Fundação Getulio Vargas
ESCOLA DE POS-GRADUAÇÃO EM ECONOMIA

Andre Espozel Pinheiro da Silva

Testing Dynamic Agency Predictions to Corporate Finance

Rio de Janeiro
22 de março de 2017
Testing Dynamic Agency Predictions to Corporate Finance

Dissertação submetida à Escola de Pós-Graduação em Economia como requisito parcial para a obtenção do grau de Mestre em Economia.

Orientador: Felipe Saraiva Iachan

Rio de Janeiro
22 de março de 2017
Silva, Andre Espozel Pinheiro da
37 f.

Dissertação (mestrado) - Fundação Getulio Vargas, Escola de Pós-Graduação em Economia.
Orientador: Felipe Saraiva Iachan.
Inclui bibliografia.


CDD – 332
TESTING DYNAMIC AGENCY PREDICTIONS TO CORPORATE FINANCE.

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

Data da defesa: 22/03/2017.

ASSINATURA DOS MEMBROS DA BANCA EXAMINADORA

Felipe Saraiva Iachan
Orientador (a)

Luis Henrique Bertolino Braido

Igor Felizatti C. da Cunha
Abstract

This paper tests theoretical predictions concerning agent compensation, debt structure and investment in the models of dynamic agency in DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006) and DeMarzo, Fishman, He and Wang (2012). The results related to agent compensation are consistent with the patterns predicted in the models, indicating that the firm-years that the models would have as more likely to pay dividends are indeed the ones more likely to pay; also, among firms that pay dividends, more profits generate higher dividend payments and higher executive compensation, as predicted in the models. The prediction that firms that go well and reach a payment threshold present marginal $q$ equal to average $q$, and thus after controlling for average $q$ cash flows would not explain investment is also supported by the tests in here. On the other hand, predictions related to the role of the credit line and to the debt structure are not compatible with the results in here. The credit line doesn’t seem to be the provider of financial slack that protects the firm from low cash flows and also doesn’t seem to have the dynamics of being paid when profits are high and being more used when profits are low.

Keywords: credit line, debt structure, executive compensation, investment, dynamic agency, financial contracting
Contents

1 Introduction 7

2 Theoretical Framework 9
   2.1 DeMarzo and Sannikov (2006) 9
   2.2 DeMarzo, Fishman, He, and Wang (2012) 16

3 Empirical Tests 22
   3.1 Payments to the agent 23
   3.2 Debt and Credit 28
   3.3 Investment 32

4 Concluding Remarks 35
List of Figures

1. The principal’s value function $b(W)$. ........................................ 12
2. The optimal contract with low volatility. ................................. 15
3. Investors scaled value function $p(w)$ as a function of the agent’s scaled continuation payoff $w$. ................................. 19
4. Average $q_a$ and marginal $q_m$. ........................................ 21

List of Tables

1. Comparative Statics for the Optimal Contract .......................... 15
2. Summary Statistics ........................................................... 22
3. Summary Statistics - firms with credit lines .......................... 23
4. Results - Payment of dividends ............................................ 24
5. Results - Payment of dividends, alternative dummies ................ 25
6. Results - Compensation ..................................................... 26
7. Results - Payments in payment threshold ............................. 27
8. Results - Total debt capacity .............................................. 29
9. Results - Credit line or debt .............................................. 31
10. Results - Dynamics of credit line ....................................... 32
11. Results - Investment ....................................................... 34
1 Introduction

According to Modigliani and Miller (1958), under certain assumptions, capital structure is irrelevant, that is, does not affect the value of the firm. However, some of these assumptions such as the absence of taxes and bankruptcy costs; the nonexistence of asymmetric information between insiders and outsiders; and the absence of agency costs seem to be unrealistic, what motivated and still motivates more theoretical work related to the subject. Our focus is on the last assumption, the absence of agency costs, and we develop tests to verify the accuracy of predictions from agency models on capital structure, more specifically, on choices over debt and credit. However, these models also deal with agent compensation and investment decisions, and this paper also focuses on developing tests in order to verify whether predictions over these two other subjects hold.

A literature starting in DeMarzo and Fishman (2007) has developed theoretical agency models applied to corporate financing. In their model, an entrepreneur with limited liability needs external resources to finance a long-term project that is profitable, but has risky cash flows. The agency problem arises from the possibility the agent has of misreporting cash flows enjoying a fraction of the deviated resources with private consumption, optimal contracts are developed conditioning transfers of firm cash flows made to the agent and a termination decision on the history of reported cash flows.

DeMarzo and Sannikov (2006) presents a continuous time version of the discrete time model in DeMarzo and Fishman (2007), both papers offer general predictions concerning cash flows that the agent receives from the firm and propose a specific implementation of the optimal contract that involves three securities: a credit line, long-term debt, and equity. In the optimal contract, there is a state variable that represents both the value the contract has to the agent and the financial slack of the firm, this variable increases with high cash flows until a threshold where the agent begins to receive transfers, and decreases with low ones until a point in which termination occurs (in the case of DeMarzo and Fishman (2007) termination is randomized). In the specific formulation, the balance in the credit line works as the state variable, and when drawn amount is zero the agent receives cash through dividends, conversely, when the credit line is overdrawn, the termination occurs (or is randomized). DeMarzo, Fishman, He and Wang (2012) extend the continuous time model to an environment with a firm size that varies over time, with endogenous investment decisions also conditioned on reported cash flows. Due to the agency problem, their model predicts a marginal \( q \) - the incremental impact of a unit of capital on firm value - equal to average \( q \) - the ratio between firm value and capital stock - only at termination and payments thresholds.

Despite the theoretical advances brought by these models, their implications still haven’t
been empirically tested, and to test them is the purpose of this paper. To do that, reduced-form regressions are run with panel data with the intention to test predictions that are more general to the optimal contracts and also others that are more specific to the proposed implementation. Tests are divided in three groups: the first concerning to the agent compensation; the second that relates to debt structure; and the third focuses on investment decisions.

Results are mixed. The ones related to agent compensation are consistent with the patterns predicted in the models, indicating that the firm-years in which agents in the model pay dividends are indeed the ones more likely to have dividend payments; also, among firms that pay dividends, more profits generate higher dividend payments and higher executive compensation, as predicted in the models. The prediction that firms in the payment threshold present marginal $q$ equal to average $q$, and thus, after controlling for average $q$, cash flows would not explain investment is also supported by the tests in here. On the other hand, predictions related to the role of the credit line and to the debt structure are not compatible with the results in here. The credit line doesn’t seem to be the provider of financial slack that protects the firm from low cash flows and also doesn’t seem to have the dynamics of being paid when profits are high and being more used when profits are low.

The data consists on a panel of US-based public companies and covers years from 2001 (first year of available credit line data) until 2016. Data from credit lines, executive compensation and credit rating come from S&P’s Capital IQ, the rest of the data is from Compustat. The paper is structured with a theoretical description of the model after this introduction, then a section with the presentation of the tests that were run and their respective results, and it comes to an end with the concluding remarks.
2 Theoretical Framework

2.1 DeMarzo and Sannikov (2006)

DeMarzo and Fishman (2007) introduce a discrete-time model of an entrepreneur that needs investors to finance a long-term risky project. In their environment, there is an agency problem arising from the uncertainty of cash flows and the possibility the agent has to misreport earnings and deviate an amount for personal use. The description of these model here will be brief, as a continuous-time version of the model - DeMarzo and Sannikov (2006) - will be analyzed in more detail.

The authors solve for an optimal contract involving the possibility of termination and transfer payments between principal and agent conditional on reported cash flows. In this solution incentives are provided to ensure truth telling, that is, reported cash flows are the true ones. The only state variable turns out to be expected value to the agent, that responds to reported cash flows for incentive reasons, and there are two thresholds for this variable. The lower one is reached when reported cash flows are low, and it determines the possibility of termination, that is random with higher probability for greater distance to the threshold. The higher one determines transfers to the agent, that chooses to consume them.

A particular implementation of the contract involving three securities is suggested. The first one is equity, which is used to provide incentives for true reports; the second one is a credit line that provides the necessary financial slack; and the third is a long-term debt, that is useful for providing earlier consumption for the impatient agent. In this solution higher cash flows are first used to pay down the credit line. When its balance reaches zero, dividends start being paid. Conversely, lower cash flows provoke draw-downs on the credit line and, when it is over-drawn, liquidation is randomized.

The aforementioned continuous-time version of this model, in DeMarzo and Sannikov (2006), has some additional desirable features. First, its characterization of the optimal contract is cleaner, allowing an analytic determination of capital structure and computation of comparative statics and security prices. Second, the termination decision is no longer stochastic.

