Labor Earnings Dynamics in Post-Stabilization Brazil

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Abstract

This paper analyzes both the levels and evolution of wage inequality in the Brazilian formal labor market using administrative data from the Brazilian Ministry of Labor (RAIS) from 1994 to 2009. After the covariance structure of the log of real weekly wages is estimated and the variance of the log of real weekly wages is decomposed into its permanent and transitory components, we verify that nearly 60% of the inequality within age and education groups is explained by the permanent component, i.e., by time-invariant individual productive characteristics. During this period, wage inequality decreased by 29%. In the first years immediately after the macroeconomic stabilization (1994−1997), this decrease is explained entirely by reductions in the transitory component, suggesting that the end of the macroeconomic instability was a relevant factor in reducing inequality. In the second sub-period (1998−2009), the decrease is mostly explained by reductions in the permanent component. Finally, we show that education and age account for a sizable share of the permanent component (54% on average).

Keywords: Earnings dynamics, wage inequality, formal labor market, variance decomposition

JEL: J3, J6, O5

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1. Introduction

It has been well documented that Latin American countries have experienced decreased income inequality over the most recent two decades and that these decreases are mostly associated with decreased inequalities in labor earnings (e.g., Lopez-Calva and Lustig (2010) [31]; Azevedo et al. (2013) [2]). However, the underlying forces behind these trends remain the subject of debate. Some authors argue that such trends are mainly explained by structural changes in the economic fundamentals of these countries. For instance, relying on studies of Argentina, Brazil, and Mexico, Lustig et al. (2013) [33] explain the decrease in inequality by the decrease in the observed skill premia. Manacorda et al. (2010) [35] also find a decline in the skill premia and argue that it is largely explained by the sharp rise in the supply of secondary-level educated workers relative to primary-level educated workers in the 1980s and 1990s. Additionally, for selected countries in Latin America, there is evidence that openness to trade reduced wage differentials in the 1990s (e.g., Gonzaga et al. (2006) [22] and Ferreira et al. (2007) [18] for Brazil and Robertson (2004 14, 2007 45) for Mexico). However, other authors argue that the decrease in inequality may be attributable to more transient factors. For instance, Gasparini et al. (2011) [21] argue that the increase in the supply of more educated workers cannot account for the decrease in inequality. Other factors including changes in terms of trade (e.g., the commodities boom) and changes in trade and industrial policies are likely to be more important. The authors also suggest the possibility that skill mismatches reduce the productivity of highly educated workers. Additionally, Gasparini et al. (2009) [20] posit the hypothesis that the observed compressions in wage distributions may be associated with realignments after the strong shocks of the 1990s in the region.

We aim to shed some light on this debate by examining the relative importance of these two groups of factors in a unified framework. Specifically, we evaluate the relative roles played by the permanent and transitory components of wages in the changes in wage inequality in an important Latin American
country Brazil in the 1990s and 2000s. We rely on a large body of literature that discusses the causes of changes in wage inequalities in the United States and other developed countries. In particular, there is a branch of this literature that addresses this issue by quantifying the relative importance of transitory and permanent shocks over time. By examining the relative importance of each, it is possible to understand the underlying factors that might explain the evolution of inequality, which is helpful for understanding the functioning of the labor market and for designing appropriate public policies. This literature is well suited to help clarify the debate on the underlying causes of the decreases in wage inequality in the Latin American region.

In contrast to developed countries, Latin American countries (including Brazil) have recently experienced decreases in inequalities in their labor earnings. As we document below, the decreases in wage inequality also occur within groups by education level and age (at least in the country we analyze). To the best of our knowledge, there is no systematic data and analysis available regarding the evolution of unobserved skills and their returns in the region or on the relative importance of transitory and permanent shocks in explaining the decline in the region’s wage inequality observed during the post-stabilization period. This paper aims to fill this gap by exploring a unique panel dataset from Brazil that covers an extended period of time.

The permanent wage component relates to workers’ productivity characteristics (such as human capital), whereas the transitory component is associated with noise caused by economic instability. In other words, the permanent

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component is associated with individuals’ wage profiles during their lifetimes, whereas the transitory component represents stochastic fluctuations. A down-trend or uptrend in inequality may be explained by different factors. A decrease in inequality resulting mainly from a decrease in the permanent component is associated with a mobile labor market in which workers are able to change positions in wage distribution during their lifetimes. Individuals with lower lifetime earnings have relatively larger increases in their earnings. A change in demand in favor of less skilled workers would cause a shift in wage distribution; thus, the position occupied by each worker would change, which would lead to a decrease in the variance of the permanent component and, ultimately, in the variance of inequality. This result, for example, would be consistent with openness to trade where less-skilled workers are relatively more abundant.

By contrast, a decrease in inequality resulting from a decrease in the transitory component is related to lower mobility. In this case, changes occur in wage distribution, but the positions occupied by workers remain unchanged; there are no systematic changes in their lifetime earnings. Some possible causes for changes in the transitory component are macroeconomic shocks that affect workers differently, changes in competition and weaker worker-firm attachment (Haider (2001) [24]), changes in regulations or in union activities and even changes in the demand for temporary jobs and for self-employment (Moffitt and Gottschalk (2012) [40]).

Understanding which component is responsible for the variation in inequality is important both to check the impact on social welfare (given that permanent and transitory shocks have different effects) and to design public policies that are appropriate for the proper functioning of the labor market. If the permanent component is predominant, policies oriented toward enhancing worker education and qualifications and toward better matching between workers and firms are more appropriate. However, if the transitory component is stronger, policies should be adopted that mitigate the effects of any negative shocks workers may suffer in the labor market. Wage instability particularly when it results from an increase in the transitory component may lead to temporary changes in
consumption. Because individuals' welfare depends on their capacity to smooth consumption over their lifetimes (Friedman (1957) \[19\]), transitory shocks are more easily absorbed than permanent shocks and are therefore associated with reduced social welfare loss and reduced impact on inequality.


To the best of our knowledge, there is no such decomposition exercise for a developing country and finding evidence of transitory and permanent inequalities for a developing country is interesting in and of itself. Additionally, we have a rare opportunity to exploit a long-period longitudinal dataset from a representative country in Latin America to investigate possible explanations for the decreases in wage inequality experienced throughout the region in recent decades. This paper assesses the changes in formal wage inequality in Brazil between 1994 and 2009 using variance decomposition, as referenced in the labor economics literature. After the covariance structure of the log of real weekly wages is estimated and the variance of the log of real weekly wages is decomposed into permanent and transitory components, we verify that nearly 60% of the inequality within age and education groups is explained by the permanent component, i.e., by time-invariant individual productive characteristics. We also show that education accounts for a sizable share of this component (54%, on average).

