borrow everything from the market and benefit from the partial insurance offered. Note that an increase in b^M will increase insurance (by its effect on $\tilde{z}$);

2. $\forall x > \underline{x}$, $b^F > 0$
   Now, the dominating effect is that increasing the risk premium increases payments and therefore decreases consumption. As the sovereign is taking more loans from IMF, insured interval decreases.

3. *For a $\bar{x}$ large enough, $b^M = 0$. The insurance is too expensive. In this case, having an insurance requires paying too much in return. The foreign insurer shows that it is only willing to participate in such contract if its compensation is enough to account for the risk which, in this occasion, is too high in their point of view. The sovereign finds it better to borrow solely from the IMF, as there's no risk premium charged.*

Proof of the above found on Appendix D.

The intuition behind Proposition 1 is that for a small enough interval of the risk premium the price of the insurance is low, and the advantage of taking on the full amount desired from the foreign investors is that the size of the insurance increases with the amount borrowed. Hence, borrowing everything from the market makes up for higher insurance than would otherwise happen had the sovereign shared its proceedings from the foreign investors and the IMF. However, as the price of the insurance increases the IMF now poses a benefit as it lends at a smaller interest. In such situation, the opportunity cost is: borrowing from the market provides insurance - albeit an expensive one - or taking from the IMF at lower cost, without insurance. Such opportunity cost will cause the sovereign to borrow from both sources. However, at a level of risk premium $\bar{x}$, the insurance is too costly. Taking resources from the foreign investors, even with the insurance, results in a lower consumption than borrowing everything from the IMF.

2.2.3 Reputational Equilibrium

We now suppose the sovereign is not able to commit to a repayment function. The sovereign's problem will be to maximize (1) subject to (2), (4) and (5). However, the expectation of the service function to the market will depend on the central government's reputation:

$$E_{s,t}[S_{t+1}(z_{t+1})] = \begin{cases} R^{M*}(z_{t+1}), \quad t = -1 \\ R^{M*}(z_{t+1}), \ \forall t \geq 0 \quad if\ s^M_{t-j} = R^{M*}(z_{t-j})\ \forall j = 1, ..., k \\ 0, \quad otherwise \end{cases} \tag{2.19}$$

$$R^{M*}(z_{t+1}) \geq 0 \tag{2.20}$$

Where $R^{M*}(z_{t+1})$ is the utility maximizing repayment function, which the market is able to observe. Since z_t is stationary and the analysis abstracts from the sovereign's wealth changes between periods, equation (20) is constant in time. Equation (19) tells us that the sovereign begins period zero with a solid reputation, which means that investors expect it to repay the maximizing function from the start. From then on, its credibility is built according to its actions. If it fulfills the investors' expectations in period 1 and remains doing so throughout history, creditors will continue to believe its intention to remunerate debt. However, if the sovereign ever breaches the market's expectations, it will no longer count on a reputation as investors will anticipate default at all times in the future and henceforth there will be no lending to this nation. Note that equation (20) also allows for excusable default.

The inability to commit will lead to a game theoretic equilibrium in which the nation's strategy is to either fulfill the investor's expectations and continue to be a part of the contingent debt markets or default, keeping the proceedings from lenders, however being excluded from further borrowing.

Another condition reputational equilibrium must meet is the incentive compatibility condition. That is, repaying the creditors and accessing the market for debt must be better to the government than repudiating debt. Since the utility function discounts the future, if the government were to deny following repayment function it would occur in the first period. We must study one stage deviation in the sense that:

$$U^R \geq U^0 \quad \forall z_t \in [z_L,\, z_H],\; \forall t = 0, ... \tag{2.21}$$

$$U^R = u[F(B^*) + z_0 - R^{M*}(z_0) - R^F] + \sum_{i=1}^{\infty} E_{z,t}\Big\{\beta^i u[F(B^*) + z_i - R^{M*}(z_i) - R^F]\Big\} \tag{2.22}$$

$$U^0 = u[F(B^*) + z_0] + \sum_{i=1}^{\infty} E_{z,t}\Big\{\beta^i u[z_i]\Big\} \tag{2.23}$$

Notice that U^R is the utility of repaying the market and the IMF in period zero and continuing to comply with expectations - which means having access to the debt market forever. However, U^0 is the utility of defaulting in period zero and from then on only being able to consume the stochastic shock. Also, $B*$ is the optimal amount borrowed, which the sovereign can choose to take amounts from the foreign investors and the IMF. Equation (21) poses that continuing in the market servicing debt must bear an advantage to simply defaulting on the first try.

Summing up, the reputational equilibrium will solve:

$$max_{\{b^{M*}, b^{F*}, R^{M*}(z_t)\}} \quad U^R \tag{2.24}$$

$$\begin{aligned} subject\ to \quad R^F &= (1+\rho)b^{F*} \\ U^R &\geq U^0 \\ E_{z,t}[R^{*M}(z_{t+1})] &= (1+\rho+x)b^{M*} \end{aligned} \tag{2.25}$$

Remember from the full commitment cases that $\tilde{R}^M(z_t)$ was the utility maximizing repayment function. When the sovereign is able to commit to the repayment function, it maximizes utility subject to the fact that it must follow such repayment. If we allow for the incentive compatibility equation to not be binding, and if it is the case that:

$$U^{\tilde{R}} > U^0 \tag{2.26}$$

We can accordingly state that the reputational equilibrium can be secured when the sovereign acts toward delivering the full commitment repayment function - either the full or partial insurance cases - even when it can give no guarantee that it will do so (where $U^{\tilde{R}}$ is the utility of repayment using the full commitment function). Since we're attempting to find a repayment function that sustains reputational equilibrium, we know that it'll be the case that if (26) is valid, we'll be able to uphold such equilibrium.