In DeMarzo and Sannikov (2006) a risk-neutral entrepreneur has a profitable project with risky cash flows. He needs external resources to finance an initial investment $K$ and cover possible stochastic losses to continue his project. This financial support is provided by a principal in a dynamic contract, but an agency problem emerges due to the following moral hazard problem: the agent can covertly deviate cash flows for personal use and, once that is done, he can enjoy a fixed fraction ($\lambda \leq 1$) of this cash flow. The task is to find an optimal dynamic contract that involves payments to the agent and an early termination possibility,
both conditioned on related cash flows.

The project generates cash flows, which accumulate according to the following positive drifted Brownian motion:

\[ dY_t = \mu dt + \sigma dZ_t \]  

where \( \mu \) is the drift, \( \sigma \) represents the volatility and \( Z \) follows a standard Brownian motion.

Since the principal cannot observe true cash flow, a contract is described by announced cash flows that are received by the principal \((d\hat{Y}_t)\), compensating payments made by the principal to the agent \((dI_t)\), and a termination decision \((\tau)\). The last two are based on the history of cash flow reports \( \{\hat{Y}_t; t \geq 0\} \). By the revelation principle, it’s possible to focus only on contracts that induce the agent to tell the truth, i.e. report the true cash flow. It’s important to notice that the agent always has the option of diverting cash flow till the point of termination and receive his outside option, so the contract must provide incentives to avoid this situation.

The problem is solved by maximizing the project value to the principal subject to the incentive constraint and to the constraint that it must be given an exogenous value to the agent \((W_0)\). This value represents the agent’s “bargaining-power” and leads to the following participation constraint:

\[ W_0 \leq E \left[ \int_0^\tau e^{-\gamma s} dC_s + e^{-\gamma \tau} R \right] \]  

The agent can save or consume his income, but the hypothesis of saved funds growing with rate \( \rho \leq r \) is made, so the authors solve the problem ruling out savings and then they show that the obtained contract remains efficient when saving is possible. Without savings, agent’s consumption is given by:

\[ dC_t = dI_t + \lambda (dY_t - d\hat{Y}_t) \]  

The authors find an optimal contract by maximizing the principal’s value, denoted by \((b_0)\), that is:

\[ b_0 = E \left[ \int_0^\tau e^{-r s} (d\hat{Y}_s - dI_s) + e^{-r \tau} L \right] \]  

subject to participation constraint and incentive constraint, where the latter says that \( \hat{Y} = Y \).
maximizes agent’s value, given by:

\[ W_t(\hat{Y}) = E_t \left[ \int_t^\tau e^{-\gamma(s-t)} dC_s + e^{-\gamma(\tau-t)} R \right] = E_t \left[ \int_t^\tau e^{-\gamma(s-t)} (dI_t + \lambda(dY_t - \hat{Y}_t)) + e^{-\gamma(\tau-t)} R \right] \quad (5) \]

The next step taken relates to the incentive constraint and is to show that there is a sensitivity \( \beta_t(\hat{Y}) \) of the agents promised value toward his report such that:

\[ dW_t = \gamma W_t dt - dI_t + \beta_t(\hat{Y})(\hat{Y}_t - \mu dt) \quad (6) \]

Therefore, we have that if the agent steals \((dY_t - \hat{Y}_t)\), he has an immediate gain of \(\lambda(dY_t - \hat{Y}_t)\), but loses \(\beta_t(dY_t - \hat{Y}_t)\), so now we can write agent’s gain from lying for a while as:

\[ W(\hat{Y}) - W(Y) = E \left[ \int_0^\tau e^{-\gamma t} \lambda(dY_t - \hat{Y}_t) - \int_0^\tau e^{-\gamma t} \beta_t(dY_t - \hat{Y}_t) \right] \quad (7) \]

this equation is non-positive with \(d\hat{Y} = dY\) if \(\beta_t \geq \lambda\), for all \(t\), so if \(\beta_t < \lambda\) for a set of positive measure, then the contract is not incentive compatible.

The next task is to find the best way the principal has to deliver value to the agent, characterizing the principal’s value function. That is, find the highest profit that can be obtained by the principal while giving \(W\) to the agent, this profit level is denoted by \(b(W)\).

Once the principal has the option to provide value to the agent by simply giving him \(dI\), we know that:

\[ b(W) \geq b(W - dI) - dI \quad (8) \]

What gives us \(b'(W) \geq -1\). By the concavity of \(b\), we have that there is a value \(W^1\) that is the lowest value such that \(b'(W) = -1\), and so it’s optimal to pay the agent all that exceeds \(W^1\), that is:

\[ dI = max(W - W^1, 0) \quad (9) \]

After using Itos lemma, we have the following Hamilton-Jacobi-Bellman (HJB) equation for the value function:

\[ rb(W) = \max_{\beta \geq \lambda} \mu + \gamma W b'(W) + \frac{1}{2} \beta^2 \sigma^2 b''(W) \quad (10) \]

We already know that, for incentive reasons, the sensitivity \(\beta_t(\hat{Y})\) of the agents promised value toward his report is higher or equal than the proportion of diverted cash flows he can
enjoy \((\beta_t \geq \lambda)\). Turns out that this constraint binds, so we have \(\beta = \lambda\). The intuition for this result is that having the agent bear risk brings a cost related to the threat of inefficient terminations demanded by the agent that has limited liability. Therefore, the exposure of the agent to volatility is necessary for incentive reasons, but must be the lowest possible, in order to reduce the distortion brought by the agent’s limited liability leading to inefficient terminations.

Now we need the boundary conditions to pin down a solution to this equation and the boundary of transfers \(W^1\). As the principal needs to terminate the contract once the agent’s value falls to \(R\), we gave \(b(R) = L\). We also have the “smooth pasting” condition that the first derivatives must agree at the boundary, thus we have \(b'(W^1) = -1\). The last condition is the “super contact” condition for optimality of \(W^1\), requiring second derivatives being the same at the boundary, that is, \(b''(W^1) = 0\), what is equivalent to \(rb(W^1) + \gamma W^1 = \mu\).

Figure 1: The principal’s value function \(b(W)\). The principal’s value function starts at \((R, L)\), and obeys the differential equation (12) until the point \(W^1\), and then continues with slope -1.

Source: DeMarzo and Sannikov (2006, p.2691)

The authors summarize the their findings till this point by the following proposition:
PROPOSITION: The contract that maximizes the principals profit and delivers the value $W^0 \in [R, W^1]$ to the agent takes the following form: $W_t$ evolves according to:

$$dW_t = \gamma W_t dt - dI_t + \lambda (dY_t - \mu dt) \quad (11)$$

When $W_t \in [R, W^1), dI_t = 0$. When $W_t = W^1$, payments $dI_t$ cause $W_t$ to reflect at $W^1$. If $W^0 > W^1$, an immediate payment $W^0 - W^1$ is made. The contract is terminated at time $\tau$, when $W_t$ reaches $R$. The principals expected payoff at any point is given by a concave function $b(W_t)$, which satisfies

$$rb(W) = \mu + \gamma W b'(W) + \frac{1}{2} \lambda^2 \sigma^2 b''(W) \quad (12)$$

on the interval $[R, W^1]$, $b'(W) = -1$ for $W \geq W^1$, and boundary conditions $b(R) = L$ and $rb(W^1) = \mu - \gamma W^1$ (DeMarzo and Sannikov, 2006, p.2691-2692)

Note that the distance to termination can be represented by the state value $W$ as $W - R$. This represents the capacity the firm has to take low cash flows without liquidating. We denominate this capacity as “financial slack”.

After solving for the optimal contract, the authors describe a way in which it could be implemented. This way involves three securities: inside equity, long-term debt and credit line. Inside equity is defined as an equity that has no rights in the event of termination, thus it would only bring the benefits of dividends. Long-term debt is implemented as a consol bond requiring continuous coupon payments of a fixed amount $x$ and with interest rate $r$, thus it’s face value is $D = \frac{x}{r}$. The credit line involves a limit $C^L$ and an interest rate $r^c = \gamma$ in which amounts already drawn from it ($M$) are charged.

