

FUNDAÇÃO GETÚLIO VARGAS

DOCTORAL THESIS

Essays on Monetary Theory

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*A thesis submitted in fulfillment of the requirements
for the degree of Doctor of Philosophy*

Escola de Pós-Graduação em Economia

March 3, 2017

Barros Júnior, Fernando Antônio de

Essays on monetary theory / Fernando Antônio de Barros Júnior. – 2017.

102 f.

Tese (doutorado) - Fundação Getulio Vargas, Escola de Pós-Graduação em Economia.

Orientador: Ricardo de Oliveira Cavalcanti.

Inclui bibliografia.

1. Política monetária. 2. Inflação. 3. Moeda. I. Cavalcanti, Ricardo de Oliveira. II. Fundação Getulio Vargas. Escola de Pós- Graduação em Economia. III. Título.

CDD – 332.4

FERNANDO ANTÔNIO DE BARROS JUNIOR

“ESSAYS ON MONETARY THEORY”

Tese apresentada ao Curso de Doutorado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Doutora em Economia.

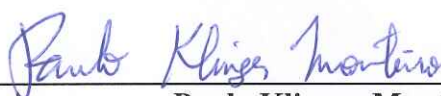
Data da defesa: 30/01/2017

Aprovada em:

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Orientador (a)



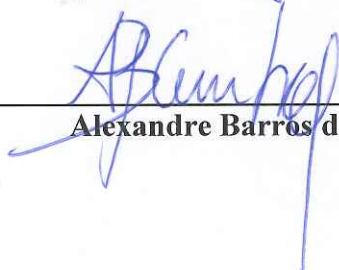
Paulo Klinger Monteiro



José Santiago Fajardo Barbachan



Jefferson Donizeti Pereira Bertolai



Alexandre Barros da Cunha

“... I kind of believe that some of the best economics consists of counterexamples. People think A is true, and someone builds a little model whose ingredients don't seem particularly weird and it implies not A. And so you've got to confront that.”

Neil Wallace

Acknowledgements

First and foremost, I would like to thank God, whose many blessings have made me who I am today.

I am forever grateful to my advisor Ricardo de Oliveira Cavalcanti for his patience, guidance and support in my thesis. I am also indebted to Jefferson Donizeti Pereira Bertolai for his suggestions and help during the development of this thesis.

I would like to thank many colleges for comments and stimulating discussions. Specially Bruno Ricardo Delalibera, Valdemar Rodrigues de Pinho Neto, Rodrigo Bomfim Andrade and André Victor Doherty Ludovice. I have learned a lot from them and many other students at EPGE-FGV.

I also thank my girlfriend Regiane Paes, who has supported me throughout my academic trajectory, and my family.

Finally, I thankfully acknowledge financial support of Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and Graduate School of Economics at Fundação Getúlio Vargas (EPGE-FGV).

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Introduction

In this thesis, we apply numerical methods to study some model where money is *essential*. We follow the mechanism-design approach to monetary theory and search for settings where people can experience the higher welfare.

In chapter 1, we pursue more general simulations of the Shi-Trejos-Wright model with lump-sum transfers of fiat money, as well as develop a model of intermediation in tripartite meetings, to demonstrate the following implications of lack of commitment in matching models of money: savings are inefficiently low; inflation has a negative effect on self-insurance; and although lump-sum transfers should be avoided in many specifications, positive inflation can be optimal with inside money.

In chapter 2, we study the optimality of inflationary policies in a model of inside money. Our numerical findings indicates that inflation is optimal in matching models of money due to higher benefits of money creation than its effects on the return of money. This results holds for a board set of parameters, however, when people are patient and the fraction of monitored people is high enough, then inflation is not part of the optima.

In chapter 3, we introduce ‘open market operations’ in a matching model of money where inflationary policies are optimal - the social benefits of inflation are higher than its costs. Using the average utility as welfare criteria, we numerically solve the model for the best implementable allocation and find that having bonds in the economy can enhance social welfare. In our model, bonds are immune to inflation, thus people value them more than money in equilibrium. Therefore, bond holders can consume more, but they also produce less than money holders. In steady state, both bonds and money circulate in the economy, which increases welfare due to a better distribution of consumption among people. In addition, we present numerical examples to show maturity of bonds and monitoring affect the optimal allocation.

Chapter 1

A paradox of expansionary policies

1.1 Introduction

A recent consensus in monetary theory is that money is an imperfect mechanism.¹ Absent interventions, it is a poor store of value. And the dynamics of monetary trades produces an uneven distribution of money, exposing traders to liquidity risks. The main lesson seems to be that a monetary program for creating liquidity can lessen these imperfections, hurting one dimension in order to improve the other.

A benchmark experiment involves models of pure currency, with people facing the risk of running out of money due to a particular sequence of trade opportunities. It is then asked whether lump-sum transfers of money improve welfare, relative to a zero-inflation equilibrium. Using variations of the Bewley (1980) model, Levine (1991) and Kehoe, Levine, and Woodford (1992) have tackled this question, thereby constructing an important counterexample to the Friedman (1953) and Friedman (1969) rule. Using particular assumptions, they focus, however, on Markov equilibria in which nominal payments are invariant to the inflation rate.

Keeping the distribution of money tractable and still capturing a negative effect of inflation on output is certainly a step forward. In this paper, however, we argue that there is a key drawback in this forced simplification, because another negative effect of inflation is ignored. It is robust and can change the role of expansionary policies. Using matching models, we ask what happens when inflation is allowed to reduce savings. We then find a paradox: lump-sum transfers can actually make the risk-sharing problem worse when the distribution of money is sufficiently rich. Our point about monetary policy is that it should

¹This is a joint work with Ricardo Cavalcanti and Caio Teles.

be assessed by similar considerations related to crowding-out of self-insurance [see, for instance, the non-monetary model with private insurance markets and limited enforcement by Krueger and Perri (2011)].

Illustrating our point requires removing simplifications adopted by existing work, forcing us to appeal to numerical methods. Instead of a Bewley model, we use some variations of the basic random-matching model and provide a careful explanation why previous work has missed this paradox. Part is due to the misconception that main issue with inflation is the so called ‘hot potato effect’. One can indeed easily see in a cash-in-advance model that inflation reduces the return of money, producing a distortion as the real quantity of goods traded falls. But the ignored effect we are talking about is only present when the quantity of nominal savings is important, and this is not the case with a representative agent, or when the model allows for a quick rebalancing of money balances. In order to study the paradox, it is important to be able to exacerbate the role of nominal savings.²

The fact is that in matching models with fiat money, better incentives to save can prevent a negative externality from taking place: when sellers receive a high nominal payment they become less inclined to trade goods for money in the future, aside from the fact that each unit of money may have an inadequate return. In this paper, we illustrate this fact from different but related angles. First, we explain why extensions of the models in Shi (1995) and Trejos and Wright (1995), pursued by Molico (2006) and Deviatov (2006), find a role for inflation: it is because the former does not allow for allocations that provide better insurance to poor money-holders, but which could introduce non-convexities, while the latter restricted the upper-bound on holdings in such a way that inflation was not reducing savings when expansionary policies mattered. Second, we find a way of making savings more important in a commodity-money version of Lagos and Wright (2005). In particular, we allow for tripartite meetings, highlighting the importance of money holdings by intermediaries. Third, we revisit fiat-money exercises with this tripartite structure, discussing restrictions to ‘core’ allocations and the assumption that intermediaries can now be regulated. We then document the fact that, for many parameter specifications, savings are particularly low with core

²Bewley models (see recent application by Lippi, Ragni, and Trachter (2014)) allow poor and rich traders to find a relatively fast regression to mean holdings because they participate in the same market every period. The speed of such redistribution, by contrast, is much slower in general matching models. This difference has not been emphasized in the literature (see Wallace, 2014a for other novelties), but is key for understanding the paradox and may as well be present when inflationary transfers are used to pay interests on money (see Wallace, 2014b). Such policies, although also interesting, lie outside the scope of this paper.

allocations, but regulation of intermediaries can lessen the problem.

If Levine (1991) and Bewley (1980) are offering counterexamples to the Friedman rule, the reader may ask, to what previous research our results give more support? The related literature is a voluminous one. In the conclusion we make few remarks on the Wallace, 2014b conjecture that positive inflation in pure-currency economies should be optimal with the possible exception of knife-edge specifications. In section 2 we mention some additional literature, while in section 6 we highlight passages of Bagehot (1873) as early references to the importance of keeping a proper distribution of money. Finally, section 7 concludes and the appendix contains proofs as well as auxiliary information about simulations.

1.2 A preview

Our counterexamples are selected from steady states, and focus on how traders can privately (self-) insure against shocks using currency, given that meetings are heterogeneous and random, while gains from trade can in principle be divided in a variety of ways, according to wealth profiles in each meeting.³ They also complement findings by Deviatov (2006), and Deviatov and Wallace (2014), about optimal policy with outside and inside money.⁴ Before describing novel ways to reinforce concerns with monetary savings (starting first with a quasi-linear model with commodity money, in the next session, and then with fiat money and more general preferences), we give now an overview of basic simulations to motivate that idea that even the simple pairwise-meetings model can give rise to robust counterexamples.

³The idea that policy evaluation depends crucially on self-insurance and thus on available forms to store wealth motivates the empirical study of Krueger and Perri (2008). There is also a large literature featuring numerical work in Bewley models (such as Imrohoroglu (1992)). While in matching models (see an extensive survey by Lagos, Rocheteau, and Wright (2014)) negative effects of inflation on self-insurance have not been singled out, there is a remote relationship between our findings and the notion of constrained inefficiency (see Geanakoplos and Polemarchakis (1986)): an inefficiently high level of capital holdings in the Aiyagari (1994) model, for instance, generates an externality in the form of more labor-income risk (see Davila et al. (2012) and references therein). Finally, an empirical assessment of how redistributive effects of inflation depend on spending possibilities, measured by the maturity structure of nominal assets, is the subject of Doepke and Schneider (2006).

⁴Instead of using a Bewley model, Cavalcanti and Nosal (2009) propose another way of making flat expansions appealing, using a simple matching model with just 0-1 holdings of money, and adding a seasonal pattern. In particular, they specify a utility jump for people specializing in consumption at a particular season and finding welfare gains when expansions target that season (see also Wallace, 2014a and references therein). But negative effects of inflation in their setting are also narrowly defined (in their model policies are introduced to correct the problem that savings are too high in low seasons).

There are two key observations in these simulations. First, because consumption and production of a perishable good takes part in pairwise meetings, the social planner is not restricted to the ‘law of one price’ as in Bewley models. This means that allocations involving poor traders can be target so as to provide insurance with attractive ‘prices’ for this group, an alternative to inflationary policies affecting the whole population. Second, self-insurance in the form of money holdings depends on how wealth can be stored. In particular, in numerical simulations, if the upper bound on holdings is too small, expansionary policies gain an artificial edge over self-insurance.

Our simulations (presented in detail later) use lotteries, as in Berentsen, Molico, and Wright (2002), having the interpretation that people are spending (and receiving from the government, when expansionary policies are in effect) non-integer amounts of money. This approach is complemented by a proxy of inflation suggested by Li (1994) and Li (1995): the inflation tax is captured by some random confiscation of money holdings. As in many money models, these economies have people meeting randomly in pairs and trading perishable goods for fiat money as in Shi (1995) and Trejos and Wright (1995). Expected utilities associated with money holdings, v , must be consistent with payments and output produced in single-coincidence meetings. A social planner chooses from stationary allocations according to average utility. For an affordable numerical cost, we assume that money is indivisible and that goods are traded for lotteries on money holdings, with support $\{0, 1, 2, 3, 4\}$. There is a large set of possible stationary distributions of money and terms of trade: output and lotteries for each type of meeting, as indexed by traders’ wealth. Exchanges must be better than autarky in meetings for both traders and belong to the core (defined formally later). We have assembled, in Table 1, selected features of optimal allocations.

TABLE 1.1: Insurance when optimal inflation is zero: taxing consumers in meetings with the poorest producer

β	0.9	0.8	0.7	0.6	0.5
i	v / tax	v / tax	v / tax	v / tax	v / tax
0	0.4514 / -	0.0263 / -	0.0089 / -	0.0018 / -	0.0013 / -
1	1.2638 / 0.0000	0.5940 / 0.0000	0.3948 / 0.0000	0.2282 / 0.0000	0.2253 / 0.0000
2	1.5692 / 0.4067	0.7824 / 0.0000	0.5166 / 0.0000	0.3663 / 0.0502	0.2738 / 0.0698
3	1.7538 / 0.2271	0.8794 / 0.0000	0.5675 / 0.0000	0.3892 / 0.0000	0.2861 / 0.0000
4	1.9027 / 0.0552	0.9291 / 0.3964	0.5851 / 0.2921	0.4107 / 0.0000	0.3010 / 0.0000

Based on Table 9. v represents the expected discounted utility for each level of holdings i before meetings take place; In meetings where the consumer holds i and the producer holds 0, ‘tax’ is computed as $1 - y/x$, where y is actual output and x is the cutoff level determining zero surplus for the producer, keeping fixed the optimal payment.

By letting a key parameter, the discount factor β , take different values, we allow the discrepancy between private and social objectives to vary: as we shall see later on, as β is increased, concerns about savings loses importance, so that there is more flexibility with respect to how taxation is levied. For now, some interesting insights are revealed by measures of maximum output that poorest producers could be asked to hand out, given the expected utilities they are actually receiving as payments (not displayed). The relative differences between these ceilings and quantities actually delivered, labeled *tax*, represents a loss to consumers that varies with their holdings of money. According to Table 1, for all β , the poorest producer is always receiving a surplus in some meetings (identified by positive taxes). The precise wealth i of whom is taxed depends on β .

Now there are two remarks on previous work in connection with features of Table 1. First, in attempts to reproduce Levine, 1991 findings by Molico (2006) and Deviatov (2006), corresponding tax statistics are typically zero. In the former case, the tax is always zero due to a bargaining assumption. In the latter, as shown below, for examples with positive inflation the tax is zero because the support used was too small (a 2-unit upper bound), leading to a flatter v (less affected by Inada conditions). Second, we did allow for a lump-sum transfer of money followed by inflation in the same fashion as Deviatov, 2006 did, but no improvements were found. The reason, as mentioned above, is that inflation interacts adversely with self-insurance.

1.3 Adding more risk

We now describe how to make savings even more important in matching models. Adopting the mechanism-design perspective of Hu, Kennan, and Wallace (2009) and Cavalcanti and Puzzello, 2010, for the model of Lagos and Wright (2005), the idea is to require a form of intermediation in meetings, highlighting the importance of the distribution of money and, later, new effects when policies include regulation of inside money.

In this section, for simplicity, we avoid the discussion of interventions in the quantity of money. The goal for now is to show externalities in savings decisions. We model commodity money in a version of quasi-linear economies due to Cavalcanti and Puzzello (2010). Random meetings are preceded by a subperiod in which people must decide how much to consume or to save in the form of a durable commodity. Ideally, society would have traders

choosing high savings at that moment. But since traders cannot commit there is an instance of the Jacklin (1987) problem: after people learn their types, and corresponding opportunity costs of carrying money holdings, they tend to choose lower levels of savings.⁵ It is then shown how consumption taxes can improve welfare relative to the alternative of giving all the surplus to consumers at the meetings stage.

In order to avoid complicated dynamic effects, we assume that commodity money is valued according to separable, linear utility. Relevant histories of savings are entirely captured by recent preference shocks. When people trade in pairs taxation is not needed and consumers keep all gains from trade. But when intermediation is considered, and meetings include a third trader that can lend to consumers, we find that savings by intermediaries are important and that some interventions can provide a better allocation of risk.

1.3.1 A commodity-money environment

Time is discrete and each period is divided into two subperiods. The economy has a large population living forever and experiencing random meetings in the first subperiod, and preference shocks in the second. Preference shocks are realizations of an *iid* process. There is a durable good called money that can be consumed and produced in the second subperiod, according to an idiosyncratic marginal utility θ drawn every date from a uniform distribution. For simplicity, we assume a discrete support $\{\theta_1, \dots, \theta_n\}$ and let F , such that $F(\theta_i) = \frac{i}{n}$ for all i , denote the cumulative distribution of θ . In addition, we normalize its mean, setting $\sum_i \frac{1}{n} \theta_i = 1$.

Money is hence a commodity, produced and consumed when people are by themselves, according to linear utility that is the realization of a preference shock. Money balances are planned in order to reach ideal savings, for each θ , for use as a medium of exchange in the next period, first subperiod, when random meetings take place. Money holdings are observable in meetings but trade histories are private information and people cannot commit to future actions.

There is no discounting between first and second subperiods, but there is discounting at the common factor β across dates. We assume θ_i is increasing in i with $\theta_1 > \beta$, so that savings

⁵In Jacklin (1987), allowing traders to exchange claims on bank deposits eliminates risk sharing from deposit contracts. Hence trading after types are assigned allows people to avoid taxation. In monetary models, when a bank is not providing the final allocation of goods, it is necessary to discuss if trading is amplified by policy, in a version of the Lucas (1976) critique.

are always costly. There is also the standard specialization of production and consumption in meetings. We assume that every meeting is formed by three people: a producer, an intermediary and a consumer. We assume that a person has equal probability of taking part in a meeting in any of these three occupations. And that the meeting is a single-coincidence meeting, when the first person can produce a perishable good for the third one, with probability 3α , where $\alpha \leq \frac{1}{3}$. With probability $1 - 3\alpha$ there are no potential gains from trade. The utility of consuming $c \in \mathbb{R}_+$ units of the perishable good is $u(c)$, and the utility of producing c units of the perishable good is $-c$. We assume that $u(0) = 0$ and that u is continuous, concave, differentiable and such that $u'(0) = +\infty$ and $u(c) < c$ for c sufficiently large.

We assume that the only feasible trade in a meeting has the intermediary transferring money to the producer, as loan to the consumer, in exchange for goods produced. Then, after production takes place and the producer leaves the meeting, the consumer is able to receive goods and to pay out the loan with the intermediary.

In this economy, the planner's problem is to maximize the present value of average utility by choice of incentive-compatible allocations that provide a suitable level of insurance against shock θ and exchange risk. Following Cavalcanti and Puzello, 2010, we restrict attention to stationary allocations. We also anticipate that, due to the quasi-linear structure, optimal allocations are not functions of past histories. A meeting is a vector $m = (m_1, m_2, m_3)$ describing holdings of money of the producer, m_1 , the intermediary, m_2 , and the consumer, m_3 . An allocation is a list (s, x, y, z) describing saving plans s in the second subperiod, as a function θ , and trade plans (x, y, z) in the first subperiod, as a function of m . Saving plans say how much money people will take with them when leaving the second subperiod, according to the realization of idiosyncratic shocks. Trade plans describe loan size x , output level y , and payment amount z . That is, z is the reduction in holdings of money suffered by the consumer, x is how much the producer receives, and $z - x$ is the intermediation profit. We require money transfers to be feasible in the sense of $x(m) \leq m_2$ and $z(m) \leq m_3$.

A plan $s : \{\theta_1, \dots, \theta_n\} \rightarrow \mathbb{R}_+$ generates a distribution of money μ on \mathbb{R}_+ . It is convenient to denote by μ^3 the distribution of meetings on \mathbb{R}_+^3 generated by μ , and by μ^2 its marginal distribution on \mathbb{R}_+^2 when one coordinate of m is fixed. The welfare criteria corresponds to

the utility of an ex-ante representative agent and can be written as

$$w(s, y) = - \int (\theta - \beta) s(\theta) dF(\theta) + \alpha \beta \int (u(y(m)) - y(m)) d\mu^3(m). \quad (1.1)$$

Notice that the welfare function w does not depend on monetary payments. This is so because expected utility, when leaving meetings, as a function of after-trade holdings, is the same for all traders. Hence, no matter how money is divided by trade, the average discounted value attached to after-trade holdings is $\beta \sum_i \frac{1}{n} s(\theta_i)$.

1.3.2 Implementable allocations

We also follow the notion of implementability adopted by Cavalcanti and Puzello, 2010, that is, that traders agree with (s, x, y, z) , given μ associated to s , if autarky in meetings would not make them better off, and if there are no other saving choices that could improve individual utility given (x, y, z) and μ . We shall leave the discussion of group deviations for the fiat money environment of following sections. But while in Cavalcanti and Puzello, 2010 it is optimal to give all surplus to consumers, here this is not so due to an externality associated to savings decisions of people who end up in position to make loans.

In order to be implementable, an allocation must satisfy incentive constraints. Trade incentive constraints are given by

$$y(m) \leq x(m), x(m) \leq z(m) \text{ and } z(m) \leq u(y(m)). \quad (1.2)$$

These inequalities ensure that trade surpluses are nonnegative in all meetings. The saving incentive constraint is that $s(\theta)$ must solve the problem of maximizing $-(\theta - \beta)k + \alpha \beta v(k)$ by choice of money holdings k , where the expected gain from trade $v(k)$ is defined by

$$v(k) = \int (u(y(a, a', k)) - z(a, a', k) + z(a, k, a') - x(a, k, a') + x(k, a, a') - y(k, a, a')) d\mu^2(a, a'). \quad (1.3)$$

Because the distribution of money μ in turn must be generated by s , an incentive-compatible savings plan is a fixed point for each (x, y, z) . Allocations that are feasible and incentive compatible are called implementable.

1.3.3 Welfare bounds

We are now ready to make two basic points about this intermediation economy. In this subsection we show that there is essentially no friction leading to low savings if intermediation is either removed or if the distribution of shocks is degenerate. If constraints associated with intermediation are relaxed then we find no wedge between private incentives to save and the planner's problem. In addition, the solution can be computed recursively: for a given distribution of savings, welfare is maximized by giving all trade surpluses to consumers; and given the implied rate of exchange between money and output, savings are chosen optimally in a separable way according to the costs of supplying money (see proof of next proposition). In the next subsection, we show how a tax system can be used to perturb this kind of allocation in order to increase the incentives to save according to a transfer plan that rewards intermediation.

With intermediation, part of money held by relatively rich consumers cannot be used to weaken incentive constraints of producers. For a given stock of money, average utility in meetings depends only on consumption and production, not on how money is divided among traders. But if intermediation activity receives no compensation, incentives to save can be suboptimal.

Let (s^*, x^*, y^*, z^*) denote the solution of the planner's problem of maximizing $w(s, y)$ in the set of implementable allocations. Let us first turn off the intermediation constraint (the cash-in-advance requirement $x(m) \leq m_2$), denoting by $(\hat{s}, \hat{x}, \hat{y}, \hat{z})$ a solution of the corresponding relaxed problem. Notice that, in this case, the incentive constraints $y(m) \leq x(m)$ and $x(m) \leq z(m)$, together with the feasibility constraint $z(m) \leq m_3$, imply the inequality $y(m) \leq m_3$. It turns out that $w(\hat{s}, \hat{y})$ is the optimal welfare of a random-matching model without intermediaries.

In the following proposition, a comparison is made with another relaxed problem, obtained by imposing $x(m) \leq m_2$ but ignoring saving incentive constraints, as if s can be imposed on individuals. If $(\tilde{s}, \tilde{x}, \tilde{y}, \tilde{z})$ denotes the solution of this second problem, the following holds.

