COORDINATION FAILURES IN BUSINESS CYCLES
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Tese submetida à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para a obtenção do título de Doutor em Economia.

Orientador: Bernardo de Vasconcellos Guimarães

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ABSTRACT

Coordination failures are often said to play an important role in business cycles. If agents' incentives of taking a given action depend on the amount of other agents expected to take the same action, coordination failures can often arise. Firms may not invest because they do not expect others to invest, confirming their initial expectations. Similarly, banks may not lend because they do not expect others to lend. This dissertation analyzes different environments in which crises arise as a result of coordination failures.

The first chapter analyzes an economy that is subject to a dynamic coordination problem. Because of aggregate demand externalities, firms' incentives to increase their production depend on expected demand, which in turn depends on the amount produced by other firms. The problem is dynamic since firms do not take investment decisions at the same time, implying that a firm deciding today is trying to forecast what other firms will decide in the future. This opens the possibility of dynamic coordination traps: firms do not invest today because they do not believe others will invest tomorrow, generating lower incentives for firms to invest at future dates. This chapter focuses on the following questions: In economies subject to dynamic coordination traps, what is the optimal stimulus policies? Should policy makers provide higher incentives to production in times of low economic activity? The answer is that a constant subsidy implements the first-best in an economy where beliefs are endogenously determined. The reason is that, although it is harder to coordinate in times of low economic activity, agents are naturally more optimistic about the future in times of poor economic activity and reasonably good fundamentals. This optimism arise from the fact that in bad times negative shocks do not change the level of economic activity, while positive shocks may end a recession.

The second chapter proposes a model to study unusually deep financial crises. Previous empirical work has found that financial crises are very deep and persistent on average, but there is a lot of heterogeneity across different episodes. Some financial crises feature a very distressed financial sector, but little distress on the real sector, while others are real macroeconomic disasters. In light of this evidence, I propose a model in which there is a highly non-linear feedback between the real and the financial sector. Disaster episodes arise from the dynamic interaction of two frictions: coordination frictions and financial frictions. When banks have weak balance sheets they do not intermediate much capital. This causes firms to get trapped in a self-reinforcing regime with low aggregate demand, which ends up provoking further damage to banks’ balance sheets. I use the model as a laboratory to study unusually deep financial
crises and the effects of some policies. It is shown that the effects of disasters go far beyond what we observe during those episodes: they imply very low asset prices, economic growth and welfare, even in good times and when their probability is very small. Policies that protect the financial sector from those episodes can be very beneficial. Moreover, higher risk-taking in bad times may improve economic growth, welfare and financial stability.

The third chapter studies the policy trade-off of a regulator that wants to avoid coordination failures, but at the same time does not want to generate distortions arising from moral hazard. Banks have investment opportunities with an expected return that depends positively on the amount of other banks undertaking similar investments, opening room for coordination failures. At the same time, banks may risk-shift to projects with smaller expected return but higher volatility. By providing guarantees in case of failures, a regulator can enhance coordination, but that leads banks to switch to worse projects. It is shown that in some states a regulator will provide no guarantees, even if it means allowing a coordination failure to happen. Moreover, the possibility of risk-shifting reduces the amount of guarantees needed to avoid a coordination failure.

**Keywords:** coordination failures, fiscal stimulus, demand externalities, financial frictions, financial crises, bank runs, moral hazard.

**JEL Classification:** D84, E32, E62, G10, G21, G28.
RESUMO

Com frequência argumenta-se que falhas de coordenação têm um papel importante no ciclo de negócios. Se os incentivos dos agentes a realizar determinada ação depende da quantidade esperada de outros agentes que tomarão a mesma ação, falhas de coordenação podem acontecer. Empresas podem não investir porque não esperam que outras empresas irão investir, confirmando suas expectativas iniciais. De maneira similar, bancos podem não conceder empréstimos porque eles não esperam que outros bancos irão fazer o mesmo. Esta tese analisa diferentes ambientes onde crises surgem como o resultado de falhas de coordenação.

O primeiro capítulo analisa uma economia que está sujeita a falhas de coordenação dinâmicas. Por causa de externalidades de demanda agregada, os incentivos para uma dada firma aumentar sua produção dependem da demanda esperada, que por sua vez depende da quantidade produzida por outras firmas. O problema é dinâmico porque as firmas não tomam decisões de investimento ao mesmo tempo, implicando que uma firma tomando decisões hoje está tentando prever o que outras firmas decidirão no futuro. Isso abre a possibilidade de falhas de coordenação dinâmicas: firmas não investem hoje porque elas não acreditam que outras firmas investirão amanhã, gerando incentivos menores para outras firmas investirem no futuro. Este capítulo focal nas seguintes questões: Em economias sujeitas a este problema de coordenação dinâmico, qual a política de estímulo ótima? O governo deveria prover mais estímulos em épocas de baixa atividade econômica? A resposta é que um subsídio constante implementa o ótimo nesta economia. O motivo é que, embora seja mais difícil coordenar em tempos de baixa atividade, os agentes estão naturalmente mais otimistas sobre o futuro em tempos de baixa atividade e fundamentos razoavelmente bons. Este otimismo surge do fato que em tempos ruins choques negativos não alteram o nível de atividade econômica, mas choques positivos podem acabar com uma recessão.

O segundo capítulo desta tese propõe um modelo para estudar crises financeiras mais severas que o usual. Trabalhos empíricos prévios mostram que, em geral, crises financeiras são muito profundas e persistentes, mas também que há muita heterogeneidade entre diferentes episódios. Algumas crises financeiras causam enormes danos no sistema financeiro, mas pouco dano no setor real, enquanto outras são verdadeiros desastres macroeconômicos. À luz desta evidência, esta tese propõe um modelo onde há um feedback extremamente não linear entre o setor financeiro e o setor real. Desastres surgem através da interação dinâmica de duas fricções: fricções de coordenação e fricções financeiras. Quando os bancos estão com problemas em seus balanços, eles optam por intermediar menos capital. Isso leva as firmas a entrar em
um regime com baixa demanda agregada, que causa ainda mais dano ao capital dos bancos. Este modelo é utilizado como um laboratório para estudar crises financeiras muito severas e o efeito de algumas políticas. É mostrado que os efeitos de desastres econômicos vão muito além do que observamos durante estes episódios. Eles levam à queda dos preços de ativos, baixo crescimento e perdas de bem-estar, mesmo que a probabilidade destes eventos seja muito pequena. Finalmente, quando os bancos tomam mais risco em tempos ruins, podemos ter um aumento de crescimento, bem-estar e estabilidade financeira.

O terceiro capítulo estuda o trade-off enfrentado por um regulador que quer evitar falhas de coordenação, mas ao mesmo tempo não quer gerar distorções que surgem por conta de risco moral. Os bancos possuem oportunidades de investimento cujo retorno esperado depende positivamente da quantidade de outros bancos investindo em projetos similares, abrindo espaço para a possibilidade de falhas de coordenação. Ao mesmo tempo, bancos podem escolher investir em projetos com menor retorno esperado e maior volatilidade. Ao prover garantias em caso de falha de um banco, um regulador pode melhorar a habilidade que estes têm de coordenar, mas ao mesmo isto pode levar os bancos a tomarem risco excessivo. É mostrado que em alguns estados o regulador não proverá garantias, mesmo que isso implique permitir que uma falha de coordenação aconteça. Ainda, a possibilidade dos bancos tomarem risco excessivo reduz a quantidade de garantias necessárias para evitar uma falha de coordenação.

**Palavras-chave:** falhas de coordenação, estímulos fiscais, externalidades de demanda, fricções financeiras, crises financeiras, corridas bancárias, risco moral.

**Classificação JEL:** D84, E32, E62, G10, G21, G28.
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Chapter 1

Dynamic coordination and the optimal stimulus policies*

Abstract
This paper studies stimulus policies in a simple macroeconomic model featuring a dynamic coordination problem that arises from demand externalities and fixed costs of investment. In times of low economic activity, firms face low demand and hence have lower incentives for investing, which reinforces their low-demand expectations. In a benchmark case with no shocks, the economy might get trapped in a low-output regime and a social planner would be particularly keen to incentivize investment at times of low economic activity. However, this result vanishes once shocks are considered.

Keywords: coordination failures, fiscal stimulus, timing frictions, demand externalities.
Jel Classification: D84, E32, E62.

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*This chapter is coauthored by Bernardo Guimaraes.
1.1 Introduction

Coordination is often said to play a role in recessions. This idea is captured by models with demand externalities that generate strategic complementarities in production.\(^1\) In times of low economic activity, a firm faces low demand and thus has low incentives for investment. In a dynamic setting, this feedback effect may trap the economy in a low-output regime: lower investment today implies lower economic activity and lower investment tomorrow. What is the optimal stimulus policy for an economy subject to this dynamic problem? Is there a special reason for stimulus at times of low economic activity?

This paper develops a model to answer these questions. Investment is modeled as a fixed cost that increases production capacity. Investment decisions are staggered. Producers of each variety receive investment opportunities according to a Poisson clock, a simple way to capture production decisions that cannot adjust overnight. This assumption implies that investment decisions are not synchronized and economic activity is a state variable.

Returns to investment depend on future demand and hence on whether producers with subsequent investment opportunities choose to take them as well. Thus investment decisions are strategic complements, and producers have to form expectations about others’ future decisions when deciding about investment.

Returns to investment depend not only on demand but also on productivity. If the increase in production resulting from investing is large enough, then investing is a dominant strategy. Likewise, if productivity is very low, investing is a dominated strategy. In an intermediate range, a producer’s decision depends on his expectations about the actions of others: investing is the optimal decision if agents expect others to do so, but refraining from investment is the best choice in case of pessimistic beliefs.

We first consider a benchmark model with no shocks. In this case, there are multiple equilibria. In order to close the model, we need some assumption on beliefs. Assuming either ‘pessimistic’ or ‘optimistic’ beliefs, the solution to the planner’s problem differs from the decentralized equilibrium in two ways: (i) the planner requires lower productivity to invest because it internalizes the benefits to consumers from cheaper prices (monopoly distortion); and (ii) the difference between the planner’s solution and the decentralized equilibrium is larger at times of low economic activity.

The second point highlights an inefficiency related to the dynamics of the economy. Agents might get stuck in a situation where economic activity is low, hence there is low demand and firms prefer not to invest even though productivity would be high enough to encourage

\(^1\)Seminal papers in this literature are Shleifer (1986), Cooper and John (1988), Kiyotaki (1988) and Murphy, Shleifer and Vishny (1989).
investment if demand was high. In this situation, a firm would like others to invest, so that demand would increase and generate incentives for it to invest as well. However, nobody wants to be the first to invest and the economy is trapped in a situation with low economic activity. The planner would be particularly keen to stimulate investment in this situation.

We then consider the model with shocks to aggregate productivity. As in Frankel and Pauzner (2000), a unique equilibrium arises in the model. The model generates a unique set of rationalizable beliefs about others’ actions. Intuitively, fully pessimistic beliefs are not rationalizable in a region where a small shock to productivity would make it dominant for all firms to invest. Likewise, fully optimistic beliefs are not rationalizable in a region where a small shock to productivity takes the economy to a region where investing is a dominated strategy. Agents know all others will reason like this and try to anticipate what others will do. This process yields a unique rationalizable set of strategies and beliefs.\(^2\)

The main result of this paper is that differently from the case with multiple equilibria, there is no special reason for subsidies at times of low economic activity. The maximum amount of investment subsidies the planner is willing to provide at times of high and low economic activity is exactly the same.

The result holds even when the variance of productivity shocks is arbitrarily small. The only meaningful difference between arbitrarily small shocks and no shocks at all is that a unique set of beliefs is pinned down by the model in the former case. Hence the beliefs that arise in equilibrium exactly offset the dynamic inefficiency, so that there is no special reason for stimulus at times of low economic activity.

What are equilibrium beliefs like? Consider an agent indifferent between investing or not in a state of low economic activity. She understands that if fundamentals get a bit worse, firms will still be refraining from investing but there will be no major change in the state of the economy. Conversely, a slight improvement in fundamentals will trigger a recovery because firms will choose to invest and that will push the economy to a situation where investing is profitable for everyone. Owing to larger demand, firms will then have more incentives to invest, so it will take a large negative productivity shock to offset the benefit from increased demand and stop the recovery. The fundamental asymmetry is that bad news basically leaves the economy parked in a region of inaction, while good news drives the economy to a different state.

That does not mean that agents are usually more optimistic at times of low economic activity at times of low economic activity.

\(^2\)Uniqueness of equilibrium does not preclude coordination failures – actually, the equilibrium is typically inefficient in static and dynamic coordination games with a unique equilibrium (examples include Carlsson and Van Damme (1993), Morris and Shin (1998) and most papers in the literature of global games). The equilibrium is also inefficient in our model.
activity; it means that an agent that is indifferent between investing or not expects a positive change at times of low economic activity. Likewise, in a situation with high economic activity and relatively low productivity, agents understand that small shocks might trigger an investment slump.

In the benchmark case with multiple equilibria, the beliefs of agents that are indifferent between investing or not might be ‘optimistic’ (they expect others will invest) or ‘pessimistic’ (they expect others will not invest), depending on which equilibrium is considered. The key feature of the model with shocks is that beliefs in the indifference region are ‘optimistic’ when economic activity is low and productivity is relatively high and ‘pessimistic’ when economic activity is high and productivity is relatively low.

Related literature. This paper is related to the theoretical contributions in Frankel and Pauzner (2000) and Frankel and Burdzy (2005). They show there is a unique rationalizable equilibrium in a class of dynamic models with time-varying fundamentals and timing frictions. Our macroeconomic model fits in their framework so their results can be used to show equilibrium uniqueness in our model. However, neither of these papers solve the social planner’s problem.

The demand externalities that play a key role in this paper are in the seminal contributions by Blanchard and Kiyotaki (1987), Kiyotaki (1988) and Murphy, Shleifer and Vishny (1989). When others produce more, the demand for a particular variety shifts to the right, and its producer finds it optimal to increase production. In Kiyotaki (1988), multiple equilibria arise because of increasing returns to scale. The model in this paper also gives rise to multiple equilibria in the absence of shocks to fundamentals, owing to the assumption of a fixed cost that increases production capacity.

A branch of the literature takes expectations to be driven by some “sunspot” variable, or simply, in the words of Keynes, by “animal spirits”. Depending on agents’ expectations, coordination failures might arise and an inefficient equilibrium might be played. Early examples include Cooper and John (1988), Benhabib and Farmer (1994) and Farmer and Guo (1994). Recent research on business cycles has explored the implications of equilibrium multiplicity in a variety of settings.

Benhabib, Schmitt-Grohé and Uribe (2001) show that once the zero lower bound is considered, Taylor rules lead to multiple equilibria. Building on this insight, Evans and Honkapohja (2005) study the implications of replacing perfect foresight with learning in this environment; Mertens and Ravn (2014) show that government spending succeeds in stimulating output in case of a fundamental-driven liquidity trap, but fails to do so in case of a confidence-driven liquidity trap; and Aruoba, Cuba-Borda and Schorfheide (2013) quantitatively assess the importance of sunspot shocks for the recent recessions in the US and in Japan. In Benigno and Fornaro (2015), the zero lower bound also play a key role and the model features two steady states owing to the interplay between productivity growth and aggregate demand.

The literature has shown other possible channels that generate multiplicity of equilibria, exploring a variety of feedback loops: in a search and matching model, Farmer (2012) replaces the assumption of Nash bargaining over the wage with the assumption that firms produce as many goods as are demanded; Chamley (2014) presents a model with decentralized trade and credit constraints where pessimistic expectations lead to precautionary savings; in Benhabib, Wang and Wen (2015), firms make production decisions based on expected demand and households choose consumption and labor supply based on their expected income; and in Kaplan and Menzio (2016), larger unemployment implies people spend more time searching for lower prices, which reduces firms’ incentives to produce more. In Heathcote and Perri (2015), multiple equilibria arise only when the level of household wealth is low, because demand is more sensitive to unemployment expectations in this case, owing to a stronger precautionary motive. Bacchetta and van Wincoop (2016) argue that when economic integration is large enough, self-fulfilling panics cannot be limited to one country. Rendahl (2016) considers an economy at the zero lower bound with search and matching frictions and studies how government spending can put a halt to a downward spiral of self-reinforcing thrift.

Another branch of this literature considers coordination and strategic complementarities in business cycles models with a unique equilibrium. Angeletos and La’O (2010) and Angeletos and La’O (2013) show in an environment with noisy and dispersed information how self-fulfilling fluctuations can emerge. Their model has a unique equilibrium, but features some key aspects of sunspot models. Angeletos, Collard and Della (2014) attempt to quantitatively assess the role of confidence in business cycles. Nimark (2008) builds a model where pricing complementarities together with private information help to explain the inertial behavior of inflation due to the inertial response of expectations (see also Angeletos and La’O, 2009). In Schaal and Taschereau-Dumouchel (2016a), if firms expect low aggregate demand, they post

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5See also Howitt and McAfee (1987).
fewer vacancies, produce less and aggregate demand is indeed low (as in Howitt and McAfee, 1992). Firms heterogeneity restores equilibrium uniqueness.

Some of this work uses the global games methodology to understand the effects of stimulus packages on coordination. Sákovics and Steiner (2012) build a model to understand who matters in coordination problems: in a recession, who should benefit from government subsidies? They conclude that the government should subsidize sectors that have a large externality on others but that are not much affected by others’ actions. Guimaraes, Machado and Ribeiro (2016) study how government spending affects coordination in a static model. Closer to our paper, Schaal and Taschereau-Dumouchel (2016b) build a dynamic macroeconomic model with coordination failures and a unique equilibrium. Firms’ choices of capacity utilization at every period is subject to coordination failures and is modelled as a global game. Households’ consumption-saving decisions affect coordination among firms and the dynamics of the economy. Although their environment is substantially different from ours, simple subsidies can also implement the first best in their paper.

Differently from models that employ the global games methodology to obtain equilibrium uniqueness in macroeconomic settings with strategic complementarities, our results do not rely on noisy heterogeneous information – all information is common knowledge here. The key ingredients to resolve indeterminacy in this model are timing frictions and shocks to fundamentals. Our framework is particularly suitable to understand the dynamic interplay between economic activity, productivity and beliefs that arise in equilibrium.7

The framework used here is similar to the one in my master’s thesis (see Machado, 2013). However, while that work calibrated the model and solved it numerically to study the optimal timing of stimulus policies, here an analytical solution to the planner’s problem is provided; the results are contrasted with a similar environment with multiple equilibria; the optimality of a constant subsidy is proved; and an extension with endogenous hazard rates is developed.

The paper is organized as follows. Section 1.2 presents the model. Section 1.3 shows results for the benchmark case without shocks, while Section 1.4 considers the model with shocks and explain the beliefs that arise in equilibrium. Section 1.5 drops the assumptions of fixed cost of investing and random switching opportunities and lets firms choose a switching rate at every moment. Our main result also holds in this extension: there is no special reason for stimulus

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7Expectations also play a key role in the literature of news-driven business cycles (e.g., Beaudry and Portier (2006)), but here expectations about future productivity depend solely on the current state of the economy. In the models of Lorenzoni (2009) and Eusepi and Preston (2011), it is noisy information about current variables that leads to excessive optimism or pessimism about the future.
at times of low economic activity. Section 1.6 concludes. All proofs are in Appendix 1.A.

1.2 Model

1.2.1 Environment

Time is continuous. A composite good is produced by a perfectly competitive representative firm. At time $t$, $Y_t$ units of the composite good are obtained by combining a continuum of intermediate goods, indexed by $i \in [0, 1]$, using the technology

$$Y_t = \left( \int_0^1 y_{it}^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}, \quad (1.1)$$

where $y_{it}$ is the amount of intermediate good $i$ used in the production of the composite good at time $t$ and $\theta > 1$ is the elasticity of substitution. The zero-profit condition implies

$$\int_0^1 \hat{p}_{it} y_{it} di = P_t Y_t, \quad (1.2)$$

where $P_t$ is the price of the composite good and $\hat{p}_{it}$ is the price of good $i$ at time $t$.

There is a measure-one continuum of agents who discount utility at rate $\rho$. An agent’s instantaneous utility at time $t$ is given by $U_t = C_t$, where $C_t$ is her instantaneous consumption of the composite good. The assumption of linear utility implies that policies will be concerned with inefficiencies in production but will not aim at providing insurance to the household.

Agent $i \in [0, 1]$ produces intermediate good $i$. Since $y_{it}$ is the quantity produced by agent $i$ at time $t$, her budget constraint is given by

$$P_t C_t \leq \hat{p}_{it} y_{it} \equiv w_i P_t.$$

Prices are flexible and each price $\hat{p}_{it}$ is optimally set by agent $i$ at every time. Since goods are non storable, supply must equal demand at any time $t$.

The assumptions on technology aim at modelling staggered investment decisions in a simple and tractable way. There are 2 production regimes, a High-capacity regime and a Low-capacity regime. Agents get a chance to switch regimes according to a Poisson process with arrival rate $\alpha$.\(^8\) Once an individual is picked up, she chooses a regime and will be locked in this regime

\(^8\)Real world investments require a lot of planning and take time to become publicly known, so investments from different firms are not synchronized. The Poisson process generates staggered investment decisions in a simple way. As an implication, investment decisions depend on expectations about others’ actions in the near
until she is selected again. Choosing the Low regime is costless. Choosing the High regime costs $\psi$ units of the composite good.\(^9\)

An agent in the Low regime can produce up to $y_{Lt}$ units at zero marginal cost at every time $t$, and an agent in the High regime can produce up to $y_{Ht}$ units at zero marginal cost, with $y_{Ht} = A_t x_H$ and $y_{Lt} = A_t x_L$, where $x_H > x_L$ are constants and $A_t$ is a time-varying productivity parameter.

The High regime can be interpreted as the use of frontier technology, while the Low regime corresponds to a less productive technology. The cost $\psi$ can be thought of as the cost difference between each technology and the difference $y_{Ht} - y_{Lt}$ as the resulting gain in productivity. Agents are locked in a regime until the next opportunity arises. In one interpretation, the equipment will break after some (random) time and the firm will then decide again between a more productive or a less productive technology. Alternatively, that might capture attention frictions.\(^10\)

Investment requires agents to acquire a stock $\psi$ of composite goods, which cannot be funded by their instantaneous income, so we assume agents can trade assets and borrow to invest. Owing to the assumption of linear utility, any asset with present value equal to $\psi$ is worth $\psi$ in equilibrium. For example, an agent might issue an asset that pays $(\rho + \alpha)\psi dt$ at every interval $dt$ until the investment depreciates ($\rho \psi dt$ would be the interest payment and $\alpha \psi dt$ can be seen as an amortization payment since debt is reduced from $\psi$ to 0 with probability $\alpha dt$). Since agents are risk neutral, other types of assets would deliver the same results.

Let $a_t = \log(A_t)$ vary in time according to\(^11\)

$$da_t = \sigma dZ_t,$$

where $\sigma > 0$ and $Z_t$ is a standard Brownian motion. In order to ensure that agents face a coordination problem, we assume that $\sigma^2 < 2(\rho + \alpha)$.\(^12\)

\(^9\)Investment is thus a binary decision. As shown in Gourio and Kashyap (2007), the extensive margin accounts for most of the variation in aggregate investment, so a binary choice set can capture much of the action in investment.

\(^10\)In another possible interpretation, $\psi$ is the cost of hiring a worker that cannot be fired until his contract expires. In this case, the fixed cost may not be paid at once, but that makes no difference in the model.

\(^11\)The main results also hold if $a_t$ has a constant drift and, under some additional technical assumptions, if $a_t$ follows a process with mean reversion. See the Online Appendix.

\(^12\)Intuitively, an agent’s expected gain from investing is infinite if $a_t$ is too volatile. The expected growth rate of the utility flow from investing in this model is $\sigma^2/2$ (Ito’s Lemma). This has to be smaller than agents’ effective discount rate (which is $\rho + \alpha$).
1.2.2 The agent’s problem

The composite-good firm chooses its demand for each intermediate good taking prices as given. Using (1.1) and (1.2) and defining \( p_{it} \equiv \tilde{p}_{it} / P_t \), we get

\[
p_{it} = y_t^{-1/\theta} Y_t^{1/\theta},
\]

(1.3)

for \( i \in [0,1] \). Since marginal cost is zero and marginal revenue is always positive, an agent in the Low regime will produce \( y_{Lt} \), and an agent in the High regime will produce \( y_{Ht} \). Thus at any time \( t \), there will be two prices in the economy, \( p_{Ht} \) and \( p_{Lt} \) (associated with production levels \( y_{Ht} \) and \( y_{Lt} \), respectively). Hence the instantaneous income available to individuals in each regime is given by

\[
w_{Ht} = p_{Ht} y_{Ht} = y_{Ht}^{\theta-1} Y_t^{\theta-1},
\]

(1.4)

and

\[
w_{Lt} = p_{Lt} y_{Lt} = y_{Lt}^{\theta-1} Y_t^{\theta-1}.
\]

(1.5)

Moreover, using (1.1),

\[
Y_t = \left( h_t y_{Ht}^{\theta-1} + (1 - h_t) y_{Lt}^{\theta-1} \right)^{\theta/1-\theta},
\]

(1.6)

where \( h_t \) is the measure of agents locked in the High regime.

Let \( \pi(a_t, h_t) \) be the difference between instantaneous income of agents locked in the High regime (\( w_{Ht} \)) and income of agents locked in the Low regime (\( w_{Lt} \)) when the economy is at \((a_t, h_t)\). Combining (1.4), (1.5), (1.6) and using \( y_{Lt} = e^{a_t} x_L \) and \( y_{Ht} = e^{a_t} x_H \) leads to

\[
\pi(a_t, h_t) = e^{a_t} \left( h_t x_H^{\theta-1} + (1 - h_t) x_L^{\theta-1} \right)^{\theta/1-\theta} \left( x_H^{\theta-1} - x_L^{\theta-1} \right).
\]

(1.7)

Function \( \pi \) is increasing in both \( a_t \) and \( h_t \). The effect of \( a_t \) captures the supply-side incentives to invest: a larger \( a_t \) means a higher productivity differential between agents who had invested and those who had not. The effect of \( h_t \) captures the demand-side incentives to invest: a larger \( h_t \) means a higher demand for a given variety. The equilibrium price of a good depends on how large \( y_{it}/Y_t \) is, so a producer benefits from others producing \( y_{Ht} \) regardless of how much she is producing. Nevertheless, since \( \theta > 1 \), an agent producing more reaps more benefits from a higher demand.

One key implication of (1.7) is that investment decisions are strategic complements: the higher the production level of others, the higher the incentives for a given agent to increase her production level.
A strategy is as a map \( s(a_t, h_t) \mapsto \{\text{Low, High}\}. \) An agent at time \( t = \tau \) that has to decide whether to invest will do so if

\[
\int_{\tau}^{\infty} e^{-(\rho + \alpha)(t-\tau)} E_\tau[\pi(a_t, h_t)] \, dt > \psi;
\]

and will not invest if the inequality is reversed. In words, investing pays off if the discounted expected additional profits from choosing the High regime are larger than the fixed cost \( \psi \). Future profits \( \pi(a_t, h_t) \) are discounted by the sum of the discount rate and depreciation rate \((\rho + \alpha)\).