The overall wage inequality decreased over the 1994−2009 period. This wage inequality decrease is also observed within formal and informal sectors. In the years immediately following the stabilization of the Brazilian economy (1994−1997), the decrease in formal wage inequality is explained entirely by
the decline in the transitory component of wage inequality. By contrast, in
the second sub-period (1998–2009), the decrease in formal wage inequality is
mostly explained by the decrease in the permanent component of wage inequal-
ity. These findings suggest, first, that (the end of) economic instability is an
important element in the explanation of the decrease in inequality, and its ef-
teffects can be immediately sensed in the labor market. Second, changes in the
fundamentals also have an important role in explaining the decrease in inequal-
ity following economic stabilization. Notably, because we use wage inequality
within education and age groups, the changes in the fundamentals are associated
with factors beyond education and demographic composition changes.

The paper is organized in seven sections, including this introduction. The
second section presents stylized facts about the economic performance and evo-
lution of wage inequality in Brazil over recent decades. The third section
presents the empirical models and estimation method. The fourth section de-
scribes the dataset and the sample used. The fifth section presents the results,
and the sixth section presents a more detailed discussion of certain results. The
seventh section concludes.

2. Economic Instability and Wage Inequality in Brazil

Latin American countries, including Brazil, experienced strong macroeco-
nomic instability in the 1980s and 1990s. This period included a sequence of
negative economic shocks that led to high inflation and volatile growth. Af-
fter four failed attempts to control inflation, the Brazilian government finally
launched a successful stabilization plan, the Real Plan, in 1994. Concurrently,
Brazil and the region implemented market structural reforms such as imple-
menting economic openness to trade, government reforms, privatization, dereg-
ulation and liberalizations in the financial sector throughout the 1990s (see, e.g.,

Brazil faced hyperinflation in the beginning of the 1990s, and inflation rates
were well above 1,000% a year in 1992 and 1993. The stabilizing Real Plan was
launched based on a pegged exchange rate in mid-1994. A new currency was established (the Real), and inflation was finally controlled. From 2,477% a year in 1993, the inflation rate decreased to a single-digit rate in 1996 and remained near the single-digit range during most of the subsequent period. Economic growth was volatile in the 1990s, ranging from 4% a year to no growth in the second part of the decade. In the 2000s, Brazil experienced better economic performance with higher growth rates (an average of approximately 4% annually, with the exception of 2009, the year of the global financial crisis). Adult unemployment rose after the price stabilization from approximately 5% to 10% in 2003 but decreased to approximately 7−8% by the end of the first decade of the 2000s.

Using the Brazilian National Household Survey (PNAD/IBGE) for selected years, we note that wage inequality decreased throughout the period for all waged workers and for both formal and informal waged workers separately. The average proportion of formal waged workers is 71% and the standard deviation is 0.02. Figure 1 presents the evolution of the variance of the log of real wages, and Figure 2 presents the variance of the residual of the regression of the log of real wages on age and education dummies.

We note that wage inequality decreased between 1993 and 2009. This pattern is observed whether considering the variance of the unconditional log of real wages or the variance of residuals. Figure 1 shows that there was a decrease in unconditional wage inequality during the period. Figure 2 shows that there was a decrease in within-group (age and education level) wage inequality during the period.

These trends are not sensitive to the inequality measure used; we found the
same patterns using the Gini coefficient and the Theil measure. Regardless of the inequality measure used, there is evidence of a decrease in wage inequality among both formal waged workers and informal waged workers. Understanding the factors that explain this decrease is the objective of the present paper.
The changes in the economic environment and in wage inequality may be linked in a number of ways. On the one hand, the structural changes in the Brazilian economy in recent decades may have caused permanent shocks in the labor market. Openness to trade, skilled-biased technological changes, and demographic and skill composition changes in the workforce may have changed the distribution of skills, occupations and their returns in the market.

On the other hand, alongside the structural changes, the end of economic instability may have contributed to the decrease in wage inequality by reducing the frequency of negative transitory shocks. The Brazilian economy experienced a number of external shocks in the 1990s but enjoyed a positive external environment in the 2000s with favorable terms of trade and a commodities boom. Moreover, before inflation was curbed, Brazilians had been living under high and volatile inflation for more than a decade. As a result, they had developed mechanisms to work around inflation, such as indexation mechanisms, specific financial contracts, and formal and informal labor agreements. When these mechanisms are unevenly distributed across individuals, inflation can exacerbate inequality. Indeed, Cysne et al. (2005) show formally that in an economy in which (i) the higher the inflation the more monetary assets are substituted by shopping time and (ii) the poor have more restricted access to financial assets, a positive link can be found between inflation and inequality as long as the productivity of the interest-bearing assets in the transacting technology is sufficiently high. In fact, there is evidence from Argentina that people do take more time to shop during an economic crisis (Mackenzie and Scharf, 2011). Additionally, there is evidence from Brazil that inflation and wage inequality are positively correlated (Cardoso et al. (1995), Barros et al. (2000) and Souza (2002)). Under very low inflation, this inequality-enhancing mechanism is no longer relevant.

In this paper, we will not examine one or some specific mechanisms directly. Instead, we will apply an earnings dynamics model that evaluates the relative importance of the permanent and transitory components associated with wage inequality changes during the post-stabilization period in Brazil.
3. Models and Estimation Methods

Variance decomposition models decompose covariance into two types of components: permanent and transitory. This type of model is commonly used in studies whose objective is to explain cross-sectional inequality and its evolution over time. By modeling a labor income function based on permanent and transitory components, the variance and covariance of earnings can be analyzed over time, which allows the importance of each component to be assessed in the cross-sectional variance as well as in their changes over time. To avoid participation self-selection bias issues, this study (like most studies) considers only economically active prime-age males.

