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Addressing important econometric issues on how to construct theoretical based exchange rate misalignment estimates

Emerson Fernandes Marçal
Beatrice Zimmermann
Diogo de Prince
Giovanni Merlin

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Emerson Fernandes Marçal*
Beatrice Zimmermann†
Diogo de Prince‡
Giovanni Merlin§

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Abstract

Exchange rate misalignment assessment is becoming more relevant in recent period particularly after the financial crisis of 2008. There are different methodologies to address real exchange rate misalignment. The real exchange misalignment is defined as the difference between actual real effective exchange rate and some equilibrium norm. Different norms are available in the literature. Our paper aims to contribute to the literature by showing that Behavioral Equilibrium Exchange Rate approach (BEER) adopted by Clark & MacDonald (1999), Ubide et al. (1999), Faruqee (1994), Aguirre & Calderón (2005) and Kubota (2009) among others can be improved in two following manners. The first one consists of jointly modeling real effective exchange rate, trade balance and net foreign asset position. The second one has to do with the possibility of explicitly testing over identifying restrictions implied by economic theory and allowing the analyst to show that these restrictions are not falsified by the empirical evidence. If the economic based identifying restrictions are not rejected it is also possible to decompose exchange rate misalignment in two pieces, one related to long run fundamentals of exchange rate and the other related to external account imbalances. We also discuss some necessary conditions that should be satisfied for disrcarding trade balance information without compromising exchange rate misalignment assessment. A statistical (but not a theoretical) identifying strategy for calculating exchange rate misalignment is also discussed. We illustrate the advantages of our approach by analyzing the Brazilian case. We show that the traditional approach disregard important information of external accounts equilibrium for this economy.

JEL codes: F31, F32, F41, C51.

Keyword: exchange rate misalignment, cointegration, identification.

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*Head of Center for Applied Macroeconomic Research at Sao Paulo School of Economics and CSSA-Mackenzie. Corresponding author: emerson.marcal@fgv.br

†PhD Candidate at Sao Paulo School of Economics

‡Researcher of the of Center for Applied Macroeconomic Research at Sao Paulo School of Economics

§PhD Candidate at Sao Paulo School of Economics

1 Introduction

Exchange rate assessment is becoming more relevant in recent period. In Europe, for example, many countries have current account deficits in recent period while Germany has a current account surplus. In contrast, a group of Asian countries has current account surplus for a long time. These imbalances rise questions about the existence of significant over and undervaluation of the currencies. On the other hand, the adoption of unusual monetary policy in United States may generate side effect such as under or over valuation of the dollar. Exchange rate misalignment may be linked to financial crisis events. It can also be a sign of macroeconomic imbalances. Persistent misaligment may also generate permanente effect on trade flow. Summing up, it is important to develop better ways of calculating the real exchange rate misalignment in this environment.

There are different methodologies to calculate real exchange rate misalignment. The real exchange rate misalignment is defined as the difference between actual real effective exchange rate and some equilibrium norm. Different norms are available in the literature.

The main goal of our paper is to suggest a new way of calculating the exchange rate misalignment using the Behavioral Equilibrium Exchange Rate approach (BEER). Under this approach a set of fundamentals is choosen from a theoretical intertemportal macroeconomic model and then a estimate is obtained from a econometric model that links real effective exchange rate to these fundamentals in the long-run. Examples of this approach are Aguirre & Calderón (2005), Faruqee (1994), Clark & MacDonald (1999), Ubide et al. (1999) and Kubota (2009). These authors opt to construct an econometric model based on the relationship between real exchange rate, net foreign asset and other set of fundamentals. But they don't incoporate the trade balance information implied by a solvency condition that links trade balance to net foreign asset position (NFA).

We want to highlight that there is no reason to suppress the trade balance information unless the statistical evidence allows the analyst to discard this information. This paper also suggests that the traditional intertemporal macroeconomic model used as the theoretical base of BEER empirical analysis implies in identifying restrictions in the long run parameters that should be tested. If these restrictions are not rejected, we can state that restrictions provided by the theory are not falsified by the empirical evidence. The possible non rejection of the null hypothesis strengths the exchange rate misalignment estimate and also allows the analyst to decompose the exchange rate misalignment estimates into economic meaningful pieces, giving them a reasonable economic interpretation. These restrictions can be tested from congruent time series econometric model.

This paper is organized in five sections. The first is this introduction. The second section provides a brief review of exchange rate misalignment literature. The third section describes the behavioural approach to estimate exchange rate misalignment based on fundamentals. Here, we also present a comparative analysis of the traditional approach and our joint modelling approach, that includes trade balance information. Besides that, we also discuss some other points: the possibility of explicitly testing over identifying restrictions derived by a theoretical model; some necessary conditions that should be satisfied for disregarding trade balance information without compromising exchange rate misalignment assessment; and a pure statistical identifying strategy for calculating exchange rate misalignment. The fourth section presents an illustrative example using the data of the Brazilian economy. Finally, the fifth section concludes the paper.

2 A short review of exchange rate misalignment literature

Currency misalignments can be measured by econometric methods of time series and panel data models. In these analyses, what is important is the range of variation and whether misalignments show significant and persistent undervaluation or overvaluation. Large changes in an exchange rate always generate debate on whether the movements are "excessive", reflect "fundamentals", or are "rational". Empirical studies have developed models to assess the long-term determinants of real exchange rates. Many studies have attempted to construct more accurate estimates of the magnitude and sign of exchange rate misalignment.

Exchange rate misalignment is defined as the difference between a measure of the real exchange rate and some equilibrium norm. Taking into consideration this definition, discussions on exchange rate misalignment can be divided into two levels. The first focuses on which is the best norm to evaluate exchange rate equilibrium. Economic models are constructed to provide a better understanding of the determinants of the real exchange rate. These models attempt to determine the best set of fundamentals that may explain real effective exchange rates in the long run.

The second level of the debate is about the best empirical strategy to measure exchange rate equilibrium norms. This is an econometric debate. Empirical studies need to choose between a time series or panel approach. The time series approach has the advantage of allowing a particular structure to be estimated for each country. However, the approach does not allow a broader set of variables to be analyzed at the same time because the available macroeconomic samples are not long enough. Panel techniques allow analysts to enlarge the spectrum of variables, but at the cost of imposing untested similarities between the parameters of different countries' models.

Exchange rate misalignment is an unobservable variable and must be estimated. The estimates of misalignment may differ for two reasons: the equilibrium norm that the analysts may choose and the econometric strategy adopted to implements the norm. Some analysts opt to use a "desirable" current account target as norm. This notion of equilibrium is closed related to macroeconomics, that is, the analysts may be concerned with issues like real, financial and global instability. This is a natural choice for institutions like International Monetary Fund (IMF) among others. However, it is not clear what are the current account targets and what criteria is used for choosing among different targets. Under this approach, the equilibrium real exchange rate can be defined as the level of real effective exchange rate that guarantees some target level of current account at a certain time horizon.

The target can be a "desirable" level defined by the analyst or can be obtained by an econometric model[1]. Some authors opt to define the equilibrium as the level of current account and real exchange rate that stabilizes the net foreign asset position at some "desirable" level. The strength of this notion of equilibrium is that it may help to avoid great macroeconomic imbalances and it is a good guide for macroeconomic policy formulation. The weakness of this approach lies on the need of estimating elasticity of trade (exports and imports) to real exchange rate and the subjectivity in defining "desirable" level of current account. So, the exchange rate misalignment estimate is not robust to different choices.