Proposition 1. *For m in the support of distributions of meetings, output is $\hat{y}(m) = m_3$ when intermediation is relaxed, and $\tilde{y}(m) = \min\{m_2, m_3\}$ when savings need not be incentive compatible. In*

these relaxed problems, moreover, welfare satisfies $w(\hat{s}, \hat{y}) \geq w(\bar{s}, \bar{y}) \geq w(s^*, y^*)$, with inequalities replaced by equalities when there is a single type of trader.

Proof. See appendix. □

1.3.4 Taxes and financial profits

In the absence of intermediation, if consumers extract all surpluses in meetings then $g(k) = \beta[u'(k) - 1]$ is the marginal private gain from bringing an additional unit of money to meetings when savings is k (see Cavalcanti and Puzello, 2010). The next proposition explores the fact that, with such terms of trade and intermediation, incentive-feasible savings satisfy $\theta - \beta = \alpha F(\theta)g(s(\theta))$.⁶ From a social perspective, however, each additional unit saved also affects, with probability α , the volume of resources lent to rich consumers, so that if higher savings could be imposed a welfare gain would follow.

When all surpluses are given to consumers, output in meetings is given by $y(m) = \min\{m_2, m_3\}$ and idle holdings $m_3 - m_2$, when positive, remain with consumers. Taxes can improve savings without reducing intensive margins of consumption for a fixed m . To see this, consider the following perturbation with transfers that increase the incentives to save for all types, except the richest one.

We let $y(m) = x(m) = \min\{m_2, m_3\}$ for all m but allow part of $m_3 - \min\{m_2, m_3\}$ to be transferred to intermediaries. Let $(\bar{s}_1, \dots, \bar{s}_n)$ denote incentive-compatible savings for the no-taxation allocation. It is straightforward to show that the saving problem is convex and that $\bar{s}_i > \bar{s}_j$ whenever $\theta_i < \theta_j$. The new allocation is constructed as follows. First a quantity limit $\varepsilon > 0$ and interest rate $r > 0$ are fixed. Then, when consumer with m_3 holdings meets an intermediary with m_2 , for $m_3 > m_2$ and $|m_3 - m_2| > \varepsilon$, then some interest rx is paid to this intermediary if he or she is providing $x \in (\bar{s}_j, \bar{s}_j + \varepsilon)$ in loans. Hence, in the profit allocation, $z(m) = y(m) + rm_2$ in meeting m such that m_2 is discretely lower than m_3 . The values of ε and r are chosen sufficiently small so that idle money $m_3 - m_2$ in such meetings is greater than the extra payment rm_2 , and also to insure that each type does not envy savings designed for another type.

⁶In the proposition, the first-order condition for the savings problem is written in terms of left derivatives. For numerical examples, incentive-compatible saving s_1 , for instance, is found assuming that all other type-1 people are saving a bit more than s_1 and then finding the interior solution $\theta_1 - \beta = \frac{\alpha}{n}g(s_1)$. More generally, savings are still found independently for all grid points.

Proposition 2. *If there is more than one type of trader then welfare is increasing in the profit rate r in a neighborhood of zero, so that it is not optimal to give all surpluses to consumers.*

Proof. See appendix. □

A numerical illustration of welfare gains promoted by taxation is as follows.⁷ Table 1.2 displays basic statistics of allocations as r varies. We find that, for a broad range of values of r , taxes are strictly less than $m_3 - m_2$.

TABLE 1.2: Savings according to profit rates

r	s_1	s_2	s_3	s_4	s_5	Welfare
0%	8.5235	4.5027	1.9184	1.0917	0.7832	3.4733
5%	8.5235	4.5621	1.9361	1.0991	0.7876	3.4758
10%	8.5235	4.6228	1.9541	1.1067	0.7921	3.4783
15%	8.5235	4.6850	1.9724	1.1143	0.7966	3.4807

(1) Values multiplied by 100.

(2) If r is increased to 16% then the richest saver is willing to change behavior (to avoid paying next-type profits).

The proof of proposition 2 can be strengthened. Uniform distributions are not needed but we have omitted a more general treatment for ease of exposition. The fact that, with quasi-linear preferences, a third trader is needed to show that taxation has a role may explain why this point has not appeared formally in the literature. The proof, of course, is simplified by the absence of wealth effects: due to the quasi-linear structure, money transferred to intermediaries (or producers) does not change adversely their incentives to produce in the future. In order to allow for such effects we have unfortunately to resort to numerical methods when we discuss economies with fiat money.

1.4 Adding inflation

Fiat and commodity money have in common an externality: after types are learned, consumers may spend too much fiat money, or traders may save too little commodity money, ignoring that their actions can facilitate trades of others. In the case of the commodity-money model they can help by lending money. In the case of the fiat-money model this too can be important but, in addition, there is the problem that a large payment to someone

⁷We set $u(y) = \sqrt[4]{y}$, $\beta = .6$ and $\alpha = .2$. We let the support of θ be $\{.614, .675, .877, 1.215, 1.619\}$ and set $\varepsilon = 1.854 \times 10^{-3}$.

makes that person less willing to produce in the future (capturing this requires dropping the quasi-linearity assumption).

In this section we depart from the quasi-linear structure, taking the tripartite construct to a fiat-money specification with discrete holdings and lotteries. Risk can now be affected by government transfers and a random confiscation of money that resembles the inflation tax. Except in extreme cases, that we interpret as perfect monitoring, money is in exogenous supply (it is *outside* money) and is fiat (it does not provide direct utility). For tractability, we restrict attention to steady states. We consider persistent occupation in sectors, group deviations and a limit case with inside money. To understand why expansionary policies are not needed in some cases, we explore simulations with a smaller support (an upper bound of 2), that introduces corner solutions and cases of optimal intervention on the supply of money.

1.4.1 The fiat-money environment

A steady-state allocation is now $(\mu, y, \lambda, \tau, \pi)$, where μ is a distribution of money, y defines output for each meeting $m \in M$, λ defines payments in terms of lotteries, also for each meeting m , $\tau = (\tau^n, \tau^b)$ describes occupation-dependent transfers, and π is a measure of inflation. People start each period carrying 0, 1 or 2 units of money, so that $M = \{0, 1, 2\}^3$, either in the *bank* (intermediation) sector or in the complement, the *nonbank* sector. The cases of pairwise meetings are straightforward modifications of the specification with intermediation.

After trades occur, holdings of money evolve according to a stochastic process reflecting inflationary transfers. Bank and nonbank occupations are idiosyncratic shocks evolving according to a first-order Markov process. In particular, the probability that bank people keep their occupation in the next period is ρ , and that for nonbank people is $\frac{1+\rho}{2}$. As a result, in a steady state, the bank sector is always composed by one-third of the population.

We let $m = (m_1, m_2, m_3)$ to denote that money holdings are m_1 for the producer, m_2 for the intermediary, and m_3 for the consumer. The ex ante probability that a nonbank person becomes a consumer or a producer in a meeting is $\frac{\alpha}{2}$. Intermediaries, like nonbank people, take part in a no-coincidence meeting with probability $1 - \alpha$. We denote by μ_i^b the fraction of people starting a period in the bank sector and holding i , and by μ_i^n that in the nonbank

sector and also holding i , where $i \in \{0, 1, 2\}$. In what follows, we often omit the qualification ‘coincidence meeting’ about m whenever it is clear from the context.

Consumer and producer utilities are again $u(c)$ and $-c$, respectively, and the discount factor is also β . In meeting m , output is deterministic and often denoted by $y(m)$, while there is a probability distribution $\lambda(m)$ defining transfers of money among the three traders. More specifically, for $i = 1, 2, 3$, we let $\lambda_i^j(m)$ denote the (marginal) probability that ‘person i ’ (the person starting with m_i) leaves the meeting holding $j \in \{0, 1, 2\}$ units of money. Hence $\lambda_1^j(m)$ denotes the probability that the producer leaves the meeting holding j units of money. In what follows, Bellman equations are more easily expressed by having $\lambda(m)$ written as a vector, so that $\lambda_i^j(m)$ as a particular coordinate of $\lambda(m)$ (see appendix for more details).

We assume initially that no money can be created or destroyed in meetings, and there are physical restrictions on money flows in meetings dictated by intermediation (this assumption is eventually modified when inside money is discussed later). For now, we say that an (outside-money) allocation is *feasible*, reflecting intermediation frictions of the previous section, if for all $m \in M$ two flow conditions are satisfied. As a first condition, we require that $\lambda_1^{m_1+p}(m) = \lambda_2^{m_2-p}(m)$ for all $p \in \{0, 1, 2\}$ and, moreover, if $p > \min\{m_2, 2 - m_1\}$ then $\lambda_1^{m_1+p}(m) = \lambda_2^{m_2-p}(m) = 0$. That is, if payment to producer has mass on p then the intermediary transits to state $m_2 - p$ with the same probability that the producer transits to $m_1 + p$. Likewise, as a second condition, for every realization p for this payment, we require that $\lambda_2^{m_2-p+q}(m) = \lambda_3^{m_3-q}(m)$ for all $q \in \{0, 1, 2\}$ and, moreover, if $q > \min\{m_3, 2 - m_2 + p\}$ then $\lambda_2^{m_2-p+q}(m) = \lambda_3^{m_3-q}(m) = 0$. That is, if a payment to an intermediary has mass on q then the consumer transits to state $m_3 - q$ with the same probability that the intermediary transits to $m_2 - p + q$.

After meetings, but before the period ends, money holdings are affected by policy and new occupation draws take place. We describe policy as transition matrices detailed in the appendix. First there is an inflation shock: a matrix with parameter π is constructed to capture the probability that money disappears, regardless of occupation. A person with one unit has holdings transiting to 0 with probability π , and not transiting with probability $1 - \pi$. A person with two units has holdings transiting to 1 with probability $2(1 - \pi)\pi$, and to 0 with probability π^2 . After the π -shock holdings are updated by a transfer matrix with parameter $\tau = (\tau^b, \tau^n)$. After-inflation holdings j transit to state $j + 1$ with probability τ^b (τ^n), and

remain in state j with probability $1 - \tau^b (1 - \tau^n)$ if $j < 2$, for people in the bank (nonbank) sector. If $j = 2$, the probability of transition is zero.

We say that an allocation is *stationary* if, given λ and (τ, π) , $\mu = (\mu^b, \mu^n)$ is a time-invariant distribution of money (see details in the appendix).

Notice, for a given λ , the effect on μ of increasing (τ^b, τ^n, π) above $(0, 0, 0)$ is to reduce the mass of people with holdings in $\{0, 2\}$, in exchange for an increase in the mass of people holding one unit. In principle this policy improves extensive margins, although it now has a potentially negative effect on the return of money, that can reduce y . As we shall see, however, one must account for changes in λ that are incentive-compatible with saving/spending decisions and which can worsen extensive margins as well. For this we need to describe incentive constraints, according to continuation values, defined as follows.

1.4.2 Welfare criteria and rationality constraints

We now present the welfare criteria and incentive constraints, whose details are also included in the appendix. At the beginning of a period, the expected discounted utility of a person with i units of money in bank and nonbank sectors take, respectively, the following form

$$v_i^b = (1 - \alpha)w_0^b(i) + \alpha \sum_{\{m:m_2=i\}} \mu_{m_1}^n \mu_{m_3}^n w_2^b(m),$$

and

$$v_i^n = (1 - \alpha)w_0^n(i) + \frac{\alpha}{2} \left(\sum_{\{m:m_1=i\}} \mu_{m_2}^b \mu_{m_3}^n w_1^n(m) + \sum_{\{m:m_3=i\}} \mu_{m_2}^b \mu_{m_1}^n w_3^n(m) \right),$$

where w_0^b and w_0^n results from transitions after a no-coincidence meeting, while w_1^n results from transitions after a meeting as a producer, w_3^n results from transitions after a meeting as a consumer, and w_2^b results from transitions after a meeting as an intermediary. In the appendix it is presented the system defining value functions in detail. In particular it is shown that for $m \in M$, $w(m)$ takes the form

$$\begin{aligned} w_1^n(m) &= -y(m) + \beta \lambda_1(m) A^n v \\ w_2^b(m) &= \beta \lambda_2(m) A^b v \\ w_3^n(m) &= u(y(m)) + \beta \lambda_3(m) A^n v \end{aligned}$$

where A^n and A^b are transition matrices reflecting current occupation. Likewise,

$$\begin{aligned} w_0^b(i) &= \beta A_{0i}^b v \\ w_0^n(i) &= \beta A_{0i}^n v \end{aligned}$$

where A_{0i}^b and A_{0i}^n are particular matrices for those holding i units of money in no-coincidence meetings. For a given (μ, λ, y) and policy (τ, π) this system has a contraction property and features an unique solution v .

The welfare criteria is given by average utility, corresponding to an inner product of $\mu = (\mu^b, \mu^n)$ and $v = (v^b, v^n)$, which amounts to

$$w(y, \mu) = \mu \cdot v = \frac{\alpha}{1 - \beta} \sum_{m \in M} \mu_{m_1}^n \mu_{m_2}^b \mu_{m_3}^n [u(y(m)) - y(m)]. \quad (1.4)$$

Remark 1. Lotteries λ and policy parameters (τ, π) have only indirect effects on w . The same can be said about β , since it does not change preference orders over stationary outcomes from the social perspective.

The previous remark implies that the savings friction is relevant since, ex post, consumers take discounting into account when ranking alternative actions. This kind of *excessive spending* can be singled out in simulations, according to the following objects.

We assume that individuals can deviate during trades from what is proposed for a particular meeting, taking as given value functions and the law of movement for aggregate variables. They can deviate individually, by choosing autarky in the meeting, or in groups, by seeking a trade bundle that dominates the proposed allocation for the meeting, without making trade partners worse off. Given such notion of rationality, implementable allocations must satisfy inequalities corresponding to individual-rationality and (static) core requirements. Trade weakly dominates autarky in meeting m for an intermediary if

$$w_2^b(m) \geq w_0^b(m_2), \quad (1.5)$$

and for producer and consumer if

$$w_1^n(m) \geq w_0^n(m_1) \text{ and } w_3^n(m) \geq w_0^n(m_1). \quad (1.6)$$

Individuals can also consider group deviations in a meeting. One way to define the requirement that trade belongs to the core in meeting m is to allow the consumer to search for an alternative output/lottery pair $(\bar{\lambda}, \bar{y})$, subject to intermediation constraints with preservation of money holdings defined above, so as to find

A feasible and stationary allocation $(\mu, y, \lambda, \tau, \pi)$ is *implementable* if associated values (v, w) satisfy individual-rationality (1.5-1.6) and core constraints

$$w_3^n(m) \geq \bar{w}_3^n(m) \quad (1.7)$$

for all $m \in M$.⁸

Remark 2. An intuitive description of constraint (1.7) can be given with pairwise meetings (no intermediation), differentiable value functions (divisible money) and degenerate lotteries. First-order necessary conditions for an interior solution to (2.14) can be shown to imply, in this case,

$$v'(m_3 - p) = u'(y)v'(m_1 + p)$$

where v' is the derivative of the value function, $m_3 - p$ is after-trade consumer holdings of money, $m_1 + p$ is after-trade producer holdings of money, and y is output. Notice that, according to this condition, money payment p is inversely related to output level y when v is concave. In particular, if β is low and, in turn, individual rationality requires low output, then due to the core requirement p must be high. Average trades therefore feature high spending when β is low.

In summary, the core requirement imposes to the planner a level of spending in meetings most favorable to consumers for a given producer surplus. Turning off the core requirement leads therefore to a savings rate more advantageous in terms of ex-ante average utility and to a natural definition of excessive spending.

1.5 Measuring the savings friction

In this section we report the solution of the planner's problem for many specifications, including also the removal of the intermediation friction (which is a particular case of the environment presented in the previous section). We give special attention on how the planner obtains better results if people could commit to not deviate in groups (that is, when

⁸Our algorithm (see appendix) is written with a more general formulation for (1.7).

the core requirement is removed). This should give an useful interpretation of numerical results: expansionary policies are either compensating for low self-insurance in the form of outside money, or creating credit with inside money.

We compute three sets of simulations with intermediation. The first two concern outside-money economies exactly as described in section 4. The third set describes results for extreme values of occupation persistence, allowing for an inside-money interpretation of the model. In terms of parameters introduced in the previews section, we set $\alpha = 1$ and $u(y) = y^{2/10}$. Since results for intermediation with the upper bound of 2 units are easier to interpret (there are fewer output levels), it is convenient to leave the discussion of pairwise meetings with the upper bound of 4 units to subsection 5.3.

1.5.1 Outside-money inflation

In the first set of simulations we put $\beta = .9$, while in the second set we put $\beta = .5$. In both cases, we vary the parameter ρ that determines how persistent the intermediation occupation is. In these outside-money examples we find that at most one unit is transferred and take advantage of this fact, reporting in Tables 1.5 and 1.6, the probability λ that the consumer pays a unit of money. Also, in almost every meeting, the payment from intermediaries to producers is equal to the payment from the consumers to the intermediaries; an exception may occur in meeting (1, 1, 2). When that happens, consumers pay exactly one unit and we report with entry ‘profit (1, 1, 2)’ the probability that the intermediary is leaving the meeting with two units. Finally, we report y relative to $\arg \max_x \{u(x) - x\}$, which for our specification is $y^* = .1337$.

We notice first that without persistence in intermediation occupation (*iid* case), as in the pairwise economy of Deviatov, 2006 (see appendix), inflationary interventions are only optimal when the discount factor β has a low value. Hence this corner condition, with consumers saving zero, is robust to the introduction of intermediation. Corners are easily hit because, with the small support for holdings, value functions are relatively flat and Inada conditions do not help generating positive savings. In these corners, velocity effects are turned off and stop imposing welfare losses when people have a low propensity to save.

By contrast, when $\beta = .9$, as in Table 1.3, the distribution of money can be considered a good one, as about 56% of people have one unit of money, without any redistributive intervention. Hence it becomes a good thing to have zero inflation, and an average monetary

TABLE 1.3: Outside money, $\beta = .9$ and core on

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	1.0000 / 0.19	1.0000 / 0.19	1.0000 / 0.24
(0,1,2)	4.4824 / 1.00	4.4211 / 1.00	1.9903 / 1.00
(0,2,1)	1.0000 / 0.19	1.0000 / 0.19	1.0000 / 0.24
(0,2,2)	4.4824 / 1.00	4.4211 / 1.00	1.9903 / 1.00
(1,1,1)	0.2229 / 0.14	0.2266 / 0.14	0.2842 / 0.19
(1,1,2)	1.0000 / 0.71	1.0000 / 0.72	0.3987 / 1.00
(1,2,1)	0.2229 / 0.14	0.2266 / 0.14	0.2842 / 0.19
(1,2,2)	1.0000 / 0.77	1.0000 / 0.67	1.0000 / 0.65
profit (112)	0	0	0.74
μ_0^n / μ_0^b	0.1452 / 0.1452	0.1455 / 0.1455	0.2277 / 0.0311
μ_1^n / μ_1^b	0.5550 / 0.5550	0.5581 / 0.5581	0.4938 / 0.1084
μ_2^n / μ_2^b	0.2998 / 0.2998	0.2964 / 0.2964	0.2785 / 0.8605
v_0^n / v_0^b	0.0910 / 0.0780	0.1167 / 0.0875	0.4589 / 0.5998
v_1^n / v_1^b	0.9087 / 0.7789	0.9489 / 0.7117	1.1372 / 0.6395
v_2^n / v_2^b	1.1150 / 0.9900	1.2024 / 0.9018	1.3683 / 0.6456
π	0	0	0.0385
τ^n	0	0	0
τ^b	0	0	0.7794

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and $\mu_i^k-v_i^k$ is the measure-value pair of people in sector k holding i units of money.

spending of just .14 in meetings (1, 1, 1) and (1, 2, 1) allows this distribution of money to remain stationary.

Now, if $\beta = .5$ then core constraints, together with producer incentive constraints, push allocations towards low savings and negative effects of inflation are reduced, yielding a measure of optimal inflation of about .22, as we can see in Table 1.4. Meetings (1, 1, 1) and (1, 2, 1), that are key for keeping a good distribution of money without inflation, feature no savings at all. The .22-inflation expansion prevents a very bad distribution of money from taking place, so that about 38% of people hold one unit of money.

Effects of velocity and discounting on saving rates become evident when the core constraint is turned off. If this is done for $\beta = .9$ then about 77% of the population are always holding one unit of money in a better distribution relative to the case with core on. This is due to a smaller monetary payment of .02 on average becomes implementable in meetings (1, 1, 1) and (1, 2, 1). If $\beta = .5$, turning off the core requirement allows spending in these meetings to fall from maximum levels to .02, delivering a good distribution with about 74% of the population holding one unit, without inflation.

Notice that this pattern is robust to specifications with low persistence in intermediation occupation. When persistence parameters is set as 1/3 (*iid* case) or 2/3 (*Markov-low* case),

TABLE 1.4: Outside money, $\beta = .5$ and core on

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,1,2)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,2,1)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,2,2)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(1,1,1)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,1,2)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,2,1)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,2,2)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
profit (112)	0	0	0
μ_0^n / μ_0^b	0.2357 / 0.2357	0.3053 / 0.1221	0.3479 / 0.0366
μ_1^n / μ_1^b	0.3826 / 0.3826	0.3426 / 0.2427	0.3637 / 0.1252
μ_2^n / μ_2^b	0.3816 / 0.3816	0.3521 / 0.6351	0.2885 / 0.8382
v_0^n / v_0^b	0.0149 / 0.0597	0.0096 / 0.0542	0.0385 / 0.0264
v_1^n / v_1^b	0.1471 / 0.0642	0.2067 / 0.0593	0.2732 / 0.0282
v_2^n / v_2^b	0.1639 / 0.0652	0.2413 / 0.0601	0.3166 / 0.0286
π	0.2241	0.1576	0.2042
τ^n	0	0.0128	0.1751
τ^b	1	1	1

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and $\mu_i^k - v_i^k$ is the measure-value pair of people in sector k holding i units of money.

inflation appears only when $\beta = .5$ and the core requirement is on. When $\beta = .9$ or the core requirement is off, low spending in meetings (1, 1, 1) and (1, 2, 1) suffices to generate a good extensive margin. We still find, nevertheless, that a small but robust inflation appears when persistence in intermediation occupation is high. Even when $\beta = .9$ and the core is off, a case of good spending limits, we see the necessity of an inflation measure of .028. Here, however, the intermediation friction is adding a role for expansionary policies that is different from the usual insurance explanation.