3.1. The Covariance Structure

Consider the following canonical model\(^5\)

\[
y_{it} = \mu_{i} + v_{it} \tag{1}
\]

where \(y_{it}\) is a measure of individual \(i\)'s earnings at \(t\) and corresponds to the sum of a time-invariant individual component \(\mu_{i}\), which is uncorrelated between individuals, and of a serially uncorrelated transitory component (white noise), \(v_{it}\). We also have that:

\[
\mu_{i} \sim iid(\mu_{i}, \sigma_{\mu}^{2}) \tag{2}
\]

\[
v_{it} \sim iid(0, \sigma_{v}^{2}) \tag{3}
\]

Assuming both components are orthogonal, such that \(Cov(\mu_{i}, v_{it}) = 0\), we obtain:

\(^5\)This model is commonly used in the related literature. It was first reported by Lillard and Willis (1978) \(29\) and described in further detail by Ramos (2003) \(42\).
\[ Cov(y_{it}, y_{is}) = \begin{cases} \sigma^2_\mu + \sigma^2_v & \text{for } t = s \\ \sigma^2_\mu & \text{for } t \neq s \end{cases} \] (4)

The variance of the permanent component \((\sigma^2_\mu)\) represents the effect of individuals’ characteristics whether observable or not that do not change over time, fully determining the covariance between different time periods, as the transitory component corresponds to white noise.

Because this approach has strict constraints on the dynamic structure, a number of authors have proposed more sophisticated and complex approaches. The permanent component may be modified by including time-varying factor models (factor loads), and the transitory component may be treated as a time series model (ARMA\((p, q)\))\(^6\). In matrix notation, we have:

\[ y_{it} = \rho_i' \mu_i + v_{it} \] (5)

\[ a(L)v_{it} = m(L)\epsilon_{it} \] (6)

where \(\rho_i' = (\rho_{1t}, \ldots, \rho_{gt})\) is a \(1 \times g\) vector of coefficients that may or may not be known, \(\mu_i' = (\mu_1, \ldots, \mu_g)\) is a \(1 \times g\) vector of time invariant disturbances with:

\[ E(\mu_i, \mu_j') = \begin{cases} \sigma^2_\mu & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \] (7)

and \(L\) is a lag operator such that the roots of \(m(L) = 0\) lie outside the unit circle by hypothesis, and \(\epsilon_{it}\) is white noise with:

\[ E(\epsilon_{it}, \epsilon_{jt'}) = \begin{cases} \sigma^2_\epsilon & \text{if } i = j \text{ and } t = t^* \\ 0 & \text{otherwise} \end{cases} \] (8)

In this case, the covariance matrix is given by:

$$\Omega = E(y_{it}y_{it}^\prime) = \rho \sigma^2 \rho' + E(v_{it}v_{it}^\prime)$$  \hspace{1cm} (9)$$

Depending on the ARMA($p,q$) model used for $v_{it}$, the covariance matrix changes via $E(v_{it}v_{it}^\prime)$. We tested the following different specifications: AR(1), ARMA(1,1), ARMA(1,2), AR(2), ARMA(2,1) and ARMA(2,2). AR(1) was the specification with the best fit, and this model is described below. Assuming $v_{it}$ follows an AR (1) process, we have:

$$y_{it} = \rho_t \mu_i + v_{it}$$ \hspace{1cm} (10)

$$v_{it} = \phi v_{it-1} + \epsilon_{it}$$ \hspace{1cm} (11)

where:

$$\mu_i \sim iid(\bar{\mu}_i, \sigma^2_\mu)$$ \hspace{1cm} (12)

$$\epsilon_{it} \sim N(0, \sigma^2_\epsilon)$$ \hspace{1cm} (13)

$$E(\epsilon_{it}\epsilon_{jt}^\prime) = \begin{cases} 
\sigma^2_\epsilon & \text{if } i = j \text{ and } t = t^* \\
0 & \text{otherwise} 
\end{cases}$$ \hspace{1cm} (14)

$$\text{Cov}(\mu_i, v_{it}) = 0$$ \hspace{1cm} (15)

Including parameter $\sigma^2_{v_{-1}}$, which corresponds to the variance of the transitory component ($v_{it}$) in the period immediately before the initial period, it is possible to identify the variance of $y_{it}$ as:

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7Specifications that involve an autoregressive process of order 2 were tested by assuming stationarity.
\[ \text{Var}(y_{it}) = \rho^2 \sigma_{\mu}^2 + \phi^2(t+1)\sigma_{v_{-1}}^2 + \sum_{j=0}^{t} \phi^{2(t-j)} \sigma_{\epsilon}^2 \]  

(16)

Additionally, the covariance between periods \( t \) and \( t+s \) is:

\[ \text{Cov}(y_{it}, y_{it+s}) = \rho_t \rho_{t+s} \sigma_{\mu}^2 + \phi^{2t+s+2} \sigma_{v_{-1}}^2 + \sum_{j=0}^{t} \phi^{2(t+j)} \sigma_{\epsilon}^2 \]  

(17)

Inequality in the labor market may be decomposed as a whole (by directly analyzing wages) and/or within age and education groups. To do so, the same analysis is conducted using the residuals of the regression of the log of wages on the age and education indicator variables. Because we aim to investigate the trends in unobserved skills and to control for changes in workers’ demographic and education compositions within the formal sector, we first present the results for the residuals of the log of wages. We also performed the same decomposition for unconditional wages, and the results, which are presented succinctly in subsection 6.2 below, are qualitatively similar.

3.2. Estimation

Using individual panel data on the log of real weekly wages over \( T \) periods, it is possible to calculate observed covariance matrices \( C \), and the covariance matrix of observed covariance, \( V \). As shown in the previous subsection, the elements of the covariance matrix for the log of wages can be modeled in various and distinct ways. Consider any model that depends on a vector of parameters \( b \) (whose size is smaller than \( T \)) such that \( m = f(b) \). We can estimate \( b \) using minimum distance methods in which the following expression is minimized:

\[ (m - f(b))' A (m - f(b)) \]  

(18)

where \( A \) corresponds to a positive definite weighting matrix (Dickens (2000)). Hence, \( b \) is chosen to reduce the distance between the observed moments.

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8 These results are available upon request.
9 The calculation method is available in Appendix A.
and the theoretical moments predicted by the model \( f(b) \) as close to zero as possible. For the specific case of the AR(1) model with factor loads, the expression to be minimized is indicated in Appendix B.