Investment decisions depend on expected profits. Producers will decide to invest not only if productivity is high, but also if they are confident they will be able to sell their varieties at a good price. Hence investment decisions crucially depend on demand expectations, which in turn are determined by expectations about the path of \( a_t \) and \( h_t \).

### 1.2.3 The planner’s problem

The planner maximizes expected welfare, given by

\[
E_\tau(W) = E_\tau \int_{\tau}^{\infty} e^{-\rho(t-\tau)} (Y(a_t, h_t) - \alpha \psi I(t)) \, dt,
\]

where \( Y(a, h) \) is given by (1.6) and \( I(t) \in [0, 1] \) is the decision of the planner about investing at time \( t \) (the proportion of those who got an investment opportunity at \( t \) that will invest).

Suppose the optimal investment decision at date \( \tau \) implies \( I(\tau) < 1 \) and consider the following deviation: the planner chooses \( I(\tau) = 1 \) today and keeps investment choices for every realization of the Brownian path in the future unchanged. This deviation cannot be profitable. Investing extra \( dI \) units today raises \( h_\tau \) by \( \alpha dI \), but this depreciates at rate \( \alpha \). Hence

\[
\frac{dh_\tau}{dI(\tau)} = \alpha e^{-\alpha(t-\tau)}.
\]

Therefore, this deviation is not profitable if

\[
\int_{\tau}^{\infty} e^{-\rho(t-\tau)} E_\tau \left[ \frac{\partial Y(h_t, a_t)}{\partial h} \alpha e^{-\alpha(t-\tau)} dI \right] \, dt - \alpha \psi dI \leq 0.
\]

To simplify the exposition, we present the definition of a Markovian strategy, but our results do not rely on that restriction on the strategy space.

Since there is no interaction with other players and no intrinsic time-inconsistency in the planner’s preferences, there is no commitment issue in the planner’s problem.
Since
\[
\frac{\partial Y(h_t, a_t)}{\partial h} = e^{a_t} \frac{\theta}{\theta - 1} \left( h_t x_H^{\frac{\theta - 1}{\theta}} + (1 - h_t) x_L^{\frac{\theta - 1}{\theta}} \right)^{\frac{1}{\theta - 1}} \left( x_H^{\frac{\theta - 1}{\theta}} - x_L^{\frac{\theta - 1}{\theta}} \right),
\]
(1.10)
if the planner chooses not to invest \((I(\tau) = 0)\), it must be that
\[
\int_0^\infty e^{-(\rho + \alpha)(t-\tau)} E_\tau \left[ \frac{\theta}{\theta - 1} \pi(h_t, a_t) \right] dt \leq \psi.
\]
(1.11)
An analogous reasoning shows that if at date \(\tau\) the planner chooses to invest \((I(\tau) = 1)\), it must be that:
\[
\int_0^\infty e^{-(\rho + \alpha)(t-\tau)} E_\tau \left[ \frac{\theta}{\theta - 1} \pi(h_t, a_t) \right] dt \geq \psi.
\]
(1.12)

The expressions in (1.11) and (1.12) are necessary conditions for optimality. Note they are analogous to the corresponding necessary conditions for a Nash Equilibrium in the agents’ game (which are also sufficient conditions in that case). The only difference between (1.8) and (1.12) is the constant multiplying \(\pi(a, h)\) in (1.12). Thus, finding candidates for the planner’s solution is equivalent to finding the equilibrium set of a modified game. It will be shown that in some cases, these necessary conditions are also sufficient.\(^{15}\)

1.3 Benchmark case: no shocks

We first consider the case where the fundamental \(a\) does not vary over time, \(\sigma = 0\).

1.3.1 Equilibria

Consider an agent with optimistic beliefs, i.e., she expects all others will choose to invest in the future. An agent is indifferent between investing or not at \((a_{\text{opt}}^*, h_0)\) if \(a_{\text{opt}}^*(h_0)\) solves
\[
\int_0^\infty e^{-(\rho + \alpha)t} [\pi(a_{\text{opt}}^*(h_0), h_t^U)] dt = \psi,
\]
(1.13)
where \(h_t^U = 1 - (1 - h_0)e^{-\alpha t}\). If the agent holds pessimistic beliefs (if she expects all others will choose not to invest in the future), she is indifferent between investing or not at \((a_{\text{pes}}^*, h_0)\) if \(a_{\text{pes}}^*(h_0)\) solves
\[
\int_0^\infty e^{-(\rho + \alpha)t} [\pi(a_{\text{pes}}^*(h_0), h_t^D)] dt = \psi,
\]
(1.14)
\(^{15}\)Agents takes others’ strategies as given and anticipate other agents will be choosing according to (1.8). The planner can decide on the path of \(h\), but it also anticipates its future selves will satisfy (1.12).
where \( h_t^D = h_0 e^{-\alpha t} \).

Proposition 1.1 characterizes the conditions for multiple equilibria.

**Proposition 1.1 (No Shocks).** Suppose \( \sigma = 0 \). There are strictly decreasing functions \( a_{\text{opt}}^*: [0, 1] \mapsto \mathbb{R} \) and \( a_{\text{pes}}^*: [0, 1] \mapsto \mathbb{R} \) with \( a_{\text{opt}}^*(h) < a_{\text{pes}}^*(h) \) for all \( h \in [0, 1] \) such that

1. If \( a < a_{\text{opt}}^*(h_0) \) there is a unique equilibrium, agents always choose the Low regime;
2. If \( a > a_{\text{pes}}^*(h_0) \) there is a unique equilibrium, agents always choose the High regime;
3. If \( a_{\text{opt}}^*(h_0) < a < a_{\text{pes}}^*(h_0) \) there are multiple equilibria, that is, both strategies High and Low can be long-run outcomes.

Figure 1.1 illustrates the result of Proposition 1.1. If the productivity differential is sufficiently high, agents will invest as soon as they get a chance and the economy will move to a regime where \( h = 1 \) (and there it will rest). If the productivity differential is sufficiently low, the gains from investing are offset by the fixed cost, so not investing is a dominant strategy. In an intermediate area, there are no dominant strategies, the optimal investment decision depends on expectations about what others will do and there are multiple equilibria.

We say the economy is in the ‘good equilibrium’ when agents choose High and expect others to do so whenever that is rationalizable. Conversely, we say the economy is in the ‘bad equilibrium’ when agents choose Low and expect others to do so whenever that is rationalizable.

The left threshold (good equilibrium) is the set of \((a, h)\) where an agent is indifferent between investing or not assuming all others will invest, and it is given by (1.13). For the right threshold (bad equilibrium), the assumption is that no other agent will ever choose to invest, as in (1.14). The good equilibrium Pareto dominates the bad equilibrium.

Cycles are possible in this economy, but their existence depends on exogenous changes in beliefs. Demand expectations are not pinned down by the parameters that characterize the economy and its current state.
1.3.2 Optimal policy

One important question is about whether inefficiencies are more pronounced at times of low economic activity. For instance, suppose the economy is stuck in a regime with low $h$. Is this situation particularly inefficient? Would a social planner be particularly inclined to stimulate investment in this case?

In the case with $\sigma = 0$, the planner chooses between always investing and never investing (any other option is dominated by one of these alternatives). Thus the planner is indifferent between investing and not investing at $(a^*_P(h_0), h_0)$ when $a^*_P(h_0)$ is given by

$$\int_0^{\infty} e^{-\rho t} \left( Y(a^*_P(h_0), h^U_t) - \alpha \psi \right) dt = \int_0^{\infty} e^{-\rho t} \left( Y(a^*_P(h_0), h^D_t) \right) dt. \quad (1.15)$$

This expression pins down a decreasing threshold $a^*_P$ such that the planner will invest when $a > a^*_P(h_0)$ and not invest when $a < a^*_P(h_0)$.

In order to compare the social planners’ solution with the decentralized economy and characterize optimal policies, we need to select an equilibrium in the model. One possible way is to assume agents coordinate in the good equilibrium whenever possible. An alternative way is to assume the worst equilibrium is played whenever it exists and search for policies that eliminate this equilibrium. Proposition 1.2 shows both ways lead to similar conclusions.

Proposition 1.2. Suppose $a$ is fixed and the economy is either in the ‘good’ or in the ‘bad’ equilibrium. In both cases:

1. [Planner’s choices depend less on $h$] The distance between the planner’s threshold at $h = 0$ and at $h = 1$ is smaller than the equivalent distance in both decentralized equilibria:

$$a^*_P(0) - a^*_P(1) < a^*_{opt}(0) - a^*_{opt}(1) \quad \text{and} \quad a^*_P(0) - a^*_P(1) < a^*_{pes}(0) - a^*_{pes}(1).$$

2. [More subsidies at low $h$] The planner’s solution can be implemented by investment subsidies. At the planner’s threshold, the amount of subsidies required to coax agents to invest is higher when $h = 0$ than when $h = 1$.

The equilibria of the model and the planner’s threshold are illustrated in Figure 1.2.

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This assumption is common in the literature. Examples include Allen and Gale (1998), Zawadowski (2013) and Boissay, Collard and Smets (2016).

Policy analysis in models with multiple equilibria often take this route. Examples include Chang and Velasco (2000), Aghion, Bacchetta and Banerjee (2004), Benhabib, Schmitt-Grohe and Uribe (2002) and some of the work building on the latter (e.g., Schmitt-Grohe and Uribe (2016) and Benigno and Fornaro (2015)).
There are important differences between the planner’s solution and the decentralized equilibrium. First, the solution for the planner’s problem is unique but there are multiple self-fulfilling equilibria. More importantly, regardless of whether we assume optimistic or pessimistic beliefs, the equilibrium threshold is further away from the planner’s threshold for low values of $h$. The difference in slopes reflects the planner’s willingness to pay higher subsidies when $h$ is low.

At the heart of this problem lies an inefficiency related to the dynamics of the economy. Agents only take into account the benefit of investment until their next decision, which means they effectively discount the future at rate $\alpha + \rho$. In contrast, the planner takes into account the effect that investing has on the whole path of the economy, and thus discounts the future at the much smaller rate $\rho$. In a region with low $h$ and $a$ just below the threshold, investment in the short run would drive the economy to the high regime and the planner takes that into account. Agents don’t find it profitable to invest while demand is still low, but would be happy to sign a contract forcing everyone to take investment opportunities in the short run, as these losses for some would imply gains for all in the future.

In the absence of investment subsidies, the economy might be stuck in a recession trap (low $h$, not so low $a$). The root of the problem is a self-reinforcing lack of demand (and, consequently, investment) when $h$ is low: no investment in the past leads to low demand and no investment today, which in turn leads to low demand and no investment in the future and so on.

The recession trap does not stem from the multiplicity of rationalizable beliefs, since it holds under the assumption that agents coordinate on the good equilibrium whenever possible. This result thus suggests that in a world with demand externalities and an inaction region for investment, subsidies to take the economy out of a recession are warranted. However, beliefs are exogenous in this reasoning. We now consider the model with $\sigma > 0$. 

Figure 1.2: The case with no shocks
1.4 The model with shocks

We now turn to the general case where productivity varies over time. We say that an agent is playing according to a threshold \( a^* : [0, 1] \rightarrow \mathbb{R} \) if she chooses \textit{High} whenever \( a_t > a^*(h_t) \) and \textit{Low} whenever \( a_t < a^*(h_t) \). Function \( a^* \) is an equilibrium if the strategy profile where every player plays according to \( a^* \) is an equilibrium.

1.4.1 Equilibrium

The model satisfies the assumptions of the framework in Frankel and Pauzner (2000). Hence we can apply their results to show there is a unique rationalizable equilibrium where agents play according to a decreasing threshold \( a^*(h) \).

**Proposition 1.3** (Frankel and Pauzner, 2000). Suppose \( \sigma > 0 \). There is a unique rationalizable equilibrium in the model. Agents invest if \( a > a^*(h) \) and do not invest if \( a < a^*(h) \), where \( a^* \) is a decreasing function.

In order to understand how shocks affect the set of rationalizable strategies, consider a situation where productivity is relatively low, so a firm is only willing to invest if the probability the following firms will also invest is very high. In Figure 1.1, that would correspond to a point in the multiplicity region but close to its left boundary. In a world with shocks, the economy might cross to the region where investing is a dominated strategy. That imposes a cap on the probability that others will invest in the near future – the belief that they will certainly invest is not rationalizable. In consequence, some dominated strategies are eliminated, which imposes further limits on beliefs agents can hold. Iterating on this process leads to a unique equilibrium.\(^{18}\)

This reasoning highlights a fundamental difference between the benchmark model with no shocks and this one. The ‘good equilibrium’ and the ‘bad equilibrium’ of the model in Section 1.3 are derived under the assumption that agents know what others will do (different beliefs would be rationalizable). Those equilibria would not survive the inclusion of some uncertainty about others’ future actions. Here, in contrast, uncertainty about the path of \( a \) opens the door to uncertainty about the actions of others. This is arguably an important component of an economy prone to dynamic coordination failures and plays a key role in determining the equilibrium. The iterative process that leads to the elimination of a large set of strategies can be interpreted as agents trying to forecast the forecast of others.\(^{19}\)

\(^{18}\)Since \( a \) follows a Brownian motion, shocks to \( a \) in a small period of time are potentially unbounded, but this is not important for the results. Uniqueness stems from the iterative elimination of dominated strategies, not from unlikely large shocks.

\(^{19}\)For more on higher order beliefs in dynamic coordination games with timing frictions, see Morris (2014).
The equilibrium is characterized by a threshold. A larger $h$ implies that agents are willing to invest for lower values of $a$, as in Figure 1.3. Beliefs about others’ investment decisions are pinned down by fundamentals ($a$) and history ($h$). Shocks to $a_t$ and movements in $h_t$ might affect expectations about others’ actions.

Figure 1.3: Equilibrium with shocks

Let $V(a, h, \tilde{a})$ be the utility gain from choosing High obtained by an agent in state $(a, h)$ that believes others will play according to threshold $\tilde{a}$. Then

$$V(a, h, \tilde{a}) = \int_{0}^{\infty} e^{-(\rho+\alpha)t} E[\pi(a_t, h_t)|a, h, \tilde{a}] dt - \psi,$$

(1.16)

where $E[\pi(a_t, h_t)|a, h, \tilde{a}]$ denotes the expectation of $\pi(a_t, h_t)$ of an agent in state $(a, h)$ that believes others will play according to a threshold $\tilde{a}$. An agent choosing when $a = a^*(h)$ and believing all others will play according to the cutoff $a^*$ is indifferent between High and Low, which means that $V(a^*(h), h, a^*) = 0$, for every $h$.

### 1.4.2 Optimal policy

Proposition 1.3 shows there is a unique rationalizable equilibrium in the model. Although agents face a dynamic coordination problem, a unique set of rationalizable beliefs emerges and, from the point of view of an individual firm, pins down the optimal decision. It is then natural to ask about the beliefs that arise in equilibrium and, in particular, about the inefficiencies that might exist in the model.

The key implications for the optimal stimulus policies are in Proposition 1.4.

**Proposition 1.4.** Optimal policy:

1. **[Optimality of a constant subsidy]** The planner’s solution can be implemented by a constant subsidy of $\psi/\theta$ whenever an agent invests.

2. **[Parallel shift of the threshold]** The planner invests according to a threshold $a^*_P$, such that
for any $h \in [0, 1]$,

$$a_p^*(h) = a^*(h) - \log \left( \frac{\theta}{\theta - 1} \right),$$

where $a^*$ is the threshold for the decentralized equilibrium.

Figure 1.4: Planner’s problem

In principle, it is difficult to characterize the planner’s solution because expectations about the path of $(a, h)$ have to be taken into account when solving for the optimal decision, but the path of $h$ will be optimally chosen by the planner. However, mathematically, the planner’s problem is similar to the agent’s problem in the decentralized equilibrium. At every point in time, there is investment if (1.12) holds, taking into account that the path of $h$ in the future will be determined by a similar choice. The planner thus chooses according to a threshold $a_p^*$ such that (1.12) holds with equality at $a_p^*(h)$ for $h \in [0, 1]$. The only difference is that the planner and agents follow different decision rules.

In the decentralized equilibrium, agents choose according to (1.8). Optimal policy boils down to making agents decide according to the expression in (1.12). The only difference between these expressions is the term $\theta/(\theta - 1)$ multiplying the benefit from investing in (1.12). Hence, the extra incentive for investment the planner would like to provide is a constant proportion of the flow payoff. Alternatively, the planner would like to reduce the investment cost, multiplying it by $(\theta - 1)/\theta$.

Since the investment cost is fixed, a constant investment subsidy equal to $\psi/\theta$ implements the planner’s solution. As an alternative, the planner could top up firms’ revenues, paying $\pi(h_t, a_t)/(\theta - 1)$ for firms in the $\textit{High}$ regime at every time $t$. The bottom line is that there is no special reason for incentivizing investment at times of low economic activity (low $h$) – regardless of how investments are incentivized. These policies disregard any costs imposed by taxation, required to fund subsidies, but in Section 1.4.5 we assume that every unit of subsidy has a small welfare cost and obtain similar results.\textsuperscript{20}

\textsuperscript{20}The result is also robust to the inclusion of some monitoring costs. If the planner faces a cost $c$ to monitor the investment it subsidizes, with $c < \psi/\theta$, an argument similar to the one in Proposition 1.4 shows that the optimal policy can be implemented by a constant subsidy and leads to a different translation of the threshold, but no rotation.
From a social point of view, the problem in the decentralized equilibrium is that investment requires an excessively high benefit. The key result in Proposition 1.4 is that this problem is not more severe when \( h \) is low (or high). Hence there is no special reason for incentivizing investment at times of low (or high) economic activity.

Proposition 1.4 also shows that the planner’s threshold is a translation of the equilibrium threshold, as in Figure 1.4. The slope of the threshold affects the likelihood of a recession and its expected duration.\(^{21}\) Hence Proposition 1.4 also shows that the planner has no reason to affect the expected duration of recessions.

The planner’s solution prescribes no extra stimulus for investment when \( h \) is low because equilibrium beliefs offset the dynamic inefficiency highlighted in Section 1.3. In order to understand this point, it is instructive to look at the case with very small shocks. In the case with no shocks, there are multiple equilibria, so beliefs outside the dominance regions are not determined by the model. In the case with arbitrarily small shocks, productivity \( a \) behaves in a very similar way, but a unique set of beliefs is pinned down by the model. This comparison allows us to understand the beliefs that arise in equilibrium and how they affect policy.

### 1.4.3 The case with very small shocks

The uniqueness result and the expressions for the equilibrium and planner’s thresholds hold for any \( \sigma > 0 \). However, the expressions for the thresholds depend on beliefs about the path of \((a, h)\). In general, these are complicated objects, but in case \( \sigma \to 0_+ \), these beliefs can be determined.

For any \( h_0 \in [0, 1] \), suppose the economy is at the threshold, i.e., at \((a^*(h_0), h_0)\). Where will the economy go? This mathematical problem is studied by Burdzy, Frankel and Pauzner (1998) and their main result is that when \( \sigma \to 0_+ \), the economy will instantaneously move up in the direction of \((a^*(h_0), 1)\) with probability \( 1 - h_0 \) and will move down in the direction of \((a^*(h_0), 0)\) with probability \( h_0 \). This result determines agents’ beliefs at the equilibrium threshold.

In order to understand this result, suppose \( h_0 = 0.1 \). If the economy is just at the right of the threshold, 90% of the agents that get an opportunity to switch will change from the Low to the High regime (and the remaining 10% will stay in the High regime); while if the economy is just at the left of the threshold, 10% of the agents that get an opportunity to switch will change from the High to the Low regime (and the remaining 90% will stay in

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\(^{21}\)If the threshold is close to a vertical line, \( h \) will start to fall when productivity is below some \( a^\dagger \) but firms will resume investing whenever \( a_t > a^\dagger \). A rotation of the threshold that reduces \( a^*(1) \) but raises \( a^*(0) \) implies there will be less occasions where productivity will cross the threshold to the left of \( a^* \) when \( h \) is large, but when that happens, \( h \) is likely to go further down and it will take longer for \( h \) to increase again.
the Low regime). Hence, at the right of the threshold, the economy moves up with speed proportional to $90\% = 1 - h_0$, and at the left of the threshold the economy moves down with speed proportional to $10\% = h_0$. Burdzy, Frankel and Pauzner (1998) show that the probabilities the economy move up or down are proportional to the speeds at each side of $(a^*(h_0), h_0)$. Intuitively, in a very short period of time, shocks to $a$ will make the economy move around the threshold. Since the threshold is negatively sloped, an economy that moves up very quickly when it is at the right of the threshold is likely to find itself sufficiently above the threshold very soon, so that negative shocks to $a$ cannot bring it back to the left side of the threshold.

Using the beliefs implied by the result in Burdzy, Frankel and Pauzner (1998) and the equilibrium condition in (1.16), we get that for any $h_0 \in [0, 1]$, $a^*(h_0)$ solves

$$
(1 - h_0) \int_0^\infty e^{-(\rho + \alpha)t} [\pi(a^*(h_0), h_t^U)] dt + h_0 \int_0^\infty e^{-(\rho + \alpha)t} [\pi(a^*(h_0), h_t^D)] dt = \psi. \quad (1.17)
$$

The solution to the planner’s problem is similar. Using (1.12), we get that for any $h_0 \in [0, 1]$, the planner’s threshold $a^*_P(h_0)$ solves

$$
(1 - h_0) \int_0^\infty e^{-(\rho + \alpha)t} [\pi(a^*_P(h_0), h_t^U)] dt + h_0 \int_0^\infty e^{-(\rho + \alpha)t} [\pi(a^*_P(h_0), h_t^D)] dt = \psi - \frac{\psi}{\theta}. \quad (1.18)
$$

This expression seems very different from (1.15) but yields the same results. For the planner, there is no difference between the cases with no shocks ($\sigma = 0$) or vanishing shocks ($\sigma \to 0_+$). Beliefs about the future are basically the same and the planner can effectively choose the path of $h$.

Figure 1.5: The case with very small shocks

---

22The irrelevance of vanishing shocks for the planner’s solution highlights the point that very small fluctuations in $a$ are not intrinsically important. Their effects on the decentralized equilibrium stem from the determination of beliefs in case $\sigma \to 0_+$. 

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Figure 1.5 summarizes the main results of this paper. The planner chooses to invest if the economy is at the right of $a^*_p$. In case $\sigma \to 0_+$, agents choose according to the threshold $a^*$, which solves (1.17). In case $\sigma = 0$, there are multiple equilibria. In the ‘good equilibrium’, agents believe all others will choose to invest. This is an equilibrium as long as the economy is at the right of $a^*_{opt}$. In the ‘bad equilibrium’, agents believe all others will not invest. This is an equilibrium as long as the economy is at the left of $a^*_{pes}$.

The result in case $\sigma \to 0_+$ is thus completely different from the case with $\sigma = 0$. In case $\sigma \to 0_+$, the slopes of the planner’s and the agents’ thresholds are the same. The slopes of both $a^*_{opt}$ and $a^*_{pes}$ are very different.

Since the productivity parameter moves very slowly when $\sigma \to 0_+$, the difference between the cases $\sigma \to 0_+$ and $\sigma = 0$ must stem from the difference in beliefs around the thresholds. As shown in Figure 1.5, $a^*(0)$ and $a^*_{opt}(0)$ coincide. Hence, beliefs at $(a^*(0), 0)$ and $(a^*_{opt}(0), 0)$ must be the same. Indeed, in the model with shocks, when the economy is at $(a^*(0), 0)$, agents believe others will choose to invest (when given an opportunity). Likewise, $a^*(1)$ and $a^*_{pes}(1)$ also coincide, as agents believe others will not invest when the economy is at $(a^*(1), 1)$ in case $\sigma \to 0_+$. \(^{23}\)

Intuitively, in a neighborhood of the equilibrium threshold $a^*$, optimistic beliefs (i.e., beliefs that agents with a switching opportunity will invest) make perfect sense at $h = 0$, but no sense whatsoever at $h = 1$. Suppose the economy is at $(a^*(0), 0)$. The economy will stay around there as long as $a < a^*(0)$, but any shock that moves $a$ above $a^*(0)$ leads agents to invest and drives the economy up in Figure 1.5. Since the slope of the threshold is negative, as soon as the economy is at $h > 0$, it is at the right of the threshold and hence agents have more incentives to invest. Thus the economy moves up in the direction of $(a^*(0), 1)$. The fundamental asymmetry here is that a small negative shock basically leaves the economy where it is, while a small positive shock drives the economy up in the direction of $h = 1$. The same reasoning implies that in a neighborhood of the equilibrium threshold at $(a^*(1), 1)$, a regime switch is also expected (beliefs are pessimistic).

Agents are indifferent between investing or not at $(a^*(0), 0)$ and at $(a^*(1), 1)$, but for different reasons. Around $(a^*(0), 0)$, productivity is relatively high but the economy is parked in a region of no investment; around $(a^*(1), 1)$, productivity is relatively low but everyone is producing at full capacity. The key feature of the model with shocks is that beliefs at the equilibrium threshold are ‘optimistic’ when economic activity is low and productivity is relatively high, and ‘pessimistic’ when economic activity is high and productivity is relatively low. This point

\(^{23}\)An implication of Burdzy, Frankel and Pauzner (1998) is that in case $\sigma \to 0_+$, the equilibrium threshold connects the ‘good-equilibrium’ threshold at $h = 0$ and the ‘bad-equilibrium’ threshold at $h = 1$ as in Figure 1.5.
does not apply to the model with multiple equilibria and no shocks, where beliefs of agents that are indifferent between investing or not might be ‘optimistic’ or ‘pessimistic’ for any value of \( h \), depending on which equilibrium is considered. When shocks are considered, a unique set of beliefs arises in equilibrium and exactly offsets the dynamic inefficiency, so that there is no special reason for stimulus at times of low economic activity.

The explanation so far has considered the case \( \sigma \to 0^+ \) but Proposition 1.4 shows the result holds for any \( \sigma > 0 \). The intuition for the case with \( \sigma \) bounded away from zero is very similar. When economic activity \( (h) \) is low and agents are around the equilibrium threshold, it is likely that others will soon start to invest. Again, the key asymmetry here is that a movement of \( a \) to the left does not significantly affect the state of the economy, but a movement of \( a \) to the right affects the mass of agents investing, raising demand in the economy and incentives for the following agents with investment opportunities to take them. The recovery is just waiting for a small piece of good news.\(^{24}\)

That does not mean that agents are more optimistic in recessions, in general. The empirical counterpart of the theoretical implication about beliefs is that ‘confidence’ (as measured by surveys) is a leading indicator, as agents anticipate the economy is about to leave the inaction region (or about to enter an investment slump).\(^{25}\)

The binary set of actions is a tractable way to capture non-convexities in firms production. The fundamental assumption here is that these non-convexities generate an inaction region.\(^{26}\) Different (non-convex) technologies should also imply that good news will have large effects on a firm’s production only if the current level of production is low.

