Economic growth and complementarity between stages of human capital*

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Economic growth and complementarity between stages of human capital*

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Abstract

We study the impact of the different stages of human capital accumulation on the evolution of labor productivity. We add early childhood education to a standard continuous time life cycle economy and assume complementarity between educational stages. Agents are homogenous and choose the intensity of preschool education, how long to stay in formal school, labor effort and consumption, and there are exogenous distortions to these four decisions. The model is calibrated to the U.S. from 1961 to 2008 and matches the data very well and closely reproduces the paths of schooling, hours worked, relative prices and GDP. We find that early childhood education can explain a very large part of the observed increase of years of schooling in the U.S. since 1961, and it was as important as formal education for the increase of labor productivity in the period. Furthermore, we show that small reallocations of public expenditures from formal education to early childhood education would have sizable impacts on income per capita and productivity.

Keywords: Educational investments. Economic Distortions. Economic Growth.

JEL: O11; O40; O57.

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1 Introduction

Several recent studies on educational investments in early childhood have shown that expenditures in this stage of life increase the cognitive and non-cognitive development of children and increase the investment return in later stages of their lives. For instance, Rolnick and Grunewald (2003) and Schweinhart (2004) show that the private return on investments in preschool is high as it increases the marginal productivity of individuals. However they show there is also a sizable external effect due to improvement in the socioeconomic conditions of these persons in the form of, for instance, less crime, less time in jail and a smaller use of social services.

Carneiro and Heckman (2003) assess the Perry Preschool Program\footnote{The Perry Preschool Program, conducted in the United States in the 1960s, treated for two years a group of poor 3-year-old children. This treatment included daily classes and weekly visits of the teacher to the student’s home. The program collected data from this treatment group and a control group and followed them until they were 40 years old.} and find that measured through age 27, the program returns $5.7 for every dollar spent. When returns are projected for the remainder of the lives of program participants, the return on the dollar rises to $8.7. Also analyzing the Perry Preschool Program, Heckman et al. (2010) document that the social return of the program is between 7% and 10%, considered high compared to other investments. The education, skills and competences acquired at this stage of life facilitate learning in the rest of the student’s life.

Other studies emphasize that the early years are crucial for the formation of brain connections that capture the different impulses of the environment in which the child is located, impacting their intellectual development, personality and social behavior (Young and Mundial, 1996; Myers and de San Jorge, 1999; Knudsen et al., 2006; Irwin et al., 2007). According to Heckman (2011), the economic and intellectual inequality among individuals begins in early childhood because different investments in this stage lead to inequality in cognitive and non-cognitive skills in adulthood.

This article studies the impact of the different stages of human capital accumulation on the evolution of labor productivity in the U.S. between 1961 and 2008. More specifically, we add early childhood education to a standard continuous time life-cycle economy and assume complementarity between the two types of educational stages, in accordance with the evidence from the empirical literature (Heckman and Cunha, 2007; Cunha et al., 2010). In the model, there are three sectors: the goods sector, the early childhood sector and the formal education sector. Agents are homogenous and choose the intensity of preschool education\footnote{In this paper, we use early childhood and preschool education as synonyms.}, how long to stay in formal school\footnote{In this paper, we use formal education, late education and years of schooling as synonyms.}, labor effort and consumption. Individuals do not work when they are in school. Retirement and the early childhood timespan are exogenous.

The model is calibrated to reproduce the American economy in 2008 and the trajectory of key variables since 1961. Some variables, such as distortions of the prices of early childhood education, formal education, labor and capital, are calibrated to match targets of U.S. data using the simulated method of moments. The model fits the data very well and closely reproduces the paths of schooling, hours worked, relative prices and GDP. We use the model to estimate endogenously early childhood education from 1961 to 2008, circumventing the lack of data, especially in the initial years.

Early childhood education plays a central role in our results. Although the estimated value of its weight in the human capital function was very small, the complementarity in human capital formation between both stages of education ends up amplifying the impact of early childhood education through its effect on formal schooling. That is, a rise of early childhood education positively affects the return to formal education, leading to an increase in schooling. In our simulations, we found
that the expansion of early childhood education in the 1960-2008 period could explain around 60% of the observed increase in schooling in the U.S. Moreover, it was as important as formal education in explaining GDP per capita and productivity.

We extend the model in a later section by introducing government expenditures in both stages of education. The model was recalibrated and the main result here is that a small reallocation of educational resources for early childhood education (an addition of 0.3% of public expenditures) would have increased income per capita by 0.36%. This finding reinforces our previous results and calls for a reassessment of educational policies in favor of more attention and expenditures in the early stages of human capital formation.

This article extends and improves the previous literature in several directions. First, we relate to the literature that studies human capital accumulation in a dynamic macroeconomic framework (Rangazas, 2000, 2002; Lee and Wolpin, 2010; Restuccia and Vandenbroucke, 2013a,b; Castro and Coen-Pirani, 2014; You, 2014). None of these articles has studied early childhood education. You (2014) uses the Ben-Porath (1967) model of human capital formation and find that one-fifth of U.S. labor quality growth between 1967 and 2000 was due to the rise in educational expenditure. The main factor in explaining schooling attainment is the skill price. In Lee and Wolpin (2010) and Castro and Coen-Pirani (2014), skill price is also an important force in explaining educational attainment. Therefore, these papers address distinct but complementary sets of mechanisms to explain schooling attainment.

The second body literature that relates to our study investigates the links between government incentives, preschool education and economic development. For instance, similarly to this paper, Abington and Blankenau (2013) and Blankenau and Youderian (2015) embody the results of Cunha et al. (2010) and Cunha et al. (2010) and Heckman and Cunha (2007) in their human capital accumulation function and study the reallocation of government resources between educational stages and how it could affect aggregate income. The first article is a theoretical study, and neither paper is concerned with the evolution of labor productivity across time, as we are.

Our paper is also related to the body of literature that studies cross-country income differences and human capital (Klenow and Bils, 2000; Erosa et al., 2010; Schoellman, 2012; Córdoba and Ripoll, 2013; Restuccia and Vandenbroucke, 2014; Schoellman, 2014; Manuelli and Seshadri, 2014). Although Manuelli and Seshadri (2014) and Córdoba and Ripoll (2013) include preschool education in their models, they do not consider the complementarity between early childhood and later education stages, as documented by Cunha et al. (2010) and Heckman and Cunha (2007). Schoellman (2014) documents, using data of refugees living in the U.S., that adult outcomes are independent of age at arrival to the U.S., up to age 6. He interprets this finding, using a simple model of human capital accumulation, as evidence that the differences across countries in early childhood education are not important in explaining development differences. Del Boca et al. (2014) use a standard life-cycle model to study the child quality investment trade-off: Parents work more to have more money to invest in their children, but in turn, less time is allocated to child development. They find that for early investments, parents input is more important than monetary investments. Although it is clear that parental input is also important, we ignore this dimension because in our model fertility is exogenous.

This paper proceeds as follows. In Section 2, we present the stylized facts. In Section 3, we present the model. In Sections 4 and 5, we describe the calibration strategy and report how well the model fits U.S. data, respectively. In Section 6, we discuss results regarding the effect of both

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4Castro and Coen-Pirani (2014) analyze the evolution of educational attainment using a model with an inter-cohort with heterogeneous learning ability, in which the variation in schooling is given by changes in skill prices, tuition, education quality and ability.
education stages in the evolution of human capital, GDP and U.S. development. In Section 7, we modify the model, introducing public expenditures in education and, in section 8 we analyze the robustness of our results. Section 9 concludes.

2 The Evolution of Early Childhood Education

As put by Vinovskis (1993), there were few efforts to provide preschool training during the first half of the twentieth century, and most of them did not last long or had limited impact. Moreover, during the 1950s and early 1960s, policymakers and the general public paid little attention to preschool education. Only in the sixties does early childhood development gets more attention from the American society, not only in the education dimension but also on health, special education, research, etc. (Shonkoff and Meisels, 1990). In the early sixties studies by Benjamin Bloom, among others, argued that early childhood education was key to improving later performance in the schools and this view became widely accepted among experts.