Under some conditions, it'll be the case that (26) is supported. For the full insurance case:

$$\left[\frac{\alpha}{(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}\left(1-\frac{1}{\alpha^{\alpha}}\right) > \frac{z_H - z_L}{2}\left[1-\frac{(1+\rho)}{(1+\rho+x)}\right] \tag{2.27}$$

Note that (27) can be easily achieved if we have $x \to 0$. For the partial insurance case, we must have:

$$\begin{aligned}
&\frac{\beta}{1-\beta}\left[\frac{z_H-\tilde{z}}{z_H-z_L}u[F(b^F+b^M)+\tilde{z}-(1+\rho)b^F]-u(\bar{z})\right] \\
&> u[F(b^F+b^M)+z_0]-u[F(b^F+b^M)+z_0-(1+\rho)b^F-\tilde{R}^M(z_0)] \\
&-\sum_{i=1}^{\infty}\beta^i\left\{\frac{\tilde{z}-z_L}{z_H-z_L}\left[u[F(b^F+b^M)+z_i-(1+\rho)b^F]\right]\right\}
\end{aligned} \tag{2.28}$$

It is easy to observe that (28) will definitely hold if $\beta \to 1$. In Appendix E we show how we got to (27) and (28).

2.2.4 Welfare Discussion

We now address the issue whether the presence of the IMF is positive for the borrower country. In order to do so, we compare the sovereign's expected utility under two situations:

1. The scenario when the international market is the only lender available and it charges a risk premium to share the stochastic shock risk (we'll call this case's expected utility U^{MKT});
2. When there's the IMF lending available, a senior lender charging the risk free rate (U^{IMF})

We do not address the case where private lenders charge only the risk free rate and shares risk, as it is straightforward that the IMF's presence is irrelevant in such situation. We are interested in situations in which:

$$U^{IMF} > U^{MKT} \tag{2.29}$$

That is, instances where IMF presence in the market is welfare improving to the sovereign government.

An important difference between both scenarios is the efficiency conditions for borrowing. When the sovereign government can only borrow from financial markets, its efficiency scenario for when the market is willing to provide full insurance poses (as shown in Appendix F):

$$F'(\bar{B}^{MKT}) = (1+\rho+x) \tag{2.30}$$

$$F'(\underline{B}^{MKT}) = (1+\rho+x)\frac{u'[c(\tilde{z})]}{E_z[u'(c)]} \tag{2.31}$$

Equation (30) shows the scenario where the parameters allows for full insurance (as shown in Appendix F) and $\bar{B}^{MKT}$ is the amount borrowed, while equation (31) shows the case for partial insurance and its amount borrowed $\underline{B}^{MKT}$. However, as per equation (17), the efficiency equation when IMF is present is given by:

$$F'(B^{IMF}) = (1 + \rho) \tag{2.32}$$

As the investment function is concave, we know that marginal productivity is decreasing. Hence:

$$B^{IMF} \geq \bar{B}^{MKT} > \underline{B}^{MKT} \tag{2.33}$$

The intuition behind (33) is the following: when the only lender available is the private market, this source charges a risk premium. Hence when we add the IMF to this the market, it is possible to borrow the same amount as before at a lower rate. However, the IMF is a senior agent. There's both an income effect (which allows the sovereign to borrow the same amount at a lower rate) and a substitution effect (in which the sovereign must substitute insured debt from uninsured) at play here. The end result is that the sovereign can borrow more - depending on the size of the risk premium as seen in the previous sections. Also, we know that $\bar{B}^{MKT} > \underline{B}^{MKT}$ because the full insurance contract requires higher debt.

From the above comparison, it's clear that the IMF's presence - whether it provides full insurance or not - will be unambiguously beneficial when the parameters only allow for the market to provide partial insurance, if $x > 0$. When IMF offers the full insurance, it is clear that (29) will be valid. The case when both markets offer partial insurance, (29) is a straightforward consequence of the fact that IMF's borrowing is higher.

Hence, there are three other cases to be analysed:

1. Both scenarios allow for full insurance;
2. Both scenarios allow for partial insurance;
3. Only the IMF scenario allows for full insurance.

This discussion is followed below:

2.2.4.1 Both Scenarios with Full Insurance

In the event that the sovereign government is able to borrow while securing full insurance from the market, we'll have:

$$B^{IMF} \geq \bar{B}^{MKT} \geq \underline{b}^{M} \tag{2.34}$$

$$\Rightarrow u(c^{IMF}) \geq u(c^{MKT}), \quad \forall t \tag{2.35}$$

Moreover, we have binding (34) and (35) if the markets decide to pose $x = 0$. Hence, in this instance, the IMF's presence is irrelevant to the sovereign, which strengthens our discussion in section 5.1. Since the foreign investors are willing to provide full insurance and charge no risk premium while doing it, the IMF's advantage as a cheaper credit lender disappears. However, if the foreigners decide to pose $x > 0$, there will be an advantage of enjoying cheaper credit from the Fund, while still borrowing the minimal amount necessary to guarantee full insurance with the risk neutral market.