Another approach to calculate exchange rate misalignment is based on fundamentals. The analyst estimates a real exchange rate equation obtained from a reduced form of an economic model. In this case, the real effective exchange rate is in line with fundamentals obtained from a theoretical model. The analyst calculates the exchange rate misalignment by decomposition of the real effective exchange rate series between permanent and transitory components. The strengths of this approach lies on the possibility of choosing the set of fundamentals by using modern econometric specifications and model selection techniques. This approach can be criticized because the equilibrium norm will only prevail in the long run. It is also not easy to discover the causes driving the real exchange rate misalignment.

One of the most popular approaches to address exchange rate equilibrium is the Purchasing Power Parity (PPP). The equilibrium level of exchange rate is the one that equalizes purchasing power parity among two or more currencies. The benchmark can be an important currency like dollar or a world basket. This approach has the advantage of transparency and simplicity. There is a discussion about which is the best price index choice. The main criticism to PPP doctrine is

[1] Peterson Institute uses the first option while IMF focuses on the second approach.

that the equilibrium will prevail only in the very long run.

It is also possible to choose a pure statistical approach. This approach is named in the literature as "Atheoretical Approach" and lies on the use of univariate permanent and transitory decomposition technique to calculate misalignment. One of the most usual choice is the Beveridge & Nelson (1981) decomposition but many other options are available. This approach has two criticisms. The first is the fact that there is no economic foundation for explaining misalignment. Another negative point of this approach is that a very long reliable time series dataset is mandatory to obtain valuable results, as it is in PPP approach.

Another point that must be highlighted has to do with the definition of real exchange rate. If analysts are concerned with macroeconomic equilibrium, then a natural choice is the real effective exchange rate. But the bilateral real exchange rate must be the focus to analyze trade issues such as those regarding Word Trade Organization (WTO). If this is the case, there must be a methodology to map effective exchange rate misalignment obtained from macroeconomic approach to bilateral real exchange rate disequilibrium. Peterson Institute and some authors - such as Ubide et al. (1999) - try to do this.

In last years, the International Monetary Fund (IMF) started to systematically disseminate the results of staff research effort to measure the exchange rate misalignment in several countries members of the Fund. Since mid-1990s, the Consultative Group on Exchange Rate Issues (CGER) has provided exchange rate assessments. Since 2003, the IMF's CGER assessments have used the equilibrium exchange rate (ERER) approach. More recently, in 2012 Pilot External Sector Report, they published a new approach – External Balance Assessment (EBA) - that is being developed by the research department as a refinement of the CGER/ERER approach. Results from both methodologies are used as inputs to country desks for the assessment of current accounts and real exchange rates that are available in the External Sector Report. Desk judgments are also considered for the IMF's estimates.

The ERER approach estimates directly an equilibrium real exchange rate for each country as a function of medium-term fundamentals such as the net foreign asset (NFA) position of the country, relative productivity differential between the tradable and nontradable sectors, and the terms of trade. Some controls, such as government consumption, trade restrictions and share of administered prices, are also included in the panel estimation using Dynamic Ordinary Least Squares (DOLS) technique. Sample data covers 48 countries and the year 1980 onwards. IMF's EBA methodology is based on two panel estimations, for current account (CA) and real effective exchange rate (REER) indices , and an External Sustainability (ES) approach, which is not based on regression analysis.

The basic idea of the panel estimation is that current account and REER can be written as a function of the output gap, real interest rate differential and factors that may affect saving, investment, current account, capital flows and changes in foreign currency reserves. The explanatory variables included in the EBA model are: commodity terms of trade, trade openness, share of administered prices, VIX , share of own currency in world reserves, financial home bias, population growth, expected Gross Domestic Product (GDP) growth over the next 5 years, productivity and changes in foreign reserves. Policy-related regressors are also included: health expenditure to GDP, foreign exchange interventions, real short-term interest rate differential, private credit to GDP and capital controls. Sample data covers 40 countries and the period of 1990-2010. With the estimation of the REER equation, the misalignment can be directly calculated as the sum of the residuals with the net contributions of policy related gaps. The policy gap is a measure of a cyclical gap (over a benchmark) on six policy areas: fiscal balance, capital controls, social spending, foreign exchange market intervention, financial policies and monetary policy. The gap is calculated by the difference of the actual level of the variable and their "desirable" level, times the value of the estimated coefficient. The "desirables" levels are supplied by each IMF's countries desks. In the CA regression-based analysis, however, the REER misalignments are obtained indirectly, by calculating the required change in the REER to achieve the equilibrium in the Current Account. To do so, they need to estimate a semi-elasticity of CA to REER, based on the export and imports elasticity and the trade openness. The same is true for the ES approach (that it's not a regression method) which calculate the necessary change in the REER to allow the sustainability, in the medium term, of the Net Foreign Asset (NFA) to GDP ratio, due to Current Account adjustments.

Since 2010, Cline and Williamson, from Peterson Institute for International Economics, started to publish a biannual policy brief with the results of their simulation of "fundamental equilibrium exchange rate" (FEER). FEER is defined as an exchange rate that is expected to be indefinitely sustainable on the basis of existing policies. In short, the authors calculate the necessary change in the real exchange rate to get a sustainable current account deficit/surplus (+3% and - 3% of GDP). The model is based on Cline (2008), and the idea is pretty simple, they use the export price elasticity and the share of exports in GDP to estimate the impact of the exchange rate on current account. Then, based on IMF projections for CA for the next five years, they calculate the change in real exchange rated needed to achieve the "Target Current Account" - which is bounded in +/-3% of GDP, for countries with projected surpluses/deficits higher than +/-3% of GDP - instead the projected current account. This approach is closely related with IMF's EBA Current Account and ES analysis.

3 Behavioural exchange rate approach based on fundamentals

One possible approach to estimate exchange rate misalignment consists of implemeting an empirical strategy to estimate long run fundamentals of real effective exchange rate. The works of Aguirre & Calderón (2005), Faruqee (1994), Ubide et al. (1999) and Kubota (2009) are good examples of this strategy. These works choose the exchange rate fundamentals based on a theoretical economic model. Following Lane & Milesi-Ferretti (2003) we can obtain the steady state equations given by (1) and (2) from different variants of an intertemporal macroeconomic model:

$$\overline{tb} = -r * \overline{NFA} \tag{1}$$

$$\overline{RER} = -\phi\overline{tb} + \lambda\overline{X} \tag{2}$$

From equation (1), one country can not have in the long-run a trade balance surplus (tb) if the net revenues from NFA are not positive and large enough to cover this deficits. The second equation shows how real exchange rate is related to trade deficit and other long-run determinants. The vector X accounts for any other factor affecting equilibrium RER such as Balassa-Samuleson Effect (Balassa 1964, Samuelson 1964) or a terms of trade effect.

By merging both equations, we obtain:

$$\overline{RER} = \phi r^* \overline{NFA} + \lambda\overline{X} \tag{3}$$

The traditional papers investigate if there is evidence of cointegration between real effective exchange, net foreign asset and a list of fundamentals based on equation (3). The traditional approach consists of estimating a Vector Error Correction Model with the following set of variables: real exchange rate, net foreign asset and some group of fundamentals that does not contain trade balance series. Based on this econometric model, the real effective exchange rate is decomposed into permanent and transitory components to address exchange rate misalignment. The usual choice in the literature is the Gonzalo & Granger (1995) decomposition.[2] This approach assumes that equation (1) holds and net foreign asset contains all relevant information regarding external accounts sustentability.