New social demands, evolution of scientific production and a larger public interest in early childhood, gave this topic a greater role in the debate of socio-economics inequalities. As stated by Shonkoff and Meisels (1990, p.15) “these included President Kennedy’s interest in mental retardation, the political impact of the civil rights movement, and President Johnson’s commitment to wage war on the sources and consequences of poverty”.

From the mid-sixties on several programs were created aiming at increasing early childhood attention and care. Among them, the Perry Pre School Project\(^5\) and the Head Start Program\(^6\), which were created in 1962 and 1965, respectively; the 94-142 law of 1975 that provided funds for states to care for children as young as 3 years old; and the 99-457 law of 1986 that reinforced incentives for states to serve 3 to 6-years-olds children, and established a discretionary program providing service to newborn children up to 3 years of age.

The improvements were fast. For instance, in 1965 only eighteen states had public kindergarten and in 1970 less than 80% of five-year-old children attended public kindergarten. In 2000 all states funded public kindergarten and 98% of five-year-old children attended it (Kamerman and Gatenio-Gabel, 2007). The Head Start Program, which started in 1965 as a summer program with 561,000 children enrolled was transformed in the next year in a nine-month program and in 2015 it served nearly one million children. Since its inception, Head Start has served more than 33 million children and funding per student in real value increased 6.46 times between 1965 and 2008 (Head Start Bureau, 2012).

Table 1 presents the evolution of the absolute number of students and gross enrolment rate\(^7\) (in the brackets) of pre-primary, primary and secondary education, from 1970-2011. The progress of enrolment in pre-primary was striking in the period, especially if one takes into account that the figures for 1960 were close to zero. Indeed, the number of children enrolled in pre-primary increased 2.09 times between 1971 and 2011. In absolute terms, total enrolment increased more in pre-primary than in primary and secondary education. Pre-primary education had 4707 thousand

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\(^5\)The Perry School Project, provided, from 1962 to 1967, high-quality preschool education to three-and four-year-old African-American children living in poverty and assessed to be at high risk of school failure.

\(^6\)The Head Start Program is a federally funded preschool program, largely half day, targeted on poor children and serving 3-4 year olds primarily. It provides comprehensive education, health, nutrition, social and other services.

\(^7\)The United Nations Educational, Scientific and Cultural Organization(UNESCO), defines Gross Enrolment Ratio as the total enrolment within a country in a specific level of education, regardless of age, expressed as a percentage of the population in the official age group corresponding to this level of education. Thus, it is possible to have a percentage that is higher than 100%.
of new enrolments in the period, while primary and secondary had just 2394 and 3691 thousand, respectively. The increase of gross enrollment rate is also impressive, it goes from 37% in 1970 to twice that much forty years later, reducing considerably the distance with respect to primary and secondary education. In summary, the evidence indicates, since the mid-sixties, fast and continuous expansion of pre-school education. We will show later that this has had a significant impact on formal schooling, human capital and labor productivity.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre - Primary</th>
<th>Primary</th>
<th>Secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>4300 (37.7%)</td>
<td>22037 (88.49%)</td>
<td>20593 (83.84%)</td>
</tr>
<tr>
<td>1981</td>
<td>5163 (52.54%)</td>
<td>20420 (98.7%)</td>
<td>21585 (89.53%)</td>
</tr>
<tr>
<td>1991</td>
<td>7300 (65.17%)</td>
<td>22429 (104.42%)</td>
<td>19270 (90.88%)</td>
</tr>
<tr>
<td>2001</td>
<td>7538.72 (63.22%)</td>
<td>25297.6 (103%)</td>
<td>23087.04 (93.95%)</td>
</tr>
<tr>
<td>2011</td>
<td>9007.2 (74.17%)</td>
<td>24431.66 (100%)</td>
<td>24214.3 (94.67%)</td>
</tr>
</tbody>
</table>

Source: UNESCO.
Note: Numbers in brackets are the gross school enrolment ratio

Table 1: Enrolment in pre-primary, primary and secondary education for different years.

### 3 The Model

We consider a simple model that can be used analytically to examine the impact of early childhood education, and its complementarity to formal education, on human capital formation and development. The key elements in this economy will be the intensity of childhood education and the optimal time to leave formal school and the dynamic consequences of these decisions on human capital and growth.

#### 3.1 Production

There are three sectors in this economy: Sector one produces consumption and capital goods, sector two supplies early childhood education, and sector three supplies formal education. The output in the goods sector, \( Y_1 \), is a function of physical capital service, \( K \), and skilled labor, \( H_1 \).

The technology of both educational sectors takes into account that the production of educational services is labor intensive. For instance, the share of gross capital formation in total government expenditure in education was only 9.89% in 2010, according to the OCDE database. In this sense, it is assumed that output in educational sectors uses only skilled labor as input. We therefore have

\[
Y_i = \begin{cases} 
A_i K^{\alpha} (H_i)^{1-\alpha}, & \text{if } i = 1 \\
A_i H_i, & \text{if } i \in \{2, 3\} 
\end{cases}
\]

(1)

where \( A_i \) is the sector’s total factor productivity, and \( \alpha \) is the capital’s share in output. Skilled labor is given by:

\[
H_i = L_i h_i
\]
where $L_i$ is raw labor, and $h_i$ is the human capital of workers.

Let good 1 be the numeraire and $q_1$ and $q_2$ be the relative prices of early childhood educational services and schooling services. Given free mobility of factors across sectors, we have:

$$\begin{align*}
(1 - \alpha)A_1 \kappa^\alpha &= q_{i-1}A_i = w, \ i \in \{2, 3\} \\
\alpha A_1 \kappa^{\alpha-1} &= R, \ \text{com} \ \kappa = \frac{K}{L_1 h_1}
\end{align*}$$

(2) \hspace{1cm} (3)

where $R$ is the rental price of capital, and $w$ is wage rate of raw labor.

After algebraic manipulations, the per capita output supply of each sector is given by:

$$y_i^s = \begin{cases} A_i \kappa^\alpha l_i h_i, & \text{if } i = 1 \\ A_i l_i h_i, & \text{if } i \in \{2, 3\} \end{cases}$$

(4)

where $l_i$ is the number of workers per capita, and $l_i$ is the sector share of workers to the total number of workers in the economy.

### 3.2 Household

Time is continuous, and individuals live for $T$ years. In the first part of the life cycle, $T_i$, individuals stay in preschool and then go to school for $T_s$ years. After leaving school, they join the labor market, and once they leave school, they cannot return. Retirement is mandatory, and active life ends after $T_w$ years working.

Heckman and Cunha (2007) document that human capital consists of investments in different stages of life and that early childhood education is complementary to formal education. They show that for a disadvantaged child, the return is higher on the former than on the latter. In this sense, we follow Heckman and Cunha (2007) and use a CES function for human capital formation, taking into account these two stages of life ($T_i$ and $T_s$). This formulation will be calibrated so that there is complementarity between the first stage ($T_i$), which is fixed at 6 years, and the second stage ($T_s$), which is an individual choice.

In the model, individuals begin the second stage of cognitive development with the human capital that they accumulated in the first stage. Total human capital will be a composite function of early childhood education and formal schooling:

$$h(T_s, x) = \theta \left( \lambda h_E(x)^\sigma + (1 - \lambda) T_s^\sigma \right)^{\frac{1}{\sigma}}$$

(5)

where $T_s$ is the time spent with primary, secondary and tertiary education and $h_E$ is early childhood human capital which is given by:

$$h_E(x) = \int_0^{T_i} x dt = xT_i$$

In the above equations, $\theta$ is a normalization parameter, $\lambda$ is the weight of early childhood education on the human capital function, and $\sigma$ is the parameter that characterizes the elasticity of substitution. It is easy to see that cross derivatives are positive so that the higher $x$, the longer individuals will stay in formal school.

At each instant of time, individuals choose consumption. During the education period, they decide how much to invest in early childhood education, $x$ (intensive margin), and schooling time,
$T_s$ (extensive margin). Then, in the next period the household decides how much work effort to supply. The preference of a household of cohort $s$ is given by

$$\int_s^{s+T} e^{-r(t-s)} c(t, s) dt + \beta \int_{s+T}^{s+T_y} e^{-r(t-s)} \ln(1 - l(t, s)) dt, \beta > 0$$  \hspace{1cm} (6)$$

where $T_y = T_i + T_s$, $c(t, s)$ is the consumption in time $t$ of cohort $s$, $l(t, s)$ is the labor offer in time $t$ of cohort $s$, $\rho$ and $\beta$ are, respectively, the discount rate and the leisure preferences in terms of consumption.