In this solution described by the authors, the project is terminated once a payment of the long-term debt is not made or the credit line is overdrawn. Under this contract, because of the interest rates $r < \gamma$ and $r^c = \gamma$ charged in the long term debt and in the credit line, respectively, the agent endogenously chooses to first direct the totality of cash flows to fully repay the credit line and only then he uses all the remaining cash flows in the payments of dividends and consumes the whole part of dividends concerned to him. In other words, this contract induces the desired behavior of delaying agent’s consumption till a threshold $W^1$ - that in this implementation means no balance drawn on the credit line - of the value the contract brings to him. Note that in this contract firm is terminated when the credit line is overdrawn, and, as we need termination when $W_t = R$, we must have that when the credit line is fully drawn, that is, when $M_t = C^L$. 

13
To induce truth telling, the fraction of inside equity held by the agent, that is, his exposure to cash flows, is \( \lambda \). As the cash flows are first used entirely on payment of the credit line, and considering the boundaries exposed in the last paragraph we have that the equation \( W_t = R + \lambda (C^L - M_t) \) relates agent’s value to the balance on the credit line, and \( C^L = \lambda^{-1}(W^1 - R) \) sets the length of the credit line.

The role of long term debt is to distribute the value between agent and principal so that the equation \( W_t = R + \lambda (C^L - M_t) \) is satisfied. Too high debts would lead to too low agents payoff making it optimal for the agent to draw down the credit line and promote liquidation. On the other hand, too low debts would induce the agent to accumulate cash reserves after the credit line was paid off in order to reduce termination risk. One interesting feature of this contract is that the long-term debt may be actually negative, in other words, there may exist a compensating balance required by the bank in order to issue a long credit line. This balance pays interest to the firm at rate \( r \). This balance cannot be withdrawn by the firm, and, if liquidation occurs, it is seized by creditors.

The authors summarize this implementation with the following proposition:

**PROPOSITION:** Consider a capital structure in which the agent holds inside equity for fraction \( \lambda \) of the firm, the credit line has interest rate \( r^c = \gamma \), and debt satisfies \( rD = \mu - \frac{\gamma R}{\lambda} - \gamma C^L \). Then it is incentive compatible for the agent to refrain from stealing and to use the project cash flows to pay the debt coupons and credit line before issuing dividends. Once the credit line is fully repaid, all excess cash flows are issued as dividends. Under this capital structure, the agents expected future payoff \( W_t \) is determined by the current draw \( M_t \) on the credit line:

\[
W_t = R + \lambda (C^L - M_t)
\]  

This capital structure implements the optimal contract if, in addition, the credit limit satisfies

\[
C^L = \lambda^{-1}(W^1 - R)
\]  

(DeMarzo and Sannikov, 2006, p.2693-2694)

In this implementation we have the following equation for total debt capacity \( D + C^L \):

\[
D + C^L = \frac{\mu}{\gamma} - \frac{R}{\lambda} + \left(1 - \frac{r}{\gamma}\right)D
\]  

This implies that, when \( \gamma \) is close to \( r \), \( D + C^L \approx \frac{\mu}{\gamma} - \frac{R}{\lambda} \), making total debt capacity insensitive to volatility and liquidation value.
Figure 2: The optimal contract with low volatility. For $L = 25$, $R = 0$, $\mu = 10$, $\sigma = 5$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 1$, $K = 30$.

Source: DeMarzo and Sannikov (2006, p.2696)

The authors calculate the comparative statics for the model and show their results in Table 1. In this table, $W^0$ is the agent’s initial value when the lenders are competitive, that is $W^0$ maximizes the agent’s value with $b(W^0) = K$. Conversely, $b(W^*)$ is principal’s value when the agent has no “bargain power”.

Table 1: Comparative Statics for the Optimal Contract

<table>
<thead>
<tr>
<th></th>
<th>$dC_L$</th>
<th>$dD$</th>
<th>$dW_0$</th>
<th>$dW^*$</th>
<th>$db(W^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dL$</td>
<td>$-$</td>
<td></td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$dR^a$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$d\gamma$</td>
<td>$-$</td>
<td>$\pm$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$d\mu$</td>
<td>$+$</td>
<td></td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$d\sigma^2$</td>
<td>$+$</td>
<td></td>
<td>$-$</td>
<td>$\pm$</td>
<td>$-$</td>
</tr>
<tr>
<td>$d\lambda$</td>
<td>$-$ (if $R = 0$)</td>
<td>$+$</td>
<td></td>
<td>$-$</td>
<td>$\pm$</td>
</tr>
</tbody>
</table>

These are for the case when the projects value to investors can exceed $L$, which implies that $b(R) > 0$.

Source: DeMarzo and Sannikov (2006, p.2700)

Some of the intuitions are as following. Credit length decreases with L because liquidation becomes less costly, making financial slack less important. The intuition with the volatility
\( \sigma \) is similar, more uncertain cash flows make high losses more probable, and thus increases the need of financial slack in order to avoid termination, lengthening optimal credit line. If profitability \( \mu \) increases, it becomes more valuable to avoid termination, making it profitable to increase financial slack, therefore increasing the credit length. Also, with higher \( \mu \), it’s possible to extract more cash flow from the agent, so debt also increases. An increase in the agent’s discount factor \( \gamma \) makes delaying in his consumption costlier, reducing credit length. The interaction between \( \gamma \) and debt is more subtle: when the parameter is small, debt increases in it because the agent is able to borrow more through debt given the smaller credit line, whereas for large \( \gamma \), agent’s impatience makes the project less profitable, allowing the agent to borrow less through debt. The intuitions related to \( W^0 \) and \( b(W^*) \) are straightforward, greater \( L \) reduces distortions, and greater \( \mu \) makes the project more profitable, so both increase project’s value, raising \( W^0 \) and \( b(W^*) \). On the other hand, \( \sigma; \lambda \) and \( R \) worsen the agency problem while \( \gamma \) reduces value, so an increase in any of these parameters lowers \( W^0 \) and \( b(W^*) \).

After these comparative statistics, the authors proceed to investigate securities values. One particularly interesting result, tough not surprising given the negative relation between volatility and \( W^0 \), is that when debt is risky, that is, \( L < D + C^L \), the value of equity decreases with an increase in volatility. In other words, equity holders prefer low volatility. This result contradicts the usual presumption that equity holders would like risk, given their limited liability and option to default.

2.2 DeMarzo, Fishman, He, and Wang (2012)

DeMarzo, Fishman, He and Wang (2012), a neoclassical theory of investment with adjustment cost is added to the model of DeMarzo and Sannikov (2006). In their model, firm production is proportional to capital stock and termination allows investors to recover a fixed proportion \( (\ell \geq 0) \) of the value of the capital currently employed in the firm. The agency problem is modeled in the same way as in DeMarzo and Sannikov (2006), and therefore the agent can covertly deviate cash flows for personal use and enjoy a fraction \( \lambda \leq 1 \) of this deviated cash flow. The difference in the contract is that here, besides specifying the compensation of the agent and whether the project is terminated, it also specifies the amount invested.

The equation for accumulation of capital is standard, given by:

\[
dK_t = (I_t - \delta K_t)dt
\]

where \( K \) is capital, \( I \) investment and \( \delta \) the depreciation rate.
There is an adjustment cost $G(I, K)$ assumed homogeneous of degree one in $I$ and $K$, so we can write:

$$I = G(I, K) \equiv c(i)K$$

(17)

where $i = I/K$ and $c(i)$ represents the cost per unit of capital necessary for the firm to grow at rate $i$.

In this model we have that productivity per unit of capital ($A$) evolves just like cash flows in the model of DeMarzo and Sannikov (2006):

$$dA_t = a_t \mu dt + \sigma dZ_t$$

(18)

But now the agent enjoys private benefits with rate $\lambda(1 - a_t)\mu dt$, with $a_t$ being non-observable to the principal. Here, $a_t$ can be directly interpreted as an effort level exercised by the agent or, like in DeMarzo and Sannikov (2006), it can be seen as a fraction of non-deviated cash flows.

In this model production is linear in capital, so we have that cash flows, after possible diversion made by the agent and the cost of investment, are given by:

$$dY_t = K_t(dA_t - c(i_t)dt)$$

(19)

The process to solve the agency model is not very different to the one in DeMarzo and Sannikov (2006) and here it’s still optimal to deal with incentive problems setting the sensitivity of agent’s value to $\lambda$. Now there are two state variables, the agent’s value ($W_t$) and the stock of capital ($K_t$), so we can write principal’s value as $P(W_t, K_t)$, but given the homogeneity of technology we can state that $P(W_t, K_t) = p(w_t)K_t$ and the problem is reduced to one state variable.