To see this, notice that such inflation rate arises but the distribution of money among the nonbank public experiences relatively small changes. If $\beta = .9$ and the core is on then spending in meetings (1, 1, 1) and (1, 2, 1) hits .19 and the nonbank sector fraction holding one unit becomes about 49%. It is the distribution of money in the intermediation sector that experiences a significant change: the fraction of intermediaries without money falls from 14% in the low persistence case to about 3% in the high one. Inflation thus appears with high persistence because money transferred to intermediaries stays in the bank sector for a while, solving in a similar way the externality problem addressed with taxation in

TABLE 1.5: Outside money, $\beta = .9$ and core off

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.9387 / 1.00	0.9402 / 1.00	0.9556 / 1.00
(0,1,2)	2.6305 / 1.00	2.6746 / 1.00	1.9684 / 1.00
(0,2,1)	0.9387 / 1.00	0.9402 / 1.00	0.9556 / 1.00
(0,2,2)	2.6305 / 1.00	2.6746 / 1.00	1.9684 / 1.00
(1,1,1)	0.0509 / 0.02	0.0524 / 0.02	0.1297 / 0.08
(1,1,2)	1.0000 / 0.34	1.0000 / 0.33	0.4720 / 1.00
(1,2,1)	0.0509 / 0.02	0.0524 / 0.02	0.1297 / 0.08
(1,2,2)	1.0000 / 0.34	1.0000 / 0.33	1.0004 / 0.58
profit (112)	0	0	0.73
μ_0^n / μ_0^b	0.0637 / 0.0637	0.0634 / 0.0634	0.1709 / 0.0390
μ_1^n / μ_1^b	0.7735 / 0.7735	0.7748 / 0.7748	0.5724 / 0.1703
μ_2^n / μ_2^b	0.1628 / 0.1628	0.1618 / 0.1618	0.2567 / 0.7907
v_0^n / v_0^b	0.5735 / 0.4916	0.6120 / 0.4590	0.7285 / 0.3451
v_1^n / v_1^b	0.9839 / 0.8433	1.0266 / 0.7700	1.1546 / 0.5469
v_2^n / v_2^b	1.4433 / 1.2371	1.4994 / 1.1246	1.6662 / 0.7893
π	0	0	0.0280
τ^n	0	0	0
τ^b	0	0	0.4653

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and $\mu_i^k-v_i^k$ is the measure-value pair of people in sector k holding i units of money.

section 3.⁹

We also notice that a financial profit exists in some cases in meeting (1, 1, 2). It occurs when persistence is high. It is followed by improvements in the distribution of money in the nonbank sector. Absent profit outcomes in meeting (1, 1, 2), lottery realizations would leave either the producer or the consumer with two units of money, excluding this person from some trades next period. Although profits make consumption goods more expensive, when persistence is sufficiently high the positive effect on the distribution of money across traders is dominating.¹⁰

1.5.2 Inside-money inflation

In our last set of simulations, we consider specifications displaying no transitions for intermediation occupations ($\rho = 1$). In this case, the planner is not constrained by intermediation incentives. In this case, even without an explicit description of how intermediaries can be

⁹Transfers directed to intermediaries when persistence is low find a quick inflow into the nonbank sector. Giving money first to intermediaries reduce negative effects on producer constraints.

¹⁰Giving profits to intermediaries holding one unit in other meetings would hurt the distribution of nonbank money. Having intermediaries with 2 units is not important: meetings (0, 2, 2) feature just one unit spent and the same output as (0, 1, 2). Hence profits perform the money destruction feature of inside-money economies discussed later.

TABLE 1.6: Outside money, $\beta = .5$ and core off

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.5318 / 1.00	0.6395 / 1.00	0.8788 / 1.00
(0,1,2)	0.5318 / 1.00	0.6395 / 1.00	0.8826 / 1.00
(0,2,1)	0.5318 / 1.00	0.6395 / 1.00	0.8781 / 1.00
(0,2,2)	0.5318 / 1.00	0.6395 / 1.00	0.8826 / 1.00
(1,1,1)	0.0097 / 0.02	0.0112 / 0.02	0.0344 / 0.10
(1,1,2)	0.4233 / 1.00	0.4936 / 1.00	0.1997 / 1.00
(1,2,1)	0.0097 / 0.02	0.0112 / 0.02	0.0344 / 0.10
(1,2,2)	0.4233 / 1.00	0.4936 / 1.00	0.3508 / 1.00
profit (112)	0	0	0.43
μ_0^n / μ_0^b	0.0644 / 0.0644	0.0650 / 0.0650	0.2221 / 0.0233
μ_1^n / μ_1^b	0.7412 / 0.7412	0.7404 / 0.7404	0.5246 / 0.0810
μ_2^n / μ_2^b	0.1944 / 0.1944	0.1946 / 0.1946	0.2533 / 0.8957
v_0^n / v_0^b	0.0000 / 0.0000	0.0000 / 0.0000	0.0016 / 0.0276
v_1^n / v_1^b	0.1718 / 0.0711	0.1953 / 0.0488	0.2617 / 0.0323
v_2^n / v_2^b	0.3195 / 0.1278	0.3451 / 0.0865	0.3577 / 0.0325
π	0	0	0.0458
τ^n	0	0	0
τ^b	0	0	1

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and $\mu_i^k - v_i^k$ is the measure-value pair of people in sector k holding i units of money.

monitored, it is reasonable to assume that the planner can ask intermediaries to finance any spending levels. The economy then gains an inside-money interpretation in the spirit of Cavalcanti and Wallace (1999) and Williamson (1999).

In Tables 1.7 and 1.8, we display results for inside-money economies and according to two scenarios. The first scenario has the core requirement turned off. One view here is that intermediaries are essential for all conceivable transactions. As a result, the ability to perfectly control them implies that producer and consumers cannot deviate as a group (in a sense, therefore, monitoring is removing the core requirement).

The second scenario leaves the producer-consumer pair with the option of not using the intermediary, and preventing this option from being exercised is in fact a constraint imposed to the planner. That is, although the intermediary is perfectly controlled, it is constrained to do financing only in ways that improve what producer and consumer get by themselves. Although this scenario is in a way a change in the environment (since money can flow freely from consumer to producer off the equilibrium path), it is instructive for checking out how robust our conclusions are.¹¹

¹¹While we thank Neil Wallace for suggesting examination of this second scenario, we did not find previous work discussing how monitoring of a subset of traders can change the set of core allocations.

TABLE 1.7: Inside money and core off

β	.5	.6	.7	.8	.9
m	y / λ	y / λ	y / λ	y / λ	y / λ
(0,0)	0.06 / 0.25	0.08 / 0.22	0.10 / 0.22	0.14 / 0.21	0.18 / 0.20
(0,1)	0.25 / 1.00	0.35 / 1.00	0.48 / 1.00	0.66 / 1.00	0.92 / 1.00
(0,2)	0.40 / 1.87	0.53 / 2.00	0.72 / 2.00	0.88 / 1.68	0.98 / 1.16
(1,0)	0.01 / 0.04	0.01 / 0.09	0.02 / 0.10	0.40 / 0.12	0.06 / 0.15
(1,1)	0.10 / 0.59	0.18 / 1.00	0.24 / 1.00	0.32 / 1.00	0.40 / 0.97
(1,2)	0.18 / 1.00	0.18 / 1.00	0.24 / 1.00	0.32 / 1.00	0.41 / 1.00
b_{00}	0.25	0.22	0.22	0.21	0.20
b_{01}	0.00	1.00	1.00	1.00	1.00
b_{02}	0.87	1.00	1.00	0.68	0.16
b_{10}	0.04	0.09	0.10	0.12	0.15
b_{11}	-0.41	1.00	1.00	1.00	0.97
b_{12}	0.00	0.00	0.00	0.00	0.00
μ_0	0.7262	0.6424	0.6430	0.6428	0.6412
μ_1	0.2373	0.3156	0.3142	0.3150	0.3182
μ_2	0.0364	0.0421	0.0427	0.0422	0.0406
v_0	0.2295	0.2868	0.3982	0.6435	1.3914
v_1	0.2692	0.3518	0.4989	0.7678	1.5571
v_2	0.2996	0.3932	0.5234	0.8014	1.5945
π	0.1854	0.3537	0.3543	0.3505	0.3445

π is the inflation rate, μ_i is the fraction of people in nonbank sector holding i units of money, b_m is the probability that one unit of money be created in meeting m and v_i is the value associated to people holding i units of money.

In these two scenarios, each meeting is fully described by a pair $m = (m_1, m_2)$, where m_1 (m_2) denotes money holdings of the producer (consumer). In Tables 1.7 and 1.8 we report, for each meeting, output y , relative to y^* , as well as a measure of money transferred to the producer λ . In some meetings, the producer is paid 2 units with positive probability, and hence a reported value $\lambda > 1$ indicates that a two-unit payment has probability $\lambda - 1$, while a single-unit payment has probability $2 - \lambda$. We also indicate, using positive values for b_m , the probability that one unit is created by the intermediary in meeting m . When b_m is negative then $|b_m|$ is the probability that a unit of money of the consumer is destroyed (extracted from the consumer but not transferred to the producer).

We find that in simulations leading to Tables 1.7 and 1.8 there is no use of transfers to the nonbank sector, and hence there is no need to report τ in these tables.

In Table 1.7 we find that inflationary policies are implemented in all configurations. Since there is the option to create credit with perfect control according to its social impact, the concern with distributions of holdings is less important in comparison with the outside-money case. The mass of nonbank people with two units is reduced by inflation. The high magnitude of inflation, of 35%, is necessary to remove money created in credit operations.

TABLE 1.8: Inside money and core on

β	.5	.6	.7	.8	.9
m	y / λ	y / λ	y / λ	y / λ	y / λ
(0,0)	0.05 / 0.28	0.06 / 0.19	0.09 / 0.22	0.12 / 0.22	0.16 / 0.16
(0,1)	0.16 / 1.00	0.32 / 1.00	0.39 / 1.00	0.54 / 1.00	1.00 / 1.00
(0,2)	0.23 / 2.00	0.35 / 1.22	0.51 / 1.49	0.59 / 1.15	1.00 / 1.00
(1,0)	0.00 / 0.05	0.01 / 0.05	0.01 / 0.06	0.02 / 0.07	0.05 / 0.13
(1,1)	0.07 / 1.00	0.14 / 1.00	0.23 / 1.00	0.32 / 1.00	0.34 / 1.00
(1,2)	0.07 / 1.00	0.14 / 1.00	0.23 / 1.00	0.32 / 1.00	0.38 / 1.00
$b_{0,0}$	0.28	0.19	0.22	0.22	0.16
b_{01}	0.00	0.00	0.00	0.00	1.00
b_{02}	0.00	0.22	0.49	0.15	0.00
b_{10}	0.05	0.05	0.06	0.07	0.13
b_{11}	0.00	0.00	0.00	0.00	0.00
b_{12}	0.00	0.00	0.00	0.00	0.00
μ_0	0.7752	0.7467	0.7395	0.7286	0.6406
μ_1	0.1954	0.2159	0.2198	0.2290	0.3184
μ_2	0.0294	0.0374	0.0407	0.0424	0.0410
v_0	0.2146	0.2821	0.4059	0.6508	1.375
v_1	0.2673	0.3633	0.4931	0.7545	1.5714
v_2	0.2846	0.3921	0.5376	0.8091	1.6073
π	0.1892	0.1210	0.1387	0.1235	0.2433

π is the inflation rate, μ_i is the fraction of people in nonbank sector holding i units of money, b_m is the probability that one unit of money be created in meeting m and v_i is the value associated to people holding i units of money.

In Table 1.8, monetary policy becomes less expansionary. When the producer-consumer pair can deviate as a group, spending increases and generates larger distortions on extensive margins, forcing the planner to create less credit. As a result a lower inflation rate emerges. We find that inside-money inflation is associated to more efficient insurance overall, compensating for negative velocity effects. Computed increases in consumption are in line with simulations reported by Deviatov and Wallace, 2014 for inside-money economies with pairwise meetings and 0-1 holdings of money.

1.5.3 Zero inflation with pairwise meetings

We have anticipated some basic results of economies without intermediation (pairwise meetings) in section 2. We have included in the appendix a test case when the upper bound is 2 units. There is no optimal inflation with the core off, but expansionary policies are welfare improving when the core is on and the discount factor is low, so that in meeting type (1, 1) consumers are spending all their holdings (output is 12% of first-best level or less).

Tables 9 and 10 show the effects of increasing the upper bound of outside money, from 2 to 4. Lump-sum transfers are not optimal, regardless of β or the core requirement. In

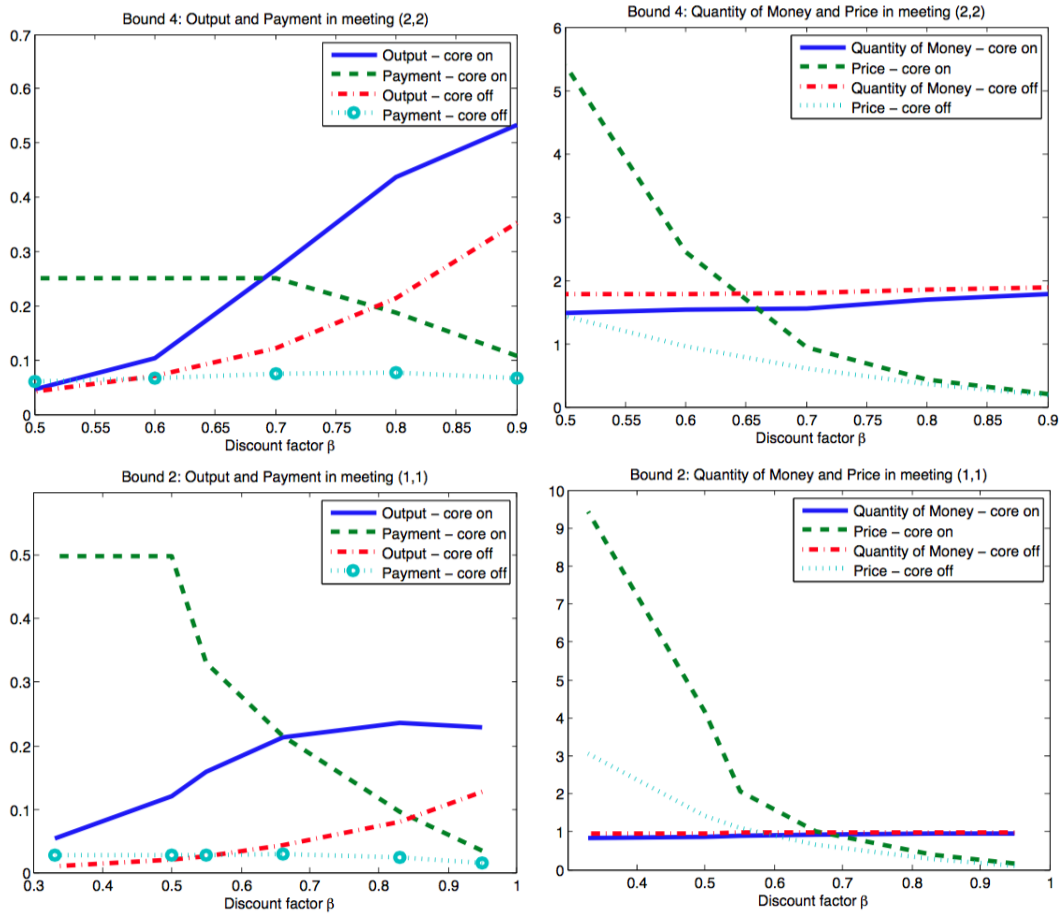


FIGURE 1.1: Salient features of pairwise trades

addition to taxes discussed in section 2, when consumers meet the poorest producer, we also find taxation in meeting (1, 3), when the producer has one unit and the consumer has three, but only for very high β (of .9). So the main lesson is that with more divisibility of money traders are more conservative in terms of spending money and consumer taxes prove to be more efficient in terms of providing insurance.

Some basic effects of changes in the upper bound are displayed in Figure 1. The top two panels refer to the 4 bound, while the bottom two refer to the 2 bound. On the left we can see the effects of β on output and average payment in meeting (2, 2) with the 4 bound (top panel), and in meeting (1, 1) with the 2 bound (bottom panel). On the right, curves represent now the mean of the distribution of holdings and the ratio between average payment and output (called ‘price’) for the 4 bound (top panel) and the 2 bound (bottom panel).¹²

¹²Payment statistics used in Figure 1 are defined as the average transfer of money in meeting (2, 2) (for bound 4) or (1, 1) (for bound 2), paid by consumers to producers, divided by the corresponding bound.

TABLE 1.9: Outside money in pairwise meetings: core on

β	0.9	0.8	0.7	0.6	0.5
m	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$
(0,1)	1.0007 / 0.18 (1)	1.0007 / 0.29 (1)	1.0067 / 0.50 (1)	1.0000 / 0.78 (1)	0.8377 / 1.00 (1)
(0,2)	3.1900 / 1.00 (1)	3.3972 / 1.00 (1)	2.0209 / 1.00 (1)	1.2184 / 1.00 (1)	0.7794 / 1.00 (1)
(0,3)	4.2274 / 1.00 (1)	3.3972 / 1.00 (1)	2.0209 / 1.00 (1)	1.2850 / 0.90 (2)	1.0000 / 0.90 (2)
(0,4)	5.1675 / 1.00 (1)	2.2917 / 0.49 (2)	1.8818 / 1.00 (2)	1.6358 / 1.00 (2)	1.0194 / 1.00 (2)
(1,1)	0.2947 / 0.14 (1)	0.2521 / 0.22 (1)	0.2364 / 0.37 (1)	0.1967 / 0.56 (1)	0.1473 / 0.81 (1)
(1,2)	1.0000 / 0.49 (1)	1.0000 / 0.89 (1)	0.6372 / 1.00 (1)	0.3500 / 1.00 (1)	0.1810 / 1.00 (1)
(1,3)	1.8766 / 1.00 (1)	1.1272 / 1.00 (1)	0.6372 / 1.00 (1)	0.3500 / 1.00 (1)	0.1810 / 1.00 (1)
(1,4)	2.0553 / 1.00 (1)	1.1272 / 1.00 (1)	0.9035 / 1.00 (2)	0.4533 / 1.00 (2)	0.2274 / 1.00 (2)
(2,1)	0.1571 / 0.13 (1)	0.1099 / 0.19 (1)	0.0793 / 0.30 (1)	0.0426 / 0.41 (1)	0.0269 / 0.58 (1)
(2,2)	0.5333 / 0.43 (1)	0.4368 / 0.75 (1)	0.2663 / 1.00 (1)	0.1025 / 1.00 (1)	0.0464 / 1.00 (1)
(2,3)	0.5333 / 0.43 (1)	0.4368 / 0.75 (1)	0.2663 / 1.00 (1)	0.1025 / 1.00 (1)	0.0464 / 1.00 (1)
(2,4)	1.2423 / 1.00 (1)	0.5804 / 1.00 (1)	0.2663 / 1.00 (1)	0.1990 / 1.00 (2)	0.1017 / 1.00 (2)
(3,1)	0.1197 / 0.12 (1)	0.0479 / 0.16 (1)	0.0209 / 0.23 (1)	0.0396 / 0.41 (1)	0.0337 / 0.61 (1)
(3,2)	0.4076 / 0.41 (1)	0.1892 / 0.64 (1)	0.0898 / 0.97 (1)	0.0965 / 1.00 (1)	0.0553 / 1.00 (1)
(3,3)	0.4076 / 0.41 (1)	0.1892 / 0.64 (1)	0.0927 / 1.00 (1)	0.0576 / 0.59 (1)	0.0509 / 0.92 (1)
(3,4)	1.0007 / 1.00 (1)	0.2969 / 1.00 (1)	0.0927 / 1.00 (1)	0.0965 / 1.00 (1)	0.0553 / 1.00 (1)
μ_0	0.0720	0.1171	0.1862	0.2263	0.2695
μ_1	0.3572	0.3708	0.3584	0.3164	0.2801
μ_2	0.3437	0.2806	0.2450	0.2459	0.2415
μ_3	0.1712	0.1605	0.1337	0.1163	0.1083
μ_4	0.0559	0.0710	0.0767	0.0951	0.1006
v_0	0.4514	0.0263	0.0089	0.0018	0.0013
v_1	1.2638	0.5940	0.3948	0.2282	0.2253
v_2	1.5692	0.7824	0.5166	0.3663	0.2738
v_3	1.7538	0.8794	0.5675	0.3892	0.2861
v_4	1.9027	0.9291	0.5851	0.4107	0.3010

$\lambda(x)$ is the optimal probability of transferring x units of money (and paying $x-1$ units with probability $1-\lambda(x)$).

Meetings (1, 1) and (2, 2) are important meetings in terms of their impacts on the distribution of money for, respectively, bounds 2 and 4. Curves for output, representing consumption relative to first-best levels, indicate that the economy with 4 units has more trade going on. Curves for payments show that people save more for a large set of preferences with the 4 bound, an indication that the Inada condition has produced steeper value functions around 0 holdings. In this sense, money becomes more valuable (output doubles at high β).

With the 2 bound payments hit the ceiling of consumers' holdings in meeting (1, 1) at low β such that positive inflation is optimal. Core effects are important for low β , confirming what has been remarked above. When the core is off, output is smaller but both the meeting's payment and the quantity of money vary little with β .

It is fair to say that the curve changing the most with the bound is the one reflecting optimal payments in these 'critical' meetings. Based on Figure 1, a reasonable conjecture is that output and payments vary less with the discount factor as the bound is increased further,

TABLE 1.10: Outside money in pairwise meetings: core off

β	0.9	0.8	0.7	0.6	0.5
m	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$	$y / \lambda(x)$
(0,1)	0.8856 / 1.00 (1)	0.7345 / 1.00 (1)	0.5969 / 1.00 (1)	0.4981 / 1.00 (1)	0.4084 / 1.00 (1)
(0,2)	1.1316 / 1.00 (1)	1.1668 / 1.00 (1)	1.0299 / 1.00 (1)	0.8773 / 1.00 (1)	0.6305 / 1.00 (1)
(0,3)	2.5093 / 1.00 (1)	2.2064 / 1.00 (1)	1.4570 / 1.00 (1)	0.9895 / 0.04 (2)	0.9058 / 1.00 (2)
(0,4)	3.2446 / 1.00 (1)	2.2745 / 0.07 (2)	2.1144 / 1.00 (2)	1.4009 / 1.00 (2)	0.9058 / 1.00 (2)
(1,1)	0.2266 / 0.12 (1)	0.1249 / 0.12 (1)	0.0718 / 0.11 (1)	0.0419 / 0.10 (1)	0.0254 / 0.09 (1)
(1,2)	1.0007 / 0.55 (1)	1.0000 / 0.97 (1)	0.6574 / 1.00 (1)	0.4293 / 1.00 (1)	0.2745 / 1.00 (1)
(1,3)	1.8325 / 1.00 (1)	1.0337 / 1.00 (1)	0.6574 / 1.00 (1)	0.4293 / 1.00 (1)	0.2745 / 1.00 (1)
(1,4)	1.8325 / 1.00 (1)	1.0337 / 1.00 (1)	1.0000 / 0.84 (2)	0.6904 / 1.00 (2)	0.4473 / 1.00 (2)
(2,1)	0.1137 / 0.09 (1)	0.0516 / 0.07 (1)	0.0254 / 0.06 (1)	0.0135 / 0.05 (1)	0.0082 / 0.05 (1)
(2,2)	0.3530 / 0.27 (1)	0.2132 / 0.31 (1)	0.1227 / 0.30 (1)	0.0711 / 0.27 (1)	0.0419 / 0.24 (1)
(2,3)	1.0000 / 0.76 (1)	0.6941 / 1.00 (1)	0.4099 / 1.00 (1)	0.2610 / 1.00 (1)	0.1728 / 1.00 (1)
(2,4)	1.3126 / 1.00 (1)	0.6941 / 1.00 (1)	0.4099 / 1.00 (1)	0.2610 / 1.00 (1)	0.1728 / 1.00 (1)
(3,1)	0.0464 / 0.05 (1)	0.0157 / 0.04 (1)	0.0060 / 0.03 (1)	0.0030 / 0.02 (1)	0.0015 / 0.02 (1)
(3,2)	0.0965 / 0.09 (1)	0.0396 / 0.09 (1)	0.0172 / 0.08 (1)	0.0082 / 0.07 (1)	0.0045 / 0.06 (1)
(3,3)	0.2378 / 0.23 (1)	0.1369 / 0.31 (1)	0.0868 / 0.41 (1)	0.0606 / 0.50 (1)	0.0411 / 0.57 (1)
(3,4)	1.0000 / 0.98 (1)	0.4361 / 1.00 (1)	0.2132 / 1.00 (1)	0.1212 / 1.00 (1)	0.0711 / 1.00 (1)
μ_0	0.0361	0.0377	0.0365	0.0335	0.0307
μ_1	0.2906	0.3165	0.3366	0.3443	0.3433
μ_2	0.4432	0.4301	0.4341	0.4455	0.4580
μ_3	0.2030	0.1893	0.1692	0.1558	0.1471
μ_4	0.0271	0.0264	0.0236	0.0209	0.0209
v_0	0.7894	0.2035	0.0706	0.0229	0.0068
v_1	1.2715	0.5723	0.3489	0.2394	0.1754
v_2	1.5437	0.0745	0.4745	0.3351	0.2489
v_3	1.7387	0.8610	0.5527	0.3932	0.2951
v_4	1.8896	0.9339	0.5934	0.4201	0.3141

$\lambda(x)$ is the optimal probability of transferring x units of money (and paying $x-1$ units with probability $1-\lambda(x)$).

indicating that more insurance is provided. This perhaps can explain that the quantity of money less than doubles as the bound is increased from 2 to 4: as the insurance problem is alleviated, the planner reduces the quantity of money, relatively, in order to improve the return of money (and the intensive margin of consumption).¹³

Confirming what we have seen with intermediation, the core requirement has a strong effect on exchange risk. This is quite evident in the relationship between β and the optimal distribution of money across tables. When the core is off (Table 10) the distributions do not change much with β and display a bell shape. When the core is on (Table 9), by contrast, reductions in β (and thus in saving rates) have a remarkable effect on the mass of people with low holdings of money, up to the point that the distribution approaches the uniform case. This is consistent with a strong velocity effect that makes expansionary policies sub-optimal. In addition, consumers in meeting (2, 4) spend more than one unit of money when

¹³Explaining why turning the core off leads to a higher quantity of money and taxation of poor consumers is more challenging. Focusing on poor consumers instead of richer ones helps with the return of money, which perhaps needs more attention when the quantity of money is increased.