The expenditure side of the budget constraint consists of the consumption and payment of both school tuitions over time. The revenue is composed of wages from labor services, rents from capital, and lump-sum transfers. Therefore, the intertemporal budget constraint is given by

$$\int_s^{s+T} e^{-r(t-s)} c(t, s) dt + (1 + \tau_{Hc}) \int_s^{s+T_i} e^{-r(t-s)} xq_1 dt + (1 + \tau_H) \int_{s+T_i}^{s+T_i+T_s} e^{-r(t-s)} q_2 dt =$$

$$\int_s^{s+T} e^{-r(t-s)} c(t, s) dt + (1 - \tau_L) \int_{s+T_i+T_s}^{s+T_i+T_w} e^{-r(t-s)} w(s, T_s, x) l(t, s) dt$$  \hspace{1cm} (7)$$

where $w(s, T_s, x) = wh(T_s, x)$ is the wage rate of a worker of cohort $s$ with $x$ units of investment in early childhood education and $T_s$ years of schooling; $r$ is the interest rate; $\tau_{Hc}$ and $\tau_H$ are, respectively, the distortions or subsidy to formal and early childhood school tuition$^8$; $\tau_L$ is the distortion on wages; and $\chi$ is government transfers.

The optimal consumption level is obtained solving equation (6), subject to budget constraint (7). To simplify, we assume that the interest rate is equal to the discount rate, $r = \rho$. With this assumption, the growth rate of consumption and labor supply is zero, so that, the optimum paths of consumption and labor are constant throughout the life cycle$^9$. Furthermore, there are two decisions concerning education. Individuals choose the optimal educational investment in the first period of their life, $x$, and then the optimal time, $T_s$, to leave school. In the latter decision, individuals consider that the longer they stay in school, the shorter their productive life, $T_w$, as retirement, $T_r$, is exogenous. In this way, we have $T_i + T_s + T_w = T - T_R$ is independent of the schooling-years decision. In this case, the first-order conditions are:

$$\bullet [l]: l = 1 - \frac{c\beta}{(1 - \tau_L)wh(T_s, x)}$$  \hspace{1cm} (8a)$$

$$\bullet [c]: c = (1 - \tau_L)wh(T_s, x)le^{-r(T_i+T_s)} \frac{1 - e^{-rT_i}}{1 - e^{-rT_i}} + \chi - (1 + \tau_{Hc})xq_1 \frac{1 - e^{-rT_i}}{1 - e^{-rT_i}}$$

$$(1 + \tau_H)q_2e^{-rT_i} \frac{1 - e^{-rT_i}}{1 - e^{-rT_i}}$$

$$\bullet [T_s]: (1 - \tau_L)wh(T_s, x)l(\frac{1 - e^{-rT_i}}{r}) = (1 - \tau_L)wh(T_s, x)l + (1 + \tau_H)q_2$$  \hspace{1cm} (8c)$$

$^8$In section 3.5 we discuss the distortions.

$^9$The assumption that $r = \rho$ is common in literature. For instance, see Restuccia and Vandenbroucke (2013a).
Equations (8a), (8b), (8c) and (8d) are the first order conditions with respect to labor, consumption, formal education and early childhood education, respectively. Equations (8a) and (8b) are common in the literature. For instance, in the labor equation the human capital term reflects the opportunity cost of leisure and at the same time it positively affects consumption.

Given the early childhood investment, equation (8c) equates the marginal contribution of schooling to lifetime earnings, on the left-hand side, to its marginal cost, on the right-hand side. The latter is the opportunity cost of not working plus the tuition cost at the stopping time. The former is the impact on the present value of the time endowment of staying in school one additional unit of time, which implies higher wages in the future due to higher human capital. Equation (8d) has a distinct interpretation because the investment in early childhood is given in intensive terms, and there is no opportunity cost of time. Therefore, this equation equates the increase in earnings only to the increase in tuition costs.

### 3.3 Demography

At each instant, mass \( me^nt \) of homogeneous agents is born. As the total population in period \( t \) is the sum of all generations living in \( t \), total population is given by

\[
\int_{t-T}^{t} me^{na} da = m\left(1 - e^{-nT}\right)e^{nt}
\]

where \( t - T \) is the oldest generation, and \( t \) is the newest generation.

Assuming \( m = \frac{n}{1 - e^{-nT}} \), total population at \( t \) period is \( e^{nt} \), and the population growth rate is \( n \).

For a generation \( s \in [t - T, t] \), let \( N(s, t) \) be the share in the total population of the s-cohort in period \( t \). In this way,

\[ N(s, t) = \frac{me^{ns}}{e^{nt}} = me^{-n(t-s)} \]

At instant \( t \), an individual of the s-cohort is \( \varepsilon = t - s \geq 0 \) years old \(^{10}\). Therefore, let \( N(\varepsilon) = me^{-n(\varepsilon)} \) and define

\[
N_i = \int_{0}^{T_i} me^{-n\varepsilon} d\varepsilon = \frac{1 - e^{-nT_i}}{1 - e^{-nT}}
\]

\[
N_s = \int_{T_i}^{T_i+T_s} me^{-n\varepsilon} d\varepsilon = e^{-nT_i}\frac{1 - e^{-nT_s}}{1 - e^{-nT}}
\]

\[
N_w = \int_{T_s}^{T_s+T_w} me^{-n\varepsilon} d\varepsilon = e^{-nT_s}\frac{1 - e^{-nT_w}}{1 - e^{-nT}}
\]

\[
N_R = \int_{T_s+T_w}^{T_y+T_y+T_R} me^{-n\varepsilon} d\varepsilon = e^{-n(T_s+T_w)}\frac{1 - e^{-nT_R}}{1 - e^{-nT}}
\]

where \( N_i, N_s, N_w \) and \( N_R \) are, respectively, the population of children, formal students, workers, and retirees as a share of the total population\(^{11}\).

\(^{10}\)Note that \( \varepsilon \in [0, T] \).

\(^{11}\)Note that the sum \( N_i + N_s + N_w + N_R = 1 \).
3.4 Aggregate Consumption and Total Labor Supply

The total flow of labor services per capita in this environment is the sum of all individual’s labor supply for those in labor market. Therefore, using equations (8a) and (9c), we have

\[ l^s = \int_{s + T_y}^{s + T_y + T_w} N(\varepsilon)l(\varepsilon) d\varepsilon = e^{-nT_y} 1 - e^{-nT_w} \left( 1 - \frac{\beta c}{(1 - \tau_L)w(T_s,x)} \right) \]  

(10)

The aggregate consumption is the sum of all individual’s consumption at instant \( t \). Therefore,

\[ C = \int_0^{T_i} N(\varepsilon)c(\varepsilon)d\varepsilon + \int_{T_i}^{T_i + T_s} N(\varepsilon)c(\varepsilon)d\varepsilon + \int_{T_i}^{T_y + T_w} N(\varepsilon)c(\varepsilon)d\varepsilon + \int_{T_y + T_w + T_R} N(\varepsilon)c(\varepsilon)d\varepsilon \]  

(11)

However, with the assumption \( \rho = r \), we have \( c(\varepsilon) = c \) for all \( \varepsilon \in \{0, T\} \). Therefore, \( C \) is given by equation (8b).

3.5 Policy Distortions

In this economy, there are distortions or subsidies to the choice of human capital, labor supply and capital. These distortions are represented by tax on early childhood education, formal education, labor effort and the rental rate of capital. Net tax revenue is equally distributed to individuals through a lump-sum transfers, \( \chi \).

Although several factors may cause these distortions, we consider a generic formulation that encompasses many possible distortions. For example, some factors that are not modeled here can be thought of as distortions in the choice of human capital. Among them, we highlight credit constraints, which are an important factor in educational choices, mainly for poor families; information problems regarding the importance of early childhood education in the formation of children’s cognitive development; myopic behavior; and the lack of altruism (Restuccia and Urrutia, 2004; Heckman and Cunha, 2007; Córdoba and Ripoll, 2013; Attanasio, 2015). Thus, distortion here are generic factors causing the misallocation of resources.