Let \( R^{-1} \) be the inverse matrix of \( R = PP' \), where \( V \) is the covariance matrix of the log of wage covariance, and \( P = I - F(F'AF)^{-1}F' \), where \( F = F(b^*) \) is the Jacobian matrix assessed in \( b^* \) (the estimated value of \( b \)):

\[
F(b^*) = \frac{\partial f(b)}{\partial b}(b^*)
\]  

(19)

Inference is made based on the following statistic, which, under the null hypothesis that \( m = f(b^*) \) (equivalent to the specification used being correct), has a chi-squared asymptotic distribution (as demonstrated by Newey (1985) [41]):

\[
n[(m - f(b^*))'R^{-1}(m - f(b^*))] \sim \chi^2_h
\]  

(20)

where \( R^{-1} \) corresponds to the generalized inverse of matrix \( R \), and \( h \) is the number of degrees of freedom, representing the difference between the size of \( m \) and the rank of the Jacobian matrix assessed in \( b^*(F(b^*)) \). This statistic may be used to test the general structure of the model and to compare different methods.

The standard errors of the estimated parameters can be obtained by calculating the asymptotic variance of \( b^* \), thus allowing for \( t \) tests to be conducted for each of the estimated coefficients. According to Chamberlain (1984) [12], under some conditions, \( b^* \) converges to the true value of \( b \) and \( \sqrt{n}(b^* - b) \) converges in distribution to \( N(0, \Omega) \), where \( \Omega = (F'AF)^{-1}F'AVAF(F'AF)^{-1} \). We can treat \( b^* \) as if:

\[
b^* \sim N(b, \Omega/n)
\]  

(21)

such that the asymptotic variance of \( b^* \) can be estimated using:

\[10\] Rao (1962) [43].
\[ AVar(b^*) = \frac{((F'AF)^{-1}F'AVAF(F'AF)^{-1})}{n} \tag{22} \]

The choice of A determines the type of minimum distance to be estimated. One alternative would be the optimal minimum distance method, where \( A = V^{-1} \) and where \( V^{-1} \) corresponds to the inverse matrix of V (covariance matrix of covariance), thereby minimizing:

\[ (m - \hat{f}(b))^\prime V^{-1}(m - \hat{f}(b)) \tag{23} \]

This method is referred to as optimal because it minimizes the asymptotic variance of \( b^* \), reducing it to \( \Omega = (F'V^{-1}F)^{-1} \).

Another alternative is the equally weighted minimum distance, where \( A = I \), minimizing \( (m - \hat{f}(b))^\prime (m - \hat{f}(b)) \). In this case, the weight attributed to each moment is the same. However, because an unbalanced panel is used and different numbers of observations are thus utilized for each estimated moment, giving equal weights to different moments may not be appropriate.

Following Haider’s (2001) suggestion, we choose \( A = \Pi^{-1}\Pi^{-1} \), where:

\[
\Pi = \begin{bmatrix}
\frac{n_1}{n} & 0 & \cdots & 0 \\
0 & \frac{n_2}{n} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{n_L}{n}
\end{bmatrix} \tag{24}
\]

and \( n_l \) corresponds to the number of observations used to calculate moment \( l \), and \( n \) is the total number of observations. Therefore, a larger weight is attributed to moments for which there are proportionally more available observations.

4. Data

The decomposition exercise requires the availability of a long period panel dataset of workers. Brazil has such a dataset. We use data from the Annual Reports of Social Information (RAIS) from the Brazilian Ministry of Labor.
and Employment. Firms that hire workers formally report information about the wages and occupations of all their workers to the Ministry on an annual basis in addition to individual characteristics such as education, gender and age. Because each worker has a unique identification number that comes from registering in the Brazilian Social Integration Program (PIS), it is possible to follow individuals over time as long as they stay in the formal labor market.

The advantage of this panel dataset is that it follows a large number of workers over a relatively long period of time. In particular, it covers a previous period of macroeconomic instability that is followed by a period of stability. The drawback is that it does not include informal workers. However, decomposing wage inequality into permanent and transitory components among formal workers in a developing country is interesting in and of itself, and, more importantly, it still sheds light on the roles played by economic instability and lifetime earnings distribution in changing wage inequality in the Latin American region.

There were more than 54 million PIS identification numbers from male individuals in the RAIS administrative database that appeared at least once in the period between 1994 and 2009. A 5% random sample was chosen from the list of the PIS identification numbers of all male individuals. Furthermore, the sample was restricted to male individuals aged 25 to 64 years old with reported information regarding at least one valid wage.

We begin in 1994 to minimize measurement error problems. Before 1994, wages were reported in different currencies. Because of the high inflation, nominal wages changed monthly or even weekly, and it is not clear that wage changes were reported to the Ministry correctly. We end in 2009 because it is the last year with available information to us.

\[\text{11}\] All formal employers in Brazil must provide information on their employees to the Ministry of Labor and Employment (MTE), pursuant to Executive Order n. 76.9000 of September 23, 1975.

\[\text{12}\] The decision to investigate wage inequality only among males was based on the fact that the participation of females in the labor market has peculiarities that are outside of the scope of this study.
Table 1 reports the number of available observations in each stage of this sample selection process.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Individuals (Male)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appeared at least once between 1994 and 2009</td>
<td>54,344,896</td>
</tr>
<tr>
<td>Sample of 5%</td>
<td>2,717,245</td>
</tr>
<tr>
<td>Sample of 5%, 25 to 64 years old</td>
<td>2,094,360</td>
</tr>
<tr>
<td>Sample of 5%, 25 to 64 years old with information reported for at least one wage</td>
<td>1,870,098</td>
</tr>
</tbody>
</table>

Table 1: Number of Observations in Each Stage. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

An unbalanced panel dataset is formed from this sample. Table C.1 in Appendix C presents the information availability matrix. It shows the number of observations available each year as well as the number of PIS identification numbers that are matched for each pair of years. There were approximately 550 thousand observations in 1994 and more than 1 million observations in 2009, which reflects the increase in formal jobs observed during this period. Moreover, approximately 250 thousand observations were found in both 1994 and 2009.