β is .5 or .6 and the core is on, but less than one unit when the core is off (which reduces the dispersion of money).

1.6 A classic case for high returns

We have shown that inflation reduces self-insurance, as people trade more consumption in the present for less insurance in the future, so that lump-sum transfers produce a crowding-out effect. Although for a large set of parameters it is best to just avoid expansionary policies altogether, society can provide better insurance with less distortions by supporting inside money with some level of monitoring.

The prescription of preserving the value of money can in fact be linked to old views about the importance of having money flowing into the financial sector, as it can be concluded from introductory passages in Bagehot (1873). In his classic description of money markets and the operation of central banking 150 years ago, the distribution of money seems to play a central role.

“Everyone is aware that England is the greatest moneyed country in the world; everyone admits that it has much more immediately disposable and ready cash than any other country. But very few persons are aware how much greater the ready balance—the floating loan-fund which can be lent to anyone or for any purpose—is in England than it is anywhere else in the world. [...] Of course the deposits of bankers are not a strictly accurate measure of the resources of a Money Market. On the contrary, much more cash exists out of banks in France and Germany, and in all non-banking countries, than could be found in England or Scotland, where banking is developed. But that cash is not, so to speak, ‘monymarket money’: it is not attainable. [...] But the English money is ‘borrowable’ money. [...] Concentration of money in banks, though not the sole cause, is the principal cause which has made the Money Market of England so exceedingly rich, so much beyond that of other countries.”

This is a simple argument that money needs to be in the right hands to exercise its full potential. From the perspective of models in this paper, there is also a flip side that needs to be emphasized: when money is spent then this potential is lost, unless some special mechanism

is in place to make liquidity flow back to its origin. Bagehot finds that the financial sector has a delicate function, considered fragile to some extent. We have modeled this tension in a rudimentary way by noting that in some cases intermediation activity needs to be properly compensated. One avenue for future research is to disentangle the advantage of having money ready for lending when it is concentrated in the hands of the banking sector, from the possibility that a well developed financial system can be more responsive to contractive policies when spending needs to be controlled. Bagehot in fact witnessed a gold standard facing episodes of adverse liquidity shocks. His well-known policy recommendation of high interest rates in moments of bank panic, coupled with broad measures of liquidity provision by discounting of bills, are geared at discouraging people from withdrawing funds from the financial sector.

1.7 Conclusion

Mechanism design offers the advantage of replacing definitions of incompleteness by primitives such as imperfect monitoring and lack of commitment to future actions, which are actually necessary for restricting credit transactions that would leave money without a role. With these assumptions, steady states feature a dispersion in money holdings. In this paper, we have explored a particular implication of lack of commitment that has not been singled-out by the literature to date: steady-state savings in matching models are inefficiently low. Since the planner is maximizing *ex-ante* utility, there is a trade-off between immediate average-utility gains from output traded in meetings, and the distribution of money that results from savings decisions. Good distributions result from high savings, but after people receive idiosyncratic shocks their incentives no longer coincide with social ones.

We have discussed a way to measure this new friction. One can ask how much consumers agree to spend if the alternative is autarky in the meeting (a high punishment), in contrast to the more natural assumption of allowing consumers to select trades as long as they keep producers indifferent (with such a threat of deviation by the pair, the implied lower punishment forces the planner to accept lower savings). As it turns out, with low discount factors, the implied allocations are very different because the commitment problem becomes more severe. Expansionary policies are sometimes needed in our examples precisely because self-insurance (savings in the form of outside money) are too low.

But what low savings have to do with expansionary policies? Since the distribution of money is endogenous, resulting from savings decisions, we have seen that inflation further reduces self-insurance. In our simulations, considering a large range of discount factors, this finding seem robust in versions with standard pairwise meetings (without intermediation) as well. Due to the high dimensionality of the numerical problem, we cannot offer a definitive answer of whether expansionary policies can be ruled out as a genuine feature of richer outside-money specifications.

We have seen that tripartite meetings are convenient for showing what happens when some monitoring is introduced, so that intermediaries can lessen the problem. We find that monitoring considered by Cavalcanti and Wallace (1999) can potentially promote savings, and inflation with inside money is more likely to be optimal than with outside money.

Wallace, 2014b notes that if the environment has only pure currency (no monitoring whatsoever) then the usual taxation of money holdings is not feasible, but the return of money of richer traders can be improved by a regressive policy that pays interests on holdings above a certain cutoff. His conjecture is that model details should tell whether such policy dominates the lump-sum transfers studied here, that target poor traders. He in fact builds on Kehoe, Levine, and Woodford (1992) to construct an example in which both regressive and lump-sum policies have positive marginal effects on welfare, starting from a no-intervention, zero-inflation equilibrium (without saying which alternative attains the best outcome).

While the conjecture that some intervention (positive inflation) is better than no intervention (zero inflation) seems reasonable for this kind of Bewley model, our results for matching models indicate that assumptions leading to Markov allocations (the result that buyers spend all holdings of money) are not neutral. Of course, this is not to say that matching models in which regressive policies are optimal cannot be constructed. We conjecture instead that for a set of parameters the social desire for consumption smoothing should remain strong, and that in this case monetary policy can reach a corner of no inflation due to negative effects on self-insurance.

In conclusion, with low savings, negative effects of inflation on self-insurance can dominate. Rich effects of expansionary policies already appear in market economies with overlapping generations (see Wallace, 1992), but with matching models the analysis is more complex because risk is in part endogenous: trades are heavily constrained by the distribution

of money and therefore by savings behavior more generally.

Chapter 2

On the optimality of inside-money inflation in random-matching models

2.1 Introduction

Inflationary policies due to lump-sum transfers can generate welfare gains in some models of outside money.¹ In those models, transfers have a beneficial effect on extensive margins by changing the money holdings of those who trade in a way that more than offsets their harmful effect on intensive margins implied by the decrease in the return on money. In models of inside money, the study of effects of inflationary policies is incipient. We address this problem using a random-matching model of money, essentially a version of Cavalcanti and Wallace (1999), CW hereafter. Using numerical simulations, we show that inflation can be optimal for broad set of specifications. However, for some extreme cases inflationary policies remains in the corner of inaction.

The search for settings in which the presence of money allows good outcomes to be achieved, or in Hahn (1987) terminology, settings in which money is *essential*, have a conclusion in the mechanism design approach to monetary economics.² Imperfect monitoring, some privacy of the history of individual actions, is necessary for essentiality of money. However, there is no general necessary and sufficient conditions for essentiality of money. In matching model of money, in the spirit of Kiyotaki and Wright (1989), money is essential

¹See Levine (1991), Kehoe, Levine, and Woodford (1992), Green and Zhou (2005), Molico (2006) and Deviatov (2006) for some examples where inflationary policies improve welfare in outside-money economies. Also, see chapter 1 for a critique of the approach of some of those studies. There, we argue that there are trade mechanisms that can implement a higher welfare when inflationary policies are kept in the corner of inaction. Assumption like Markov equilibria or bargain prevent other trade mechanisms to be used by the planner and, therefore, allow a welfare gain due to inflationary policies.

²See, for example, Ostroy (1973), Townsend (1989) and Kocherlakota (1998).

due to a combination of no monitoring, discounting (that is not taken to the limit of no discounting), a large number of agents, some background absence-of-double-coincidence, and no durable objects other than money.

CW propose an extension of Kiyotaki and Wright (1989) where inside money circulates in the economy as media of exchange. In their model, a fraction of the population is perfectly monitored and the remaining people remain perfectly anonymous. Also, every person has a printing press that allows her to produce identical, indivisible, and durable objects. Trading among monitored people can be done using credit arrangements and does not require use of money, but exchanges between non-monitored people remain as before, consumers should give money to get production from their partner. Even more interesting relations take place when a monitored person meets a non-monitored partner, where the former can print money and deliver it to the latter. People believe in the promise of future consumption embedded in such objects since society can punish monitored people if they refuse to redeem it. As a result, this inside money is actually used in all types of meetings as a perfect substitute to ordinary money.

Studying steady-state allocations, Deviatov and Wallace (2014) show that optimum involves positive inflation and, therefore, the Friedman rule is not optimal in the CW model. They present numerical examples limiting money holdings to the set $\{0, 1\}$. Although they do not search for parameters for which positive inflation generates welfare gains, they provide a robust counter-example to the view that inside-money economies should be regulated so as to avoid inflation. Optimality of inflation is due to a net outflow of money from monitored to non-monitored people, i.e., it is due to inside money creation. Since producers' rationality constraints bind in the optima, inflation works as a social tax that keeps the quantity of money constant in steady state and allows a redistribution of consumption in the economy. Bertolai, Cavalcanti, and Monteiro (2012) contribute to this discussion by studying the role played by transitions in the Deviatov and Wallace (2014) model. They show that although optimality of inflation in transitions comes from the same source as the optimality of inflation in steady state, when the planner is able to choose non-stationary sequences of distributions of money, inflation plays a much smaller role in the optimum.

We extend the Deviatov and Wallace (2014) model by adopting a higher upper bound of money holdings. This extension provides a rich setting in which we can study the optimality of inflation when there is a higher heterogeneity of the distribution of money. We study

optima by running numerical simulations of optimal allocations for a range of parameters on the discount factor and the fraction of monitored agents in population. In addition, we provide some characterization of implementable allocation with respect to individual rationality and group deviations.

The rest of the paper is organized as follows. Section 3.2 presents the model. Section 2.3 specifies the concept of steady-state allocations and the planner's problem. There is a discussion about the numerical problem and how we solve it in section 2.4. Section 2.5 presents some properties of implementable allocations and describes how those properties can reduce the numerical problem of solving the planner's problem. Section 3.4 shows the results of our simulations and section 3.5 presents the conclusions. In the appendix D we present a replication of Deviatov and Wallace (2014) as robustness test for our algorithm.

2.2 Environment

The environment is a version of a Shi-Trejos-Wright economy, which we borrow from CW.³ Time is discrete and infinite and there is nonatomic measure of people each of whom maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. The period utility is given by $u(y) - x$, where y denotes consumption, x denotes production of an individual, u is strictly increasing, strictly concave, twice continuously differentiable, $u(0) = 0$ and $u'(0)$ is arbitrary large. Also, let $y^* = \operatorname{argmax}_y [u(y) - y]$ be positive. Production is perishable, trades occur in pairwise meetings and matches are random. We can summarize the random matching process as follows: a person looks forward to being a consumer or a producer with probability θ and looks forward to no pairwise meeting with probability $1 - 2\theta$, where $\theta \leq 1/2$.⁴

As in CW, people in the model are ex ante identical, but a fraction α become permanently monitored, which we refer as banks, while the rest are permanently nonmonitored (nonbanks hereafter). Histories are common knowledge for banks and they are private for nonbanks. However, the monitoring status and consumer-producer status of people in a pairwise meeting are common knowledge. Only the planner can commit to future actions.

³Wallace (2010) and Deviatov and Wallace (2014) also build on this environment.

⁴The process of random matching describe lead us to the same results as the standard one where people are specialized in production and consumption of one of K types of goods and each person meets other person at random each period.

Each person and the planner have printing presses capable of turning out indivisible and durable objects. Those turned out by the printing press of any one person are distinguishable from those turned out by other peoples' printing presses. That allows us to prevent a bank who defects from issuing additional money. In addition, each person's holding of money issued by others is restricted to be in $\{0, \dots, M\}$, where $M \in \mathbb{N}$. In a pairwise meeting, the money holdings are observable.

Finally, each unit of money can disappear with probability π . This is a simple form to model inflation is borrowed from Li (1994) and Li (1995). Also, the planner can make monetary transfers, which we also model as a stochastic policy: a bank receives a unit of money with probability τ^b and a nonbank person receives a unit of money with probability τ^n . Let $\tau = (\tau^n, \tau^b)$. The sequence of events in an arbitrary period is: first, there are pairwise random meetings between people, that is when trade can occur, then the meetings are dissolved and people are separated; second, after separation inflation occurs; third, the planner can make monetary lump-sum transfers for each group of people.

2.3 Implementable steady state allocations

Due to our numerical limitation - better explained below - we limit our attention only to *steady-state* allocations. Moreover, we search for steady-state allocations that are *symmetric* in the following senses: (i) all people in the same situation take the same action, and (ii) each unit of money issued by the planner or by a bank who have not defect are treated as perfect substitutes. Also, we assume that all monies issued by nonbanks are worthless.

Let an individual's state be $s \in S = \{b, n\} \times \{0, \dots, i, \dots, M\}$, $s = (s_1, s_2)$, where b (n) indicates that a person is a bank (nonbank) and i represents her money holdings. A (s, s') meeting is a meeting between a potential producer in state s and a potential consumer in state s' . For every meeting, let $y(s, s')$ represents the output traded. In addition, we allow people to run lotteries on money transfers. Let $\lambda(s, s') = (\lambda_p(s, s'), \lambda_c(s, s'))$ be a function mapping meetings to money lotteries for producer and consumer, respectively, $\lambda_p(s, s')$ and $\lambda_c(s, s')$, where $\lambda_k(s, s') = (\lambda^{s_1}(0), \dots, \lambda^{s_1}(M))$ for $k \in \{p, c\}$ and $\lambda^{s_1}(i)$ represents the probability of a person in monitoring state s_1 leaves the meeting holding i units of money. By allowing people to run lotteries on money transfers, we attenuate the friction of money indivisibility. Additionally, let $\mu = (\mu_0^n, \dots, \mu_M^n, \mu_0^b, \dots, \mu_M^b)$ be a distribution of money, where

$\mu_{s_2}^{s_1} = \mu(s)$ represents a fraction of people in state s

An *allocation* is a set $(y, \lambda, \mu, \pi, \tau, V)$, where V is a column vector of value function for all $s \in S$. An element of V must satisfy

$$v(s) = \theta \sum_{s' \in S} \mu(s') [p(s, s') + c(s', s)] + (1 - 2\theta)h(s), \quad (2.1)$$

where

$$p(s, s') = -y(s, s') + \beta \lambda_p(s, s') \Pi \Upsilon^{s_1} V^{s_1} \quad (2.2)$$

$$c(s', s) = u(y(s', s)) + \beta \lambda_c(s', s) \Pi \Upsilon^{s_1} V^{s_1} \quad (2.3)$$

$$h(s) = \beta e_{s_2} \Pi \Upsilon^{s_1} V^{s_1} \quad (2.4)$$

and

$$\Pi = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ \pi & 1 - \pi & 0 & \dots & 0 \\ \pi^2 & 2\pi(1 - \pi) & (1 - \pi)^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \binom{M}{0} \pi^M & \binom{M}{1} \pi^{M-1} (1 - \pi)^1 & \binom{M}{2} \pi^{M-2} (1 - \pi)^2 & \dots & \binom{M}{M} (1 - \pi)^M \end{pmatrix}, \quad (2.5)$$

$$\Upsilon^{s_1} = \begin{pmatrix} 1 - \tau^{s_1} & \tau^{s_1} & 0 & \dots & 0 & 0 \\ 0 & 1 - \tau^{s_1} & \tau^{s_1} & \dots & 0 & 0 \\ 0 & 0 & 1 - \tau^{s_1} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 - \tau^{s_1} & \tau^{s_1} \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \text{ for } s_1 \in \{b, n\}. \quad (2.6)$$

$V^{s_1} = (v(s_1, 0) \ v(s_1, 1) \ \dots \ v(s_1, M))'$ and e_{s_2} is a canonical vector in \mathbb{R}^{M+1} in the direction $s_2 + 1$. Thus,

$$V = \begin{pmatrix} V^n \\ V^b \end{pmatrix}. \quad (2.7)$$

The matrices Π and Υ^{s_1} are squared matrices with dimension $M + 1$ implied by the monetary policy. The inflation matrix Π is a lower triangular matrix that describes the stochastic

process of money disintegration. Υ^{s_1} stands for the lump-sum transfers of money for people with monitoring status s_1 . Note that, $p(s, s')$, $c(s', s)$ and $h(s)$ are payoffs (including continuation values): in state s a person who looks forward to be a producer [consumer] and meets a consumer [producer] in state s' has a payoff $p(s, s')$ [$c(s', s)$]; $h(s)$ is a payoff for not being in a meeting and being in state s .

A steady state allocation must satisfy a transition for the distribution of money such that

$$(\mu_0^{s_1}, \dots, \mu_M^{s_1})T^{s_1} = (\mu_0^{s_1}, \dots, \mu_M^{s_1}), \quad (2.8)$$

where, T^{s_1} is a transition matrix implied by lotteries, monetary policy and the distribution of money. A element of T^{s_1} is defined as

$$t^{s_1}(i, j) = \theta \sum_{s' \in S} \mu(s') [\lambda_p((s_1, i), s') + \lambda_c(s', (s_1, i))] \Pi \Upsilon^{s_1}(j) + (1 - 2\theta)e_i \Pi \Upsilon^{s_1}(j) \quad (2.9)$$

where $\Upsilon_{s_1}(j)$ is the $(j + 1)$ -th column of Υ^{s_1} . More precisely, $t^{s_1}(i, j)$ is the probability that a person in state (s_1, i) starts next period holding j units of money.

We assume that individuals can deviate during trades from what is proposed for a particular meeting, taking as given value functions and the law of motion for aggregate variables. Therefore, for nonbanks, the individual rationality constraints for meetings are

$$p((n, s_2), s') \geq h((n, s_2)), \quad (2.10)$$

$$c(s', (n, s_2)) \geq h((n, s_2)) \quad (2.11)$$

and for banks

$$p((b, s_2), s') \geq h((n, 0)), \quad (2.12)$$

$$c(s', (b, s_2)) \geq h((n, 0)), \quad (2.13)$$

because punishment of a bank is loss of bank-status, the defection payoffs, the right-hand sides of equations (2.12) and (2.13), are always expected discounted utilities for nonbanks without money. Note that if a allocation satisfies the constraints (2.11) to (2.13), all people in the economy are individually better off in accepting that allocation than not participating in any trade. However, our model deals with anonymous people who trade in pairs and

individuals can also consider group deviations in a meeting. Therefore, an *implementable* allocation not only have to be immune to individual defections, but it also have to be immune to pairwise defection.

One simple way to avoid group deviations in allocations is set those allocations that belong to the core of all meetings. We can find those allocations by allowing the consumer to search for an alternative output/lottery pair $(\bar{\lambda}, \bar{y})$, subject to not make his trade partner worse off, taking aggregate variables as given. More precisely, a allocation in the core solves the following problem:

$$\begin{aligned} \underset{\bar{\lambda}, \bar{y}}{\text{Max}} \quad & u(\bar{y}) + \beta \bar{\lambda}_c(s, s') \Pi \Upsilon^{s_1} V^{s_1} \\ \text{s.t.} \quad & -\bar{y} + \beta \bar{\lambda}_p(s, s') \Pi \Upsilon^{s_1} V^{s_1} \geq K(s, s'), \end{aligned} \quad (2.14)$$

where $K(s, s')$ is some positive value consistent with the producer payoff from a trade. Although it is not explicit, relevant rationality constraints also apply to the problem above.⁵

Finally, as welfare criteria we choose the average utility, corresponding to an inner product of μ and V , which amounts to

$$W = \mu \cdot V = \frac{\theta}{1 - \beta} \sum_{s \in S} \sum_{s' \in S} \mu(s) \mu(s') [u(y(s, s')) - y(s, s')]. \quad (2.15)$$

The planner's problem is to choose an allocation to maximize welfare subject to relevant constraints. We create two groups of problems. For the first one, we ask the planner to search allocations that satisfy rationality constraints but that are not necessarily in the core of meeting, i.e., we solve the planner's problems without constraints implied by the problem described in (2.14). We call these allocations as **core-off** allocations. For the other group of allocation, we solve the planner's problems constrained by allocations in the core of meetings. We call these allocations as **core-on** allocations.

2.4 Numerical problem

We rely on numerical methods to unveil some properties of the the solution of the Planner's problem. The challenge of using the mechanism design approach on monetary economics is not a trivial one. As pointed by Bertolai, Cavalcanti, and Monteiro (2016), increasing

⁵The core set is formed after varying $K(s, s')$, as a contract curve.

the upper bound on money holding while taking into account every heterogeneity in the economy, such as the distribution of money, can become intractable very quickly. Likewise, a full analytical characterization of the mechanism is unfeasible due to the complexity of the objects that compose it. Therefore, we select some simple examples to explore CW economies.

The optimum problem is inside a class of numerical problems named as constrained nonlinear programming problems. We search for the maximum of a nonlinear function - the welfare function - subject to: (a) linear constraints (equalities and inequalities), such as lotteries summing to unit, (b) upper and lower bound (except for value functions, all variables are limited) and (c) nonlinear constraints (equalities and inequalities), for example stationarity constraints, core and rationality constraints. Fortunately, there are some algorithms that can handle this class of problem. In our numerical simulations, we use KNITRO solver, which is a solver designed for the solution of large linear, nonlinear, and mixed integer optimization problems (Byrd, Nocedal, and Waltz, 2006). Our routine is based on an active-set algorithm - designed to search for local solutions. We overcome the problem of local solutions executing the algorithm for thousands of start points. The routine keeps all local solutions found by active-set algorithm and select the one which give the higher welfare as the global optimum. Although, we can not prove that this routine will always attain a global solution, we have some evidence about the success of the routine: (i) for several initial points, the algorithm finds the same solution; (ii) we can reproduce the results of Deviatov and Wallace (2014) as a test case.⁶ For more details see appendix D.