2.2.4.2 Both Scenarios with Partial Insurance

Suppose that we have:

$$\underline{b}^M > B^{IMF} \tag{2.36}$$

$$\frac{\bar{z} - z_L}{(1 + \rho + x)} > \left[\frac{(1 + \rho + x)}{\alpha}\right]^{\frac{1}{\alpha - 1}} \tag{2.37}$$

$$\rightarrow \alpha^{\frac{1}{\alpha}} (\bar{z} - z_L)^{\frac{\alpha - 1}{\alpha}} > (1 + \rho + x) \tag{2.38}$$

As per (33), (36) implies that there will be partial insurance on the case without the IMF as well.

Our discussion now must return to Proposition 1. The benefit derived from the presence of the IMF will be dependent on the size of the risk premium. We showed that if $x \in [0, \underline{x}]$, the sovereign government will find it better to borrow zero from the IMF. Being subject to the conditions of the market means that IMF's presence is not welfare improving, as insurance increases with the amount borrowed from the foreign investors. Hence, we will not have (29).

On the other hand, since we show on Appendix D that the amount of borrowing with the IMF is increasing with the risk premium, there will be an interval in which the government decides to borrow both with market and IMF, $x \in (\underline{x}, \bar{x}]$. Now the sovereign enjoys the possibility of borrowing at a cheaper rate, while still enjoying partial insurance offered by the market. Since the insurance increases with the amount borrowed, it might be the case that insurance is higher when there's no Fund to borrow from. However, Proposition 1 implies that the government finds it better to borrow more at the cost of lowering insurance than borrowing everything from the more expensive source. As a consequence of Proposition 1, we have that the IMF's presence is better, $U^{IMF} > U^{MKT}$.

The more straightforward implication from Proposition one is when the insurance costs $x > \bar{x}$. We show on Appendix D that in this scenario, the sovereign government will find it optimal to borrow everything from the Fund, finding insurance too expensive. Considering this, it is clear that the IMF is welfare improving, since in his absence the sovereign will have to purchase the expensive insurance. Ergo, (29) is true.

2.2.4.3 Only the IMF allows for Full Insurance

The intermediary event occurs when:

$$B^{IMF} \geq \underline{b}^M > \bar{B}^{MKT} > \underline{B}^{MKT} \tag{2.39}$$

Note that, for this to be true, we must have $x > 0$. If not, we'd have $B^{IMF} = \bar{B}^{MKT}$ and we'd be back to case 5.4.1. With a strictly positive risk premium in mind and condition (39) above, only when the IMF is participating in the market will the sovereign government be able to forge a full insurance contract. In this case, the utility comparison must consider that IMF lending will require seniority:

$$c^{IMF} = F(B^{IMF}) + \bar{z} - (1+\rho)b^F - (1+\rho+x)b^M \tag{2.40}$$

On the other hand, there will be uncertainty when there's no IMF:

$$c_t^{MKT} = \begin{cases} F(B^{MKT}) + z_t, & \text{if} \quad z_t < \tilde{z} \\ F(B^{MKT}) + \tilde{z}, & \text{if} \quad z_t \geq \tilde{z} \end{cases} \tag{2.41}$$

Note that we must have:

$$\tilde{z} = z_H - [2(z_H - z_L)(1+\rho+x)B^{MKT}]^{\frac{1}{2}} \tag{2.42}$$

Hence, so that (29) is valid, we must have:

$$\frac{u(c^{IMF})}{1-\beta} > u(c_t^{MKT}) + \sum_{i=1}^{\infty} \beta^i E_{z,t}[u(c_{t+i}^{MKT})] \tag{2.43}$$

Suppose we have $\beta \to 1$. To arrive at $u(c^{IMF}) > u(c_t^{MKT})$, it must be the case that:

$$c^{IMF} > c_t^{MKT} \quad \text{for a given} \quad z_t \in [z_L, z_H] \tag{2.44}$$

So that we can use the fact that utility is increasing. Suppose that we have a shock smaller than $\tilde{z}$. Equation (44) requires that:

$$F(B^{IMF}) - F(B^{MKT}) > z_t - \bar{z} + (1+\rho)B^{IMF} + xb^M \tag{2.45}$$

Note that the left-hand side is positive. A similar condition will be needed if the shock is above $\tilde{z}$:

$$F(B^{IMF}) - F(B^{MKT}) > \tilde{z} - \bar{z} + (1+\rho)B^{IMF} + xb^M \tag{2.46}$$

Note however that an increase in x will cause the left-hand side to increase (per (31), there will be a decrease in B^{MKT}). On the right-hand side, there will be a decrease in b^M, which will be offset by the increase in x on the last term of the equation. However, (45) and (46) will have different implications as in (46), an increasing x forges a decrease in $\tilde{z}$ (as shown before). It'll be easier to achieve to achieve (45), as the sovereign will only borrow the minimum amount possible to get the full insurance. This effect will be intensified in (46). The decrease in $\tilde{z}$ means that the insurance is closer to complete that it was before. However, since the same effect that led to an increase in insurance causes the amount borrowed to decrease, the effect will be offset.