The age profile of this sample is presented in Table C.2 of Appendix C. The first columns of Table C.2 show the annual percentages of individuals by age. The percentages of individuals who were 25 to 34 years old and 35 to 44 years old decreased between 1994 and 2009, whereas the shares of individuals who were 45 to 54 years old and 55 to 64 years old increased. In other words, we notice that the proportion of older individuals increased during the period analyzed. Nonetheless, the great majority of individuals were 25 to 44 years old and encompassed 71% of the sample in 2009. There were increases in both the mean and the standard deviation during this period. The average age increased from 37.45 years old in 1994 to 38.89 in 2009. The standard deviation increased from 9.37 years in 1994 to 10.08 in 2009.
The sample’s education profile is presented in Figure 3 and shows the percentage of individuals by their levels of schooling. The percentage of individuals with at most elementary education decreased during the period. This reduction is attributable mainly to the decline in the percentage of individuals who had not finished the first cycle (up to 5th grade) of elementary education and of individuals who managed to finish this cycle only. In addition, the percentage of individuals who had finished high school increased significantly during the period, which is consistent with the increase in the educational level of the workforce observed during the period.

The RAIS reports individuals’ monthly wages in December of every year in minimum wage units for the period analyzed (1994 to 2009). Because minimum wage values vary across years, workers’ wages were converted to Brazilian currency units (Reais) using the nominal value of the minimum wage from December of the corresponding year. Real values were obtained using the Brazilian Consumer Price Index (INPC) from the Brazilian Census Bureau (IBGE). The weekly real wage is obtained by dividing the monthly wage by contracted weekly working hours. The weekly real wage is reported in 2009 Reais (R$). Table C.3 in Appendix C presents descriptive statistics of the log of real weekly wages and shows that the mean of the log of the real weekly wage increased during the period as did its minimum and its maximum values. On average, wages increased 0.17 log−points between 1994 and 2009.

Given these wage data, it is possible to compute the covariance matrix of the (within-group age and education level) log of real weekly wage for December of each year. They are obtained as residuals from cross-sectional regressions of the log of real weekly wages on the age and education indicator variables. In this case, because information on age and education is also necessary, some observations with missing age or education information were lost, leaving 1,869,446 individuals with valid information. The covariance matrix is presented in Table C.4 in Appendix C.

Figures 4 and 5 presents variances and selected covariances (one-, five- and ten-year lags) for the log of the real weekly wage (Figure 4) and the residuals
Figure 3: Descriptive Statistics: Education. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

Wage inequality decreased during the period, as measured by the RAIS administrative dataset during the period. This trend is similar to that observed
using Brazilian National Household Surveys (PNAD/IBGE). Our goal is to explain what factors led to this inequality decrease by considering the relative importance of the permanent and transitory components associated with these changes. The trends depicted in Figures 4 and 5 can give us some hint of that. As time passes, the effects of transitory shocks eventually disappear. It is plausible to consider that the larger the interval for the covariance, the more important is the role played by the permanent component. Figures 4 and 5 illustrate the covariance for some numbers from previous periods (lags). It is notable that covariance values remain high and relatively close to the variance values after a number of periods, indicating that the permanent component is playing an important role in explaining inequality.
5. Results

5.1. Wage Inequality within Age and Schooling Groups

5.1.1. Model Selection

We tested multiple specifications to estimate the covariance structure of the log of the real weekly wage within age and schooling groups.\footnote{Tested specifications include the following: the canonical model, the model with factor load, the AR(1) model with factor load, and the ARMA(1,1) model with factor load.} The estimations were obtained using the modified weighted minimum distance method (equations 18 and 24). The different models can be compared by their chi-squared statistics. Table 2 presents the chi-squared statistics for all tested models.

The AR (1) specification with factor loads has the lowest chi-squared statistic\footnote{The chi-squared statistic values indicate rejection of the general model structures, suggesting that estimating more embracing models would be preferable. Ramos (2003)} For this reason, we present the results for this model. Estimation results

Figure 5: Covariance of the residuals. Behavior of Inequality Measured by Variance and Covariance. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).
are shown in Table 2.

Note that all the included parameters are significantly different from zero at 1%. Moreover, the model is stationary. The estimated parameter $\phi$ is such that $|\phi| < 1$. Notably, the estimated returns of the permanent component ($\rho$) first present an upward trend until 1997 and then decrease thereafter.

5.1.2. The Decomposition of the Levels of Wage Inequality

Based on the model, we decompose the cross-sectional wage inequalities into permanent and transitory components. The results are presented in Figure 6. The permanent component accounts for the largest share of total cross-sectional variance, which is 60% between 1994 and 2009, on average. The share of the permanent component ranges from 43% in 1994 to 64% in 2002. In the final year of the sample, this component accounted for 59.05% of the within-group wage inequality.

The overall within-group wage inequality for age and education groups decreases over time. The permanent component increases between 1994 and 1997, but this increase is followed by a decline in subsequent years, particularly from 2003 onwards. Changes in the permanent component are attributable to changes in its returns. Thus, the model estimates that the return on individual permanent abilities increased until 1997 and remained relatively stable until 2002.

---

Table 2: Chi-Squared Statistics of the Tested Models. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

<table>
<thead>
<tr>
<th></th>
<th>Canonical</th>
<th>Factor Loads</th>
<th>AR(1) with Factor Loads</th>
<th>ARMA(1,1) with Factor Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-squared statistic</td>
<td>751,281.51</td>
<td>795,808.02</td>
<td>221,557.38</td>
<td>192,498,409,553.30</td>
</tr>
</tbody>
</table>

---

comments that the rejection of the null hypothesis at conventional significance levels is a common finding in this literature, and the chi-squared statistic is thus used to compare different models.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimation Results</th>
<th>Parameter</th>
<th>Estimation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_0$</td>
<td>0.286*</td>
<td>$\rho_{2001}$</td>
<td>1.098*</td>
</tr>
<tr>
<td>(0.001)</td>
<td></td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{v_{-1}}$</td>
<td>0.505*</td>
<td>$\rho_{2002}$</td>
<td>1.098*</td>
</tr>
<tr>
<td>(0.001)</td>
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<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_\epsilon$</td>
<td>0.078*</td>
<td>$\rho_{2003}$</td>
<td>1.071*</td>
</tr>
<tr>
<td>(0.000)</td>
<td></td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.775*</td>
<td>$\rho_{2004}$</td>
<td>1.078*</td>
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<tr>
<td>(0.000)</td>
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<td>(0.001)</td>
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</tr>
<tr>
<td>$\rho_{1995}$</td>
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<td>$\rho_{2005}$</td>
<td>1.053*</td>
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<td>(0.001)</td>
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<td>(0.001)</td>
<td></td>
</tr>
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<td>$\rho_{1996}$</td>
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<td>$\rho_{2006}$</td>
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<tr>
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<td>(0.002)</td>
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<tr>
<td>$\rho_{1997}$</td>
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<td>$\rho_{2007}$</td>
<td>1.023*</td>
</tr>
<tr>
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<td>(0.001)</td>
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</tr>
<tr>
<td>$\rho_{1998}$</td>
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<td>$\rho_{2008}$</td>
<td>1.009*</td>
</tr>
<tr>
<td>(0.001)</td>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{1999}$</td>
<td>1.105*</td>
<td>$\rho_{2009}$</td>
<td>0.990*</td>
</tr>
<tr>
<td>(0.001)</td>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{2000}$</td>
<td>1.096*</td>
<td>$\chi^2$ Statistics</td>
<td>221.397.31</td>
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<tr>
<td>(0.001)</td>
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<td></td>
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</tbody>
</table>

Table 3: Estimation of the AR(1) Model with Factor Load for Wage Variance within Age and Education Groups. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS). Note: Standard errors in brackets. The sign * indicates significance at 1%.

where it declined steadily thereafter.