In this paper, we want to explore and show that important information regarding exchange rate misalignment can be missed when the analysts opt not to jointly modeling net foreign asset and real exchange rate in the system. Assuming that all variables in the system of equations given by (1) and (2) are integrated of order one - I(1) - then we have at least two cointegration relationships. The evolution of trade balance can provide important information regarding the long run level of NFA and consequently to the long run level of the real exchange rate. This approach is discussed in the next subsection.

3.1 Exchange misalignment estimates: Comparing traditional and joint modeling strategy

In this subsection, we want to propose a different strategy to estimate exchange rate misalignment. We compare the results obtained from both approaches: the traditional one and our joint modelling approach. The first one does not use the trade balance information whereas the second incoporates this information into the analysis. Our results suggest that important information regarding exchange rate misalignment can be missed without analysing jointly trade balance, real effective exchange and net foreign asset position in an econometric model. Using our approach, we can test if a particular econometric model satisfies over identification restrictions suggested by equations (1) and (2). Our approach can also provide information regarding the causes that drive real effective exchange misalignment. All these points are addressed below.

3.1.1 How to compute exchange rate misalignment under BEER approach

Let's assume that the local data generator process for the variables trade balance, real effective exchange rate and net foreign asset position is given by the following vector autoregressive model:

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + ... + \Gamma_{k-1} \Delta Y_{t-k+1} + \alpha \beta' Y_{t-1} + \mu + \varepsilon_t \tag{4}$$

where ε_t are random normal and not correlated errors, Ω denotes the variance and covariance matrix

[2] Levtchenkova et al. (1998) provides a survey on different ways of decomposing a series into permanent and transitory components.

of the errors which do not vary with time and $\theta = [\Gamma_1, ...\Gamma_{k-1}, \alpha, \beta, \mu]$ contains the parameters of the model. The vector Y_t contains the real effective exchange rate and the set of fundamentals chosen by the analyst.

In order to address real exchange rate misalignment, we need to compute a permanent and transitory decomposition. Several decompositions have been proposed to decompose the series into transitory and permanent components. In general, the decomposition takes the following form:

$$Y_t = [c_\perp(\beta' c_\perp)^{-1}\beta' + \beta_\perp(c'\beta_\perp)^{-1}c]Y_t \tag{5}$$

The existence of this decomposition is not always guaranteed, because the matrix $c'\beta_\perp$ may not have full rank. Gonzalo & Granger (1995) proposed $c = \alpha_\perp$. This representation always exists for a model with a VECM of zero order. Johansen (1995) suggests $c = \alpha_\perp(\Gamma_1 + ... + \Gamma_k - I)$. This decomposition always exists, provided that there are variables in the system with an order of integration of at most one. Kasa (1992) proposes $\beta_\perp$. Another possibility is to generate forecasts from the VECM estimated for each point. The values on which the series converge are called fundamentals.

The decomposition of Gonzalo & Granger (1995) is widely used in exchange rate misalignment empirical literature[3]. In their decomposition, the transitory component do not Granger cause in the long run changes of the permanent component. In other words, misalignment (defined as the transitory component of the real exchange rate in a multivariate equation system) does not contain relevant information for predicting the changes of the permanent component in the long term.

Using the parameters from (4), it is possible to calculate the transitory (T_{it}) and permanent (P_{it}) components from the following equations:

$$P_t = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp Y_t \tag{6}$$

$$T_t = \alpha(\beta'\alpha)^{-1}\beta' Y_t \tag{7}$$

[3] Ubide et al. (1999) and Kubota (2009).

The estimative of exchange rate misalignment is the component associated with the position of the real exchange rate in vector Y_t. Assuming that the real exchange rate is in the first position of the vector, and using the value of the error correction mechanism centered on their own means, we can calculate the misalignment using the following equation:

$$mis_t \equiv [\ 1 \quad 0 \quad \ldots \quad 0\ ]\alpha(\beta'\alpha)^{-1}(\beta' Y_t - E(\beta' Y_t)) \tag{8}$$

3.1.2 Our starting point

Unlike the traditional approach, we will start our analysis from a model that contains the following variables: real effective exchange rate, trade balance and net foreign asset position as a share of GDP. The starting point will be a time series econometric model given by equation (4). We will investigate the number of cointegrated relationships. The theoretical model suggests the existence of at least two cointegrated vectors depending on the number of added fundamentals. The latter variables can also cointegrate among themselves. After that, the cointegration space must be identified and the theoretical model suggests some identifying restrictions.

3.1.3 How to test that real exchange rate is not a variable with drift?

The economic theory suggests that real exchange rate is not a variable with drift. Contrary to GDP, where there is some rationality for assuming that a drift may exist, there is little space for assuming that real exchange rate can have a drift.

The model given by equation (4) does not preclude that the variables of system contain a drift. However, Johansen (1995) show how to test that none of the series in the system will "drift away". The restriction that the drift term of model given by (4) should satisfy is:

$$\mu = \alpha\varphi \tag{9}$$

where φ is a matrix of order r x 1 and r is the number of cointegrated relationship in the system. The unrestricted drift vector term μ is contains p parameters and the restricted drift vector given by (9) contains $r<p$ parameters.

3.1.4 How to check identification restrictions implied by theoretical model?

The set of equations given by (1) and (2) implies identification restrictions that can be imposed on cointegration space. First, these equations indicate the existence of two cointegrated vector in a system with real effective exchange, net foreign asset and trade balance. If the constant enters restricted in the cointegrated space as suggested by (9), then all the variables will not have a time trend in the level. But equation (1) also implies that the constant term should be excluded from the first cointegrated relationship as well as the coefficient associated with the real effective exchange rate must be zero. These restrictions provide more identifying restrictions than it is required to identify the first relationship. The second vector can be exactly identified by imposing that the trade balance is not part of the second relationship and the coefficient of real exchange is normalized to one.

If the analyst wants to investigate exchange rate misalignment using a broader set of fundamentals, a similar restriction must hold. Assume that the analyst wants to start from a model that contains real effective exchange, net foreign asset, trade balance and a variable to address the possible Balassa-Samuelson effect.[4] The vector of variables X must appear in the second cointegrated relationship but not in the first. The inclusion of the X provides more identifying restrictions.

For example, assume that the following sets of variable are modelled:

$$Y_t = \begin{bmatrix} rer & tb & nfa & Fund1 & Fund2 \end{bmatrix}'$$

where $Fund1$ and $Fund2$ are any variables related to two fundamentals.

If overidentifying restrictions hold, then the cointegrated vectors satisfy (10):

$$\beta_{r'} = \begin{bmatrix} 1 & 0 & b_{31} & b_{41} & b_{51} \\ 0 & 1 & b_{32} & 0 & 0 \end{bmatrix}' \tag{10}$$

where $\gamma = \{b_{31}, b_{32}, b_{41}, b_{51}\}$ are the coefficients of the long run relationship to be estimated.

The cointegrated space will be overidentified using restrictions obtained from a theoretical model. Since the analysts will work with a overidentifyng cointegrated space, it is possible to test jointly the validity of all these assumptions. A detalied description of how to implement and test these restrictrions can be founded in Johansen (1995) and Juselius (2006).

[4] See Balassa (1964) and Samuelson (1964)

3.1.5 Decomposing the real exchange rate misalignment in two pieces: trade imbalances and long-run factors

If the structure given by (1) and (2) is validated, the next step is to calculate the exchange rate misalignment using a time series decomposition into permanent and transitory components. Assuming that there are two cointegrated vectors properly identified, then it is possible to decompose the exchange rate misalignment from (8) in two pieces:

$$mis_t \equiv \begin{bmatrix} 1 & 0 & ... & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} (\beta'\alpha)^{-1}([\beta_1\beta_2]'Y_t - E([\beta_1\beta_2]'Y_t)) \tag{11}$$

where β_1 and β_2 are respectively the first and second cointegration vectors and α_1 and α_2 are the first and the second vector of the loading matrix respectively.