For the best of our knowledge, this is the first work that provides some examples for CW economy where $M > 1$ where inflation and transfers are allowed.

2.5 Properties of implementable allocations

In this section, we describe some properties of implementable allocations. First, we present some results that give us some advantages for posterior numerical work. More precisely, these results help us reduce the numerical effort. Yet, it does not mean that the problem

⁶Deviatov and Wallace (2014) solve the optimum problem using the General Algebraic Modeling System (GAMS), which consists of a language compiler and a large menu of solver. They choose a Branch-And-Reduce Optimization Navigator (BARON) solver - a global solver.

solved is a trivial one. We begin with a results about meetings where at least one person is a bank.

Claim 1. *Consider an allocation satisfying rationality constraints. If for a (s, s') meeting, the producer or consumer is a bank, then the allocation is in the core of this meeting.*

A sketch of the proof of claim 1 is the following. Consider an allocation satisfying the rationality constraints. Remember that the planner can punish a bank by banning his monitoring status and making all of his printed monies useless, because all information about banks are available and monies are identified by the bank who printed them. If the bank is a consumer, then the producer has no incentive to deviate because all the money received from that bank will be worthless in the next period. Therefore, there is no group deviation in this case. On the other hand, if the bank is a producer, he has no incentive to deviate, because no money from the consumer can not make him better off and deviation implies losing his monitoring status forever. Again, there is no group deviation in this case. We can conclude that all allocation that satisfy the rationality constraints are in the core set of the meetings where at least one person is a bank.

This results is important to numerical work, because it allow us to reduce the number of constraints the solver must deal with. Next, we describe a claim due to Wallace (2010), that also gives us some advantage to solve numerically the planner's problem.

Claim 2 (Wallace (2010)). *If an allocation satisfy rationality constraints, then there is another allocation that satisfy rationality constraints with the same production and consumption in which banks start every period without money.*

This result allow us to reduce even more the size of our numerical problem, because we do not need to track money holdings of banks, i. e., we can assume that whenever a bank receives an unit of money, he destroys it.

2.5.1 Core constraints

Now we show how simple inequalities equations can be used as constraints in the planner's problem to compute core on allocations. We start with the most simple case as an illustration. Suppose that $M = 1$. In this economy, the planner cares about group deviations only in one meeting: the one where the producer has no money, the consumer has one unit and

both of them are nonbanks. For this meeting, a core allocation solves the following problem

$$\begin{aligned} \underset{\bar{\lambda}, \bar{y}}{\text{Max}} \quad & u(\bar{y}) + \beta(\bar{\lambda}\hat{v}^{n,0} + (1 - \bar{\lambda})\hat{v}^{n,1}) \\ \text{s.t.} \quad & -\bar{y} + \beta(\bar{\lambda}\hat{v}^{n,1} + (1 - \bar{\lambda})\hat{v}^{n,0}) \geq K_{M1}, \end{aligned} \quad (2.16)$$

where

$$\begin{pmatrix} \hat{v}(n, 0) \\ \hat{v}(n, 1) \end{pmatrix} = \Pi \Upsilon^n V^n. \quad (2.17)$$

Note that $\hat{v}(n, i)$ is the expect lifetime utility for a person holding i units of money after the trade and before monetary policy. Also, in the current context $\bar{\lambda}$ is the probability that a unit of money is transferred from consumer to producer.

Lemma 1. Fix $M = 1$ and suppose that money is valuable in equilibrium, i.e., $\hat{v}(n, 1) > \hat{v}(n, 0)$. Let $\tilde{\lambda}$ be the probability of one unit of money be transferred from consumer to producer in the trade meeting $((n, 0), (n, 1))$ and \tilde{y} the output traded in this meeting. Therefore, a allocation is in the core of the meeting $((n, 0), (n, 1))$ if, and only if,

$$(1 - \tilde{\lambda})(u'(\tilde{y}) - 1) \leq 0. \quad (2.18)$$

Proof. For necessity, let δ be the Lagrange multiplier associated with the constraint of the problem described at (2.16) and consider the first order condition (FOC) with respect to \tilde{y}

$$u'(\tilde{y}) - \delta = 0, \quad (2.19)$$

equality comes from our assumption about u . From equation (2.19), we have $\delta = u'(\tilde{y})$. Now, consider the FOC with respect to $\tilde{\lambda}$,

$$\beta(\hat{v}(n, 0) - \hat{v}(n, 1)) + \delta\beta(\hat{v}(n, 1) - \hat{v}(n, 0)) \stackrel{\geq}{\leq} 0, \quad (2.20)$$

given that $\hat{v}(n, 1) > \hat{v}(n, 0)$ and substituting the Lagrange multiplier we get

$$u'(\tilde{y}) - 1 \stackrel{\geq}{\leq} 0. \quad (2.21)$$

Given the linearity of the Lagrangian in $\tilde{\lambda}$, a corner solution arises unless $u'(\tilde{y}) = 1$. If

$u'(\tilde{y}) > 1$ (< 1), then, $\tilde{\lambda} = 1$ ($= 0$). Otherwise $\tilde{\lambda} \in [0, 1]$. This result is summarized by equation (2.18). The sufficiency part of the proof is given by the assumptions about the preferences. \square

One implication of this lemma is that as long as the output in the meeting where people can deviate in groups is lower than y^* , people would always prefer expend all the money they have. Only if the output is such that the marginal utility of consumption and marginal cost of production are equal, then consumer would save some money.

Now we state a more general results about core allocations.

Proposition 3. Consider $M \in \mathbb{N}$ and let

$$\begin{pmatrix} \hat{v}(n, 0) \\ \hat{v}(n, 1) \\ \vdots \\ \hat{v}(n, M) \end{pmatrix} = \Pi \Upsilon^n V^n.$$

and also let $\lambda^{s, s'}(i)$ be the probability of transferring i units of money from consumer to producer in meeting (s, s') . Therefore, an allocation is in the core of meeting (s, s') , where $s_1 = s'_1 = n$, if, and only if,

$$\lambda^{s, s'}(i) \left(\text{Max}_j \{L_j^{s, s'}\} - L_i^{s, s'} \right) \leq 0 \text{ for all } i \in \{0, \dots, \text{Max}(s_2, s'_2 - M)\}, \quad (2.22)$$

where $L_i^{s, s'} = \beta [\hat{v}(n, s'_2 - i) - \hat{v}(n, s'_2)] + u'(y(s, s'))\beta [\hat{v}(n, s_2 + i) - \hat{v}(n, s_2)]$ is a FOC, with respect to $\lambda^{s, s'}(i)$, of the Lagrangian associated to the problem in equation (2.14), and $\lambda^{s, s'}(0) = 1 - \sum_i \lambda^{s, s'}(i)$.

The proof of this proposition is similar to the proof of lemma 1. The main idea is that an allocation in the core of a meeting features a combination of output and a money transfer which satisfy the first order conditions of core problem. Linearity again play an important role: the lottery degenerates in the direction where the FOC with respect to the probability of i units being transferred is higher. There is randomization in the payments only if two or more FOCs of the Lagrangian are equal.

2.6 Numerical simulations

In this section we show the numerical simulations we run for the model. In the simulations below we vary the upper bound of money holding, the fraction of banks, the discount factor and the core requirement to the planner's problem. Unfortunately, we can only solve the planner's problem for $M \in \{1, 2, 3\}$ due to a dimensionality problem. For example: for $M = 3$ our routine must deal with 57 variables and more than 70 constraints. We set $u(y) = 1 - e^{10y}$ and fix $\theta = 1/3$ as Deviatov and Wallace (2014). Our motivation to do so is to keep a good test case for our code. In appendix D we present a careful replication of their results. This simple exercise show us that our routine can deal with the planner's problem, at least when the problem's dimensionality is not very large.

We start by showing when core constraints are binding in the planner's problem. In figure 2.1 we compute the relative optimum welfare between a core-off allocation and a core-on allocation. For figure 2.1 we set $\alpha = 1/4$ e vary β and M . For $M = 1$ core constraints do not bind in any of our simulation, i.e., the best implementable welfare is the same for core-on and core-off allocations, because of that we do not report the results for $M = 1$ in figure 2.1. In contrast, if the upper bound of money holdings is greater the 1, core constraints are bind for some specifications. For $M = 2$, this effect happens only if β high enough, but for $M = 3$ there is a difference in the highest implementable welfare for all simulations we run.

We notice a difference between core-ff and core-on allocations with respect to the expenditure in meetings, i.e., with respect to money lotteries. First, note that there are some meetings that do not change the distribution of money. For example: in $((n, 0), (n, 1))$ meeting only one unit of money can be transferred from consumer to producer and after trade only one person holds this unit, which means that the distribution of money do not change when money goes from one person to another in this meeting. However, there are some meetings that can change the distribution. For example: in $((n, 1), (n, 1))$ meeting if any money is transferred from consumer to producer, after trades one person will have two units of money and the other will have no money, which is very different from the initial money holdings. In table 2.1 we show different patterns for money payments for core-on and core-off allocations. Table 2.1 reports the average payments for allocations where

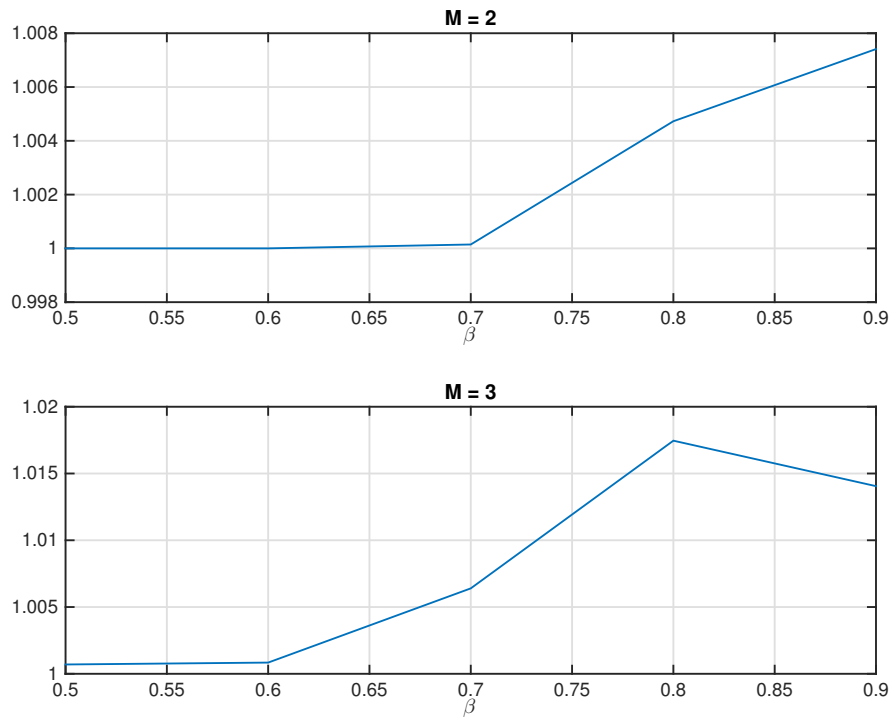


FIGURE 2.1: Relative Optimum Welfare: core-off/core-on

$M = 3$ (the results for $M = 2$ are similar). Note that, specially but not only, in meetings $((n, 1), (n, 1)), ((n, 2), (n, 1)), ((n, 2), (n, 2))$ the average monetary payments diverge for core-off and core-on allocations, it is more clear if the discount factor is high. We observe higher payments in core-on allocation where there is a divergence with core-off allocation. More than that, this higher payment implies in a distribution of money more disperse, i.e., a greater fraction of nonbanks holding no money and 3 units of money than holding 1 or 2 units.

Figure 3.1a displays the distribution of money for best implementable allocations. For $M = 1$, if the discount factor is high enough, then the distribution of money is constant on β : it consists on $2/3$ of the nonbanks holding no money and the others holding money. It is easy to check that if the output is equal y^* (maximizes the intensive margin) for all trade meetings, given that $\alpha = 1/4$, then the distribution of money that maximizes the extensive margin is exactly $(\mu_0^n, \mu_1^n) = (1/2, 1/4)$. For $M > 1$, we do not find a similar result. Core-off allocation presents less nonbanks on the lower and upper bounds of the money holdings when compared to core-on allocation. Notice that the higher is the discount factor, the difference between core-on and core-off allocations distributions increases as a response

TABLE 2.1: Average Payment

core off					
β	0.50	0.60	0.70	0.80	0.90
((n,0),(n,1))	1.0000	1.0000	1.0000	1.0000	0.4574
((n,0),(n,2))	1.0000	1.0000	1.0000	1.0000	1.0000
((n,0),(n,3))	2.0000	2.0000	1.1343	1.0000	1.0000
((n,0),(m,0))	2.0000	1.3521	1.0000	1.0000	1.0000
((n,1),(n,1))	1.0000	1.0000	0.8119	0.5740	0.4850
((n,1),(n,2))	1.3737	1.0000	1.0000	1.0000	0.8956
((n,1),(n,3))	2.0000	2.0000	1.7699	1.0000	1.0000
((n,1),(m,0))	2.0000	2.0000	1.0000	1.0000	0.8998
((n,2),(n,1))	0.9049	0.4050	0.0000	0.0807	0.2410
((n,2),(n,2))	1.0000	1.0000	1.0000	1.0000	0.7059
((n,2),(n,3))	1.0000	1.0000	1.0000	1.0000	1.0000
((n,2),(m,0))	1.0000	1.0000	1.0000	0.9323	0.7269
((m,0),(n,0))	0.0000	0.0000	0.0000	0.0000	0.0000
((m,0),(n,1))	0.0000	0.0000	0.0000	0.0000	0.0000
((m,0),(n,2))	1.0000	0.5899	0.8298	1.0000	1.0000
((m,0),(n,3))	0.0585	0.4012	0.0000	1.0000	1.0000
core on					
beta	0.50	0.60	0.70	0.80	0.90
((n,0),(n,1))	1.0000	1.0000	1.0000	1.0000	0.4388
((n,0),(n,2))	1.0000	1.0000	1.0000	1.0000	0.8917
((n,0),(n,3))	2.0000	2.0000	1.0883	1.0000	1.0000
((n,0),(m,0))	2.0000	1.0043	1.0000	1.0000	0.9683
((n,1),(n,1))	1.0000	1.0000	1.0000	1.0000	0.5997
((n,1),(n,2))	1.8051	1.0000	1.0000	1.0000	0.9123
((n,1),(n,3))	2.0000	2.0000	2.0000	1.0000	1.0000
((n,1),(m,0))	2.0000	2.0000	1.0000	1.0000	0.9572
((n,2),(n,1))	1.0000	1.0000	1.0000	1.0000	0.6558
((n,2),(n,2))	1.0000	1.0000	1.0000	1.0000	1.0000
((n,2),(n,3))	1.0000	1.0000	1.0000	1.0000	1.0000
((n,2),(m,0))	1.0000	1.0000	1.0000	1.0000	0.7728
((m,0),(n,0))	0.0000	0.0000	0.0000	0.0000	0.0000
((m,0),(n,1))	0.0000	0.0000	0.0000	0.0000	0.0072
((m,0),(n,2))	1.0000	0.0037	0.1760	0.1653	0.8906
((m,0),(n,3))	0.7525	0.8392	0.1511	1.0000	1.0000

Average payment for a meeting is calculated as $\sum_i i\lambda(i)$, where i is a possible transference for a trade meeting and $\lambda(i)$ represents the probability of transferring i units of money from consumer to producer.

the the different payments of this allocations. In other words, there is a kind of conflict between public and private interests, as chapter 1 and Barros (2014) point. Although, in these works that effect is more severe if people are more impatient, here the opposite occurs. This is due to the nature of inside money in the model. Here, the interaction between banks and nonbanks permits the planner to use a group of meeting to change the distribution of money. Notice in table 2.1 that in the meetings where a bank is a consumer there is a higher average payment, than in those meetings where a bank is a producer. Moreover, note that the money payments of banks is decreasing in the money holding of nonbank producers,

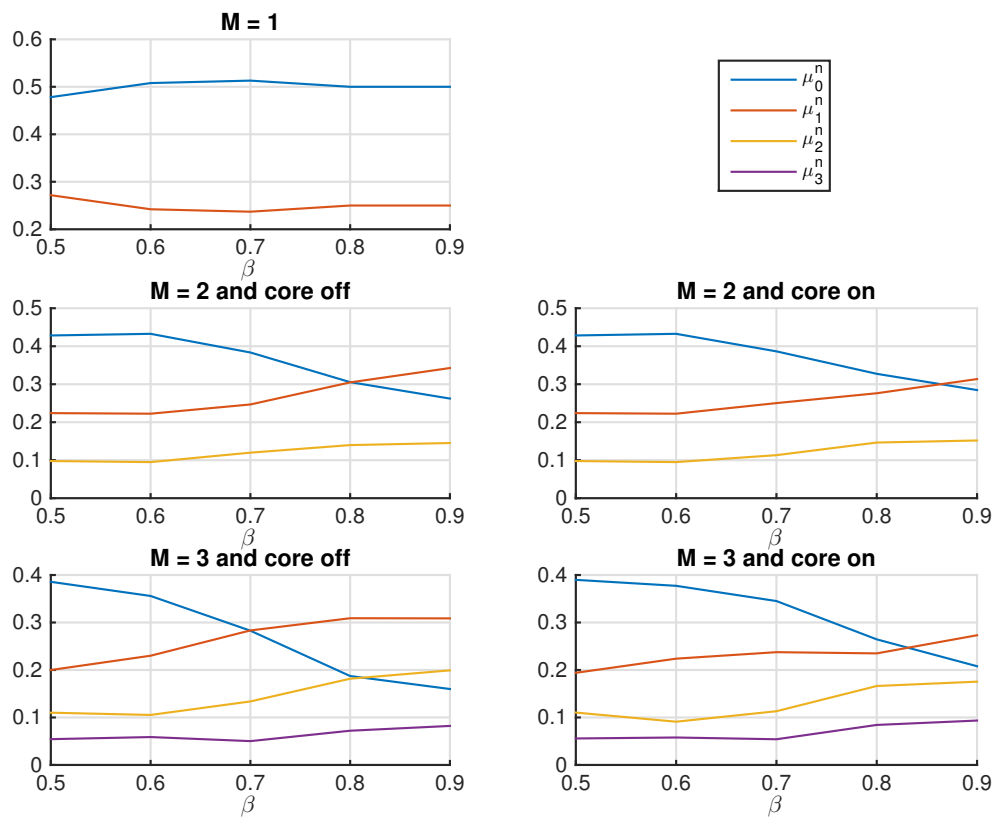


FIGURE 2.2: Distribution of money

i.e., the money creation from banks is a progressive policy. As a result, money expending with inside money has minor effect over the distribution of money than in a outside money economy.

Also, there is no lump-sum transfer in all the simulations we run, because the progressive money creation by banks is more effective. Money is created in the economy only when there is some production and can be directed to meetings where the producer has less money, which provides a better insurance for nonbanks.⁷

The net injection of money from banks to nonbanks is counterbalanced by inflation in the economy. As pointed in Deviatov and Wallace (2014) optimal positive inflation occurs because the influx of money from banks to nonbank is higher than the outflow into nonbanks money holding. The alternative to this inflationary policy is reduce inside-money creation by banks (asking lower payments to them), but that would reduce welfare because nonbank

⁷Only for $M = 1$ and $\beta = 0.9$ we find a positive lump-sum transfer, but that happens because a the linearity of value function. Notice that the same allocation of $\beta = 0.8$ could be implemented.

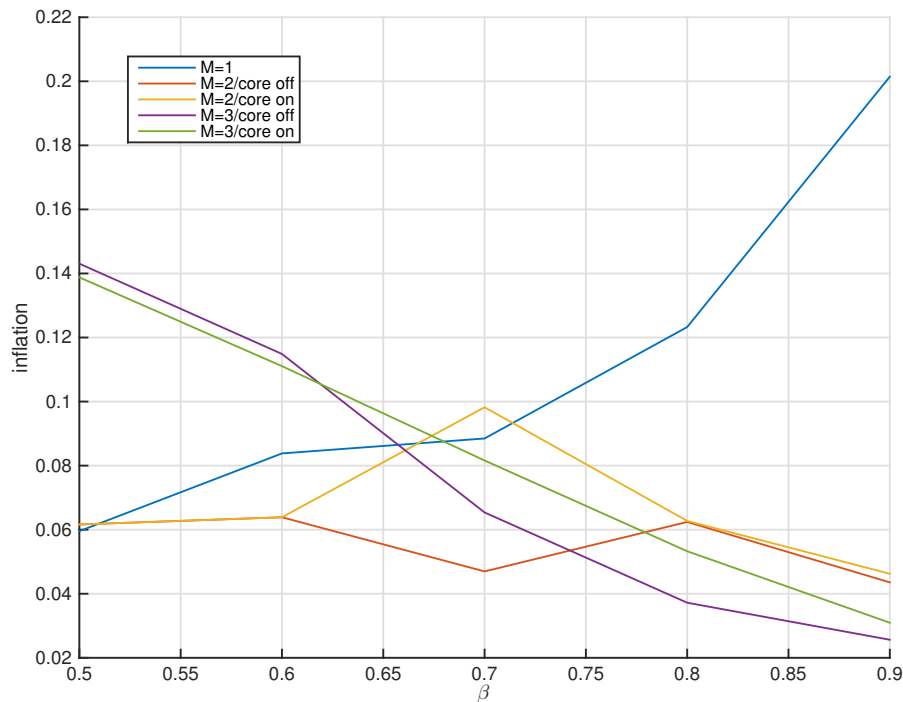


FIGURE 2.3: Optimal inflation

producers are binding in optima and a reduction in payments would imply a lower output in those trades. We strengthen their results by increasing the upper bound of money holding and we also show that even for core-off allocations positive inflation can be optimal for a boarder set of parameters. In figure 2.3 we plot optimal inflation for a some different specification of our economy. For this set of simulations we fix the fraction of banks in $1/4$ and find positive inflation due the (progressive) money creation by banks.