By contrast, the transitory component presents a different pattern. It decreases in the beginning of the analyzed period and remains stable from 2000 onwards. This pattern seems to reflect the decreasing instability and fewer shocks in the Brazilian economy in the 1990s. Indeed, Brazil’s macro stabilization plan, the Real Plan, was launched in 1994 and succeeded in bringing
inflation under control, from an annual rate of 2.694% in January 1994 to 5.92% annually in January 2001. Concurrently, a number of structural reforms were undertaken during the same period, including privatization, economic openness, restructuring of the financial sector, and restructuring of fiscal and monetary policy.

Figure 6 shows the evolution of the covariances between $t$ and $t - 1$ as predicted by the model. The estimated covariances decreased during the period. The permanent component accounts for 51%, 68% and 65% of the total covariance in 1995, 2000 and 2009, respectively.

Figures 7 presents five-year lag period covariances. The permanent component accounts for 80%, 85% and 85% of the total covariance in 2000, 2005 and 2009, respectively.

Finally, Figure 8 shows ten-year lag covariances. Covariance in the 10th lag also decreased during the period from 1995 onwards. The patterns of the permanent and transitory components follow the
path described for variance and covariance (previous period and 5\textsuperscript{th} lag).

5.1.3. Changes in Wage Inequality

Having estimated each of the covariance components, it is now possible to verify the extent to which each contributed to the decrease in inequality between 1994 and 2009. Table \ref{table:decomposition} contains this information.

\begin{table}[h]
\centering
\begin{tabular}{l|c|c|c}
\hline
 & Decrease in Estimated Variance & Contribution of the Permanent Component & Contribution of the Transitory Component \\
\hline
Wages within Age and Schooling Groups & 28.77\% & 2.89\% & 97.11\% \\
\hline
\end{tabular}
\caption{Decomposition of Wage Variance Changes from 1994 to 2009. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).}
\end{table}
Comparing 1994 with 2009, there is a 28.77% decline in estimated total within-group variance. The change in the permanent component accounts for 2.89% of that decrease, whereas the transitory component change accounts for 97.11%. Thus, when comparing the two extreme years, the decline in wage inequality is explained almost entirely by the decrease in transitory inequality.

Transitory inequality declined throughout the period. However, permanent inequality presented different patterns in different periods. First, it increased between 1994 and 1997, then became stable until 2002, and decreased thereafter. Although the general picture seems to highlight the finding that the decrease in wage inequality is mostly explained by the decrease in the transitory component, the interaction between the permanent and transitory components suggests a more subtle and nuanced story. To shed light on these interactions, we further split the overall period (1994–2009) into two sub-periods and analyze them separately to check each component’s contributions to the reduction in inequality.
in each sub-period.

Table 5 shows the decomposition results for each sub-period. The first sub-period ranges from 1994 to 1997, and the second ranges from 1998 to 2009. Although this division might seem ad hoc, we chose these sub-periods according to the estimated trends in the permanent component. The first sub-period is associated with an increase in the permanent component, and the second sub-period is associated with its decrease.

Between 1994 and 1997, there is an 11.22% decrease in the estimated variance, which the transitory component fully accounted for because the permanent component experienced an increase during the period (24.88% from its initial value) that was more than compensated for by the decline in the transitory component (38.34% from its initial value). Table 5 illustrates this finding by means of the negative contribution of the permanent component to the decline in variance and by means of the contribution of the transitory component, which
was greater than 100%. If there were no decrease in transitory inequality, total inequality would have increased based on the increase in permanent inequality. Alternatively, if there were no increase in permanent inequality, the total inequality decrease would have been twice as great.

There is a different story after 1997. The (within age and education groups) estimated variance of log-wages decreased by 16.90% between 1998 and 2009. Our model predicts that the permanent component accounts for 74.96% of that decline and the transitory component is associated with the remaining 25.04%

Table 5: Decomposition of Wage Variance by Periods within Age and Schooling Groups. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

<table>
<thead>
<tr>
<th>Period</th>
<th>Decline in Variance</th>
<th>Contribution of the Permanent Component</th>
<th>Contribution of the Transitory Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994 to 1997</td>
<td>11.22%</td>
<td>-95.10%</td>
<td>195.10%</td>
</tr>
<tr>
<td>1998 to 2009</td>
<td>16.90%</td>
<td>74.96%</td>
<td>25.04%</td>
</tr>
</tbody>
</table>

Notably, when the 1994–2009 period is taken as a whole, the decline in wage inequality can almost be entirely attributed to the reduction in the transitory component. However, when this period is split into two sub-periods, this finding does not hold separately for each. The decrease in inequality between 1994 and 1997 results from the reduction in the transitory component alone, which seems to be related to the decline in economic instability after implementation of the Real Plan in Brazil. By contrast, the decrease in inequality observed in the 2000s is mainly associated with the decrease in the permanent component through decreases in returns on workers’ time-invariant productive characteristics over and above age and education. This finding is related to the existence of a more stable scenario in which there is less room for reduction of the transitory component. In conjunction with a more stable macroeconomic environment, other forces seem to have been in place that led to the decrease in the returns on individuals’ time-invariant characteristics beyond age and education. Hence,
there is evidence that the decline in inequality in terms of schooling is not solely accountable for the decrease in wage inequality resulting from the permanent component.

6. Discussion

6.1. Mobility

The permanent component is directly associated with earnings mobility. Increases in this component are associated with less mobility and with a strict labor market in which individuals seldom change positions within the wage distribution over their lifetimes. Conversely, declines in this component are associated with greater mobility.