The cyclical behavior of beliefs offsets the dynamic inefficiency shown in the case with no shocks (Figure 1.2). For \( \sigma > 0 \), the distance between the planner’s and the agents’ threshold is independent of \( h \), so subsidies do not depend on \( h \) as well. The only difference between the planner’s solution and the decentralized equilibrium is the monopoly distortion in the investment decision. When beliefs are uniquely determined by the model, planner and agents solve a very similar problem. At every \((a, h)\), investment is undertaken if its expected return pays off and the equilibrium (or planner’s) threshold is a fixed point. “Pays off” means different things for agents and planner but the ratio is constant since the only difference is the externality from market power.

\(^{24}\)Graphically, for \( \sigma \) bounded away from 0, the planner’s and agents’ equilibrium thresholds would be parallel to each other as in Figure 1.5, but \( \alpha^* \) would not touch \( \alpha_{opt}^* \) and \( \alpha_{pes}^* \) at any point.

\(^{25}\)Confidence variables appear in many leading indicators indexes, such as those in the OECD System of Composite Leading Indicators.

\(^{26}\)For a model where this inaction region arises as a result from fixed adjustment costs and may give rise to coordination failures, see Guimaraes, Machado and Ribeiro (2016).
1.4.4 The case with vanishing frictions

In case of vanishing frictions \((\alpha \to \infty)\), the economy moves very fast from \(h = 0\) to \(h = 1\) but agents’ horizons also become very short.\(^{27}\) In the model with no shocks, agents take \(h\) into account in their decisions, regardless of whether we assume fully optimistic or fully pessimistic beliefs. In contrast, the planner does not take \(h\) into consideration. Moving the economy to a different regime takes very little time and hence the transition is unimportant. The threshold from the planner’s problem converges to a vertical line, as in Figure 1.6. This result holds for any \(\sigma > 0\).

Figure 1.6: The case with vanishing frictions

An implication of Proposition 1.4 is that in equilibrium agents also play according to a vertical threshold. History thus becomes irrelevant. For a large \(\alpha\), an agent at the equilibrium threshold and \(h = 0\) knows the economy will move up with probability 1, while an agent at the equilibrium threshold and \(h = 1\) is sure the economy will move down. They don’t know their ‘position in the queue’, i.e., when they will have the next opportunity for revising their behavior (and thus how much time will have elapsed and the value of \(h\) when they can choose again).

On the one hand, the agent at \(h = 0\) will experience a lower range of values of \(h\) than the agent at \(h = 1\) (before their next opportunity to choose again): the one starting at \(h = 0\) will experience \(h\) from 0 to \(h^1\), where \(h^1\) is uniformly distributed in \([0, 1]\); the agent starting at \(h = 1\) will experience \(h\) from 1 to \(1 - h^1\). On the other hand, for the agent at \(h = 0\), the economy moves up very quickly at lower values of \(h\), but slowly as \(h\) approaches 1, so the last firms to change their decision will spend relatively more time at high values of \(h\). As it turns out, both effects exactly cancel each other, so the agents at \(h = 0\) and \(h = 1\) are indifferent between investing and not investing for the same value of \(a\).

This result might sound obvious owing to a simple but incorrect intuition: the economy

\(^{27}\)When taking the limit \(\alpha \to \infty\), we fix the user cost of capital \((\rho + \alpha)\psi\), not the investment cost \(\psi\).
can move very quickly to the $h = 1$ regime, so $h$ does not affect the result. However, this intuition would also apply to the model with multiple equilibria, and the results in that case are completely different, $h$ does matter as $\alpha \to \infty$. That is because although the economy can move very quickly from a low $h$ to a high $h$, the next opportunity to invest also comes very quickly, so only the very short run matters for an agent’s decision.

1.4.5 Implementation

Proposition 1.4 shows that a constant subsidy implements the first best. However, in a large set of states, much less generous subsidies would be enough to coax agents to invest. This leads to the following question: what if every unit of subsidy has a small welfare cost $\varepsilon \approx 0$? It is not difficult to show that the government will use minimal spending policies, as in Definition 1.1.

**Definition 1.1.** Let $a^*$ be an equilibrium of the game and $a_p^*$ a continuous function such that $a_p^*(h) < a^*(h)$, for every $h$. Let $\hat{a}$ be the boundary where an agent is indifferent between High and Low when others are playing according to $a_p^*$. The function $\varphi(a, h)$ is the minimal spending policy that implements $a_p^*$ if

$$\varphi(a, h) = \begin{cases} 
\psi - \int_0^\infty e^{-(\rho+\alpha)t}E[\pi(a_t, h_t)|a, h, a_p^*]dt & \text{if } a_p^*(h) \leq a \leq \hat{a}(h), \\
0 & \text{otherwise}. 
\end{cases} \quad (1.19)$$

Figure 1.7 shows three thresholds: $a_p^*$ is the threshold implemented by the policy, $\hat{a}$ is the best response of a player that believes others will play according to $a_p^*$ and $a^*$ is the equilibrium threshold without intervention. By definition, $a^*$ is the best response to others playing according to $a^*$. Now, the sheer change in beliefs affects agents’ strategies: once they believe others will play according to $a_p^*$, they will be indifferent between High and Low at a threshold $\hat{a}$ such that $\hat{a}(h) < a^*(h)$ for all $h \in [0, 1]$.

Figure 1.7: Example of minimal spending policy

A government following a minimal spending policy is committed to give an investment
subsidy to each agent in the region between $a_p^*$ and $\hat{a}$ (the gray area in figure 1.7). The subsidy $\varphi(a, h)$ makes her indifferent between choosing High and Low given others will play according to $a_p^*$. Under those beliefs, playing according to $a_p^*$ is a best response under this policy, so $a_p^*$ is an equilibrium. Interestingly, no subsidies are needed in the area between $\hat{a}$ and $a^*$.\footnote{The equilibrium under the minimal spending policy is no longer unique. If agents believe others will play according to $a^*$ their best response is to play according to $a^*$ as well, thus the policy has no effect at all. The amount of subsidies required to coax agents to invest depends on whether they expect others to respond to the stimulus policy. However, the government could implement this allocation through a contingent subsidy that would be essentially equivalent to a minimal spending policy: a large subsidy contingent on others not investing, and the subsidy prescribed by the minimal spending policy in case others invest as well.}

Proposition 1.5 shows that minimum spending policies do not affect the main result of the paper.

**Proposition 1.5.** In the model of Section 1.2 with minimal spending policies, the maximum optimal subsidy is $\psi/\theta$ for all $h \in [0, 1]$.

The result is intuitive. Under minimal spending policies, investment subsidies are equal to the minimum between how much the planner is willing to pay and how much is required to coax agents to invest. The result from Proposition 1.5 thus follows from the planner’s willingness to subsidize being independent of $h$ (Proposition 1.4).

### 1.5 The model with endogenous hazard rates

The model so far has considered an exogenous rate $\alpha$ for the arrival of switching opportunities. We now modify the model in order to endogenize the switching rate.

As before, a firm produces $A_t x_H$ in the High regime and $A_t x_L$ in the Low regime. At each moment, a firm in the Low regime chooses the hazard rate of switching opportunities $\alpha_L t \in [0, \overline{\alpha}]$, with $\overline{\alpha} < \infty$, subject to a cost $c(\alpha_L t)$. The cost function is increasing, continuous and convex. This assumption replaces the fixed cost of investing in the previous model. A firm in the High regime switches to the Low regime with an exogenous hazard rate $\alpha^H$.

In one interpretation, $c(\alpha_L t)$ is the amount of resources a firm spends in the search for new ideas of production methods, and a useful idea appears at rate $\alpha_L t$. In another interpretation, equipments break or become obsolete at rate $\alpha^H$ and are immediately replaced, but the time until a new equipment can be used in production is stochastic and depends on the amount of resources allocated to this end, $c(\alpha_L t)$. Alternatively, as in Howitt and McAfee (1992), a firm in state Low searches for a match and the cost $c(\alpha_L t)$ is increasing in the search intensity. Owing to attrition, matches are broken at rate $\alpha^H$.
To ease notation, define:

\[
g(h_t) = \left( h_t x_H^{\theta - 1} + (1 - h_t) x_L^{\theta - 1} \right)^{\frac{1}{\theta - 1}} \left( x_H^{\theta - 1} - x_L^{\theta - 1} \right)
\]

In order to find the decentralized equilibrium, we can apply the results in Frankel and Burdzy (2005). Their Theorems 4 and 5 imply that the relative value of being in the High regime is given by:

\[
\Delta V_\tau = E_\tau \int_\tau^\infty e^{-\int_\tau^v (\rho + \alpha_L^L + \alpha_H^H) dv} \left[ e^{\alpha_L^L g(h_s) + c(\alpha_L^L)} \right] ds,
\]

agents choose \( \alpha_L^L \) in order to maximize

\[
\alpha_L^L E_\tau \int_\tau^\infty e^{-\int_\tau^v (\rho + \alpha_L^L + \alpha_H^H) dv} \left[ e^{\alpha_L^L g(h_s) + c(\alpha_L^L)} \right] ds - c(\alpha_L^L),
\]

and this problem yields essentially a unique equilibrium.

The expression in (1.20) shows that firms’ optimal choice of hazard rates depends on the expected gains from switching. The relative value of being in the High regime is increasing in expected values of \( a \) and \( h \), which depend positively on their current values. Hence, larger \( a \) and larger \( h \) will induce higher hazard rates.

We now proceed to solve the planner’s problem. Since the cost function is convex, the planner chooses the same hazard rate for every firm in regime Low. Welfare in this economy is thus given by

\[
E_\tau(W) = E_\tau \int_\tau^\infty e^{-\rho(t-\tau)} \left[ Y(a_t, h_t) - (1 - h_t)c(\alpha_L^L) \right] dt,
\]

where \( Y(a, h) \) is given by (1.6). Suppose the planner is following the optimal plan and consider the following deviation: change \( \alpha_L^L \) to \( \tilde{\alpha} \) at \( \tau \) for an infinitesimal period \( dt \) and keep future choices for every realization of the Brownian path in the future unchanged. This affects current costs and output net of switching costs for all \( s > \tau \). Since there are \( 1 - h_t \) agents at the Low state, costs change by

\[
[c(\tilde{\alpha}) - c(\alpha_L^L)](1 - h_t)dt
\]

and the immediate effect on \( h_t \) is

\[
dh_\tau = (\tilde{\alpha} - \alpha_L^L)(1 - h_t)dt.
\]

This effect dies out in time:

\[
dh_s = dh_\tau - \int_\tau^s dh_v \left( \alpha_L^L + \alpha_H^H \right) dv,
\]
which implies that
\[ dh_s = dh_\tau e^{-\int_\tau^s (\alpha_L^z + \alpha_H^z) \, dv}. \] 

(1.23)

The effect on output net of switching costs for \( s > \tau \) is
\[ E_\tau \int_\tau^\infty e^{-\rho(s-\tau)} \left\{ \left[ \frac{\partial Y(h_s, a_s)}{\partial h_s} + c(\alpha_s^L) \right] dh_s \right\} ds. \]

(1.24)

This deviation cannot be profitable. Hence, putting together (1.21), (1.22), (1.23) and (1.24) and using (1.10) it must be that
\[ (\tilde{\alpha} - \alpha_L^L) E_\tau \int_\tau^\infty e^{-\int_\tau^s (\rho + \alpha_L^z + \alpha_H^z) \, dv} \left[ \frac{\theta}{\theta - 1} e^{\alpha_s^z g(h_s)} + c(\alpha_s^L) \right] ds - \left[ c(\tilde{\alpha}) - c(\alpha_L^L) \right] \leq 0, \]

for any \( \tilde{\alpha} \in [0, \alpha_L^L] \). That is equivalent to stating that a necessary condition for the planner’s solution is that \( \alpha_L^L \) must maximize
\[ \alpha_L^L E_\tau \int_\tau^\infty e^{-\int_\tau^s (\rho + \alpha_L^z + \alpha_H^z) \, dv} \left[ \frac{\theta}{\theta - 1} e^{\alpha_s^z g(h_s)} + c(\alpha_s^L) \right] ds - c(\alpha_L^L). \]

(1.25)

This expression is very similar to (1.20), the only difference is the term \( \theta/(\theta - 1) \) in the integral. Mathematically, finding the solution to the planner’s problem is thus equivalent to finding a solution to a game played by agents that maximize (1.25). We can thus apply the results in Frankel and Burdzy (2005) and obtain a result analogous to Proposition 1.4.

**Proposition 1.6.** The planner’s solution can be implemented by a subsidy equal to \( c(\alpha_L^L)/\theta \).

As in Proposition 1.4, the optimal policy prescribes a subsidy equal to a constant fraction of the switching cost (or, equivalent, a payment of \( e^{\alpha_s^z g(h_s)}/(\theta - 1) \) to all firms in the High regime at every \( t \)). The important implication is that the planner is not particularly concerned about incentivizing investment when \( h \) is low.

1.6 Concluding remarks

This paper proposes a macroeconomic model that captures in a simple way the dynamic coordination problem arising from demand externalities and fixed investment costs. From a substantive point of view, the main result of the paper is the absence of a special reason for subsidies at times of low economic activity – a constant subsidy implements the planner’s solution.
From a methodological point of view, the paper highlights the importance of understanding beliefs that arise in equilibrium for policy analysis. The main result of the paper relies on the link from the business cycle to agents’ beliefs about economic activity.

1.A Proofs

1.A.1 Proof of Proposition 1.1

Consider an agent deciding at time normalized to 0 who believes that every agent that will get an opportunity to change regime will choose Low. He assigns probability 1 that the path of \( h_t \) will be \( h_t^D = h_0 e^{-\alpha t} \), which is independent of \( a \). Thus, choosing High raises his payoff by

\[
\bar{U}(a, h_0) = \int_0^\infty e^{-(\rho + \omega)t} \pi(a, h_t^D) dt - \psi
\]

\[
= e^a \left( \frac{\theta-1}{H} - \frac{\theta-1}{L} \right) \int_0^\infty e^{-(\rho + \omega)t} \left( h_t^{U} \frac{\theta-1}{H^2} + (1 - h_t^D) \frac{\theta-1}{L^2} \right) \frac{1}{\pi} dt - \psi.
\]

Therefore this agent will choose High if \( \bar{U}(a, h_0) \geq 0 \). Now, \( \bar{U}(a, h_0) \) is continuous and strictly increasing in \( a \), \( \lim_{a \to \infty} \bar{U}(a, h_0) = \infty \), and \( \lim_{a \to -\infty} \bar{U}(a, h_0) = -\psi \). Thus for any \( h_0 \), there is \( a = a_{pes}^*(h_0) \) such that \( \bar{U}(a, h_0) = 0 \). Since \( \bar{U}(a, h_0) \) is strictly increasing in \( a \), for any \( a' > a_{pes}^*(h_0) \) we have \( \bar{U}(a', h_0) > 0 \) and thus choosing High is a strictly dominant strategy (any other belief about the path of \( h_t \) will raise the relative payoff of choosing High). Notice that \( \bar{U}(a, h_0) \) is strictly increasing in both \( a \) and \( h_0 \) and thus \( a_{pes}^*(h_0) \) is strictly decreasing.

A similar argument proves that there exists a strictly decreasing threshold \( a_{opt}^*(h_0) \) such that if \( a < a_{opt}^*(h_0) \), Low is a dominant action. Consider an agent who believes others will choose High after him. He believes that the motion of \( h_t \) will be given by \( h_t^U = 1 - (1 - h_0)e^{-\alpha t} \), so choosing High instead of Low raises his payoff by

\[
U(a, h_0) = \int_0^\infty e^{-(\rho + \omega)t} \pi(a, h_t^U) dt - \psi
\]

\[
= e^a \left( \frac{\theta-1}{H} - \frac{\theta-1}{L} \right) \int_0^\infty e^{-(\rho + \omega)t} \left( h_t^U \frac{\theta-1}{H^2} + (1 - h_t^U) \frac{\theta-1}{L^2} \right) \frac{1}{\pi} dt - \psi.
\]

This agent will choose Low whenever \( U(a, h_0) < 0 \) and, as in the previous case, we can show that there exists a strictly decreasing threshold \( a_{opt}^* \) such that if \( a < a_{opt}^*(h_0) \), Low is a dominant action. Since for every \( h_0 \) and \( t > 0 \) we have \( h_t^U > h_0 > h_t^D \), \( U(a, h_0) > U(a, h_0) \). This implies \( a_{pes}^*(h_0) > a_{opt}^*(h_0) \).

Take a pair \( (a, h_0) \) such that \( a_{opt}^*(h_0) < a < a_{pes}^*(h_0) \). Since \( a < a_{pes}^*(h_0) \), if an agent
believes that the path of \( h_t \) will be \( h_t^D \), then \( U(a,h_0) < 0 \) and thus his optimal strategy is to play \( Low \). Therefore this belief is consistent and the strategy profile where every player plays \( Low \) is an Nash equilibrium. Likewise, since \( a > a_{opt}^*(h_0) \) the strategy profile where every player plays \( High \) is also a Nash equilibrium. Hence, there is multiplicity in this set. \( \square \)

1.A.2 Proof of Proposition 1.2

Since the planner’s expected discounted payoff (given a chosen threshold) is a continuous function of \( \sigma \), solving the planner’s problem with \( \sigma = 0 \) or solving it with \( \sigma > 0 \) and then taking the limit of the solution when \( \sigma \to 0 \) must yield the same planner’s threshold. Proposition 1.4 (to be proved next) implies that for any \( \sigma > 0 \), the planner’s threshold is equivalent to the agent’s threshold if the investment cost were \( \psi - \psi / \theta \) (instead of \( \psi \)). It follows from Theorem 2 in Burdzy, Frankel and Pauzner (1998) that when \( \sigma \to 0 \), the limit of the agents threshold is given by the indifference condition of an agent that believes \( h_t \) will either go up forever (i.e., \( h_t = h^U_t \), for every \( t \)) with probability \( 1 - h_0 \) or go down forever (i.e., \( h_t = h^L_t \), for every \( t \)) with probability \( h_0 \). Therefore, the planner’s threshold \( a^*_p(h_0) \) is given by the solution to equation (1.26) in Section 1.4.3. Rearranging, that expression becomes

\[
e^{\alpha^*_p(h_0) + \log(\pi^*_h)} \left[ (1 - h_0) \int_0^\infty e^{-(\rho + \alpha)t} g(h_t^U)dt + h_0 \int_0^\infty e^{-(\rho + \alpha)t} g(h_t^D)dt \right] = \psi, \tag{1.26}
\]

where \( g(h) \) is such that \( \pi(a, h) = e^{\alpha} g(h) \).

**First statement:** The expression in (1.26) implies that the translation \( \hat{a}^*_p(h_0) \equiv a^*_p(h_0) - \log(\theta/\theta - 1) \) of the planner’s threshold is a curve that lies between \( a_{opt}^*(h_0) \) and \( a_{pes}^*(h_0) \). Moreover, \( \hat{a}^*_p(h_0) = a_{opt}^*(h_0) \) only for \( h_0 = 0 \) and \( \hat{a}^*_p(h_0) = a_{pes}^*(h_0) \) only for \( h_0 = 1 \).

Since \( a_{opt}^*(h) < a_{pes}^*(h) \) for any \( h \in [0,1] \),

\[
\hat{a}^*_p(0) = a_{opt}^*(0) < a_{pes}^*(0) \quad \text{and} \quad a_{opt}^*(1) < \hat{a}^*_p(1) = a_{pes}^*(1),
\]

which yields

\[
\hat{a}^*_p(0) - \hat{a}^*_p(1) < a_{opt}^*(0) - a_{opt}^*(1) \quad \text{and} \quad \hat{a}^*_p(0) - \hat{a}^*_p(1) < a_{pes}^*(0) - a_{pes}^*(1),
\]

and since \( \hat{a}^*_p(h) = a^*_p(h_0) - \log(\theta/\theta - 1) \), we get the claim.

**Second statement:** Since \( \pi(a, h) \) is increasing in \( h \) and the planner cares about the discounted sum of agents wealth minus the investment cost, the planner’s threshold must lie entirely to the left of the of \( a_{opt}^*(h_0) \). If agents that believe everyone will invest in the future are willing
to invest, so must be the planner, due to the positive externality of investment.

Suppose agents always play according to the best equilibrium (i.e., the economy is always in the ‘good equilibrium’). Let \( \epsilon \) be the subsidy required to coax agents to invest at some state \((a_0, h_0)\) to the left of the agents threshold \(a^*_\text{opt}(h_0)\). The subsidy \( \epsilon \) must be such that it makes agents indifferent between investing or not:

\[
e^{a_0} \int_0^\infty e^{-(\rho+\alpha)t}g(h_t^U)dt + \epsilon = \psi = e^{a^*_\text{opt}(h_0)} \int_0^\infty e^{-(\rho+\alpha)t}g(h_t^U)dt,
\]

where the last equality follows from the fact that agents are indifferent at the threshold \(a^*_\text{opt}(h_0)\).

From the equations above we get

\[
\epsilon = \psi \left(1 - e^{-|a_0 - a^*_\text{opt}(h_0)|}\right),
\]

and thus, the subsidy \( \epsilon \) is a increasing function of the distance \(|a_0 - a^L(h_0)|\). Since the distance between the planner’s threshold and \(a^*_\text{opt}(h_0)\) is larger at \(h = 0\) than at \(h = 1\), we obtain the result.

The same argument applies if we assume agents will always play according to the ‘bad equilibrium’.

\[\square\]

1.A.3 Proof of Proposition 1.3

Proposition 1.3 follows from Frankel and Pauzner (2000), but to apply their results we need to show that for sufficiently high (low) \(a\) choosing the High (Low) regime is the optimal choice, regardless of the actions of others. Consider an agent deciding at some state \((a_0, h_0)\) that expects \(h_t = 0\), for every \(t \geq 0\), with probability one. Her relative gain of choosing regime High is given by

\[
U(a_0) = x^L \left(\frac{a^{-1}}{x_H^2} - \frac{a^{-1}}{x_L^2}\right) \int_0^\infty e^{-(\rho+\alpha)t}E_0[e^{a_0}] dt - \psi.
\]

Since \(a_t|a_0 \sim N(a_0, \sigma^2 t)\), \(E_0[e^{a_0}] = e^{a_0 + 0.5\sigma^2 t}\). Therefore

\[
U(a_0) = x^L \left(\frac{a^{-1}}{x_H^2} - \frac{a^{-1}}{x_L^2}\right) e^{a_0} \int_0^\infty e^{-(\rho+\alpha-0.5\sigma^2)t} dt - \psi,
\]
which implies that there is \( \tilde{a} \) such that \( U(a_0) > 0 \) for every \( a_0 > \tilde{a} \) and every belief over \( h_t \). A similar reasoning shows that there exists \( \underline{a} \) such that for every \( a_0 < \underline{a} \) the relative gain of choosing regime \( \text{Low} \) is negative.

### 1.A.4 Proof of Proposition 1.4

**First statement:** The solution to the planner’s problem prescribes investment if the condition in (1.12) is satisfied (and no investment if the inequality in (1.12) is reversed). Multiplying both sides of (1.12) by \((\theta - 1)/\theta\) yields the condition for an agent to invest in (1.16) in an economy where the cost for investing is \( \psi - \psi/\theta \).

**Second statement:** Since \( \pi(a_t, h_t) \) can be written as \( e^{a_t g(h_t)} \), for some function \( g(\cdot) \), we can rewrite condition (1.12) as

\[
\int_{\tau}^{\infty} e^{-(\rho+\alpha)(t-\tau)} E_{\tau} [e^{a_t + \log(\frac{\theta}{\theta - 1})}] g(h_t) dt > \psi. \tag{1.27}
\]

Define \( b_t = a_t + \log(\theta/(\theta - 1)) \) and consider the planner’s problem in the \((b, h)\)-space. The expression for the planner’s decisions is identical to the expression in (1.16) for the agents’ decisions in the decentralized equilibrium (in the \((a, h)\)-space). Moreover, the law of motion for \( b_t \) is exactly the same as the law of motion for \( a_t \). Therefore, the solution for the problem must be the same as well.

We know there is a unique decentralized equilibrium given by a threshold \( a^* \), hence \( a^* = b^* \), which implies \( a^*(h) = a^*_p(h) + \log(\theta/(\theta - 1)) \) and yields the claim.

### 1.A.5 Proof of Proposition 1.5

For a given \( h \in [0, 1] \) the maximum amount of subsidies the planner have to pay to implement his threshold \( a^*_p(h) \) is \( \varphi(a^*_p(h), h) \), since agents payoffs are increasing in \( a \). But from Proposition 1.4 we know that an agent will be indifferent between investing and not investing at \((a^*_p(h), h)\) if the cost is \( \psi - \psi/\theta \) and he believes that the others will play according to \( a^*_p(h) \). Thus, \( \varphi(a^*_p(h), h) = \psi/\theta \), which is independent of \( h \).

### 1.A.6 Proof of Proposition 1.6

The solution to the planner’s problem prescribes a switching rate \( \alpha^L_\tau \) that maximizes (1.25). Multiplying (1.25) by \((\theta - 1)/\theta\) yields the agents’ objective function for an economy where the cost is \( c(\alpha^L_\tau)(\theta - 1)/\theta \) (see the expression in (1.20)).

\[30\]
Chapter 2

Financial crises, coordination failures and disasters

Abstract

Why do some financial crises lead to macroeconomic disasters, while others barely affect the real economy? How should policy makers deal with such extreme events? This paper proposes a model to study unusually deep financial crises. Disaster episodes arise as the consequence of demand-driven coordination failures on the productive sector, and weak balance sheets on the financial sector. There is an endogenous dynamic feedback between intermediaries’ balance sheets and coordination. Coordination failures alone have small effects, but once one takes into account the dynamic feedback from the financial sector, they have a large negative impact on asset prices, investment and welfare, even if the economy is in good times and they rarely happen. Macroprudential policies that increase intermediaries’ returns during disasters greatly improve welfare, growth and financial stability, almost mitigating the negative effects of coordination failures.

Keywords: coordination failures, financial frictions, financial crises.
Jel Classification: D84, E32, G10.

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2.1 Introduction

Some financial crises have little effect on economic activity, while others are real economic disasters with deep and long-lasting effects. A recent example of the latter is the 2007-2009 crisis. The financial turmoil quickly spread to the real economy and by 2013, US output was 13% below its trend path from 1990 to 2007. Many macroeconomic aggregates have not fully recovered yet.\footnote{Hall (2015) nicely documents the aftermath of the great recession.} Such events are rare and seem to build up in the background, with no apparent shock that justifies them at the time they happen. But their immediate consequences are startling and evident: very low economic activity and a deeply harmed financial sector. If agents somehow anticipate such severe crises, their effects may go far beyond what we observe during those episodes, affecting agents’ decisions even in tranquil times. This paper raises the following questions: What are the consequences of those episodes for asset prices, economic growth and welfare? What role does the financial sector play in such events and how should it be regulated? To tackle these and other questions, a theory of endogenous disasters based on coordination failures is presented. Recessions start with financial distress, and sometimes spread strongly to the real sector, deeply affecting economic activity.