Defining F by:

$$F \quad \equiv \quad [F_1F_2] = \begin{bmatrix} \begin{bmatrix} 1 & 0 & ... & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} V_1 & \begin{bmatrix} 1 & 0 & ... & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} V_2 \end{bmatrix} \tag{12}$$

where $\begin{bmatrix} V_1 & V_2 \end{bmatrix} = (\beta'\alpha)^{-1}$ and V_i has dimension 2x1.

Then:

$$mis_t \equiv F_1(\beta_1'Y_t - E([\beta_1'Y_t]) + F_2(\beta_2'Y_t - E([\beta_2'Y_t]) \tag{13}$$

The first term in (13) refers to the contribution related to the first cointegrated vector of the model whereas the second term refers to the contribution of the second cointegrated vector. It is possible to decompose the exchange rate misalignment into a factor related to external account issues and another related to long run determinants of the real exchange rate. The decomposition can be quite helpful to better understand the causes of exchange rate misalignment.

3.1.6 When the trade balance information is not useful?

The main point of this paper is that there is not priori reason for not using the information contained in trade balance. We want to discuss now when this information is not useful to calculate exchange rate misalignment.

Using a pure statistical approach, we can conclude that the trade balance is not useful if two conditions hold. The first is that the trade balance variable can be excluded from the cointegrated space. The second is that the system satisfies the condition for the weak exogeneity of the long run parameters to trade balance as defined by Hendry (1994) . One should note that in this case, the restrictions of the theoretical model are not satisfied.

Let's assume that identifying restrictions given by (10) hold and the loading matrix (α) has the form given by (14). The cointegrated vector related to (2) enters in all equations of system excepting trade balance and net foreign asset equations and the cointegrated vector related to (3) enters only in the trade balance equation.

Under this case $Y_t = \begin{bmatrix} rer & tb & nfa & Fund1 & Fund2 \end{bmatrix}'$ and the long run matrices are

$$\alpha' = \begin{bmatrix} \alpha_{11} & 0 & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ 0 & \alpha_{21} & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

and

$$\beta' = \begin{bmatrix} 1 & 0 & b_{31} & b_{41} & b_{51} \\ 0 & 1 & b_{32} & 0 & 0 \end{bmatrix}' \tag{15}$$

The trade balance will have no effects on misalignment. This can be seen by using (13), (14) and (15). The parameters of interest $\theta = \{\alpha_{11}, \alpha_{31}, \alpha_{41}, \alpha_{51}, b_{31}, b_{41}, b_{51}\}$ to calculate misalignment are not coming from trade balance equation. It must be noted that this is a very special case and there is no reason to assume that these conditions hold a priori. They can and should be tested.

It is possible to show that $F_1 = \frac{\alpha_{11}}{(\alpha_{11}+\alpha_{41}b_{41}+\alpha_{51}b_{51})}$ and $F_2 = 0$ if the analyst opts to use Gonzalo and Granger decomposition.

The matrices given by (14) and (15) are not the only possibility to discard the information of trade balance equation to calculate exchange rate misalignment when theoretical identifying restriction holds. For example, assume that the long run matrices have the following configuration:

$$\alpha' = \begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{31} & \alpha_{41} & \alpha_{51} \end{bmatrix} \tag{16}$$

and

$$\beta' = \begin{bmatrix} 1 & 0 & b_{31} & b_{41} & b_{51} \\ 0 & 1 & b_{32} & 0 & 0 \end{bmatrix} \tag{17}$$

It is possible to show that $F_1 = 1$ and $F_2 = 0$. In this case the first vector can be directly interpreted as exchange rate misalignment.

3.1.7 What if theoretical identifying restrictions fail? Try a pure statistical approach

The set of restrictions implied by (1) and (2) guaranties that the cointegration space is overidentified but they can be rejected by the data. Under these circunstances others possible strategies to identify the cointegration space can be tried. In the case where the number of cointegrating relationships is higher than 1, it is possible to test whether or not there is evidence that one vector enters only in real exchange rate while the other enters in all other equations but not in the real exchange rate equation. If this is the case, the error corrrection mechanism that enters in the real exchange rate equation can be directed interpreted as exchange rate misalignment and the cointegration space is identifed. This statistical approach to identify cointegrated vector by imposing restrictions on loading matrix is discussed in Juselius (2006).

In order to better understand the identification strategy, one should note that the long run matrices can be rewriten as:

$$\pi = \alpha\beta' = \alpha\Psi^{-1}\Psi\beta' \tag{18}$$

where is Ψ is a any rxr full rank matrix.

One can opt to choose Ψ in way that

$$\widetilde{\alpha} = \alpha\Psi^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & I_{r-1} \\ \widetilde{\alpha}_1 & \widetilde{\alpha}_2 \end{bmatrix} \tag{19}$$

where $\widetilde{\alpha}_1$ has dimension (p-r)x1, $\widetilde{\alpha}_2$ has (p-r)x(r-1) matrix. In (19) only a rotation was performed. In addition, if one opts to impose that $\widetilde{\alpha}_1 = 0$, then the cointegration space will be overidentified.

Under this case of the orthogonal alpha matrix is given by

$$\widetilde{\alpha}_\perp = \begin{bmatrix} 0 \\ D \end{bmatrix} \tag{20}$$

where D is a (p-1)x(p-r) matrix and $\widetilde{\alpha}'_\perp \widetilde{\alpha} = 0$.

The common trends as defined in Johansen (1995) page 41 is given by

$$\alpha'_{\perp} \sum_{i=0}^{t} \varepsilon_{t-i} \tag{21}$$

By using the Granger and Johansen representation theorem[5], it is possible to show that the shocks that hit only exchange rate (the first equation of the system) do not have long run effects on the system.

Another possibility to identify the long run relationship is to test hypothesis directly on elements of β. Firstly, analysts should check wheter real effective exchange rate can be excluded from long run cointegrated space. This hypothesis can be easily tested by Johansen and Juselius tests.[6] If the null hypothesis that the real exchange rate can be excluded from the long run relationship is rejected, it is possible to proceed the identification process.

The goal of the analyst is to estimate a long run relationship linking real effective exchange rate to some set of fundamentals. If there is evidence of more than one long run relationship, we may check if exchange rate and some set of selected fundamentals enter only in one of the cointegrated relationship. This strategy is illustrated in equation (22). The first element of the system is the real effective exchange rate.

$$\beta' = \begin{bmatrix} 1 & b_{21} & b_{31} & b_{41} & b_{51} \\ 0 & 1 & b_{32} & 0 & 0 \end{bmatrix}' \tag{22}$$

Additionally, one can test whether element $b_{i1} = 0$ for some i $\in \{2, 3\}$. If this is the case, the second cointegrated vector of the system can not be directly seen as fundamentals but it provides relevant informational of logn run value of the fundamentals. This is exactly the role that net foreign asset position is playing in the model given by equations (1) and (2) but in a pure statistical approach view any of these fundamentals can play this role.