Therefore, the per capita transfers is the sum of all revenue weighted by the share of each \( s \)-cohort in total population \( \epsilon^n \). Thus, per capita transfers are

\[ \tau_H x q_1 N_i + \tau_H q_2 N_s + \tau_L w(T_s,x) N_w + \tau_K R K = \chi \]  

(12)

where \( N_i, N_s \) and \( N_w \) are given by demography equations, and \( k \) is per capita capital of economy.

3.6 Equilibrium

In early childhood education, individuals consume \( x \) units of education, while in formal education, they consume 1 unit of education. Thus, with the demography structure in mind, we have

\[ y_d^e(q) = \int_0^{T_i} x(q)N(\varepsilon)d\varepsilon = x(q)N_i \]  

(13)

\[ y_d^f(q) = \int_{T_i}^{T_i + T_s(q)} N(\varepsilon)d\varepsilon = N_s(q) \]  

(14)

\[ y_d^f(q) = \int_{T_i}^{T_i + T_s(q)} N(\varepsilon)d\varepsilon = N_s(q) \]  

(15)
where $N_i$ and $N_s$ are given by equations (9a) and (9b), respectively.

The aggregate demand of sector 1 is given by the sum of capital and consumption aggregate demands:

$$y^d_1(q) = C(q) + (\delta + n)k$$  \hspace{1cm} (16)

where $C(q)$ is given by equation (11), and $k$ is the capital per capita.

The steady-state equilibrium of this economy is given by prices $\{q_1, q_2, r, w\}$, government transfers $\chi$ and allocations $\{c^*_t, l^*_t\}_{t=1}^T$, $\{T^*_s, x^*\}$ and $k^*$, such that

i) Given a list of distortions $\{\tau_H, \tau_{Hc}, \tau_L, \tau_K\}$, prices $\{q_1, q_2, r, w\}$ and transfers $\chi$, $\{c^*_t, l^*_t\}_{t=1}^T$ and $\{T^*_s, x^*\}$ are optimum allocations of the consumer problem.

ii) Given a list of prices $\{q_1, q_2, r, w\}$, $k^*$ is an optimum allocation of the problem of the firm.

iii) Net tax revenue is equally distributed to individuals through a lump-sum transfer, $\chi$.

iv) The equilibrium in the educational market is:

$$y^*_2 = xN_i$$  \hspace{1cm} (17)

$$y^*_3 = N_s$$  \hspace{1cm} (18)

v) The equilibrium in the goods market is:

$$y^*_f = C + (\delta + n)k$$  \hspace{1cm} (19)

vi) The equilibrium in the asset market is:

$$r = (1 - \tau_k)R - \delta = (1 - \tau_k)\alpha A_1 \kappa^{\alpha - 1} - \delta$$  \hspace{1cm} (20)

## 4 Calibration

Our calibration strategy involves choosing parameters so that the steady-state implications of the model are consistent with observations for the United States. We do the calibration for the following years\footnote{Data restrictions explain the choice of 1961 as the first year. The last year is 2008 to avoid the financial crisis and its consequences.}: \{1961, 1970, 1980, 1990, 2000, 2008\}.

There is a group of parameters that are constant over time and are observed in the data. We follow the standard procedure of employing data from national accounts, the U.S. Census and values commonly used in the literature. Table 2 presents these parameters.

The population growth rate, $n$, is the average U.S. population growth between 1900 and 2010, obtained from the \textit{U.S. Census}. The National Income and Product Accounts (NIPA) provides the
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Log(1.0125)</td>
<td>U.S. Census</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Log(1.066)</td>
<td>NIPA</td>
</tr>
<tr>
<td>$r$</td>
<td>Log(1.04)</td>
<td>Standard</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Log(1.04)</td>
<td>Assumption $\rho = r$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>NIPA</td>
</tr>
<tr>
<td>$T_i$</td>
<td>6</td>
<td>Standard</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.5</td>
<td>Heckman et al. (2010)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>

Table 2: Observed parameters

depreciation rate and the capital share in the goods sector, which are set to 6.6% and 40%, respectively. The interest rate is set at 4%, and the parameter $\rho$ has the same value. The first stage of the life cycle, $T_i$, is the pre-school period and is set at 6.

When studies on economic growth disregard early childhood education in the human capital accumulation, they implicitly assume that the two human capital stages are substitutes. However, according to Heckman and Cunha (2007), an optimal allocation of human capital resources would be a distribution of investment between these stages. Thus, there is some degree of complementarity in the equation (5), i.e., parameter $\sigma$ should be negative. Cunha et al. (2010) confirm the conclusion of Heckman and Cunha (2007). Indeed, with a nonlinear factor model with endogenous input, Cunha et al. (2010) estimate that the stages of human capital formation are complementary. Thus, the parameter that characterizes the complementarity of human capital function, $\sigma$, is set at -0.5, which is an intermediate value of Heckman et al. (2010) estimation.

Demography structure in this economy is given by life expectancy and retirement. Life expectancy at birth $T$ for 1961, 1970, 1980, 1990, 2000 and 2008 was set to 70.27, 71.11, 74.01, 75.37, 76.74 and 77.94, respectively, which are the values reported in the World Development Indicators database. Additionally, the age of retirement was set to 61, 65, 63, 62, 63 and 63.45 for 1961, 1970, 1980, 1990, 2000 and 2008, respectively, which are the values reported in the Current Population Survey, 1962-2012 of the U.S. Bureau of the Census for man. Thus, the retirement time is the gap between life expectancy at birth and the age of retirement.

The remaining parameters to be calibrated are:

$\{A_1, A_2, A_3, \tau_K, \tau_L, \tau_H, \tau_{Hc}, \lambda, \beta, \}$

For each year, we have different distortions and TFP parameters but the same parameters of technology, $\lambda$, and preference, $\beta$. Thus, in total, we have 44 parameters to calibrate. To reduce the number of parameters and simplify the calibration procedure, we normalize $A_2$ to 1 and assume that some parameters have a constant growth rate. Thus, we set

$\xi(s) = \xi^{2008}e^{\xi(s-2008)}$, for $\left\{ \xi \in \{A_1, A_3, \tau_L, \tau_H, \tau_{Hc}\} \right\}$


---

13 For the depreciation rate, we use the long-run average for the investment/capital ratio.
14 In section 8 we conduct sensitivity analyses with this parameter.
where the $g$ superscript is the growth rate, and $s$ is the year. With this simplification, we still need to calibrate 13 parameters:

$$
\theta = \{A_1^0, A_1^g, A_3^0, A_3^g, \tau_L^0, \tau_L^g, \tau_H^0, \tau_H^g, \tau_K, \lambda, \beta\}
$$

These parameters are calibrated so that in equilibrium, the model economy matches the following targets from the U.S. data:

i) GDP per capita in 2008 that is normalized to 1.

ii) Investment share of GDP in 2008 set to 0.2208 (Penn World Table).

iii) Share of early childhood sector workers in 2008 set to 0.0034 (OCDE statistics).

iv) Share of formal school sector workers in 2008 set to 0.0378 (OECD statistics).

v) Share of average worked hours in 2008 set to 0.3067 (OECD statistics).

vi) Average years of schooling in 2008 set to 13.2867 (Barro and Lee database).


xiii) Mean of GDP per capita growth rate between 1961 and 2008 set to 2.25% per year (World Development Indicators)

In this model, the output per capita of the economy is given by:

$$
y = y_1 + q_1 y_2 + q_2 y_3 \tag{21}
$$

Thus, we use $y$ to match GDP per capita. The investment share of GDP is equal to $(\delta + n)k$ and the share of workers in each sector\textsuperscript{15} is given by equation (4).

\textsuperscript{15}It is represented by $l_i$ for $i \in \{1, 2, 3\}$. 

As agents are homogeneous in our model, the average years of schooling is the optimum value of $T_s$ that is implicitly set in the schooling decision, given by equation (8c) and (8d).