After that the authors set $\overline{w}$ as the smallest agent’s value with $p'(w) = -1$, and this is the threshold of agent consumption. When $w_t$ is between agent’s termination value and $\overline{w}$ it evolves according to the following equation:

$$dw_t = (\gamma - (i_t - \delta))w_t dt + \lambda(dA_t - \mu dt)$$

(20)

As in DeMarzo and Sannikov (2006), using Ito’s Lemma, it can be found a differential equation for $p(w)$ when $w \in [0, \overline{w}]$, but now it’s needed to optimize in investment. The equation is:

$$rp(w) = sup_i(\mu - c(i)) + (i - \delta)p(w) + (\gamma - (i - \delta))wp'(w) + \frac{1}{2}\lambda^2\sigma^2p''(w)$$

(21)
Deriving the right side of this equation with respect to $i$ and equalizing to zero we have that investment can be defined implicitly by the following equation:

\[ c'(i(w)) = p(w) - wp'(w) \quad (22) \]

This equation states that the marginal cost of investing equals the principal’s marginal benefit of investment, that is, the per unit of capital value of the firm to him, less the value he loses by the decrease in value of the other units of capital.

After that, the optimal contract is found by solving the differential equation for $w$ determined by (21) and (22), with boundary conditions that are analogous to the ones in DeMarzo and Sannikov (2006) They summarize the contract contract in their version with agent’s termination value set to zero with the following proposition:

PROPOSITION: The investors’ value function $P(K,W)$ is proportional to capital stock $K$, in that $P(K,W) = p(w)K$, where $p(w)$ is the investors scaled value function. For $w \in [0, w]$, $p(w)$ is strictly concave and uniquely solves the ODE

\[ rp(w) = \sup_i (\mu - c(i)) + (i - \delta)p(w) + (\gamma - (i - \delta))wp'(w) + \frac{1}{2}\lambda^2 \sigma^2 p''(w) \quad (23) \]

with boundary conditions $p(0) = \ell$, $p'(\overline{w}) = -1$, and $p''(\overline{w}) = 0$. For $w > \overline{w}$, $p(w) = p(\overline{w})(w/\overline{w})$. The agents scaled continuation payoff $w$ evolves according to,

\[ dw_t = (\gamma - (i_t - \delta))w_t dt + \lambda(dA_t - \mu dt) \quad (24) \]

for $w_t \in [0, w]$. Cash payments $du_t = dU_t/Kt$ reflect $w_t$ back to $w$, and the contract is terminated at the first time $\tau$ such that $w_\tau = 0$. Optimal investment is given by $I_t = i(w_t)K_t$, where $i(w)$ is defined in:

\[ c'(i(w)) = p(w) - wp'(w) \quad (25) \]

(DeMarzo, Fishman, He and Wang, 2012, p.2307)
Figure 3: *Investors scaled value function* $p(w)$ *as a function of the agent’s scaled continuation payoff* $w$. We illustrate scenarios in which the liquidation value is high ($\ell_1$) and low ($\ell_0$).

Source: DeMarzo, Fishman, He and Wang (2012, p.2308)

The optimal contract is pretty similar to the one in DeMarzo and Sannikov (2006), the only novelties here are that now variables are usually scaled by capital, and that an investment decision needs to be taken. However, some aspects of the contract are worth citing here. First, the agent’s exposure to variations in profits is still $\lambda$, what can be interpreted as him having as inside equity a $\lambda$ fraction of total equity. Second, the agent’s value of continuing the project per unit of capital ($\omega_t$) grows with high realized profits per unit of capital, till a limiting point in which all the extra profit starts being paid as dividends. In the opposite situation, in which profits are low, agent’s continuation value drops till the point it reaches his outside option, when the project is terminated.

In a contract in the terms of the one constructed in DeMarzo and Sannikov (2006) the amount of short-run losses needed to cover the difference between thresholds of dividend payment and termination can be seen as the total extension of the credit line, while financial slack, the largest amount of short-run losses that the firm can sustain without termination can be seen as non-used credit line. Obviously, the amount of short-run losses needed to cover the difference between dividends threshold and current agents value can be seen as used credit line. The most important to our purposes here is that it’s shown that financial slack is given by the agents continuation value scaled by capital divided by his exposure to
firm performance $\omega_t/\lambda$ (its considered here that the agents outside option has 0 value), and thus it’s proportional to $\omega_t$.

Some predictions concerning agent compensation in these models are as follows. After high realized profits, the firm reaches a threshold where payments to the agent begin to be made, after this point is reached these payments come proportionally to the difference current cash flow minus mean cash flow. According to the proposed specific implementation this threshold is reached when the balance in the credit line is zero, and, once total debt is composed by a consol bond and the drawn amount in the credit line, this happens when total debt reaches it’s minimum. This conclusion of reaching the threshold once debt is minimum could also be made in a more general thinking. As only after this point the agent receives part of the cash flow, while agent is not receiving any payments all cash flow is being directed to pay the creditors. Therefore, while the threshold is not reached, the agent needs to pay down some debt and only after that he can receive payments.

Also, the models predict that the value of cash flow is decreasing in the financial strength of the firm. We have that total value of the project is given by $b(W) + W ((p(w) + w)K)$, and that, while the response of $W$ to cash flow is constant and equal to $\lambda$, the function $b$ ($p$) is concave, therefore the sensitivity of total value (before paying the agent) to cash is decreasing in the state variable. As the state value dynamics are guided by realized profits, it is predicted that low past cash flows increase the sensitivity of total value to current cash flow. In the same way of thinking as in the last paragraph, we can associate higher cash flow sensitivity of total project value to more used credit line (in the specific implementation) or higher total debt.

Now let’s take a closer look at investment and $q$. If there were no agency problem, that is, if we had $\lambda = 0$ or $\sigma = 0$, the investment problem would be the same as in Hayashi (1982), hence we would have average $q$ ($q_a$) equal to marginal $q$ ($q_m$). However for the model with agency problem, this equality no longer holds. We have:

$$q_a(\omega) = \frac{P(W, K) + W}{K} = p(\omega) + \omega \quad (26)$$

$$q_m(\omega) = \frac{\partial(P(K, W) + W)}{\partial K} = c'(i(w)) = p(\omega) - \omega p'(\omega) \quad (27)$$

We can see that $q_a \geq q_m$ and that the difference between them is non-monotonic. This happens, because fixing $W$ investment lowers agent’s scaled value, inducing a worse agency problem, therefore having an effect that reduces total firm value ($P(K, W) + W$).

Combining both equations, we have $q_a(\omega) - q_m(\omega) = \omega(1 + p'(\omega))$. In the termination point $\omega = 0$ and both are equal. When $\omega$ reaches dividends threshold ($\omega$), $p'(\omega) = -1$, and they are the same once again. We have that for intermediate values $\omega > 0$ and $p'(\omega) > -1$,
thus $q_a > q_m$.

Figure 4: *Average $q_a$ and marginal $q_m$.* The left panel shows a geometrical illustration of the determination of $q_a$ and $q_m$. The right panel plots $q_a$ and $q_m$ with the first-best $q^{FB}$.

Source: DeMarzo, Fishman, He and Wang (2012, p.2310)

Given the non-monotonicity of the difference between average and marginal $q$, it’s problematic to empirically verify all the predicted behavior of investment. However, the model says that firms with high agent’s value present marginal $q$ equal to average $q$, hence present investments that, once cash flow is high enough to maintain the firm in $\overline{w}$, are unresponsive to cash flow. This can be tested by verifying if firms that are predicted to be and stay in the payment threshold present investment that, after controlling for average $q$, do not respond to cash flow.
3 Empirical Tests

The tests are divided into three groups: the ones concerning payments to the agent; choices over debt and credit; and to investment. Different formulations are analyzed, some considering the specific solution proposed in the theoretical papers, others using a broader view.

Most part of the data comes from Compustat, the credit line data, the rating data and the executive remuneration data are from S&P’s Capital IQ. The sample consists of public companies and, as usual in the related literature, it disregards observations from financial institutions, utilities, not-for profit organizations and governmental enterprises. The data covers years from 2001 (first year of available credit line data in Capital IQ) to 2016 and covers 3084 firms. Net Plant Property and Equipment is used as the capital variable, the variable for executive compensation consists of total compensation (cash and non-cash) paid in the year for the executive that received the highest amount. Summary statistics are displayed in the following tables.