We estimate that the permanent component accounts for 60% of within-group wage inequality by age and education level, on average. This estimate implies that individuals’ time-invariant characteristics beyond age and education largely account for their positions in the wage distribution.

The fact that the permanent component increased between 1994 and 1997 and decreased thereafter indicates that the initial position occupied by individuals in the wage distribution plays a crucial role in their potential future positions over their lifetimes. Nonetheless, the continuous decrease in the permanent component throughout the 2000s suggests that long-run mobility has increased in Brazil’s formal labor market.

6.2. The Role of Age and Education

We have documented thus far the evolution of wage inequality within age and education groups in Brazil. A number of studies have highlighted the fact that (unconditional) wage inequality in Latin America and, in particular, in Brazil, is largely explained by a few observable productivity characteristics, such as workers’ age and educational attainment and their returns (e.g., dos Reis and Barros (1991) \[16\] and Lustig et al. (2013) \[33\]). We can shed some light on this discussion by estimating the relative importance of age and education
distribution on the wage distribution. Specifically, we can estimate how much of the permanent inequality is explained by observable characteristics such as age and education.

To undertake this estimation, we perform the same permanent-transitory decomposition for the (unconditional) log of real weekly wages using the same preferred model from the previous section\textsuperscript{15}. Table 6 presents the decomposition results. The patterns are similar to those found when considering wage inequality within age and education groups.

<table>
<thead>
<tr>
<th>Period</th>
<th>Decline in Variance</th>
<th>Contribution of the Permanent Component</th>
<th>Contribution of the Transitory Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994 to 2009</td>
<td>25.81%</td>
<td>29.73%</td>
<td>70.27%</td>
</tr>
<tr>
<td>1994 to 1997</td>
<td>9.70%</td>
<td>-73.08%</td>
<td>173.08%</td>
</tr>
<tr>
<td>1998 to 2009</td>
<td>15.92%</td>
<td>94.99%</td>
<td>5.01%</td>
</tr>
</tbody>
</table>

Table 6: Decomposition of Wage Variance by Periods (Unconditional). Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

The unconditional log wage variance declined by almost 26% between 1994 and 2009. Among all this change, the change in the transitory component accounts for 70.27% of the total change. As with the results of the log of wage residual decomposition, the change in the transitory component entirely accounts for the inequality decrease in the sub-period immediately after the economy stabilized. By contrast, the change in the permanent component accounts for 95% of the decrease in inequality in the 2000s.

Additionally, we can evaluate how much the observable characteristics of age and education explain permanent wage inequality. Let $P_G$ be the permanent component of the within-group wage inequality for age and education, and let $P$.

\textsuperscript{15}In this analysis, we calculate the covariance structure of the unconditional log of the real weekly wage rather than using the residuals of the regression of the log of the real weekly wage on age and formal education dummies. Results are available in Appendix D.
be the permanent component of inequality of the unconditional log-wage. The share of the permanent component attributed to the age and education variables can be obtained by:

$$s = 1 - \frac{P}{P}$$  \hspace{1cm} (25)

Figure 10 depicts the evolution of the share of the permanent component attributed to age and education calculated by $s$.

![Figure 10: Evolution of the Permanent Component Share Attributed to Age and Schooling. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).](image)

The dispersion of the wages among age and education groups represents more than 50% of the permanent component of the log-wage variance. Notably, this share declines sharply between 1994 and 1997 (from 59 to 53%), becomes stable between 1998 and 2005, and increases – albeit slightly – from 2006 onward.

Although the portion of the permanent component associated with age and education accounts for a sizable share of the permanent inequality, other time-invariant individual factors beyond age and education are also important. On
average, 46% of the permanent inequality is associated with these other factors. Figure 11 presents the returns on permanent attributes in both situations (unconditional wages and residual wages). The returns have similar declining patterns throughout the period, which suggests that the returns on productive characteristics associated with education and age as well as with other time-invariant characteristics presented the same behavior. In other words, there appears to be a decrease in the average returns of several dimensions of human capital (observable and non-observable) in the Brazilian formal labor market.

Figure 11: Behavior of the Returns on the Permanent Component. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).

7. Conclusion

In this study, we investigate the roles played by the permanent and transitory wage components in the evolution of wage inequality in Brazil’s formal labor market between 1994 and 2009. Using the variance of the log of the real weekly wage as a measure of inequality, we document that within age and education group inequality decreased by 28.77%. Applying variance decomposition
methods to assess the relative importance of permanent and transitory wage components, we find evidence that the decrease in wage inequality stems from the transitory component, which accounts for 97.11% of the reduction in the variance, whereas the permanent component accounts for only 2.89% of this decrease between 1994 and 2009. Qualitatively similar results are obtained using the unconditional log of real weekly wages. Therefore, the decline in wage inequality might be attributed to the decrease in economic instability that was observed during the period.

Moreover, by separately assessing the evolution of inequality between 1994 and 1997 and between 1998 and 2009, we also document that the decrease in wage inequality observed in the first sub-period results from the decrease in the transitory component alone, a result that is likely to be associated with the macroeconomic stability attained during this first sub-period. By contrast, the decrease in inequality observed in the second sub-period is mainly associated with the reduction in permanent inequality. This decrease is explained by the decline in market returns on individuals’ observable and unobservable productivity characteristics.

These findings are comparable with those of other studies on the evolution of wage inequality in developed countries. With respect to the United States, a number of studies document an increase in wage inequality between the late 1960s and the early 1990s. The permanent and transitory components each account for approximately 50% of this increase in inequality. Haider (2001) [25] investigates this period as a whole and attributes the results to skill-biased technological changes, to the expansion of international trade, and to the effect of education, which accounts for roughly one-third of the permanent component. Moffitt and Gottschalk (2011) [39] analyze the 1970s and 1980s and also find that both components contributed equally to the increase in inequality. In another study, Moffitt and Gottschalk (2012) [40] extend the study period to 2004. The authors highlight the role of the transitory component in explaining the increase in wage inequality that was observed in the 1970s and 1980s — associating it with economic instability — and emphasize the role of the per-
manent component, which had a remarkable increase after the mid-1990s and accounted for two-thirds of the increase in wage inequality that was observed thereafter. Studies on the United Kingdom also show that both components contributed equally to the increase in inequality between the 1970s and 1990s (Dickens (2000) [15]). In the early 1990s, the permanent component predominated but later lost ground in favor of the transitory component during the same decade (Ramos (2003) [42]). Finally, in a study of Canada for the period between 1976 and 1992, Baker and Solon (2003) [4] also demonstrated that both components were equally important for the increase in wage inequality.