2.6.1 Inflation and monitoring

Now we investigate the properties of optimal implementable allocation when the fraction of banks varies. For the examples in figure 2.4 and in table 2.2, we fix the upper bound of money holding to 3 units and vary α . In figure 2.4 we report optimal inflation of simulations using two different discount factors and two different patterns arise. First, for high discount inflation is increasing on the fraction of banks. However, if people are more patient, inflation is increasing on the fraction of banks only if the fraction of banks is small. After a threshold, optimal inflation is decreasing on the fraction of banks. Moreover, these two patterns are independent of core constraints. Again, average payments in meetings presents an

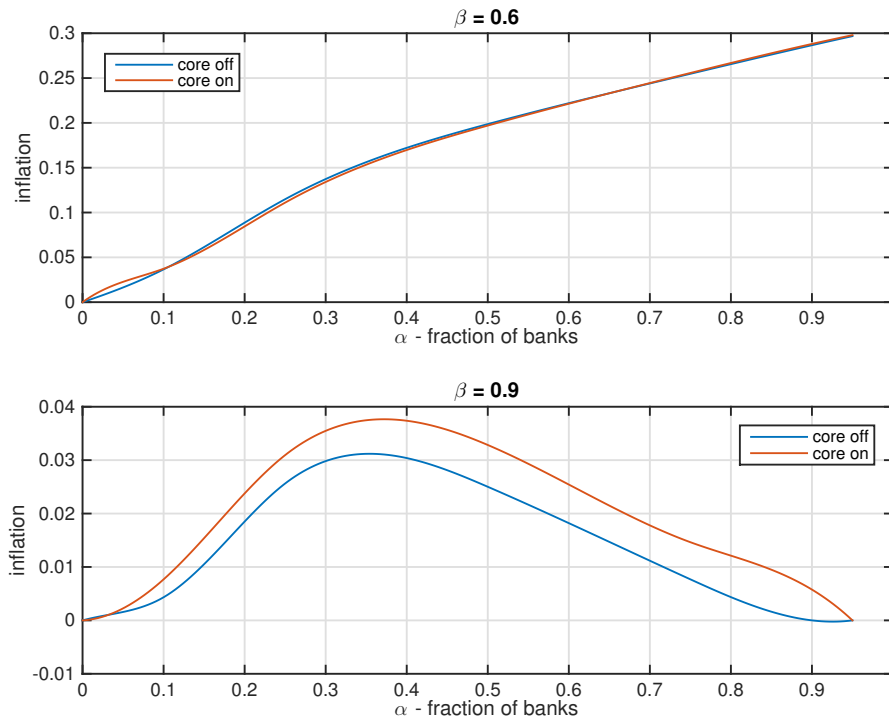


FIGURE 2.4: Inflation varying the fraction of banks

explanation for these different patterns.

TABLE 2.2: Selected Average Payment

$\beta = 0.6$							
α	0.05	0.10	0.25	0.50	0.75	0.90	0.95
$((n,0),(m,0))$	1.4738	1.1527	1.0043	1.5452	1.8096	1.9277	1.9724
$((n,1),(m,0))$	1.9909	1.2986	2.0000	2.0000	2.0000	2.0000	2.0000
$((n,2),(m,0))$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$((m,0),(n,1))$	0.0000	0.1155	0.0000	0.0000	0.0000	0.0000	0.0000
$((m,0),(n,2))$	0.7501	0.4549	0.0037	0.0000	0.0000	0.0000	0.0000
$((m,0),(n,3))$	0.5999	0.8002	0.8392	0.1005	0.0000	0.0000	0.0000
$\beta = 0.9$							
$((n,0),(m,0))$	1.0000	1.0000	0.9683	0.4397	0.3396	0.5369	0.4482
$((n,1),(m,0))$	0.6519	0.7908	0.9572	0.9071	0.8304	0.8594	0.7880
$((n,2),(m,0))$	0.6317	0.5678	0.7728	0.7697	0.6010	0.5666	0.5580
$((m,0),(n,1))$	0.0017	0.0002	0.0072	0.0000	0.0000	0.0000	0.0000
$((m,0),(n,2))$	0.9971	1.0000	0.8906	1.0000	1.0000	1.0000	1.0000
$((m,0),(n,3))$	1.0000	0.9996	1.0000	1.0000	1.0000	0.9977	1.0000

Average payment for a trade meeting is calculated as $\sum_i i\lambda(i)$, where i is a possible transference for a trade meeting and $\lambda(i)$ represents the probability of transferring i units of money from consumer to producer.

Table 2.2 display the average payments of core-on allocation in those meetings where one of the traders is a bank. First, note that the average payment in meeting where a bank

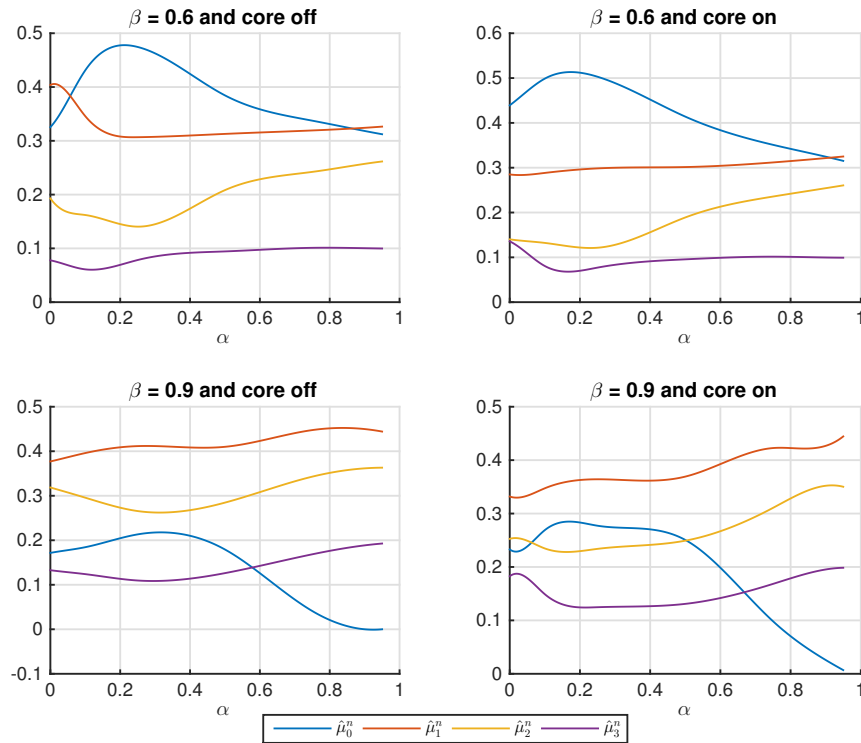


FIGURE 2.5: Distribution of money - monitoring

is a consumer are higher for $\beta = 0.6$ compared to the same meeting for $\beta = 0.9$. Also, for meetings where a bank is a producer the average payment is higher when people are more patient in general. This is due to the rationality constraints. When people are more impatient, a higher payment loosens the constraint of the producer and permit a higher output for trades.⁸ Thus, when the producer is a nonbank and is impatient, an implementable allocation features more money creation by banks compared to allocation for more patient people. These pattern of payments have implications for: (i) the implementable distribution of money (ii) the optimal monetary policy.

In figure 2.5 we plot the distribution of money relative to the fraction of nonbanks, i.e., we plot $\hat{\mu}_i^n = \mu_i^n / \sum_j \mu_j^n$. Note that for a low β the best implementable distribution of money features a relative high proportion of people holding no money. Even if the fraction of banks is close to one, we still find a distribution of money where the relative fraction of nonbanks without money is big. On the other hand, if β is high enough, the planner can implement a distribution where the fraction of nonbanks without money goes to zero as

⁸In the online appendix we present the full allocation where you can see that the output trade is lower when people are more impatient, even those allocation featuring higher average payments.

the fraction of banks goes to one. If the fraction of banks is high enough and people are patient, then the planner can implement a allocation where there is less expending between nonbanks (see online appendix) a distribution of money with few people holding no money and reduce the money creation by banks (and inflation, which increases the return of money in all meetings).

2.7 Final Remarks

We study the optimality of inflationary policies in a random-matching model of inside money. Using numerical simulations we extend the results of Deviatov and Wallace (2014) for upper bound of money holdings higher than one. Our findings indicates that for a board set of parameters inflation is optimal in this model due to higher benefits of money creation by banks than its effects on the return of money. However, when people are patient enough and the fraction of monitored people is high, then inflation is not part of the optima. In those extreme cases, the planner can implement an allocation where the fraction of agents holding no money goes to zero and then the benefits of inflation do not exceed its costs on intensive margin. Moreover, some transfers are made from banks to nonbanks in order to redistribute some benefits of inside-money creation.

Chapter 3

Computing benefits of 'open market operations' tied to inflationary policies in matching models

3.1 Introduction

Inflation¹ can be optimal in some models as the mechanism-design approach to monetary theory has shown.² In monetary economies, we can distinguish effects of inflation on the *extensive margin* and the *intensive margin*. The last one is a decrease in the value of money, i.e., in the presence of inflation a fix amount of money can purchase less goods and services over time, which prejudices consumption smoothing from those who save using money (compared to a zero-inflation economy). However, inflationary policies can be used to redistribute money in the economy via progressive transfers, allowing more people to consume - an extensive margin effect.

In chapter 1, we argue that inflationary policies also have an intensive margin effect with consequences to the extensive margin. If people can react to inflationary policies spending more money, the benefits of inflation can be diminished because the extra spending creates a worse distribution of money than in the case of no intervention. However, they show that inflationary policies are more attractive in economies where inside money circulate. It happens because the creation of money can be directed, i.e., once the money is created by some agents inside the economy, it is possible to transfer money for specific agents. This

¹This is a joint work with Ricardo Cavalcanti

²See Levine (1991), Kehoe, Levine, and Woodford (1992) and Deviatov (2006) for examples where inflationary policies due to lump-sum transfer can improve welfare.

generates a welfare gain due to better distribution of consumption even with people spending more money and a decreasing the return of money. Barros (2017) extends the work of Deviatov and Wallace (2014) and show that inflation is optimal in Cavalcanti and Wallace (1999) model of inside money. He run simulations for a broad range of parameters and finds that inflation is optimal except for extreme cases where people are patient and there are a big fraction of monitored people.

In this paper, we introduce bonds in a version of Shi (1995) and Trejos and Wright (1995) economy, where a fraction of people are monitored as in Cavalcanti and Wallace (1999). In our model, bonds are real assets - assets that do not lost value with inflation - that mature into money over time. Bonds, as well as money, are issued by private agents (who we call banks) and cannot be traded in competitive markets. This simple way to introduce bonds in the economy is inspired in Aiyagari, Wallace, and Wright (1996) model, where there is a group of government agents that issues securities (money with a different color) stochastically. They show that there exists a higher-welfare equilibrium with two assets, compared to the unique equilibrium with only money. Although the two kinds of money differ only in their color, one of them is more valuable in the better equilibrium. By valuing two kinds of money differently, it endogenously generates a richer set of asset holdings, and leads to a welfare improvement.

In our model, we allow for this structure, but we show that there are welfare gains due to other mechanism. Our simulations show that bonds are a better assets than money in terms of individual consumption smooth over time, then they are more valuable in equilibrium. Due to this, bond holders can have a higher consumption, but they are less willing to work in exchange of assets. Therefore, the creation of bonds is moderated, allowing only a small fraction of people to hold this asset in equilibrium. The benefits of bonds in our simulation relies on the fact that, as explained above, inflation is optimal because it allow a better distribution of consumption in the economy. Then, the inflationary policies tightened to creation of bonds, and not only inside money, can implement an allocation where there is a better distribution of consumption in the economy.

The literature explored some mechanism where the coexistence between money and other assets improves social welfare. Kocherlakota (2003) argues that the coexistence between illiquid bonds and money can improve welfare, while liquid bonds cannot. In his

model, the coexistence of money and bonds is achieved by assuming cash-in-advance constraint. People receive an idiosyncratic marginal utility shock in their first period of life and trade in a competitive markets. Also, people can adjust their portfolios after the shock realizes. Agents with higher marginal utility can borrow from agents with lower marginal utility by transferring purchasing power from the former to the latter using illiquid bonds. This implicit borrowing and lending enabled by illiquid bonds improves welfare. Zhu and Wallace (2007) show that there exists a trading mechanism under which trade outcomes are in pairwise core and both money and bonds are held by people. The trading mechanism shares the gains from trade depending on the proportion of money in consumers' portfolios. Specifically, it benefits those who held some amount of money and forgo the interest paid by bonds. Hu and Rocheteau (2013) build on Lagos and Wright (2005) model, where they extend the setting by adding physical capital. Physical capital works as an input for production in one subperiod and also is a mean of payment in another subperiod. Hence, physical capital can be accumulated too much to supplement money as a medium of exchange. Under the optimal trading mechanism, physical capital have higher rate of return than money to prevent the overaccumulation of capital. So, coexistence of money and assets with higher return (physical capital) is necessary to achieve welfare improvement.

The rest of the paper is organized as follows. Section 3.2 presents the model. Section 3.3 describes the concept of implementable allocations and welfare criteria. Section 3.4 presents our numerical approach to the problem we and the results of our simulations. Finally, section 3.5 presents conclusions. Appendix E contains an additional numerical exercise to show that welfare gains do not come entirely from an increase in heterogeneity of trades due to the introduction of bonds.

3.2 Environment

The environment is a version of a Shi-Trejos-Wright economy, which we borrow from Cavalcanti and Wallace (1999). Time is discrete and infinite and there is nonatomic measure of people each of whom maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. The period utility is given by $u(y) - x$, where y denotes consumption, x denotes production of an individual, u is strictly increasing, strictly concave, twice continuously differentiable, $u(0) = 0$ and $u'(0)$ is arbitrary large. Also, let $y^* = \operatorname{argmax}_y [u(y) - y]$ be positive. Production

is perishable, trades occur in pairwise meetings and matches are random. We can summarize the random matching process as follows: a person looks forward to being a consumer or a producer with probability θ and looks forward to no pairwise meeting with probability $1 - 2\theta$, where $\theta \leq 1/2$.³

As in Cavalcanti and Wallace (1999), people in the model are ex ante identical, but a fraction α become permanently monitored, whom we refer as banks, while the rest are permanently nonmonitored (nonbanks hereafter). Histories are common knowledge for banks and they are private for nonbanks. However, the monitoring status and consumer-producer status of people in a pairwise meeting are common knowledge. Also, only the planner can commit to future actions.

Each person and the planner have printing presses capable of turning out indivisible and durable objects. Those turned out by the printing press of any one person are distinguishable from those turned out by other peoples' printing presses. That allows us to prevent a bank who defects from issuing additional assets. Banks are able to print two different kinds of durable objects. The first one, which we call money, can disappear with probability π , as in Li (1994) and Li (1995), i.e., π is the inflation rate of the economy. The other kind of durable object (bonds) is turned into money with probability ξ , i.e., each unit of bond has a *maturity rate* of ξ . We can think each unit of bond as a unit of money with protection against inflation, but it is lost with probability ξ .⁴

This particular way to introduce bonds in random matching models is similar to Aiyagari and Wallace (1997) and Aiyagari, Wallace, and Wright (1996). In those papers, there are a type of agent, which they call government agent, that randomly issue securities and in other circumstances 'redeem' matured securities, i.e., they exchange matured securities by money. Here, instead of creating a new group of people, we allow banks to create both of types of assets and do not require the owner of a bond to meet a bank to redeem it. The maturity is stochastic and instantaneously convertible, once a bond matures, it is turned into money.

In addition, each person's holding of assets issued by others is restricted to be at most two units. We assume that the asset holdings are observable in a pairwise meeting. The

³The process of random matching described in the text implies the same results as the standard one: people are specialized in production and consumption in one of K types of goods and each person meets other person at random.

⁴Bonds are real assets that mature into nominal assets. The maturity of bonds presents a geometric distribution with parameter ξ , which results in a average maturity of $1/\xi$.

planner can make monetary transfers, which we also model as a stochastic policy: a bank receives a unit of money with probability τ^b and a nonbank person receives a unit of money with probability τ^n . Let $\tau = (\tau^n, \tau^b)$. The sequence of events in an arbitrary period is: first, there are pairwise random meeting between people, this is when trade can trade, then the meetings are dissolved and people are separated; second, after separation inflation occurs; third, the planner can make monetary lump-sum transfer for each group of people; fourth, bonds turn into money stochastically. Finally, the period ends.

3.3 Implementable steady state allocations

We compute the model's *steady state* allocations that are *symmetric* in the following senses: (i) all people in the same situation take the same action, and (ii) each unit of asset issued by the planner or by a bank who have not defect are treated as perfect substitutes. Also, we assume that all monies issued by nonbanks are worthless.⁵

Let $Z = \{b, n\}$ be the set monitoring status of people, where b (n) stands for banks (non-banks), and $A = \{(m, d) \in \mathbb{N}^2; m + d \leq 2\}$ be the set of asset holdings of people, where m (d) denotes the money (bonds) holdings. Notice that A has cardinality of 6. An individual's state is $s = (z, a) \in S = Z \times A$ and a (s, s') meeting is a meeting between a potential producer in state s and a potential consumer in state s' . For every meeting, let $y(s, s')$ represents the output traded. In addition, we allow people to run lotteries on assets transfers/exchange. Let $\lambda(s, s') = (\lambda_p(s, s'), \lambda_c(s, s'))$ be a function mapping meetings to asset lotteries for producer and consumer, respectively, $\lambda_p(s, s')$ and $\lambda_c(s, s')$, where $\lambda_k(s, s') = (\lambda^{a_1}, \dots, \lambda^{a_6})$ for $k \in \{p, c\}$ and λ^{a_i} represents the probability of ending the meeting in state $a_i \in A$ of asset holdings. By allowing people to run lotteries on money transfers, we attenuate the friction of money indivisibility. Additionally, let $\mu = (\mu_{a_1}^n, \dots, \mu_{a_6}^n, \mu_{a_1}^b, \dots, \mu_{a_6}^b)$ be a distribution of money, where $\mu_a^z = \mu(s)$ represents a fraction of people in state $s = (z, a)$.

In accord to Wallace (2010), in a similar economy except for money being the only asset in the economy, we can reduce states of banks: they start every period with no money and $y(s, s')$ is still implementable in all (s, s') meeting.⁶ Therefore, we assume that banks always

⁵We compute only steady-state allocations due to numerical tractability.

⁶The argument relies on the fact that banks can print money without any costs. In our environment we keep this assumption for money and bond. Therefore, Wallace's argument also apply for our economy.

start a period in state $s = (b, 0)$. Also, we set $\tau^b = 0$. Additionally, we sort A as following: $A = \{(0, 0), (1, 0), (2, 0), (0, 1), (1, 1), (0, 2)\}$. With this ordination, we can easily make reference to elements of A . For example, $a_2 = (1, 0)$.

An *allocation* is a set $(y, \lambda, \mu, \pi, \tau, \xi, V)$, where V is a column vector of value function for all $s \in S$. An element of V must satisfy

$$v(s) = \theta \sum_{s' \in S} \mu(s') [p(s, s') + c(s', s)] + (1 - 2\theta)h(s), \quad (3.1)$$

where, for a nonbank person,

$$p(s, s') = -y(s, s') + \beta \lambda_p(s, s') \Pi \Upsilon \Xi V^n \quad (3.2)$$

$$c(s', s) = u(y(s', s)) + \beta \lambda_c(s', s) \Pi \Upsilon \Xi V^n \quad (3.3)$$

$$h(s) = \beta e_a \Pi \Upsilon \Xi V^n \quad (3.4)$$

and, for a bank person,

$$p(s, s') = -y(s, s') + \beta v(b, 0) \quad (3.5)$$

$$c(s', s) = u(y(s', s)) + \beta v(b, 0) \quad (3.6)$$

$$h(s) = \beta v(b, 0). \quad (3.7)$$

$$(3.8)$$

Also,

$$\Pi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \pi & 1 - \pi & 0 & 0 & 0 & 0 \\ \pi^2 & 2\pi(1 - \pi) & (1 - \pi)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \pi & 1 - \pi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3.9)$$

$$\Upsilon = \begin{pmatrix} 1 - \tau & \tau & 0 & 0 & 0 & 0 \\ 0 & 1 - \tau & \tau & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - \tau & \tau & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3.10)$$

$$\Xi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \xi & 0 & 1 - \xi & 0 & 0 \\ 0 & 0 & \xi & 0 & 1 - \xi & 0 \\ 0 & 0 & \xi^2 & 0 & 2\xi(1 - \xi)^2 & (1 - \xi)^2 \end{pmatrix}, \quad (3.11)$$

$V^n = (v(n, a_1) \ v(n, a_2) \ \dots \ v(n, a_6))'$, $v(b, 0)$ is the value function of a bank and e_a is a canonical vector in \mathbb{R}^6 in the direction i for all a_i . Notice that,

$$V = \begin{pmatrix} V^n \\ v(b, 0) \end{pmatrix}. \quad (3.12)$$

The matrices Π , Υ and Ξ are squared matrices with dimension of 6 implied by the monetary police. The inflation matrix Π describes the stochastic process of money disintegration, Υ stands for the lump-sum transfers of money and Ξ describes the process of bonds maturation. Note that, $p(s, s')$, $c(s', s)$ and $h(s)$ are payoffs (including continuation values): in state s a person who looks forward to be a producer [consumer] and meets a consumer [producer] in state s' has a payoff $p(s, s')$ [$c(s', s)$]; $h(s)$ is a payoff for not being in a meeting and being in state s .

A steady-state allocation must satisfy a transition for the distribution of assets for non-banks such that

$$(\mu_{a_1}^n, \dots, \mu_{a_6}^n)T = (\mu_{a_1}^n, \dots, \mu_{a_6}^n), \quad (3.13)$$

where, T is a transition matrix implied by lotteries, monetary policy and the distribution of money. A element of T is defined as

$$t(i, j) = \theta \sum_{s' \in S} \mu(s') [\lambda_p((n, a_i), s') + \lambda_c(s', (n, a_i))] \Pi \Upsilon \Xi(j) + (1 - 2\theta) e_{a_i} \Pi \Upsilon \Xi(j) \quad (3.14)$$

where $\Xi(j)$ is the (j) -th column of Ξ . More precisely, $t(i, j)$ is the probability that a person in state (n, a_i) starts next period in state (n, a_j) .

We assume that individuals can deviate during trades from what is proposed for a particular meeting, taking as given value functions and the law of movement for aggregate variables. Therefore, for nonbanks, the individual rationality constraints for meetings are

$$p((n, a), s') \geq h(n, a), \quad (3.15)$$

$$c(s', (n, a)) \geq h(n, a) \quad (3.16)$$

and for banks

$$p((b, 0), s') \geq h(n, a_1), \quad (3.17)$$

$$c(s', (b, 0)) \geq h(n, a_1), \quad (3.18)$$

because punishment of a bank is loss of bank-status, the defection payoffs, the right-hand sides of equations (3.17) and (3.18), are always expected discounted utilities for nonbanks without any assets. Note that if a allocation satisfies the constraints (3.15) to (3.18), then all people in the model individually support the allocation. However, our model deals with anonymous people who trade in pairs and individuals can also consider group deviations in a meeting. Therefore, an *implementable* allocation not only have to be immune to individual defections, but it also have to be immune to pairwise defection.