Table 2: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Capitalization</td>
<td>31659</td>
<td>1.620</td>
<td>1.566</td>
</tr>
<tr>
<td>Total Equity</td>
<td>35231</td>
<td>0.428</td>
<td>0.581</td>
</tr>
<tr>
<td>Total Debt</td>
<td>35362</td>
<td>0.337</td>
<td>4.102</td>
</tr>
<tr>
<td>Used Credit</td>
<td>35028</td>
<td>0.118</td>
<td>7.016</td>
</tr>
<tr>
<td>Unused Credit</td>
<td>35028</td>
<td>0.030</td>
<td>0.923</td>
</tr>
<tr>
<td>Credit Line Length</td>
<td>35028</td>
<td>0.148</td>
<td>7.166</td>
</tr>
<tr>
<td>Inventory</td>
<td>35362</td>
<td>0.129</td>
<td>0.15</td>
</tr>
<tr>
<td>Receivables</td>
<td>35362</td>
<td>0.148</td>
<td>0.125</td>
</tr>
<tr>
<td>Cash</td>
<td>35362</td>
<td>0.168</td>
<td>0.189</td>
</tr>
<tr>
<td>Operating Income</td>
<td>34665</td>
<td>0.014</td>
<td>2.356</td>
</tr>
<tr>
<td>Capital</td>
<td>35362</td>
<td>0.198</td>
<td>0.19</td>
</tr>
<tr>
<td>Investment</td>
<td>31858</td>
<td>0.005</td>
<td>0.077</td>
</tr>
<tr>
<td>Dividends</td>
<td>35347</td>
<td>0.016</td>
<td>0.355</td>
</tr>
<tr>
<td>Executive Compensation</td>
<td>19929</td>
<td>0.063</td>
<td>4.338</td>
</tr>
</tbody>
</table>
Table 3: Summary Statistics - firms with credit lines

For this table, only firms that actually have a credit line are considered. Credit line data come from S&P’S Capital IQ, total assets data come from Compustat. The sample consists of public companies and disregards observations from financial institutions, utilities, not-for profit organizations and governmental enterprises. The data covers years from 2001 (first year of available credit line data in Capital IQ) to 2016 and covers 3084 firms. In this table, all variables are scaled by total assets.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Credit</td>
<td>14760</td>
<td>0.211</td>
<td>10.465</td>
</tr>
<tr>
<td>Unused Credit</td>
<td>14760</td>
<td>0.061</td>
<td>0.775</td>
</tr>
<tr>
<td>Credit Line Length</td>
<td>14760</td>
<td>0.272</td>
<td>10.492</td>
</tr>
</tbody>
</table>

### 3.1 Payments to the agent

The specific implementation suggested in DeMarzo and Sannikov (2006) predicts that the payments are made through dividends once credit line is fully paid. On the other extreme, being most lenient, we can predict that agent receives payments when things go well. The proposed tests will go from one edge to the other.

We begin with tests that contain a dummy variable that indicates whether the firm is supposed to be in a state that involves payments made to the agent. As these dummies rely on ad hoc thresholds, for robustness, we run regressions with two different thresholds for each of these dummies, the alternative thresholds are displayed in parentheses. First, the most strict test, is a conditional fixed-effects logit regression explaining payment of dividends by the balance in the credit line being close to zero. This test is run with two specifications. The first explains just by the credit line balance being close to zero, to control for this, a dummy variable indicating drawn amounts being less or equal to 20%(10%) of the credit line length is used. The second specification, besides the dummy variable, also uses current operating profit scaled by capital. As exposed in the review of the theoretical model, we expect zero balance drawn in the credit line and high operating profits to increase the likelihood of dividend payment.

A less strict way to test for this is to run the same tests but replacing the low balance drawn from the credit line variable by low debt in relation to the sample mean (within-firm). Debt is scaled by total assets, and the variable that accounts for low debt is a dummy that has value 1 if debt is less or equal to the 40\textsuperscript{th} percentile (30\textsuperscript{th} percentile) of the sample mean. The rationale behind the prediction of payment made to the agent when total debt is low is explained in the previous chapter.

It is predicted that credit line is paid down or debt is low when past profits were high, so in an alternative formulation high past profits is the explanatory dummy variable that
replaces low debt or low drawn on the credit line. Past profits are considered high if the last four years mean operating profit scaled by capital is higher than the 60th percentile (70th percentile) of the sample mean (within-firm) of operating profits.

All these regressions are run with year fixed-effects and are controlled for firm age. Standard errors are clustered at firm-level.

Table 4: Results - Payment of dividends

Regressions are run with conditional fixed-effects logit model, columns differ from each other by the right hand variables in the regression. Low Drawn Credit is a dummy variable indicating whether drawn amounts are less or equal to 20% of the credit line length. Low Debt is a dummy that has value 1 if debt (scaled by total assets) is less or equal to the 40th percentile of the sample mean. High Past Profits is a dummy with value 1 if the last four years mean operating profit scaled by capital is higher than the 60th percentile of the sample mean (within-firm) of operating profits. The variable Operating income is operating profit scaled by capital. All regressions are run with year fixed-effects. Standard errors are clustered at firm-level. Significance levels 1%, 5% and 10% are represented by ***, **, *, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Drawn Credit</td>
<td>-0.05358</td>
<td>-0.05955</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.40)</td>
<td>(-0.44)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Debt</td>
<td></td>
<td></td>
<td>0.33002***</td>
<td>0.31784***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.46)</td>
<td>(4.26)</td>
<td></td>
</tr>
<tr>
<td>High Past Profit</td>
<td></td>
<td></td>
<td></td>
<td>0.87213***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(9.97)</td>
<td></td>
</tr>
<tr>
<td>Operating Income</td>
<td>0.00637</td>
<td></td>
<td>0.00446</td>
<td>0.00236</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td></td>
<td>(1.25)</td>
<td>(1.26)</td>
<td></td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.02897</td>
<td>0.02957</td>
<td>0.11143***</td>
<td>0.10909***</td>
<td>0.14511***</td>
</tr>
<tr>
<td></td>
<td>(0.69)</td>
<td>(0.70)</td>
<td>(3.68)</td>
<td>(3.56)</td>
<td>(4.39)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>4804</td>
<td>4788</td>
<td>13099</td>
<td>12802</td>
<td>12261</td>
</tr>
<tr>
<td>Number of firms</td>
<td>573</td>
<td>570</td>
<td>1006</td>
<td>990</td>
<td>943</td>
</tr>
<tr>
<td>Pseudo R-sqr</td>
<td>0.102</td>
<td>0.103</td>
<td>0.099</td>
<td>0.103</td>
<td>0.133</td>
</tr>
</tbody>
</table>
Table 5: Results - Payment of dividends, alternative dummies

Regressions are run with conditional fixed-effects logit model, columns differ from each other by the right hand variables in the regression. Low Drawn Credit is a dummy variable indicating whether drawn amounts are less or equal to 10% of the credit line length. Low Debt is a dummy that has value 1 if debt (scaled by total assets) is less or equal to the 30\textsuperscript{th} percentile of the sample mean. High Past Profits is a dummy with value 1 if the last four years mean operating profit scaled by capital is higher than the 70\textsuperscript{th} percentile of the sample mean (within-firm) of operating profits. The variable Operating income is operating profit scaled by capital. All regressions are run with year fixed-effects. Standard errors are clustered at firm-level. Significance levels 1%, 5% and 10% are represented by ***, ***, * respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Drawn Credit</td>
<td>0.33761***</td>
<td>0.33833***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.35)</td>
<td>(3.34)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Debt</td>
<td></td>
<td></td>
<td>0.33313***</td>
<td>0.32233***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.47)</td>
<td>(4.28)</td>
<td></td>
</tr>
<tr>
<td>High Past Profits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.75424***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(7.94)</td>
</tr>
<tr>
<td>Operating Income</td>
<td>0.00462</td>
<td></td>
<td>0.00449</td>
<td>0.00282</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.25)</td>
<td></td>
<td>(1.25)</td>
<td>(1.15)</td>
<td></td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.10697***</td>
<td>0.11011***</td>
<td>0.11245***</td>
<td>0.10999***</td>
<td>0.14472***</td>
</tr>
<tr>
<td></td>
<td>(3.48)</td>
<td>(3.57)</td>
<td>(3.72)</td>
<td>(3.59)</td>
<td>(4.41)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>12943</td>
<td>12780</td>
<td>13099</td>
<td>12802</td>
<td>12261</td>
</tr>
<tr>
<td>Number of firms</td>
<td>1001</td>
<td>990</td>
<td>1006</td>
<td>990</td>
<td>943</td>
</tr>
<tr>
<td>Pseudo R-sqr</td>
<td>0.098</td>
<td>0.101</td>
<td>0.099</td>
<td>0.103</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Table 4 and Table 5 present mixed results. In both, coefficients for the dummies of low debt and high past profits present the expected sign and are highly significant, indicating that firms in fact pay dividends when the model would predict them to pay. Also these coefficients present similar magnitude in both tables, indicating robustness. However, the coefficient for the low drawn debt dummy is only significant and with the expected sign in Table 5. On one hand this goes in favor to the model, in the sense that it’s predicted that firms pay dividends when the drawn amount in the credit line is zero, and Table 5 used a lower threshold for the dummy; on the other hand this could indicate that the results are not robust. Also, looking at the columns that refer to this variable we note, by the number of firms in the regressions, that more firms passed from using more than 10% of the credit line to less than 10% than passed from using more than 20% of the credit line to less than 20%, this should be explained by the zero lower bound. In both tables, the coefficients for operating income always present the expected positive sign but they are never statistically significant.