The life-cycle average labor supply of an individual is given by the labor supply throughout the life-cycle divided by the time that the individual could allocate to work. Therefore,

$$
\bar{l} = \frac{1}{T - T_y} \int_{T_y}^{T_y + T_w} l(\varepsilon) d\varepsilon
$$

This statistic matches the average hours worked. For this, we assume that there is a total of 16 hours of discretionary time per day, which implies a total of 5840 (365*16) hours per year. Therefore, in 2008, we find that this statistic targeted a 1791/5840 ratio, where 1791 is the average hours worked in 2008.

Formally, the calibration strategy can be described as follows. Given a value for $\theta$, we compute an equilibrium for each year $s \in \{1961, 1970, 1980, 1990, 2000, 2008\}$ and define the following objects:

$$
J_1(\hat{\theta}) = \begin{bmatrix}
V(y^s(\hat{\theta})) - 0.0636 \\
E(y^s(\hat{\theta})) - 0.6763 \\
V(\bar{T}^s(\hat{\theta})) - 9.185E-05 \\
E(\bar{T}(\hat{\theta})) - 0.3171 \\
V(T_s^s(\hat{\theta})) - 2.4782 \\
E(T_s^s(\hat{\theta})) - 11.6987
\end{bmatrix}
$$

where $V(.)$ and $E(.)$ indicate the variance and mean for years $\{1961, 1970, 1980, 1990, 2000, 2008\}$. The second object that we define is

$$
J_2(\hat{\theta}) = \begin{bmatrix}
y^{2008}(\hat{\theta}) - y^{1961}(\hat{\theta}) - e^{0.0225(46)} \\
y^{2008}(\hat{\theta}) - 1 \\
\bar{l}^{2008}(\hat{\theta}) - 0.3067 \\
T_s^{2008}(\hat{\theta}) - 13.2867 \\
l_2^{2008}(\hat{\theta}) - 0.0034 \\
l_3^{2008}(\hat{\theta}) - 0.0378 \\
\bar{l}^{2008}(\hat{\theta}) - 0.2208
\end{bmatrix}
$$

where the superscript indicates the year in which the respective variable was taken.

The first component of this vector indicates that between 1961 and 2008, the GDP per capita grew at a rate of 2.25% per year. The other terms are GDP per capita, life-cycle labor supply, years of schooling, share of workers in secondary and tertiary sectors and investment/GDP ratio, in this order.

Then, to find $\theta$, we solve the following minimization problem using simulated method of moments with identity matrix as weighting matrix:

$$
\min_{\theta} J_1^T(\theta) J_1(\theta) + J_2^T(\theta) J_2(\theta)
$$

Table 3 summarizes the results of our endogenous calibration. The model matches the targets very well. For instance, GDP per capita in 2008 is the worst match, but in this case, the error is
only 2%. Furthermore, the objective function value of the minimization problem is 8.2E-04, and the average percentage error—the gap between the U.S. data and the model—is 0.48%. Thus, the model reproduces the U.S. data very well.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>U.S. data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference</td>
<td>β</td>
<td>Mean of average hours worked to sample</td>
<td>0.317</td>
<td>0.316</td>
</tr>
<tr>
<td>Weight of early childhood in Human Capital Technology</td>
<td>λ</td>
<td>Mean of schooling to sample</td>
<td>11.7</td>
<td>11.67</td>
</tr>
<tr>
<td>Initial Labor Tax</td>
<td>τ_L</td>
<td>Average hours worked in 2008</td>
<td>0.307</td>
<td>0.308</td>
</tr>
<tr>
<td>Capital Tax</td>
<td>τ_K</td>
<td>Investment share in 2008</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Initial Schooling Tax</td>
<td>τ_H</td>
<td>Years of schooling in 2008</td>
<td>13.29</td>
<td>13.33</td>
</tr>
<tr>
<td>Initial Early Childhood Education Tax</td>
<td>τ_Hc</td>
<td>Worker share in sector 2 in 2008</td>
<td>0.0034</td>
<td>0.0034</td>
</tr>
<tr>
<td>TFP Initial Goods sector</td>
<td>A_1</td>
<td>GDP per capita in 2008</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>TFP Initial Schooling sector</td>
<td>A_3</td>
<td>Worker share in sector 3 in 2008</td>
<td>0.0378</td>
<td>0.0378</td>
</tr>
<tr>
<td>Trend of TFP goods sector</td>
<td>τ_A</td>
<td>Variance GDP per capita to sample</td>
<td>0.0636</td>
<td>0.0629</td>
</tr>
<tr>
<td>Trend of TFP schooling sector</td>
<td>τ_A3</td>
<td>Mean GDP per capita growth rate between 1961 to 2008</td>
<td>0.0225</td>
<td>0.0228</td>
</tr>
<tr>
<td>Trend of labor tax</td>
<td>τ_L</td>
<td>Variance of average hours worked to sample</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Trend of schooling tax</td>
<td>τ_H</td>
<td>Variance of schooling to sample</td>
<td>2.478</td>
<td>2.477</td>
</tr>
<tr>
<td>Trend of early childhood education tax</td>
<td>τ_Hc</td>
<td>Mean GDP per capita to sample</td>
<td>0.676</td>
<td>0.678</td>
</tr>
</tbody>
</table>

Note: This sample is given by \{1961, 1970, 1980, 1990, 2000, 2008\}.

Table 3: Endogenous parameters

5 Results

Figure 1 shows the results for schooling, output and hours worked. In this simulation, we use the calibrated parameters to compute the equilibrium for each year between 1961 and 2008. The dynamics of the model is given by changing parameters \{A_1, A_3, τ_L, τ_H, τ_Hc\} and the demographic distribution, which is given by retirement age and life expectancy at birth.

![Figure 1](image1.png)

(a) Years of schooling  
(b) GDP per capita  
(c) Average Hours of Work

Figure 1: Model and U.S. data - 1961 to 2008
The blue and red lines are, respectively, the model’s results and U.S. data. The model results are very close to the U.S. data. The average model errors in absolute terms for school, GDP and hours worked are 0.76%, 1.92% and 1.43%, respectively. It is interesting to note that the fit for schooling and hours worked is surprisingly good because we use little information for the first years of these series. Although we do not use information to 1961 as a target, in the 1960s the average error in absolute terms was 0.23% and 1.96%, for school and hours worked, respectively. Furthermore, for 1961, the first year of our series, the average error for these series were 0.68% and 1.87%, in this order.

As robustness check of our calibration, we also calibrate and simulate the model for other two set of years: i) $\text{Set}_1 = \{1961, 1962, 1963, 2006, 2007, 2008\}$; and ii) $\text{Set}_2 = \{1961, 1965, 1970, 1980, 2000, 2005, 2008\}$. Table 8 in the appendix shows that the parameters are very close to the benchmark calibration, and a simulation shows that the model is still good to fit the U.S. data. Indeed, the average error in absolute terms for schooling, GDP and hours worked is, respectively, 0.78%, 1.99% and 1.42% for $\text{Set}_1$ and 0.89%, 1.83% and 1.42% for $\text{Set}_2$.

Figure 2 presents the estimation for early childhood education. This variable does not have a direct counterpart in the data, and was generated endogenously in the model. This variable shows a large increase between 1961 and 2008. Indeed, early childhood education in the model increased 6.46 times between 1961 and 2008, and its share of human capital increased 3.88 times. Although it is not directly comparable to the statistics in Table 1, it mimics the observed increase of the pre-school education in the period.