We select allocations that belong to the core of meetings avoiding group deviations. Allocations that are immune to group deviations solve the following consumer's problem for a given (s, s') meeting

$$\begin{aligned} & \underset{\bar{\lambda}, \bar{y}}{\text{Max}} u(\bar{y}) + \beta \bar{\lambda}_c(s', s) \Pi \Upsilon \Xi V^n \\ & \text{s.t.} \quad -\bar{y} + \beta \bar{\lambda}_p(s, s') \Pi \Upsilon \Xi V^n \geq K(s, s'), \end{aligned} \quad (3.19)$$

where $\bar{\lambda}$ and \bar{y} represents a alternative lottery and output respectively and $K(s, s')$ is some positive value consistent with the producer payoff from a trade. Although it is not explicit, relevant rationality constraints also apply to this problem.⁷

The planner's problem is to choose an allocation to maximize a welfare function subject to relevant constraints - stationarity, rationality and core constraints. As welfare criteria we choose the average utility, corresponding to an inner product of μ and V , which amounts to

$$W = \mu \cdot V = \frac{\theta}{1 - \beta} \sum_{s \in S} \sum_{s' \in S} \mu(s) \mu(s') [u(y(s, s')) - y(s, s')]. \quad (3.20)$$

As chapter 2 shows, banks have no incentive to deviated from allocations that satisfy their rationality constraints, then we set core constraints only for meetings between nonbanks. For those meeting, we set necessary and sufficient inequality conditions. Let $\lambda(a_i)$ represents the probability of transferring all the assets of a nonbank in state $a_i \in A$, then a allocation is a solution of (3.19) if, an only if,

$$\lambda(a_i) \left(\text{Max}_j \{L_j^{s, s'}\} - L_i^{s, s'} \right) \leq 0 \text{ for all } i, \quad (3.21)$$

where $L_i^{s, s'}$ is the FOC of the Lagrangian associate with problem in equation (3.19) with respect to the probability of transferring all the assets of a nonbank in state a_i .

3.4 Simulations

In this section we briefly describe our numerical routine and present the results of the simulations. Besides of showing why there are benefits of bonds in the economy, we also describe how monitoring and maturity of bonds influence the optimal allocation.

We rely on numerical methods to unveil some properties of the solution of the Planner's problem described in the last section. In our simulations, we use KNITRO - a solver designed for the solution of large linear, nonlinear, and mixed integer optimization problems (Byrd, Nocedal, and Waltz, 2006). Our routine is base on an active-set algorithm, which is designed to search for local solutions. We overcome this problem of local solutions executing the algorithm for thousands of start points. Our routine keeps all local solutions found by the active-set algorithm and select the one that attains the highest welfare as the global

⁷The core set is formed after varying $K(s, s')$, as a contract curve.

optimum. Although, we can not prove that this routine will always attain a global solution, for several initial points the algorithm finds the same solution. We consider it as a evidence that the problem is well behaved, in the sense that a the problem is concave for a big part of the domain of the welfare function, thus a search for local solution is equivalent to a search for a global one.⁸

For all simulation below we set $u(y) = 1 - e^{10y}$ and $\theta = 1/3$. We use those preferences and matching parameter because they are used in other numerical studies with matching models of money. Thus, we can compare our results we those findings.

3.4.1 Benefits of bonds

In the first set of simulations, we fix $\alpha = 1/4$ and vary the discount factor. The later is a key parameter to determine output in pairwise meetings. It is easy to see that if people are very impatient, then producers' rationality constraints are more tight and y^* is not implementable. When that happens, planner's trade off of in implementing an inflationary policy is straitened. Although a inflationary policy allow the planner to improve the distribution of assets, it also tightens even more producers' rationality constraints. Then, when we vary the discount factor we can understand what happens with the optimal allocation when that trade off is stronger.

A robust feature of the optimal allocations computed is that the distribution of asset has a positive mass of people holding money or bonds as figure 3.1 shows. Moreover, for all simulation inflation is positive, which implies that money and bonds are, in fact, different assets. Although we present money and bonds as different assets in text, we do not impose to the planner that they should be strictly different or that they should be used in a steady-state equilibrium. The planner could make money and bonds be the same asset by setting inflation to zero, this would result in a economy where money is the only media of exchange.⁹ In addition, the planner could implement an allocation with only money and inflation by choosing a distribution of assets with the mass of nonbanks in states a_4 , a_5 and

⁸We select start points by solving a relaxed planner's problem - by relaxed problem we refer to the planner's problem presented in section 3.3 without the core inequalities constraints - and then we make a perturbation on the solution by adding a Gaussian noise. Finally, we use this perturbed solution as a start point to the original problem. This method improves the success of our routine in finding feasible solutions to the optimization problem. We check the robustness of this kind of selection of start points by solving the model using completely random start points for a smaller set of parameter then present here. The global maximum we find with this alternative procedure is the same, but the time spent in computation is much bigger than the time spent with 'perturbation method.'

⁹In this case, we also could call the asset in the economy as bond, it would be a matter of nomenclature.

a_6 equal to zero and banks issuing only money. In 3.1a we report the optimal distribution of assets holdings of nonbanks relative to the mass of nonbank, i.e., we show $\mu_{a_i}^n/(1-\alpha)$ for all $a_i \in A$.

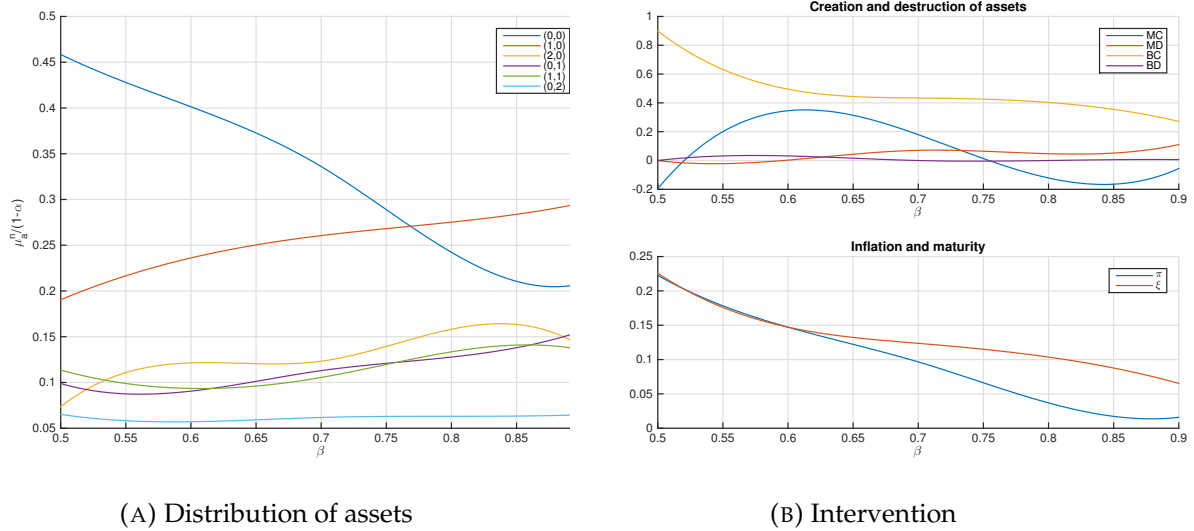


FIGURE 3.1: Optimal allocation varying the discount factor

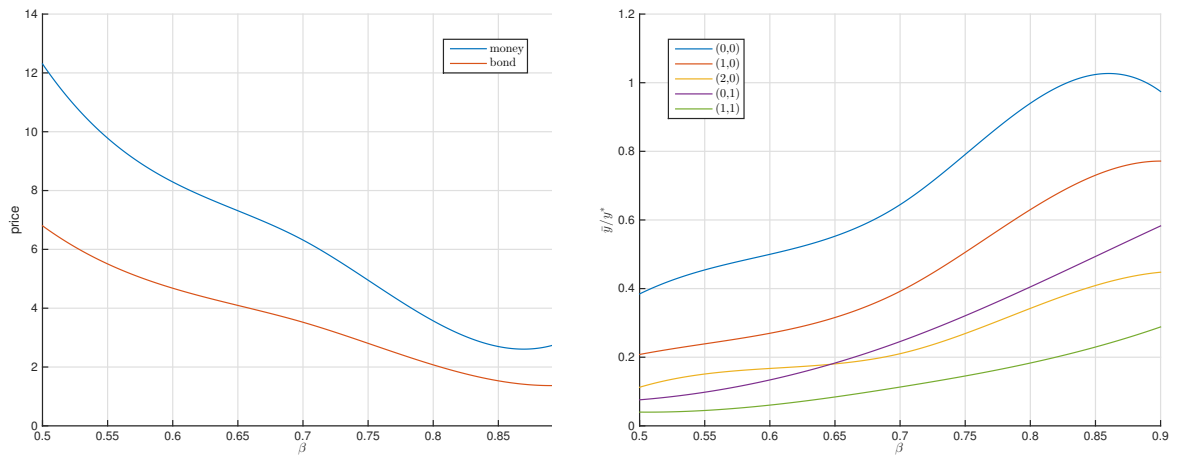
The benefits of bonds can be separated in three categories. First, bonds can partially mitigate negative effects of inflation on the return of savings, at least on those who have some bonds. As we can see in figure 3.1b positive inflation is optimal in all simulations. As in Deviatov and Wallace (2014) examples, inflation is optimal because the creation of assets by banks is greater than the destruction of assets by banks, therefore inflation keeps the amount of constant and allow for a higher output in meetings.¹⁰ We compute aggregates variables to show that the net creation of assets by banks is positive: Money Creation (MC) and Bond Creation (BC) are, respectively, the average amount of money and bonds that banks use as payment when they are consumers in meeting with nonbanks;¹¹ Money Destruction (MD) and Bond Destruction (BD) are, respectively, the average amount of money and bonds that banks receive when they are producers in meeting with nonbanks. In figure 3.1b we can notice that there is a positive net creation of asset.

Inflation also has some effects on optimal behavior of agents. It makes consumers willing to spend more money, which has a negative effect on the distribution of money. In addition, inflation reduces the value of money relative to bonds. Therefore, when paid with

¹⁰We assume that banks always start a period holding no assets, then we use the term ‘destruction of assets’ to refer to the assets that are discarded by banks after producing for nonbanks.

¹¹These aggregates can be negative due to asset exchange. For example, MC can be negative if banks exchange money for bonds more frequently than they use money for paying nonbanks.

bonds producers give a higher output relative to trades where they are paid with money. In figure 3.2a we plot the price of goods in term of bonds and money. We select the price (asset/output) for two meetings where the producer (a nonbank) has no asset: one meeting where the consumer has a unit of money and the other where the consumer holds one bond. Although it is an extreme case, the same price relation applies to all other meetings (see online appendix).¹² However, bond holders have a tighter rationality constraint when producers, because they are ‘richer’ than money holders. As a consequence, the output of nonbanks holding bonds is lower than the output produced by money holders. This pattern is presented in figure 3.2, where we show the average output of nonbanks relative to y^* - the expected output is computed as $y^{*-1} \sum_{s'} \mu(s')y(s, s')$.



(A) Prices

(B) Average output

FIGURE 3.2: Optimal allocation: selected output and price

Moreover, the fact that bond holders produce less relative to money holders also explains the shape of the distributions of assets implemented by the planner. Although bonds allow people - banks and nonbanks - to consume more, nonbanks are less willing to produce when holding bonds. Thus, the planner has to balance people’s production and consumption changing the distribution of assets. As a consequence, the optimal allocation presents a higher proportion of people holding money (or no asset) than people holding bonds. It happens because the planner is worried only with the average utility and this distribution of assets creates a good set of producers and consumers.

¹²In the simulations we select to plot the price, producers’ rationality constraints are binding except for the case where $\beta = 0.9$. Thus, the prices are not distorted to give surplus to consumers.

A second benefit of bonds is that they allow the planner to tax more the banks. Bonds buy more output relative to money, therefore banks consume more printing bonds and not money, this increases their value function. Then, the planner can ask them to produce more to nonbanks, even in cases where there is no payment, as in pairwise meetings where nonbanks consumers have no asset, but they receive a positive output in the optimal allocation. Third, when there are more assets in the economy, the planner can take advantage of the high marginal utility in zero and induce a positive mass of people with bonds in order to create artificially a higher welfare. In the appendix E, we present a numerical exercise that shows that these effects exist, but it does not account for all welfare gains of introducing bond in our inside-money economy.

Finally, we can not identify a mechanism similar to Zhu and Wallace (2007) in our simulations. The surplus of trades do not go to people with money. In general, all of the trade surplus stays with consumers independently of asset holding (see only appendix). Moreover, for some of our simulations consumer with money do not keep all of the trade surplus in some meeting with producers in specific states, but bond holders, meeting with producers in the same states, keep all the trade surplus in those meetings.

3.4.2 The role of maturity

In all simulations above, we notice that the maturity rate of bond is interior (see figure 3.1b), i.e., ξ is neither zero nor one, which would generate perpetual or 1-period bonds respectively. For a better understanding of this fact, we run the following experiment. We fix $\beta = 0.6$ and solve the planner's problem varying exogenously the maturity rate of bonds.¹³ Our results show that the welfare function presents a unique peak with respect to the maturity rate of bonds, as we can see in figure 3.3. From our previous numerical examples, the optimal maturity rate, ξ^* , is 0.1473 for a discount factor of 0.6.

Why optimal maturity rate is interior? On one hand, a higher maturity rates implies that bonds turn into money more easily - they are assets more similar to money. Then, the higher 'purchase power' they presented in relation to money would be reduced. Consider the extreme case where the maturity rate is equal to one, then bonds only last for the period that banks issue them and are not used as a media of exchange in meetings. It partially eliminate the first benefit we present previously, because only banks benefit from the higher

¹³We keep $\alpha = 1/4$ in the simulations where we vary exogenously the maturity rate.

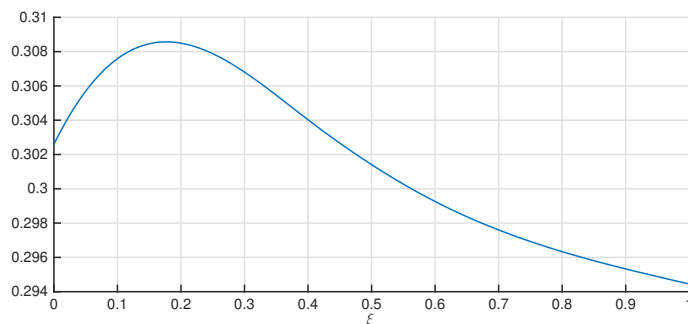


FIGURE 3.3: Optimal welfare

value of bonds. On the other hand, if maturity rate is very low, bonds do not mature into money easily. Although in that case bonds can offer a better protection against inflation and would allow people to consume even more, bond holders have a tighter producer's rationality constraint (creating bad producers) and the distribution of assets would become less sensitive to monetary policy. If maturity rate is low, bonds circulate in the economy for more time on average. Then, the planner can not create use inflation the change the distribution of money in the case presented above. Therefore some benefits of asset creation by banks are reduced. For example, consider the case where $\xi = 0$, in this case a bonds circulates in the economy until a nonbank use it to pay a bank. Then, the planner implements an allocation where banks do not print bonds frequently, which generate a distribution of assets with less people starting a period holding bonds then in an allocation that the planner can choose the maturity rate.

Figure 3.4 displays some features of the optimal allocation for a exogenous set of maturity rate of bonds. In figure 3.4a we show the distribution of assets. Notice that the distribution of assets has no people starting a period holding bonds when the $\xi = 1$, but we still have 1-period bonds in the economy, as we can see in figure 3.4b that banks issue bonds for all maturity rate. There is a peak in states where people have bonds and a vale in states where people have money (and also no asset) in the point where maturity rate is equal to ξ^* , i.e., when the planner can choose the maturity rate to balance the benefits and costs of bonds, there are more bonds in the economy.

The inflation rate also present a peak if the maturity rate is equal to ξ^* (see figure 3.4b). An allocation where there are more bonds, can have a higher change in the distribution of assets by inflation, because part of the negative effects of the inflationary police is mitigated by bonds.

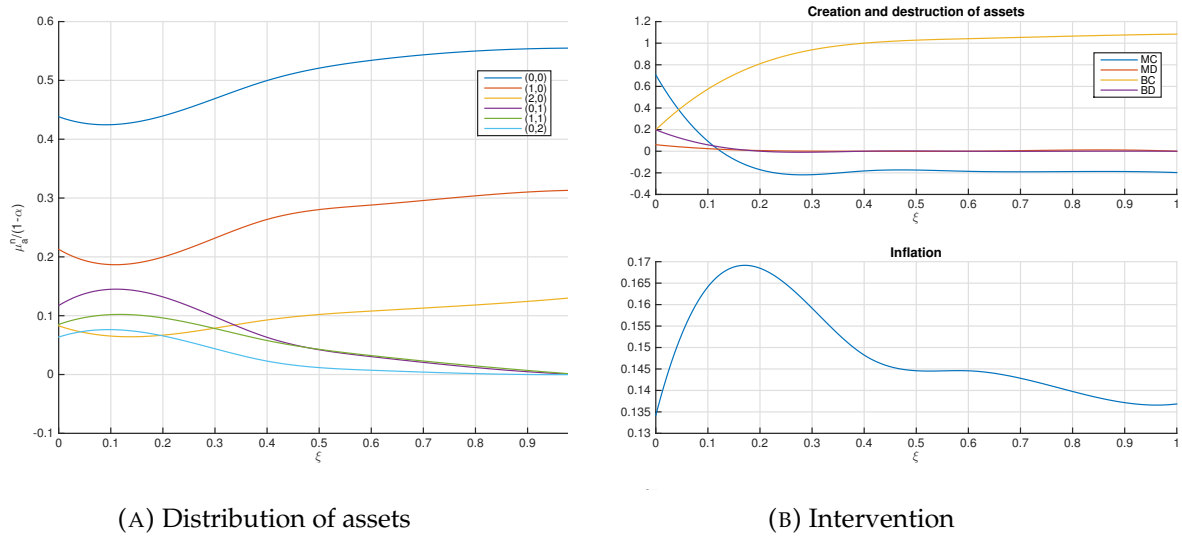


FIGURE 3.4: Optimal allocation varying bond's maturity

3.4.3 The role of monitoring

Until this point, we keep the fraction of banks constant in our simulation. Now we study how monitoring affects the optimal allocation in our model. We compute numerical examples where we vary the fraction of banks. We set $\beta = 0.6$ and solve the planner's problem for $\alpha \in \{0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95\}$. We select some features of the optimal allocation and show them in figure 3.5.

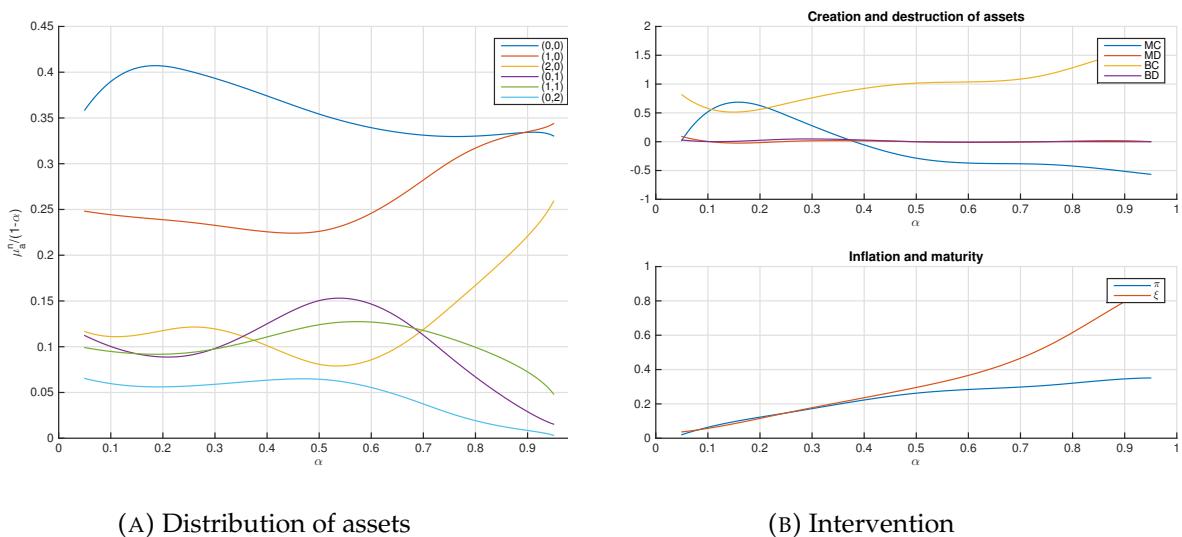


FIGURE 3.5: Optimal allocation varying monitoring

Figure 3.5a displays the distribution of assets for different fractions of banks. We notice that for all fraction of banks the optimal distribution of assets features some people holding bonds and some people holding money. Therefore, the results in the previous simulations,

where we set $\alpha = 1/4$, are robust to changes in the fraction of banks in the economy. However, the fraction of nonbanks holding bonds goes to zero as the fraction of banks goes to one. Although the creation of bonds by banks is increasing in the fraction of banks, inflation and maturity rates also are increasing in the fraction of banks, as figure 3.5b shows. In other words, when the fraction of banks increases there is an increasing in creation of assets (specially bonds) that is compensated with higher inflation and maturity of bonds. Although high π and ξ can prejudice nonbanks, when there are more banks in the economy, nonbanks are more likely to receive output as a gift (notice that MD and BD are almost zero in figure 3.5b), which compensate the negative effect of higher inflation and maturity rate of bonds. The less people holding bonds, the higher the output that banks receive when consumers (issuing bonds). This increases banks value function and allow the 'gifts' for nonbanks. In summary, when there are more banks in the economy, the planner implements an allocation where there is more taxation of banks and also more intervention: assets creation and inflation.

3.5 Conclusion

In this paper, we study the benefits of introducing bonds in Cavalcanti and Wallace (1999) model of inside money with inflationary policies as Deviatov and Wallace (2014). In our model, bonds are assets immune to inflation that mature into money over time. This asset is issued by banks - a group of people which all actions are recorded -, who also issue inside money. This simple method to introduce bonds is inspired in Aiyagari, Wallace, and Wright (1996) model, where there is a different kind of agent who issues securities stochastically. In our simulations we find benefits in terms of welfare gains, when the planner implements allocations where both bond and money circulate in the economy. In addition, we study how bonds maturity and monitoring influences optimal allocations.

In the model we build on, inflationary policies can improve welfare because the gain of redistribution in consumption is higher than the costs on self insurance and the distribution of money. Giving the average utility as welfare criteria, the best implementable allocation presents a good dispersion of consumption among the agents, i.e., the planner worries not only with a high production, but also how it is distributed in the economy. As we argue in chapter 1, inflationary policies besides reducing the average output in meetings, an

intensive-margin effect, they also make people more willing to spend money. Also called as *hot-potato effect* of inflation, a higher spending due to inflation increases the dispersion of money (and consumption) in the economy, an extensive margin effect. However, asset creation by banks can be used as a progressive policy, which compensate negative effects of inflation on intensive and extensive margins, and improves social welfare.

We show that a higher welfare can be implemented by allowing banks to issue both money and bonds. In our model, bonds are a better assets than money in terms of individual consumption smooth over time, then they are more valuable in equilibrium, i.e., people are more willing to incur costs of production in exchange for bonds compared to money. Thus, when banks transfer bonds to nonbanks the economy can have a higher consumption, at least of a fraction of the people holding bonds. However, bonds also create a 'income effect' on production. Bond holders are less willing to work in exchange of assets, because they have a good asset to smooth consumption. Therefore, the creation of bonds has to be moderated, allowing only a small fraction of people to hold this kind of asset in equilibrium. The benefits of bonds in our simulation relies on the fact that, as explained above, inflation is optimal because it allow a better distribution of consumption in the economy. Then, the inflationary policies tightened to creation of bonds, and not only money, can implement an allocation where there is a better distribution of consumption in the economy.