Finally, it is possible to investigate whether the cointegrating relationship satisfies the condition for long run separability. Separation in cointegration, introduced by Konishi et al. (1993), Konishi & Granger (1992) and later extended by Granger & Haldrup (1997) implies that common trends can be extracted from sub-systems of I(1) time series. These authors consider situations where subsets of cointegration relationships exist between economic time series which have no variables in common. Under complete separation, the common trends extracted from a sub-group in a sub-

[5]See Johansen (1995) page 49.
[6]See Johansen (1995) and Juselius (2006)

system analysis correspond to those that would have been extracted from the complete system.

Assume that cointegration space has the following form which satisfies the conditions for long run separability:

$$\beta' = \begin{bmatrix} 1 & b_{21} & b_{31} & 0 & 0 \\ 0 & 0 & 0 & 1 & b_{52} \end{bmatrix}' \tag{23}$$

Under this case, there are variables that neither explain real effective exchange rate nor the fundamentals. They are part of the second cointegrated relationship. The set of variables of the second cointegrated relationship can be disregarded if the goal is to analyze the fundamentals of real exchange rate.

Finally the analyst can use an automatic algorithm to identify cointegrated relationship. One procedure is suggested by Omtzigt (2002). Another possibility to identify the cointegration space is the concept of irreducible cointegration relantionship (IC) developed by Davidson (1998a). A procedure to identify IC is discussed in the paper. In the paper the author proposes to use the test developed in Davidson (1998b) to evaluate the existence of IC relationships.

4 One illustrative example: Brazilian economy case

Our sample covers the period from 1979 to 2013. The frequency of the data is annual. The data was collected at the following sources: a) International Financial Statistics - International Monetary Fund; b) World Bank Developtment Indicators - World Bank; c) Bureau of Economic Analysis and d) Lane & Milesi-Ferretti (2007) database. We opt to work with end of period data to avoid problems related to time aggregation.

We address the exchange rate misalignment of Brazil as an illustrative example, although it can be applied to any country where the data is available. Brazil is an emerging market economy that had faced macroeconomic instability, current account crisis and cronic inflation during most part of the period of the sample. Figure 1 shows the time evolution of the variables.

We collected the data for real effective exchange rate (RER), share of net foreign asset position (NFA) and trade balance (TB) to Gross Domestic Product, ratio between real per capita GDP growth (BSGDP) and the terms of trade (TOT). The letter L denotes logarithm.

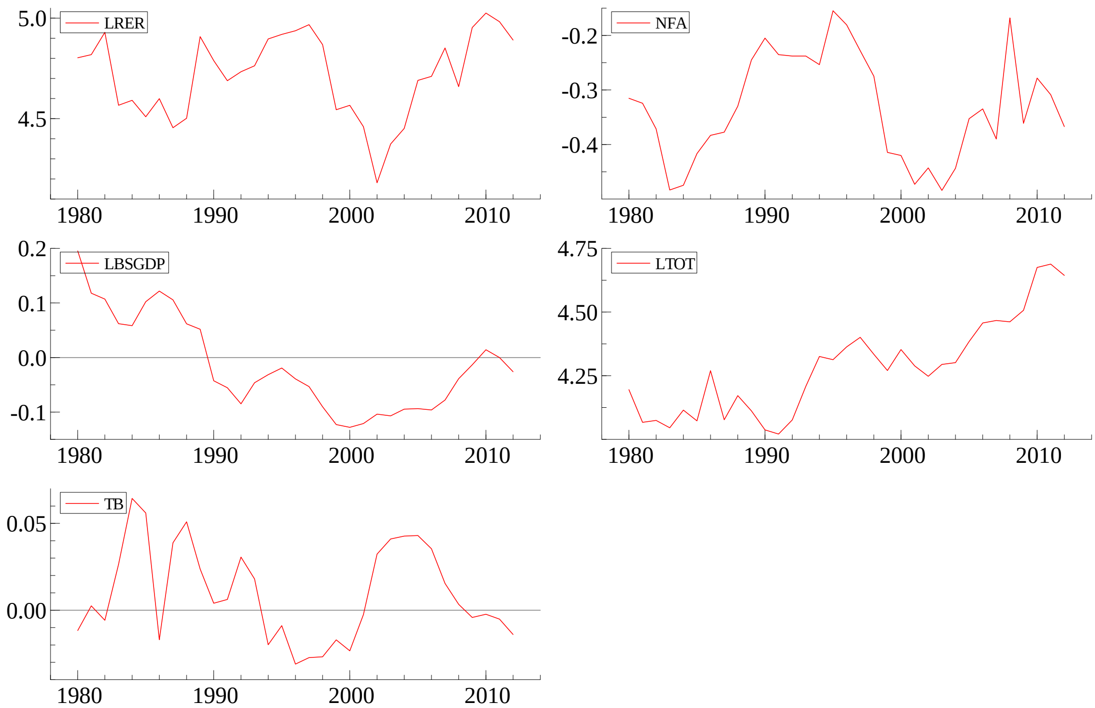

Figure 1: Plot of Brazilian Data.

4.1 Results

We start the analysis by investigating the existence of cointegration. We run cointegration tests for the following systems:

System BRA I: LRER, TB, NFA, LTOT, LBSGDP

System BRA-II: LRER, NFA, LTOT, LBSGDP

System BRA-III: NFA, TB

The results of the cointegration tests can be seen at Tables 1 and 2. Table 1 shows the results of Cheng & Phillips (2009) semi parametric test for cointegration and Table 2 shows the results of Johansen & Juselius (1990) cointegration test. From Johansen cointegration test we can see that there is evidence of two cointegrated vectors for system I, one for system II and two for system III in both tests considering p-value of 10%. The evidence collected from the analysis of system III suggests that trade balance and net foreign asset may be both stationary. The system II shows evidence in favour of existence of one vector. This can also be due the possible stationarity of net foreign asset or the existence of cointegration among variables if the net foreign asset is integrated of order one. The system I shows evidence of two vectors. This can also be consistent with the evidence of stationarity of trade balance and net foreign asset or cointegration between these variables and the fundamentals. Similar results are obtained from the analysis of Cheng and Phillips test. We opt to work with system I and assume that there are two cointegrated relationships and then investigate which variables are part of the cointegrated space. We also investigate if trade balance and net foreign asset can be seen as stationary variables.

System I* LRER, NFA, TB, LTOT, LBSGDP						System II* LRER, NFA, LTOT, LBSGDP						System III* TB, NFA					
rank **	Log-likelihood	Information Criteria SC		HQ		rank **	Log-likelihood	Information Criteria SC		HQ		rank **	Log-likelihood	Information Criteria SC		HQ	
0	248.67	-14.11		-14.26		0	162.20321	-9.1265		-9.2448		0	119.6268	-6.8294		-6.8886	
1	265.78	-14.18		-14.60		1	178.76504	**-9.3747**	<	**-9.7001**	<	1	127.5219	-6.9827		-7.1306	
2	278.30	**-14.19**	<	**-14.81**	<	2	182.78271	-9.0925		-9.5658		2	131.67072	**-7.123**	<	**-7.3005**	<
3	284.10	-14.02		-14.78		3	185.46406	-8.939		-9.5011							
4	286.58	-13.85		-14.71		4	186.12761	-8.8744		-9.466							
5	287.90	-13.82		-14.71													

* All system were estimated from a VAR(1).
** The number of cointegrated vectors is selected using the best information criteria.

Table 1: Cheng & Phillips (2009) cointegration test.