Another way to make these tests less dependent on the proposed implementation is to
replace dividend payment by payments made directly to the agent (executive compensation disclosed in financial statements). For now we look at the full sample, but the model predicts payments made only by firms that are at the dividend threshold. To make this test, it’s proposed a fixed-effects model with agent compensation as the left hand variable, past profits and current profits as right hand variables. Alternatively, the regression is run with a dummy for low debt or low balance drawn in the credit line \(^4\) in place of the past profits variable.

Table 6: Results - Compensation

Regressions are run with the fixed-effects model. Low Drawn Credit is a dummy variable indicating whether drawn amounts are less or equal to 20\% of the credit line length. Low Debt is a dummy that has value 1 if debt (scaled by total assets) is less or equal to the 40\textsuperscript{th} percentile of the sample mean. Past Profits is the last four years mean of operating profit scaled by capital. Operating Income is the operating income scaled by capital. All regressions are run with year fixed-effects. Standard errors are clustered at firm-level. Significance levels 1\%, 5\% and 10\% are represented by ***, ***, *, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th>Credit and profit</th>
<th>Debt and profit</th>
<th>Past and current profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Drawn Credit</td>
<td>0.42114</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.20)</td>
<td></td>
</tr>
<tr>
<td>Low Debt</td>
<td>0.25621</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.08)</td>
<td></td>
</tr>
<tr>
<td>Past Profits</td>
<td>0.01867**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.87)</td>
<td></td>
</tr>
<tr>
<td>Operating Income</td>
<td>-0.00346</td>
<td>-0.01800</td>
</tr>
<tr>
<td></td>
<td>(-0.08)</td>
<td>(-1.27)</td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.08878</td>
<td>0.12570</td>
</tr>
<tr>
<td></td>
<td>(0.72)</td>
<td>(1.31)</td>
</tr>
<tr>
<td>Number of observations &amp; 8092 &amp; 19715 &amp; 19155</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of firms &amp; 1989 &amp; 2889 &amp; 2826</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-sqr &amp; 0.003 &amp; 0.047 &amp; 0.097</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results in Table 6 present coefficients for the dummies related to the state of the firm that, as expected, are of positive sign, but they are not statistically significant, thus failing to provide evidence that firms give higher payments to executives when they present low debt or low drawn amount in the credit line. The coefficient concerning to past profits

\(^4\) The criteria for low value for these variables were the same as in the previous regressions. The results for credit line drawn amount being less or equal to 20\% of the credit line length and debt scaled by total assets being less or equal to the 40\textsuperscript{th} percentile of the firm sample are displayed in Table 6. Regressions considering 10\% and the 30\textsuperscript{th} percentile for credit line drawn amounts and debt in firm sample, respectively, were also run, with similar results that were omitted for brevity.
is statistically significant (at levels 10%, 5% and 1%), and, as expected positive, indicating that higher past profits bring higher compensation. The coefficients related to current cash flow, tough not statistically significant, present unexpected negative signs.

The next tests aim to verify if agent compensation, after the firm reaches it’s payment threshold, responds to operating income as predicted in the model. For these tests, the sample is restricted to firm-years that had positive dividend payments and are preceded by an year of also positive dividend payment, this is done in order to restrict the sample to firms that were already in the dividend payment threshold and are still in this situation.

\[
\text{Dividend}_{it} = \xi_1 \text{OperatingIncome}_{it} + \xi_2 \text{Markettobook}_{it} + \xi_3 \text{Age} + \Gamma \text{Year}_t + \epsilon_{it} \quad (28)
\]

\[
\text{Compensation}_{it} = \xi_1 \text{OperatingIncome}_{it} + \xi_2 \text{Markettobook}_{it} + \xi_3 \text{Age} + \Gamma \text{Year}_t + \epsilon_{it} \quad (29)
\]

Compensation and operating profits are scaled by capital, regressions are run with the fixed-effects model, they are controlled for firm-age and year fixed-effects are added.

Table 7: Results - Payments in payment threshold

Regressions are run with the fixed-effects model. The sample is restricted for firm-years with positive dividend payment and preceded by an year with also positive dividend payment. The column names refer to their left hand variables. Operating Income is the operating income scaled by capital. Market-to-book is defined as total assets less book value of equity plus market value of equity, all divided by total assets. All regressions are run with year fixed-effects. Standard errors are clustered at firm-level. Significance levels 1%,5% and 10% are represented by ***,**,*, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Dividends</th>
<th>Dividends</th>
<th>Compensation</th>
<th>Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Income</td>
<td>0.15182***</td>
<td>0.15190***</td>
<td>0.00369***</td>
<td>0.00368***</td>
</tr>
<tr>
<td></td>
<td>(601.96)</td>
<td>(750.83)</td>
<td>(62.37)</td>
<td>(61.15)</td>
</tr>
<tr>
<td>Market-to-book</td>
<td>0.85887</td>
<td>-0.06994</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.24)</td>
<td>(-0.83)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.05122</td>
<td>0.02925</td>
<td>-0.00404</td>
<td>-0.00266</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(0.78)</td>
<td>(-0.33)</td>
<td>(-0.20)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>9644</td>
<td>9271</td>
<td>5571</td>
<td>5416</td>
</tr>
<tr>
<td>Number of firms</td>
<td>1139</td>
<td>1068</td>
<td>992</td>
<td>933</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.74420</td>
<td>0.74575</td>
<td>0.31157</td>
<td>0.32302</td>
</tr>
</tbody>
</table>

As strong evidence is found of positive relation to agent remuneration through direct compensation or through dividend payment when the firm is paying dividends, results in Table 7 are in favor of the model prediction related to agent compensation after the payment threshold is reached. These results are more meaningful than the ones in Table 6, because
the model predicts that payments to the agent are made and respond to cash flow only since the payment threshold is reached.

### 3.2 Debt and Credit

This subsection focuses on predictions related to debt and consists on empirical tests that rely on the specific implementation presented on the theoretical papers. As regressions here depend directly on the credit line, we restrict our data to observations where drawn credit is less or equal to total debt. This restriction is imposed because, as amounts drawn on the credit line are part of the debt, drawn credit higher than total debt indicates poor data. Also, the data is restricted to firms that actually have a credit line, so we discard observations with zero values both on drawn and undrawn credit. The two first tests focus on parameters that are supposed constant (liquidation value of assets, volatility of cash flows and mean cash flow) in the theoretical models that we are analyzing here, so, for these regressions, we take means across years and work with cross-sectional data.

In the specific implementation suggested in DeMarzo and Sannikov (2006), total debt capacity, when $\gamma$ is close to $r$ is predicted to be insensitive to volatility and liquidation value. To test for this, we propose:

\[
\text{UnusedCredit}_i + D_i = \xi_1 \text{Tangibility}_i + \xi_2 \text{Volatility}_i + \xi_3 \mu_i + \Gamma \text{Industry}_i + \epsilon_i \quad (30)
\]

\(\epsilon\) represents the firm, the regression is run with industry fixed-effects represented by the variable Industry in the equation, that considers two digit SIC codes (three digit SIC codes would lead to few observations per industry). The dependent variable in total debt reported in the financial statements plus the part of the credit line that is not being used, all divided by total assets. Variables represent the firm means across years.

Tangibility is used to address liquidation value. As more tangible assets should be easier to reallocate, it is expected that higher tangibility of assets would imply higher termination value. The measure used here is the one constructed in Berger, Ofek and Swary (1996) and used in Almeida and Campello (2007), given by:

\[
\text{Tangibility}_i = \text{Cash}_i + 0.715 \cdot \text{Receivables}_i + 0.547 \cdot \text{Inventory}_i + 0.535 \cdot \text{Capital}_i \quad (31)
\]

as in Almeida and Campello (2007), this variable is scaled by total assets.