The environment consists of a standard stochastic growth model, with two additional ingredients that are at the heart of the proposed mechanism: financial frictions and coordination frictions. Financial frictions force financial intermediaries to invest in risky assets while funding themselves through risk-free debt. That implies that they use leverage in equilibrium, exposing their balance sheets to aggregate risk and requiring them to cut down intermediation when negative shocks hit. Coordination frictions affect the productive sector. Due to demand externalities, a firm’s decision to increase its production depends on its expectation of aggregate demand, which in turn depends on the production of other firms. Firms may want to scale up their production only if they believe others will do the same. This opens the possibility of the economy getting trapped in a self-fulfilling regime with low economic activity and low aggregate demand. Disaster episodes, understood as periods where output is significantly below its potential, are times of low demand expectations and weak intermediaries’ balance sheets. The dynamic interaction between financial frictions of intermediaries and coordination frictions of producers turns out to be a powerful propagation mechanism.

How are financial frictions and coordination frictions linked? When capital is efficiently allocated, firms are more productive and more likely to increase production, enhancing coordination. Financial intermediaries have expertise in channeling capital from households to firms. Thus, when intermediaries have strong balance sheets and intermediate a lot of capital, firms have good projects and are more likely to coordinate on the high regime. If
intermediaries are not channeling funds from households to firms, good projects do not find the required capital and there are a lot of firms with bad projects out there that do not have much to gain by increasing the utilization rate of capital. Even if a producer happens to have good projects and capital available, she may cut down production because she knows others are not likely to do the same. Demand expectations are low, coordination is hard.

Importantly, coordination failures also affect the financial sector, creating an endogenous two-way feedback between intermediaries’ balance sheets and firms’ decisions. When the economy enters a coordination trap, firms reduce their demand for capital services. Intermediaries’ capital returns and asset prices fall, harming intermediaries’ balance sheets. Moreover, it turns out that in equilibrium only intermediaries’ capital returns are affected by coordination failures, which contributes to a larger relative reduction in their capital demand. Therefore, intermediaries respond to coordination failures by intermediating even less capital, further discouraging coordination.

The dynamic feedback is also very important: coordination failures at a given date lower intermediaries’ expected net worth in the future, further lowering intermediation and asset prices in the present. Similarly, the mere expectation of a coordination failure at some future date reduces asset prices and intermediation in the present. If intermediaries expect a coordination failure in the future, they cut down their demand for capital, reducing intermediation and confirming their initial expectations. In equilibrium, intermediaries’ precautionary behavior ends up precipitating a coordination failure.

The model is solved globally, building on techniques similar to those in Brunnermeier and Sannikov (2014). Differently from that paper, here there are strategic complementarities among financial intermediaries and their returns depend directly on what other intermediaries are doing. If intermediaries expect others to hold a lot of capital, firms coordinate on high capacity, capital returns are high, balance sheets are strong, and consequently intermediaries are happy to hold capital. The converse is also true and therefore expectations are self-fulling. It implies that on top of the multiple equilibria on firms’ capacity decisions, there are multiple dynamic equilibria on the financial sector. Firms multiplicity can be dealt with using global games selection, as in Morris and Shin (2001). But as long as some mild restrictions are satisfied, the main results do not depend on the specific selection procedure. To deal with multiplicity on the financial sector, a numerical procedure to compute the two extreme equilibria is developed. It is verified that the two are close and the results are similar regardless of the equilibrium selected.

Two types of financial crises emerge from the model. The first type is interpreted as mild financial crises. Financial intermediaries’ balance sheets are weak and some of the capital in
the economy does not find its first-best use, but firms still have projects good enough to be able to coordinate on the high regime. The second kind is associated with coordination failures. Intermediaries deeply reduce their investments in risky assets, capital is poorly allocated, firms have low demand expectations and operate way below its full potential. The propagation from the financial sector to the real sector is non-linear, generating some heterogeneity across financial crises that is consistent with the empirical evidence.\(^2\)

Coordination failures episodes introduce some non-monotonic local effects in the model. As negative shocks hit the economy, financial intermediaries assets lose value and their leverage increases, even though they are also borrowing less. Initially, they also require a higher risk premium to hold risky assets, increasing capital returns. But as soon as the economy enters a coordination trap, capital returns for intermediaries fall, leading intermediaries to be willing to hold a smaller amount of risky assets and reducing their leverage. Sharpe ratios follow a pattern similar to leverage. Asset prices are very depressed in a coordination trap, reducing investment and growth.

To analyze the global effects of disasters, the model is calibrated for the US economy. The model is able to capture well the empirical regularities observed in the data, matching not only standard deviations and correlations, but also the long-run growth of many variables.

In equilibrium, disasters have a huge negative impact on investment, assets prices, output and welfare. Asset prices are much lower than in the model without coordination failures, even in good times. Low asset prices harm investment and growth, ultimately reflecting in lower welfare. Moreover, coordination failures increase asset price volatility, raising the probability of bad states. Surprisingly, coordination failures significantly reduce welfare even when they are extremely unlikely. The model without coordination failures is compared to a sequence of models where the expected time to reach a coordination trap gets extremely large (say, a thousand years). Those terrible states where firms produce way below their potential affect prices, allocations and welfare even in tranquil times. The welfare losses operate through lower asset prices (in bad and good times) and through banks’ precautionary behavior that reduces their intermediation. Coordination failures alone have small effects, but they become large once the feedback from the financial sector is taken into account. Global effects matter.

The model has many sources of inefficiencies and there is room for policy interventions. The inefficiencies are deeply related to two market failures: firms’ monopoly power and missing markets for aggregate risk (no equity issuance). Firms do not internalize the positive impact that increasing their production has on other firms profits. Banks do not internalize the

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\(^2\)Looking at approximately forty financial crises across many countries, Krishnamurthy and Muir (2016) document that while the average three-year contraction in those episodes is 2.6%, the standard deviation of this rate is 8.5%.
positive effect that higher leverage has on coordination, nor the impact that leverage has on aggregate volatility. Macroprudential policies that increase banks’ returns in disaster states can greatly increase welfare, almost mitigating the inefficiencies caused by coordination failures.

Previous work, such as Phelan (2016), often finds that in models with financial frictions intermediaries take too much leverage in bad times. The story goes like this. Leverage amplifies the effect of negative shocks on intermediaries’ wealth, increasing aggregate volatility and the probability of bad states. Intermediaries do not internalize this, and policies that reduce their leverage in bad times can improve welfare. Here, instead, a policy that increases banks’ leverage in bad times and keeps it unchanged in good times increases welfare. The reason is that the effect of leverage on volatility goes on the other direction in this paper. There are two effects at play: On one hand, leverage can increase volatility by making intermediaries more exposed to shocks. On the other hand, by leveraging up intermediaries can avoid a coordination failure (disaster states). The latter contributes to lower volatility, since coordination traps happen less often and sudden movements in asset prices are avoided. The effect of leverage on the occurrence of coordination failures dominates, and higher leverage in bad times can increase not only welfare, but also financial stability.

Literature. The kind of coordination problem studied here is present in the seminal contributions of Kiyotaki (1988), Cooper and John (1988) and Murphy, Shleifer and Vishny (1989). Recent papers along this line, such as Guimaraes and Machado (2017) and Schaal and Taschereau-Dumouchel (2016b), use equilibrium selection techniques and are able to generate strong and long-lasting recessions.\textsuperscript{3}

In Schaal and Taschereau-Dumouchel (2016b), there is a feedback between capital accumulation and coordination. When savings are low, firms are less likely to coordinate on high capacity, which further decreases capital accumulation. In Guimaraes and Machado (2017) firms adjust their capacity sequentially and the economy may get stuck in a coordination trap simply because no firm wants to be the first to switch to a high regime. Schaal and Taschereau-Dumouchel (2016a) explore the interaction of demand externalities and labor market frictions: high aggregate demand leads to more vacancy posting, which leads to lower unemployment and higher demand.\textsuperscript{4} Here, the feedback comes from banks’ balance sheets to coordination. The choice of coordination failures to generate endogenous disasters is motivated by Schaal and Taschereau-Dumouchel (2016b), who show in their RBC model that coordination

\textsuperscript{3}While Schaal and Taschereau-Dumouchel (2016b) use a global games approach, Guimaraes and Machado (2017) use the techniques of Frankel and Pauzner (2000). See also Guimaraes, Machado and Ribeiro (2016).

\textsuperscript{4}Similarly, in Kaplan and Menzio (2016) people spend more time searching for lower prices when unemployment is high, which reduces firms’ incentives to increase production.
failures offer a good explanation for the aftermath of the Great Recession.

The consequences of coordination frictions and multiple equilibria in macroeconomics have been explored through many different channels. A branch of the literature has relied on “sunspots” to drive agents’ expectations in real business cycles models, creating the possibility of agents coordinating on an inefficient equilibrium. Early contributions along that line include Benhabib and Farmer (1994) and Farmer and Guo (1994). Multiple equilibria in macroeconomics can arise from the interplay of productivity growth and aggregate demand (Benigno and Fornaro, 2016); Taylor rules and the zero lower bound (Benhabib, Schmitt-Grohé and Uribe, 2001); decentralized trading and precautionary savings (Chamley, 2014); to name a few.

There is a large literature that incorporates financial frictions in macroeconomic models, as in the seminal papers of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). As in this literature, this paper has some kind of friction that prevents agents from issuing fully state-contingent financial claims. More specifically, intermediaries are not allowed to issue new equity and need to fund themselves exclusively through debt. This is meant to capture some incentive problem that requires banks to have some “skin in the game”, as in Townsend (1979) and Hart and Moore (1994), among others. Due to the highly non-linear nature of coordination failures, the model needs to be solved away from the steady-state, and the continuous time approach of Brunnermeier and Sannikov (2014) is employed. From a technical point of view, the main difference from Brunnermeier and Sannikov (2014) is that here coordination failures introduce externalities across banks, since capital returns depend directly on the amount of capital held by other banks.

Gertler and Kiyotaki (2015) develop a macroeconomic model in which bank runs are possible if intermediaries’ balance sheets are weak. Using a mechanism similar to theirs, Clymo (2016) generates banking crises using the continuous time approach of Brunnermeier and Sannikov (2014). Although the economic mechanism here is very different, the consequences of banking crises are similar to those of coordination failures. Output and asset prices fall, and capital is inefficiently allocated. Differently from those papers, though, sunspot shocks are not needed to generate crises here, coordination failures can happen exclusively as the result of negative fundamental shocks. Along this line, Boissay, Collard and Smeets (2016)
offer a macroeconomic model of banking crises based on interbank market freezes. Roughly speaking, the model here presented could be interpreted as capturing different types of disasters. Extending the analysis of this paper to other types of events such as bank runs and market freezes seems like an interesting avenue for future research.

There is a large empirical literature that tries to explain the equity premium puzzle through rare disasters (e.g., Rietz, 1988 and Barro, 2006). That literature is concerned with the effect of exogenous disasters on asset prices and as such, their definition of disaster is broader than the one used here. It includes not only deep financial crises, but also wars and natural disasters. In this paper, the goal is to understand how disasters happen endogenously and what are their consequences for investment, growth, welfare and asset prices. Therefore, the term disaster here refers exclusively to deep financial crises, such as the Great Recession and the Great Depression. Still in the disasters literature, but more related to this paper, is the work of Tiu and Yoeli (2013), who study endogenous disasters in a model with production externalities and labor market frictions.

**Layout.** The remainder of the paper is organized as follows. Section 2.2 presents the model. Section 2.3 characterizes the equilibrium and discusses some of its properties. Section 2.4 characterizes the welfare and the first-best. Section 2.5 calibrates the model, and Section 2.6 discusses the global effects of coordination failures. Section 2.7 analyzes some policy interventions and Section 2.8 concludes. All proofs are relegated to the appendix.

### 2.2 Model

#### 2.2.1 Preliminaries

Time is continuous and indexed by $t$. There are three types of agents: banks (financial intermediaries), households and firms. Households and banks are in a unit-measure continuum each and they can hold capital and a risk-free bond in their portfolios. Capital is rented to firms in a competitive market. There are two types of firms, a unit-measure continuum of intermediate goods producers and a competitive final good producer. Intermediate goods firms use capital and labor to produce differentiated varieties, which are aggregated by the competitive final producer. The final good is the numeraire.

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to generate coordination failures and are not considered. Thus, coordination failures will happen exclusively as the result of negative fundamental shocks. Gertler, Kiyotaki and Prestipino (2016a) offer an intuitive way of modeling the bank run probability in their model.
Capital held by banks \((k^b_t)\) and households \((k^h_t)\) evolves according to
\[
dk^i_t = g^i_t k^i_t \, dt + \sigma k^i_t \, dZ_t,
\]
where \(i \in \{h, b\}\), \(g^b_t\) and \(g^h_t\) are the capital growth rate (investment) for banks and households, respectively, \(dZ_t\) are aggregate Brownian shocks and \(\sigma > 0\) is the exogenous volatility. Investment \(g^i_t\) implies standard adjustment costs \(\iota(g^i_t)\), where \(\iota'(\cdot) > 0\) and \(\iota''(\cdot) > 0\). The total capital of the economy is \(K_t = k^b_t + k^h_t\).

The price of capital \(q_t\) follows a diffusion process:
\[
dq_t = \mu^q_t q_t \, dt + \sigma^q_t q_t \, dZ_t,
\]
where \(\mu^q_t\) and \(\sigma^q_t\) will be determined in equilibrium.

### 2.2.2 Households

Households own banks and firms, and have net worth \(n^b_0\) at the initial date. Households are risk neutral and can have negative consumption, so that the equilibrium risk-free rate is constant and equal to households’ discount rate \(\rho\).\(^8\) Capital invested in firms by households pays a rental rate \(R^h_t\) (to be determined in equilibrium), while the risk-free bond pays an interest rate \(r_f\). Using Ito’s Lemma, we can write the household’s capital return \(dr^{kh}_t\) as
\[
 dr^{kh}_t = \frac{R^h_t - \iota^h_t}{q_t} \, dt + \left( g^h_t + \mu^q_t + \sigma^q_t \right) \, dt + \left( \sigma^q + \sigma^q_t \right) \, dZ_t.
\]
The households’ problem is given by
\[
\max_{c_t, \varphi^b_t, \varphi^h_t, \xi_t, l_t} \quad \mathbb{E}_\tau \left[ \int_\tau^\infty e^{-\rho(t-\tau)} (c_t - \xi l_t) \, dt \right]
\]
subject to
\[
\begin{align*}
&dn^h_t = \varphi^h_t n^h_t \, dr^{kh}_t + (1 - \varphi^h_t) n^h_t r_f \, dt + (\zeta_t + \Pi_t + w_t l_t - c_t) \, dt \\
&n^h_t \geq 0,
\end{align*}
\]
where \(c_t\) denotes consumption, \(l_t\) denotes hours worked, \(w_t\) is the wage, \(\varphi^b_t\) is the share of the net worth \(n^b_t\) invested in capital, \(\zeta_t\) denotes dividends paid by banks, \(\Pi_t\) denotes dividends paid

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\(^8\)One can think of negative consumption as a backyard technology: households can produce the final good in their backyard at a disutility that equals the amount produced. Alternatively, one could think of a small open economy that has access to inelastic lending at the global interest rate \(\rho\). In equilibrium, aggregate consumption will always be positive.
by firms and $\xi > 0$ denotes the marginal disutility of labor. Since agents are risk-neutral, any inefficiency that arises in equilibrium comes from distortions in production, capital allocation and/or investment, not from consumption smoothing.

### 2.2.3 Banks

Each bank is managed by a banker. Bankers maximize the discounted dividends they pay during their lifetime, but they are subject to mortality risk: in each interval $dt$, they die with probability $\lambda dt$. When a banker dies, it is immediately replaced by a new banker who inherits her assets. As usual in the macro-finance literature, this kind of assumption is necessary to prevent banks from accumulating too much net worth.\(^9\)

Banks dividend payouts $\zeta_t$ cannot be negative, implying that banks cannot issue new equity. Thus, the only way banks can obtain extra funds to invest in capital is through risk-free debt, having to absorb all the risk in their balance sheets. This restriction is standard in macro-finance models and it can be justified by some incentive problem of bankers that requires them to have some “skin in the game”, as extensively shown in the corporate finance literature.\(^{10}\)

Banks earn a rental rate $R^b_t$ on capital. Therefore, similarly to households’, banks’ capital returns are

$$dr^{kb}_t = \frac{R^b_t - \mu^b_t}{q_t} dt + \left( q^b_t + \mu^q_t + \sigma^q_t \right) dt + (\sigma + \sigma^q_t) dZ_t.$$  

The banks’ problem is given by

$$\max_{\zeta, \phi^b, \sigma^b} \quad \mathbb{E}_\tau \left[ \int_{\tau}^\infty e^{-(\rho+\lambda)(t-\tau)} \zeta_t dt \right]$$

subject to

$$dn^b_t = \phi^b_t n^b_t dr^{kb}_t + (1 - \phi^b_t) n^b_t r^f dt - \zeta_t dt$$

$$n^b_t \geq 0, \zeta_t \geq 0.$$

Despite the fact that households have additional sources of income (dividends and wages), the main difference between banks’ and households’ problems is the non-negativity constraint $\zeta_t \geq 0$. One can interpret it as a solvency constraint: given that the bank cannot issue new equity, whenever a banker’s net worth drops to zero she will be wiped out.

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\(^9\)Mortality risk guarantees a non-degenerate stationary distribution of wealth.

\(^{10}\)See Sannikov (2013) for a survey of this literature.
2.2.4 Firms

There are two types of firms: a continuum of intermediate goods producers $j \in [0, 1]$, each producing a differentiated good, and a competitive final good producer.

Final good producer

The competitive final good firm uses intermediate goods to produce the final good $Y_t$, according to the production function

$$Y_t = \left( \int_0^1 y_{j,t}^{\varepsilon} \, dj \right)^{\frac{1}{\varepsilon - 1}},$$

where $\varepsilon > 1$ is the elasticity of substitution and $y_{j,t}$ is the quantity of good $j$ used as input. This technology implies the usual demand for each variety:

$$p_{j,t} = \left( \frac{Y_t}{y_{j,t}} \right)^{\frac{1}{\varepsilon}} P_t,$$

where $P_t$ is the price of the final good, which is normalized to one.

Intermediate goods producers

Intermediate goods firms rent capital and hire labor to produce their variety. It is assumed that banks are better at intermediating capital. This assumption is meant to capture banks’ expertise on screening and monitoring projects. In the model, it means that banks rent their capital to projects that have a higher return. A simple way to capture this is by assuming that each unit of capital intermediated by households contributes to production only a fraction $\beta < 1$ of what it would contribute if intermediated by banks. Let $k_{j,t}$ denote the amount of capital rented by a firm in efficiency units. It is given by $k_{j,t} = k_{j,t}^b + \beta k_{j,t}^h$, where $k_{j,t}^b$ and $k_{j,t}^h$ are the amounts of capital rented from households and banks, respectively. Firms’ production function is a standard Cobb-Douglas:

$$y_{j,t} = u_{j,t} A k_{j,t}^{\alpha} l_{j,t}^{1-\alpha},$$

where $u_{j,t}$ is a capacity utilization variable and $\alpha \in (0, 1)$ denotes the capital share. These assumptions justify why the capital rental rate will be different for banks and households. I denote the rental rate per efficiency unit of capital as $R_t$, implying that in equilibrium we must have $R_t^b = R_t$ and $R_t^h = \beta R_t$. 

In order to add non-convexities to the model and generate a coordination problem, I follow Guimaraes and Machado (2017) and Schaal and Taschereau-Dumouchel (2016b). It is assumed that capacity can take only two values, $u_t \in \{u_L, u_H\}$. That is the simplest way to model coordination failures and, as shown in Schaal and Taschereau-Dumouchel (2016b), a binary capacity choice is enough to match several features of the Great Recession. I normalize $u_L$ to 1 and set $u_H > 1$. Choosing the low capacity $u_L = 1$ can be interpreted as inaction. Adopting high capacity entails a fixed cost $f_t > 0$ (in units of the final good).

To prevent firms from growing out of the fixed cost, it is assumed that fixed costs scale with aggregate capital. The fixed cost firms pay to increase capacity is written as $f_t = \chi K_t$, where $\chi > 0$. Therefore, fixed costs are proportional to firms’ size. If $f_t$ was constant, in the long run firms would always coordinate on high capacity, since they would become very large relative to the fixed cost. This linear specification is specially useful, since it preserves the scale invariance properties of the economy. Moreover, this linear specification is common in the investment and corporate finance literature.\footnote{See Cooper and Haltiwanger (2006), Bolton, Chen and Wang (2011) and Decamps et al. (2016).}

Firms take the wage $w_t$ and the rental rate $R_t$ as given. They choose capacity $u_{j,t}$, capital $k_{j,t}$ (in efficiency units) and labor $l_{j,t}$ to maximize profits

$$\pi_t = p_{j,t} y_{j,t} - w_t l_{j,t} - R_t k_{j,t} - 1_{\{u_{j,t} = u_H\}} f_t,$$

subject to the demand schedule (2.1) and the production function (2.2). To ease the exposition, the firms’ problem is split in two stages. In the first stage, firms choose their capacity $u_{j,t}$. In the second stage, taking as given the proportion of firms with high capacity, denoted by $x_t$, they choose capital and labor.

### 2.3 Equilibrium

Having described each agent’s problem, the equilibrium definition is standard. An equilibrium is a set of stochastic processes for prices and quantities defined on some filtered probability space adapted to the Brownian motion $\{Z_t, t \geq 0\}$ such that: (i) banks, households and firms solve their problems and (ii) markets clear.

Given banks’ and households’ capital ($k^b_t$ and $k^h_t$), we can find the equilibrium on the real sector. Therefore, first the static equilibrium of firms is characterized for a given $k^b_t$ and $k^h_t$. Then, the map from banks’ and households’ capital to firms’ decisions is used as an input to solve for the banks’ and households’ portfolio choices and asset prices (dynamic equilibrium).
2.3.1 Static equilibrium

In this section I describe how to derive the static equilibrium. The equilibrium is solved backwards. First, firms’ production decisions are computed for a given proportion $x_t$ of firms with high capacity (second stage). Then, firms’ capacity decisions are determined (first stage).

Second stage equilibrium

Since firms with the same capacity make the same decisions in equilibrium, sometimes firms’ subscripts $j$ are replaced by $L$ or $H$, where $L$ is used for firms in regime low and $H$ for firms in regime high.

Firms optimality. Remember that firms take the wage $w_t$, the rental rate $R_t$ and the aggregate demand $Y_t$ as given. The aggregate demand $Y_t$ shifts the demand schedule each firm faces. Let $mc_{j,t}$ denote a firm’s marginal cost. It can be shown that

$$mc_{j,t} = \frac{1}{u_{j,t} A} \left( \frac{R_t}{\alpha} \right)^\alpha \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha}.$$

Firms with high capacity have a lower marginal cost (with $mc_{L,t} = (u_H / u_L) mc_{H,t}$), but have to pay the fixed cost $f_t$. Due to the market power, firms will optimally set their prices above the marginal cost, with $mc_{j,t} = \frac{\varepsilon - 1}{\varepsilon} p_{j,t}$. Using the demand schedule (2.1), it implies that

$$mc_{j,t} = \frac{\varepsilon - 1}{\varepsilon} \left( \frac{Y_t}{y_{j,t}} \right)^{\frac{1}{2}}, \tag{2.4}$$

which pins down a firms’ choice of $y_{j,t}$. Capital expenditures represent a share $\alpha$ of total costs and therefore

$$k_{j,t} = \alpha \frac{y_{j,t} mc_{j,t}}{R_t} \quad \text{and} \quad l_{j,t} = (1 - \alpha) \frac{y_{j,t} mc_{j,t}}{w_t}.$$

Market clearing. Due to the linear disutility of labor, the labor supply is flat and the labor market always clears at the wage $w_t = \xi$. Capital and goods market clearing imply

$$x_t k_{H,t} + (1 - x_t) k_{L,t} = k_t^b + \beta k_t^h$$

and

\footnote{The details are provided in the proof of Proposition 2.1.}
\[ Y_t = \left( x_t \frac{\varepsilon - 1}{\varepsilon} + (1 - x_t) \frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{\varepsilon - 1}}. \]

Using firms’ optimality and market clearing, we find the equilibrium in stage two, for a given \( x_t \). Proposition 2.1 characterizes the second stage equilibrium. The variable \( \psi_t \) is the share of total capital intermediated by banks, \( \psi_t \equiv k_t^b / K_t \).

**Proposition 2.1** (second stage equilibrium). Let the (weighted) average capacity \( \bar{u}_t \) and the endogenous TFP \( \bar{A}_t \) be given by

\[
\bar{u}_t \equiv \left( x_t u_{H}^{\varepsilon - 1} + (1 - x_t) u_{L}^{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}} \quad \text{and} \quad \bar{A}_t \equiv A \bar{u}_t \left( \psi_t (1 - \beta) + \beta \right)^{\alpha}.
\]

Then, in equilibrium wages and capital returns are

\[
w_t = \xi \quad \text{and} \quad R_t = \alpha \left( \frac{\xi}{1 - \alpha} \right)^{\frac{\alpha - 1}{\alpha}} \left[ \left( \frac{\varepsilon - 1}{\varepsilon} \right) A \bar{u}_t \right]^{\frac{1}{\alpha}}.
\]

Aggregate labor and gross output are

\[
L_t = \left[ \left( \frac{1 - \alpha}{\xi} \right) \left( \frac{\varepsilon - 1}{\varepsilon} \right) \bar{A}_t \right]^{\frac{1}{\alpha}} K_t \quad \text{and} \quad Y_t = \bar{A}_t K_t^\alpha L_t^{1 - \alpha}.
\]

For \( j \in \{L, H\} \), firms production and profits are

\[
y_{j,t} = \left( \frac{u_{j,t}}{\bar{u}_t} \right)^{\varepsilon} Y_t \quad \text{and} \quad \pi_{j,t} = \frac{1}{\varepsilon} \left( \frac{u_{j,t}}{\bar{u}_t} \right)^{\varepsilon - 1} Y_t - I_{\{u_{j,t}=u_H\}} f_t.
\]

Note that I refer to \( Y_t \) as the gross output, since it ignores the fixed costs paid. Remember that the final good is also used as an input to pay for firms fixed costs. Thus, the actual output available for investment and consumption is given by \( Y_t - x_t f_t \).

Proposition 2.1 shows that the endogenous TFP of the economy depends on the average capacity \( \bar{u}_t \) and on the share of capital intermediated by banks \( \psi_t \). The effect of \( \psi_t \) on \( \bar{A}_t \) summarizes the role of capital allocation on output, as in the seminal contribution of Kiyotaki and Moore (1997). The effect of firms’ capacity choices on output is summarized by the variable \( \bar{u}_t \). This captures the coordination effect, as in Schaal and Taschereau-Dumouchel (2016b).