Figure 2: Early Childhood Education - Calibrated model

5.1 Relative Prices

In the model it is observed a rise in the relative prices of education which is supported by U.S. data. The main mechanism is the relative increase in the labor productivity of the goods sector. The estimated reduction of the TFP of sector 3 and the increase in sector 1 over time help explain the increase.\footnote{The variance of these errors are 6.14-E05, 2.66-E04 and 7.92-E05, respectively.  
17This is supported by results in Hanushek and Rivkin (1997). Indeed, Hanushek and Rivkin (1997, p.37) stat “The consideration of expenditure in education or other labor-intensive activities typically refers to the cost implications of differential technological change and productivity growth, a discussion that implicitly assumes efficient choices. The well-known theory, originally suggested by Scitovsky and Scitovsky (1959) and subsequently developed by Baumol and Bowen (1965) and Baumol (1967), concentrates on the cost disadvantages of a sector, of which education appears to be a prime example, that experiences little technological change while other sectors undergo regular productivity improvements. Because wages rise roughly in proportion with the average growth rate of labor productivity in all sectors, the technologically stagnant sector faces increased labor costs.”}
Figure 3: Model and Data prices - 1978 to 2008

Figure 3 shows the relative prices of the model and the price index for the U.S. economy. To relate the prices of our model (Figure 3a) with the data prices (Figure 3b) note that the variables "all items", "durables goods" and "housing" are associated with the goods sector, while the variables "services", "educational books and supplies", "tuition other school fees and childcare", "college tuition and fees and elementary" and "high school tuition and fees" are associated with the education sector. It is clear from Figure 3b that the prices associated with education sector increased faster than the prices associated with the goods sector, which supports the results of our model. Indeed, in the U.S. data, the index price of "all items", "durables goods" and "housing" increased 3.3, 1.62 and 3.46 times, respectively, while the index price of "services", "educational books and supplies", "tuition, other school fees and childcare", "college tuition and fees and elementary" and "high school tuition and fees" increased 4.2, 7.3, 8.73, 9.55 and 9.62 times, respectively. In our model, the relative prices of formal and early childhood education increased 93.07% and 47.9%, respectively.

We conduct an experiment of the model with TFP parameters to measure the importance of the relative prices in our quantitative results. Basically, we keep the values of the TFP parameters ($A_1$ and $A_3$) constant at their 2008 levels, changing the others.

When we keep constant the TFP parameters at their 2008 levels, the prices of education for the other years are higher than in the calibrated model. This increased price would have an expressive impact on educational choices. For all years, formal education and early childhood education would be lower in the experiment. In 1961, when we have the biggest gap between the experiment and the calibrated model, formal and early childhood education would be 60.48% and 55.18% below the results of the benchmark model, respectively.\textsuperscript{18}

In Restuccia and Vandenbroucke (2013a), the growth of wages, which is given exogenously, is the main factor explaining the evolution of years of schooling in the U.S. economy. In our model, however, the growth of wages is given endogenously, and TFP parameters are the exogenous force affecting the wages. What is important in our model for schooling choices is the growth of wages relative to tuition costs\textsuperscript{19}, $w/q_i$ for $i \in \{1, 2\}$. In this way, the impact of the TFP of the goods sector in schooling is insignificant because it affects only the level of wages and does not affect the ratio of wages to tuition costs.

\textsuperscript{18}In 1961 the TFP experiment increased the prices of early childhood education and formal education by 1.85 and 2.8 times, in this order, with respect to the benchmark model.

\textsuperscript{19}See equation (8c).
6 Economic Development and Education

The theory we propose is convenient to study the relative contribution of early childhood and formal schooling to the variation, in the recent past, of human capital and income per capita in the U.S.. The model includes, in addition to the two education decisions, saving and labor decisions and three sectors of production. In this general equilibrium framework, there are transmission mechanisms that are not in general present in the literature. For instance, early childhood decision does not affect directly the decision to join the labor market, so one could think that its impact on the income path along the life-cycle is not large. However, it indirectly affects the return to formal schooling - they are complement - and hence individuals' decision to stay longer in school and consequently its total effect on human capital and labor productivity can be quite high.

In the exercises presented in Figure 4 we simulate the model holding constant at its 1961 level one of the two educational stages. In this way, we can isolate and measure the effect of early childhood education or formal education on endogenous variables.

(a) Years of schooling
(b) Human Capital
(c) GDP per capita

Figure 4: Constant level at 1961 for education - 1961 to 2008

Panel 4a presents the calibrated path of the benchmark calibration and simulated path of formal schooling – years of education – when early childhood education is kept constant at its initial level (the blue and green lines, respectively). Note that the graph shifts down markedly in the counterfactual simulation, and formal education by the end of the period would increase just 17.7% had early childhood education remained constant, in contrast to the observed increase of 45.3%. Thus, according to our model 61% of the variation of schooling in the U.S. from 1961-2008 can be explained by the expansion of early childhood education alone.

From Figure 4b one can see that the impact of formal years of education and early childhood education on human capital were almost the same in the period: had any of them remained constant, human capital would be 31% lower than observed. This result is surprising given that the weight of early childhood education in the human capital function, $\lambda$, is very small – just 0.013 – and
the schooling weight is, consequently, 0.987. Although this implies that the direct return on early childhood investment is small, the complementarity in human capital formation of both types of educational stages end up causing the total effect of early childhood education on human capital to be very high. Increases in \( x \) and \( h_E \), as one can see from the human capital function, boost the gains of schooling and hence leads to more years of education, further increasing human capital.

Early childhood education is as important as formal education in explaining the evolution of GDP per capita in the period, as one can see from Figure 4c. Indeed, if in 2008 early childhood were at the same level as it was in 1961, income per capita would be 29% below its observed level, while it would decrease 28% if we had kept years of schooling at its 1961 level. This is not too surprising after one considers that the effect of both educational stages in human capital (Figure 4b), and hence on labor productivity, are similar in magnitude. The impact of early childhood on labor productivity is high due to its interaction with formal education, so that the expansion of early childhood in the period can explain a large part of the increase in GDP per capita and labor productivity.

We also made an experiment to evaluate the contribution of life expectancy. Although life expectancy affects schooling, its contribution is not too large. For instance, if in 2008 the life expectancy had been 70.27 years – the 1961 level – schooling would have been 13.09. Thus, life expectancy explained 6.07% of observed increase in the years of schooling to U.S. This result is consistent, such as, in Hazan (2009) and Restuccia and Vandenbroucke (2013b), who also find that life expectancy is not an important exogenous force to explain schooling evolution.

### 7 Introducing Public Expenditures on Education

One interpretation of the distortions/incentives for human capital accumulation is that they are a measure of, among other things, the impact of government policies. It would be interesting, hence, to incorporate a direct role of the government in affecting education. This is done in this section by adding public investment to the human capital functions (equation (5)).

We consider that investment in education now has private and public components. The human capital function of early education is now given by:

\[
h_1(x, g_1) = \left( (x T_i)^{\eta} + (g_1 T_i)^{\eta} \right)^{\frac{1}{\eta}}
\]

(22)

where \( g_1 \) is government investment in early education, and \( \eta \) is the parameter that characterizes the elasticity of substitution between private and public goods\(^{20}\). The human capital in the second stage is now given by:

\[
h(T_s, x) = \left( \lambda h_1(x, g_1)^{\sigma} + (1 - \lambda) h_2(T_s, g_2)^{\sigma} \right)^{\frac{1}{\sigma}}
\]

(23)

where \( h_2 \) is:

\[
h_2(T_s, g_2) = \left( T_s^{\eta} + (g_2 T_s)^{\eta} \right)^{\frac{1}{\eta}}
\]

\(^{20}\)This formulation of human capital is similar to Blankenau and Youderian (2015); thus, we set \( \eta = 0.48 \), which is the calibrated value from this article.
In the function above, \( g_2 \) is government investment in formal education. Note that as in the previous case, the human capital accumulated in the first stage is a component of the human capital function of the second stage. Equation (12), the distortions/budget constraint equation of the government, is modified to:

\[
\tau_L w h(T_s, x) N_w + \tau_K \kappa + (\tau_H x - g_1) q_1 N_I + (\tau_H - g_2) q_2 N_S = \chi
\]  

(25)

There are no data on government expenditure on pre-primary education before the nineties, so we estimate this series. We first calculate the average rate of the share of government expenditure on pre-primary to pre-primary through secondary education between 1998 and 2008, which is available in the UNESCO database. We then use pre-primary through secondary education expenditures as percentage of GDP, available in the US Government Spending database, for the years prior to 1998, and keep constant the estimated share of government expenditure on pre-primary education to estimate the government expenditure of pre-primary education between 1971 and 1997. Finally, we use student enrollment in both pre-primary and primary + secondary education, also available in the UNESCO database, to calculate the ratio of government expenditure per student to GDP per capita in each educational stage.

Figure 5 presents the ratio of government expenditure on primary through secondary to pre-primary between 1971 and 2008. The red and blue lines are, respectively, that ratio of gross and per student expenditures. As we can see, government expenditure on primary through secondary is approximately 10 times higher than government expenditure on pre-primary. However, when we look for government expenditure per student this ratio is less than 2. Indeed, the highest value that we observe of the per student series is 1.88 times, while the minimum value of the gross series is 9.30 times.