Similarly to Sufi (2009) the measure for volatility is the sample standard deviation of the first difference of operating profit scaled by capital. Two measures related to $\mu$ are proposed.
The first is the market-to-book ratio, defined as total assets less book value of equity plus market value of equity, all divided by total assets. This measure is chosen because the market value of equity is the expected present value of dividends, or the value of future cash flows after liabilities are paid. The second relies more directly on the model and consists on the sample mean of operating profits, scaled by capital.

The model does not make clear what determines the discount factor, \( \gamma \), and why it is higher than the interest rate, \( r \), so the proposed interpretation is that \( \gamma \) reflects the cost of capital to the firm. Following this, the sample is restricted to investment grade firms, that are supposed to have low cost of credit.

Table 8: Results - Total debt capacity

Variables represent firm means across years, observations with drawn credit higher than total debt or without a credit line are discarded. We further restrict the data to firms with investment grade credit rating. The dependent variable is unused credit plus total debt, all divided by total assets. \( \text{Tangibility}_i = \text{Cash}_i + 0.715 \cdot \text{Receivables}_i + 0.547 \cdot \text{Inventory}_i + 0.535 \cdot \text{Capital}_i \). Volatility is the sample standard deviation of the first difference of operating profit scaled by capital. Market-to-book is defined as total assets less book value of equity plus market value of equity, all divided by total assets. Mean profits is the sample mean of operating profits, scaled by capital. Regressions are run with industry fixed-effects (two digit SIC). Standard errors are robust to heteroskedasticity. Significance levels 1%, 5% and 10% are represented by ***, ***, * respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Mean profits</th>
<th>Market-to-book</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tangibility</strong></td>
<td>-0.23021*</td>
<td>-0.22805*</td>
</tr>
<tr>
<td></td>
<td>(-1.82)</td>
<td>(-1.79)</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>-0.04771</td>
<td>-0.01546</td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td>(-0.37)</td>
</tr>
<tr>
<td><strong>Mean profits</strong></td>
<td>0.00168</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td><strong>Market-to-book</strong></td>
<td></td>
<td>-0.02342*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.73)</td>
</tr>
<tr>
<td><strong>Number of observations (firms)</strong></td>
<td>210</td>
<td>208</td>
</tr>
<tr>
<td><strong>R-sqr</strong></td>
<td>0.334</td>
<td>0.367</td>
</tr>
</tbody>
</table>

The regressions summarized in Table 8 present coefficients for tangibility that are significant only at 10% level and coefficients for volatility that are not statistically significant. These results do not show evidence against the model, since it predicts that firms with low rate of discount for time have debt capacities that are relatively insensitive to liquidation value or cash flow volatility. The numbers of observations are much lower than the ones in the past regressions because now there’s only one observation per firm and firms are required to have their credit rated with an investment grade by Standard & Poor’s.
The next goal here is to test whether firms choose between debt and credit lines in a manner that is consistent with what is predicted in DeMarzo and Sannikov (2006). To this purpose, the regressions will rely on a ratio of credit line length to debt as dependent variable, and a set of other variables as explanatory variables.

As seen before, the model predicts a more extensive use of credit related to debt in cases of higher risks emerging from the threat of losses due to inefficient terminations. Naturally, this threat of loss is guided by the probability of default and the loss given default, in the model, the first one derives from volatility of cash flow and agent’s termination value (non-observable); the second is given by liquidation value. The relation induced by profitability ($\mu$) is more subtle, since a higher $\mu$ increases both credit line length and debt. The intuition related to credit line raises no concerns, and it is that a more profitable project should be more protected from termination. However, the reason why higher profitability raises debt is that it makes it possible to extract more cash from the agent, and it relies on the hypothesis that the value that the contract must give to the agent is exogenous, therefore independent from the value of the project.

Another parameter that guides debt and credit is the rate in which the agent discounts time. Credit line is predicted to be decreasing in this rate, because it delays consumption to the agent. The effect on debt is the opposite for low values of $\gamma$ due to replacement of credit line for debt. As the agent’s discount rate is not observable, the strategy to control for this variable is to add dummy variables for the firm credit rate.

All that said, we have the following equation to be estimated:

$$\frac{CL_i}{D_i} = \xi_1Tangibility_i + \xi_2Volatility_i + \xi_4\mu_i + \Gamma_1CreditRating_i + \Gamma_2Industry_i + \epsilon_i \quad (32)$$

where $i$ refers to the firm and the variables (except the dummies for rate) represent firm means across years. The dependent variable is the length of the credit line (used credit plus unused credit) divided by total debt. Industry fixed-effects are used (2 digit SIC Code). Standard errors are robust to heterokedasticity.
Variables represent firm means across years, observations with drawn credit higher than total debt or without a credit line are discarded. The dependent variable is the length of the credit line (used credit plus unused credit) divided by total debt. 

\[ Tangibility_i = Cash_i + 0.715 \cdot Receivables_i + 0.547 \cdot Inventory_i + 0.535 \cdot Capital_i. \]

Volatility is the sample standard deviation of the first difference of operating profit scaled by capital. Market-to-book is defined as total assets less book value of equity plus market value of equity, all divided by total assets. Mean profits is the sample mean of operating profits, scaled by capital. Regressions are run with industry fixed-effects (two digit SIC) and dummies for the S&P’s credit rates. Standard errors are robust to heteroskedasticity. Significance levels 1%, 5% and 10% are represented by ***, ***, *, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th>Mean profits</th>
<th>Market-to-book</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangibility</td>
<td>9.61431*</td>
</tr>
<tr>
<td></td>
<td>(1.87)</td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.04485</td>
</tr>
<tr>
<td></td>
<td>(-1.37)</td>
</tr>
<tr>
<td>Mean profits</td>
<td>0.05831*</td>
</tr>
<tr>
<td></td>
<td>(1.93)</td>
</tr>
<tr>
<td>Market-to-book</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of observations (firms)</td>
<td>539</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.091</td>
</tr>
</tbody>
</table>

The results displayed in Table 9 are not consistent with the model. The coefficients for tangibility and volatility are of the opposite sign in relation to what was expected, with the coefficients for tangibility being significant at 10% level. This indicates that the specific solution proposed to the model does not accurately describe the driving forces for the usage of credit line and debt.

The last test explores the dynamics of debt structure. The suggested implementation predicts a fixed amount of debt and a fixed length of the credit line. As it’s predicted that amounts are drawn from the credit line when performance is poor and the credit line is paid-down when profits are high, we expect the ratio drawn credit to total debt to be decreasing in realized profits. To test for this, observations preceded by a period with zero drawn amount in the credit line are discarded and the following regression is used:

\[
\Delta Drawntodebt_{it} = \xi_1 RelativeOperatingProfit_{it} + \xi_2 DeltaMarkettoBook_{it} + \xi_3 Age_{it} + \epsilon_{it}
\]  

(33)

Where \( \Delta Drawntodebt_{it} = (DrawnCredit_{it}/Debt_{it}) - (DrawnCredit_{it-1}/Debt_{it-1}) \), and
$\Delta Marketto\text{book}_{it} = Marketto\text{book}_{it} - Marketto\text{book}_{it-1}$. Operating profits represent the operating profit divided by capital less the firm mean across years of operating profit divided by capital. There is a control for firm-age, year fixed-effects are used and standard errors are clustered at firm-level.

Table 10: Results - Dynamics of credit line

Observations with drawn credit higher than total debt, observations preceded by a period with zero drawn amount in the credit line, and observations without a credit line are discarded. The dependent variable is $\Delta Drawnto\text{debt}_{it} = (DrawnCredit_{it}/Debt_{it}) - (DrawnCredit_{it-1}/Debt_{it-1})$. Relative Operating Income represent the operating profit divided by capital less the firm mean across years of operating profit divided by capital. $\Delta Marketto\text{book}_{it} = Marketto\text{book}_{it} - Marketto\text{book}_{it-1}$, where Market-to-book is defined as total assets less book value of equity plus market value of equity, all divided by total assets. Year fixed-effects are used. Standard errors are clustered at firm-level. Significance levels 1%, 5% and 10% are represented by ***, **, *, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Operating Income</td>
<td>-0.00024</td>
<td>0.00001</td>
</tr>
<tr>
<td></td>
<td>(-1.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\Delta$ Market-to-book</td>
<td></td>
<td>0.00116</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.31)</td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.00029</td>
<td>0.00029</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>6797</td>
<td>6246</td>
</tr>
<tr>
<td>Number of firms</td>
<td>1402</td>
<td>1265</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.009</td>
<td>0.011</td>
</tr>
</tbody>
</table>

The results summarized in Table 10 also do not provide evidence in favor to the model predictions related to debt structure. The coefficient relating the cash flow to the ratio drawn balances in the credit line to total debt are of the expected sign, but not statistically significant. This fails to provide evidence in favor of the predicted dynamic of a debt that is unresponsive to realized cash flow and a credit line that is drawn down when earnings are low and paid-down when earnings are high.