System I LRER, NFA, TB, LTOT, LBSGDP				System II LRER, NFA, LTOT, LBSGDP				System III TB, NFA			
Rank	Log-likelihood	Trace Test [pvalue]	Max Test [pvalue]	Rank	Log-likelihood	Trace Test [pvalue]	Max Test [pvalue]	Rank	Log-likelihood	Trace Test [pvalue]	Max Test [pvalue]
0	264.6245	**78.07 [0.008]** ***	**32.79 [0.064]** *	0	173.6067	**49.77 [0.031]** **	**36.72 [0.002]** ***	0	118.5959	**20.76 [0.006]** ***	**13.98 [0.054]** *
1	281.0181	**45.29 [0.084]** *	**28.31 [0.037]** **	1	191.9677	13.05 [0.887]	6.810 [0.951]	1	125.5837	**6.780 [0.009]** ***	**6.780 [0.009]** ***
2	295.1737	16.98 [0.650]	10.61 [0.693]	2	195.3731	6.240 [0.671]	5.770 [0.648]	2	128.9758		
3	300.4769	6.370 [0.656]	4.770 [0.769]	3	198.2589	0.470 [0.493]	0.470 [0.493]				
4	302.8637	1.600 [0.207]	1.600 [0.207]	4	198.4937						
5	303.6614										

***,**,* significance levels: 1%, 5% and 10%, respectively.

Table 2: Johansen & Juselius (1990) cointegration test.

4.1.1 Calculating misalignment from our approach

After having defined the rank of long run matrix, we run some tests to evalutate if the cointegration space satisfies restrictions suggested by theory. Table 3 contains the results of the tests, the estimated cointegrated vectors and loading matrices for different identification strategies. The hypothesis that trade balance and net foreign asset position are stationary variables are imposed in model 3 but they are strongly rejected. Model 1 uses the statistical identification strategy. These restrictions are easily accepted from the analysis of the likelihood ratio test.

We opt to work with model 2. All estimated coefficients have the correct expected sign with plausible magnitude. In equilibrium, the Brazilian economy must run a surplus due the fact that it has a negative foreign asset position. The error correction mechanism implied by the model and properly normalized are shown in equations (24) and (25). A simple exercise using equation (24) suggests that a level of -35% of GDP in Brazilian net foreign position will require a trade balance of 1.85% per year (=-35%*(0.052797)) to be sustainable. At end of the sample Brazil was running a trade balance deficits. The first vector is in line with Fundamental Equilibrium Exchange Rate approach. It moves the economy towards a sustainable level of external account in the long run. The second vector is inline with the BEER approach that links real effective exchange rate to a set of fundamentals.

$$ECMBRA2_t = RER_t \quad -1.3494 * NFA_t - 1.2642 * LBSGDP_t - 0.60179 * LTOT_t - 2.2089 \tag{24}$$

$$ECMBRA1_t \quad = \quad TB_t + 0.052797 * NFA_t \tag{25}$$

LR Statistics = 10.021 with $\chi^2(3)$ p-value: 1,84%

In addition to the direct link of cointegrated vectors to the economic theory, our approach can also shed light on the problem of determining the long run sustainable result of current account. The FEER is criticized by many authors to rely on a great degree of subjectivity when defining the target current account level. So, our approach can help to reduce this degree of subjectivity.

Long-run Paramaters of different specifications and LR ratio statistics *

Unrestricted Model

β'	rer	tb	nfa	bsgdp	tot	Constant
Vector 1	1	2.1648	-1.141	-1.2892	-0.56297	-2.3706
Vector 2	-0.04523	1	0.22403	0.034923	0.06974	-0.04147

α'	Δrer	Δtb	Δnfa	Δbsgdp	Δtot
Vector 1	-0.627	-0.07021	0.08476	0.053633	0.29145
Vector 2	5.2464	-0.71351	1.1446	0.58282	2.3438

Model 2: Theoretical Identification

β'	rer	tb	nfa	bsgdp	tot	Constant
Vector 1	1	0	-1.3494	-1.2642	-0.60179	-2.2089
Vector 2	0	1	0.052797	0	0	0

α'	Δrer	Δtb	Δnfa	Δbsgdp	Δtot
Vector 1	-0.81546	-0.04661	0.034752	0.031293	0.21507
Vector 2	1.9531	-0.50591	1.6116	0.58355	1.7685

LR Statistics	10.021
Distribution	χ2(3)
p-value	1.84%

Model 1: Statistical Identification

β'	rer	tb	nfa	bsgdp	tot	Constant
Vector 1	-0.88425	3.8961	1.952	1.0177	0.72615	1.3288
Vector 2	-0.04232	-0.86059	-0.07323	0.070741	-0.00712	0.20163

α'	Δrer	Δtb	Δnfa	Δbsgdp	Δtot
Vector 1	1	0	0	0	0
Vector 2	0	1	0.97222	0.35121	0.75858

LR Statistics	0.18655
Distribution	χ2(3)
p-value	97.97%

Model 3: Stationarity

β'	rer	tb	nfa	bsgdp	tot	Constant
Vector 1	0	0	1	0	0	-0.02498
Vector 2	0	1	0	0	0	0.41137

α'	Δrer	Δtb	Δnfa	Δbsgdp	Δtot
Vector 1	3.6618	-0.62169	1.2522	0.51514	2.3641
Vector 2	0.6563	-0.11037	-0.0353	-0.00183	0.5352

LR Statistics	27.121
Distribution	χ2(6)
p-value	0.01%

* Likelihood Ratio Statistics is calculated by comparing unrestricted and Model i.

Table 3: Long Run Matrices and Likelihood Ratio test.

The final vector error correction model is given by equations (26) to (30):

$$\text{DLRER} \quad = \quad -\underset{(0.154)}{0.5897}\ ECMBRA2_{t-1} + \underset{(0.854)}{2.124}\ \text{ECMBRA1}_{t-1} \tag{26}$$

$$\text{DTB} \quad = \quad -\underset{(0.0272)}{0.009329}\ ECMBRA2_{t-1} - \underset{(0.15)}{0.4722}\ ECMBRA1_{t-1} \tag{27}$$

$$\text{DNFA} \quad = \quad \underset{(0.0922)}{0.1372}\ \text{ECMBRA2}_{t-1} + \underset{(0.511)}{1.22}\ \text{ECMBRA1}_{t-1} \tag{28}$$

$$\text{DLBSPIB} \quad = \quad \underset{(0.0399)}{0.01338}\ ECMBRA2_{t-1} + \underset{(0.221)}{0.4047}\ ECMBRA1_{t-1} \tag{29}$$

$$\text{DLTOT} = \underset{(0.101)}{0.1153}\ \text{ECMBRA2}_{t-1} + \underset{(0.561)}{1.447}\ \text{ECMBRA1}_{t-1} \tag{30}$$

$AR\ 1-2\ test:$	$F(50,80):$	1.5832	[0.0329]$*$
$Normality\ test:$	$Chi^2(10)$	11.178	[0.3438]
$ZHetero\ test:$	$F(20,80)$	2.0731	[0.0118]$*$
$RESET23\ test:$	$F(35,86)$	1.0011	[0.4825]