Banks’ and households’ rental rates are given by \( R_t^b = R_t \) and \( R_t^h = \beta R_t \). Proposition 2.1 shows that the rental rate of capital increases when firms coordinate on high capacity. The intuition is straightforward: when firms coordinate, they demand more capital, raising rental rates. Thus, coordination increases banks’ and households’ incentives to hold capital.
Define $\Delta \pi_t = \pi_{H,t} - \pi_{L,t}$ as the relative gain of being in regime high (relative profits) and let $\Delta \tilde{\pi}_t \equiv \Delta \pi_t / K_t$ be the scaled relative profits. Using Proposition 2.1, we can write it as

$$\Delta \tilde{\pi}_t = B_t \Omega_t (\psi_t (1 - \beta) + \beta) - \chi,$$

where $B > 0$ is a function of parameters and $\Omega \equiv 1 + 1/\alpha - \varepsilon$. One can see that the return of paying the fixed cost is affected by two endogenous variables, $\psi_t$ and $\pi_t$. Therefore we can write $\Delta \tilde{\pi}_t = \Delta \tilde{\pi}(\psi_t, \pi_t)$. Note that relative profits are increasing in $\psi_t$. When capital is efficiently allocated, firms are more productive and therefore have higher incentives to pay the fixed cost and scale up their production.

As for average capacity, $\Delta \pi_t$ increases with $\pi_t$ when $\Omega > 0$. In this case, firms’ capacity decisions are strategic complements, i.e., a firm has higher incentives to increase its production when others do the same. When $\Omega < 0$, we have the opposite and firms capacity decisions are strategic substitutes. This ambiguity arises because there are two effects at play. On one hand, an increase in average capacity raises the aggregate demand $Y_t$, shifting up the demand schedule each firm faces. This high demand provides incentives to pay the fixed cost and scale up production. On the other hand, an increase in capacity raises the demand for capital, and consequently its rental rate, reducing firms’ absolute profits and their incentives to expand. Which effect dominates depends on the sign of $\Omega$. Following Angeletos and La’O (2010), $\Omega$ is named the degree of complementarity of the economy. This tension between demand externalities and competition in the market for factors is a common characteristic of models with demand externalities. To focus on coordination failures, from now on it is assumed that $\Omega > 0$. The case with $\Omega \leq 0$ is presented in Appendix 2.C.

**First stage equilibrium**

In the first stage, firms play a capacity game. Proposition 2.2 characterizes firms capacity decisions in equilibrium.

**Proposition 2.2** (first stage equilibrium). Assume that $\Omega > 0$ and let $\psi$ and $\tilde{\psi}$ be such that $\Delta \tilde{\pi}(\psi, u_H) = \Delta \tilde{\pi}(\tilde{\psi}, u_L) = 0$. Then,

1. if $\psi_t < \psi$, there is a unique equilibrium and all firms choose low capacity ($x_t = 0$);
2. if $\psi_t > \tilde{\psi}$, there is a unique equilibrium and all firms choose high capacity ($x_t = 1$);
3. if $\psi_t \in [\psi, \tilde{\psi}]$ there are three equilibria: $x_t = 0$, $x_t = 1$ and $x_t = h(\psi_t) \in [0, 1]$, where $h(\cdot)$ is a decreasing function of $\psi_t$. 

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It is easy to see that $\tilde{\psi} > \psi$, but Proposition 2.2 does not ensure that the cutoffs $\psi$ and $\tilde{\psi}$ are in the unit interval, and thus some of the cases above may not be possible. To guarantee that the three cases can happen, we need to impose two parametric assumptions: $\psi > 0$ and $\tilde{\psi} < 1$. Figure 2.1 illustrates the equilibrium set in that case.

The cutoff $\psi$ is the lower dominance threshold, since for $\psi_t < \psi$ it is dominant to choose low capacity. Similarly, $\tilde{\psi}$ is the upper dominance threshold, since for $\psi_t > \tilde{\psi}$ high capacity is the dominant strategy for firms. For $\psi_t \in [\psi, \tilde{\psi}]$, there is no dominant strategy and firms are willing to scale up their production only if they are confident that a sufficient number of firms will do the same. Choosing low capacity in the interval $[\tilde{\psi}, \tilde{\psi}]$ is interpreted as a coordination failure, since everyone would be better off if firms coordinated on the high regime.

**Equilibrium selection**

Firms play a coordination game with multiple equilibria. In some regions of the state space, firms may fail to coordinate on high capacity. A central message of the global games literature is that multiplicity on coordination games usually relies on strong assumptions about agents’ knowledge of the behavior of others in equilibrium. Some equilibria can only be sustained if agents face no strategic uncertainty about the aggregate action of others. A small departure from this benchmark may lead to equilibrium uniqueness.\(^\text{13}\)

The global game approach of the continuous time debt-run model of Morris and Shin (2001) is applied. I briefly explain the procedure here, a formal derivation is left to the appendix. The main idea is to consider a slightly perturbed version of the model where firms have private information about $\psi_t$. More specifically, I consider a discrete-time approximation of the model in which time evolves in intervals of length $\nu > 0$. At a given date $t$, firms observe only: (i) the history of all variables up to date $t - \nu$; (ii) a private signal about the productivity shock $\Delta Z_t \equiv Z_t - Z_{t-\nu}$. In the limit where private signals become very precise, the equilibrium is essentially unique.\(^\text{14}\) Then, we can go back to continuous time by taking the limit when the

\(^{13}\)For instance, assume that a firm is making its choice knowing for sure that $\psi_t$ is slightly above $\tilde{\psi}$. But it knows that other firms may have observed the state with a small error, and thus it is not entirely sure that other firms know that $\psi_t > \tilde{\psi}$. If others think that $\psi_t < \tilde{\psi}$, they will certainly choose low capacity. Thus, it seems unreasonable that a firm will choose high capacity in this case, since it has a lot to lose if others do not follow, but not much to gain if others follow.

\(^{14}\)“Essentially” here means that players actions are uniquely determined almost everywhere.
time interval vanishes. A technical requirement for this procedure to work is the existence of dominance regions, meaning that \( \tilde{\psi} > 0 \) and \( \tilde{\psi} < 1 \) must be assumed.\(^{15}\)

As shown in Appendix 2.D, in the unique equilibrium that survives this process, firms choose high capacity if \( \psi_t \geq \psi_{GG} \) and low if \( \psi_t < \psi_{GG} \) (a cutoff strategy). The threshold \( \psi_{GG} \) is given by the indifference condition of a firm that has a uniform belief over the proportion of firms that will choose high capacity. It is given by\(^{16}\)

\[
\psi_{GG} = \left(\frac{1}{1-\beta}\right) \left[ \frac{\chi}{\alpha} \left( \frac{\xi}{1-\alpha} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{1}{A \varepsilon - 1} \right)^{\frac{1}{\alpha}} \left( u_H^{\frac{1}{\alpha}} - u_L^{\frac{1}{\alpha}} \right)^{-1} \right].
\]

Notice that \( \psi_{GG} \) is increasing on the fixed cost, which is a desirable property (the higher the fixed cost, the more difficult it is to coordinate). Figure 2.2 describes the equilibrium.

Different selection devices would lead to the same qualitative conclusions. For instance, if we select the best equilibrium, in which firms choose the high capacity whenever \( \psi_t \geq \psi \), the equilibrium outcome is still inefficient, as shown in Section 2.4. The dynamics of the economy would be similar, but coordination failures would happen less often. When I calibrate the model, the scaled fixed cost \( \chi \) is chosen to match a given probability of coordination failures. Different selection devices that still select a cutoff strategy that is increasing in \( \chi \) will only lead to a different choice of \( \chi \) in the calibration. Thus, what is key for the results is that agents play a cutoff strategy (no sunspots, for instance). Nonetheless, following Guimaraes, Machado and Ribeiro (2016) and Schaal and Taschereau-Dumouchel (2016b), the global games equilibrium seems like the natural choice.

### 2.3.2 Households and banks equilibrium

Given the firms equilibrium, we can solve for the dynamic equilibrium of the economy. Coordination affects banks’ and households’ portfolio decisions through the rental rate of capital. Moreover, portfolio decisions matter for capital allocation, possibly creating a two-way feedback between bank’s balance sheets and coordination.

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\(^{15}\)For a survey on global games, see Morris and Shin (2003).

\(^{16}\)Agents action are not determined when they are indifferent between the two actions, but that happens with zero probability. Without loss of generality, it is assumed that agents choose high capacity whenever they are indifferent, which implies that they invest when \( \psi_t = \psi_{GG} \).
The state of the economy can be summarized by the banks’ share of total wealth:

\[ \eta_t \equiv \frac{n_t^b}{q_t K_t}. \]

It must follow a diffusion process:

\[ d\eta_t = \mu^n\eta_t dt + \sigma^n\eta_t dZ_t, \]

where \( \mu^n_i = \mu^n(\eta_t) \) and \( \sigma^n_i = \sigma^n(\eta_t) \) must be determined in equilibrium. I will look for Markov equilibria where banks always borrow from households, and prices and allocations are functions of \( \eta_t \):

\[ \psi_t = \psi(\eta_t) \quad \text{and} \quad q_t = q(\eta_t). \]

The non-standard feature of the model is the endogeneity of the capital rental rates \( R^b_t \) and \( R^h_t \), since those depend on how much capital banks and households have on their balance sheets. This will create \textit{strong complementarities} in banks’ capital holdings, generating multiple dynamic equilibria, even when the firms equilibrium is uniquely determined.\(^{17}\)

The rental rates \( R^b_t \) and \( R^h_t \) can be represented as functions of the state of the economy \( \eta_t \). Thus, I write \( R^b_t = R^b(\eta_t) \) and \( R^h_t = R^h(\eta_t) \). First, the equilibrium is computed for an arbitrary function \( R(\eta_t) \), where banks and households rental rates are \( R^b(\eta_t) = R(\eta_t) \) and \( R^h(\eta_t) = \beta R(\eta_t) \). This is the \textit{partial dynamic equilibrium} of the economy. Then, I turn to the general equilibrium where rental rates are jointly determined by the \textit{static equilibrium} of firms and the \textit{partial dynamic equilibrium} of banks and households.

\textbf{Partial dynamic equilibrium}

Before characterizing the equilibrium, households’ and banks’ optimality conditions are briefly discussed. To ease notation, time subscripts may be omitted if no confusion arises.

\textbf{Optimal investment.} Let \( g_t^{i*}, i \in \{h, b\} \), denote banks’ and households’ choices of \( g_t \). It is given by the marginal Tobin’s q relationship:

\[ \iota'(g_t^{i*}) = q_t. \]

\(^{17}\)The equilibrium set could further expand if we allow for non-Markov equilibria or equilibria where banks are not always leveraged. A full characterization of the equilibrium set is beyond the scope of this paper and thus I restrict attention to the equilibrium class most studied in the literature.
Investment only depends on $q$ and we can write $g_i^* = g_i^*(q_t)$. Since it is the same for households and banks, superscripts are dropped from now on and I simply write

$$g(\eta) = g^* (q(\eta)) \text{ and } \iota(\eta) = \iota (g(\eta)).$$

**Households optimality.** Households are happy to invest whenever $E[dr_{kh}] = r_f = \rho$. They must either be indifferent between capital and consumption, or not holding capital at all:

$$\frac{\beta R(\eta) - \iota(\eta)}{q(\eta)} + g(\eta) + \mu^q(\eta) + \sigma^q(\eta)\sigma = r_f \text{ if } 1 - \psi(\eta) > 0,$$

$$\frac{\beta R(\eta) - \iota(\eta)}{q(\eta)} + g(\eta) + \mu^q(\eta) + \sigma^q(\eta)\sigma \leq r_f \text{ if } 1 - \psi(\eta) = 0. \quad (2.6)$$

**Banks optimality.** Let $v(n^b, \eta)$ denote the bank’s value function. It depends on the aggregate state of the economy $\eta$ and on individual net worth $n^b$. The homogeneity of the problem implies linearity in net worth. We can write:

$$v(n^b, \eta) = n^b \theta(\eta), \quad (2.8)$$

where the function $\theta(\eta)$ represents the banks’ marginal utility of net worth. Notice that $\theta(\eta) \geq 1$, since banks can always pay their total net worth out as dividends at a given date. Writing down the bank’s Hamilton-Jacobi-Bellmann equation and using (2.8), we get that banks are happy to hold capital whenever

$$\frac{R(\eta) - \iota(\eta)}{q(\eta)} + g(\eta) + \mu^q(\eta) + \sigma^q(\eta)\sigma = \eta (\sigma + \sigma^q(\eta)) \frac{\theta'(\eta)}{\theta(\eta)} = r_f. \quad (2.9)$$

Note that although banks are risk-neutral, they may require a risk premium to hold capital if the marginal utility $\theta(\eta)$ is time-varying.\(^{18}\) Moreover, the dividend policy must be consistent with:

$$\zeta(\eta) \begin{cases} = 0 & \text{if } \theta(\eta) > 1, \\ \geq 0 & \text{if } \theta(\eta) = 1. \end{cases} \quad (2.10)$$

Therefore, whenever $\theta(\eta) = 1$, banks start paying dividends, which implies a reflecting boundary $\eta^*$ on banks’ wealth share ($\eta_t \leq \eta^*$ for every $t$).

\(^{18}\)This happens because banks fear reaching zero net worth and losing good investment opportunities in the future.
Let $q$ denote the price of capital if banks are wiped out and households have to hold it forever. This is given by the Gordon growth formula:

$$q = \max_g \frac{\beta R - \nu(g)}{\rho - g},$$

where $R$ is the capital rental rate when $x_t = 0$. Proposition 2.3 characterizes the partial dynamic equilibrium as a system of differential equations.

**Proposition 2.3** (partial dynamic equilibrium). For a given function $R(\eta)$, the partial dynamic equilibrium is characterized as follows:

1. For a given $\theta(\eta)$, $\theta'(\eta)$, $q(\eta)$ and $q'(\eta)$, the functions $\mu(\eta)$, $\sigma(\eta)$, $\mu(\eta)$ and $\psi(\eta)$ are given by equations (2.15) to (2.19) in Appendix 2.A.

2. The functions $\theta(\eta)$ and $q(\eta)$ satisfy the following boundary value problem with unknown parameter $\eta^*$:

$$\lambda \theta(\eta) = \eta \mu(\eta) \theta'(\eta) + \frac{1}{2} (\eta \sigma(\eta))^2 \theta''(\eta),$$

$$\mu(\eta) = \mu(\eta) \eta q'(\eta) + \frac{1}{2} \sigma(\eta)^2 \eta q''(\eta),$$

with boundary conditions

$$q(0) = q, \quad \theta(\eta^*) = 1, \quad \lim_{\eta \to 0} \theta(\eta) = \infty, \quad q'(\eta^*) = 0 \quad \text{and} \quad \theta'(\eta^*) = 0.$$

Equation (2.11) is obtained from banks’ HJB equation and banks’ optimality (2.9). Equation (2.12) is obtained by applying Ito’s Lemma to $q(\eta)$. The remaining variables are obtained using households’ and banks’ optimality conditions (2.6), (2.7) and (2.9), together with the law of motions for $\eta_t$ and $q_t$ that can be found using Ito’s Lemma.\(^{19}\)

**General dynamic equilibrium**

Now, I need to jointly determine the functions $\psi(\eta)$ and $R(\eta)$. The partial dynamic equilibrium pins down the function $\psi(\eta)$ for a given function $R(\eta)$. Similarly, for a given function $\psi(\eta)$, we pin down $R(\eta)$ using the static equilibrium. In general equilibrium, $R(\eta)$ and $\psi(\eta)$ must both be consistent with the static equilibrium and the partial dynamic equilibrium.

Let $R^*(\psi)$ be the function that determines the rental rate for each $\psi$, according to the static equilibrium computed in Section 2.3.1. We can define a dynamic general equilibrium as follows.

\(^{19}\)See the proof in the appendix for details.
Definition 2.1 (general equilibrium). The functions \( \mu^q(\eta), \sigma^q(\eta), \mu^n(\eta), \sigma^n(\eta), \theta(\eta), q(\eta), \psi(\eta) \) and \( R(\eta) \) are a general dynamic equilibrium if: (i) they are a partial dynamic equilibrium given the rental rate \( R(\eta) \); (ii) \( R(\eta) \) is consistent with the static equilibrium, i.e., \( R(\eta) = R^*(\psi(\eta)) \).

Therefore, the set of functions \( \psi(\eta) \) that are part of a general equilibrium can be represented by the fixed point of the transformation \( T[\psi(\eta)] = \mathcal{P}[R^*(\psi(\eta))] \), where \( \mathcal{P} \) be is the operator that takes a function \( R(\eta) \) and returns \( \psi(\eta) \) in the partial dynamic equilibrium. It turns out that there may be more than one fixed point, meaning that we may have multiple dynamic equilibria.

If banks expect other banks to hold a lot of capital, then firms are more likely to coordinate. Coordination increases the rental rate of capital for banks and households, but it turns out that in general equilibrium banks end up with more capital relative to households, confirming banks’ expectations and creating a self-fulfilling prophecy. What is key here is that, although households and banks rental rates increase by the same factor when firms coordinate, only banks’ capital returns \( (dr_{kb}) \) increase, since in equilibrium households cannot earn more than the risk-free rate. Multiplicity does not arise from multiplicity in firms’ decisions, since we are already selecting a unique static equilibrium using global games.

There is no obvious equilibrium selection device we can use here. Differently from firms, banks and households are price-takers. In equilibrium, agents are indifferent between holding any amount of capital. In the firms’ problem, it seemed unreasonable to assume that firms would choose the high capacity in states where their gain would be very small if others followed, but the losses would be huge if others did not follow. Global games took care of it, by iteratively eliminating this kind of unreasonable equilibria. In the dynamic equilibrium, however, capital always pays the required risk premium for banks to be happy to hold any amount of it. There is no penalty for taking the wrong action and betting on the wrong scenario (for instance, betting that banks will hold a lot of capital when they do not).

In Appendix 2.A.5 I provide an algorithm to numerically compute the two extreme equilibria. The good equilibrium is such that in any other equilibria we have \( \psi(\eta) \leq \psi_G(\eta) \) for every \( \eta \), where \( \psi_G(\eta) \) is the bank’s share of total capital in the good equilibrium. The same is true for the bad equilibrium, with the inequality reversed.

In what follows, the good equilibrium is always assumed. This is the best equilibrium in terms of household welfare. The results for the bad equilibrium are very similar. In fact, for the parameters used in the calibration the two extreme equilibria practically coincide. For firms, we could also assume that they play the best or worst equilibrium instead of the global games one. When the model is taken to the data, the parameter \( \chi \) is chosen to match a given probability of coordination failures. Different selection devices would end up delivering similar
dynamics, but implying a different choice of $\chi$. What is important, though, is that sunspots are not allowed. All the risk in the economy comes from the Brownian shocks $dZ_t$.

### 2.3.3 Equilibrium description

Before calibrating the model and turning to the main results of the paper, it is useful to understand the workings of the model through some stark numerical examples, in which coordination failures are particularly strong. The goal here is to understand some qualitative properties of the model and the local effects of disasters on equilibrium outcomes. The analysis of the global effects and the welfare analysis are left to Section 2.6. The results are reported for the best equilibrium only, since the results for the worst equilibrium are very similar. Unless otherwise stated, the qualitative properties of the model do not seem to depend on the choice of parameters.

Figure 2.3 shows the share of total capital intermediated by banks and their leverage. We can divide the state space in three regions. In the interval $[0, \hat{\eta})$, banks are poor, intermediation is low and firms do not coordinate on high capacity. I interpret states in which $\eta_t \in [0, \hat{\eta})$ as financial crises with coordination failures. On the interval $[\hat{\eta}, \eta^\psi)$, banks are not sufficiently well capitalized to hold all the capital of the economy, but still intermediate capital enough to avoid a coordination failure. The economy is in a financial crisis, but firms are still coordinating on high capacity. In the region $[\eta^\psi, \eta^*)$ banks are rich enough to be willing to hold all capital and no allocative inefficiency arises. As in Brunnermeier and Sannikov (2014), the reflecting boundary $\eta^*$ denotes the stochastic steady state of the economy, since the economy spends a lot of time near it. If we had assumed the bad equilibrium both the boundaries $\hat{\eta}$ and $\eta^\psi$ would move to the left, but usually not much.

The amount of banks’ capital holdings decreases continuously as the economy approaches the boundary $\hat{\eta}$ from above, until it jumps down when $\hat{\eta}$ is reached. When the economy hits $\hat{\eta}$, both banks’ and households’ capital rental rates decrease in the same proportion. But it turns out that it does not affect households’ demand for capital, since in equilibrium households’ capital returns (that also includes capital gains) are always equal to the risk-free rate. As opposed to households’, banks’ capital returns experience a sudden fall, leading them to demand less capital (see Figure 2.4). This sudden and fast unloading of capital on the household sector can be interpreted as market-run, as in Bernardo and Welch (2004).

There has been some discussion in the literature about whether intermediary leverage increases or decreases during financial crises. Standard macro-finance models (e.g., He and

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The parameters used are $\rho = 0.04$, $\lambda = 0.03$, $\sigma = 0.1$, $\beta = 0.7$, $u_H = 1.3$, $u_L = 1$, $\epsilon = 4$, $\xi = 1$, $\alpha = 0.3$, $\chi = 0.785$, $A = 1.1$. Moreover, $\iota(g) = (g + 0.1) + \frac{3}{2}g^2$. 

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Figure 2.3: Intermediation and leverage

Krishnamurthy, 2012) predict that it should undoubtedly increase, while some empirical evidence suggests otherwise (e.g., Adrian and Shin, 2010). Whether it increased or decreased in the Great Recession depends on which kind of data one looks at (market-value leverage vs. book-value leverage) and on the sector analyzed. Yet, the model here presented is able to generate movements that are consistent with both narratives. As shown in Figure 2.3, leverage increases when the economy leaves the stochastic steady-state $\eta^*$ and crosses $\eta^\psi$ (starting a financial crises). Then it stabilizes and even decreases slowly as we move from $\eta^\psi$ to $\hat{\eta}$ (depending on parameters it may keep increasing in that region). The important thing is that if negative shocks keep arriving, a coordination failure starts and suddenly banks reduce their capital holdings, paying off part of the debt and deleveraging. It happens because financial intermediaries are no longer willing to take all that risk now that capital is not paying as much. For many sectors such as broker dealers, leverage rose at the beginning of the crisis and then suddenly dropped at some point in 2008. If we interpret the fourth quarter of 2007 as the period where the financial crisis started ($\eta_t$ fell below $\eta^\psi$), and then a few quarters later as when the crisis intensified ($\eta_t$ fell below $\hat{\eta}$), the model can explain this up and down pattern of leverage. How large is this effect depends, among other things, on how large is the reduction in capital returns caused by the coordination failure. In fact, in the calibrated version of the model to be presented, this effect is small.\footnote{Experimenting with some different models suggested that this effect is very large if banks’ rental rate increases by a larger factor than households when firms coordinate, which is not the case here.}

Figure 2.4 shows the stationary distribution and other equilibrium functions. Notice that the economy spends a large amount of time in coordination failures (as shown by the density of $\eta$). In the absence of coordination failures, the stationary distribution of $\eta$ would have a U-shape, with a thinner left-tail. This is because coordination failures are very persistent. As
Panel E shows, the drift of the state variable drops sharply when the economy approaches the coordination failure region from above. The same happens with the volatility term, meaning that the forces pushing the economy away from the coordination failure get weaker.

But, depending on parameters, things can get much worse. Panel F shows the drift of $\eta$ when we increase the importance of the coordination channel (in that example, it means a higher $u_H$) and the exogenous volatility $\sigma$. There are two main differences between panels E and F. In the latter the drift $\mu^\eta$ increases when the economy enters the coordination failure region. But most importantly, in Panel F the drift of the state variable is negative when the economy is approaching the coordination failure boundary $\hat{\eta}$ from above. As soon as the economy gets in a coordination failure, the drift becomes positive, pushing the economy away from it. But as the economy is pushed out of the coordination failure, it is pulled back in again, since $\mu^\eta$ is negative to the right of $\hat{\eta}$. In the absence of large productivity shocks, the economy wanders around the cutoff $\hat{\eta}$, being in and out the coordination failure for a long time. Although intermediaries net worth is growing in that region – they are not paying dividends and capital returns are positive in the absence of shocks – the total net worth of the economy is growing faster. Even though asset prices are decreasing to the right of that boundary ($\mu^q < 0$), the drift of the total net worth $q_tK_t$ is also affected by $\sigma\sigma^q_t$, which can be large. If $u_H$ is small (the gains from coordination are not so large), then the economy looks more like Panel E. In fact, in the calibration $\mu^\eta$ will never be below zero, but coordination failures are still persistent since they slow down the growth of banks’ net worth share.

Finally, note that the Sharpe ratio follows a pattern similar to leverage. Even though asset price volatility falls once the economy enters the coordination failure region, capital returns (and spreads) also fall. The fall in capital returns is more significant than the fall in price volatility, causing the Sharpe ratio to decrease. An important message of the model is that there may be a lot of non-monotonic relationships between credit spreads, Sharpe ratios, intermediaries leverage and economic activity. Empirical work trying to use these variables to predict financial crises should be aware of it, at the risk of missing relevant effects due to misspecification.

### 2.4 Welfare

The welfare of the representative household is given by

$$ W(K_t, \eta_t) = \mathbb{E} \left[ \int_0^\infty e^{-pt} (C_t - \xi L_t) \, dt \right], $$
Figure 2.4: Equilibrium functions

(a) Stationary density

(b) Asset price volatility

(c) Asset price drift

(d) Wealth share volatility

(e) Wealth share drift I

(f) Wealth share drift II

(g) Asset price

(h) Sharpe ratio

(i) Returns
where $C_t$ is the aggregate consumption. By market clearing, $C_t = Y_t - \iota_t K_t - x_t f_t$.

Let $\Lambda_t$ denote the household flow utility per unit of capital.\footnote{Using the static equilibrium and the market-clearing condition $C_t = Y_t - \iota_t K_t - f_t$, one can easily verify that flow utility scales with capital.} Using the static equilibrium (Propositions 2.1 and 2.2), we can write it as functions of $\psi_t$ and $\iota_t$. But we know from the dynamic equilibrium that $\psi_t$ and $\iota_t$ are functions of $\eta$. Hence, we can write $\Lambda_t = \Lambda(\eta_t)$. Next Proposition shows how to write the welfare of the household as a boundary value problem.

**Proposition 2.4 (welfare).** For a given initial $K$ and $\eta$, the welfare of the representative household is given by $W(K, \eta) = KW(\eta)$, where $W(\eta)$ solves the following boundary value problem:

$$(\rho - g(\eta)) W(\eta) = \Lambda(\eta) + (\eta \mu^n(\eta) + \eta \sigma^n(\eta) \sigma) W'(\eta) + \frac{1}{2} \eta \sigma^n(\eta)^2 W''(\eta),$$

with boundary conditions $W(0) = \frac{\Lambda(0)}{\rho - g(0)}$ and $W'(\eta^*) = 0$. The flow utility per unit of capital is given by

$$\Lambda(\eta) = \begin{cases} 
Du^A (\psi(\eta) (1 - \beta) + \iota(\eta)) & \text{if } \psi(\eta) < \psi_{GG} \\
Du^H (\psi(\eta) (1 - \beta) + \iota(\eta) - \chi) & \text{if } \psi(\eta) \geq \psi_{GG} 
\end{cases},$$

where $D \equiv \left( \alpha + \frac{1}{\varepsilon - 1} \right)^{\frac{1 - \alpha}{\alpha}} \left( \frac{1 - \alpha}{\varepsilon} \right)^{\frac{1 - \alpha}{\alpha}} \left( A^{\frac{1 - \alpha}{\varepsilon}} \right)^{\frac{1 - \alpha}{\alpha}}$.