We now analyse the case when we have $0 > \beta > 1$. We use the Jensen inequality for a concave function.

$$\frac{u(c^{IMF})}{1-\beta} > u(c_t^{MKT}) + \sum_{i=1}^{\infty} \beta^i u[E_{z,t}(c_{t+i}^{MKT})] \tag{2.47}$$

$$\geq u(c_t^{MKT}) + \sum_{i=1}^{\infty} \beta^i E_{z,t}[u(c_{t+i}^{MKT})] \tag{2.48}$$

Note that:

$$E_{z,t}(c_{t+i}^{MKT}) = p(z_{t+i} < \tilde{z})[F(B^{MKT}) + z_{t+i}] + p(z_{t+i} \geq \tilde{z})[F(B^{MKT}) + \tilde{z}] \tag{2.49}$$

However, if it is the case that (46) is valid, we know that:

$$u(c^{IMF}) > E_{z,t}(c_{t+i}^{MKT}) \tag{2.50}$$

Which means that using the fact that utility is strictly increasing, we'll have (47). In such scenario, we can claim that the IMF's presence is welfare improving.

2.3 Conclusion

We have presented a model in which the International Monetary Fund is a senior agent and hence is able to offer cheaper credit to a borrowing benevolent government. IMF seniority is a consequence of empirical observation that the IMF is always paid by its member countries. Even nations in abnormal situations such as civil wars or at economic turmoil that poses risk of default have paid the IMF in full, considering also nations that have delayed payment for a certain amount of time.

In such model, the private market of contingent debt offers wither full or partial insurance however charges a higher lending rate exogenously given to do so. The sovereign government's debt is used for investing as well as repayment, in an effort to smooth out consumption since the government is hit by a stochastic shock. The sovereign must choose the amount borrowed and its creditor, considering both the seniority of the IMF and the insurance offered by the foreign market. We have showed when it would be optimum to make a linear combination of the Fund's and the market's proceedings, as well as the implications for the insurance offered. When the government is able to borrow at full insurance, for a low enough risk premium the Fund is irrelevant as its cheaper credit - and seniority - comes at the cost of full insurance.

Similarly, when partial insurance is at stake we established that if the risk premium is low enough (below a $\underline{x}$) it will be optimum for the sovereign to borrow only with the private agents and hence enjoy its insurance. However, as this safety's cost increases, the IMF's cheaper option becomes more and more inviting, and we show that IMF borrowing is increasing with risk premium.

We have also discussed the welfare implications from this model. It surfaces that the Fund's presence will be welfare improving also depending on the size of the risk premium. Again, for a large enough risk premium it is better for the sovereign government to borrow from the IMF. If the insurance's cost doesn't reach that threshold, IMF's offering funds will be irrelevant (as the sovereign can always choose to deal only with the foreign debt market).

Our work disregards the possibility of risk averse foreign lenders. In this regard, a risk premium would follow as a consequence of risk aversion as the market will share the risk of the stochastic shock. Although we leave this for future research, we believe our paper has helped understand that when a senior agent such as the IMF is allowed to participate in debt markets there will be a possibility of welfare improvement to the sovereign government.

2.4 Appendices

Appendix A

Since private investors are risk neutral, it must be the case that:

$$E_{z,t}[R_{t-1}(z_t)] = (1+\rho+x)b^M_{t-1}$$

Therefore, we search for a $\tilde{z}$ which would make:

$$\int_{\tilde{z}}^{z_H} (z_t - \tilde{z})[f(z_t)]dz_t = (1+\rho+x)b^M_{t-1}$$

Which is:

$$\int_{\tilde{z}}^{z_H} \frac{(z_t - \tilde{z})}{z_H - z_L} dz_t = (1+\rho+x)b^M_{t-1}$$

Ergo, $\tilde{z}$ will solve:

$$\frac{1}{z_H - z_L}\Big[\frac{z_H^2 + \tilde{z}^2}{2} - \tilde{z}z_H\Big] = (1+\rho+x)b^M_{t-1}$$

By verefying the solution on MatLab, we find:

$$\tilde{z} = z_H - \sqrt{2}[b^M_{t-1}(1+\rho+x)(z_H - z_L)]^{\frac{1}{2}}$$

Note that by construction, $\tilde{z} \in [z_H \ z_L]$. Therefore, the repayment function the government will be able to commit itself to is:

$$\tilde{R}^M_{t-1}(z_t) = \begin{cases} 0, \text{ if } z_t < \tilde{z} \\ z_t - \tilde{z}, \text{ if } z_t \geq \tilde{z} \end{cases} \tag{PI}$$

However, suppose that the amount desired to borrow from the market is such that:

$$\tilde{b}^M_{t-1} = \frac{\bar{z} - z_L}{(1+\rho+x)}, \ \bar{z} \geq z_L$$

Substituting it into $\tilde{z}$:

$$\tilde{z} = z_H - \sqrt{2}[(z_H - z_L)(1+\rho+x)\tilde{b}^M_{t-1}]^{\frac{1}{2}}$$
$$= z_L$$

And so:

$$\tilde{R}^M_{t-1}(z_t) = \begin{cases} 0, \text{ if } z_t < z_L \\ z_t - z_L, \text{ if } z_t \geq z_L \end{cases}$$

However:

$$z_L = \bar{z} - (1+\rho+x)\frac{\bar{z} - z_L}{(1+\rho+x)}$$
$$= \bar{z} - (1+\rho+x)\tilde{b}^M_{t-1}$$

Following:

$$\tilde{R}^M_{t-1}(z_t) = z_t - \bar{z} + (1+\rho+x)\tilde{b}^M_{t-1}, \text{ for } z_t \in [z_L,\ z_H] \quad \text{(FI)}$$

And then we'll have full insurance as the consumption will be constant. The market will bear the full risk over z_t. If the amount desired to borrow is higher than $\tilde{b}^M_{t-1}$, there's no reason for the investors to give constant consumption at a level bigger than $\bar{z}$, as the government consumption is already smoothed out. In fact, the amount repayment function used will continue to be (FI).