![Figure 5: Ratio of government expenditure on primary through secondary to pre-primary - Total and per student government expenditure](image)

We calibrate this model to fit the U.S. economy following the previous procedure discussed in the calibration section\(^{21}\). Table 6 summarizes the result. As before, the model matches the targets very well. GDP per capita in 2008 is the worst match, but in this case, the error is only 1.05%.

Note that the calibration is not too sensitive to these changes. Indeed, only the distortion to formal education and the weight of early childhood education in the human capital function (\( \lambda \)) changed significantly. This happened because the government expenditure per student on late education increased over time relative to early childhood education. Therefore, distortions in late

\(^{21}\)Due to data restrictions, we use only the sample \{1971, 1980, 1990, 2000, 2008\).
Table 4: Endogenous parameters - Government expenditure per student as percentage of GDP per capita

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>US</th>
<th>BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference</td>
<td>β</td>
<td>0.58 Mean of average hours worked to sample</td>
<td>0.3137</td>
<td>0.3115</td>
</tr>
<tr>
<td>Weight of early childhood in Human Capital Technology</td>
<td>λ</td>
<td>0.031 Mean of schooling to sample</td>
<td>12.25</td>
<td>12.31</td>
</tr>
<tr>
<td>Initial Labor Tax</td>
<td>τ_L</td>
<td>0.73 Average hours worked to 2008</td>
<td>0.3067</td>
<td>0.3087</td>
</tr>
<tr>
<td>Capital Tax</td>
<td>τ_C</td>
<td>0.44 Investment share to 2008</td>
<td>0.2208</td>
<td>0.2208</td>
</tr>
<tr>
<td>Initial Schooling Tax</td>
<td>τ_S</td>
<td>0.24 Years of schooling to 2008</td>
<td>13.29</td>
<td>13.21</td>
</tr>
<tr>
<td>Initial Early childhood education Tax</td>
<td>τ_H</td>
<td>1.34 Worker share in sector 2 to 2008</td>
<td>0.0034</td>
<td>0.0034</td>
</tr>
<tr>
<td>Initial Goods sector PTF</td>
<td>A_L</td>
<td>0.33 GDP per capita to 2008</td>
<td>1</td>
<td>1.105</td>
</tr>
<tr>
<td>Initial Schooling sector PTF</td>
<td>A_S</td>
<td>1.54 Worker share in sector 3 to 2008</td>
<td>0.0378</td>
<td>0.0378</td>
</tr>
<tr>
<td>Tendency goods sector PTF</td>
<td>A_f</td>
<td>0.0095 Variance GDP per capita to sample</td>
<td>0.0473</td>
<td>0.0472</td>
</tr>
<tr>
<td>Tendency schooling sector PTF</td>
<td>A_H</td>
<td>-0.016 Mean GDP per capita growth rate</td>
<td>2.0763</td>
<td>2.0738</td>
</tr>
<tr>
<td>Tendency labor tax</td>
<td>τ_L</td>
<td>-0.003 Variance of average hours worked</td>
<td>4e-05</td>
<td>4e-05</td>
</tr>
<tr>
<td>Tendency schooling tax</td>
<td>τ_H</td>
<td>-0.048 Variance of schooling to sample</td>
<td>0.9554</td>
<td>0.9557</td>
</tr>
<tr>
<td>Tendency early childhood education tax</td>
<td>τ_HC</td>
<td>-0.078 Mean GDP per capita to sample</td>
<td>0.7464</td>
<td>0.7403</td>
</tr>
</tbody>
</table>

Note: This sample is given by \{1971, 1980, 1990, 2000, 2008\}.

childhood education capture this fact and, thus, are lower than in the benchmark model. In the same way the parameter \(\lambda\) is higher because there is relatively more investment in formal education.

The largest share of total government expenditure on pre-primary education was observed in 2000, which was 9.7% of government expenditure in pre-primary, primary and secondary education. The lowest was 9.27% in 1971. To evaluate the impact of marginal increases of public expenditures on early education vis-à-vis formal schooling we simulated the economy, keeping constant total government expenditure on education but increasing by 10%, for all years, the share of total government expenditure on pre-primary education.

In this experiment the government would spend, on average, 10.36% more with pre-primary students and 0.99% less with late childhood students. Income per capita would be higher in all years, 0.36% greater on average. This result reinforces the importance of early childhood education and its effect on labor productivity that was found in previous exercises.

Following Blankenau and Youderian (2015), we change (and recalibrate) our model to use in the human capital function, as they do, government expenditure on education as a percentage of GDP instead of expenditure per student. Therefore, if in 2008 the government expenditure on pre-primary education had been 0.004 higher (+0.4% of GDP), income per capita would have been 8.62% higher, which is near what they found in their article. Moreover, if the government reallocated expending such that it were equal among pre-primary, primary and secondary education, income per capita would be 2.19% higher. This very strong result is due to their use of total public expenditure and disappears once one uses public expenditure per student.

8 Sensitivity

In this section, we analyze the sensitivity of the results with respect to the complementarity parameter \(\sigma\). Studies on human capital formation found that investments across stages of cognitive development are complementary, i.e., early investment increases the productivity of later investment (Cunha et al., 2006; Heckman and Cunha, 2007; Heckman et al., 2010). We simulate the
model with the following values: $\sigma \in \{-1, -0.75, -0.25, -0.01, 0.01\}$, and recalibrate the model for each different $\sigma$. The parameters are shown in Table 5.

<table>
<thead>
<tr>
<th>Complementarity</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\gamma^L_T$</th>
<th>$\gamma^H_T$</th>
<th>$\gamma^L_K$</th>
<th>$\gamma^H_K$</th>
<th>$A^L_0$</th>
<th>$A^H_0$</th>
<th>$A^L_1$</th>
<th>$A^H_1$</th>
<th>$\tau^L_T$</th>
<th>$\tau^H_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma=-1$</td>
<td>0.64</td>
<td>0.003</td>
<td>0.71</td>
<td>0.45</td>
<td>0.57</td>
<td>1.15</td>
<td>0.41</td>
<td>2.20</td>
<td>0.0087</td>
<td>-0.011</td>
<td>-0.00009</td>
<td>-0.029</td>
</tr>
<tr>
<td>$\sigma=-0.75$</td>
<td>0.65</td>
<td>0.006</td>
<td>0.70</td>
<td>0.45</td>
<td>0.61</td>
<td>1.25</td>
<td>0.42</td>
<td>2.25</td>
<td>0.0086</td>
<td>-0.011</td>
<td>-0.00009</td>
<td>-0.030</td>
</tr>
<tr>
<td>$\sigma=-0.5$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.42</td>
<td>2.30</td>
<td>0.0078</td>
<td>-0.009</td>
<td>-0.00012</td>
<td>-0.024</td>
</tr>
<tr>
<td>$\sigma=-0.25$</td>
<td>0.65</td>
<td>0.028</td>
<td>0.70</td>
<td>0.45</td>
<td>0.61</td>
<td>1.14</td>
<td>0.43</td>
<td>2.39</td>
<td>0.0068</td>
<td>-0.008</td>
<td>-0.00016</td>
<td>-0.020</td>
</tr>
<tr>
<td>$\sigma=-0.01$</td>
<td>0.67</td>
<td>0.06</td>
<td>0.69</td>
<td>0.45</td>
<td>0.67</td>
<td>1.21</td>
<td>0.45</td>
<td>2.55</td>
<td>0.0061</td>
<td>-0.012</td>
<td>-0.00012</td>
<td>-0.028</td>
</tr>
<tr>
<td>$\sigma=0.01$</td>
<td>0.66</td>
<td>0.062</td>
<td>0.70</td>
<td>0.45</td>
<td>0.66</td>
<td>1.10</td>
<td>0.45</td>
<td>2.55</td>
<td>0.0063</td>
<td>-0.012</td>
<td>-0.00011</td>
<td>-0.029</td>
</tr>
</tbody>
</table>

Table 5: Sensitivity - Calibrated parameters

There is a negative correlation between $\lambda$ – the weight of early childhood education in human capital function – and $\sigma$. Moreover, $\lambda$ fluctuates considerably with $\sigma$. This makes sense because investment in formal education is more attractive when the elasticity of the substitution of human capital increases (when $\sigma$ increases) because the individual would rather save during early childhood and invest their savings in later stages of education. However, remember that we have the same target for schooling. Thus, the weight of schooling, $1-\lambda$, decreases as it compensates for the increase in the return of schooling, i.e., the weight of early childhood education increases.