### 2.4.1 Inefficiencies and first-best

Before discussing the inefficiencies that arise in equilibrium, let’s first define the first-best economy. Next proposition characterizes the prices and allocations that maximize welfare.

**Proposition 2.5 (first-best).** If $\tilde{\psi} < 1$ (firms want to choose high capacity if they believe everyone will do the same) then in the first-best we have that:\footnote{The condition $\tilde{\psi} < 1$ ensures that we are in the interesting case where coordination failures are possible.}

1. Total welfare is given by $W^{FB} = q^{FB} K_0$, where the first-best Tobin’s $q$ is given by

$$q^{FB} = \max_g \frac{\Lambda^{FB}_g}{\rho - g}, \quad \text{with} \quad \Lambda^{FB}_g \equiv (Au_H)^{1/\alpha} \left( \frac{1 - \alpha}{\xi} \right)^{\frac{1}{\alpha}} - \chi - \iota(g).$$

2. Firms always choose high capacity and set prices equal to the marginal cost, i.e., $u_{j,t} = u_H$ and $p_{j,t} = mc_{j,t}$.

3. Banks intermediate all the capital (i.e., $\psi_t = 1$) and investment satisfies $\iota'(g^{FB}) = q^{FB}$.\footnote{The condition $\tilde{\psi} < 1$ ensures that we are in the interesting case where coordination failures are possible.}
The economy has a lot of externalities and is far from the first-best. Two market failures are responsible for this: (i) firms market power and (ii) missing market for aggregate risk (no equity issuance).

On the productive sector, firms do not internalize the positive impact that choosing high capacity has on other firms’ demand (coordination failures). Moreover, producers set prices above the marginal cost (monopoly distortion). A social planner would choose high capacity in more states and set lower prices.

If banks could issue equity, the price of capital would be constant and larger than the equilibrium price. Higher asset prices would lead to higher investment, boosting economic growth. The equity constraint creates externalities that would not be present otherwise. Banks do not internalize the impact that leveraging up has on asset price volatility and on the probability of bad states.\textsuperscript{24} They also do not internalize the positive impact that higher leverage (and consequently, higher intermediation) has on firms coordination and capital returns.

As will be shown in Section 2.7, some macroprudential policies can bring the economy closer to the first-best.

\section{2.5 Calibration}

Unless otherwise stated, I use quarterly data from 1973Q1 to 2016Q1. The growth rates refer to the annualized growth rates between two consecutive quarters. The model is simulated using an Euler scheme and then the obtained data is aggregated quarterly. Data sources and further details are presented in Appendix 2.B. I assume quadratic adjustment costs on net investment with depreciation $\delta$:

$$\iota(g) = (g + \delta) + \frac{\kappa}{2}g^2.$$ 

Table 2.1 shows the parameters used.

\subsection{2.5.1 Parametrization}

The parameters $\rho$ (household discount rate), $\alpha$ (capital share), $\delta$ (depreciation rate), $\kappa$ (adjustment cost) and $\varepsilon$ (elasticity of substitution) are taken from the literature and are relatively standard. The marginal disutility of labor ($\xi$) and the capacity in the low regime

\textsuperscript{24} As will be further discussed in Section 2.7, in this model higher bank’s leverage does not necessarily imply higher volatility.
Table 2.1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Literature/normalized parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$ Discount rate</td>
<td>0.05</td>
<td>Schaal and Taschereau-Dumouchel (2016b)</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate</td>
<td>0.1</td>
<td>Schaal and Taschereau-Dumouchel (2016b)</td>
</tr>
<tr>
<td>$\kappa$ Adjustment costs</td>
<td>3</td>
<td>He and Krishnamurthy (2014)</td>
</tr>
<tr>
<td>$\alpha$ Capital share</td>
<td>0.3</td>
<td>Schaal and Taschereau-Dumouchel (2016b)</td>
</tr>
<tr>
<td>$\epsilon$ Elasticity of substitution</td>
<td>3</td>
<td>Schaal and Taschereau-Dumouchel (2016b)</td>
</tr>
<tr>
<td>$u_L$ TFP shifter (low regime)</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\xi$ Marginal disutility of labor</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td><strong>Panel B: Other parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ Capital quality shocks</td>
<td>0.011</td>
<td>GDP volatility</td>
</tr>
<tr>
<td>$u_H$ TFP shifter (high regime)</td>
<td>1.02</td>
<td>$\Delta$ Capacity after 2007</td>
</tr>
<tr>
<td>$A$ Productivity</td>
<td>1.5445</td>
<td>Investment-to-capital rate</td>
</tr>
<tr>
<td>$\beta$ Household’s relative efficiency</td>
<td>0.94</td>
<td>$\Delta$ Output-to-capital after 2007</td>
</tr>
<tr>
<td>$\lambda$ Bank’s mortality risk</td>
<td>0.004</td>
<td>Probability of financial crisis</td>
</tr>
<tr>
<td>$\chi$ Scaled fixed cost</td>
<td>0.0096</td>
<td>Probability of coordination failure</td>
</tr>
</tbody>
</table>

$(u_L)$ are normalized to one. The choice of the remaining parameters is discussed below.

The volatility of the productivity shocks $\sigma$ is chosen to match the average standard deviation of output growth of 3.2% in my sample. As shown in the next section (Table 2.2), the model matches almost perfectly the volatility of output, hours, investment, consumption and intermediary equity, even though only the volatility of output was targeted. Not surprisingly, the volatility of capacity utilization is much lower in the model than in the data (due to the binary capacity assumption).

To calibrate $u_H$, I follow a procedure similar to the one used by Schaal and Taschereau-Dumouchel (2016b) and use the Federal Reserve Board index of capacity utilization. According to the Board, this index “tries to conceptualize the idea of sustainable maximum output, which is defined as the highest level of output a plant can sustain within the confines of its resources”. Within the model, the “sustainable maximum output” can be interpreted as the output of a firm with the maximum capacity $u_H$. Therefore, the index is mapped into the ratio $y_{L,t}/y_{H,t} = (u_L/u_H)^\epsilon$. It is assumed that after 2007Q2 the economy shifted to a regime with low capacity. In the data, the post-2007Q2 average of this index is 6.56% lower than its level in 2007Q2, implying a parameter $u_H = 1.022$. If we compare it with a post-2009Q4...
average (as in Schaal and Taschereau-Dumouchel, 2016b), the implied parameter is 1.019.\textsuperscript{25} Therefore, an average of those two values is chosen, implying $u_H = 1.02$, which is pretty close to the value of 1.0182 used in Schaal and Taschereau-Dumouchel (2016b).\textsuperscript{26}

The productivity parameter $A$ is set to get an average investment-to-capital ratio near 10%, consistent with the usual estimates in the investment literature (see Bachmann, Caballero and Engel, 2013). The average investment-to-capital ratio in the model is 11.5%.

A tricky parameter to calibrate is the household’s relative efficiency on capital intermediation ($\beta$). Everything else constant, it controls the fall in output per unit of capital after the economy leaves its stochastic steady state and enters a coordination trap. Output per unit of capital services fell approximately 8% from 2007 to 2009. I set $\beta = 0.94$, which implies that households earn on average a rental rate 6% smaller than financial intermediaries, and that expected output per unit of capital, conditional on being in a coordination failure, is 7.8% lower than its level at the stochastic steady state.

Two key parameters are banks’ mortality risk $\lambda$ and the fixed cost parameter $\chi$. The mortality risk controls how prone to take risks banks are. If $\lambda$ is low, banks are very patient and they would be willing to accumulate a lot of net worth, reducing their leverage and making financial crises very unlikely. Therefore, $\lambda$ affects how much time the economy spends in financial crises. The parameter $\chi$ affects how hard it is for firms to coordinate on high capacity, affecting the probability of coordination failures. In the data, there have not been many financial crises in the US. Moreover, there is not a clear procedure to identify coordination failures. I make some conservative choices here. I target a probability of coordination failure of 8%, and a probability of financial crises of 12% (remember that every coordination failure is also a financial crisis in the model). The choices of $\lambda$ and $\chi$ imply a probability of coordination failure of 8.3% and a probability of financial crisis of 12.4% (under the measure given by the stationary distribution). The expected time to reach a coordination failure when the economy is on the stochastic steady-state is 77 years, while the expected time for a financial crisis to happen is 24 years. This is roughly consistent with the interpretation that two coordination failures happened from 1929 to 2008.

\textsuperscript{25}This is justified by the fact that a model with a binary capacity choice cannot explain the deep and short lived fall in capacity after 2007Q4, but can explain the lowest level in which it stabilized after the crisis.

\textsuperscript{26}Since capacity in 2007Q2 was close to its pre-2007Q4 average, it does not matter whether we use the index value in 2007Q4 or its average prior to the crisis. The value used here is slightly different from Schaal and Taschereau-Dumouchel (2016b) because the sample includes more recent data.
2.5.2 Equilibrium properties

Figure 2.5 shows the stationary distribution of the state variable and some equilibrium functions. Those are very similar to the ones presented in Section 2.3.3. The main difference is that the left tail of the state variable density and the discontinuities around \( \hat{\eta} \) are much smaller. This comes from the fact that here I set a low probability of coordination failures, to capture the idea that those are rare events. From panels B and C, we can get an idea of the magnitude of the amplification mechanism. When the economy enters the low regime, output falls about 5.5% at once. Due to the fall in asset prices, the investment-to-capital rate also fluctuates a lot as the economy moves toward the low regime, falling from its steady-state level of 12% to less than 6%.

Table 2.2 compares some moments of the model and the data. All the statistics refer to the annualized growth rate of the variables. The calibration targeted the volatility of output, but the model does a great job capturing the volatility of consumption, hours, investment and intermediary equity. Not surprisingly, the volatility of capacity utilization is much lower in the
model than in the data. Since this is a binary variable in the model, its growth rate equals zero in many quarters and therefore the model misses a lot of high frequency movements in capacity. As for the correlations, the model predicts correlations usually a bit higher than those observed in the data.

Interestingly, the model matches well the long-run growth rate of most variables, even though it was not targeted in the calibration. In the data, the average growth rate of output, consumption and investment are 1.6%, 1.9% and 1.5%, respectively. In the model, these numbers are 1.5%, 1.5% and 1.6%.

Table 2.2: Dynamic properties of the data

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Hours</th>
<th>Investment</th>
<th>Equity</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Standard deviations (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>3.2</td>
<td>2.7</td>
<td>3.3</td>
<td>8.7</td>
<td>41.6</td>
<td>6.0</td>
</tr>
<tr>
<td>Model</td>
<td>3.2</td>
<td>2.7</td>
<td>3.5</td>
<td>8.9</td>
<td>37.5</td>
<td>1.8</td>
</tr>
</tbody>
</table>

|                  |        |             |       |            |        |          |
| **Panel B: Correlation with output** |        |             |       |            |        |          |
| Data             | 1      | 0.67        | 0.72  | 0.73       | 0.24   | 0.71     |
| Model            | 1      | 0.95        | 0.99  | 0.78       | 0.40   | 0.69     |

|                  |        |             |       |            |        |          |
| **Panel C: Correlation with intermediary equity** |        |             |       |            |        |          |
| Data             | 0.24   | 0.31        | 0.12  | 0.28       | 1      | 0.25     |
| Model            | 0.41   | 0.28        | 0.40  | 0.58       | 1      | 0.24     |

Notes: all variables refer to the annualized growth rate between two consecutive quarters.

2.6 The effects of endogenous disasters

The goal of this section is to study the impact of coordination failures (disasters) on equilibrium outcomes and welfare. While the analysis of sections 2.2 and 2.5 showed how asset prices, capital returns, output, and many other variables respond when the economy enters a coordination trap, the effects of disaster episodes may go far beyond what we observe during those crises. To fully understand the impact of endogenous disasters on welfare and long-run dynamics, we need to look at the global effects and not only local effects. As will be shown in this section, the global effects are large and affect the system even if coordination failures are very unlikely.
To analyze the impact of coordination failures, we need to compare the baseline model to an economy without coordination failures, which I call the second-best economy. In the second-best economy, financial frictions still exist but firms always coordinate on high capacity. That is a natural choice, since the main goal of this paper is not to study the impact of financial frictions, but instead to analyze the role of endogenous disasters that arise as a consequence of financial frictions. Following the related literature, I think of financial frictions as coming from some incentive problem of banks that are not easy to overcome.\footnote{See Townsend (1979) and Carlstrom and Fuerst (1997), to name only a few.} To exemplify, assume that banks’ returns are not verifiable and that a banker can divert assets at a very low cost. Thus, any contract that allows the banker to transfer some risk to its investors will not be used, since bankers would misreport its returns and investors would end up empty-handed. Banks need to have “skin in the game”, and debt contracts are used in equilibrium. I also assume that in the second-best, firms still set prices as monopolists. Assuming otherwise would only reinforce the effects presented here.\footnote{We could shut down the monopoly distortion by subsidizing a fraction $\frac{1}{\varepsilon}$ of firms’ expenditures on labor and capital. That would induce firms to set prices that are equal to the marginal cost, which would further increase welfare and asset prices.} Thus, the second-best is identical to the model of Section 2.2, except for the fact that firms play according to a cutoff \( \hat{\psi} \leq 0 \) (they always coordinate). This could be achieved by imposing a lump-sum tax on households and using the revenue to subsidize firms’ fixed cost \( f_t \).

As will be shown below, coordination failures have a large impact on the behavior of the system, even when coordination failures are extremely unlikely.

### 2.6.1 Global responses to disasters

Here I compare agent’s behavior in an economy with and without coordination failures. Disaster episodes affect agents’ behavior and prices even in states where they are very unlikely. Their local effects in bad times spread through the system, affecting the economy even in tranquil times.

Figure 2.6 shows the endogenous responses of some variables. I start by analyzing the two top panels. Note that asset prices are much lower in the calibrated economy with coordination failures, at any point of the state space. Asset prices get depressed because banks fear the disaster states, sharply reducing their demand for capital. In principle, banks can adjust their capital at every period and one could expect that a coordination failure that is not expected in 77 years would not have a large impact on their capital demand. At a given date \( t \), banks’ demand for capital depends on the dividend yield of capital and on the change of asset prices from \( t \) to \( t + dt \). Thus, the demand and the price of capital at \( t \) will be affected by the expected

\[ \frac{1}{\varepsilon} \]
Figure 2.6: Effects of coordination failures on equilibrium

(a) Asset prices
(b) Capital growth
(c) Capital share held by banks
(d) Leverage
(e) Total volatility
(f) Wealth share volatility

Baseline Second-best

- Baseline
- Second-best

σ + σ^2
η
0 0.05 0.1 0.15 0.2

σ^2
η
0 0.05 0.1 0.15 0.2

η
0 0.05 0.1 0.15 0.2

ησ
0 0.02 0.04 0.06

η
0 0.05 0.1 0.15 0.2

η
0 0.05 0.1 0.15 0.2

η
0 0.05 0.1 0.15 0.2
price of capital at $t + dt$, which in turn will be affected by the price of capital at $t + 2dt$ and so on, ultimately being affected by asset prices in distant bad states. If a coordination failure is expected $T$ years from now, where $T$ is large, agents expect very low asset prices at that date. The important thing is that asset prices suffer contagion from bad to good times, even when bad times seem very remote. Of course, the sooner a disaster is expected, the larger the effect on asset prices. Low asset prices imply a low growth rate of capital, reducing economic growth.

The middle panels show that banks intermediate much less capital in the economy with coordination failures. Moreover, they take on much less risk, as reflected by their lower leverage in bad states. This happens because, as negative shocks hit, banks anticipate low capital returns and asset prices, and start to unload their capital on the household sector, fearing coordination failures. But that is precisely what causes a coordination failure. Banks’ deleveraging anticipates disasters.

Interestingly, higher leverage in the economy without coordination failures does not imply higher volatility. As the bottom panels show, the total volatility of capital returns ($\sigma + \sigma^2$) can be more than twice as large in the economy with coordination failures, even though banks use less leverage. Volatility of banks’ wealth is higher in the baseline economy too. Since at the stochastic steady-state positive shocks do not improve the state of the economy, but bad shocks make things worse, higher volatility is harmful and it increases the probability of bad states.

Table 2.3 reports some statistics for both economies. Notice that coordination failures greatly increases the frequency of financial crises, given banks’ preemptive behavior when confronted with bad shocks. Output grows much less in the economy with coordination failures, while being much more volatile.

<table>
<thead>
<tr>
<th>Table 2.3: Likelihood of bad states</th>
<th>Baseline</th>
<th>Second-best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of financial crisis</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>Time to reach financial crisis</td>
<td>24</td>
<td>84</td>
</tr>
<tr>
<td>Output growth</td>
<td>1.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Output volatility</td>
<td>3.2%</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

Notes: Time to reach financial crisis (coordination failure) is the expected time to reach the cutoff $\eta^0 (\hat{\eta})$, when $\eta_0 = \eta^*$. Output volatility refers to the volatility of the annualized growth rate.
2.6.2 Welfare losses

We have seen a lot of channels through which coordination failures may negatively affect welfare. Coordination failures induce firms to produce below its full potential, lowering the endogenous productivity of the economy. By reducing banks’ capital returns in bad times, it reduces banks’ demand for capital, leading to lower asset prices. Lower asset prices reduce investment and growth. But there could be potential benefits too. As shown by Phelan (2016), in models with financial frictions banks may use too much leverage in bad times. By depressing banks’ returns precisely in bad times, coordination failures could serve as a discipline device (market discipline, as in Eisenbach, 2014). The analysis so far hints that it is not the case in this model, since banks’ higher leverage in the second-best actually helps to reduce volatility and increase economic growth.

Figure 2.7 shows the welfare in each economy and the welfare gains of avoiding coordination failures. The results are reported in welfare and consumption equivalent units, for each initial value of $\eta$.

Figure 2.7: The welfare costs of disasters

In the calibrated economy, the welfare gains of avoiding disasters can be equivalent to a 15% permanent increase in consumption when the economy is on the stochastic steady-state and agents expect a coordination failure to happen in 77 years. In welfare units, the gain is near 34%. Remember that the economy experiences an instantaneous output loss of 5.5%.

\[ \text{Due to the scale invariance properties of the model, the initial level of capital does not matter to compute welfare gains, since welfare is proportional to the initial capital. Notice also that welfare is defined even when } \eta_0 > \eta^*, \text{ since in this case intermediaries pay a lot of dividends and the economy instantaneously jumps to } \eta^*. \]
when it enters the low regime, and productivity falls only 2% (ignoring the benefit of not having to pay the fixed cost). Thus, it seems surprising that coordination failures can generate such a large welfare loss. Some back-of-the-envelope calculations suggest that if we ignored any general equilibrium effects that the possibility of coordination failures have on banks’ and households’ decisions, and dramatically assumed that productivity falls to zero in disaster states, the welfare loss should be of an order of magnitude similar to the probability of those states. But in fact, the welfare losses are much larger and this is due to the much lower asset prices caused by banks’ precautionary behavior.

Surprisingly, the perverse effects of disasters propagate through the state space even if they are extremely unlikely. In order to show that, Figure 2.8 compares the second-best economy to a sequence of economies that get closer and closer to the second-best. These economies are identical to the calibrated model but with different values of $\hat{\psi}$ (firms’ cutoff). As $\hat{\psi}$ approaches zero, the expected time to reach a coordination failure becomes very large, i.e., agents on the stochastic steady-state expect them to almost never happen. One could interpret lowering $\hat{\psi}$ as reducing firms’ fixed costs by providing a partial subsidy to producers that is not large enough to induce firms to always coordinate.

Figure 2.8 plots the welfare gains when the economy is at the stochastic steady-state of the second-best economy for many different values of $\hat{\psi}$. Note that this is a lower bound for the welfare gains, since lower initial values of $\eta$ lead to higher welfare gains, as coordination failures become more likely. Instead of reporting the value of $\hat{\psi}$ on the horizontal axis, the expected time to reach a coordination failure associated to each $\hat{\psi}$ is reported. For instance, in the economy with a time to reach near 1350 years, we have $\hat{\psi} = 0.06$ and an unconditional probability of a coordination failure below 1%.
Coordination failures that are not expected in almost 1500 years still have a significant impact on welfare, as measured in consumption equivalents. One should expect that events that happen in a so distant future would have no impact on equilibrium outcomes, unless agents’ discount rate was absurdly low. But even a tiny chance of disasters can have important global effects, although the economy almost never reaches those states.

To sum up, extreme events that are very unlikely can have important effects on welfare, even in good times. These effects operate through changes in asset prices and volatility that harm investment and economic growth in all states.

The main message of this section is that small-probability events may have huge and perverse effects on the global behavior of the system – and it happens through banks’ precautionary behavior. Banks require higher returns to hold capital, and welfare, asset prices and investment are very low. Policies that eliminate this kind of bad states can have positive effects that go far beyond the output and efficiency losses that they avoid at those specific episodes. That is what I analyze next.

2.7 Policies

In the last section, I showed that even rare coordination failures may have huge impacts on the behavior of the economy, even in tranquil times. The fall in asset prices is not restricted to bad states. It spreads through the system, harming investment and economic growth. Moreover, banks’ preemptive behavior increases the frequency of crises. In this section, policies that can alleviate that problem are discussed.

2.7.1 Subsidies to producers

A simple way to solve the coordination problem of the economy is to provide subsidies to firms that operate in high capacity, as in Guimaraes and Machado (2017) and Schaal and Taschereau-Dumouchel (2016b). The regulator could set a lump-sum tax $T_t$ on the household and use that to pay firms fixed cost $f_t$. For a high enough subsidy, firms would be happy to always operate in high capacity and coordination failures would never happen. We would be in the benchmark case analyzed in Section 2.6, and thus the welfare gains of such policy are huge.

In practice, such subsidies may be hard to implement. For this policy to work, the regulator must provide the subsidy conditional on firms choosing high capacity, which may be hard to verify. The productive sector is very large and unregulated, and it may be very difficult for a
regulator to successfully implement such policies on a large number of firms. In what follows I focus on policies targeted at financial intermediaries instead.

2.7.2 Insurance against disasters

A simple thing a regulator can do is to protect financial institutions from these economic disasters that happen from time to time. Since those events are very rare, the cost of this intervention may be low. One could think that large-scale asset purchases or equity injections undertaken by monetary authorities during and after the Great Recession were attempts to increase financial intermediaries’ returns in bad times.

In the calibrated model, the rental rate of intermediaries’ capital is 15.4% in the high regime and 14.4% in the low regime. Thus, rental rates do not change much, but as we have seen, this is enough to have a large impact on banks’ behavior. In this section I consider a policy in which the regulator guarantees a rental rate in bad times that is equal to the rental rate in good times (using a subsidy, for instance). The policy is broad and the subsidy is paid to anyone that is intermediating capital, including households. Restricting the subsidy to banks would only reinforce the effects presented here. One should think of this as a reduced form for any policy that tries to increase banks’ returns in bad times, such as those implemented by the Fed during the 2007-2009 crisis.

By protecting financial intermediation from the effects that coordination failures have on capital returns, this exercise helps disentangle the inefficiencies caused by the interplay of coordination failures and financial frictions from those directly caused by coordination failures. As will be shown below, the direct effects of coordination are very small relative to the overall effects.

Notice that the policy does not require the regulator to insure banks against bank-specific losses, which could imply large moral hazard costs that are not modeled here. It simply insure banks against terrible aggregate disasters, but they still have to bear any losses if their assets loose value \( d(q_t k_t) < 0 \). Policies that totally insure banks could worsen the not modeled moral hazard frictions that justify banks having to bear some risk in the first place.
Figure 2.9: Output and TFP

Under this policy, banks behave exactly like in the economy without coordination failures – the second-best economy of Section 2.6. All variables for banks look exactly like the dashed lines in Figure 2.6 and therefore are not reported here. The only difference is that in this economy, output and the endogenous TFP still fall sharply when \( \eta \) gets low. This is shown in Figure 2.9, which compares the baseline economy with the one where the policy is implemented. Note that coordination failures happen much less often (\( \hat{\eta} \) is lower), since banks do not behave preemptively and therefore do not hasten coordination failures. Table 2.4 shows that the policy reduces the frequency of financial crises and coordination failures considerably.

<table>
<thead>
<tr>
<th></th>
<th>Policy</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to reach financial crisis</td>
<td>84</td>
<td>24</td>
</tr>
<tr>
<td>Time to reach coordination failure</td>
<td>290</td>
<td>77</td>
</tr>
<tr>
<td>Probability of financial crisis</td>
<td>0.052</td>
<td>0.124</td>
</tr>
<tr>
<td>Probability of coordination failure</td>
<td>0.023</td>
<td>0.083</td>
</tr>
</tbody>
</table>

Notes: Time to reach financial crisis is the expected time to reach the cutoff \( \hat{\psi} \) when \( \eta_0 = \eta^* \) (in years). Probabilities are computed using the stationary distribution.

Figure 2.10 shows the welfare consequences of this policy, by comparing the world in which the policy is implemented with the second-best. Notice that for high values of \( \eta \), the policy brings the economy very close to the second-best, almost mitigating the effects of coordination failures. For low values of \( \eta \) there is still some room for improvement, but the welfare gains of
avoiding coordination failures are much lower than in the baseline model. The policy is very effective.

From Panel B of Figure 2.6, we can conclude that the policy considered here increases banks’ leverage in bad times. Moreover, the policy increases both welfare and stability (see the bottom panels of Figure 2.6). This is at odds with some previous literature. In Phelan (2016), banks take too much leverage in bad times, and policies that reduce their leverage on those states are often welfare improving. The reason is that banks do not internalize the perverse effects that higher leverage has on volatility and on the probability of bad states. The story here is quite different. On one hand, by increasing their leverage in mildly bad states, banks also become more exposed to negative shocks, which contributes to higher volatility. On the other hand, by leveraging up banks help avoid really terrible states, which helps reduce volatility. Of course, banks do not internalize any of those effects. But as it turns out, the positive effect of leverage on financial stability dominates and the policy ends up increasing not only efficiency, but also making the system more stable.

To summarize, low leverage in bad times is a bad deal, since it throws the economy into coordination traps and make the system globally volatile. By increasing banks’ returns in bad times, the regulator alleviates the perverse two-way feedback between the financial and the real sector, almost mitigating the negative effects of coordination failures.

### 2.8 Conclusion

This paper proposes a dynamic model of endogenous disasters that are associated with financial distress and coordination failures. In the model, there is an endogenous two-way
feedback between intermediaries’ balance sheets and firms’ production decisions.