Although our results rely on extreme assumptions as: (i) limits to asset holdings, (ii) indivisibility of assets and (iii) lack of asset markets, we think that the mechanism in our simulations that shows a benefit of the coexistence of money and real financial assets is not a extreme one. Relaxing hypotheses (i) and (ii) should no change the results, once the standard problem of central planner would be present in a economy where assets are continuous and there is no upper bound for asset holding. Besides that, (iii) is only a extreme assumption about financial frictions. As long as there some friction in the asset market that exclude some agents from it, the mechanism presented here must induce a higher welfare.

Appendix A

Proof of the propositions

Proposition 1 For m in the support of distributions of meetings, output is $\hat{y}(m) = m_3$ when intermediation is relaxed, and $\tilde{y}(m) = \min\{m_2, m_3\}$ when savings need not be incentive compatible. In these relaxed problems, moreover, welfare satisfies $w(\hat{s}, \hat{y}) \geq w(\tilde{s}, \tilde{y}) \geq w(s^*, y^*)$, with inequalities replaced by equalities when there is a single type of trader.

Proof. That optimal welfare $w(s^*, y^*)$ is bounded above by $w(\tilde{s}, \tilde{y})$ is trivial. Consider now optimal allocations with pairwise trades described by Cavalcanti and Puzzello, 2010. They show that consumers should spend all their holdings and keep all surplus from trade. Such allocations are implementable in our setting when $x(m) \leq m_2$ is relaxed and intermediaries can make loans matching holdings of money by consumers, without profits. Since profits do not affect aggregate welfare (1.1), we conclude that such allocations solve the relaxed problem with $\hat{y}(m) = m_3$. Cavalcanti and Puzzello, 2010 also show that in their economy, since holdings of money by producers do not matter for output, then individuals do not care about the distribution of money when making saving choices: if they become producers their surplus is zero, and if they become consumers they do not care whether they meet with rich or poor producers. This means that savings are decided on the basis of realizations of idiosyncratic shocks and on output obtained when consumers, without consideration of how others are choosing money holdings. As a result, the distribution of money can be computed residually, after an incentive-compatible savings function associated to \hat{y} is found. This proves the inequality $w(\tilde{s}, \tilde{y}) \leq w(\hat{s}, \hat{y})$. More generally, Cavalcanti and Puzzello, 2010 show that the pairwise optimum solves a relaxed problem with no wedge between private and social savings. Although allocation $(\tilde{s}, \tilde{x}, \tilde{y}, \tilde{z})$ is obtained by ignoring private incentives for saving, these ignored constraints do not bind in the associated pairwise problem. In

addition, if there is a single type then in all meetings consumers and intermediaries have exactly the same money holdings. In this case, the cash-in-advance requirement $x(m) \leq m_2$ is irrelevant and $w(s^*, y^*) = w(\tilde{s}, \tilde{y}) = w(\hat{s}, \hat{y})$ must hold. Finally, $\tilde{y}(m) = \min\{m_2, m_3\}$ follows from an application of the upper-bound construction of Cavalcanti and Puzzello, 2010: if saving incentives can be ignored, the arrangement that maximizes $w(s, \cdot)$ for a given savings function has all trade surplus going to consumers, and has consumers spending all their holdings up to the bound dictated by intermediation. \square

Proposition 2 If there is more than one type of trader then welfare is increasing in the profit rate r in a neighborhood of zero, so that it is not optimal to give all surpluses to consumers.

Proof. For sufficiently small r , incentive-compatible savings s_i for type i satisfies

$$\theta_i - \beta = \alpha\beta[u'(s_i) - 1]\frac{i}{n} + \alpha\beta r \frac{i-1}{n}.$$

The term $u'(s_i) - 1$ reflects the utility gain of consumption from bringing an extra unit of money when the consumer receives all the surplus from trade, net of the expected opportunity cost of carrying money, which has been assumed equal to the unity. The expression $\theta_i - \beta = \alpha g(s_i)$, for $g(k) = \beta(u'(k) - 1)$, was obtained by Cavalcanti and Puzzello, 2010. With intermediation, $\alpha g(s_i)$ must be adjusted by the probability that the consumer is paired with an intermediary carrying at least the same quantity of money, $\frac{i}{n}$ (otherwise there is no marginal effect of an extra unit saved). The term $\alpha r \frac{i-1}{n}$ represents the marginal, expected payment of profits from richer consumers. Now the derivative of the welfare function with respect to r , w' , satisfies

$$w' = \sum_{i=1}^n \left[-\frac{\theta_i - \beta}{n} s'_i + \frac{\alpha}{n^2} \left(\sum_{j \geq i} g(s_j) s'_j + \sum_{j < i} g(s_i) s'_i \right) \right]$$

where s'_i denotes the derivative of s_i with respect to r . Using now that for $r = 0$

$$g(s_j) = \frac{(\theta_j - \beta)n}{\alpha j}$$

then

$$w'|_{r=0} = \sum_{i=1}^n \left[-\frac{\theta_i - \beta}{n} s'_i + \frac{\alpha}{n^2} \left(g(s_i) s'_i + \sum_{j>i} g(s_j) s'_j + \sum_{j<i} g_i(s_i) s'_i \right) \right] = \frac{1}{n} \sum_{i=1}^n \left(\sum_{j>i} \frac{\theta_j - \beta}{j} s'_j + \sum_{j<i} \frac{\theta_i - \beta}{i} s'_i \right).$$

Now, since u is concave then s'_i is positive if $i > 1$ and r is sufficiently small. Therefore, under the assumption that $n > 1$, w' is positive for such r . \square

Appendix B

Test case

In this appendix we present some facts about economies without intermediation, like those studied by Deviatov, 2006. Reproducing his numerical experiments is useful as a test case (see more on our numerical approach in the second part of the appendix), and for showing that inflationary interventions can appear just to repair effects of lack of commitment in corner situations. In order to do that, we need to change the timing of monetary policy in the model presented above. In Deviatov, 2006 money transfers occur first, followed by the inflationary process that keeps the quantity of money constant. We should remark, by the way, that we tried other configurations to make sure the relationship between positive inflation and corner outcomes is robust to timing specifications (there are small changes in allocations overall).

The first table below is produced with Deviatov's specification (and the same utility function as in the study of intermediation). It turns out that in his economy the consumer never spends more than one unit of money. Hence we can just report λ_{ij} , defined as the probability that one unit is transferred in meeting (i, j) — henceforth a meeting in which the producer has i and the consumer has j units of money — in addition to reporting output relative to first-best output y^* .

In Table B.1 we can notice two effects taking place as the discount factor falls: money spent in meeting $(1, 1)$ increases and, consequently, holdings are scattered as the set of people holding one unit loses mass (μ_1 falls). This is relevant because people holding one unit can be both producers and consumers. When discount factors β are very low, expansionary policies are needed, but these are also corner cases in which velocity effects are absent. To see that interventions would not be necessary if spending is sufficiently controlled we turn off the core requirement preventing group defections, as in Table B.2.

TABLE B.1: Pairwise meetings and core on

β	.95	.83	.66	.55	.50	.33
y_{01}	1.0000*	1.0000*	1.0000*	1.0000*	0.4353*	0.2109*
y_{02}	3.7614	2.4951	1.7038*	1.0486*	0.4353*	0.2109*
y_{11}	0.2296*	0.2356*	0.2124*	0.1593*	0.1204*	0.0531*
y_{12}	1.0000*	1.0000*	0.4928*	0.2415*	0.1204*	0.0531*
λ_{01}	0.0915 [†]	0.2565 [†]	0.5872 [†]	0.9538 [†]	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0682 [†]	0.1921 [†]	0.4306 [†]	0.6607 [†]	1 [†]	1 [†]
λ_{12}	0.2972 [†]	0.8155 [†]	1	1	1	1
μ_0	0.1932	0.2600	0.3232	0.3679	0.3686	0.3810
μ_1	0.6553	0.5314	0.4302	0.3769	0.3970	0.4016
v_0	1.0169	0.0783	0.0000	0.0000	0.0217	0.0088
v_1	2.5514	0.7040	0.3417	0.2524	0.1890	0.1348
v_1	3.0240	0.9008	0.4405	0.3105	0.2174	0.1476
Inflation	0	0	0	0	0.1763	0.2018
Transfers	0	0	0	0	0.2498	0.2795

* Producer's incentive constraint is binding.

[†] The core constraint is binding.

When the core is off, the planner manages to implement a better distribution of money. In this case, the average payment in meeting (1, 1) is low even when a low β tightens up the producer constraint and reduces output.

TABLE B.2: Pairwise meetings and core off

β	.95	.83	.66	.55	.50	.33
y_{01}	0.9634	0.9132	0.8751	0.8265	0.7449*	0.3328*
y_{02}	3.6649*	2.3717*	1.3552*	0.9237*	0.7449*	0.3328*
y_{11}	0.1279*	0.0792*	0.0426*	0.0254*	0.0194*	0.0089*
y_{12}	1.0000*	1.0000*	0.7501*	0.4577*	0.3567*	0.1623*
λ_{01}	1	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0282	0.0458	0.0563	0.0557	0.0551	0.0545
λ_{12}	0.2212	0.5778	1	1	1	1
μ_0	0.1299	0.1619	0.1738	0.1774	0.1800	0.1882
μ_1	0.7484	0.6997	0.6774	0.6782	0.6789	0.6785
v_0	1.9821	0.2728	0.0435	0.0067	0.0000	0.0000
v_1	2.4065	0.6532	0.3154	0.2290	0.1992	0.1336
v_1	3.0414	0.9310	0.4658	0.3392	0.2946	0.1988
Inflation	0	0	0	0	0	0
Transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

Appendix C

Auxiliary objects and numerical approach

We describe below in more detail objects used in our simulations. The probability distribution of after-trade holdings, λ , is in fact a 3×3 matrix, $\lambda(m) = (\lambda_1(m); \lambda_2(m); \lambda_3(m))$. In particular, $\lambda_i(m) = (\lambda_i^0(m), \lambda_i^1(m), \lambda_i^2(m))$ is a line vector for $i = 1, 2, 3$, where $\lambda_i^j(m)$ denotes the (marginal) probability that ‘person i ’ (the person starting with m_i) leaves the meeting holding $j \in \{0, 1, 2\}$ units of money. For example, $\lambda_1^j(m)$ denotes the probability that the producer leaves the meeting holding j units of money.

The state space can be written as $\{n, b\} \times \{0, 1, 2\} = \{(n, 0), (n, 1) \dots, (b, 1), (b, 2)\}$. The value function can be written in vector notation as $v = (v_0^n, v_1^n, v_2^n, v_0^b, v_1^b, v_2^b)'$. For this configuration of states, monetary policy implies two transition matrices. The inflation matrix P is

$$P = \begin{bmatrix} \Pi & \mathbf{0}_3 \\ \mathbf{0}_3 & \Pi \end{bmatrix}, \quad (\text{C.1})$$

where

$$\Pi = \begin{pmatrix} 1 & 0 & 0 \\ \pi & 1 - \pi & 0 \\ \pi^2 & 2\pi(1 - \pi) & (1 - \pi)^2 \end{pmatrix} \quad (\text{C.2})$$

and $\mathbf{0}_3$ is a 3×3 matrix of zeros. Money transfers imply the following matrix

$$T = \begin{bmatrix} \Psi^n & \mathbf{0}_3 \\ \mathbf{0}_3 & \Psi^b \end{bmatrix} \quad (\text{C.3})$$

where

$$\Psi^k = \begin{pmatrix} 1 - \tau^k & \tau^k & 0 \\ 0 & 1 - \tau^k & \tau^k \\ 0 & 0 & 1 \end{pmatrix} \quad \text{for } k \in \{b, n\}. \quad (\text{C.4})$$

For occupation shocks, let

$$\Lambda = \begin{pmatrix} \frac{1+\rho}{2} & \frac{1-\rho}{2} \\ 1-\rho & \rho \end{pmatrix}. \quad (\text{C.5})$$

The transition matrix generated by occupation shocks can be written as $S = \Lambda \otimes \mathbf{I}_3$, where \mathbf{I}_3 is the 3×3 identity matrix and \otimes represents the Kronecker product.

Now, let $\mathcal{I}^n = [\mathbf{I}_3, \mathbf{0}_3]$ and $\mathcal{I}^b = [\mathbf{0}_3, \mathbf{I}_3]$, then we can write the matrices A^n and A^b described in the text as:

$$A^n = \mathcal{I}^n P T S \quad (\text{C.6})$$

$$A^b = \mathcal{I}^b P T S. \quad (\text{C.7})$$

Finally, let \mathbf{e}_k be a canonical vector in direction k of \mathbb{R}^3 , $\mathbf{f}_i^n = [\mathbf{e}_{i+1}, (0, 0, 0)]$ and $\mathbf{f}_i^b = [(0, 0, 0), \mathbf{e}_{i+1}]$. Then,

$$A_{0i}^n = \mathbf{f}_i^n P T S \quad (\text{C.8})$$

$$A_{0i}^b = \mathbf{f}_i^b P T S. \quad (\text{C.9})$$

If we denote by $\sigma^m(i, j)$ the joint probability that after-meeting holdings of the producer-consumer pair is precisely (i, j) then

$$\lambda_1^i(m) = \sum_j \sigma^m(i, j) \quad (\text{C.10})$$

$$\lambda_3^j(m) = \sum_i \sigma^m(i, j) \quad (\text{C.11})$$

$$\lambda_2^k(m) = \sum_{(i,j):i+j+k=\bar{m}} \sigma^m(i, j) \quad (\text{C.12})$$

where $\bar{m} = \sum_\ell m_\ell$.

Now we rewrite participation constraints as

$$\Pi_1(m) = -y(m) + \beta \sum_i \lambda_1^i(m) (\mathbf{f}_i^n - \mathbf{f}_{m_1}^n) PTSv \geq 0 \quad (\text{C.13})$$

$$\Pi_2(m) = \beta \sum_k \lambda_2^k(m) (\mathbf{f}_k^b - \mathbf{f}_{m_2}^b) PTSv \geq 0 \quad (\text{C.14})$$

$$\Pi_3(m) = u(y(m)) + \beta \sum_j \lambda_3^j(m) (\mathbf{f}_j^n - \mathbf{f}_{m_3}^n) PTSv \geq 0 \quad (\text{C.15})$$

and, using σ ,

$$\Pi_1(m) = -y(m) + \beta \sum_i \sum_j \sigma^m(i, j) (\mathbf{f}_j^n - \mathbf{f}_{m_1}^n) PTSv \geq 0 \quad (\text{C.16})$$

$$\Pi_2(m) = \beta \sum_k \sum_{(i, j): i+j+k=\bar{m}} \sigma^m(i, j) (\mathbf{f}_k^b - \mathbf{f}_{m_2}^b) PTSv \geq 0 \quad (\text{C.17})$$

$$\Pi_3(m) = u(y(m)) + \beta \sum_j \sum_i \sigma^m(i, j) (\mathbf{f}_j^n - \mathbf{f}_{m_3}^n) PTSv \geq 0. \quad (\text{C.18})$$

As a result, the problem defining the core in meeting m is

$$\text{Max}_{y(m), \sigma^m} \Pi_3(m)$$

$$\text{s.t. } \Pi_1(m) \geq \gamma_1(m) \text{ and } \Pi_2(m) \geq \gamma_2(m).$$

for some (meeting-specific) $\gamma_1(m)$ and $\gamma_2(m)$ consistent with participation constraints.

Let $\zeta_1(m)$ and $\zeta_2(m)$ be Lagrange multipliers associated to the restrictions in the problem above. It is easy to see that $\zeta_1(m) = u'(y(m))$. Also, let $L_{ij}(m)$ denote the derivative of the Lagrangian with respect to $\sigma^m(i, j)$. As a consequence of the linearity of Π 's in σ 's, the solution must satisfy

$$\sigma^m(i, j) \left(\text{Max}_{i', j'} L_{i'j'}(m) - L_{ij}(m) \right) = 0, \quad \forall i, j. \quad (\text{C.19})$$

The numerical problem is to find allocations maximizing (1.4) subject to constraints dictated by rationality, stationarity, core and feasibility, given value-function definitions and bounds on money holdings. In particular, there are bounds necessary to guarantee that measures of people across states add up to one, and transition probabilities defined by lotteries

also add up to one, so that in the outside-money case money is not created nor destroyed in meetings.

Our approach is to guess and verify that value functions are increasing and concave (that is, $0 \leq v_0^k < v_1^k < v_2^k$ and $v_2^k - v_1^k < v_1^k - v_0^k$ for $k = n, b$). We also restrict lotteries associated to people with intermediation occupations, due to incentive constraints, and transform (C.19) into inequality constraints. This way the numerical problem fits into conventional non-linear maximization routines. We then resort to the *KNITRO* solver. Issues related to local optima are handled by considering of many alternative initial conditions.

We now resort to an example of how (C.19) is handled, and how some lotteries can be eliminated in the outside-money case. Let us fix $\tilde{m} = (0, 2, 1)$. Since the consumer never ends with two units of money, we put $\sigma^{\tilde{m}}(i, 2) = 0$ for $i \in \{0, 1, 2\}$. Also, we can impose $\sigma^{\tilde{m}}(0, 0) = 0$, since money cannot be destroyed. In addition, the intermediary would not entertain an allocation with less than two units after trade, so that $\sigma^{\tilde{m}}(2, 0) = \sigma^{\tilde{m}}(1, 1) = \sigma^{\tilde{m}}(2, 1) = 0$. It remains to be determined just two transition probabilities for this meeting, that is, choices of $\sigma^{\tilde{m}}(1, 0)$ and $\sigma^{\tilde{m}}(0, 1)$. Hence we can write

$$\begin{aligned}
L_{01}(\tilde{m}) &= (\mathbf{f}_1^n - \mathbf{f}_1^n)PTSp + (\mathbf{f}_1^n - \mathbf{f}_1^n)PTSp = 0. \\
L_{10}(\tilde{m}) &= (\mathbf{f}_0^n - \mathbf{f}_1^n)PTSp + \zeta_1(\tilde{m})(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSp \\
&= (\mathbf{f}_0^n - \mathbf{f}_1^n)PTSp + u'(y(\tilde{m}))(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSp. \tag{C.20}
\end{aligned}$$

Given that $\sigma^{\tilde{m}}(1, 0) + \sigma^{\tilde{m}}(0, 1) = 1$, the core constraint for meeting \tilde{m} becomes

$$\begin{aligned}
\left(u'(y(\tilde{m})) - \frac{(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSp}{(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSp} \right) \sigma^{\tilde{m}}(1, 0) &\geq 0 \\
\Leftrightarrow (u'(y(\tilde{m})) - 1) \sigma^{\tilde{m}}(1, 0) &\geq 0. \tag{C.21}
\end{aligned}$$

Appendix D

A replication of Deviatov and Wallace (2014)

In this appendix, we replicate the numerical results of Deviatov and Wallace (2014). The only purpose of the replication is to increase the reliability of our algorithm. In the table D.1 we report the results of our replication. Output is relative to y^* .

TABLE D.1: Deviatov and Wallace (2014) replication

α	1/4	1/2	3/4
$y((n, 0), (n, 1))$	0.6054	0.6627	0.7389
λ	1.0000	1.0000	1.0000
$y((n, 0), (b, 0))$	0.6054	0.6627	0.7389
λ	1.0000	1.0000	1.0000
$y((b, 0), (n, 0))$	0.2941	0.1403	0.1058
λ	0.0000	0.0000	0.0000
$y((b, 0), (n, 0))$	0.7159	0.8177	0.9104
λ	1.0000	1.0000	1.0000
$y((b, 0), (b, 0))$	0.7159	0.8177	0.9104
λ	-	-	-
μ_0^n	0.5060	0.3142	0.1505
μ_1^n	0.2440	0.1858	0.0995
π	0.0821	0.1034	0.1136
$v(n, 0)$	0.1000	0.1122	0.1318
$v(n, 1)$	0.3574	0.4007	0.4571
$v(b)$	0.3794	0.4314	0.4871
Welfare	0.2326	0.3254	0.4306

λ represents the probability of transferring a unit of money from consumer to producer in the trade meeting described in the output immediately above.

Appendix E

Heterogeneity and welfare gains

In this appendix, we present a simple exercise showing that the gains from introducing bonds in the economy do not come only from the increasing in the heterogeneity - number of different trades. We fix $\xi = 1$ - this means the bonds have the smallest maturity in the model - and run simulations for three different economies: (1) money is the only asset in the economy, (2) Bonds and money can be used in the economy as assets and (3) the last one is a special case of the economy (2) where money and bonds can be used as assets but the planner is constrained to implement allocations having only the meetings of the economy (1).¹ This simple exercise shows that although some of the benefits of introducing bonds in the economy come from the increasing heterogeneity, but part of the benefits occurs for other mechanism. In the text, we advocate that one of benefits of bonds introducing bonds in this economy is a better distribution of consumption. Finally, for the simulations in this appendix, we fix $\alpha = 1/4$, $\theta = 1/3$ and $u(y) = 1 - e^{-10y}$.

Our numerical exercise becomes simpler when we fix the maturity rate of bond equal to 1. If bonds always turn into money in the end of a period, in the beginning of each period there is no people holding bonds, then there are less objects to compute compared to the simulations presented in the main text. Also, the gains of introducing more heterogeneity are potentially bigger, because of the people only start trade with money - less heterogeneity than in a economy with bonds. This simplification in the numerical problem allows us to run simulations for larger upper bounds of assets holdings. In table E.1 we present the implementable welfare for economies where the upper bound of assets holding vary from 1 to 3. The extension of the model presented in sections 3.2 and 3.3 is trivial for a higher

¹In the examples there is one additional meeting in economies of type 2: people in the upper bound of the distribution of money can produce to bank and exchange money for bonds. Therefore, in the economies of type 3, although people can exchange money for bonds, nonbanks in the upper bound of the distribution of money are not allowed to do so.

TABLE E.1: Comparing Welfare

UB = 1					
β	0.50	0.60	0.70	0.80	0.90
Money	0.1576	0.2424	0.3633	0.5581	1.1162
Money and bonds	0.1640	0.2520	0.3789	0.5918	1.2062
M & B - restricted	0.1587	0.2434	0.3637	0.5581	1.1162
UB = 2					
Money	0.1857	0.2895	0.4423	0.7135	1.4940
Money and bonds	0.1906	0.2944	0.4493	0.7243	1.5191
M & B - restricted	0.1893	0.2924	0.4448	0.7145	1.4983
UB = 3					
Money	0.1944	0.2999	0.4668	0.7851	1.6745
Money and bonds	0.1978	0.3049	0.4716	0.7889	1.6871
M & B - restricted	0.1971	0.3043	0.4705	0.7861	1.6764

Money: simulations where money is the only asset in the economy. Money and bonds: simulations where banks can print money and bonds. M & B - restricted: simulations where banks can print money and bonds, but the planner cannot create additional meetings with respect to a economy where money is the only asset available.

upper bound of asset holdings. In this particular set of examples we do not include core constraints.

The results in table E.1 show that there are gains of welfare for introducing bonds in the economy. The welfare is higher in economies where bonds and money circulate than in economies where money is the only asset available. We notice that, in fact, part of the gains of welfare comes from the increase in the number of productive meetings - meeting where some output is traded -, but even in the economy of type 3, a higher welfare can be attained. For simulations with upper bound of assets holding equal to one, the benefits only occur if people are impatient enough. However, for upper bounds higher than one, the benefits of introducing bonds do not depend on the discount factor.

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