3.3 Investment

The model predicts that firms with high agent’s value present marginal $q$ equal to average $q$, hence once cash flow is high enough to maintain the firm in $w$, investments are unresponsive to cash flow. This can be tested by verifying if firms that are predicted to be and stay in the
payment threshold present investment that, after controlling for average \( q \), does not respond to cash flow.

The procedure is to restrict the sample to firms likely to be in the threshold and see if the regression present the coefficient related to cash flow not statistically different from zero.

\[
Investment_{it} = \xi_1 Q_{it-1} + \xi_2 OperatingProfit_{it-1} + \xi_3 Age_{it} + \Gamma_1 Industry_i + \Gamma_2 Year_t + \epsilon_{it} \tag{34}
\]

Investment is scaled by capital, the variable for operating profit consists on operating profit divided by capital less the sample mean (within-firm) for operating profit scaled by capital. \( Q \) is given by total assets less book value of equity plus market value of equity, all divided by total assets. Because of our sample selection of firms supposed to have marginal \( q \) equal to average \( q \) we have few observations for each firm (many firms with only one observation), so firm fixed-effects are not used and, instead, regressions are run with industry fixed-effects (three digit SIC Code). Time fixed-effects are also added and regressions are controlled for firm age. Standard errors are clustered at firm-level.

Four alternative criteria for selecting the firms in the sample in a given year are proposed. Firms with currently low debt to total asset ratio relative to the sample of this variable for this firm across years (under percentile 20); firms with low balance in the credit line relative to the length of the credit line (less than 20%); firms with high profits (scaled by capital) for the last periods relative to the firm sample (above percentile 70). Also, in all these three criteria, it’s required that the firm present current operating profit scaled by capital higher than its sample percentile 60. The last one requires only high dividend payment for the year (scaled by common equity, above percentile 70 within-firm).

The implementation that relies on credit line and the one that looks at payment of dividends rely on more strict views of the model, and consider the specific proposed solution. This happens because we need the specific solution to associate the payment threshold to zero balance on the credit line or payments of dividends. The ones that consider the level of total debt and high realized profits do not rely on the specific solution and are expected to work with other model solutions.

Also, following Almeida and Campello (2007) there is a data selection consisting on eliminating firm-years for which the value of capital stock is less than $5 million, those displaying asset growth exceeding 100\%, and those with negative \( Q \) or with \( Q \) in excess of 10.
Table 11: Results - Investment

In each column a different criterion is used in order to restrict the sample to firm-years supposed to have average $q$ equal to marginal $q$, these criteria are explained in the text. The left hand variable is investment scaled by capital. Operating profit consists on operating profit divided by capital less the sample mean (within-firm) for operating profit scaled by capital. Q is given by total assets less book value of equity plus market value of equity, all divided by total assets. The explicative variables $Q$ and Operating Profit are lagged. Industry fixed-effects (three digit SIC code) and year fixed-effects are used. Standard errors are clustered at firm-level. Significance levels 1%, 5% and 10% are represented by ***, ***, *, respectively; t-statistics are displayed in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Low Drawn Credit</th>
<th>Low Debt</th>
<th>High Past Profits</th>
<th>High Dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{t-1}$</td>
<td>0.03753***</td>
<td>0.05682***</td>
<td>0.06619***</td>
<td>0.04567***</td>
</tr>
<tr>
<td>(2.32)</td>
<td>(5.64)</td>
<td>(7.83)</td>
<td></td>
<td>(4.21)</td>
</tr>
<tr>
<td>Operating Profit$_{t-1}$</td>
<td>0.00150</td>
<td>0.00011</td>
<td>0.00142</td>
<td>-0.00012</td>
</tr>
<tr>
<td>(1.00)</td>
<td>(0.14)</td>
<td>(0.75)</td>
<td></td>
<td>(-0.01)</td>
</tr>
<tr>
<td>Firm Age</td>
<td>-0.01692***</td>
<td>-0.02082***</td>
<td>-0.01019***</td>
<td>-0.00691</td>
</tr>
<tr>
<td>(-3.02)</td>
<td>(-3.24)</td>
<td>(-2.78)</td>
<td></td>
<td>(-0.47)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>5744</td>
<td>1016</td>
<td>2182</td>
<td>3576</td>
</tr>
<tr>
<td>Number of firms</td>
<td>1707</td>
<td>742</td>
<td>1268</td>
<td>988</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.035</td>
<td>0.269</td>
<td>0.184</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Table 11 shows results that are consistent with the model’s prediction related to investment that says that, after certain threshold, investment is insensitive to cash flow after controlling for average $Q$. The results for all sample selections indicate with statistically significant coefficients the usual end expected positive relationship among investment opportunities proxied by average $Q$, and coefficients for operating profit that are not statistically significant. This is consistent with the proposition that after a threshold of firm financial strength, average $Q$ equals marginal $Q$, therefore being a good proxy for investment decisions and making cash flow irrelevant to explain investment.
4 Concluding Remarks

In this papers, predictions concerning corporate finance developed in the dynamic agency models of DeMarzo and Sannikov (2006); and DeMarzo, Fishman, He and Wang (2012) were tested by reduced form regressions. The tests covered predictions involving payments made to the agent; debt structure and investment decisions. Some of these tests relied more closely on the specific implementation of an optimal contract consisting on three securities: credit line; perpetual debt and equity proposed in DeMarzo and Sannikov (2006), while others were developed to work more generally on optimal contracts in the context of these models.

These models predict that payments are made to the agent only after realized cash flows are high enough to reach a threshold of firm financial strength, and after this point all extra cash flows are paid as dividends, bringing the firm back to the threshold. In the specific solution, this threshold is reached when drawn balances in the credit line reaches zero. The tests here found evidence that firms supposed to be in the dividend threshold - with low debt, credit line mostly unused or with high realized profits - are more likely to pay dividends, supporting the model’s prediction. Also, evidence was found that, when firm pays dividends and thus is supposed to be in the threshold, payments made to the agent responds positively to operating income. This positive response also supports the model and was found both looking at agent payments disclosed in financial statements as the compensation paid to the main executive; and at dividends payments.

Investment decisions in DeMarzo, Fishman, He and Wang (2012) are guided by a marginal $q$ ($q_m$) that, in intermediate states of the contract detaches from the observable average $q$ ($q_a$) in a non-monotonic way. In their model, there is a threshold reached when the firm goes well where extra cash flows are paid to the agent and at this point average $q$ equals marginal $q$, making the investment decisions unresponsive to high cash flows after controlling for $q_a$. The tests conducted here found statistically insignificant responses for investment to operating income in subsamples chosen by different criteria of firms supposed to be in the payment threshold, thus supporting the model prediction.

The predictions related to debt structure developed in the specific implementation were not supported here. In this context, the credit line is offered to provide financial slack, and firms that carry more risk, due to more dramatic loss caused by an inefficient default, or due to a higher probability of default caused by more volatile earnings, should rely more on a lengthy credit line in relation to long-term debt. However, no evidence of earning volatility bringing longer credit line was found and some evidence of a more dramatic loss given default bringing less credit line was found. In addition, the prediction of a credit line that is paid down after high earnings and drawn-down after low earnings was also not supported by the
As the data from Capital IQ was not available until 2010, and before that credit line data had to be hand collected as, for example, in Sufi (2009), the literature on credit line is recent and not yet vast. However, some of these papers, despite of being focused on liquidity management, shed light on possible explanations for the credit lines not behaving as predicted in the models analyzed here. The survey on Lins, Servaes and Tufano (2010) indicates that credit line is used to provide liquidity for investing when the firm has a profitable investment opportunity, while cash is the resource used to provide the financial slack needed to cover losses. Acharya, Almeida and Ippolito (2014) shows that credit lines are more commonly used by high credit quality firms, while financially constrained firms tend to rely on cash for their liquidity management; their explanation for this behavior is that credit line is a monitored liquidity source, subject to revocation due to covenant violations. In this context, they reason that firms with high cash flow risk and less pledgeable income should use more cash and less credit, this could be an explanation for the negative sign for tangibility found here.
References


DEMARZO, P.M.; SANNIKOV, Y., (2016) Learning, Termination, and Payout Policy in Dynamic Incentive Contracts, Stanford University Graduate School of Business Research 16-31