The model offers a laboratory to study the global effects of extreme financial crises such as the Great Recession and the Great Depression. It shows that such events can have a large impact on welfare, asset prices and economic growth, even if they are extremely unlikely. Moreover, policies that increase intermediaries’ risk taking in bad times can improve not only welfare, but also increase financial stability. The model delivers several predictions about the relationship between financial and real variables.

This paper restricted the analysis to economic disasters driven by low demand expectations among producers. Yet, the techniques here developed could be useful to macroeconomists interested in modeling other non-linear phenomena related to financial crises, such as bank runs and interbank market freezes.

2.A Technical appendix

2.A.1 Proof of Proposition 2.1

I start finding firms’ optimal production decisions for a given $w_t$, $R_t$, $u_{j,t}$ and $Y_t$. Plugging the demand schedule (2.1) and the production function (2.2) into the firms’ profit, and taking first-order conditions for capital and labor we get:

$$(1 - \alpha) \frac{\epsilon - 1}{\epsilon l_{j,t}} \left(u_{j,t} A k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} = w_t \quad \text{and} \quad \alpha \frac{\epsilon - 1}{\epsilon k_{j,t}} \left(u A k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} = R_t.$$

Dividing the two equations above and rearranging, we get that under the optimal choice, $k_{j,t} / l_{j,t} = \frac{\alpha w_t}{1-\alpha R_t}$. Using this optimal capital-to-labor rate and the production function, we can write firms’ profits as a function of their production $y_{j,t}$:

$$\pi_{j,t} = \frac{\bar{y}_{j,t}^{\frac{1}{\epsilon}}}{\epsilon} - \frac{y_{j,t}}{A u_{j,t}} \left(\frac{R_t}{1-\alpha} \left(w_t \frac{1}{\epsilon} \right)^{1-\alpha} \right) - \mathbb{1}_{\{u_{j,t} = u_H\}} f_t.$$

(2.13)

Taking first-order conditions with respect to $y_{j,t}$ and using the demand schedule, one can easily verify that $p_{j,t} = \frac{\epsilon - 1}{\epsilon} mc_{j,t}$, where $mc_{j,t} = \frac{1}{A u_{j,t}} \left(\frac{R_t}{1-\alpha} \left(w_t \frac{1}{\epsilon} \right)^{1-\alpha} \right)$ is firm’s $j$ marginal cost. Taking the first-order condition with respect to $y_{j,t}$ in (2.13) and using the optimal ratio of labor and capital, we can find $y_{j,t}$, $k_{j,t}$, and $l_{j,t}$ by solving the following system of equations:

$$k_{j,t} = \alpha \frac{y_{j,t} mc_{j,t}}{R_t}, \quad l_{j,t} = \left(1 - \alpha\right) \frac{y_{j,t} mc_{j,t}}{w_t} \quad \text{and} \quad \frac{\epsilon - 1}{\epsilon} y_{j,t}^{\frac{1}{\epsilon}} = mc_{j,t}.$$

(2.14)
Let \( l_{H,t}, k_{H,t} \) and \( y_{H,t} \) denote the solution of the system above when \( u_{j,t} = u_H \), and \( l_{L,t}, k_{L,t} \) and \( y_{L,t} \) denote the solution when \( u_{j,t} = u_L \). Those are functions of \( w_t, R_t \) and \( Y_t \). Let \( \tilde{K}_t \equiv k_t^h + \beta k_t^l \) denote the total capital in the economy in efficiency units. Using the market clearing condition for capital and final goods we get:

\[
\tilde{K}_t = x_t k_{H,t} + (1 - x_t) k_{L,t}
\]

and

\[
Y_t = \left( x_t y_{H,t} + (1 - x_t) y_{L,t} \right)^{\frac{1}{1-\xi}}.
\]

Solving the system of equations above for \( R_t \) and \( Y_t \), one can find the equilibrium output and rental rate as a function of \( w_t \). Replacing \( w_t = \xi \), by the equilibrium condition in labor market, we get the equilibrium value of \( R_t \) and \( Y_t \), for a given \( \tilde{K}_t \). Plugging the equilibrium \( Y_t, R_t \) and \( w_t \) in the solution of (2.14), one can find \( k_{j,t}, y_{j,t} \) and \( k_{j,t} \), for \( j \in \{L,H\} \) as a function of \( \tilde{K}_t \). Then, one can find \( \pi_{L,t} \) and \( \pi_{H,t} \) using (2.13), and \( L_t = x_t l_{H,t} + (1 - x_t) l_{L,t} \).

To conclude the proof and get to expressions in Proposition 2.1, one should notice that \( \tilde{K}_t = (\psi_t (1 - \beta) + \beta) K_t \).

\( \square \)

### 2.A.2 Proof of Proposition 2.2

Using Proposition 2.1, we can write firms’ scaled relative profits as

\[
\Delta \hat{\pi}_t(\psi_t, \eta_t) = B \pi_t^\Omega \left( \psi_t (1 - \beta) + \beta \right) - \chi,
\]

where \( B = \left( \frac{1}{\xi - 1} \right) \left( \frac{\xi}{\xi + 1} \right) \left( \frac{1 - \alpha}{\xi} \right) A^\frac{1}{\alpha} \left( u_H^{\xi - 1} - u_L^{\xi - 1} \right) \). Firms choose high capacity if \( \Delta \hat{\pi}_t > 0 \), low capacity if \( \Delta \hat{\pi}_t < 0 \) and are indifferent if \( \Delta \hat{\pi}_t = 0 \). If \( \Omega > 0 \), \( \Delta \hat{\pi}_t \) is increasing on the average capacity. Moreover, it is increasing in \( \psi_t \). This if \( \psi_t < \hat{\psi}_t \), we have \( \Delta \hat{\pi}_t(\psi_t, u) < \Delta \hat{\pi}_t(\hat{\psi}_t, u) \), for any \( u \in [u_L, u_H] \), and thus firms can only choose low capacity. Similarly, if \( \psi_t > \hat{\psi}_t \), \( \Delta \hat{\pi}_t(\psi_t, u) > \Delta \hat{\pi}_t(\hat{\psi}_t, u) \), \( \Delta \hat{\pi}_t(\psi, u_L) = 0 \), and all firms must choose high capacity in equilibrium. If \( \psi_t \in [\hat{\psi}_t, \hat{\psi}_t] \), we have \( \Delta \hat{\pi}_t(\psi_t, u_H) > 0 > \Delta \hat{\pi}_t(\psi_t, u_L) \). Therefore, if all firms choose high capacity \( x_t = 1 \) and \( \eta_t = u_H \), it is optimal to choose high capacity, and thus \( x_t = 1 \) is an equilibrium. Similarly, if no one chooses high capacity, it is optimal to choose low capacity and \( x_t = 0 \) is an equilibrium. Finally, by the continuity and monotonicity of \( \Delta \hat{\pi}_t(\psi_t, \eta_t) \), for each \( \psi_t \in [\hat{\psi}_t, \hat{\psi}_t] \), there is a unique capacity \( \hat{u} \) such that firms are indifferent between high and low capacity. Thus, there is an \( x_t = h(\psi_t) \) (that implies an average capacity \( \hat{u} \)) that is also an equilibrium. As \( \psi_t \) increases, firms require a lower average capacity to be indifferent, and thus \( h'(\psi) < 0 \).

\( \square \)
2.A.3 Proof of Proposition 2.3

First, let me state the complete version of Proposition 2.3.

Proposition (full statement of Proposition 2.3). For a given function $R(\eta)$, the partial dynamic equilibrium is characterized as follows:

1. For a given $\theta(\eta)$, $\theta'(\eta)$, $q(\eta)$ and $q'(\eta)$, the functions $\mu^q(\eta)$, $\sigma^q(\eta)$, $\mu^\eta(\eta)$, $\sigma^\eta(\eta)$ and $\psi(\eta)$ are given by

   \[
   \eta \sigma^\eta(\eta) = -\frac{1}{2} q(\eta) \sigma' \eta(\eta) + \sqrt{(\sigma q(\eta) \theta' \eta(\eta))^2 + 4 R(\eta) q(\eta) \theta' \eta(\eta) (\beta - 1)} \frac{q'(\eta) \theta' \eta(\eta)}{q(\eta)} , \tag{2.15}
   \]

   \[
   \sigma^q(\eta) = \frac{\eta \sigma^\eta(\eta) q' \eta(\eta)}{q(\eta)} , \tag{2.16}
   \]

   \[
   \eta \mu^q(\eta) = \frac{[\eta (R(\eta) - \iota(\eta)) - \eta \sigma^\eta(\eta) q(\eta) (\sigma + \sigma^q(\eta))] \theta(\eta) - q(\eta) \theta' \eta(\eta) \sigma^q(\eta)^2 \sigma(\eta)^2}{q(\eta) \theta' \eta(\eta)} , \tag{2.17}
   \]

   \[
   \mu^q(\eta) = \frac{[1 - \eta \sigma^q(\eta) - q(\eta) R(\eta) + \iota(\eta)] \theta(\eta) - (\sigma + \sigma^q(\eta)) \eta \sigma^\eta(\eta) \theta' \eta(\eta) q(\eta)}{q(\eta) \theta' \eta(\eta)} , \tag{2.18}
   \]

   and

   \[
   \psi(\eta) = \frac{(\sigma + \sigma^q(\eta)) \eta + \eta \sigma^\eta(\eta)}{\sigma + \sigma^q(\eta)} , \tag{2.19}
   \]

   if we have $\psi(\eta) < 1$ above. Otherwise, $\psi(\eta) = 1$ and the expressions for $\sigma^q$ and $\mu^q$ are replaced by:

   \[
   \eta \sigma^\eta(\eta) = \frac{(1 - \eta) \sigma q(\eta)}{(1 - \eta) q'(\eta) + q(\eta)} , \tag{2.15'}
   \]

   \[
   \eta \mu^\eta(\eta) = \frac{[\sigma + \sigma^\eta(\eta)] \theta(\eta) + \theta' \eta(\eta) \eta \sigma^\eta(\eta)] (\sigma + \sigma^\eta(\eta)) (\eta - 1) q(\eta) + \eta \theta(\eta) (R(\eta) - \iota(\eta))}{q(\eta) \theta' \eta(\eta)} , \tag{2.17'}
   \]

2. The functions $\theta(\eta)$ and $q(\eta)$ satisfy the following boundary value problem with unknown parameter $\eta^*$:

   \[
   \lambda \theta(\eta) = \eta \mu^\eta(\eta) \theta' \eta(\eta) + \frac{1}{2} (\eta \sigma^\eta(\eta))^2 \theta'' \eta(\eta) , \tag{2.20}
   \]

   \[
   \mu^\eta(\eta) = \mu^\eta(\eta) \frac{q'(\eta)}{q(\eta)} + \frac{1}{2} \sigma^\eta(\eta)^2 \eta^2 \sigma'' \eta(\eta) , \tag{2.21}
   \]

   with boundary conditions

   \[
   q(0) = 1 , \quad \theta(\eta^*) = 1 , \quad \lim_{\eta \to 0} \theta(\eta) = \infty , \quad q'(\eta^*) = 0 \quad \text{and} \quad \theta'(\eta^*) = 0 .
   \]
Proof. I start deriving banks’ optimality condition. The Hamilton-Jacobi-Bellmann equation for banks can be written as

\[(\rho + \lambda) v(n^b, \eta) = \max_{\varphi^b(n^b, \eta), \zeta(n^b, \eta)} \left\{ \zeta(n^b, \eta) + n^b \mu^{nb}(\eta) \frac{\partial v(n^b, \eta)}{\partial n^b} + \eta \mu^v(\eta) \frac{\partial v(n^b, \eta)}{\partial \eta} \right\} \]

\[+ \frac{1}{2} \left( n^b \sigma^{nb}(\eta) \right)^2 \frac{\partial^2 v(n^b, \eta)}{\partial \eta^2} + \frac{1}{2} \left( \eta \sigma^v(\eta) \right)^2 \frac{\partial^2 v(n^b, \eta)}{\partial \eta^2} \]  

(2.22)

where,

\[\mu^{nb}(\eta) = \varphi^b(n^b, \eta) \left( \frac{R^b(\eta) - c(\eta)}{q(\eta)} + g(\eta) + \mu^q(\eta) + \sigma \sigma^q(\eta) - r_f \right) + r_f - \frac{\zeta(n^b, \eta)}{n^b} \]

and

\[\sigma^{nb}(\eta) = (\sigma + \sigma^q(\eta)) \varphi^b(n^b, \eta).\]

\[\mu^{nb}(\eta) \text{ and } \sigma^{nb}(\eta) \text{ come from the law of motion of banks wealth } n^b. \text{ To see that banks’ value function can be written as } v(n^b, \eta) = \theta(\eta)n^b \text{ consider two banks, 1 and 2, with initial wealth } n^1 \text{ and } n^2 = \vartheta n^1, \text{ respectively, where } \vartheta > 0. \text{ Let } (\hat{\zeta}^1_\lambda, \varphi^1_\lambda) \text{ and } (\hat{\zeta}^2_\lambda, \varphi^2_\lambda) \text{ denote banks’ optimal decisions at each date } t, \text{ where } \hat{\zeta}^1_\lambda = \zeta^1 / n^1_\lambda \text{ and } \hat{\zeta}^2_\lambda = \zeta^2 / n^2_\lambda, \text{ denote banks’ dividends payments per unit of wealth. Note that bank 2 is able to implement the policy } (\hat{\zeta}^1_\lambda, \varphi^1_\lambda), \text{ in which case } dn^1_\lambda / n^1_\lambda = dn^2_\lambda / n^2_\lambda, \text{ for every } t. \text{ Thus, under this policy she would get a payoff } \vartheta v(n^1, \eta). \text{ Hence, } v(n^2, \eta) \geq \vartheta v(n^1, \eta). \text{ A similar argument shows that } v(n^1, \eta) \geq \frac{1}{\vartheta} v(n^2, \eta) \Leftrightarrow v(n^2, \eta) \leq \vartheta v(n^1, \eta) \text{ and thus } v(n^2, \eta) = \vartheta v(n^1, \eta), \text{ implying that the value function is proportional to wealth. Replacing } v(n^b, \eta) = n^b \theta(\eta) \text{ on (2.22) and using the fact in equilibrium } r_f = \rho \text{ we get}
\]

\[\lambda \theta(\eta) = \max_{\varphi^b(n^b, \eta), \zeta(n^b, \eta)} \left\{ \frac{\zeta(n^b, \eta)}{n^b} (1 - \theta(\eta)) + \varphi^b(n^b, \eta) \left[ \frac{R^b(\eta) - c(\eta)}{q(\eta)} + g(\eta) + \mu^q(\eta) \right] \right\} \]  

\[+ \sigma \sigma^q(\eta) - r_f + \eta (\sigma + \sigma^q(\eta)) \sigma^q(\eta) \frac{\theta'(\eta)}{\theta(\eta)} \]  

(2.23)

It is assumed that banks are always leveraged, and thus they must hold capital. From the (2.23), one can see that banks will be indifferent between holding capital or risk-free debt whenever (2.9) is satisfied. Households’ optimality condition (2.6) and (2.7) are straightforward. Notice that in the optimum \( \frac{\zeta(n^b, \eta)}{n^b} (1 - \theta(\eta)) = 0 \) (either banks are not paying dividends or \( \theta(\eta) = 1 \)). Using \( \frac{\zeta(n^b, \eta)}{n^b} (1 - \theta(\eta)) = 0 \) and banks indifference condition (2.9), equation (2.23) becomes equation (2.11) in Proposition 2.3.
Applying Ito’s Lemma to \( q(\eta) \) we get (2.12) and

\[
\sigma^q(\eta) = \frac{\eta \sigma^q(\eta)q'(\eta)}{q(\eta)}. \tag{2.24}
\]

Applying Ito’s Lemma to \( \eta_t = n_b t + q_t k_t \), using \( \psi(\eta) = \varphi(\eta)\eta \) and the fact that banks never pay dividends below \( \eta^* \) we get:

\[
\eta \mu^q(\eta) = \frac{R^b(\eta) - \psi(\eta)}{q(\eta)} + \left( g(\eta) + \mu^q(\eta) - \sigma^2 - \sigma^q(\eta)\sigma - \sigma^q(\eta)^2 - r_f \right) (\psi(\eta) - \eta) \tag{2.25}
\]

\[
\eta \sigma^q(\eta) = (\psi(\eta) - \eta)(\sigma + \sigma^q(\eta)) \tag{2.26}
\]

Let’s start assuming in equilibrium \( \psi(\eta) < 1 \). In this case, we can solve for \( \mu^q(\eta), \sigma^q(\eta), \eta \mu^q(\eta) \) and \( \eta \sigma^q(\eta) \) as functions of \( \theta(\eta), \theta'(\eta), q(\eta) \) and \( q'(\eta) \) using (2.7), (2.9), (2.24), (2.25) and (2.26). Doing a lot of algebra, we get two solutions, associated to two different values of \( \eta \sigma^q(\eta) \):

\[
\eta \sigma^q(\eta) = \frac{1}{2} - q(\eta)\sigma \theta'(\eta) \pm \sqrt{(\sigma q(\eta)\theta'(\eta))^2 + 4R(\eta)q'(\eta)\theta(\eta)\theta'(\eta)(\beta - 1)} \tag{2.27}
\]

Next, I show how to eliminate the positive root. Notice that equation (2.26) implies \( \varphi(\eta) = 1 + \frac{\sigma^q(\eta)}{\sigma + \sigma^q(\eta)} \). Taking the limits, we get

\[
\lim_{\eta \to 0^+} \varphi(\eta) = 1 + \frac{\lim_{\eta \to 0^+} \sigma^q(\eta)}{\sigma + \lim_{\eta \to 0^+} \sigma^q(\eta)} = 1 + \frac{\lim_{\eta \to 0^+} \sigma^q(\eta)}{\sigma},
\]

where in the second equality I used the fact that when banks are wiped out \( q \) is constant. Thus, to have \( \varphi(\eta) > 1 \) for every \( \eta > 0 \) (i.e., banks are always borrowing), we must have \( \lim_{\eta \to 0^+} \sigma^q(\eta) > 0 \iff \lim_{\eta \to 0^+} \eta \sigma^q(\eta) > 0 \). Taking limits of (2.27) when \( \eta \to 0 \), we can see that the denominator is negative, since \( \theta'(0) < 0 \) and \( q'(0) > 0 \). Thus, for \( \lim_{\eta \to 0^+} \eta \sigma^q(\eta) > 0 \), we must have the numerator negative, which is not possible if the square root term is positive.

Getting rid of the positive root, the unique solution of the system is as stated in Proposition 2.3 for \( \psi < 1 \). If \( \psi = 1 \), in (2.19), we drop households indifference condition and solve the system using (2.9), (2.24), (2.25), (2.26) and \( \psi(\eta) = 1 \), which yields the unique solution stated in Proposition 2.3.

The boundary conditions are standard. The zero derivatives at \( \eta^* \) come from the fact \( \eta^* \) is a reflecting boundary. The boundary for \( q(0) \) is the price of capital when banks are wiped.
out. The condition \( \theta(\eta^*) = 1 \) comes from the fact that banks pays dividend at the reflecting boundary. To see why \( \lim_{\eta \to 0} \theta(\eta) = \infty \), notice that if a bank has one unit of capital when \( \eta = 0 \), she would face no risk (asset prices are constant) and would be able to get a capital return higher than households time preference \( \rho \) (since households would get \( dt^{kh} = \rho \) with their inferior technology). Thus the bank could borrow an infinite amount at rate \( \rho \), buy capital and make infinite profits without any risk.

2.A.4 Proof of Proposition 2.4

The expression for the flow utility per unit of capital \( \Lambda(\eta) \) can be obtained by static equilibrium, assuming \( x = 0 \) and \( x = 1 \). The HJB equation of household’s welfare can be written as

\[
\rho W(K, \eta) = \Lambda(\eta) K + \eta \mu'(\eta) \frac{\partial W(K, \eta)}{\partial \eta} + g(\eta) K \frac{\partial W(K, \eta)}{\partial K} \\
+ \frac{1}{2} (\sigma \sigma'(\eta))^2 \frac{\partial^2 W(K, \eta)}{\partial \eta^2} + \frac{1}{2} (\sigma K)^2 \frac{\partial^2 W(K, \eta)}{\partial K^2} + \eta \sigma(\eta) \sigma K \frac{\partial W(K, \eta)}{\partial \eta \partial K}.
\]

Suppose the initial capital of the economy is multiplied by \( \vartheta > 0 \), but \( \eta \) is kept constant. Since the flow utility scales with aggregate capital and a change in the initial capital does not affect the expected values of \( \psi_t \) (since it only depends of \( \eta \)), household welfare also scales with aggregate capital. We can write \( W(K_t, \eta_t) = W(\eta_t)K_t \) and plug it that in (2.28), yielding the ordinary differential equation of Proposition 2.4. The boundary condition \( W(0) = \Lambda(0) \rho - g(0) \) comes from the Gordon Growth formula. Finally, \( W'(\eta^*) = 0 \) since \( \eta^* \) is the reflecting boundary.

2.A.5 Numerical solution

First, for a given function \( R(\eta) \), the partial dynamic equilibrium can be found by solving the boundary value problem given in Proposition 2.3. That can be done either by using a standard boundary value problem solver (such as bvp4c in Matlab), or by using a bisection method as in Brunnermeier and Sannikov (2014). The Brunnermeier and Sannikov (2014) method works as follows. First, we guess a value for \( q'(0) \). Then we solve the system of ODE’s using the guessed \( q'(0) \) and the boundary conditions for \( \theta(0), \theta'(0) \) and \( q(0) \). Notice that \( \theta'(0) = -\infty \), since \( \theta(0) = \infty \). Moreover, if a function \( \theta(\eta) \) solves the system of ODE’s, then so does \( \vartheta \theta(\eta) \), where \( \vartheta > 0 \). Thus, we can normalize \( \theta(0) \) to 1. The system is integrated from \( \eta = 0 \) until some of those events happen: (i) \( \theta'(\eta) \) reaches zero or (ii) \( q'(\eta) \) reaches zero or (iii) \( q \) reaches \( \overline{q} = \max_q \frac{R - i(\eta)}{\rho - g} \), where \( \overline{R} \) is the rental rate of capital if banks hold all capital.
forever (and $\pi_t = u_H$). If both $\theta'(\eta)$ and $q'(\eta)$ are near zero at the terminal point, then we can stop and the reflecting boundary $\eta^*$ is given by the terminal point. Otherwise, we must update the guess of $q'(0)$. If (ii) happens first, we must increase $q'(0)$, otherwise we decrease it. Once we have found the functions $\theta(\eta)$ and $q(\eta)$, we must divide $\theta(\eta)$ by $\theta(\eta^*)$, so that $\theta(\eta^*) = 1$. The remaining functions can be recovered using Proposition 2.3.

When moving to the general dynamic equilibrium we need to find functions $R(\eta)$ and $\psi(\eta)$ that are consistent with the static equilibrium. To find the best general dynamic equilibrium I guess for each $\eta$ that the equilibrium is such that firms coordinate on high capacity. Thus, when solving for the partial dynamic equilibrium, I set the high rental rate $\bar{R}$ on the expressions given by Proposition 2.3 and check for each point in the state space if the implied $\psi(\eta)$ is consistent with the static equilibrium, i.e., $\psi(\eta) \geq \psi_{GG}$. If $\psi(\eta) < \psi_{GG}$, then it is assumed that firms do not coordinate, and we must set the rental $R$ associated to $\pi_t = u_L$ in Proposition 2.3. To find the worst equilibrium, the opposite is done. I start guessing that firms do not coordinate and then check if $\psi(\eta) < \psi_{GG}$. Alternatively, one could guess $\hat{\eta}^0$ is such that firms coordinate if $\eta \geq \hat{\eta}^0$ and do not otherwise. Then, the computed function $\psi(\eta)$ will imply a new value for $\hat{\eta}^1$, which is given by $\arg\min_\eta \{|\psi(\eta) - \psi_{GG}|\}$. If it is different from the initial guess $\hat{\eta}^0$, we must repeat the procedure using $\hat{\eta}^1$ as an initial guess. The good equilibrium is found by starting the iterations from the right ($\hat{\eta}^0 = 1$) and the bad equilibrium by starting it from the left ($\hat{\eta}^0 = 0$).

2.A.6 Proof of Proposition 2.5

Proposition 2.5 is a direct consequence of the First Welfare Theorem. The expression for $\Lambda_{FB}$ can be found solving the static equilibrium assuming that firms always set prices equal to the marginal cost and operate in high capacity. The steps are the same as in Proposition 2.1 and therefore, are omitted. The welfare is then computed by the Gordon Growth formula. That a social planner would set $\psi = 1$ is obvious. Also, the planner would choose high capacity if $\hat{\psi} < 1$, since firms profits are higher in the high regime. Therefore, household utility would be higher too.

2.B Data sources

Table 2.5 describes the data sources. All time series are quarterly from 1973Q1 to 2016Q1. All variables except capacity are expressed in per-capita terms. All variables are seasonally adjusted and all nominal variables are adjusted for inflation using the GDP deflator. The equity variable is the one with identifier FL793164113.Q in the Fed’s Flow of Funds. After
1996, it consists of the sum of the market value of common and preferred equity for U.S.
domestic financial corporations in CRSP (based on 2-digit SIC codes). For further details,
check the Fed’s website.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
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<tbody>
<tr>
<td>Output</td>
<td>BEA - Gross Domestic Product</td>
</tr>
<tr>
<td>Investment</td>
<td>BEA - Fixed Private Investment</td>
</tr>
<tr>
<td>Hours</td>
<td>BLS - Nonfarm Business Sector: Hours of All Persons</td>
</tr>
<tr>
<td>Consumption</td>
<td>BEA - Personal Consumption Expenditures</td>
</tr>
<tr>
<td>Equity</td>
<td>Flow of Funds - Domestic Financial Sectors; Public Corporate Equities</td>
</tr>
<tr>
<td>Capacity</td>
<td>FRB - Capacity Utilization: Total Industry</td>
</tr>
<tr>
<td>Population</td>
<td>Census - Total Population: All Ages including Armed Forces Overseas</td>
</tr>
<tr>
<td>GDP Deflator</td>
<td>BEA - Gross Domestic Product: Implicit Price Deflator</td>
</tr>
</tbody>
</table>

2.C Model with $\Omega \leq 0$

If $\Omega = 0$, then firms do not care about the average capacity $\bar{\pi}_t$ and we can write $\hat{\Delta}$ as a
function of $\psi_t$ exclusively. Thus, they all choose high capacity if $\hat{\Delta}(\psi_t) > 0$ and low capacity
if $\hat{\Delta}(\psi_t) < 0$. If $\hat{\Delta}(\psi_t) = 0$, then any $x_t \in [0, 1]$ is consistent with equilibrium. In what
follows, I focus on the case with $\Omega < 0$.

There will be essentially a unique equilibrium in the firm’s capacity game, but there are
few possibilities, depending on $\psi_t$. There are three cases:

1. If $\hat{\Delta}(\psi_t, \bar{\pi}_t) \leq 0$, for all $\bar{\pi}_t \in [u_L, u_H]$, then the unique equilibrium is the one where all
   firms choose low capacity.