Appendix B

Proof of full commitment full insurance case: If the market is willing to offer full insurance to the sovereign government, it means that throughout the maximization the government will count on (FI) in its consumption. Its problem is:

$$EU_{c,t} = u(c_t) + \sum_{i=1}^{\infty} E_{c,t}[\beta^i u(c_{t+i})]$$
$$c_t = F(b^F_{t-1} + b^M_{t-1}) - (1+\rho)b^F_{t-1} + \bar{z} - (1+\rho+x)b^M_{t-1}$$

We'll follow the maximization considering that first order conditions require that partial derivatives of $EU_{c,t}$ with respect to the choice variables are equal to zero. From $u()$ and $u'()$ continuous functions and $\bar{z}$ continuous, we have:

$$\frac{\partial EU}{\partial b^M} = E\left[\frac{\partial U}{\partial b^M}\right]$$
$$\frac{\partial EU}{\partial b^F} = E\left[\frac{\partial U}{\partial b^F}\right]$$

First order conditions for the choice variables are:

$$b^F_{t+i-1}: \quad F'(b^F_{t+i-1} + b^M_{t+i-1}) = (1+\rho)$$
$$b^M_{t+i-1}: \quad F'(b^F_{t+i-1} + b^M_{t+i-1}) = (1+\rho+x)$$

Since we can't have both equations binding at once, the sovereign will choose the efficiency condition to be binding, which is the FOC on b^F_{t+i-1}. We'll have:

$$F'(B_{t+i-1}) = (1+\rho)$$
$$B_{t+i-1} = b^F_{t+i-1} + b^M_{t+i-1}$$

The choice on allocation of debt between lenders is discussed in the Results section.

Appendix C

Proof of full commitment partial insurance case: Suppose that the amount desired by the government to borrow from the market is such that $b^M_{t-1} < \underline{b}^M$. And the government's repayment function is given by:

$$\tilde{R}^M_{t-1}(z_t) = \begin{cases} 0, \text{ if } z_t < \tilde{z} \\ z_t - \tilde{z}, \text{ if } z_t \geq \tilde{z} \end{cases}$$

The government's problem is solved by maximizing:

$$EU_{c,t} = u(c_t) + \sum_{i=1}^{\infty} E_{c,t}[\beta^i u(c_{t+i})]$$
$$c_t = F(b^M_{t-1} + b^M_{t-1}) - (1+\rho)b^F_{t-1} - \tilde{R}^M_{t-1}(z_t)$$

Where the above equation for c_t already considers the market repayment function. The choice variables are b^M_{t+i-1} and b^M_{t+i-1} as we consider that in period t choices from $t-1$ are given. First order conditions require that partial derivatives of $EU_{c,t}$ with respect to the choice variables are equal to zero. From $u()$ and $u'()$ continuous functions and $\tilde{R}^M$ continuous, we have:

$$\frac{\partial EU}{\partial b^M} = E\left[\frac{\partial U}{\partial b^M}\right]$$
$$\frac{\partial EU}{\partial b^F} = E\left[\frac{\partial U}{\partial b^F}\right]$$

And so we take the first order conditions:

$$b^F_{t+i-1}: \quad u'(c_{t+i})[F'(b^F_{t+i-1} + b^M_{t+i-1}) - (1+\rho)] = 0 \qquad \text{(i)}$$

$$b^M_{t+i-1}: \quad \begin{cases} u'(c_{t+i})[F'(b^F_{t+i-1} + b^M_{t+i-1}) - 0], \text{ if } z_{t+i} < \tilde{z} \\ u'(c_{t+i})\Bigg\{ F'(b^F_{t+i-1} + b^M_{t+i-1}) \\ \qquad -\sqrt{2}[(z_H - z_L)(1+\rho+x)]^{\frac{1}{2}} \left(\frac{b^M_{t+i-1}}{2}\right)^{-\frac{1}{2}} \Bigg\}, \text{ if } z_{t+i} \geq \tilde{z} \end{cases} = 0 \qquad \text{(ii)}$$

By taking expectations from equations (i) and (ii) above, we get:

$$b^F_{t+i-1}: \quad F'(b^F_{t+i-1} + b^M_{t+i-1}) = (1+\rho) \tag{i}$$

$$b^M_{t+i-1}: \quad \begin{aligned} &E_z[u'(c_{t+i})][F'(b^F_{t+i-1} + b^M_{t+i-1})] = \\ &p(z_{t+i} \geq \tilde{z}) \left[u'[c_{t+i}(\tilde{z})] \sqrt{2}[(z_H - z_L)(1+\rho+x)]^{\frac{1}{2}} \left(\frac{b^M_{t+i-1}}{2} \right)^{-\frac{1}{2}} \right] \end{aligned} \tag{ii}$$

In (ii), we use the fact that because of the insurance, in the interval $[\tilde{z},\ z_H]$ the consumption is constant, and so must be its marginal utility.