In Table 6 we re-evaluate the results of section 6 changing the value of complementarity parameter. We analyse the impact on formal education and income in 2008 when we keep the value of some type of education at 1961 level. When complementarity is high (low $\sigma$) early childhood education has a higher impact on formal education and income. For instance, when the complementarity parameter is equal to -1 and early childhood education is kept constant at 1961 level, formal education and income would be 36.42% and 44.6% lower than in the benchmark case, while for complementarity equal 0.01 it would be 4.99% and 13.25% lower. Although early childhood has lower impact on income than formal education when the complementarity parameter is higher than -0.5, the impact on income is still impressive. Indeed, for sigma equal to 0.01 the early childhood impact on income (13.25%) is almost three times higher than the impact of early childhood on formal education (4.99%).

In Table 7 we re-evaluate the results of last section for different values of the complementarity parameter. Thus, we kept constant total government expenditure on education but increased by 10%, for all years, the share of total government expenditure on pre-primary education for each $\sigma$. When complementarity is higher the effect of this reallocation of government expenditure is higher too. For the case where $\sigma > 0$ the impact of a reallocation in the government expenditure is small, 0.1%. Therefore, these results are robust toward several values of complementarity and it strengthens the importance of early childhood education.

9 Conclusion

This paper studies the evolution of human capital of American workers between 1961 and 2008. Its novelty is to incorporate to an otherwise standard continuous time life cycle model results from
Reduction in the formal education at 2008 (%)

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early childhood at 1961 level</td>
<td>36.42%</td>
<td>28.09%</td>
<td>19%</td>
<td>10.99%</td>
<td>5.47%</td>
<td>4.99%</td>
</tr>
</tbody>
</table>

Reduction in income at 2008 level (%)

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early childhood at 1961 level</td>
<td>44.6%</td>
<td>37.31%</td>
<td>28.86%</td>
<td>20.78%</td>
<td>14.42%</td>
<td>13.52%</td>
</tr>
<tr>
<td>Formal education at 1961 level</td>
<td>27.73%</td>
<td>27.68%</td>
<td>27.77%</td>
<td>27.87%</td>
<td>27.66%</td>
<td>27.59%</td>
</tr>
</tbody>
</table>

Table 6: Sensitivity - Contrafactual results

<table>
<thead>
<tr>
<th>σ</th>
<th>-1</th>
<th>-0.75</th>
<th>-0.5</th>
<th>-0.25</th>
<th>-0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average increase in income</td>
<td>0.56%</td>
<td>0.54%</td>
<td>0.36%</td>
<td>0.25%</td>
<td>0.11%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Table 7: Sensitivity - Contrafactual of reallocation of government expenditure in education

the early childhood literature and a human capital formulation close to that of Heckman and Cunha (2007) and Cunha et al. (2010). In this sense, early and formal education are complementary. We use the model to measure the evolution of early childhood human capital and its importance to labor productivity and formal education.

Early childhood education is shown to be very critical to understand the evolution of U.S. income. This is so because the dynamic effect of the complementarity in human capital formation leads early childhood education to be used as input to formal education, which improves the return to the latter. Indeed, early childhood education explained 60.88% of observed increase in the years of schooling in the U.S. Moreover, its impact on output is sizable and in the same order of magnitude as schooling. These results are show to be very robust and reinforce, in a general equilibrium set up, results from the labor literature that found that the first stage of human capital formation is key for future income and cognitive development of individuals and hence, nations.

A modified version of the model was used to analyze the role of government expenditures in education. The main result is that the government could have increased income per capita in the past by just reallocating expenses in favor of early childhood education. Therefore, our result calls for a reassessment of educational policies with increasing emphasis on pre-school education.

References


Head Start Bureau (2012). Head start program facts fiscal year 2012.


Appendix

<table>
<thead>
<tr>
<th>Informational set</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\tau_L^{0}$</th>
<th>$\tau_K$</th>
<th>$\tau_H^{0}$</th>
<th>$A_1^{0}$</th>
<th>$A_3^{0}$</th>
<th>$A_1^{f}$</th>
<th>$A_3^{f}$</th>
<th>$\tau_L^{f}$</th>
<th>$\tau_H^{f}$</th>
<th>$\tau_H^{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IS_1$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.19</td>
<td>0.42</td>
<td>2.30</td>
<td>0.0075</td>
<td>-0.0072</td>
<td>-0.95E-05</td>
<td>-0.019</td>
</tr>
<tr>
<td>$IS_2$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.43</td>
<td>2.30</td>
<td>0.0080</td>
<td>-0.0065</td>
<td>-3.15E-05</td>
<td>-0.022</td>
</tr>
<tr>
<td>$IS_3$</td>
<td>0.65</td>
<td>0.013</td>
<td>0.70</td>
<td>0.45</td>
<td>0.60</td>
<td>1.21</td>
<td>0.42</td>
<td>2.30</td>
<td>0.0078</td>
<td>-0.0089</td>
<td>-1.23E-04</td>
<td>-0.024</td>
</tr>
</tbody>
</table>

Table 8: Endogenous calibration - Different informational set for targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>U.S. data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference</td>
<td>$\beta$</td>
<td>Mean of average hours worked to sample</td>
<td>0.3171</td>
<td>0.3152</td>
</tr>
<tr>
<td>Weight of early childhood in Human Capital Technology</td>
<td>$\lambda$</td>
<td>Mean of schooling to sample</td>
<td>11.7</td>
<td>11.6</td>
</tr>
<tr>
<td>Initial Labor Tax</td>
<td>$\tau_L^{0}$</td>
<td>Average hours worked in 2008</td>
<td>0.307</td>
<td>0.3088</td>
</tr>
<tr>
<td>Capital Tax</td>
<td>$\tau_K$</td>
<td>Investment share in 2008</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Initial Schooling Tax</td>
<td>$\tau_H^{0}$</td>
<td>Years of schooling in 2008</td>
<td>13.3</td>
<td>13.4</td>
</tr>
<tr>
<td>Initial Early Childhood Education Tax</td>
<td>$\tau_H^{c}$</td>
<td>Worker share in sector 2 in 2008</td>
<td>0.0034</td>
<td>0.0034</td>
</tr>
<tr>
<td>TFP initial goods sector</td>
<td>$A_1^{0}$</td>
<td>GDP per capita in 2008</td>
<td>1</td>
<td>1.028</td>
</tr>
<tr>
<td>TFP initial schooling sector</td>
<td>$A_1^{c}$</td>
<td>Worker share in sector 3 in 2008</td>
<td>0.0378</td>
<td>0.0379</td>
</tr>
<tr>
<td>Trend of TFP goods sector</td>
<td>$A_1^{g}$</td>
<td>Variance GDP per capita to sample</td>
<td>0.064</td>
<td>0.063</td>
</tr>
<tr>
<td>Trend of TFP schooling sector</td>
<td>$A_3^{g}$</td>
<td>Mean GDP per capita growth rate between 1971 to 2008</td>
<td>0.0225</td>
<td>0.0228</td>
</tr>
<tr>
<td>Trend of labor tax</td>
<td>$\tau_L^{g}$</td>
<td>Variance of average hours worked to sample</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Trend of schooling tax</td>
<td>$\tau_H^{g}$</td>
<td>Variance of schooling to sample</td>
<td>2.478</td>
<td>2.476</td>
</tr>
<tr>
<td>Trend of early childhood education tax</td>
<td>$\tau_H^{c}$</td>
<td>Mean GDP per capita to sample</td>
<td>0.676</td>
<td>0.675</td>
</tr>
</tbody>
</table>

Note: This sample is given by \{1961, 1971, 1980, 1990, 2000, 2008\}.

Table 9: Endogenous parameters - Government expenditure as percentage of GDP