2. If $\hat{\Delta}(\psi_t, \bar{\pi}_t) \geq 0$, for all $\bar{\pi}_t \in [u_L, u_H]$, then the unique equilibrium is the one where all
   firms choose high capacity.

3. If $\hat{\Delta}(\psi_t, u_L) > 0$ and $\hat{\Delta}(\psi_t, u_H) < 0$, we have a unique equilibrium with $x_t \in (0, 1)$,
   where $x_t$ is given by the indifference condition $\hat{\Delta}(\psi_t, \bar{\pi}_t) = 0$.

Notice that it covers all possible cases (for $\Omega < 0$). Depending on parameters, some of the
cases above might not be possible. Figure 2.11 illustrates the static equilibrium. Notice that,
by using Proposition 2.1, we can recover $R_t$ for each $\psi$, which will be a continuous function of
Figure 2.11: Static equilibrium when $\Omega \leq 0$

$\psi$ (as opposed to the case with $\Omega > 0$). To solve for the dynamic equilibrium, a procedure similar to the one applied for $\Omega > 0$ can be used. The main difference is that now one must search for intermediaries values of $x_t$ that are consistent with equilibrium when the guesses $x_t = 1$ or $x_t = 0$ fail.

2.D Global games selection

The results here follow from Morris and Shin (2001) and Morris and Shin (2003).

Suppose time is discrete and firms only make decisions at $t \in \{\nu, 2\nu, 3\nu, \ldots\}$. Assume that the capital quality shocks follow $Z_t = Z_{t-1} + \sigma \sqrt{\nu} \omega_t$, where $\omega_t$ and normally distributed iid shocks with zero mean and unit variance. At the end of each period, firms learn the true state of the economy. But at the beginning of each date $t$, before taking their capacity decisions, all firms observe the history up to date $t - \delta$. They also know that $\psi_t = s_t(\omega_t)$, where $s_t(\omega)$ is some increasing function of $\omega$ that comes from banks’ decisions (if positive shocks hit, banks intermediate more capital). Moreover, assume that states where $\psi_t = 0$ and $\psi_t = 1$ happen with positive probability in any interval on length $\nu > 0$ (as is the case in our model). Finally, assume there are dominance regions, i.e., $\tilde{\psi} > 0$ and $\tilde{\psi} < 1$.

Therefore, firms start period $t$ with a prior $m_{t-\delta}(\psi_t)$ over $\psi_t$. Assume that before taking their capacity decision, each firm $j$ also observe a private signal about the shock $\omega$:

$$\hat{\omega}_j = \omega_t + \Gamma \Xi_j,$$

where $\Xi_j$ are iid error terms with zero mean and positive variance, and $\Gamma > 0$. This is equivalent to getting a private signal signal about $\psi_t$, since $\psi_t = s_t(\omega_t)$ (with $s_t(\cdot)$ increasing). Since the capacity game satisfies the usual assumptions in the global games literature (see Morris and Shin (2003) for details), when $\Gamma \to 0$ there is a unique equilibrium in the capacity
game played by firms, and they play according to the cutoff given by (2.5). Then we can take the limit when \( \nu \to 0 \), so that \( Z_t = Z_{t-1} + \sigma \sqrt{\nu} \omega_t \) converges to a Brownian motion. The order of the limit here matters. If we first take the limit \( \nu \to 0 \), then the prior becomes too precise relative to the private signals and multiplicity still survives.
Chapter 3

Bailouts and the trade-off between coordination and moral hazard*

Abstract

This paper analyzes the role of guarantees when a regulator wants to avoid a market freeze, but at the same time is concerned that its policies may lead to excessive risk-taking. Banks face a coordination problem in their investment decisions and are allowed to risk-shift to projects with smaller expected return and higher volatility. Government guarantees can avoid a market freeze at the cost of increasing risk-shifting behavior. When fundamentals are below a threshold, the policy maker prefers not to intervene and a market freeze happens. The possibility of risk-shifting reduces the amount of guarantees needed to avoid a coordination failure.

Keywords: bank runs, moral hazard, risk-shifting, guarantees.

Jel Classification: G21, G28.

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*This chapter is coauthored by Ana Elisa Pereira.
3.1 Introduction

Recessions are often associated with coordination failures. If agents are pessimistic about economic activity they might refrain from investing and lending, and expectations may become self-fulfilling. During the credit crunch observed in the 2007-2009 crisis, governments have adopted a series of measures such as tax cuts, banks bailouts, asset purchase programs, etc. One possible motivation to those policies was to restore market confidence, enhancing coordination. As many times argued, the flip-side of those interventions is that they might lead to excessive risk-taking, creating a policy trade-off between coordination and moral hazard. This paper studies this trade-off in a simple model where banks face strategic complementarities in their investment decisions, leaving room for coordination failures. Banks have many projects available, some riskier and with lower expected return than others, leading to the possibility of excessive risk-taking.

In our model, a continuum of banks have basically two investment opportunities: a risk-free asset and a risky project. The riskless asset yields zero net return. The expected return of the risky project depends positively on two variables: the amount of banks investing in risky projects and fundamentals. If banks are not confident others will invest, profitable projects do not find the required funding (a market freeze). Under the assumption that fundamentals are common-knowledge, we have the possibility of multiple equilibria if fundamentals are in an intermediate range. In those situations, it is not clear when and why coordination failures will occur. By assuming that agents have a small amount of private information about fundamentals, we are able to select a unique equilibrium in the model using global games techniques.\(^1\)

Instead of a single type of risky project, we assume that there is a continuum of risky projects with different idiosyncratic probability of failure. Riskier projects yield a higher return in case of success. The expected return is increasing in the probability of success and therefore, in the absence of any intervention, agents choose the less risky project.

In this environment with strategic complementarities, there is room for government interventions aiming to eliminate coordination failures. We analyze a policy in which the government commits to cover part of the losses incurred by banks in case a project fails. However, this policy has a side effect on the choice of risky projects. Moral hazard arises from the fact that, when the government commits to bail out banks in case of failure, it can lead

\(^1\)We follow Morris and Shin (2003) to deal with equilibrium selection. Following the seminal work of Carlsson and Van Damme (1993), many authors have applied what became known as global games techniques to study speculative attacks (Morris and Shin, 1998; Guimaraes and Morris, 2007), bank runs (Goldstein and Pauzner, 2005), among others. Sákovics and Steiner (2012) apply those techniques to analyze to which sectors government should target subsidies to avoid coordination failures.
them to choose worse projects, with smaller expected return. In this context, the government
needs to consider the pros and cons of a policy that aims to enhance coordination. In short,
the trade-off we are interested in is that the more guarantees the government commits to, the
smaller is the probability of a market freeze, but on the other hand, the more banks take
excessive risk. We show that these moral hazard costs can make the government prefer to
allow market freezes to happen, even knowing that banks would be better off if they were able
to coordinate and invest in risky projects. Also, the possibility of moral hazard reduces the
amount of guarantees needed to avoid a coordination failure.

Government interventions that aim to restore markets functioning have been receiving a
growing attention recently. A large part of this literature focuses on how interventions can
overcome adverse selection in asset markets (examples include Philippon and Skreta, 2012,
arises from the fact that policies that unfreeze markets harmed by adverse selection may
deteriorate the informational content of prices. Our model is more related to another branch
of the literature, that focuses on how policies can help to mitigate coordination failures, when
investment and lending decisions present some sort of strategic complementarity. Bebchuk
and Goldstein (2011) study policy interventions that aim to mitigate coordination failures
in a model of credit freezes similar to Morris and Shin (2004). Our model is very related to
theirs, but unlike in those models, here the government faces a trade-off between coordination
and moral hazard.

Cheng and Milbradt (2012), in a dynamic model built upon He and Xiong (2012), study
optimal debt maturity, debt policy and bailouts in a context of debt rollover freezes. As in
our model, managers can risk-shift, moving towards a riskier project with smaller expected
return. The authors show that debt maturity should be such that it does not lead managers to
risk-shift in good times but, at the same time, does not increase too much rollover risk. Their
work is related to ours as they consider the moral hazard issues arising from government (or,
in general, some third party’s) interventions such as bailouts, but we abstract from other issues
in their paper that not the trade-off between coordination and moral hazard, and introduce a
continuum choice set of risky projects. Our simpler and static framework allows us to find
clear analytical results.

Another work that contributes to the debate concerning possible moral hazard distortions
due to liquidity provision is Corsetti, Guimaraes and Roubini (2006). The authors study the
role played by IMF as the international lender of last resort during international financial
crises. The model contradicts the conventional view that liquidity support always induces
moral hazard. Their results suggest that the availability of liquidity funds can even encourage
government to implement desirable reforms which would otherwise be too costly and risky. In a different framework, Allen et al. (2017) study government guarantees and their effects on banks’ exposure to runs. As in our model, government guarantees affect the behavior of banks. Their model focus on how government guarantees affect the contracts banks offer to creditors, while we focus on how guarantees affect the banks’ choices of projects.

The remainder of this paper is organized as follows. In the next section we present the model. Section 3.3 presents the policy exercise and the main results. Section 3.4 concludes.

3.2 Model

There is an unit-mass continuum of risk-neutral banks, indexed by $i$. Each bank has one unit of capital to invest. They can invest it in a risk-free asset (bonds), which yields a gross return of 1, or they can invest in some project that has uncertain return. There is a continuum of projects indexed by $\pi \in (0,1]$ that require one unit of investment. We refer to those projects as risky projects or risky assets. Each risky project has a idiosyncratic probability of success increasing in $\pi$. In case a project succeeds, it yields a return given by a function $\tilde{R}(\pi, \theta)$, where $\tilde{R}: (0,1] \times \mathbb{R} \rightarrow \mathbb{R}$. In case it fails, its gross return is zero. The variable $\theta$ denote the aggregate state (fundamental) of the economy. We assume the following functional form to $\tilde{R}$:

$$\tilde{R}(\pi, \theta) = \frac{\theta}{\pi} - \frac{\pi}{2} + 1.$$  

Notice that this function is decreasing in $\pi$ for positive values of $\theta$. Thus, riskier projects yield higher return in case of success.\(^2\)

The success probability of a project depends positively on the amount of banks investing in risky projects. There are different ways to justify this assumption. If many firms (projects) in the economy get funded, that will generate a higher demand and profits for other producers (as in Cooper and John, 1988 and Kiyotaki, 1988). Alternatively, if projects produce goods that are part of a supply chain, the lack of funding for some producers may reduce one’s connections and disrupt its demand and supply of inputs (as in Taschereau-Dumouchel, 2017). The important thing here is that there are positive spillovers when profitable projects find the required funds. We denote by $l$ the proportion of banks investing in risky projects. Let $p(\pi, l)$ denote the probability of success of project $\pi$ when $l$ banks are investing in it. We assume

\(^2\)If $\theta < 0$ it is not true, but in that case no one will undertake any risky project, since they could get a higher return by investing in the risk-free asset. Thus, the case where the choice between risky projects is relevant will not include situations where $\theta < 0$.  

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that it is given by
\[
p(\pi, l) = \begin{cases} 
\pi & \text{if } l \geq b, \\
\gamma \pi & \text{if } l < b.
\end{cases}
\]

where \(0 < b < 1\) and \(0 < \gamma < 1\). We say that there is a market freeze whenever \(l < b\), and in that case the probability of success of any risk project is smaller. Letting \(R(\pi, \theta)\) be the gross return of project \(\pi\) after all uncertainty realizes, its expected return conditional on some given \(l\) can be written as
\[
E_l[R(\pi, \theta)] = \begin{cases} 
E[\theta] - \frac{\pi^2}{2} + \pi & \text{if } l \geq b, \\
\gamma (E[\theta] - \frac{\pi^2}{2} + \pi) & \text{if } l < b.
\end{cases}
\]

Notice that this function is increasing in \(\pi\) for any given \(l\). Therefore, under no interventions, banks will never invest in projects with \(\pi < 1\). Whether a bank will invest in the project \(\pi = 1\) or not depends on his beliefs about \(l\) and the value of \(\theta\). If \(\theta < 1/2\), then investing in the risk free asset is a dominant action. If \(\theta > 1/\gamma - 1/2\) then investing in the risky project is a dominant choice. Assuming that \(\theta\) is common knowledge, we have multiple equilibria if \(1/2 \leq \theta \leq 1/\gamma - 1/2\).

If we introduce some private information about \(\theta\), multiplicity is ruled out. We assume that \(\theta\) is drawn from a uniform prior on the real line and that each bank \(i\) receives two signals: a public signal \(y = \theta + \eta\), where \(\eta\) is a normal random variable with zero mean and variance \(\sigma_y^2\), and a private signal \(x_i = \theta + \epsilon_i\), where \((\epsilon_i)_{i \in [0,1]}\) are iid normal random variables with zero mean and variance \(\sigma_x^2\).

We can define a strategy in this environment as a function \(s(y, x_i)\) that take values \(I\) or \(N\), where \(I\) denotes investing in the (best) risky project and \(N\) denotes investing in the risk-free asset. We say that \(s(y, x)\) is a cutoff strategy around \(x^*\) if \(s(y, x) = I\) whenever \(x > x^*\) and \(s(y, x) = N\) whenever \(x < x^*\).

Hereafter, we will focus on the limiting case where \(\sigma_x\) goes to zero. Proposition 3.1 shows that in this case there is an essentially unique equilibrium.

**Proposition 3.1.** Let \(\theta^*\) be defined as:
\[
\theta^* = \frac{1}{1 - b(1 - \gamma)} - \frac{1}{2},
\]

In equilibrium, as \(\sigma_x \to 0\), banks play according to the cutoff \(\theta^*\) (invest if \(x_i > \theta^*\) and do not invest if \(x_i < \theta^*\)).

**Proof.** The proof follows from Proposition 2.2 in [Morris and Shin (2003)](#), which implies that the cutoff \(\theta^*\) can be computed assuming that when \(\theta = \theta^*\) agents are indifferent between the
(best) risky project and the risk-free asset if they have a uniform belief over \( l \). Therefore, given that agents will choose \( \pi = 1 \), \( \theta^* \) must be such that

\[
 b\gamma \left( \theta^* + \frac{1}{2} \right) + (1 - b) \left( \theta^* + \frac{1}{2} \right) = 1,
\]

which implies the desired result.

The result above implies that when \( \sigma_x \to 0 \) we have \( l \geq b \) if the realization of \( \theta \) is higher than \( \theta^* \), and \( l < b \) if the realization of \( \theta \) is smaller than \( \theta^* \). In other words, whenever \( \theta < \theta^* \) there is a market freeze.

Notice that \( \theta^* \in (1/2, 1/\gamma - 1/2) \). As illustrated in Figure 3.1, we say that if \( \theta \) is between \( 1/2 \) and \( \theta^* \) there are coordination failures (or a inefficient market freeze), since everyone would be better off if the risky projects were undertaken. Agents are failing to coordinate, afraid others might not undertake the project. Therefore, there is room for interventions that aim to mitigate coordination failures. The next section points out some trade-offs involving government policies in this environment.

Figure 3.1: Market freezes

<table>
<thead>
<tr>
<th>Efficient market freeze</th>
<th>Inefficient market freeze</th>
<th>No market freeze</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2 )</td>
<td>( \theta^* )</td>
<td>( \theta )</td>
</tr>
</tbody>
</table>

### 3.3 Bailouts

We assume in this section that the government commits to pay a part of the loss incurred by a bank in case its project fails. The policy analyzed here is similar to real world bailout policies that aim to rescue banks in case they suffer hard losses. We restrict attention to policies that do not cover more than the capital invested in the project. The policy is the following: before banks choose their actions, but after observing the public signal \( y \), the government commits to buy at a price \( \delta \in [0, 1) \) any risky asset after the realization of its return. Notice that to implement this policy is not necessary to assume that the regulator observes the realization of banks’ payoffs.

Agents will never invest in a project if they believe that \( \tilde{R}(\pi, \theta) < \delta \), since they could get \( 1 > \delta \) with certainty by investing in the risk-free asset. Since agents are infinitely well informed about \( \theta \), it will never happen that an agent whose project was successful sell his asset to the government. Thus, in practice, the government will just pay those guarantees in case some
project fails. The expected return becomes

\[ E_l [R(\pi, \theta)] = \begin{cases} E[\theta] - \frac{\pi^2}{2} + \pi + (1 - \pi)\delta & \text{if } l \geq b, \\ \gamma \left( E[\theta] - \frac{\pi^2}{2} + \pi \right) + (1 - \gamma\pi)\delta & \text{if } l < b. \end{cases} \]

We first study the equilibrium when there is no possibility of moral hazard. To do so, we assume that the only risky project available is the one with \( \pi = 1 \).

**Benchmark case**

Assume agents are restricted to invest either in the risk-free asset or in projects with \( \pi = 1 \). Proposition 3.2 below is the analogous of Proposition 3.1 under the government intervention.

**Proposition 3.2.** Assume that the government commits to a bailout \( \delta \in [0, 1) \) and agents are not allowed to invest in projects with \( \pi \neq 1 \). Then, the results in Proposition 1 apply, but the cutoff \( \theta^* \) is now given by

\[ \theta^*_B = \frac{1 - b(1 - \gamma)\delta}{1 - b(1 - \gamma)} - \frac{1}{2}, \]

which is a decreasing function of \( \delta \).

The proofs of Propositions 3.2 and Proposition 3.3 (to be presented) are similar to the proof of Proposition 3.2 and therefore are omitted.

**General case**

Now we turn to the case where agents choose optimally in which risky project to invest, given the government guarantee. If \( \delta > 0 \), an agent investing in the risky asset will no longer choose the higher expected return project, \( \pi = 1 \). Since government is covering some losses, banks are taking excessive risk, with their choice of projects being given by

\[ \pi^* = 1 - \delta. \]

That is, the more guarantees the government commits to, the more the banks choose worse projects. Proposition 3 characterizes the equilibrium in this situation.

**Proposition 3.3.** Assume the government commits to guarantees \( \delta \in [0, 1) \). Then, the results in Proposition 1 apply, but the cutoff \( \theta^* \) is now given by

\[ \theta^*_G = \frac{1 - b(1 - \gamma)\delta}{1 - b(1 - \gamma)} - \frac{1}{2} - \frac{\delta^2}{2}, \]  

(3.1)
which is a decreasing function of $\delta$.

Notice that it is very similar to the benchmark case, except for the term $-\frac{\delta^2}{2}$. By allowing agents to risk-shift, the effect of the intervention over $\theta^*$ is larger. The direct effect of the intervention is captured by $\frac{-b(1-\gamma)\delta}{1-b(1-\gamma)}$: banks expected payoffs of investing are higher, since government is covering part of the losses in case a project fail. The indirect effect is given by $-\frac{\delta^2}{2}$: banks now have an extra benefit due to the chance to gamble with government money and this also increases the expected payoff of investing in risky projects.

**Optimal guarantees**

A strategy to the government is a map from its signal $y$ to a level of guarantees $\delta \in [0, 1)$. Here we focus on the limit when $\sigma_y \to 0$, meaning that the government has very good information about fundamentals. Since the government has an infinitely precise signal regarding $\theta$, it assigns probability one that $y = \theta$ in the limit. Thus, to simplify the exposition we sometimes replace the public signal $y$ by the actual value of $\theta$ (one can think that the planner actually knows $\theta$).

The government has a trade-off: a higher $\delta$ induces more moral hazard, in the sense that banks move towards riskier projects with smaller expected return. But also, a higher $\delta$ reduces the values of $\theta$ for which coordination failures occur. Thus, in principle, even if $\theta$ is in the interval where coordination failures occur, the government might want not to give guarantees to avoid it, since banks would gamble with taxpayer funds by choosing projects with smaller expected return than the risk-free asset.

If the government could enforce agents to invest in the project with $\pi = 1$, whenever $\theta \in (1/2, \theta^*)$, the government would give the amount necessary to move $\theta^*$ to the left of $\theta$ and thus, make agents invest in the risky project. Now, since it is not the case anymore, the government has to take into account the losses due to the choice of worse projects.

For a given signal, the government wishes to give guarantees if the expected return (not taking into account the guarantees) of the risky projects under the intervention is higher than the return of the risk-free asset. The government seeks to maximize the total wealth generated in the economy. Precisely, under the assumption that signals are infinitely precise,

---

3The order of the limit here matters. First, we took the limit when $\sigma_x \to 0$ to derive the equilibrium for a given level of guarantees and after that we can take the limit $\sigma_y \to 0$. As shown in Morris and Shin (2003), to ensure equilibrium uniqueness, private signals need to be sufficiently precise relative to public signals.
the government objective function is

$$W = \begin{cases} 1 & \text{if } y < \theta^*_G, \\ y - \frac{(1-\delta)^2}{2} + (1 - \delta) & \text{if } y \geq \theta^*_G, \end{cases}$$

(3.2)

where the second line is just the expected return of projects when there is no failure, considering that agents will choose $\pi^* = 1 - \delta$. Since $y = \theta$, we could replace $y$ by the actual value of $\theta$ in the expression above. The government chooses $\delta$, and consequently $\theta^*_G$, to maximize (3.2), subject to $\delta \in [0, 1)$ and $\theta^*_G$ given by (3.1). Proposition 3 pins down the government decision for every realization of $\theta$.

**Proposition 3.4.** The government gives positive levels of guarantees to banks if and only if $\theta$ is in a non-empty interval $[\tilde{\theta}, \theta^*]$, where $\tilde{\theta} > 1/2$. In that case, the government commits to the smallest value of $\delta$ that implements the threshold $\theta^*_G = \theta$, which is given by

$$\delta^*(\theta) = \sqrt{\frac{1}{[1 - b(1 - \gamma)]^2} - 2\theta - \frac{b(1 - \gamma)}{1 - b(1 - \gamma)}}. \quad (3.3)$$

Proof. See Appendix 3.A. 

The results of Proposition 3.4 are illustrated in Figure 3.2. It shows that the optimal policy never implements a threshold in some neighborhood of $1/2$, in contrast to the case with no moral hazard. In that area, it is too expensive in terms of moral hazard to rescue banks, since it would be necessary a large amount of guarantees. However, when $\theta$ is higher and thus the coordination problem is not too severe, the government offers guarantees to banks, since the gain due to coordination is higher than the loss resulting from moral hazard.
For comparison purposes, the figure also shows the minimal level of guarantees the government commits to in the benchmark case, which is given by\(^4\)

\[
\delta(\theta) = 1 - \left[\frac{1 - b(1 - \gamma)}{b(1 - \gamma)}\right] \left(\theta - \frac{1}{2}\right).
\]

Notice that, in that case, the government always intervenes in the area of coordination failures. The amount of guarantees needed to avoid failures is always higher (since there is only the direct effect), but the government is always willing to do so, since agents cannot risk-shift. The possibility of “betting for resurrection” is an additional incentive for banks to finance still profitable projects. Thus, if the government is in financial distress and cannot credibly commit to a high level of guarantees, it might actually benefit from the risk-shifting behavior of financial institutions.\(^5\)

\(^4\)The government commits to, at least, the level of guarantees that implements the threshold \(\theta_B = \theta\) (the proof follows the same logic used to derive equation 3.3 and is omitted to save space). It is indifferent between announcing this minimum level of guarantees or any higher level, since there will be no coordination failure and thus all projects will succeed anyway.

\(^5\)Fiscal issues were very relevant to some countries in the Eurozone during the last financial crisis. See Allen et al. (2015).
3.4 Conclusion

This paper studies the policy trade-off of a regulator that would like to avoid a market freeze, but is concerned with the opportunistic behavior that its policies may induce. Banks face a coordination problem in their investment decisions and may not fund profitable projects if they are not confident other financial intermediaries in the economy will do the same. Government guarantees that protect investors from bad states may enhance banks’ ability to coordinate, but at the same time, it worsens the quality of projects that get funded by inducing excessive risk-taking. In intermediaries states, guarantees are still desirable, despite its moral hazard costs. For very low states, the regulator prefers not to intervene, since the moral hazard distortions would be too high, and a market freeze happens. The amount of guarantees needed to avoid a market freeze is smaller due to moral hazard.

3.A Proof of Proposition 3.4

First notice that if $\theta \not\in [1/2, \theta^*]$ the optimal choice is to give no guarantees: if $\theta > \theta^*$ the government knows that agents will invest with probability one, and thus there is no reason to give guarantees and make them choose worse projects. If $\theta < 1/2$ the government knows that the return of any risky project is smaller than one. Also, for any given $\theta$, we can show that the government will never give a guarantee level greater than $\delta_{\text{max}}(\theta) = \sqrt{2\theta - 1}$. Otherwise, banks would invest in projects with expected return smaller than 1. Therefore, for a given $\theta \in (1/2, \theta^*)$, the government problem is to verify whether he can achieve a threshold equal to $\theta$ by giving guarantees smaller than or equal to $\delta_{\text{max}}(\theta)$. Whenever $\theta \in (1/2, \theta^*)$, the government chooses to intervene and eliminate coordination failures if there is a $\delta \leq \delta_{\text{max}}(\theta)$ such that $\theta_G(\delta) \leq \theta$. Since $\theta_G(\delta)$ is a decreasing function of $\delta$, if $\theta^*(\delta) \leq \theta$ for some $\delta \in [0, \delta_{\text{max}}(\theta)]$, then $\theta^*_G(\delta_{\text{max}}(\theta)) \leq \theta$. Thus, given $\theta \in (1/2, \theta^*)$, the government gives guarantees if and only if

$$
\theta^*_G(\delta_{\text{max}}(\theta)) = \frac{1 - b(1 - \gamma)\sqrt{2\theta - 1}}{1 - b(1 - \gamma)} - \frac{1}{2} - \frac{2\theta - 1}{2} \leq \theta.
$$

Which can be expressed as

$$
\frac{1 - b(1 - \gamma)\sqrt{2\theta - 1}}{1 - b(1 - \gamma)} \leq 2\theta.
$$

(3.4)

When $\theta = 1/2$, the inequality above is violated, since $\frac{1}{1 - b(1 - \gamma)} > 1$. By continuity, we know that there is a non-empty interval $(1/2, \theta)$ in which condition (3.4) does not hold. The left-hand side is a strictly decreasing function of $\theta$ and the right-hand side is strictly increasing in $\theta$.  

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We need to show that the left- and the right-hand side are equal at some point $\theta < \theta^*$. To see if this is true, it is enough to check if $LHS < RHS$ when $\theta = \theta^*$ in condition (3.4):

$$\frac{1 - b(1 - \gamma)\sqrt{2\theta^* - 1}}{1 - b(1 - \gamma)} \leq 2\theta^*.$$ 

Substituting the expression for $\theta^*$ (with no guarantees) in the RHS, we get:

$$-\sqrt{2\theta^* - 1} < 1,$$

which is satisfied since $\theta^* > 1/2$. That is, if $\theta = \theta^*$, condition (3.4) holds with strict inequality, meaning that there is always an interval $[\underline{\theta}, \theta^*]$ in which government intervenes. $\underline{\theta}$ is the value that solves (3.4) with equality. Finally, the optimal value of guarantees, $\delta^*(\theta)$ in equation (3.3), is obtained simply imposing $\theta^*_G = \theta$ in the threshold equation given by (3.1) and solving for $\delta$. \qed
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