We work on equation (ii) now. By using equation (i) and the fact that:

$$p(z_{t+i} \geq \tilde{z}) = \frac{z_H - \tilde{z}}{z_H - z_L} = \frac{\sqrt{2}[(z_H - z_L)(1+\rho+x)b^M_{t+i-1}]^{\frac{1}{2}}}{z_H - z_L}$$

We have:

$$b^M_{t+i-1}: \quad E_z[u'(c_{t+i})](1+\rho) = u'[c_{t+i}(\tilde{z})](1+\rho+x) \tag{ii}$$

Appendix D

Proof of Proposition 1: We know that if $x = 0$, the sovereign is only willing to borrow from the private investors. Since the foreign markets share the risk of z_t, it is beneficial to the sovereign to take its loans from them as the IMF charges the same interest but bears no risk. This means that if $x = 0 \rightarrow b^F = 0$. Hence, Proposition 1's proof follows:

Suppose that $x \in [0,\ \infty)$.
Suppose, as a way of contradiction, that $\frac{\partial b^F}{\partial x} \leq 0$. From the first order conditions in Appendix B:

$$b^F_{t+i-1}: \quad F'(b^F_{t+i-1} + b^M_{t+i-1}) = (1+\rho) \tag{i}$$

$$b^M_{t+i-1}: \quad E_z[u'(c_{t+i})] = u'[c_{t+i}(\tilde{z})]\frac{(1+\rho+x)}{(1+\rho)} \tag{ii}$$

From (i) and the production function:

$$b^F + b^M = B = \left[\frac{(1+\rho)}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

Hence:

$$\frac{\partial B}{\partial x} = 0 \Leftrightarrow \frac{\partial b^M}{\partial x} \geq 0$$

Where the latter conclusion derives from our assumption about b^F. From (ii):

$$E\{u'[c_{t+i}(z_{t+i})]\} - u'[c_{t+i}(\tilde{z})]\frac{(1+\rho+x)}{(1+\rho)} \tag{iii}$$

Differentiate equation (iii) with respect to x:

$$E\Big\{\frac{\partial u'[c_{t+i}(z_{t+i})]}{\partial x}\Big\} - \frac{\partial u'[c_{t+i}(\tilde{z})]}{\partial x}\frac{(1+\rho+x)}{(1+\rho)} - u'[c_{t+i}(\tilde{z})]\frac{\partial(1+\rho+x)/(1+\rho)}{\partial x} = 0 \quad \text{(iv)}$$

Where:

$$\frac{\partial u'[c_{t+i}(z_{t+i})]}{\partial x} = \begin{cases} u''[c_{t+i}(z_{t+i})]\Big[-(1+\rho)\frac{\partial b^F}{\partial x}\Big],\ z_{t+i} < \tilde{z} \\ u''[c_{t+i}(z_{t+i})]\Big[-(1+\rho)\frac{\partial b^F}{\partial x} - \sqrt{2}[(z_H - z_L)b^M]^{\frac{1}{2}}(1+\rho+x)^{-\frac{1}{2}} \\ -\sqrt{2}[(z_H - z_L)(1+\rho+x)]^{\frac{1}{2}}(b^M)^{-\frac{1}{2}}\frac{\partial b^M}{\partial x}\Big],\ z_{t+i} \geq \tilde{z} \end{cases} \quad \text{(v)}$$

From (iv):

$$E\Big\{\frac{\partial u'[c_{t+i}(z_{t+i})]}{\partial x}\Big\} - \frac{\partial u'[c_{t+i}(\tilde{z})]}{\partial x}\frac{(1+\rho+x)}{(1+\rho)} = \frac{u'[c_{t+i}(\tilde{z})]}{(1+\rho)} > 0$$

Which means:

$$E\Big\{\frac{\partial u'[c_{t+i}(z_{t+i})]}{\partial x}\Big\} - \frac{\partial u'[c_{t+i}(\tilde{z})]}{\partial x}\frac{(1+\rho+x)}{(1+\rho)} > 0 \quad \text{(vi)}$$

Since our initial assumption is that $\frac{\partial b^F}{\partial x} \leq 0$, we separate it in two cases:

Case 1

Suppose:

$$\frac{\partial b^F}{\partial x} = 0$$

This means:

$$\frac{\partial b^M}{\partial x} = 0$$

From equation (v):

$$\frac{\partial u'[c_{t+i}(z_{t+i})]}{\partial x} = \begin{cases} 0,\ z_{t+i} < \tilde{z} \\ u''[c_{t+i}(z_{t+i})]\Big[-\sqrt{2}[(z_H - z_L)b^M]^{\frac{1}{2}}(1+\rho+x)^{-\frac{1}{2}}\Big],\ z_{t+i} \geq \tilde{z} \end{cases} \tag{2.51}$$

Substituting the above on equation (vi):

$$\begin{aligned} p(z_{t+i} \geq \tilde{z})\Big\{u''[c_{t+i}(z_{t+i})]\Big[\sqrt{2}[(z_H - z_L)b^M]^{\frac{1}{2}}(1+\rho+x)^{-\frac{1}{2}}\Big]\Big\} \\ -\frac{(1+\rho+x)}{(1+\rho)}\Big\{u''[c_{t+i}(\tilde{z})]\Big[\sqrt{2}[(z_H - z_L)b^M]^{\frac{1}{2}}(1+\rho+x)^{-\frac{1}{2}}\Big]\Big\} > 0 \end{aligned} \tag{2.52}$$

But, since:

$$p(z_{t+i} \geq \tilde{z}) < 1$$

and

$$\frac{(1+\rho+x)}{(1+\rho)} > 1$$

We know that (vii) is smaller than zero. Contradiction: $\frac{\partial b^F}{\partial x} \neq 0$

Case 2

This means:

$$\frac{\partial b^F}{\partial x} < 0$$

$$\frac{\partial b^M}{\partial x} > 0$$

From equation (v) in (vi):

$$p(z_{t+i} < \tilde{z})\Big\{u''[c_{t+i}(z_{t+i})][-(1+\rho)\frac{\partial b^F}{\partial x}]\Big\} + \underbrace{p(z_{t+i} \geq \tilde{z})\Big\{...\Big\} - \frac{(1+\rho+x)}{(1+\rho)}\Big\{...\Big\}}_{<0} > 0 \quad (2.53)$$

But, since $\frac{\partial b^F}{\partial x} < 0$, the first term of the above equation is smaller than zero, since $u''(...) < 0$. Contradiction.