The Gonzalo and Granger decomposition can be obtained from (6) and (7). In the case of Brazilian estimated VECM we have:

$$misalignmentBRA_t = -11.039 * ECMBRA1_t + 1.3868 * ECMBRA2_t \tag{31}$$

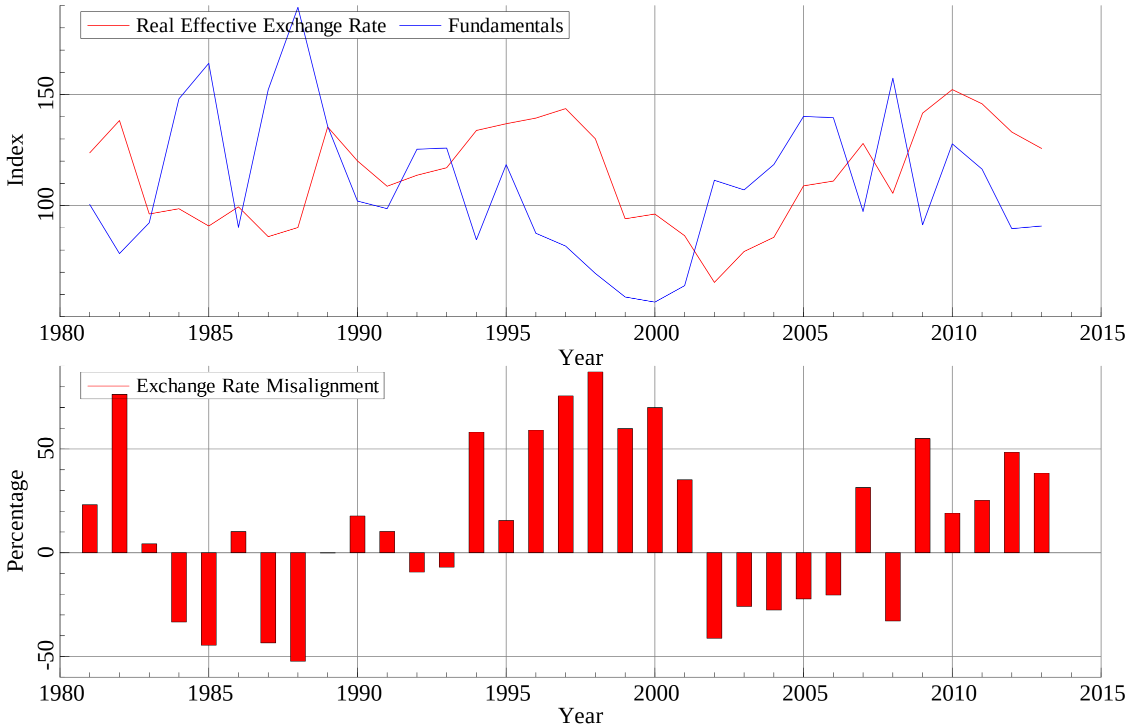

Figure 2: Exchange rate misalignment obtained from using full set of variables including trade balance.

Figure 3 shows the evolution of estimated exchange rate misalignment and its decomposition in two factors calculated from equation (31). The first factor is closed related to the deviations of Brazilian trade balance from its long run level. The other is related to the deviation of real effective exchange from the level of its long-run fundamentals and external account sustentability. At the end of the sample both factor are positive suggesting that they are contributing positively to Brazilian exchange rate overvaluation. The worsening of Brazilian trade balance at the end of the sample explains great part of Brazilian real exchange rate misalignment in the recent period. We think that this decomposition is the great strength of our methodology. This information is usually disregarded in exchange rate misalignment assessment.

Both of components have the same sign in most of time with few but quite important exceptions. For example, Brazilian economy had started to run an important current account deficits and the Central Bank started to intervene in the currency exchange market by adopting an almost fixed exchange rate regime after the Real macroeconomic stabalization plan[7] This policy remained unchanged until January of 1999 when Brazilian Central Bank were forced by the market to let Brazilian currency to go down. The component of misalignment related to long run real exchange rate determinants (blue bar in Figure 3) showed substantial overvaluation only after 1997, although Brazil was running a growing current account deficits of almost -2,8% of GDP in 1996 coming from a level of +1,6% in 1992 and reaching the level of -4,32% in 1999. Our measure of exchange rate misalignment takes this fact into account. The results suggest a great degree of imbalance from 1993 to 2001.

In the end of the sample our model suggests that Brazilian exchange rate is overvalued since 2009 due to the rising level of external account imbalances. The level of current account result was 0,11% of GDP in 2007 and reached the level of -3,6% of th GDP in 2013 after sucessive years of deterioration.

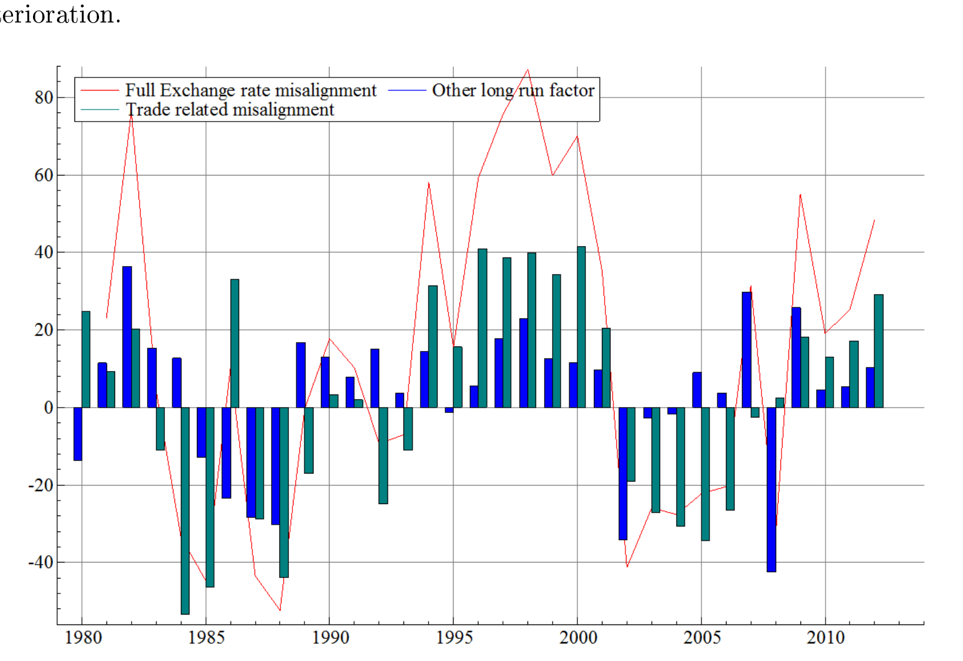

Figure 3: Exchange rate misalignment calculated from the full infomation set and its components.

[7] The macroeconomic stabilization plan was launched during Itamar Franco administration by his finance minister Fernando Henrique Cardoso in 1994. In 1995 Cardoso started his administration as Brazilian president and did a series of economic reforms to consolidate the macroeconomic stabilization.