Note that the results for Brazil during the 1994 – 2009 period differ from those reported for developed countries. Although there has been an increase in wage inequality in developed countries in recent decades, Brazil experienced a decrease in wage inequality during the same period. Notably, the proportion of change attributed to each component is not as well-balanced as it is in developed countries in which each component accounts for approximately 50% of the wage variation. When explaining the decrease in formal wage inequality in Brazil, the transitory component effect completely explains the wage inequality decrease between 1994 and 1997 (the period immediately after the stabilization of the economy), whereas the permanent component effect prevails in the 2000s.

Furthermore, the permanent component of the wage inequality within age and education groups is compared with that of unconditional wage inequality to examine the importance of the increase in schooling on inequality. We find that age and formal education account for approximately 54% of permanent inequality between 1994 and 2009.

These findings contribute to the debate on the underlying causes of the decrease in wage inequality in the Latin American region and suggest that the end of macroeconomic instability was a relevant factor in reducing inequality. Thus, the maintenance of macroeconomic stability in the region seems to be important for reasons of inequality in addition to myriad other reasons. Moreover, permanent changes in the labor market in the 2000s seem to have occurred over and above the expansion of education. The decrease in the returns on permanent
unobservable skills suggests that the relative demand for lower-skilled workers increased during the period.

References


Appendices

Appendix A

The covariance matrix of the log of wage and the covariance matrix of the elements of this matrix can be estimated following Abowd and Card (1989) [1], except for some modifications to adjust the method to fit the unbalanced panel case. Let \( y_i \) be the deviation of the log of the real weekly wage of individual \( i \) at \( t \) from the mean log of wages for that year. \( T \) time periods are taken into account such that \( \tilde{y}_i \) is a \( 1 \times T \) vector that contains information on the deviations in the log of the wage from the mean of each year for individual \( i \):
\[ \tilde{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \]  

(1)

The covariance matrix, \( C \), is a \( T \times T \) matrix whose elements, \( C_{jk} \), where \( j \in [1, T] \) and \( k \in [1, T] \), are given by:

\[ C_{jk} = \frac{1}{n_{jk}} \sum_i \tilde{y}_{ij} \tilde{y}_{ik} \]  

(2)

where \( n_{jk} \) stands for the number of individuals who contribute to the analyzed period. To define the covariance matrix of this covariance matrix, \( V \), define \( m_{ij} \) as the vector with the distinct elements of product \( \tilde{y}_{ij} \tilde{y}_{ik} \). The elements of \( C \) can be estimated again by:

\[ m_{jk} = \frac{1}{n_{jk}} \sum_i m_{ij} \]  

(3)

Finally, define \( s_{ij} = m_{ij} - m_{jk} \), such that the elements of the covariance matrix of covariance \( jk \) and \( mn \), \( V_{jkmn} \) where \( j \in [1, T] \), \( k \in [1, T] \), \( m \in [1, T] \) and \( n \in [1, T] \), are defined by:

\[ V_{jkmn} = \frac{1}{n_{jkmn}} \sum_i s_{ij} s_{imn} \]  

(4)

where \( n_{jkmn} \) stands for the number of individuals that contribute both to period \( jk \) and to period \( mn \). In other words, \( n_{jkmn} \) is the number of individuals for whom there is available information on wages at \( j, k, m \) and \( n \). Using this approach, it is possible to calculate the observed covariance matrices, \( C \), and the covariance matrix of covariance, \( V \).

Appendix B

Considering the case of the AR(1) model with factor loads, the expression to be minimized is:
\[ S'AS \]  

where:

\[
S = \begin{pmatrix}
Var(y_{1994}) - (\rho_{1994}^2 \sigma^2_\mu + \phi^2 \sigma^2_{v-1} + \sigma^2_\epsilon)
& Cov(y_{1994}, y_{1995}) - (\rho_{1994}\rho_{1995} \sigma^2_\mu + \phi^3 \sigma^2_{v-1} + \phi \sigma^2_\epsilon) \\
Cov(y_{1994}, y_{1995}) - (\rho_{1994}\rho_{1995} \sigma^2_\mu + \phi^3 \sigma^2_{v-1} + \phi \sigma^2_\epsilon)
& Cov(y_{1994}, y_{1996}) - (\rho_{1994}\rho_{1996} \sigma^2_\mu + \phi^4 \sigma^2_{v-1} + \phi^2 \sigma^2_\epsilon) \\
Cov(y_{1994}, y_{1997}) - (\rho_{1994}\rho_{1997} \sigma^2_\mu + \phi^5 \sigma^2_{v-1} + \phi^3 \sigma^2_\epsilon)
& \vdots \\
Cov(y_{2008}, y_{2009}) - (\rho_{2008}\rho_{2009} \sigma^2_\mu + \phi^{31} \sigma^2_{v-1} + \sum_{j=0}^{14} \phi^{29-2j} \sigma^2_\epsilon)
& Var(y_{2009}) - (\rho_{2009}^2 \sigma^2_\mu + \phi^{32} \sigma^2_{v-1} + \sum_{j=0}^{15} \phi^{2(15-j)} \sigma^2_\epsilon)
\end{pmatrix}
\]

Appendix C

Table C.1 shows an information availability matrix. The cells on the main diagonal display the number of available observations from each year, and the remaining cells display the number of individuals for whom there is information for both selected years.

Tables C.2 and C.3 present descriptive statistics for the age and log of the real weekly wages, respectively. Finally, Table C.4 shows the covariance matrix of the log of real weekly wages by age and education group.
<table>
<thead>
<tr>
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Table C.1: Information Availability Matrix. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).
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Table C.2: Descriptive Statistics: Age. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).
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Table C.4: Covariance Matrix of the Log of Real Weekly Wages by Age and Schooling Groups. Source: Elaborated by the authors. Annual Reports of Social Information (RAIS).
Appendix D

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Table D.1: Estimation of AR(1) Model with Factor Load for Wage Variance (Unconditional).
Source: Elaborated by the authors. Annual Reports of Social Information (RAIS). Note: Standard errors in brackets. * indicates significance at the 1% level.