Hence, we have that $\frac{\partial b^F}{\partial x} > 0$. As $b^F \in [0, B]$ and for $x = 0 \Rightarrow b^F = 0$, $\exists\ \underline{x}$ such that $\forall\ x < \underline{x}, b^F = 0$, and $\forall\ x > \underline{x},\ b^F > 0$ and $\exists\ \bar{x} \in (\underline{x},\ \infty)$ such that $\forall\ x \in [\bar{x},\ \infty)$, $b^F = B$

Appendix E

We now show the conditions so that $U^{\tilde{R}} > U^0 \quad \forall z_t \in [z_L,\ z_H] \quad \forall t$. We analyse for deviation on the period $t = 0$ for simplicity, however the conditions hold for any t.

Full Insurance case

Suppose that, from the maximization of the government, $b^M_{t-1} \geq \underline{b}^M$. We fall into the full insurance category. From Appendix B, we know that the solution is the efficient case, and then:

$$\begin{aligned}
U^{\tilde{R}} &= u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M] \\
&+ \sum_{i=1}^{\infty} E_{c,t}\{\beta^i u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M)]\} \\
&= u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M] \\
&+ \sum_{i=1}^{\infty} \beta^i u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M] \\
&= \frac{u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M]}{1-\beta}
\end{aligned}$$

On the other hand:

$$\begin{aligned}
U^0 &= u[F(b^F + \underline{b}^M) + z_0] + \sum_{i=1}^{\infty} E_{c,t}[\beta^i u(z_i)] \\
&\leq u(F(b^F + \underline{b}^M) + z_0) + \frac{\beta u(\bar{z})}{1-\beta}
\end{aligned}$$

Where the last inequality comes from Jensen's inequality for a concave function. Note that as long as $\beta \to 1$, we have that $U^{\tilde{R}} > U^0$. However, if this condition is not satisfied, it must be the case that:

$$\begin{aligned}&\beta\{u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M] - u(\bar{z})\}\\&> u[F(b^F + \underline{b}^M) + z_0] - u[F(b^F + \underline{b}^M) + \bar{z} - (1+\rho)b^F - (1+\rho+x)\underline{b}^M]\end{aligned}$$

What makes the right-hand side be positive is the following, considering utility is an increasing function:

$$F(b^F + \underline{b}^M) - (1+\rho)b^F - (1+\rho+x)\underline{b}^M > 0$$

Using the fact from Appendix B of efficiency in the amount borrowed and the definition of $\bar{b}^M$:

$$\left[\frac{\alpha}{(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\left[\frac{1}{(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} + \frac{(1+\rho)}{(1+\rho+x)}(\bar{z} - z_L) - (\bar{z} - z_L) > 0$$

Because of $\alpha < 1$, the first portion equivalent to $F(B) - (1+\rho)B$ is greater than zero. However, the second portion will depend on the size of x. The closer the risk premium is to zero:

$$x \to 0 \Longrightarrow \frac{(1+\rho)}{(1+\rho+x)} \to 1$$

Hence, the two conditions that make $U^{\tilde{R}} > U^0$ will depend on the size of x and the size of the first shock z_0 and β: the lower z_0 and the higher β the more likely the government will be to reach the reputational equilibrium with the full commitment repayment function.

Partial Insurance case

Suppose we find ourselves in the partial insurance case. We know that the repayment function to the market will be (PI). And so:

$$\begin{aligned}U^{\tilde{R}} &= u[F(b^F + b^M) + z_0 - (1+\rho)b^F - \tilde{R}^M(z_0)]\\&+ \sum_{i=1}^{\infty} E_z\{\beta^i u[F(b^F + b^M) + z_i - (1+\rho)b^F - \tilde{R}^M(z_i)]\}\end{aligned}$$

While U^0 will be:

$$\begin{aligned}U^0 &= u[F(b^F + b^M) + z_0] + \sum_{i=1}^{\infty} E_z[\beta^i u(z_i)]\\&\leq u[F(b^F + b^M) + z_0] + \frac{\beta u(\bar{z})}{1-\beta}\end{aligned}$$

If we have terms to which:

$$U^{\tilde{R}} > u(F(b^F + b^M) + z_0) + \frac{\beta u(\bar{z})}{1-\beta}$$

We have $U^{\tilde{R}} > U^0$. Then, analyse $U^{\tilde{R}}$:

$$\begin{aligned}
U^{\tilde{R}} &= u[F(b^F + b^M) + z_0 - (1+\rho)b^F - \tilde{R}^M(z_0)] \\
&\sum_{i=1}^{\infty} \beta^i \Big\{ p(z_i < \tilde{z}) \Big[u[F(b^F + b^M) + z_i - (1+\rho)b^F] \Big] \\
&+ p(z_i \geq \tilde{z}) \Big[u[F(b^F + b^M) + \tilde{z} - (1+\rho)b^F] \Big] \Big\} \\
&= u[F(b^F + b^M) + z_0 - (1+\rho)b^F - \tilde{R}^M(z_0)] \\
&\sum_{i=1}^{\infty} \beta^i \Big\{ \frac{\tilde{z} - z_L}{z_H - z_L} \Big[u[F(b^F + b^M) + z_i - (1+\rho)b^F] \Big] \Big\} \\
&+ \frac{z_H - \tilde{z}}{z_H - z_L} \left[\frac{\beta u[F(b^F + b^M) + \tilde{z} - (1+\rho)b^F]}{1-\beta} \right]
\end{aligned}$$