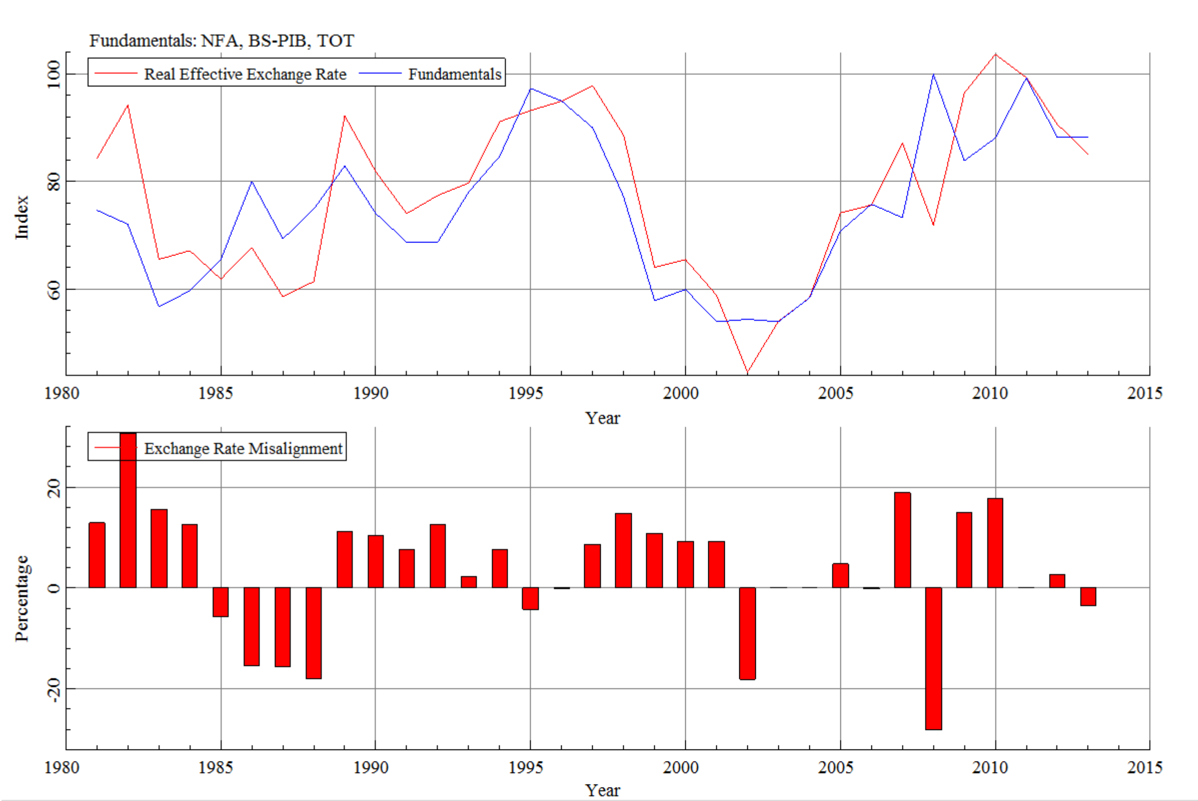

Figure 4: Exchange rate misalignment calculated from traditional methodology.

4.1.2 Comparing the two approachs

In this section, the results from two approaches are compared. Figure 5 shows the evolution of both estimates of exchange rate misalignment. Table 1 presents the descriptive statistics of the two misalignments estimates and their components. There is a positive correlation between two estimates of misalignment but not as high as expected. The correlation between the traditional methodology and the trade balance factor is positive but not high (19,7%). The high correlation is obtained from the traditional approach estimative with the component factor associated to exchange rate fundamentals (99,5%). This suggests that the information of external accounts is not well captured by the traditional approach. The amplitude (difference between the maximum and the minimum) is higher in our approach compared to the traditional approach and it may also explain why FEER and BEER results usually differs. The level of skewness is quite low for both approaches.

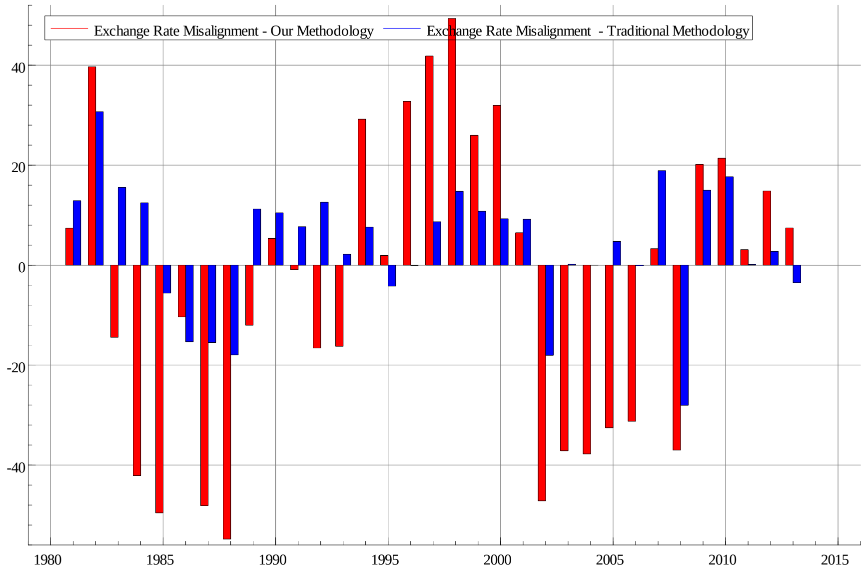

Figure 5: Comparing exchange rate misalignment from both approaches.

Another point can be highlighted. IMF Pilot Report (2012) mentioned that some methodologies can be quite sensitive to sample time span due to the fact some exchange rate misalignment methodologies uses the residuals from a regression to calculate the misalignment. These residuals have a zero mean by construction. This is not the case of our approach if the constant enters restricted in the cointegrated space. The null hypothesis that the constant enters restricted in the system can be tested from a likelihood ratio test[8]. The test was perfomed. The statistic test is 9.9277 with six degree of freedom and p-value of 12.77%.

[8] See Johansen (1995) page 80-84

	Exchange Rate Misalignment			
	Full	Component RER	Component TB	Traditional
Observations	33	33	33	33
Mean	-4.4	3.7	-8.7	3.8
Standard Deviations	29.7	15.5	22.4	12.3
Skewness	0.0	-0.6	0.0	-0.6
Excess Kurtosis	-1.1	0.1	-1.1	0.2
Minimum	-54.8	-33.9	-48.3	-28.1
Maximum	49.4	36.9	32.1	30.7
Normality test:	2.23	2.10	2.34	2.41
p-value	[0,3280]	[0,3508]	[0,3107]	[0,3003]

	Correlation matrix			
	Full	Component RER	Component TB	Traditional
Full	1			
Component RER	66.1%	1		
Component TB	88.9%	25.6%	1	
Traditional	61.3%	99.5%	19.7%	1

The sample is: 1981-2013

Table 4: Descriptive Statistics of exchange rate misalignment from both approaches.

Lane & Milesi-Ferretti (2002) state that instead of estimating the reduced form given by (3), it is better to estimate (2) and use the information of trade balance and not the foreign asset position. They state that the rate of return of net foreign asset position varies throughout the time and then estimative of the coefficients of equation (3) may not be stable and the return of different types of assets is also a problem. Although this might be the case, econometric tests can help to evaluate whether this is really a issue.

Zhang & MacDonald (2014) highlight that trade balance and net foreign asset should be cointegrated and they show that this might be the case for a sample of OECD countries they analysed. But they also suggests to use either net foreign asset or trade balance in the analysis. Based on their econometric exercise they also suggest that real exchange rate is closed related to trade balance but not to net foreign asset. But a natural extension would be to use a joint approach like ours since they also show that trade balance is related to net foreign asset position.

5 Conclusion

This paper aims to contribute to the literature of exchange rate misalignment by showing that the trade balance information, traditionally disregarded in BEER approach, can be quite useful to address exchange rate misalignment. The paper also discuss some sufficient conditions that allow the analyst to discard external account information like in standard BEER approach. We also discuss an alternative strategy to identifying exchange rate misalignment using a statistical approach not based on economic theory. If overidentifying restrictions suggested by a theoretical model are not rejected, we can decompose exchange misalignment in factors that have economic meaningful interpretation. This is important due to not only the necessity of assessing exchange rate misalignment but also to better understand its determinants.

We also ran a empirical illustration using Brazilian case. The theoretical based restrictions were not rejected and the information regarding external accounts are part of exchange rate misalignment and, during some important moments, this factor played a leading role to explain exchange rate misalignment for Brazilian economy.

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