What we need is:

$$u[F(b^F + b^M) + z_0 - (1+\rho)b^F - \tilde{R}^M(z_0)] - u[F(b^F + b^M) + z_0] \tag{A}$$

$$+ \frac{\beta}{1-\beta} \left[\frac{z_H - \tilde{z}}{z_H - z_L} u[F(b^F + b^M) + \tilde{z} - (1+\rho)b^F] - u(\bar{z}) \right] \tag{B}$$

$$+ \sum_{i=1}^{\infty} \beta^i \Big\{ \frac{\tilde{z} - z_L}{z_H - z_L} \Big[u[F(b^F + b^M) + z_i - (1+\rho)b^F] \Big] \Big\} \tag{C}$$

$$> 0 \tag{D}$$

Part (A) will always be smaller than zero, whatever z_0 is. That is because if $z_0 < \tilde{z}$ although the sovereign does not repay the market, he will still have to repay the IMF. If $z_0 \geq \tilde{z}$, although the government is insured, the shock is high enough to make default appealing on the current period. Part (C) will also be positive since total borrowed amount is efficient, $F(b^F + b^M) - (1+\rho)b^F > 0$ as shown before. Note that if $\beta \to 1$, the condition above is immediately satisfied. If we do not have that, part (B) will have to be high enough so that (B) is higher than (A) - (C).

Appendix F

In this section we show the sovereign's government maximization when there's no possibility of borrowing with the IMF, and also the international markets charge a risk premium for sharing the risk over the stochastic shock. Again, we will study the full commitment case with partial and full insurances.

Full commitment and full insurance

We now suppose that the market is willing to give full insurance to the sovereign government, and following appendices A and B, the government's problem will be to maximize:

$$U^{MKT} = EU_{c,t} = u(c_t) + \sum_{i=1}^{\infty} E_{c,t}[\beta^i u(c_{t+i})]$$
$$c_t = F(B_{t-1}^{MKT}) + \bar{z} - (1+\rho+x)B_{t-1}^{MKT}$$

The choice variable is B^{MKT}. Following:

$$\frac{\partial EU}{\partial B^{MKT}} = E\left[\frac{\partial U}{\partial B^{MKT}}\right]$$

We have:

$$\beta^i u'(c_{t+i})[F'(B_{t-1+i}^{MKT}) - (1+\rho+x)] = 0$$
$$F'(B_{t-1+i}^{MKT}) = (1+\rho+x)$$

Notice that is equivalent to:

$$\bar{B}^{MKT} = \left[\frac{(1+\rho+x)}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

And since the market is willing to give full insurance, we know that α allows for:

$$\bar{B}^{MKT} \geq \underline{b}^M$$

Which means:

$$\left[\frac{(1+\rho+x)}{\alpha}\right]^{\frac{1}{\alpha-1}} \geq \frac{\bar{z} - z_L}{(1+\rho+x)}$$
$$\Rightarrow \alpha^{\frac{1}{\alpha}}(\bar{z} - z_L)^{\frac{\alpha-1}{\alpha}} \geq (1+\rho+x)$$

Full commitment and partial insurance

In this case, following appendix A, the government's problem will be different:

$$U^{MKT} = EU_{c,t} = u(c_t) + \sum_{i=1}^{\infty} E_{c,t}[\beta^i u(c_{t+i})]$$
$$c_t = F(B_{t-1}^{MKT}) + z_t - \tilde{R}_{t-1}(z_t)^M$$

Where $\tilde{R}_{t-1}(z_t)^M$ is given by (PI) from Appendix A. But $\tilde{z}$ s:

$$\tilde{z} = z_H - \sqrt{2}[B_{t-1}^{MKT}(1+\rho+x)(z_H - z_L)]^{\frac{1}{2}}$$

Again, the choice variable is B^{MKT}. As we've seen the maximization solution in appendix C, first order condition for such problem will be:

$$B^{MKT}_{t+i-1}: \quad E_z[u'(c_{t+i})]F'(B^{MKT}_{t+i-1}) = u'[c_{t+i}(\tilde{z})](1+\rho+x)$$

We only get the efficient result if

$$E_z[u'(c_{t+i})] = u'[c_{t+i}(\tilde{z})]$$

However, since insurance is only partial, in the interval $[z_L;\tilde{z})$, consumption is lower than the one obtained in $[\tilde{z};z_H]$ (this is easy to see on Figure 3). So we know that we will not have the efficiency case in this scenario. We will have:

$$E_z[u'(c_{t+i})] > u'[c_{t+i}(\tilde{z})]$$

As marginal utility is decreasing. This makes sense as we get the result that borrowing in this case is smaller than when there's full insurance. We now have:

$$\underline{B}^{MKT}_{t+i-1} = \left[\left(\frac{u'[c_{t+i}(\tilde{z})]}{E_z[u'(c_{t+i})]}\right)\left(\frac{(1+\rho+x)}{\alpha}\right)\right]^{\frac{1}{\alpha-1}}$$

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